BOOK TWO,

CONCERNING THE CONSTRUCTION OF TELESCOPES

SECOND SECTION.

TELESCOPES OF THE SECOND KIND,

WHICH ARE CONSTRUCTED WITH A CONVEX OBJECTIVE,

REPRESENTING THE OBJECT SITUATED INVERSELY.

CHAPTER I

GENERAL PRECEPTS CONCERNING

SIMPLER TELESCOPES OF THE SECOND KIND

PREPARED FROM A SINGLE KIND OF GLASS

[Note: henceforth in this translation, we will replace the measure *digits*, or *dig*. by *inches* or *in*.]

192. Since in this section the image of objects shall always be going to be inverted, here before everything it is required to be warned in all the general formulas treated above the letter *m,* by which the magnification may be shown, to be taken negative everywhere, thus so that in these formulas, as often as *m* occurs, in its place there should be written $-m$.

PROBLEM 1

193. *To construct the most simple of this kind of telescope from two lenses made of the same kind of glass, so that the object may be represented according to the given magnification m and placed inverted.*

SOLUTION

For the proposed magnification *m* our general formulas at once provide this determination $m = \frac{\alpha}{b}$, where it is evident α to express the focal length of the objective lens, b truly of the eyepiece on account of $\beta = \infty$. Therefore since the fraction $\frac{\alpha}{b}$

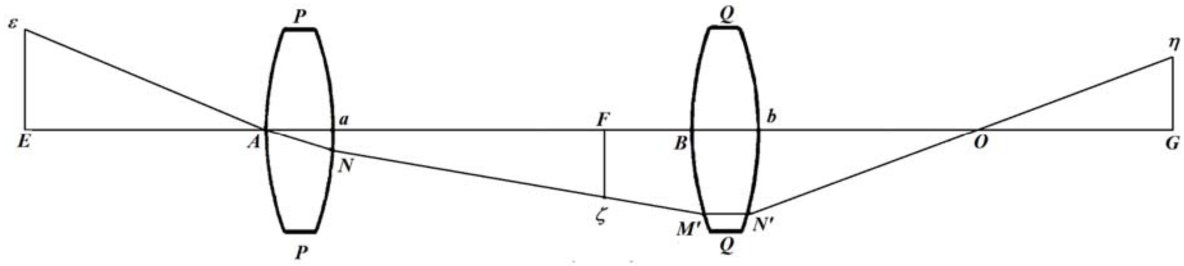

Fig. 13 (again).

here shall be positive and likewise the separation of the lenses $\alpha+b$, each distance a and b will be required to be taken positive, thus so that both lenses shall become convex and a real image may be shown at the point F (Fig. 13, Ch. 5, Book 1), which likewise is the common focus of each lens. Then truly the radius of the apparent field of view will be $\Phi=\frac{\pi}{m+1}$, but which may not be apparent unless constituted nearly in a certain place, unless the distance after the eyepiece lens is $O=\frac{\pi q}{m\Phi}$, with q denoting the focal length of the eyepiece lens, which we have seen to be $=b$. Therefore since there shall be $\pi=(m+1)\Phi$, this distance will be $O=\frac{m+1}{m}\cdot q$ and thus greater than q by a small amount. So that now objects with a given degree of clarity may become apparent, which we have called $=y$, thus so that y shall be going to measure the smaller radius of the pupil, it is evident the aperture of the objective lens must only be of such a size, so that is radius shall be $x=my$, from which it is understood now its focal length p or α certainly cannot be established smaller than $4x$. Now we may see also, how this telescope shall be going to be prepared on account of the colored margin. Since in short this may not be able to be removed, as it may be unable to happen that $0=\frac{dn'}{n'-1}\cdot\frac{\pi}{m\Phi}=\frac{dn'}{n'-1}\cdot\frac{m+1}{m}$, the confusion is much less able to be destroyed completely, since there must become $0=\frac{dn'}{n'-1}\cdot(p+q)$, where $p+q$ is the separation of the lenses. But since there it is required to be understood more, that the confusion of the first kind depending on the aperture may be rendered insensible or so that the radius of its confusion may not exceed a certain limit, which we have indicated by the letter k . Whereby from the above [§ 42, 44] this condition may be deduced :

$$+\frac{m\mu x^3}{4p^3}\left(\lambda+\frac{\lambda'}{m}\right)<\frac{1}{4k^3} \quad\text{or}\quad \frac{x^3}{p^3}\left(\mu\lambda m+\mu\lambda'\right)<\frac{1}{k^3},$$

from which we will obtain this condition for the focal length of the objective lens $p=\alpha$:

$$p>kx\sqrt[3]{(\mu\lambda m+\mu\lambda')},$$

and on account of $x=my$ there will be

$$p > kky\sqrt[3]{(\mu\lambda m + \mu\lambda')}$$

or will be able to be taken equal to the minimum p of this formula.

