Supplement.

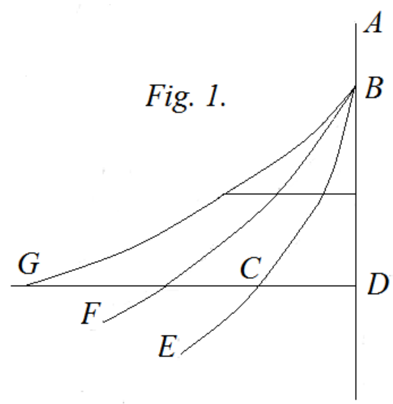
Solution of the Problem proposed by M. Leibniz in the *Nouvelles de la Republique des Lettres* for the month of September 1687. [A letter of Huygens.]

To find the curve of descent along which a heavy body descends uniformly and approaches the horizontal by equal amounts in equal time intervals.

Solution.

If one wished, that the heavy body should begin to fall along that line, since it is at rest that would be impossible.

But if the body is supposed to have some motion, however small that should be, since by that alone, as for example that which it acquired in falling through the perpendicular height AB (fig.1), then the curved line BC which is such that the cube of the perpendicular CD on AB produced, must be equal to the solid from the square of BD and from the height of $\frac{9}{4}$AB, to satisfy the problem.



But besides this line BC there will be had an infinitude of others of the same kind and easy to find, which will produce the same effect, that is to say, that the heavy body after the fall along AB fall along these lines, still will approach the horizontal equally in equal times, but more slowly, than by BC.

For if BD is double BA, the time of the descent along the part of the curve BC will be equal to the time of the fall along AB.

Huygens.

Added by M. Leibniz to the solution of his problem given by M. Huygens, article VI for the month of October 1687.

I had not considered proposing this problem to the foremost geometers, such as Monsieur Huygens, they ought rather to judge the prize, a little like the forty Academicians. However, since M. H. has found this problem worthy to be solved himself, I shall try to add something.

One asks for a curve BD(D) drawn on some plane, along which a heavy body can descent uniformly, and approach the horizontal in equal amounts in equal intervals of time, that's to say the times of the descents along BD, B(D) (fig. 2) shall be as the perpendicular heights BC, (BC) and if the heights C(C) and (C)[C] shall be equal, the time of the descents along D(D) and by (D)[D] also will be equal to each other.

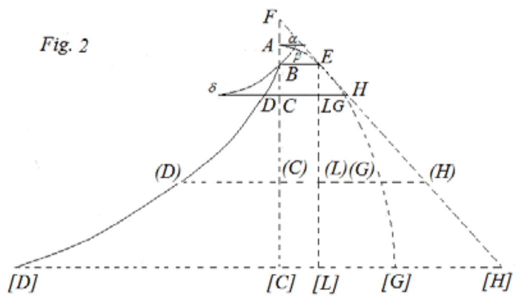


Fig. 2

I say that *the square-cubic parabola BD(D)[D] will satisfy the question and it will be the isochronous line sought, the peak of which will be B, and the square of the ordinates CD will be as the cubes of the abscissas (measured from the top) BC,* [i.e. $x^3 = ky^2$, for some constant *k*; or $\frac{x_1^3}{x_2^3} = \frac{y_1^2}{y_2^2}$. In this approach, the vertical distances can also be identified as times, as these are either in proportion or equal to each other, and Galileo's law can be used.] For example the abscissas BC, B(C) being as 1 to 4, the ordinates CD, (C)(D) can be $\frac{2}{3}$ and $\frac{16}{3}$, for the cubes of l and 4 are 1 and 64, and $\frac{2}{3}$ being to $\frac{16}{3}$ as l to 8, their squares also will be as l to 64. This curve which one can now call *Isochronous* (after the discovery of this property) in any case is well known to the geometers, and has been the first of all the curves from ordinary geometry, *for which a rule of exact equality has been given.* Now it is clear that the heavy body would not be able to descend uniformly along the curve BD when it is at rest, for if it should begin at rest, it will continue in a state of rest, that's to say, it will not have any movement. But with some speed or tardiness by which it tends to go down, there will be a way of assigning to it an infinitude of square-cubic parabolas, with one point at the peak B, the others through some point, such as D, since along which this body will continue to descend and to approach the horizontal for this same given speed or slowness. If the descent should begin uniformly from the peak B, the parameter of our isochronous parabola will be $\frac{9}{4}$ of the height or the perpendicular drop AB, which little speed has been given to the heavy body which it had at the peak B.

