§17.1. *Synopsis: Chapter Seventeen.*

Briggs demonstrates by example how any particular member of a G.P. can be found, defined only by two numbers and the given number of intervals between them. He then considers the application of logarithms to a series of problems involving compound interest, cash values of annuities, etc.

§17.2. *Chapter Seventeen.* [p.41.]

For two given numbers, together with the number of continued mean proportional intervals between the given numbers; to find any mean called for in the same series.

We show how for numbers in continued proportion from unity, where we call the root the number closest to unity, how any different root desired either nearer or further

[p.42.] from unity than the one given may be found. Now it ought be shown how between any two of the mean proportionals, any mean requested can found. But since there is no need for a new rule, I shall try to show this with some examples. Let the two numbers 729 and 15625 be taken, and let there be 5 intermediate means between the given numbers.

The logarithms of the given numbers are to be taken, and the differences of these, which is to be divided into six parts for the number of intervals between the given numbers; then if the sixth part *A* of the given difference, is multiplied by the ratio of the separation of the number sought to that given, with the same given logarithm added or taken away; the sum or difference will be the logarithm of the number sought, as we see here:

		Logarithms	Numbers in continuous proportion.
Given numbers	729. *B*	2,86272,75283,1797	
	15625. *C*	4,19382,00260,1611	944784. *B* - *D*
		1,13109,24976,9814. 6.*A*	157464.
	A	0,22184,87496,1636. 1.*A*	26444
	D	0,88739,49984,6544. 4.*A*	4374
	B	2,86272,75283,1797	729 - - *B*
5625 - - - - - - - - -		3,75012,25267,8341. *B* + *D*	1215
944784 - - - - - - - -		1,97533,25298,5253. *B* - *D*	2025 - - *C* - *D*
$120563^{22}/_{81}$ - - - - -		5,08121,50244,8155. *C* + *D*	3375
2025 - - - - - - - - -		3,30642,50275,5067. *C* - *D*	5625 - - *B* + *D*
			9375
			15625 - - *C*
			$26091^{2}/_{3}$
			$43402^{7}/_{9}$
			$72337^{26}/_{27}$
			$120563^{22}/_{81}$ C + D

[Table 17-1]

If the sixth part *A* of the difference of the given logarithms, is multiplied by 4, and the product *D* is added to the logarithm of the number *B*, then the total is 5625, the fourth continued proportion above *B*. But if is taken away from the same, it gives 944784, the fourth below the same. In the same way the numbers below and above *C* are found.

Another example.

Let 10 & 11 be given, & let there be 12 intervals between the given numbers.

		Logarithms	
Given	10. *B*	1,00000,00000,0000	
	11. *C*	1,04139,26851,5823	
		0,04139,26851,5823.	12.*A*
	A	0,00344,93004,2985	

I desire to know the 7th from the smaller between the numbers given, which is 1057172197209

	B.	1,00000,00000,0000	
		0,02414,57330,0895.	7*A*
1057172197209 - - -		1,02414,57330,0895.	*B* + 7*A*

[Table 17-2]

In the same way we can find any number between the given numbers or beyond.

We see an example[1] of this in compound interest, with 6 per cent to be added for each year: which interest was little approved by the citizens from ancient times (who disapproved of the tenth part interest on money). Let the given principal be 123 pounds: I wish to know how much ought to be added to the principal, at the end of as many years, months or days time as you please; if the profit of the whole year is distributed thus in individual months or days, with the profit added to the principal at any time you wish. The [same] ratio is always kept of the profit to the aggregate of profit and principal, in order that the profit of the whole year with 100 pounds capital amounts to only six pounds[2].

In the first place, the logarithms are taken of the given terms of the ratio, 100 & 106, and of their difference *B*, which is called the annual payment; that [in turn] divided into twelve parts gives in the quotient the monthly difference *C* ; and that [in turn] divided by $30^1/_2$, which is nearly how
p.43.] many days there are in an equal month, gives in the quotient the daily difference *D*; and which if we wish to know with greater accuracy, the difference *B* is divided by 365, the number of days in the common Julian year. Then the logarithm of the given principal is taken.

