§12.1. ***Synopsis: Chapter Twelve.***

Briggs demonstrates an improved method of subtabulation, far superior to the 'proportional parts' method of the previous chapter. The new scheme involves the use of finite differences, and can be applied when first and second order differences only need be considered, the latter being constant, or nearly so, (and negative, though the negative signs are suppressed); Briggs uses this method extensively to fill in the blank spaces in his Chiliads of logarithms: the missing Chiliades from the 31st to the 89th represents the region where this simple scheme does not work.

In modern terms, if *A* and *B* are the logarithms of two successive numbers in a table of logarithms, with first and second order differences *a* and *b*, where *b* is constant or almost so, in some region, then the logarithms of 9 equally spaced numbers z_i between the successive numbers are generated by the formula $z_i = A + x.a + [x(x-1)/2].b$, where $x = i/10$, and $1 \le i \le 9$. There is not of course an algebraic representation in the *Arithmetica* of this formula, or any other. Initially, Briggs uses a numerical scheme to calculate and add on successive difference to previously found intermediate logarithm as he proceeds; he then demonstrates the formula shown above numerically, by means of which any intermediate logarithm can be found, without the requirement of evaluating all the others up to that stage; finally, he shows how the method greatly improves on finding the square root of a number with a known logarithm.

This scheme is identical with that later formulated by Newton, and is now known as Newton's Forward Difference Method. In the notes, we give a little historical digression concerning the possible origins of this method.

§12.2. Chapter Twelve

For two given nearby numbers, together with the logarithms of these: to place nine other numbers equidistant between them; and to find the logarithms of these.

If the second differences of the given logarithms are almost equal, it will not be difficult to perform this: otherwise, if third differences shall have to be summoned, here the method will be somewhat deficient.

	47602,0016 *C*	
91235 *A*	4,96016,14763,8639 *B*	5217 *D*
	47601,4799 C	
91236 *A*	4,96016,62365,3438 *B*	5217 *D*
	47600,9582 *C*	

[Table 12-1.]

E	
1. 45 2. 35 3. 25 4. 15 5. 5	product added
6. 5 7. 15 8. 25 9. 35 10.45	product subtracted

[Table 12-2.]

Two nearby numbers *A* may be taken, and the logarithms of these *B*, together with the first and second differences of these, *C* and *D*. [Table 12-1.] If the second differences are equal, one of these shall be multiplied by the numbers in the Table E, [12-2], with the ten numbers adjoined to the first [difference]; but with the last three places of the products *F, G, H, I, K* taken away [Table 12-3] : for the first five are added, with just as many taken away, placed between *A* and *B* with the tenth part of the difference C[1]. The total remaining will be the differences of the logarithms sought; which continually added in order to the given smaller logarithm, will give the logarithms sought, as you see here. Let the given numbers be 91235, 91236, and the first difference of their logarithms 476014799 :

Abs. Number	*Logarithms*	
912350	4,96016,14763,8639	
	4760,1715	*C+F*
912351	4,96016,19524,0354	
	4760,1662	*C+G*
912352	4,96016,24284,2016	
	4760,1610	*C+H*
912353	4,96016,29044,3626	
	4760,1558	*C+I*
912354	4,96016,33804,5184	
	4760,1506	*C+K*
912355	4,96016,38564,6690	
	4760,1454	*C-K*
912356	4,96016,43324,8144	
	4760,1402	*C-I*
912357	4,96016,48084,9546	
	4760,1350	*C-H*
912358	4,96016,52845,0896	
	4760,1297	*C-G*
912359	4,96016,57605,2193	
	4760,1245	*C-F*
912360	4,96016,62365,3438	

Product		5217 *Factor*	
F - - -234	765	45	Factors remaining
G - - 182	595	35	
H - - 130	425	25	
I - - - 78	255	15	
K - - - 26	085	5	
47601479	9	$\frac{1}{10}$ C	
47601714	7	*C+ F*	
47601662	5	*C+ G*	
47601610	3	*C+ H*	
47601558	2	*C+ I*	
47601506	0	*C+K*	
47601453	8	*C - K*	
47601401	6	*C - I*	
47601349	5	*C - H*	
47601297	3	*C - G*	
47601245	1	*C - F*	

[Table 12 - 3]

If the second differences as here ∵ are not equal, the two numbers in proximity are to be added and half the sum taken for the second difference, and multiplied as before.