In order to give a rule for this motion, we suppose that the heavy body has acquired the speed which it has at the point B by falling through the perpendicular AB, and in order to represent the time of this descent, by discretion we let BE be normal to AB; then we trace the parabola AEG of which the axis shall be ABC. Furthermore one must draw a right line FEH, which is a tangent to the parabola at E and cuts the axis at F; one knows that FB is twice AB [For the tangent at (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$, etc.]. Continuing CG as far as H, and taking EL parallel to BC, cutting CH at L, I say that LH will represent the time for the fall through BD. One can dispense with the parabola, if taking FA equal to AB, one draws FEH, but the parabola serves to give the reason for this operation, for its ordinates represent the times for the straight fall along AC.

Here then is the rule : *The time LH of the uniform descent on a part BD of the isochronous curve is to the time BE of the perpendicular descent AB, which has been able to give the speed acquired at the start B of the isochronous curve which it touches, since the height BC of the isochronous descent to twice the height AB of the perpendicular descent.* For on account of the similar triangles ELH and FBE, it is seen that LH is to BE, as EL or BC is to FB, twice AB.

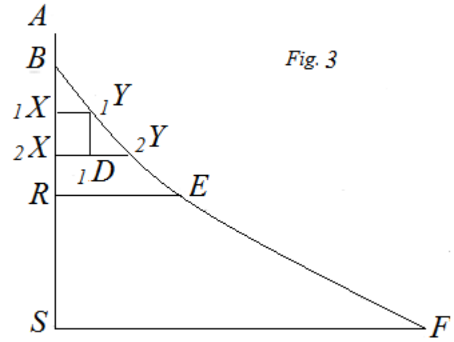
Corollary. If the height BC of the uniform descent is double the height AB of the perpendicular descent, the times LH and BE will be equal, which agrees with the remark of Mons. H. But if the time of the said perpendicular descent shall be the double of that of the uniform descent, their heights will become equal. And one can resolve likewise all the particular cases given.

But if one asks that the uniform descent does not start at the summit, then the speed at the start, or rather the height of the fall from this speed as well as the parabolic isochrone are given, it will be a matter of finding the point D where the heavy body arrives with this speed and continues its motion along the line D(D) descending uniformly.

Here then is the general rule : *"When the body falls from some height where the horizontal which passes through A, on some point D because this must be the isochronous line BD which is touched at the summit B by the perpendicular AB to the horizontal, and equal to* $\frac{4}{9}^{th}$ *of the parameter of the isochrone line; it will begin to descend uniformly along the said line from this point."* Which is sufficient to determine these questions and to construct also the lines mentioned in the figure of Mr. H., applying the convenient points of the other isochronous curves such as βBδ to the peak B of the principal line BD, so that the points A and α taken below the peaks B and β and determining the height of the fall dropping to the same horizontal Aα . That is why the weight falling from A on B will be able to fall from B along all the isochrones which intersect at B, then the points α fall on the horizontal Aα . But BD with regard to the height AB, is the *principal of the Isochrones*, which serves here from the peak and in which the weight arriving from the peak will fall uniformly with the most speed possible, and the perpendicular AB raised at the meeting point of the weight and the isochronous line touches the principal BD at the place where it intersects the other curves such as $\beta\delta$.

It is easy to give a demonstration of all these things, when they have been found already, that's why I do not wish to pause.

Analysis of the Problem of the Isochronous Curve.



The curve of isochronous descent YEF (fig. 3) is sought, along which incline a weight descending isochronically or approaches the horizontal plane uniformly, thus evidently in which the arcs BE, EF are traversed in equal times, from which arcs run through, the descents BR, RS shall be equal in length, taken to be perpendicular.

Let YY be the curve sought, the right directrix of which, along which we will measure the perpendicular descent, shall be AXX; the abscissa AX may be called x, and the ordinate XY may be called y, and $_1X_2X$ or $_1Y_1D$ will be dx and $_1D_2Y$ will be called dy.