		Logarithms	
Terms of the given account	100	2,00000,00000,0000	
	106	2,02530,58652,6477	
	B	0,02530,58652,6477	payable annual difference
	C	0,00210,88221,0540	payable monthly difference
	D	0,00006,93311,3771	payable daily difference
Principal - 123 - - - - - - - -		2,08990,51114,3940	

[Table 17-3]

With those differences prepared in this way, if it is asked how much capital ought to be added at the end of 7 years, 5 months, & 9 days: the individual differences are multiplied by their numbers: and the sum of the products is added to the logarithm of the capital; [then] the total is the logarithm of the aggregate of the principal and the profit, at the end of the agreed time.

But if the same sum of the factors are taken from the logarithm of the principal[3],

With money to appreciate	7 years difference	0,17714,10568,5339	7.B
	5 months difference	0,01054,41105,2700	5.C
	9 days difference	0,00062,39802,3939	9.D
The amount owing with compound interest for the principal in the account for the time.	The sum of the factors	0,18830,91476,1978	
	The logarithm of the principal	2,08990,51114,3940	£123
	Total	2,27821,42590,5918	1897641892
With money to depreciate.	Remainder	1,90159,59638,1962	797252637

[Table 17-4]

the remainder is the logarithm of the just amount, with which the capital may be redeemed with cash at hand, if as many years, months, and days, are counted before the day of settlement of the account. As with this example:

I assert that 123 minas [*i.e.* invested] is worth 1897641892 at the end of 7 years, 5 months, and 9 days, that is 189 - 15 - 341 . And if the day of payment [*i.e.* of a loan], by which 123 minas ought to be settled, is separated by the same total number of years, months and days: then the just price of settlement, if the money is paid out in cash, will be 797253, or 79 - 14 - 606. And with the same rate of interest kept; the principal 79 - 14 - 606 after 14 years, 10 months, and 18 days, will be worth 189 - 15 - 341.

The manner of working is the same, with any other given ratio of profit to capital : for any given interval of time, before or after the day of settlement.

2. *For any given capital and number of years profit you please: to find the ratio of the profit to the capital for one year, or month, or day.*

Let the given principal of money be 1234 pounds: [Though Briggs uses 1234, the table below has a characteristic of 4 in the original, which seems to be incorrect: hence, either the principal or the calculation has to be changed; here we have changed the characteristic from 4 to 3], In a time interval of ten years, I wish to add 766 pounds to the principal: in order that the total of the principal and the profit is 2000 pounds.

The difference of the logarithms of the principal and the total is taken: the tenth part of this difference is the annual difference, which added to the logarithm of the principal gives the logarithm of the principal with the profit gain of the year.

	Logarithms	
Principal 1234 - - - - - - - -	3.09131,51596,9723	
Principal with 10 year gain 2000	3,30102,99956,6398	
	0,20971,48359,6675	difference for 10 years
	0,02097,14835,9667	difference per annum
First year principal and added profit	3,11228,66432,9390	129$\underline{505032}$

The added profit of the principal of the first year will give 129$\underline{505032}$.

[Table 17-5]

The same annual difference added to the logarithm of 1000 gives the proper annual increment of profit above 1000.

	Logarithms	
1000	3,00000,00000,0000	
	0,02097,14835,9667	difference per annum
1049$\underline{473527}$	3,02097,14835,9667	

[Table 17-6]

[p.44.]

I assert that 1000 pounds cash with the yearly payment interval acquires $49\frac{1}{2}$ pounds

In the same way the monthly or daily interest can be found, which if it is continued for 10 years, gives with the capital added, the sum of 2000 pounds.

For the monthly interest the difference for 10 years is divided by 120, the number of months in a decade; daily, by 3652.

	Logarithms		
monthly difference	0,00174,76236,3306		
daily difference	0,00005,74246,5388		
1234 principal	3,09131,51596,9723	1234	
increased principal end of first month	3,09306,27833,3029	1238 975685	1238 - 19 - 6 16.
principal after first day	3,09137,25843,5111	1234 163177	1234 - 3 - 3 16.
First month principal added	4 - 19 - 6 16.		
First day " "	0 - 3 - 3 16.		

