§14.1. ***Synopsis: Chapter Fourteen.***

A method is presented for finding the number N corresponding to a given logarithm L, or vice versa, where either the number or the logarithm is given, and lies between tabulated values in the tables of Chiliades.

The method depends on writing N in the form: $N \sim N_0(1+q_1)(1+q_2)....(1+q_n)$, where N_0 is the largest tabulated number less than N, and the q_i are decimal fraction parts found in an iterative manner, each having a single significant digit following a string of zeros, as in Table 13-7, and being at least one order less in magnitude than its predecessor.

The value of N can readily be found from its known logarithm: for the logarithm of the largest tabulated number N_0 less than N is removed: successive known logarithms corresponding to terms of the form $1 + q_1$, $1 + q_2$, etc are then removed, until the known logarithm or the desired accuracy is reached.

To find the logarithm of N, N is divided successively by N_0 as defined already, $N_0(1+q_1)$, $N_0(1+q_1)(1+q_2)$, until N or a number with the desired accuracy is reached. The logarithms of the factors are found from the table.

§14.2. ***Chapter Fourteen***. [p. 33.]

For whatever given logarithm, to find the agreeing absolute number, and vice-versa.

In Chapter 11 we show how a number found between the Chiliads can be increased by proportionality, in order that the place of the error is hardly within the twelfth place. Now I show how with the help of this in the most recent preceding table [Table 13 - 7], through subtraction and multiplication alone, the true number sought can be found: providing the number is rational and can be described by fourteen or fewer places.

The numbers which are held here in this table, are set out in nines: which are from the left, ones, and some small fraction of one. The logarithms of these numbers are placed across on the right. The first nine are the unit numbers, designated by the letter *A*, the nine nearest ones designated by the letter *B,* are one and a unit tenth part of one; as $1\underline{1}$, $1\underline{2}$, $1\underline{3}$, etc. The next nine designated by the letter *C*, are one and the hundredths. The rest of the nines [are designated] similarly: *D* thousandths adjoining with unity, *E* tens of thousandths, etc. With the aid of these, the agreeing absolute number is found for a given logarithm. Let the given logarithm be 3,66067,57883,3852. From this (when the characteristic is dropped) the logarithm of three, 0,47712,12547,19662 , selected from the first [set of] nines, is taken away; there is left 0,18355,45336,18858: from which the logarithm of $1\underline{4}$ is taken away, which the table [14-1] shows here in the second set of nines, then from the nearest nines $1\underline{09}$ is taken away, and nothing remains. I assert the continued product from these three 3, 14, 109, namely 4578, is the number sought. The whole operation itself is performed as follows:

0,66067,57883,3852	logarithm given	14	multiplicand
0,47712,12547,19662	- - - 3	3	multiplier
0,18355,45336,18858	the remainder	42	product
0,14612,80356,78238	- - - - - 14	42	multiplicand
0,03742,64979,40620	the remainder	109	multiplier
0,03742,64979,40622	- - - - - 109	378	
0,00000,00000,00000		42	
		4578	product

[Table 14-1]

But the most advantageous [approach] is to take the next smallest logarithm given from the Chiliads, which is taken from that given logarithm, and the absolute number of this logarithm, found in the margin is noted. The rest indeed are carried out with the help of this table. For let the given logarithm be 3,48314,00744,3475 whereby the nearest logarithm in the Chiliad is found, 3,48301,64201,4413 which taken from the given logarithm, leaves 0,00012,36542,9082. Noting 3041 found in the margin, I look for the remaining logarithm in this little table, or of that nearest logarithm: which is subtracted, noting down the absolute number of the same, thus I progress as before. Here you have this method of working described. [Table 14-2].

3,48314,00744,3475	
3,48301,64201,4413	- - - - 3041
0,00012,36542,9062	
0,00008,68502,11648	- - - - 10002
0,00003,68040,78972	
0,00003,47421,68882	- - - - 100008
0,00000,20619,10090	
0,00000,17371,74453	- - - - 10000,04
3247,35637	
3040,06031	- - - - 10000,007
207,29606	
173,71779	- - - - 10000,0004
33,57827	
30,40061	- - - - 10000,00007
3,17766	
3,04006	- - - - 10000,0000,07
13760	
13029	- - - - 10000,0000,003
731	
434	- - - - 10000,0000,0001
297	
261	- - - - 10000,0000,00006
36	
34	- - - - 10000,0000,0000,08

[Table 14-2]