AX or x is the distance run through or the descent, dx is the increment of the descent, the time from A as far as to X, which may be spent in the free descent, shall be to the speed acquired up to that point or as $\sqrt[2]{x}$, therefore the increment of this time (or the time in which it may travel freely through the increment of the distance) will be as $d\sqrt[2]{x}$ [i.e. the differential of $\sqrt[2]{x}$] or $dx^{1:2}$; either $\frac{1}{2}x^{-1:2}dx$ or $dx:2\sqrt[2]{x}$. Now the time in which it actually now runs through the height $_1X_2X$ along the curve of descent by the element $_1Y_2Y$, which time we shall call dt, is to the time in which in falling through the same height freely, or to $dx:2\sqrt[2]{x}$, as $_1Y_2Y$ is to $_1X_2X$ or as $\sqrt[2]{dx^2+dy^2}$ to dx, or there becomes

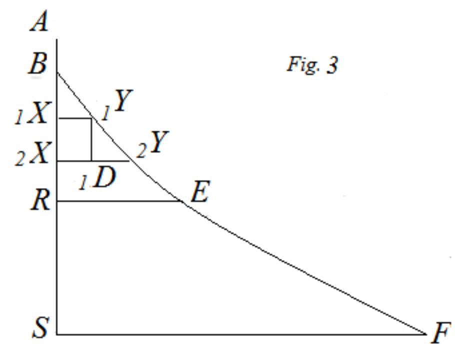


Fig. 3

$dt:\overline{dx:2\sqrt[2]{x}}::\sqrt[2]{dx^2+dy^2}:dx$; i.e. $\frac{2\sqrt[2]{x}dt}{dx}=\frac{\sqrt[2]{dx^2+dy^2}}{dx}$; indeed since the speeds of descent shall be constant (with no opposition from the inclination or freely), the times will be as the distances ; and thus there will be $dt=\frac{dx}{\sqrt{a}}$ Which is true in every curve of descent, whatever its nature shall be, truly in our case, since dt must become the element of the descent time to be as dx or the proportions from the descent becomes :

$dx=a\sqrt{dx^2+dy^2}:2\sqrt{ax}$; with *a* taken for unity, or there becomes :

$4dx^2ax=aadx^2+aady^2$, or $dy=dx\sqrt{4ax-aa}:a$ and $y=\int dx\sqrt{4ax-aa}:a$.

So that we may find which sum, we may put $z=4x-a$, and there becomes $dx=dz:4$, therefore $y=\int dz\sqrt{az}:4a=\sqrt{a}\int dz\sqrt{z}:4a$. Now $\int dz\cdot z^{1:2}=z^{3:2}:\overline{3:2}=2z\sqrt{z}:3$,

therefore $y=z\sqrt{az}:6a$ or $36ayy=z^3$ or by making $b=36a$ there becomes $byy=z^3$. Therefore we have the descent curve, which is a parabolic square-cubic, in which the square of the ordinates yy shall be as the cube of the abscissas z^3 ; but the right line b or BR will be 36a and $a=b:36$. But thus we will find AB itself: in the case of the point B there is $z=0$, therefore $0=4x-a$ or $x=a:4$ or $x=b:144$; but in this case there is $AB=x$, therefore there becomes $AB=BR:144$. And thus if the parabolic square-cubic BYY shall be erected thus, so that the directrix, of which the parts of the abscissa may have the cubes proportional to the squares of the ordinates, shall be perpendicular to the horizontal, and at its vertex B a weight falls through the height AB, which shall be the 144^{th} part of the right line, and then at B it begins to fall along the curve BYY, there its descent will be isochronous, or a weight running along the curve BYEF in equal times will come from B to E, and from E to F, with the heights BR and RS put equal.

And this is the analysis of the problem ; truly it pleases to give the synthesis as well as the method, which arrives at a common approach, for the analysis, which I have used here, will not be able to be followed except by those, who understand the principles handled by me concerning the analysis of the infinitudes [*i.e.* differential and integral calculus].

Problem.

To find the Curve of Isochronous Descent.