[Table 17-7]

3. How much is an annuity worth at the end of as many years as it pleases, and what value of redemption can it have for any number of years before the time of the first agreed settlement?

Titus must pay Sempronius [an annuity of] 57 minas ten times, the first payment to become due at the end of a 5 year period, then just as much yearly until the repayments are 570 minas. Titus wants to redeem this account with cash at hand, with the evaluation made according to the interest of 12 months at $^1/_2$% per month, which adds 6% to the capital each year.

1. In the first place, the principal is to be found which, following the given computation, gives an increase of 57 minas for a single year.

		Logarithms
proportions	6	0,77815,12504
	100	2,00000,00000
	57	1,75587,48557
principal found	- - - 950	2,97772,36053

[Table 17-8]

The logarithm of the principal found by Chapter 15 gives the principal[4] 950. Then by Prop.1 of this Chapter, the value is sought of the principal and the interest at the end of the 10 year period. The terms of the given account are to be taken, & of these the logarithms and the difference of the logarithms: which multiplied by the number of years, & added to the logarithm of the principal found, gives the logarithm of the total money, which is owed for this principal at the end of ten years, if no payment has been made meanwhile. From this sum, if the principal is taken away, the money remains which the sum of the annual payments has acquired.

		Logarithms	
Terms of the ratio	100	2,00000,00000	
	106	2,02530,58653	
		0,02530,58653	annual difference
		0,25305,86526	10 year difference
principal found		2,97772,36053	
principal increased over 10 years		3,23078,22579	170130531

[Table 17-9]

At the end of the 10 year period the increased principal is 170130531, from this sum the principal assumed earlier, 950 is taken away; when this is removed, 75130531 minas remain. Which with the payments themselves accumulating & growing, kept on prospering [5].

2. On the other hand, Sempronius (as it is not necessary for the first repayment to be made before the end of the five year period) nevertheless should expect this sum after a total of fourteen years

[p.45.] [*i.e.*, if he invests his cash pay-out at 6%]. Therefore it is asked how much the depreciation of this sum ought to become, should he desire cash at hand.

	Logarithms
Sum of all the money 170130531	2,87581,64581
difference for 14 years (assumed to depreciate at 6%)	0,35428,21136
value 3323030636	2,52153,43445

[Table 17-10]

If the difference of 14 years is taken away from the logarithm of the sum, there will remain the logarithm of the present value, 3323030636. Because in accordance with the given ratio, in 14 years this will give the sum of the money, which the payments lumped together were worth.

4. Given a sum of money, and the ratio of the increase for a given number of years, to find the annual return value.

Let the given sum of money be 300 minas, with the given annual rate of increase 100 to 106, the given time to be a ten year period, and let the first settlement be at the end of 5 years. It is asked what the annual returns may be.

With this question it would be hardly possible to be expedient in any other way than by proportion. Usually in this way.

We invest the annual return, whatever it pleases, for the time agreed, & according to Prop.3 of this Chapter. Then through proportion, counting at once the value sought of the same, it is the returns to be invested, to the given sum of money: in order that the annual return is invested to the return in question.

The annual return invested is six minas, it is the ratio for a given principal of 100 minas, which in the interval of a year becomes 106 minas.

		Logarithms	
Terms of the given account	100	2,00000,00000	
	106	2,02530,58653	
Annual difference		0,02530,58653	
10 year difference		0,25305,86526	
Given principal	100	2,00000,00000	
Increase of the account for 10 years		2,25305,86526	1790847697
Principal with interest gain is found for 10 years		1790847697	

[Table 17-11]

But with the principal taken away, the true value of the annual returns is left at the end of 10 years: 790847697. But this ten year interval begins after four years, with the first settlement to be owed after the end of the five year period. Therefore the principal is required, which in accordance with the given ratio through 14 years, gives the increase 790847697.

	Logarithms	
value of the return after 10 years 790847697	1,89809,28534	
difference for 14 years to be taken away	0,35428,21136	
log. of the remainder of principal sought	1,54381,07398	3497926995

[Table 17-12]

This principal 3497926995 increased in accordance with the given ratio over fourteen successive years, gives the same sum of money, as the annual returns of 6 minas gives for the decade.