	4,51707,8187 *C*	
9615*A*	3,98294,92885,7405 *B*	469771 *D*
	4,51660,8416 C	
9616 *A*	3,98299,44546,5866 *B*	469672*D*
	4,51613,8744 *C*	
		939443 sum
		469721 1/2 sum

Product:		469721.*Factor*	
∵ *F* 21137	445	45	Factors remaining
G 16440	235	35	
H 11743	025	25	
I - 7045	815	15	
K - - 2348	605	5	

96150	3,98294,92885,7450	¶
	45168,1979	*C+F*
96151	3,98295,38053,9429	
	45167,7282	*C+G*
96152	3,98295,83221,6711	
	45167,2585	*C+H*
96153	3,98296,28388,9296	
	45166,7887	*C+I*
96154	3,98296,73555,7183	
	45166,3190	*C+K*
96155	3,98297,18722,0373	
	45165,8493	*C-K*
96156	3,98297,63887,8866	
	45165,3796	*C-I*
96157	3,98298,09053,2662	* *
	45164,9099	*C-H*†
96158	3,98298,54218,1761	
	45164,4401	*C-G*
96159	3,98298,99382,6162	
	45163,9704	*C-F*
96160	3,98299,44546,5866	

[Table 12-4.]

469721
105
2348605
469721
49320| 705 §

E		
1.	45	Factors added.
2.	80	
3.	105	
4.	120	
5.	125	
6.	120	
7.	105	
8.	80	
9.	45	

451660841	6	$\frac{1}{10}$ C
451681979	0	C + F
451677281	8	*C+G*
451672584	6	*C+H*
451667887	4	C + I
451663190	2	*C+K*
451658493	0	*C - K*
451653795	8	*C - I*
451649098	6	*C - H*
451644401	4	*C - G*
451639704	2	*C -F*

C 4516608416
C 7
products 3161625891|2
49320|7 §
total: 3161675212
¶398294928854750
398298090532662
451660841| 6 $\frac{1}{10}$ C
product - -- 11743| 0
† remainder451649098| 6

Whereas, if you want to find any one of these logarithms, with the rest omitted[2]: the [required] number less than ten [to be] added to the given number A, shall multiply the given difference *C*, and the same adjoined number from the table *E* [Table 12-4] shall multiply the second difference D; hence with three places removed, and with a single place [taken] from there [*i.e.* the first product], the products may be added: the whole added to the given logarithm A will give the logarithm sought. As [an example], if I wish to know what the logarithm of the number 96157 shall be, the given difference 4516608416, may be multiplied by 7, [to give] the product 31616258912. Then 105 across from 7 in Table E, shall multiply the second difference 469721, the product 49320|705 with the three final places taken away is added to the previous product, from which one place has been removed will give[3] 3161625891, the total 3161675212 shall be added to the given logarithm ¶. The total 3,98298,09053,2662 will be the logarithm sought of the number 96157 * *.

If you wish to know the difference between the logarithms of this number and the next larger: the number 8 in Table [12 - 5] corresponding to G, which exceeds the seventh part by one, shall multiply the second difference 469721; the product 11743|025 with three places subtracted, taken from the tenth part of the given first difference, will be the difference sought 451649099 with the remainder[4] †.

To find the accurate proportional parts.

And here the method will find the proportional part with sufficient accuracy. For when, following that which was considered in the above chapter [11], we have transferring the given logarithm to the proper place in the last or second last Chiliad in the Tables, we will be able to add the first place of the proportional part to an absolute number found in the Chiliad, and to find the logarithm of the augmented number, through the nearby preceding, together with the difference between the same and the larger logarithm nearby. This difference will give the proportional part that we seek most accurately, as we may see by a single example.

The square root of the number 147 is sought . The given logarithm of 147 is 2,16731,73347,4814, of which the half, 1,08365,86673,7409 is the logarithm of the root sought.