The curve BYYEF (Fig.4) shall be a square-cubic of the parabolic form, of which the vertex is B, the axis BXXRS, from which with the ordinates XY drawn normal to the curve shall be the cubes of the abscissas BX shall be as the squares of the ordinates XY, I say that curve is to be sought. Surely if the curve thus shall be put in place, so that the vertex B may extend to the top, and the axis BX shall be erected perpendicular and at that produced above B, A may be taken thus, so that AB shall be the hundred and forty-fourth part of the length of the right line, then a weigh falling from the height A (freely or inclined) at B, and from B again descending along the curve BYY, it will descend along this line isochronically, thus so that the descents along the perpendicular shall be supposed proportional to the times spent, and in equal times to approach equally to the base or the horizontal plane, evidently the time in which the weight runs down from B along the curve BYY to E, will be to the time in which it travels from E to F, as BR to RS, and hence if BR and RS shall be equal, also the intervals of the time will be equal, in which it descends from B to E and from E to F; and thus the inclination gives rise to the singular line of descent, so that the weight may be moved without any acceleration of descent considered along the perpendicular, and in turn if the weight placed at F may be driven upwards along the line FEB with that speed, as it would have acquired by sliding from A to S, it will rise by an equable motion from the base FS as far as to the vertex of the line B, for although it can decrease its absolute speed continually, yet the ascents along the perpendicular are considered to be proportional to the times spent.

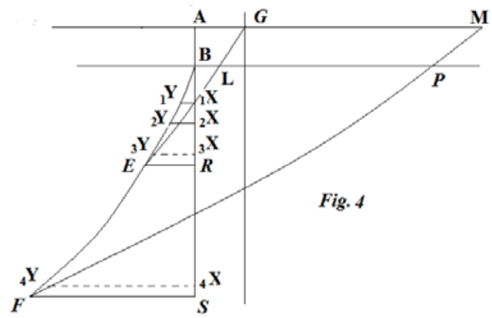


Fig. 4

Demonstration.

Two elements of the height may be taken or increments of the times of descent $_3$XR and $_4$XS, which may be put equal to each other, and to these shall correspond (although unequal to each other) the elements of the lines of descent $_3$YE and $_4$YF, I say also the elements of the time correspond to the elements of the distance or the times, in which the elements of distance are crossed over, to be equal between themselves, or the time in which $_3$YE is traversed to be equal to the time, in which $_4$YF is traversed, and thus the increments of the times of the perpendicular to be proportional everywhere.

Two right lines AGM and BLP may be drawn through A and B parallel to the horizontal, and GL (produced) touches the curve at E and MP (produced) touches the curve at F. From the nature of the motion the time, in which $_3$YE is run through, is to the time in which $_4$YF is traversed, in the ratio composed indeed directly from the right lines $_3$YE to $_4$YF or GL to MP, truly as the reciprocal of the speed at RE to the speed at SF, or the reciprocal of the square root of the heights AR and AS. Now truly (from the nature of the tangent of this curve) GL to MP will be found to be in the direct ratio of the square root AS to AR; it is therefore the ratio of the times, in which $_3$YE is traversed, to the time in which $_4$YF is traversed, composed directly from the direct ratio and the reciprocal ratio of the same terms, which is the ratio of equality; therefore these times are equal, or the descents increase equally in equal times. Which was being asserted.

[In modern terms without involving the time directly, from the principle of conservation of potential and kinetic energy of the body of mass *m* falling from rest at A, in the vertical fall $\mathrm{AB} = h$, the kinetic energy gained satisfies : $\frac{1}{2}mv_0^2 = mgh$, and by definition, this constant speed $v_o = \sqrt{2gh} = \sqrt{h}$ if we choose $g = \frac{1}{2}$ in some units. The horizontal speed v_y acquired along the isochronous curve at some subsequent point Y(*x*, *y*), if we take B as the origin, can be found from

$\frac{1}{2}mv_y^2 = mgx$ or $v_y = \sqrt{x} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = v_o\frac{dy}{dx}$; hence on integrating :

$y = \frac{2}{3v_o}x^{\frac{3}{2}}$, and the conservation of energy principle is implicit in this equation.]

Beilage.

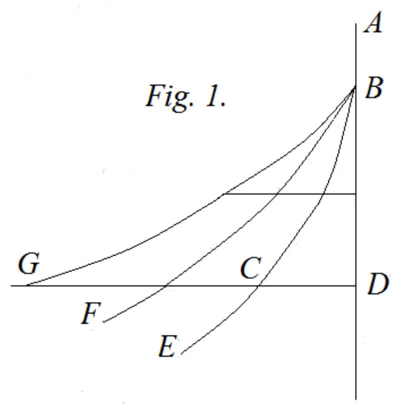
Solutione du Probleme propose par M. L. dans les Nouvelles de la Republique des Lettres du mois de Septembre 1687.