Therefore this principal is the correct value of the same annual returns, from which point I finish through proportion :

		Logarithms
Principal	3497926995	1,54381,07398
Interest	6	0,77815,12504
Principal	300	2,47712,12547
Return interest	514590502	1,71146,17653

[Table 17-13]

At last the annual returns required is found 514590502 , which should be settled at the end of the nearest five year period, & should be continued for the following nine years, if the same 300 minas value is to be paid [6].

§17.3. ***Notes for Chapter Seventeen.***

[1] We note that Briggs' foray into the financial world shows the ease of using logarithms in evaluating terms in the series $P(n) = P_0(1 + r)^n$ for compound interest, while annuities or pensions are considered according to $P(n) = P_0(1 - r)^n$. Before the advent of algebra, one had to make the mental leap from the intuitive idea of the compounded sum or difference straight to the arithmetical calculation.

[2] We can safely consider that Briggs has in mind here the English pound, or a similar currency, as he divides the decimal fractions into 20 parts or shillings, and each 20th part into a further 12 parts, the pennies. In later examples, he uses the mina, which was a Greek silver coin, which he also considers to be divided up in the same way as pounds. It is worth noting that Briggs always refers to the ratio *a* : *b* as the ratio of *b* to *a*.

[3] The first period of time considered brings the value up to the final total 189 - 15 - 3$\underline{41}$ for capital invested at 6%. On the other hand, if an annuity with an initial capital of 123 minas is entered into, and repayments made at 6% of the remaining total on a yearly basis, then the total outstanding [the just amount] after the allotted time will be as found, as the capital depreciates from the initial 123 at the same rate and length of time, the final value is 79$\underline{7253;}$ while the third period of double the time appreciating from this last value takes the value back to the original final total of 189 - 15 - 3$\underline{41.}$

[4] This is just simple proportion for the same interest rate and time.

[5] A possible scenario for this financial transactions is as follows: Titus has agreed to pay Sempronius an annuity of 57 menas payable in 10 yearly installments. Titus sets aside a principal which after 5 years amounts to 950 menas to accomplish this; for the following 10 years, invested at 6%, the interest would go to Sempronius at the end of each year, while the principal remains intact. The given calculations follow if the two men, perhaps master and servant, decide on an initial equivalent cash pay-out.

[6] Thus, according to these arrangements: if 100 minas of capital is loaned at an interest rate of 6% for 10 years, the interest gained will be $79\underline{0847697}$; this is the same as $34\underline{97926995}$ over 14 years at the same interest rate. Hence, by proportion, the amount $51\underline{4590502}$ results for 300 minas invested in the same way. Note the use of 14 years with some of these 15 year loans: no interest is due until the end of the first year. The point of this example appears to be showing how proportion can be used, after performing the calculations for a principal of 100 minas, reducing the effort a little.

§17.4. Caput XVII. [p.41.]

Datis duobus numeris, una cum numero meriorum continue proportionalium inter datos; quemlibet imperatum in eadem serie invenire.

Ostendimus quomodo in numeris ab unitate continue proportionalibus, unitati proximus quem appellamus latus, vel alius quilibet citra vel ultra datum invenire possit. Nunc ostendendum quomodo mediorum inter duos quoslibet proportionalium quilibet imperatus inveniatur. quod cum non indigeat novo praecepto, exemplo uno aut altero illustrare conabor. Sunto dati duo numeri 729 & 15625, & sint inter datos quinque intermedij.

Sumendi sunt Logarithmi datorum, & eorum differentia, quae secanda est in partes sex, pro numero intervallum inter datos numeros; deinde si *A* pars sexta datae differentiae, multiplicata pro ratione distantiae numeri quaesiti a dato, eiusdem dati Logarithmo addatur vel auferatur; totus vel reliquis erit Logarithmus numeri quaesiti. ut hic cernimus.