	Logarithms	
√147	1,08365,86673,7409	given
8192	0,91338,99436,3175	nearest complement.
N.N	0,99704,86110,0584	total.
	31548,5329	difference.
99322	0,99704,54561,5255	
	43725,6897	difference.
99323	0,99704,98287,2152	

proportions	43725	Given
	31548	differences
	10 - - -	
	7 - - -	required

Hitherto by the teaching of Chapter 11, that gives the place as 7, with the number 99322 in the table found nearby. The remainder required following that preceding

	43726,1300	
993220	0,99704,54561,5255	4403
	43725,8697	C
993230	0,99704,98287,2152	4402
	43725,2495	
993220	0,99704,54561,5255	R
	30608,0290	Q
993227	0,99704,85169,5545	M
	4372,5580	C - H
993228	0,99704,89542,1125	

C	437256897	factors
	7	
	3060798279	product
	4403	factors
	105	
	22015	
	4403	
	462|315	product
products	306079827|9	
	462|3	
Q	306080290|2	to be added
	4403	
	25	
	22|015	
	88|06	
	110|075	product to be taken away.
C	43725689|7	
H	- - - - -110|2	
C - H	43725579|6	

With the product from the given difference C with 7, added to the second difference 4403, & with the factor 105 found in table E: the total Q added to the given logarithm R will give the logarithm M, of the number 993227, between which & the nearby 991228 is the difference C - H, of which the fraction will give almost the most accurate result, if the total NN 0,99704,86110,0584, being brought together with M as shown.

NN	0,99704,86110,0584	
	1940,5039	
993227	0,99704,85169,5545	M
	4372,5580	
993228	0,99704,89542,1125	

proportions	43725580-	Differences given.
	9405039-	
	10----------	
	21509238	proportional part sought

Therefore the number corresponding to the logarithm NN 0,99704,86110,0584 is 9932272150923.

Thus if the number 8192, or the factors 8.8.8.8.2 of the same, divide the number found, the quotient 12<u>124,35565,29831,6</u> will be the root of the number 147 required and enlarged. It is thus close to the true root, given by[5] 12<u>124,35565,29824,41054.</u>

§12.3. ***Notes On Chapter Twelve.***

[1] This is an appropriate place to quote Charles Hutton, from that uniquely wide-ranging and illuminating preamble to *Hutton's Mathematical Tables* 5th Ed., (1811): *On the Construction of Logarithms,* page 71: ' …our ingenious author first of all teaches the rules of the Difference Method, in constructing logarithms by interpolation from differences. This is the same method which has since been largely treated by later authors, and in particular by the ingenious Mr. Cotes, in his *Canontechnia*. How Mr. Briggs came by it does not well appear, as he only delivers the rules, without laying down the principles or investigation of them. He delivers the method into two cases, namely when the second differences are equal or nearly equal, and when the differences run out to any length whatever. The former of these…he particularly adapts to interpolating 9 equidistant means between the two terms, evidently for this reason, that then the power of ten becomes the principal multipliers or divisors, and so the operation performed mentally. The substance of his method is this: having given two absolute numbers with their logarithms, to find the logarithms of 9 arithmetical means between the given numbers: Between the given logarithms take the 1st difference, as well as between each of them and their next or equidistant greater and less logarithm: and likewise the 2nd differences, or the two differences of these first three differences; then if these two differences be equal, multiply one of them severally by the numbers 45, 35, &c, as in the annexed table, dividing each product by 1000, that is cutting off the last three figures from each; lastly, to $^1/_{10}$ th of the 1st difference of the given logarithms to add severally the first five quotients, and subtract the other five, so shall the ten results be the respective first differences to be continually added to compose the required series of logarithms. Now this amounts to the same thing as what is at this day taught in the like case: It is known that if *A* be any

term of an equidistant series of terms, and *a*, *b*, *c*, &c, the 1st, 2nd, 3rd, &c order of differences; then the term *z*, whose distance from *A* is expressed by *x*, will be thus:

$$z = A + xa + x.\frac{x-1}{2}b + x.\frac{x-1}{2}.\frac{x-2}{3}c + \&c \qquad [12\text{-}1].$$

And if now, with our author, we make the 2nd differences equal, then *c*, *d*, *e*, &c, will all vanish, or be equal to 0, and z will become barely

$$= A + xa + x.\frac{x-1}{2}b \qquad [12\text{-}2].$$

Therefore if we take x successively to be 0/10, 1/10/ 2/10,3/10. &c, we shall have the annexed series of terms with their differences. Where it is to be observed, that our author has reduced the differences from the 1st to the 2nd form, as he thought it easier to multiply by 5 than to divide by 2. Also all the last terms $(x.\frac{x-1}{2}b)$ are set down positive, because in the logarithms *b* is negative.'