Trouver une ligne de descente dans laquelle le corps pesant descende uniformement et approche egalement de 1'horizon en temps egaux.

Solution.

Si l'on vouloit, que le corps pesant commencast a descendre dans cette ligne depuis le repos, elle seroit impossible.

Mais si le corps est suppose avoir quelque moment, quelque petit qu'il soit, comme par ex. celui qu'il aquiert en tombant de la hauteur perpendiculaire AB (fig. 1), alors la ligne courbe BC qui est telle que le cube de CD perpendiculaire sur AB prolongée, soit egal au solide du quarre de BD et de la hauteur de $\frac{9}{4}$ AB, satisfera au Probleme.



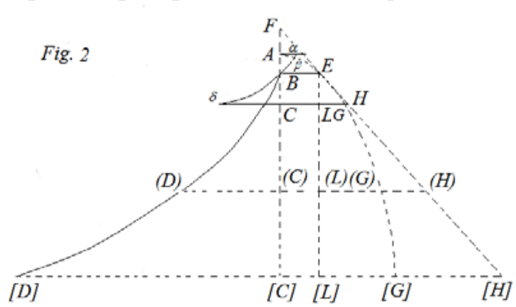
Mais outre cette ligne BC il y en aura une infinite d'autres du meme genre et aisées a trouver, qui feront le meme effet, c'est à dire, que le corps pesant apres la chute par AB descendant par ces lignes, approchera encore egalemeut de l'horizon en temps egaux, mais plus lentement, que par BC.

Que si BD est double de BA, le temps de la descente par la portion de courbe BC sera egal au temps de la chute par AB.

H. D. Z.

Addition de M. L. a la solution de son probleme donnèe par M. H. D. Z. article VI du mois d'octobre 1687.

Je n'avois garde de proposer ce probleme à des Geometres du premier rang, tels que Monsieur H. D. Z., ils doivent plustost juger des prix, a peu près comme les quarante Academiciens. Cependant puisque M. H. a trouve ce probleme digne de le resoudre luy mème, je tacheray d'adjouter quelque chose.



On demande une ligne BD(D) tracée sur quelque plan, dans laquelle un corps pesant puisse descendre uniformement, et approcher egalement de l'horison en temps egaux, c'est à dire que les temps des descentes par BD, B(D) (fig. 2) soient comme les hauteurs perpendiculaires BC, (BC) et si les hauteurs C(C) et (C)[C] estoient egales, les temps des descentes par D(D) et par (D)[D]

seroient aussi egales entre elles.
Je dis que *la Paraboloeide Quadrato-Cubique BD(D)[D] satisfera à la question et sera la Ligne Isochrone demandée dont le sommet sera B, et les quarres des ordonnées CD comme les cubes des abscisses (de la touchante du sommet) BC.* Par exemple les abscisses BC, B(C) estant 1 et 4, les ordonnées CD, (C)(D) pourront estre $\frac{2}{3}$ et $\frac{16}{3}$, car les cubes de l et 4 sont 1 et 64, et $\frac{2}{3}$ estant à $\frac{16}{3}$ comme l à 8, leurs quarres seront aussi comme l à 64. Cette ligne qu'on pourra maintenant appeller *Isochrone* (apres la decouverte de cette propriete) est assez connue d'ailleurs aux Geometres, et à *esté la premiere de toutes les lignes courbes de la Geometrie ordinaire, à qui on ait donné une droite exactement egale.* Or il est manifeste que le corps pesant ne scauroit descendre uniformement dans la ligne BD depuis le repos, car s'il commencoit par le repos, cette meme uniformité le seroit continuer ce repos, c'est a dire il n'y auroit point de mouvement. Mais avec quelque vistesse ou tardité qu'il tende de descendre, il y aura moyen de luy assigner une infinite de ces Paraboloeides Quadrato-Cubiques, l'une au sommet B, les autres dans quelque autre point, comme D, depuis lequel ce corps continuera de descendre et d'approcher de l'horison avec cette meme vistesse ou tardité. Si la descente uniforme doit commencer depuis le sommet B, le parametre de nostre *Paraboloeide isochrone* sera $\frac{9}{4}$ de la hauteur ou cheute perpendiculaire AB, qui à pu donner au corps pesant la vistesse qu'il à au sommet B.