In this operation we have twelve logarithms, the first of which is taken from the Chiliads, the nearest individual six are taken from the single sets of nines of this table, the remaining five truly from the furthest nines. Since indeed only this [amount] lies between the ensuing sets of nine and the last, because the subsequent significant places have more zeros in front; conveniently, that final place can be used in place of the subsequent [places]; if we progress further with this, we

30401	factors
10002	
3041- - - -	
6082	
30416082	product
100008	factor
30416082- - - - - -	
243328656	
30418515288656	product
1000004	factor
3041851528656 - - - - -	
12167406114624	
30418636960211	
212930459	- - - -10000,007
304186582536670	product
12167463	- - - -10000,0004
304186594704133	product
2129306	- - - -10000,00007
304186596833439	
212930	- - - - 10000,0000,07
304186597046369	
9125	- - - - 10000,0000,003
304186597055494	
304	- - - -10000,0000,0001
304186597055980	
182	- - - - -10000,0000,00006
304186597055980	
24	- - - - - 10000,0000,0000,08
304186597056004	

[Table 14-3].

may truncate more places there, because we observe the product from the foregoing. These twelve logarithms collected in one sum are equal to the given logarithm: but the numbers which are located straight across are factors, which by continued multiplication produce the number agreeing with the given logarithm. Not very much is returned from where the multiplication begins, only the product of the first by the second, multiplied by the third; and the following product by the fourth: and successively in the same way, until the individual factors left multiply the preceding product. As 30401 by 10002, the product is 30416082, which multiplied by the third 100008, makes 3041851528656, as you see here:

Finally, these factors give the number sought to us:304186597056004.

For any given absolute number, to find the corresponding logarithm.

Above we have shown, how for a given logarithm, the corresponding absolute number is found by the subtraction of the included logarithms from the preceding table, and the continued multiplication of the numbers placed across. Here the opposite is to be made clear, how for a given number the corresponding logarithm is found: by divisions of the given number, and the addition of the answering logarithms of the quotients. If the number given is written with only four places, the logarithm of this number is included among the Chiliads described. But if more places are given; a number comprising the first four places is taken (except perhaps for the sake of amusement, it may be more pleasing to be putting a number to the test either with two or three places or both, but all shall turn out the same). It divides that given number thus; as after you take away one times the divisor from the dividend, the product found from the divisor taken with the quotient, is always added to the divisor; while the logarithms of the particular quotients found in the table are added to the logarithm of the first divisor. The total of these is the logarithm of the given number sought. As, let the given number be 3041851528656, I divide this number by 3041, which I subtract from the number which is divided, noting down 1 in the quotient: there remains 851528656. Then I continue, and after three ciphers I place 2 in the quotient, and I subtract the product of 2 by the given divisor from the dividend, writing the remainder below. Afterwards I add the same product to the divisor, and I divide the remainder by the same increased amount: the product is taken away from the dividend, and adding the same to the divisor, as before. You can examine the method of the whole operation here[1].

A. 3041- - - -divisor given	3041851528656 (100028
6082 first product	3041
C. 30416082 divisor to be increased	851528656
243328656 second product	6082 - - - -first product
E. 3041851528656 divisor to be incr.	243328656
	243328656 second product
	Logarithms
A. Given divisor 3041	348301642014413
B. Second quotient 10002	0000086850211648
C. With divisor increased	3483103270355778
D. Third quotient 100008	0000034742168882
E. With divisor increased again	3483138012524660

[Table 14-4]

The description of all of these is taken from the second chapter. Indeed the given divisor *A* and a second quotient *B* are the factors of the increased divisor *C* : which is different from the divisor given, which without harm is called the second divisor. And the logarithm of this number is equal to the logarithms of *A* and *B*, by the axiom, Ch.2. Then the second divisor undertakes the same work, dividing the whole given number, and finding the third quotient 10008. For although the final place of the quotient only can multiply, and the product from the same only taken away from the remainder, yet we know the product from the unit place also to be taken away, in that division, which precedes finding this final quotient: therefore if the logarithm of this final quotient *D* is added to the logarithm of the second divisor *C*, the total is the logarithm of the divisor enlarged anew *E*: that is, of the dividend itself. Indeed this division proceeds, and increasing its divisor, while with the increase of the factors in the end it can itself become equal to the given dividend, of which the logarithm is sought. Which I will try to make clear with another example. Let the given number of which the logarithm is sought be 296682051456, this I divide by 2966. Here you can examine the whole series of operations.