(End of quote).

Note that in Table 12 - 4, Briggs calculates and adds on the successive difference to the previous logarithm as he proceeds.

Series of terms.	The differences.
A	
$A + \frac{1}{10}a + \frac{9}{200}b$	$\frac{1}{10}a + \frac{9}{200}b = \frac{1}{10}a + \frac{45}{1000}b$
$A + \frac{2}{10}a + \frac{16}{200}b$	$\frac{1}{10}a + \frac{7}{200}b = \frac{1}{10}a + \frac{35}{1000}b$
$A + \frac{3}{10}a + \frac{21}{200}b$	$\frac{1}{10}a + \frac{5}{200}b = \frac{1}{10}a + \frac{25}{1000}b$
$A + \frac{4}{10}a + \frac{24}{200}b$	$\frac{1}{10}a + \frac{3}{200}b = \frac{1}{10}a + \frac{15}{1000}b$
$A + \frac{5}{10}a + \frac{25}{200}b$	$\frac{1}{10}a + \frac{1}{200}b = \frac{1}{10}a + \frac{5}{1000}b$
$A + \frac{6}{10}a + \frac{24}{200}b$	$\frac{1}{10}a - \frac{1}{200}b = \frac{1}{10}a - \frac{5}{1000}b$
$A + \frac{7}{10}a + \frac{21}{200}b$	$\frac{1}{10}a - \frac{3}{200}b = \frac{1}{10}a - \frac{15}{1000}b$
$A + \frac{8}{10}a + \frac{16}{200}b$	$\frac{1}{10}a - \frac{5}{200}b = \frac{1}{10}a - \frac{25}{1000}b$
$A + \frac{9}{10}a + \frac{9}{200}b$	$\frac{1}{10}a - \frac{7}{200}b = \frac{1}{10}a - \frac{35}{1000}b$
$A + a$	$\frac{1}{10}a - \frac{9}{200}b = \frac{1}{10}a - \frac{45}{1000}b$

[Table 12-4.]

Briggs' first method of subtabulation is one of the earliest published applications of a finite difference method, which is now (in the general case) called Newton's Forward Difference Method: Hutton is of the opinion that Newton discovered the method for himself, and this was presumably the case; nevertheless, as Goldstine points out

(*A History of Numerical Analysis...*), p. 23, Thomas Harriot, (but not 'Sir' as Goldstine would have you believe, as Harriot was always on the wrong side of royalty, and even spent some time in the Tower of London!), used a similar method of interpolation in his investigation of rhomb lines in navigation, leading to the equi-angular or logarithmic spiral, as investigated by Lohne (J. A. Lohne, *Essays on Thomas Harriot*, Archive for the History of the Exact Sciences. Vol. 15, 1979. pp. 189 - 312), and Pepper (Jon V. Pepper, *Harriot's Calculation of the Meridional Parts*, Archive for the History of the Exact Sciences. Vol. 4, 1968. pp. 359 - 413), who have studied some of the Harriot manuscripts. However, Goldstine would appear to be in error with his assumption of Briggs and Harriot 'overlapping' at Oxford by 2 years: for by the time Briggs arrived at Oxford in 1619 or thereabouts, Harriot was a very ill man, living in semi-seclusion, trying to put his mathematical affairs into some order before his death in 1621. From Lohne's Chronology, *ibid.* p. 308, we find that Harriot was on the payroll at Gresham College in 1596 at its inception, and this is probably when and where the two men were in contact, if indeed Harriot was not responsible to some extent for Briggs being offered this position. Harriot had a strong interest in navigation and the New World, which he had visited for a year as a scientific consultant as part of an abortive colony proposed by Rayleigh, and some of this enthusiasm obviously spread to Briggs, who was to write about finding the North-West Passage, as of course did other armchair travelers such as John Dee during this period of exploration. While Harriot was to go off to be a 'gentleman scientist', living in a house in the grounds of Syon House, Isleworth, on a lucrative salary of £200 p.a. provided by Henry Percy, 9th Earl of Northumberland (The wizard earl, who unfortunately was locked up in the Tower for 16 years by James I after the Gunpowder Plot in 1605, for his suspected involvement), Briggs had accepted the more onerous position of Professor of Geometry at Gresham College, a post he held for some 25 years, at a salary of £50 p.a. Thus, the connection between Briggs and Harriot is a tenuous one without more evidence, and it seems pointless to speculate on it: what we can say is that similar subtabulation schemes, used by both men, had their origins around this time. As regards Newton and some useful references for the interested reader, a well-reasoned discourse concerning the origins of Newton's involvement with finite differences is given in the introduction to Volume 4 of D.T. Whiteside's monumental work: *The Mathematical Papers of Isaac Newton.*