Pour donner une regle de ce mouvement, supposons que le corps pesant ait acquis la vistesse qu'il a au point B en descendant par la perpendiculaire AB, et pour representer le temps de cette descente, menons a discretion BE normale a AB; puis tracons la parabole AEG dont l'axe soit ABC. De plus soit menée une droite FEH, qui touche la parabole en E et coupera 'axe en F; on sçait que FB est double d'AB. Continuons CG jusqu'en H, et menons EL parallele a BC, coupant CH en L, je dis que LH representera le temps de la descente par BD. On peut se passer de la parabole, si prenant FA egale à AB, on mene FEH, mais la parabole sert a rendre raison de cette operation, car ses ordonnées representent les temps de la cheute droite AC.

Voicy donc la regle: *Le temps LH de la descente uniforme sur une portion BD de la ligne isochrone est au temps BE de la descente perpendiculaire AB, qui a pû donner la vistesse acquise au commencement B de la ligne isochrone qu'elle touche, comme la hauteur BC de la descente isochrone au double de la hauteur AB de la descente perpendiculaire.* Car à cause des triangles semblables ELH et FBE, il est visible que LH est a BE, comme EL ou BC est à FB double d'AB.

Corollaire. Si la hauteur BC de la descente uniforme est double de la hauteur AB de la descente perpendiculaire, les temps LH et BE seront egaux, ce qui convient avec la remarque de Mons. H. Mais si le temps de la dite descente perpendiculaire estoit double de celuy de la descente uniforme, leurs hauteurs seroient egales. Et on peut resoudre de meme tous les cas particuliers donnes.

Mais si on ne demande pas que la descente uniforme commence au sommet, alors la vistesse du commencement, ou bien la hauteur de la cheute de cette vistesse aussi bien que la paraboloeide isochrone estant données, il s'agit de trouver le point D ou le corps pesant arrivant avec cette vistesse et continuant son mouvement dans la ligne D(D) descendra uniformement.

En voicy la regle generale: *"Lorsque le corps pesant tombe de quelque hauteur ou horisontale qui passe par A, sur quelque point D que ce soit de la ligne isochrone BD qui est touchée au sommet B par AB perpendiculaire à l' horisontale et egale à* $\frac{4}{9}$ *du parametre de la ligne isochrone; il commencera de descendre uniformement dans la dite ligne depuis ce point D."* Ce qui suffit à determiner ces questions et à construire aussi les lignes mentionnées dans la figure de Monsieur H., appliquant les points convenables des autres lignes isochrones comme $\beta B\delta$ du sommet B de la principale BD, en sorte qu' A et à points pris au dessus des sommets B et β et determinants la hauteur de la cheute tombent dans une meme horisontale $A\alpha$. C'est pourquoy le poids tombant d' A sur B pourra depuis B descendre dans toutes les isochrones qui se coupent en B, dont les points α tombent dans l'horisontale $A\alpha$. Mais BD à l'egard de la hauteur AB, est *la principale des Isochrones,* qui sert icy depuis le sommet et dans laquelle le poids arrivant de la hauteur AB descendra uniformement avec le plus de vistesse qu'il pourra, et la perpendiculaire AB elevée sur le point de rencontre du poids et de la ligne isochrone touche la principale BD au lieu qu'elle coupe les autres comme $\beta\delta$.

Il est aisé de donner la demonstration de toutes ces choses, lorsqu'elles sont deja trouvées, c'est pourquoy je ne veux pas m' arrester.

Analysis des Problems der isochronischen Curve.

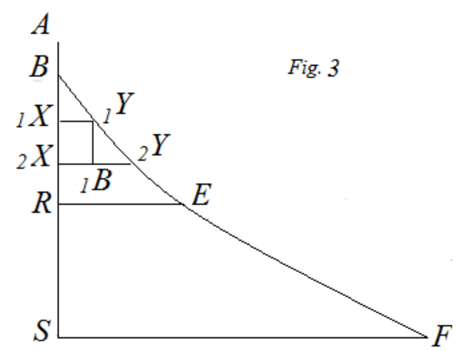


Fig. 3

Quaeritur Linea descensoria isochrona YYEF (fig. 3), in qua grave inclinate descendens isochrone seu uniformiter plano horizontali appropinquet, ita nempe ut aequalibus temporibus, quibus percurrantur arcus BE, EF, aequales sint descensus BR, RS in perpendiculari sumti.