2966 - - - - given divisor	296682051456 (10002766242
5932 1st factor *A*	2966
29665932 - - -2nd divisor	82051456 Quotients
207661524 *B* 2nd factor	A - -5932 - - - - - - - - - - - 2
2966800861524 3rd divisor	22731456
178900805 *C*	*B* - - 207661524 - -- - - - - - 7
2066818662329 4th	19653936
1780091 *D*	*C* - - 17800805169144 6
2966820442420 5th	1852230831
59336 *E*	*D* - 1780091197 - - - -6
2966820501756 6th	72139634
11867 *F*	*E* - -59336409 - - - 2
2966820513623 7th	12803225
890 *G*	*F*-11867282 - - - 4
2966820514513 8th	935943
30 *H*	*G* 890046 - - 3
2966820514543 9th	45897
15 *I*	*H* - 29668 - - 1
2966820514558 10th	16229
1 *K*	*I* - - - 14834 - -5
2966820514559 11th	1395
	K - - -1187 - -4
	208
	L - -207 - -7
	1

[Table 14-5]

	Logarithms	Quotients
Given divisor 2966 - - -	3,47217,11466,9237	
The logarithms of the quotients you see here which are added to the logarithms of the given divisors to determine the logarithm of the given number. Which certainly we can find, if we bring together the logarithms of the numbers 9072.6036.5418 which by continued multiplication, produce the given number 296682051456 - - -	0,00008,68502,11648	10002
	0,00003,03995,49597	100007
	0,00000,26057,59074	1000006
	0,00000,02605,76611	10000006
	0,00000,00086,85890	100000002
	0,00000,00017,37178	1000000004
	0,00000,00001,30288	10000000003
	0,00000,00000,04343	100000000001
	0,00000,00000,02171	1000000000005
	0,00000,00000,00174	10000000000004
	0,00000,00000,00030	100000000000007
	3,47229,12733,49534	

[Table 14-6]

Until now we have been concerned with the making and purpose of logarithms.

There follows some considerations of their use.

§14.3. *Note on Chapter 14.*

[1] We can summarise this process as follows:

$N - N_0 = r_1; r_1 - q_1 N_0 = r_2$, leading to $N\text{-}N_0(1+q_1) = r_2$;
subsequently, $r_2 - q_2 N_0(1+q_1) = r_3$; or $N - N_0(1+q_1)(1+q_2) = r_3$, *etc.*

§14.4. **Caput XIV.** [p. 33.]

Dato cuicunque Logaritho numerum absolutum convenientem invenire. & contra.

Ostendimus cap.11. quomodo numerus inter Chiliadas repertus, per partem proportionalem augeri possit, vt vix errori sit locus intra locum duodecimum. nunc ostendam quomodo ope huius in proximo praecedentis tabellae, per solam subductionem & multiplicationem numerus verus quaesitus inveniri possit: si modo sit numerus rationalis, & quatuordecim aut paucioribus notis describi poterit.

Numeri qui hac tabella continentur, distinguntur per Novenarios. qui sunt a sinistris, sunt Unitates, & unitatis aliquot particulae: a dextris e regione siti sunt eorum Logarithmi. Primi novem sunt Unitates, & signantur litera A: proximi novem signati litera B, sunt unitates, & unitatum partes decimae; ut $1\underline{1}$, $1\underline{2}$, $1\underline{3}$, &c. Proximi novem signati litera C, sunt Unitates & centesimae. reliquae Novenarii deinceps similiter, D millesimas adiiciunt unitati. E decies millesimas, &c. Horum ope, dato Logarithmo Invenitur numerus conveniens absolutus ad hunc modum. Esto datus Logarithmus 3,66067,57883,3852. ab hoc (ubi Characteristicam deposuerit) Logarithmus Ternarii 0,47712,12547,19662, e primo Novenario desumptus auferatur, restabit 0,18355,45336,18858: a quo auferri poterit Logarithmus $1\underline{4}$ quem tabella haec exhibet in secundo Novenario, deinde in proximo Novenario Logarithmus $1\underline{09}$ auferendus est, & restabit nihil. aio continue factum ab his tribus 3, $1\underline{4}$, $1\underline{09}$, nempe $4\underline{578}$ esse numerum quaesitum. Tota operatio sic se habet.

0,66067,57883,3852	logarithmus datus.	$1\underline{4}$	multiplicandus.
$\underline{0,47712,12547,19662}$	- - - 3	$\underline{3}$	multiplicator.
0,18355,45336,18858	reliquus.	$4\underline{2}$	factus.
$\underline{0,14612,80356,78238}$	- - - - - $1\underline{4}$	$4\underline{2}$	multiplicandus.
0,03742,64979,40620	reliquus.	$1\underline{09}$	multiplicator.
0,03742,64979,40622	- - - - - $1\underline{09}$	378	
0,00000,00000,00000		$\underline{42}$	
		$4\underline{578}$	factus.