		Logarithmi.	Numeri continue proportionales.
Dati	729. *B*	2,86272,75283,1797	
	15625. *C*	4,19382,00260,1611	$94\underline{4784.}$ *B* - *D*
		1,13109,24976,9814. 6.*A*	$157\underline{464.}$
	A	0,22184,87496,1636. 1.*A*	$26\underline{444}$
	D	0,88739,49984,6544. 4.*A*	$437\underline{4}$
	B	2,86272,75283,1797	729 - - *B*
5625 - - - - - - - - -		3,75012,25267,8341. *B* + *D*	1215
$94\underline{4784}$ - - - - - - - -		1,97533,25298,5253. *B* - *D*	2025 - - *C* - *D*
$120563^{22}/_{81}$ - - - - -		5,08121,50244,8155. *C* + *D*	3375
2025 - - - - - - - - -		3,30642,50275,5067. *C* - *D*	5625 - - *B* + *D*
			9375
			15625 - - *C*
			$26091^{2}/_{3}$
			$43402^{7}/_{9}$
			$72337^{26}/_{27}$
			$120563^{22}/_{81}$ C + D

Si *A* pars sexta differentiae Logarithmorum datorum, multiplicatur per 4, & factus *D* addatur Logarithmo numeri *B*, totus dabit 5625 quartum continue proportionalium supra *B*. Sin auferatur ab eodem dabit $94\underline{4784}$ quartum infra eundem. Eodem modo inveniuntur numeri supra & infra *C*.

Exemplum alterum.

Sint dati 10 & 11, & sint inter datos duocem intervalla

		Logarithmi.
Dati	10. *B*	1,00000,00000,0000
	11. *C*	1,04139,26851,5823
		0,04139,26851,5823. 12.*A*
	A	0,00344,93004,2985

Cupio scire septimum a minore intra datos. qui est $10\underline{57172197209}$

B.	1,00000,00000,0000

		0,02414,57330,0895. 7*A*
10<u>57172197209</u>	- - -	1,02414,57330,0895. *B* + 7*A*

Eodem modo invenire poterimus quemlibet intra datos vel extra.

Exemplum huius videamus in Anatocismo, cum singulis annis sorte accedant sex centesimae: quae usura, apud antiquos, (qui faenus epidecatum improbarunt) civilis & modica censebatur. Esto data sors 123 librarum: cupio scire quantum debet sorti accedere, ad finem quotlibet annorum, mensium vel dierum ; si totius anni lucrum, ita in singulus menses vel dies distribuatur, ut lucro temporis cuiuslibet sorte adiecto, ea perpetuo servetur ratio sortis ad aggregatum sortis & lucri; ut totius anni lucrum, centum nummorum sorti, sex tantum nummos adijciat.

Imprimis sumantur Logarithmi datorum terminorum rationis, 100 & 106, & eorum differentia B, quae dicatur annua, ea in partes duodecim divisa, dabit in quoto differentium C menstruam: quae divisa per $30^1/_2$, quot fere sunt dies

[p.43]

in mensa aequabili, dabit in quoto differentiam D diurnam, quam si accuratius nosse velimus, dividenda est differentia annua per 365, numerum dierum in anno communi Juliano. Deinde sumatur Logarithmus datae sortis.

		Logarithms	
Termini datae rationis.	100	2,00000,00000,0000	
	106		
		2,02530,58652,6477	
	B	0,02530,58652,6477	differentia annua.
	C	0,00210,88221,0540	differentia mentrua.
	D	0,00006,93311,3771	differentia diurna.
Sors - 123 - - - - - - - -		2,08990,51114,3940	

Istis ad hunc modum paratis, si quaeratur quantum debet sorti accedere ad finem annorum septem, mensium quinque, & dierum novem: multiplicentur singulae differentiae per suos numeros; & addatur factorum summa Logarithmo sortis; totus erit Logarithmus aggregati sortis & lucri, ad finem temporis constituti.

lucro accedere	7 annorum differentia	0,17714,10568,5339	7.B
	5 mmensium differentia	0,01054,41105,2700	5.C
Quantum in Anatocumo pro	9 dierum differentia	0,00062,39802,3939	9.D
temporis ratione debeat sorti in	Factorum summa E	0,18830,91476,1978	
pretio decedere	Logarithmus sortis	2,08990,51114,3940	123£
	Totus	2,27821,42590,5918	189<u>7641892</u>
	Reliquus	1,90159,59638,1962	79<u>7252637</u>