In modern terms, Newton's difference series for a function f(x) is the finite calculus equivalent of the Taylor series of the infinite calculus, and if the convergence and general agreement of the expansion with the left hand side is assured, may be expressed in the form (see *Concrete Mathematics*, by Graham, Knuth, and Patashnik, Addison-Wesley (1998) 2nd ed., p. 191):

$$f(a+x) = \frac{f(a)}{0!}x^{0} + \frac{\Delta f(a)}{1!}x^{\underline{1}} + \frac{\Delta^{2} f(a)}{2!}x^{\underline{2}} + \frac{\Delta^{3} f(a)}{3!}x^{\underline{3}} + \ldots \qquad (12 - 3).$$

Thus, in (12 - 2), we identify the constant term A with f(a), while the first order difference 'a '= $\Delta f(a)$ and the second order difference b = $\Delta^2 f(a)$; where $x^{\underline{1}} = x$, $x^{\underline{2}} = x(x - 1)$, etc. in (12 - 3).

2 i.e. without working out all the other intermediate values: in this case the formula used is equation (12 - 3) above with $x = 7/10$.

3 Inspection of Table 12-5 shows that the 2nd order sums of products are formed with an extra place, which is then rounded. The differences are added cumulatively in table E, and not added/subtracted to the previous difference, as in Table 12-3.

4 This follows in a straightforward manner from (12-1) & (12-2).

5 Thus, the method of the previous chapter is used to obtain an approximate value for √147 by first finding the number closest to a power of ten in the upper Chiliades by considering their logs: Briggs finds √147 × 8129 = N.N has a log that lies between 99322 and 99323. Simple proportion then gives this number more accurately to be 99322.7. However, with the power of the finite difference method available, and as the second order is sufficient as we are so close to a power of ten, the logarithms corresponding to 99322.7 and 99322.8 can be found to the full 14 figure accuracy, between which the log of N.N. is located. Hence, by simple proportion (again) the accuracy of the N.N. is increased dramatically, as Briggs shows, and so with the final value for √147.

§12.4. **Caput XII. [p.24.]**

Datis duobus numeris proximis, una cum eorum Logarithmis: intersere alios novem aequidistantes, inter eosdem; & eorum Logarithmos invenire.

Si datorum Logarithmorum differentiae secundae sint fere aequales, non erit difficile hoc praestare: alias, hic modus, si tertiae differentiae sint adhibendae, non nihil deficiet.

Sumantur duo numeri proximi A, & eorum Logarithmi B, una cum eorum differentiis primis C, & secundis D. Si differentiae secundae sint aequales, multiplicetur eorum altera per numeros in abaco E, decem primis numeris adiunctos: facti autem F G H I K demptis tribus ultimis notis, pro primis quinque addantur, pro totidem reliquis auferantur parti decimae differentiae primae C interiectae : toti & reliqui erunt differentiae Logarithmorum quaesitorum; quae dato minori Logarithmo, suo ordine continue additae, dabunt Logarithmos quaesitos. ut hic vides.
Sunto dati numeri 91235.91236. differentia prima 476014799.

