General Note on the Translation.

Each chapter of the *Arithmetica Logarithmica* is presented in four parts: -

1. A Synopsis of the chapter;

2. The main translation of the chapter in modern English;

3. Notes and Comments;

4. The original Latin text.

A brief outline of the function of each part is now given:

1. The Synopsis. Although no algebra as such is used in the original text, Briggs' intuitive arithmetical approach can be conveniently recast algebraically for the benefit of the modern reader, and presented in an introductory synopsis box, chapter by chapter. Each summary is meant as an aid to the understanding of the main text, yet kept apart from it, in order to avoid confusion with algebraic notation that was not available to Briggs in the early 1620's. The synopsis is hence a bridge between modern mathematics and the contemporary mathematical scene at that time. The first half of the book is concerned with introducing the reader to the concept of logarithms, and shows how Briggs' tables of logarithms were derived; the second half from Ch. 17 onwards, gives a number of mainly geometric applications, but deals with the Fibonacci numbers, gauging, etc.

2. The Main Translation of the Chapter. The intention of the translation is to stay as close as possible to the Latin original as understood by the translator, which is included at the end of each chapter. Thus, as little of the text has been paraphrased as possible; on the other hand, it must be presented in such a way that it is understood without too much effort by the modern reader. There are places, especially in the introductory remarks, where the original writing has been enlarged upon to convey the meaning which becomes apparent with familiarity with the work. Briggs' explanations at times are laboured, as he attempts to describe in words, operations with numbers that would now be described algebraically. Most of the actual working is presented in ubiquitous tables, which was the usual approach at the time, and these also may offer short explanations. Extra words or occasional phrases have been inserted to help the flow of the argument in English, without changing the meaning. Items in square brackets, such as table labels, *e.g.* [Table 1-1], are *not* present in the original text, and have been inserted for the reader's convenience; occasionally extra phrases are inserted in square brackets to aid the explanation without having to refer to an indexed note. The subjunctive mood, though a strong feature of Latin sentences, e.g. for expressing possible outcomes, etc., however plays no great part in modern English writing, and hence simple present, past, and future tenses are normally used in the translation, in accordance with the usual style of writing adopted in mathematical texts.

Any points of contention in the translation that arise for the reader can be referred back to the Latin original. It was, of course, difficult for early workers in the exact sciences to use Latin to explain concepts for which the language was never intended: Latin words or phrases were adapted for use in a restricted mathematical sense that only becomes apparent from an extensive reading of the original text. At the time, there was a very limited mathematical framework for new ideas outside classical Greek geometry: the reader may be disappointed by the lack of explanation offered for some of the more forward-looking concepts introduced by Briggs. Thus, there was no contemporary explanation for his square root difference algorithm that greatly aided in extracting the numerous square roots needed to find the logarithm of a number from first principles; or for the numerical limiting approximation whereby the natural and then base ten logarithms of primes were found; or again for the subtabulation schemes that he introduced to interpolate between known values to fill out the tables, etc.

It is important, however, not to fall into the trap of making anachronistic appraisals of such things: this is the original presentation, and as such it must stay. The Reader must be prepared to view the mathematical scenery presented with rather restricted vision, in order to appreciate the work. Such new ideas as there were at the time had flowed mainly from the pen of Vieta: thus, Briggs presumably discovered his numerical procedure for extracting the roots of polynomials by simplifying the process of affected cubes that Vieta had introduced. Again, Leonard and Thomas Digges, via the *Pantometria,* had an influence on the trigonometrical aspects of the work. On the other hand, the influence of Thomas Harriot, Briggs' contemporary, is hard to find: he does not get a mention in any of Briggs' work, yet he was arguably the most outstanding mathematician of the time immediately after Vieta: perhaps one has to look at religious, political, and character differences to account for this omission, as the two men appear to have had little in common. Nevertheless, Walter Warner, one of Harriot's close associates, was to make a valiant but eventually fruitless attempt to publish the missing Chiliades in Briggs' tables.

3. Notes: The indexed notes are meant to handle situations where more explanation can be provided than the author has given; also, a note may draw attention to relevant historical details that may be of interest to the reader.

4. The Latin Text This is copied straight from a facsimile of the original text. An electronic Latin parser has been used to check the spelling in the transcription. Occasionally words are abbreviated in the original, as space was at a premium: these have been restored to their full length in the transcription.