Quod si eadem factorum summa auferatur e Logarithmo sortis, reliquis erit Logarithmus iusti pretij, quo sors praesenti pecunia redimi possit, si numeretur tot annos, menses 7 dies, ante diem solutioni constitutam. ut in hoc exemplo. Aio 123 minas ad finem annorum 7, mensium 5, & dierum 9, valere 189<u>7641892</u> , id est 189 - 15 - 3<u>41</u> . Et si solutionis dies, quo123 minae sint solvendae, totidem annos mentes & dies distet ab hoc tempore: iustum redemptionis pretium si pecunia repraesentetur esse 79<u>7253</u>, vel 79 - 14 - 6<u>06</u>. Et eadem foenoris servata ratione; sortem 79 - 14 - 6<u>06</u> post annos 14 , menses 10 ,et dies 18, valere189 - 15 - 3<u>41</u> .

Idem erit operationis modus, data alia quacunque ratione sortis ad lucrum; pro dato quolibet temporis spatio, ante vel post diem solutionis.

2. *Datis sorte & quotlibet annorum lucro: quaeritur ratio sortis ad lucrum unius anni, vel mensis, vel diei*

Esto data sors nummorum 1234: cupio decennij spatio; sorti addere 766: ut summa sortis & lucri fiat 2000.

Sumatur differentia Logarithmorum sortis & summae: huius differentiae pars decima erit differentia annua, quae sortis Logarithmo adiecta, dabit Logarithmum sortis auctae lucro unius anni.

	Logarithmi.	
Sors 1234 - - - - - - - -	3.09131,51596,9723	
Sortis lucro decennali auctae 2000	3,30102,99956,6398	
	0,20971,48359,6675	differentia decennalis.
	0,02097,14835,9667	differentia annua.
Sortis auctae lucro unius anni	3,11228,66432,9390	129<u>505032</u>

Primi anni lucrum sorti additum dabit 129<u>505032</u>.

Eadem differentia annua addita Logarithmo millenarij, dabit iustum foenoris incrementum annuum supra mille.

[p.44.]

	Logarithmi.	
1000	3,00000,00000,0000	
	0,02097,14835,9667	differentia annua.
1049 473527	3,02097,14835,9667	

Aio mille nummos annuo spatio parere $49^1/_2$.

Eadem modo usura menstrua vel diurna pro data sorte inveniri poterit, quae si per decennium continuetur, dabit cum sorte nummos 2000.

pro usura menstrua dividenda est differentia decennalis per 120, numerum mensium in decennio, pro diurna, per 3652.

	Logarithmi		
differentia menstrua	0,00174,76236,3306		
differentia diurna	0,00005,74246,5388		
1234 sortis	3,09131,51596,9723	1234	
sortis auctae ad finem primi mensis	3,09306,27833,3029	1238 975685	1238 - 19 - 6 16.
sortis post primum diem	3,09137,25843,5111	1234 163177	1234 - 3 - 3 16.
primus mensis sortis adijciet	4 - 19 - 6 16.		
primus dies " "	0 - 3 - 3 16.		

3. *Annua pensio quantum valeat ad finem quotlibet annorum, & quo pretio redimi posit pro quotlibet annotum spatio ante tempus prima solutioni constitutum.*

Titius debet solvere Sempronio decies 57 minas, prima solutio fieri debet ad finem quinquennij, deinde annuatim tantundem donec solutae fuerint 570. hoc nomen redimere cupit Titus praesenti pecunia, facta aestimatione secundum usuram menstruam semicentesimam, quae singulis annis sorti addit sex centesimas.

1. Imprimis quaerenda est sors quae secundum datam rationem, unico anno dat incrementum 57 minarum.

		Logarithmi.
pro-.	6	0,77815,12504
port	00	2,00000,00000
	57	1,75587,48557
sors inventa	- - - 950	2,97772,36053

Logarithmus sortis inventus per cap. 15 dat sortem 950.