Sit linea quaesita YY, cujus recta Directrix, in qua ascensus perpendiculares metiemur, sit AXX; abscissa AX vocetur x, et ordinata XY vocetur y, et $_1X_2X$ seu $_1Y_1D$ erit dx et $_1D_2Y$ vocetur dy.

AX seu x est altitudo percursa seu descensus, dx est descensus incrementum, tempus ab A usque ad X, quod insumeretur descensu libero, foret ut celeritas eousque acquisita seu ut $\sqrt[2]{x}$, ergo incrementum hujus temporis (seu tempus quo libere percurritur incrementum spatii) erit ut $d\overline{\sqrt[2]{x}}$ seu $dx^{1:2}$ seu $\frac{1}{2}\,x^{-1:2}dx$ seu $d\bar{x}:2\sqrt[2]{x}$. Iam tempus quo nunc revera percurritur altitudo $_1X_2X$ in lineae inclinatae descensoriae elemento $_1Y_2Y$, quod tempus vocemus dt, est ad tempus quo eadem altitudo percurreretur in descensu libero seu ad $d\bar{x}:2\sqrt[2]{x}$, ut $_1Y_2Y$ ad $_1X_2X$ seu ut $\sqrt[2]{\overline{dx^2}+\overline{dy^2}}$ ad $d\bar{x}$ ad dt, seu fiet $dt:\overline{d\bar{x}:2\sqrt[2]{x}}::\sqrt[2]{\overline{dx^2}+\overline{dy^2}}:d\bar{x}$; cum enim celeritates sint aequales (non obstante inclinatione vel libertate), erunt tempora ut spatia; itaque erit $d\bar{t}\ d\bar{x} = d\bar{x}\sqrt{d\overline{x^2} + d\overline{y^2}} : 2\sqrt{x}$.. Quod verum est in omni linea descensoria, cujuscunque sit naturae, verum in nostra, cum $d\bar{t}$ elementa temporis descensorii debeant esse ut $d\bar{x}$ seu proportionalia descensibus fiet:

$dx = a\sqrt{d\overline{x^2} + d\overline{y^2}} : 2\sqrt{ax}$; assumta *a* pro unitate, seu fiet: $4d\bar{x}^2 ax = aad\bar{x}^2 + aad\bar{y}^2$ seu $d\bar{y} = d\bar{x}\sqrt{4ax - aa} : a$ et $y = \int \overline{d\bar{x}\sqrt{4ax - aa}} : a$. Quae summa ut inveniatur, ponemus $z = 4x - a$, fietque $d.\bar{x} = d\bar{z} : 4$, ergo $y = \int \overline{dz\sqrt{az}} : 4a = \sqrt{a}\int \overline{dz\sqrt{z}} : 4a$. Jam $\int \overline{dzz^{1:2}} = z^{3:2} : \overline{3:2} = 2z\sqrt{z} : 3$, ergo $y = z\sqrt{az} : 6a$ seu $36ayy = z^3$ seu faciendo $b = 36a$ fiet $byy = z^3$. Ergo habemus Lineam descensoriam, quae est Paraboloeides quadrato-cubica, in qua quadrata ordinatarum yy sint ut cubi abscissarum z^3 ; latus autem rectam b seu BR erit 36a et $a = b:36$. Ipsam autem AB sic inveniemus: in casu puncti B est z=0, ergo $0 = 4x - a$ seu $x = a:4$ seu $x = b:144$; est autem in hoc casu $AB = x$, ergo fiet $AB = BR:144$. Itaque si linea paraboloeidis quadrato-cubicae BYY sic erecta sit, ut directrix, cujus portiones abscissae habeant cubos quadratis ordinatarum proportionales, sit perpendcularis ad horizontem, et in verticem ejus B cadat grave ex altitudine AB, quae sit l44[ta] pars lateris recti, et deinde in B pergat descendere in linea BYY, erunt descensus ejus isochroni, seu grave decurrens in linea BYEF aequali tempore perveniet ex B in E, et ex E in F, posito altitudines BR et RS esse aequales.