ARITHMETICA LOGARITHMICA

§ 1.1 *Synopsis: Chapter One.*

The first chapter establishes in a general way how logarithms may be defined. By means of tabulated numerical examples (Table 1-1), Briggs shows how a set of positive numbers in continued proportion *P, i.e.* a geometric progression, can have associated with it a variety of arithmetic progressions of positive terms *L*, each element of *L* is associated with an element of *P* in a 1:1 correspondence: Briggs uses the word *adjoined* to describe this correspondence. For explanatory purposes here, we consider *P* to be an ordered finite set of N elements: $\{ p_1, p_2, p_3, \ldots \ldots, p_N \}$, with a typical member of the set defined by $p_j = r^j$, for some integer $r > 0$ in the first chapter. The corresponding set of numbers in the set *L*: $\{ l_1, l_2, l_3, \ldots, l_N \}$, may be conveniently chosen. The elements of *P* are said by Briggs to be in *continued or successive proportion,* while the corresponding members of *L* are called logarithms, the term coined by John Napier. These logarithms, or 'ratio numbers', at this early stage of the development, can be related to the index *j* in numerous ways, the simplest being *j* itself: a measure of how many times *r* has been multiplied by itself, or the 'power' of the number. Thus, the index is a number which lets you point at another number, as it were, though the notation of setting the index as a post superscript was yet to come. Also, the logarithm will continue to be defined if *j* is not a whole number, as Briggs sets out in the following chapters.

Initially then, Briggs defines logarithms in an introductory manner according to the above ideas: for any two consecutive elements in *P*, the quotient p_{j+1}/p_j is formed, to which there corresponds a constant difference of logarithms $d = l_{j+1} - l_j$, from some particular set *L* of logarithms chosen, as shown in Table 1-1. The idea of subtracting indices, (or of numbers related to the indices) for a quotient of powers, is thus fundamental to this definition of logarithms. From Ch. 6 onwards, natural logarithms of primes are worked out and changed to base 10.

<u>*Lemma 1*</u>: The difference between any two logarithms in the table is proportional to the interval between their indices, which difference itself is a multiple of *d.* In Table 1-1, the index is the number of times 2 multiplies itself, and corresponds to the rows numbered from 1 to 8 sequentially. Now, for appropriate values of *i, m*, and *n*, consider an interval to be the difference *m* corresponding to rows *i* and $i + m$; similarly, rows *i* and $i + n$ give a second interval with a difference *n*. If the logarithms l_i and l_{i+m} are given for the first interval, then the unknown logarithm l_{i+n} can be found from proportion, as *n* is known. For $(l_{i+m} - l_i)/m = (l_{i+n} - l_i)/n = d;$ in Table 1-1, $d = 1$ for columns A and B; for column C, $d = 3$; and for column D, $d = -3$.

<u>*Lemma 2*</u>: If *p, q, r,* and *s* are any four numbers such that $p - q = r - s$ then $p + s = q + r$. As Briggs shows in A2 of the next chapter, he has in mind the ratio of the numbers $a/b = c/d$, where *p, q, r*, and *s* are the respective logarithms of the positive numbers *a, b, c*, and *d.* Hence, the sum of the logarithms is the same in the products *ac* and *bd*.

§1.2 Chapter One.

Concerning the definition of logarithms and the etymology of the name.

Logarithms are numbers which, adjoined to numbers in proportion, maintain equal differences[1].

For any given numbers whatsoever, the other numbers or logarithms, which are different from these first ones, and which conveniently agree with the general definition of logarithms, can be added on, and they do offer some appreciable advantage [in performing calculations]. As an example, if the numbers in continued proportion are[2]: 1, 2, 4, 8, 16, 32, 64, 128,… then the adjoined numbers designated by *A, B, C*, or *D* or others, can act as logarithms for these, as you see here [Table 1-1]. But a single kind of logarithm is to serve from this table, in order that the differences will be equal, with the one set of logarithms either increasing or decreasing, as often as the numbers with which they are adjoined are in proportion; so that conveniently, logarithms may be called*: the equally differing companions of proportional numbers*. Hence, they seem to be called logarithms by their most distinguished inventor [John Napier], because they present numbers to us maintaining the same ratio between themselves[3].

	A	B	C	D
1	1	5	5	35
2	2	6	8	32
4	3	7	11	29
8	4	8	14	26
16	5	9	17	23
32	6	10	20	20
64	7	11	23	17
128	8	12	26	14
numbers in proportion	Log	Log	Log	Log

[Table 1-1.]

Furthermore, so that we can reach a better understanding of the purposes of these logarithms to be produced, certain Lemmas are to be considered.

First Lemma

If any numbers whatever are established increasing or decreasing equally, the differences of these are in proportion with the intervals of the same[4].