Commodissimum autem erit e Chiliadibus sumere Logarithmum proxime minorem dato, qui auferatur e dato, eiusque numerus absolutus in margine repertus, notetur. reliqua vero sunt ope huius tabella pergenda. ut esto datus Logarithmus 3,48314,00744,3475 Logarithmus proximus in Chiliade quare repertus est, 3,48301,64201,4413. qui ablatus e dato relinquit 0,00012,36542,9082. notato 3041 in margine reperto, quero in haec tabella Logarithmum relictum, vel ei proximum: quem subduco , adnotans eiusdem numerum absolutum. & progredior ut antea. operationis nodum his descriptu,m habes.

3,48314,00744,3475	
$\underline{3,48301,64201,4413}$	- - - - 3041
0,00012,36542,9062	
$\underline{0,00008,68502,11648}$	- - - - $1\underline{0002}$
0,00003,68040,78972	
0,00003,47421,68882	- - - - $1\underline{00008}$
0,00000,20619,10090	
$\underline{0,00000,17371,74453}$	- - - - $1\underline{0000,04}$
3247,35637	
$\underline{3040,06031}$	- - - - $1\underline{0000,007}$
207,29606	
$\underline{173,71779}$	- - - - $1\underline{0000,0004}$
33,57827	
$\underline{30,40061}$	- - - - $1\underline{0000,00007}$
3,17766	
$\underline{3,04006}$	- - - - $1\underline{0000,0000,07}$

13760

13029 - - - - 10000,0000,003

731

434 - - - - 10000,0000,0001

297

261 - - - - 10000,0000,00006

36

34 - - - - 10000,0000,0000,08

[p.34.]

In hac operatione habemus duodecim Logarithmos, quorum primus desumitur e Chiliadibus; sex proximi singuli e singulis novenariis huius tabella, reliqui vero quinque ex ultimo novenario. Cum enim inter subsequentes novenarios & ultimum, hoc tantum intersit; quod subsequentium notae significativae, plures habent cyphras praepositas; ultimus ille non incommode poterit loco subsequentium usurpari, si quo longius progredimur, eo plures notas amputemus: quod in antecedentibus factum fuisse observare poterimus. Hi duodecim Logarithmi, in unam summam collecti, aequantur dato Logarithmo: numeri autem e regione locati, sunt factores, qui per continuam multplicationem numerum producunt dato Logarithmo convenientem. Nec magnopere refert unde multiplicatio incipiat, modo factus a primo in secundum, multiplicetur per tertium; & factus secundus, per quartum : & deinceps eodem modo, donec singuli factores reliqui, praecedentem factum multiplicaverint. ut 30401 per10002, factus erit 30416082, qui multiplicatus per tertium 100008, facit 3041851528656, ut hic vides.

30401 } factores

10002

3041- - - -

6082

30416082 factus

100008 factor

30416082- - - - - -

243328656

30418515288656 factus

1000004 factor

3041851528656 - - - - -

12167406114624

30418636960211 factus

212930459 - - - -10000,007

304186582536670 factus

12167463 - - - -10000,0004

304186594704133

2129306 - - - -10000,00007

304186596833439

212930- - - - 10000,0000,07

304186597046369

9125 - - - - 10000,0000,003

304186597055494

304 - - - -10000,0000,0001

304186597055980

182 - - - - -10000,0000,00006

304186597055980

24 - - - - - 10000,0000,0000,08

304186597056004

[p.35.]

Tandem hi factores nobis numerum quaesitum dederunt 304186597056004.

Dato cuilibet numero absoluto, Logarithmum congruum invenire.

Superius ostendimus , quomodo dato Logarithmo numerus conveniens absolutus sit inveniendus; per subductionem Logarithmorum in praecedente tabella comprehensorum, & continuam multiplicationem numerorum e regione positorum: hic contra ostendendum, quomodo dato numero Logarithmus conveniens sit quaerendus; per dati divisionem, & Logarithmorum quotis respondentium additionem. Si datus numerus quatuor tantum notis scribatur, eius Logarithmus inter Chiliadas descriptus habetur. sin pluribus; sumatur numerus primis quatuor notis comprehensus (nisi forte animi causa, aut tentandi an utroque modo eadem omnia eventura sint, duarum aut trium notarum numerus magis arrideat). is datum numerum ita dividat, ut postquam divisorem semel subduxeris e dividendo, factus a divisore ducto in quotum inventum, additur semper divisori: quotorum autem particularium in tabella reperti Logarithmi, addantur divisoris primi Logarithmo: totus erit dati numeri quaesitus Log. ut sit datus numerus 3041851528656, hunc numerum divido per 3041, quem subduco e numero divedendo, adnotans in quoto 1: restabit 851528656 . deinde pergo , & post tres cyphras pono 2 in quoto, & factum a 2 in datum divisorem subduco a dividendo, subscribens reliquum. postea eundem factum addo divisori, & per eundem auctum, reliquum divido; factum auferens e dividendo, & eundem addens divisori: ut antea. totius operationis modum hic cernere poteris.