Atque haec est Analysis problematis; placet vero Synthesin quoque dare methodo, quae ad communem propius accedat, analysin enim, qua hic usus sum, non nisi illi assequentur, qui principia a me tradita circa Analysin infinitorum intelligunt.

Problema.

Lineam Descensoriam isochronam invenire.

Sit linea BYYEF (Fig.1) paraboliformis quadrato-cubica, cujus vertex B, axis BXXRS, unde ductis ad curvam ordinatis normalibus XY sint cubi abscisarum BX ut quadrata ordinatarum XY, dico eam esse quaesitam. Nempe si linea ita sita sit, ut vertex B summum obtineat, axisque BX sit perpendiculariter erectus et in eo producto supra B sumatur A sic, ut AB sit pars centesima quadragesima quarta lateris recti lineae, tunc grave cadens ex altitudine A (libere vel inclinate) in B, atque ex B porro descendens in linea BYY, descendet in hac linea isochrone sive aequabiliter, ita ut descensus secundum perpendiculum sumti sint temporibus insumtis proportionales, et aequalibus temporibus aequaliter appropinquetur ad basin seu planum horizontale, nempe tempus quo grave ex B in linea BYY decurret ad E, erit ad tempus quo ex E decurret ad F, ut BR ad RS, ac proinde si BR et RS sint

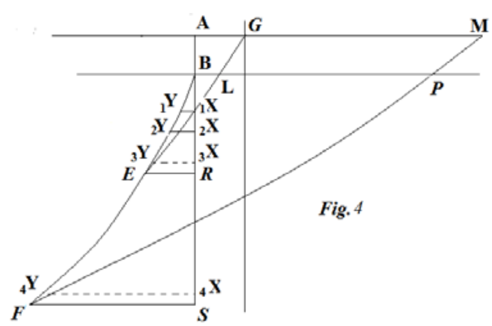


Fig. 4

aequales, etiam temporis intervalla, quibus ex B descenditur in E et ex E in F, erunt aequalia; atque ita lineae descensoriae peculiaris inclinatio efficiet, ut grave moveatur sine ulla acceleratione descensionis in perpendiculo aestimatae, et vicissim si grave in F positum sursum impellatur in linea FEB ea celeritate, quam acquirere potuisset labendo ex A ad S, ascendet motu aequabili a basi FS usque ad verticem lineae B, licet enim continue decrescat ejus celeritas absoluta, ascensus tamen in perpendiculo aestimati erunt temporibus insumtis proportionales.

Demonstratio.

Sumantur duo Elementa altitudinis seu incrementa momentanea descensus ${}_3XR$ et ${}_4XS$, quae ponantur inter se aequalia, eisque sint respondentia (licet inaequalia inter se) elementa lineae descensoriae ${}_3YE$ et ${}_4YF$, dico etiam elementa temporis elementis spatii respondentia seu tempora, quibus elementa spatii transmittuntur, fore aequalia inter se, seu tempus quo percurritur ${}_3YE$ fore aequale tempori, quo percurritur ${}_4YF$, atque ita erunt incrementa temporum incrementis descensuum perpendicularium ubique proportionalia.

Per A et B ducantur duae rectae horizonti parallelae AGM et BLP, et GL (producta) tangat curvam in E et MP in F. Ex natura motuum tempus, quo percurritur ${}_3YE$, est ad tempus, quo ${}_4YF$, in ratione composita, ex directa quidem rectae ${}_3YE$ ad ${}_4YF$ seu GL ad MP, reciproca vero celeritatis in RE ad celeritatem in SF, seu reciproca subduplicatae altitudinum AR et AS. Iam vero (ex natura tangentium hujus curvae) reperietur GL ad MP esse in directa ratione subduplicata AS ad AR; est ergo ratio temporis, quo percurritur ${}_3YE$, ad tempus, quo ${}_4YF$ percurritur, composita ex ratione directa et reciproca eorundem terminorum, quae est ratio aequalitatis; aequalia ergo sunt haec tempora seu descensus aequalibus temporibus aequaliter crescunt. Quod asserebatur.