For the first, third, and eighth numbers 35, 29, 14 can be selected from the numbers designated *D* [in Table 1-1]; between the first and third there are 2 intervals; between the third and eighth there are 5 intervals. I assert that the first to third difference 6, to be to the difference of the third and eighth 15, as two is to five.

Therefore in a series of numbers in continued proportion, from the logarithms of any two numbers given, we can find the logarithm of any other number whatever in the series. Indeed, both the intervals between these numbers themselves and the third number are given, also given is the difference of the given logarithms : therefore, for two given intervals and the difference of the given logarithms for the first interval, with the other interval to be given, the difference of the logarithms sought for the second interval will be found from proportion. As an example: let 4, 6, 9, $13^{1}/_{2}$, $20^{1}/_{4}$, $30^{3}/_{8}$, be the numbers in continued proportion, and the logarithms of the first and third given numbers are 060206 and 095424, and 035218 is the difference of these. The logarithm of the number $30^{3}/_{8}$ is sought, namely the sixth of the given numbers. The given interval between the first and the third of the numbers is two, between the third and the sixth three; the given difference of the logarithms is 035218 ; the fourth proportional number sought 052827 is the difference of the logarithms sought, which added to the logarithm of the third number gives 148251, the logarithm of the sixth number sought[5]. As this table [1-2] shows:

continued proportion	log	Logarithm		
4	7	060206		
6			difference given	035218
9	11	095424		
$13^{1}/_{2}$				
$20^{1}/_{4}$			difference sought	052827
$30^{3}/_{8}$	17	148251		

proportions		
	2----	given
	3----	intervals
	035218	given difference
	052827	sought difference

[Table 1-2]

Second Lemma

If from four numbers, the first exceeds the second as much as the third exceeds the fourth: then the sum of the first and forth will be equal to the sum of the second and third, and conversely. As, for example the numbers 9, 5, 15, 11: so the sum of the means as of the extremes is 20. See Proposition 4, Book 1, of the deductions in Bachet's *Diophantus*, [the 1621 edition].

And these two lemmas show well enough the particular purposes [i.e. properties] for a kind of logarithms.

§1.3 *Notes On Chapter One.*

[1] According to Briggs' Table 1-1, we have 1, 2, 4, 8 as four numbers in proportion, and we may take the corresponding numbers in column C as their logarithms, by way of example: 5, 8, 11, 14. The ratio of 2 to 1 is the same as the ratio of 8 to 4, or 2/1 = 8/4. With the first ratio we associate the difference of the logarithms 8 - 5 or 3, while with the second ratio we associate the difference of the logarithms14 - 3 or 3. The same above rule is found for any numbers in the table, or for any example we may construct, and for any numbers we choose as logarithms.

N.B. By continued proportions, the original writer had in mind a sequence of positive numbers *a, b, c, d,...* that satisfies a relation that we would now express as: $a/b = b/c = c/d$ ...

[2] Briggs used periods to delineate a series of numbers in continued proportion, such as 1.2.4.8.16.32.64.128,…In order to increase the readability of the text, we have changed the periods to commas.

[3] Logarithms are seen to be multiples of some common difference, as one would expect for an A.P. The word 'logarithm' is derived from two Greek words, 'logos' here meaning 'ratio' or perhaps 'reckoning' and 'arithmos', meaning 'number. An article in the *Mathematical Gazette* for 1934, pp. 92 - 205, *John Napier*, by W. R. Thomas, explores the possible origins of the word, and also sheds some light on the life of John Napier. The Introductory Chapter of this work may also be consulted.

[4] At this time there were hardly any means of describing mathematical operations on numbers, except by labourously writing out instructions. In this first lemma, a set of numbers in continued proportion may be considered to occupy the successive rows of the first column of a table: for each of these numbers there is an adjoining logarithm, in this case initially taken from the arbitrary A.P. in column D as an easy example, and subsequently using the actual logarithms from Briggs' tables of logarithms (that formed the bulk of the latter part of the *Arithmetica*, see Table 1-2, where another arbitrary A.P. is given as a guide). Briggs takes the actual row numbers as indices from which the interval between two terms can be evaluated, in an obvious way: each application of the proportionality moves to the number placed in the next row of the first column, to which there corresponds an equal increase or decrease in the logarithm in column 2 or 3 in Table 1-2. Thus, the row index can be associated with either the numbers in continued proportion, or their logarithms: the latter in this case.