	47602,0016 *C*	
91235 *A*	4,96016,14763,8639 *B*	5217 *D*
	47601,4799 C	
91236 *A*	4,96016,62365,3438 *B*	5217 *D*
	47600,9582 *C*	

E	
1. 45	addendi facti.
2. 35	
3. 25	
4. 15	
5. 5	
6. 5	auferendi facti.
7. 15	
8. 25	
9. 35	
10.45	

[p.25.]

Num. abs.	*Logarithmi.*	
912350	4,96016,14763,8639	
	4760,1715	*C+F*
912351	4,96016,19524,0354	
	4760,1662	*C+G*
912352	4,96016,24284,2016	
	4760,1610	*C+H*
912353	4,96016,29044,3626	
	4760,1558	*C+I*
912354	4,96016,33804,5184	
	4760,1506	*C+K*
912355	4,96016,38564,6690	
	4760,1454	*C-K*
912356	4,96016,43324,8144	
	4760,1402	*C-I*
912357	4,96016,48084,9546	
	4760,1350	*C-H*
912358	4,96016,52845,0896	
	4760,1297	*C-G*
912359	4,96016,57605,2193	
	4760,1245	*C-F*
912360	4,96016,62365,3438	

Facti		5217 *Factor*	
F - - -234	765	45	Factores reliqui.
G - - 182	595	35	
H - - 130	425	25	
I - - - 78	255	15	
K - - - 26	085	5	
47601479	9	$\frac{1}{10}$ C	
47601714	7	*C+ F*	
47601662	5	*C+ G*	
47601610	3	*C+ H*	
47601558	2	*C+ I*	
47601506	0	*C+K*	
47601453	8	*C - K*	
47601401	6	*C - I*	
47601349	5	*C - H*	
47601297	3	*C - G*	
47601245	1	*C - F*	

Si differentiae secundae sint inaequales ut hic ∵ addantur duae proximae, & sumatur semissis summae pro differentiae secundae, &multiplicetur at antea.

	4,51707,8187 *C*	
9615*A*	3,98294,92885,7405 *B*	469771 *D*
	4,51660,8416 C	
9616 *A*	3,98299,44546,5866 *B*	469672*D*
	4,51613,8744 *C*	
		939443 summa
		$469721\frac{1}{2}$ summae

∵

Facti:	469721.*factor*		
F 21137	445	45	reliqui factores.
G 16440	235	35	
H 11743	025	25	
I - 7045	815	15	
K - - 2348	605	5	

96150	3,98294,92885,7450	¶
	45168,1979	*C+F*
96151	3,98295,38053,9429	
	45167,7282	*C+G*
96152	3,98295,83221,6711	
	45167,2585	*C+H*
96153	3,98296,28388,9296	
	45166,7887	*C+I*
96154	3,98296,73555,7183	
	45166,3190	*C+K*
96155	3,98297,18722,0373	
	45165,8493	*C-K*
96156	3,98297,63887,8866	
	45165,3796	*C-I*
96157	3,98298,09053,2662	* *
	45164,9099	*C-H*†
96158	3,98298,54218,1761	
	45164,4401	*C-G*
96159	3,98298,99382,6162	
	45163,9704	*C-F*
96160	3,98299,44546,5866	

469721
105
2348605
469721
49320| 705 §

E		
1.	45	addasntur facti.
2.	80	
3.	105	
4.	120	
5.	125	
6.	120	
7.	105	
8.	80	
9.	45	

451660841	6	$\frac{1}{10}$ C
451681979	0	C + F
451677281	8	*C+G*
451672584	6	*C+H*
451667887	4	C + I
451663190	2	*C+K*
451658493	0	*C - K*
451653795	8	*C - I*
451649098	6	*C - H*
451644401	4	*C - G*
451639704	2	*C -F*

C 4516608416
C 7
facti 3161625891|2
49320|7 §
totus 3161675212
¶398294928854750
398298090532662
451660841| 6 $\frac{1}{10}$ C
factus - -- 11743| 0
† reliquus 451649098| 6

Quod si horum aliquem invenire cupias, reliquis omissis: numerus denario minor, numero *A* dato adiectus, multiplicet datam differentiam *C*; & numerus eidem adiunctus in abaco E, multiplicet differentiam secundam, &