A. 3041- - - -divisor datus	3041851528656 (100028
6082 factus primus	3041
C. 30416082 divisor auctus	851528656
243328656 factus secundus.	6082 - - - -factus primus.
E. 3041851528656 divisor auctus	243328656
	243328656 factus secundus
	Logarithmi.
A. Divisoris 3041	348301642014413
B. Quoti secundi 10002	0000086850211648
C. Divisoris aucti	3483103270355778
D. Quoti tertij 100008	0000034742168882
E. Divisoris aucti denuo	3483138012524660

Horum omnium demonstratio petenda est e capite secundo. sunt enim Divisor datus A, & Quotus secundus B, factores Divisoris aucti C: qui alius est a divisore dato, & non iniuria appellari poterit divisor secundus. eiusque Logarithmus aequatur Logarithmis A & B. Per 3.ax.cap.2. deinde secundus divisor

[p.36.]

idem negotium suscipit, dividens totum datum numerum, & inveniens quotum tertium 100008. nam licet ultima quoti nota multiplicet tantummodo, & factus ad eodem, solius auferatur e reliquo, tamen intelligimus factum ab unitatis nota etiam auferri, in ea divisione quae huius ultimi quoti inventionem praecessit: idcirco si huius ultimi quoti D Logarithmus, addatur Logarithmo divisoris secundi C, totus erit Logarithmus divisoris denuo aucti E: id est ipsius dividendi.

Progreditur enim haec divisio, eiusque augens divisorem, donec accessione factorum tandem fiat aequalis ipsi dividendo dato, cuius Logarithmus quaeritur. quod altero exemplo manifestum facere conabor.

Esto datus numerus cuius Logarithmus quaeritur 296682051456, hunc divido per 2966. totam operationis seriem hic cernis.

2966 - - - - divisor datus.	296682051456 (10002766242
5932 factus 1^{us} *A*	2966
29665932 - - - divisor 2^{us}	82051456 Quoti.
207661524 *B* factus 2^{us}	A - -5932 - - - - - - - - - - - - 2
2966800861524 divisor3^{us}	22731456
178900805 *C*	*B* - - 207661524 - -- - - - - - 7
2066818662329 4^{us}	19653936
1780091 *D*	*C* - - 17800805169144 6
2966820442420 5^{us}	1852230831
59336 *E*	*D* - 1780091197 - - - -6
2966820501756 6^{us}	72139634
11867 *F*	*E* - -59336409 - - - 2
2966820513623 7^{us}	12803225
890 *G*	*F*-11867282 - - - 4
2966820514513 8^{us}	935943
30 *H*	*G* 890046 - - 3
2966820514543 9^{us}	45897
15 *I*	*H* - 29668 - - 1
2966820514558 10^{us}	16229
1 *K*	*I* - - - 14834 - -5
2966820514559 11^{us}	1395
	K - - -1187 - -4
	208
	L - -207 - -7
	1

	Logarithmi.	
Divisor datus 2966 - - -	3,47217,11466,9237	Quoti.
Logarithmi Quotorum quos hic cernis, additi Logarithmo dati divisoris, constituut Logarithmum dati numeri. Quem verum esse deprehendemus , si eum conferamus cum Logarithmis numeris 9072.6036.5418 quorum continua multiplicatio, producit datum numerum 296682051456 - - -	0,00008,68502,11648	10002
	0,00003,03995,49597	100007
	0,00000,26057,59074	1000006
	0,00000,02605,76611	10000006
	0,00000,00086,85890	100000002
	0,00000,00017,37178	1000000004
	0,00000,00001,30288	10000000003
	0,00000,00000,04343	100000000001
	0,00000,00000,02171	1000000000005
	0,00000,00000,00174	10000000000004
	0,00000,00000,00030	100000000000007
	3,47229,12733,49534	

Hactenus de creatione & affectionibus Logarithmorum. Sequuntur quaedam ad eorum usum spectantia.