[5] Consider the first set of numbers: 7, 9,11,13,15,17,… to be the logs of the given continued proportions. In this case, Briggs' instructions results in: 'log' $30^3/_8$ = 'log' 9 + ('log' 9 - 'log' 4)/2 × 3= (11 - 7)/2 × 3 = 17; where 'log' is used here to avoid confusion with the base10 logarithm of a number. Note: for the actual base 10 logarithms Briggs uses no decimal point - we would normally write log 4 = 0.60206, etc. An initial zero indicates the characteristic (to be discussed in Ch.3) is zero. A similar calculation can be performed with the logarithms from the tables in column 3 of Table 1-2. We must bear in mind that the *Arithmetica* is an expository work, and the nature of logarithms is to be gradually introduced to the reader. The idea of associating an arbitrary A.P. with the numbers in a G.P. is replaced by a more sophisticated method, whereby essentially the natural logarithms of numbers are found, and changed to base 10 in an intuitive manner.

ARITHMETICA
LOGARITHMICA

§1.4. Caput Primus. [p.1.]

De Logarithmorum definitione & nominis notatione.

Logarithmi sunt numeri qui proportionalibus adiuncti aequales servant differentias.

Datis quibuscunque numeris alij numeri ijque diversi ijs adjungi poterunt qui non incommode generali Logarithmorum definitioni conveniant & usum aliquem non ingratum praebere poterunt. ut si numeri continue proportionales fuerint 1.2.4.8.16.32.64.128. poterunt ijs pro Logarithmis adjungi numeri insigniti *A*, vel *B*, vel *C*, vel *D,* ut hic vides. vel alij, modo hoc unicum servetur, ut Logarithmorum, una crescentium vel decrescentium differentiae sint aequales, quoties numeri quibus sunt adjuncti sunt proportionales. ut non incommode Logarithmi dici possint, *numerorum proportionalium comites aequidifferentes.* Qui ideo videntur a clarissimo Inventore Logarithmi nominati, quia numeros nobis exhibent eandem inter se servantes rationem.

	A	B	C	D
1	1	5	5	35
2	2	6	8	32
4	3	7	11	29
8	4	8	14	26
16	5	9	17	23
32	6	10	20	20
64	7	11	23	17
128	8	12	26	14
numeri proportion ales	Logar.	Logar.	Logar.	Logar.

Atque ut horum Logarithmorum naturum & affectiones melius noscamus, Lemmata quaedam sunt considerenda.

Lemma primum.

Si quotlibet numeri statuantur aequaliter crescentes vel decrescentes eorum differentiae sunt intervallis eorundem proportionales.

Ut in numeris *D* signatis sumantur primus. tertius, & octavus, 35.29.14 inter primum & tertium sunt duo intervalla, inter tertium & octavum sunt quinque intervalla. aio differentiam primi & tertij 6, esse ad differentiam tertij & octavi 15, ut duo ad quinque.

Idcirco in serie numerorum continue proportionium, datis duorum quorumcunque Logarithmis; poterimus alterius cuiusvis Logarithmum invenire. Datur enim horum numerorum inter se intervallum & tertij ab horum alterutro, datur etiam datorum Logarithmorum differentia. Datis ergo duobus intervallis & Logarithmorum datorum differentia, altera dati & [p.2.] quaesiti Logarithmorum differentia proportionalis inventa erit.
ut sunto 4.6.9.$13^{1}/_{2}$.$20^{1}/_{4}$.$30^{3}/_{8}$ continue proportionales, & sint Logarithmi primi & tertij numeri dati, & 060206, & 095424, eorum differentia 035218. quaeritur Logarithmus numeri $30^{3}/_{8}$, sexti nempe datorum. data intervalla inter primum numerum & tertium sunt duo, inter tertium & sextum tria; data Logarithmorum differentia 035218, quartus quaesitus numerus proportionalis 0528273 est differentia Logarithmorum quaesita, quae tertij numeri dato Logarithmo adiecta, dat 148251 Logarithmum sexti numeri quaesitum. ut hic vides.

cont. prop.	log.	Logarith.	
4	7	060206	
6			diff. data 035218
9	11	095424	
$13^{1}/_{2}$			
$20^{1}/_{4}$			diff. quart. 052827
$30^{3}/_{8}$	17	148251	

prop.: 2----, 3---- intervalla data; 035218 data, 052827 quaesita differentia

Lemma secundum.

Si e quatuor numeris, quantum primus superat secundum. tantundem tertius superet quartum: erit summa primi & quarti, aequalis summae secundi & tertij: & contra. vt 9.5.15.11 summa tam mediorum quam extremorum est 20. vide 4. prop.lib.1 porismatum Bacheti in Diaphantum.

Atque haec duo Lemmata, Logarithmorum in genere, praecipuas affectiones satis illustrant.