Deinde per 1.prop.huius capatis, quaeritur valor sortis & lucri ad finem decennij. Sumendi sunt termini rationis datae, & eorum Logarithmis, & Logarithmorum differentia: quae multiplicata per numerum annorum, et addita Logarithmo sortis inventae, dabit Logarithmum totius pecuniae quae ad finem decennij, pro hac sorte debetur, si interim nulla facta fuerit solutio. ab hac summa si sors auferatur, restabit pecunia quam annuarum pensionum coacervatio peperit.

		Logarithmi.	
Termini	100	2,00000,00000	
rationis	106	2,02530,58653	
		0,02530,58653	differentia annua
		0,25305,86526	differentia decennalis
sortis inventae		2,97772,36053	
sortis auctae per decennium		3,23078,22579	1701 30531

Ad finem decennij sors aucta erit 1701 30531, ab hac summa auferenda est sors illa prius assumpta 950, qua sublati restant 751 30531 minas, quae ex ipsis pensionibus accumulatis & auctis proveniebant.

[p.45.]

2. At Sempronio (cum non necesse sit primam solutionem fieri ante finitum quinquennium) tandem expectanda est haec summa post annos totos quatuordecim. Quaeritur igitur quanta diminutio huius summae fieri debeat, si praesentem pecuniam desideret.

	Logarithmi.
Totius pecuniae summa1701 30531	2,87581,64581
differentia pro annis quatuordecim	0,35428,21136
pretium 332 3030636	2,52153,43445

Si quatuordecim annorum differentia auferatur e Logarithmo summae, restabit Logarithmus pretij, 332 3030636 . quod secundum datum rationem, in annis 14, eandem dabit pecuniae summam, quae pensiones coacervatae valebant.

4. *data pecunia summa & ratione incrementi pro dato annorum numero, invenire reditum annuum.*

Sunto, data summa 300 minae, data ratio incrementi annui 100 ad 106, datum tempus ad decennium, & fiat solutio prima ad finem quinquennij, quaeritur quis sit annus reditus.

Haec quaesito vix aliter expediti poterit, quam per proportionalem, ad hunc fere modum.

Ponamus annuum reditum quemlibet pro tempore constituto, & per 3.prop. huius capitis, quaeritur pretium eiusdem statim numerandum. deinde per proportionem, erit pretium positi reditus, ad datam pecuniae summam: ut reditus annuus positus, ad reditum quaesitum.

Sit positus reditus annuus minarum sex. erit pro data ratione sors 100, quae annuo spatio sit 106.

		Logarithms	
Termini datae rationis	100	2,00000,00000	
	106	2,02530,58653	
differentia annua		0,02530,58653	
differentia decennalis		0,25305,86526	
sortis datae	100	2,00000,00000	
sortis auctae per decennium		2,25305,86526	$179\underline{0847697}$
Sors cum lucro decennali invenitur		$179\underline{0847697}$	

Sorte autem ablata, relinquitur verus valor annui reditus, ad finem decennij $79\underline{0847697}$. Incipit autem hoc decennium post annos quatuor; cum prima solutio fieri debeat ad finem quinquennij. quaeritur idcirco sors, quae aucta secundum datam rationem per annos quatuordecim dabit $79\underline{0847697}$.

	Logarithmi.	
valoris annui reditus ad finem decennij $79\underline{0847697}$	1,89809,28534	
differentia pro annis quatuordecim auferenda	0,35428,21136	
sortis quaesitae Logarithmus reliquus	1,54381,07398	$34\underline{97926995}$

Haec sors $34\underline{97926995}$ aucta secundum datum rationem, per annos quatuordecim continuos, eadem dabit pecuniae summam, quam dat annus reditus 6 minorum ad decennium.

Est igitur haec sors iustum pretium eiusdem annui reditus, quo invento per proportionem concludo.

			Logarithmi.
pro- port.	sors	$34\underline{97926995}$	1,54381,07398
	reditus	6	0,77815,12504
	pretium	300	2,47712,12547
	reditus	$51\underline{4590502}$	1,71146,17653

Tandem inventus est reditus annuus quaesitus $51\underline{4590502}$, qui solvendus est ad finem proximi quinquennij, & continuandus per annos novem sequentes, si 300 minae pretium eiusdem representententur.