[p.26.]

demptis hinc tribus, illinc unica nota, addantur facti: totus dato Logarithmo additus, dabit Logarithmum quaesitum. ut si scire velim quis sit Logarithmus numeri 96157, data differentia 4516608416 multiplicetur per 7 factus 31616258912. deinde105 situs e regione numeri 7 in abaco E, multiplicet differentiam secundam 469721, factus

49320|705 ablatis tribus notis ultimis, addatur priori facto, cui unica nota detracta fuerit 3161625891, totus 3161675212 addatur dato Logarithmo ¶. totus3,98298,09053,2662 erit Logarithmus numeri 96157. quaetus * *. Si differentiam inter Logarithmos huius numeri & proxime maioris scire cupias: numerus in abaco G adiunctus numerus 8 , qui datum septenarium unitate superat, multiplicet differentiam secundam 469721; factus 11743|025 ablatus tribus notis, auferatur e parte decima datae differentiae primae, reliquis 451649099 erit differentia quaersita †.

Invenire partis proportionalis accuratae.

Atque hic modus partem proportionalem inveniet satis accuratam. Cum enim secundum ea quae superiori capite tradebantur, Logarithmum datum transtulimus, a loco proportio in ultimam vel penultimam Chiliadem, poterimus primam partis proportionalis notam, numero absoluto in Chiliade reperto adijcere, & numeri aucti Logarithmum per proxime praecedentia invenire, una cum differentia inter eundem & proxime maiorem. haec differentia dabit partem proportionalem quam quaerimus accuratissimam huius rei unicum videamus exemplum.

Quaeratur latus numeri 147. Dati Logarithmus est 2,16731,73347,4814, huius semissis, 1,08365,86673,7409 est Logarithmus quaesiti lateris.

	Logarithmi.	
√147	1,08365,86673,7409	datus
8192	0,91338,99436,3175	complemento proximus.
N.N	0,99704,86110,0584	totus.
	31548,5329	differentia.
99322	0,99704,54561,5255	
	43725,6897	differentia.
99323	0,99704,98287,2152	

proport.	43725	differentia datae.
	31548	
	10 - -	
	7 - - -	quaesita.

Hucusque per praecepta cap. 11 quae dederunt notam 7, numero 99322 in abaco reperto adijciendam. reliqua quaerantur secundum ea quae proxime praecesserunt.

	43726,1300	
993220	0,99704,54561,5255	4403
	43725,8697	C
993230	0,99704,98287,2152	4402
	43725,2495	
993220	0,99704,54561,5255	R
	30608,0290	Q
993227	0,99704,85169,5545	M
	4372,5580	C - H
993228	0,99704,89542,1125	

	C 437256897	factores
	7	
	3060798279	factus
	4403	factores
	105	
	22015	
	4403	
	462|315	factus
facti	306079827|9	
	462|3	
Q	306080290|2	addendus
	4403	factores
	25	
	22|015	
	88|06	
	110|075	factus auferendus.
	C 43725689|7	
	H - - - - -110|2	auferendus.
	C - H 43725579|6	

Facto a data differentia C in 7, addatur factus a differentia secunda 4403, & 105 factore reperto in abaco E: totus Q additus dato Logarithmo R dabit logarithmum M, numberi 993227, inter quae & proximum 991228 est differentia C - H, quae partem dabit proportionalem fere accuratissimam, si totus NN 0,99704,86110,0584, conferatur cum M sic. [p.27.]

NN	0,99704,86110,0584	
	1940,5039	
993227	0,99704,85169,5545	M
	4372,5580	
993228	0,99704,89542,1125	

proport.	43725580-	Differentia datae..
	9405039-	
	10----------	
	21509238	pars proportionalis quaesita.

Est igitur numerus 99704,86110,0584 congruens Logarithmo NN 9932272150923. Quod si numerus 8192, vel eiusdem factores 8.8.8.8.2 diviserint numerum, inventum, quotus 12124,35565,29831,6 erit latus numeri 147 quaesitum & amplius. est enim latus vero proximum, 12124,35565,29824,41054.