COROLLARY 1

194. Hence it will be apparent at once, where a greater magnification may be required, there must be a greater focal length of the objective lens and thus neither also can the length of the telescope in such a simple ratio, but almost in the four thirds ratio of the magnification, evidently as $m^{\frac{4}{3}}$; and hence this length soon will have emerged so much, that at no time shall it be true, that the quantity p may become less than $4my$.

COROLLARY 2

195. The number μ will depend on the nature of the glass, from which it follows, that if that were smaller, therefore the length p may be increased. But we have seen above with the refractive index n increasing that very number μ to be diminished ; but since then the formula $\frac{dn}{n-1}$ increases and thus the colored margin is increased, it will be better to use the same kind of glass.

COROLLARY 3

196. Hence also we understand, where a greater degree of clarity y may be desired, there the quantity p must be increased more, so that also it arises, if greater distinction may be required, since then a greater value of the letter k must be attributed.

COROLLARY 4

197. But for the length of these instruments most interest thus is concerned with the construction of the objective lens, so that there may become $\lambda = 1$, certainly which is the minimum value of this letter. Whereby it may be agreed to attribute that form of this lens, as we have described above in the chapter concerning objective lenses.

COROLLARY 5

198. But we may gain little concerning the eyepiece lens, if we wish to take $\lambda' = 1$, since in most magnifications this term vanishes first ; why not rather give the figure of this kind to a lens, which shall be of the greatest aperture, since the apparent field of view depends chiefly on that; whereby this rule may be confirmed, that an eyepiece lens may be made with both sides equally convex, since then at last the letter π may be able to take the value $\frac{1}{4}$ or also even greater. Then truly there will be

I. For crown glass or $n = 1,53$:

$\lambda' = 1,60006$.

II. For common glass or $n = 1,55$:

$\lambda' = 1,62991$.

III. Finally for crystal glass or $n = 1,58$:

$\lambda' = 1,67445$.

SCHOLION 1

199. Huygens has established enough of both an incomplete theory based on incomplete experiments that the focal length of the objective lens be proportional to the square of the magnification, to which it only lacks, that I may wish to oppose so much, that rather I may wish to agree with it according to the practice at that time ; for our determination leans on this account, so that the faces of the lenses shall be formed from a perfectly spherical figure, as if the skilled craftsman may be able to effect exactly, there is no doubt, why our formula may not be in agreement with the truth, so that now indeed it may be seen to be submitted to the industry of the chief artificers ; but when the figure of the lens may depart a little from the spherical shape, the fault there is more perceptible, as the focal length of the lens will have been greater, to which therefore it cannot occur otherwise except by returning a focal length greater than according to our rule. But whether the duplicate ratio may arise precisely, it will by no means be able to be confirmed, but as any lens will have been worked on successfully, there a smaller focal length suffices for producing the same magnification, or rather the same lens will be suitable for producing a greater magnification; so that even if is required always to be observed, yet here not only with the lenses assumed to be spherical figures, but also can be given according to the given rays.

SCHOLIUM 2

200. Moreover from these observations of Huygens [*C. Hugenii Opera reliqua*, Vol. II, *Ch.* XVII] we will make use especially for the order as well as the clarity requiring to be defined, with which astronomers are accustomed to be content, even if for each either a greater or lesser order may remain to be selected. Therefore in the first place so that it reaches a degree of clarity, there is assigned an aperture of the objective lens, of which the focal length $= 20$ ft. or 240 inches, of which the radius $= 1,225$ in., and that judged suitable for a magnification $m = 89$; as the ratio also may be observed in the remaining lenses ; whereby, since here there shall be $x = 1,225$ in. and $m = 89$, on account of $x = my$ hence we may deduce $y = \frac{x}{m} = \frac{1,225}{89} = \frac{1}{73}$; whereby, since everywhere above we will have taken $y = \frac{1}{50}$, we will have gained a greater degree of clarity from these instruments and with that thus to be twice as much greater.

So that thereafter for the order of the distinction to be contained by the letter *k*, in the example proposed by Huygens, we may consider carefully to be

$p = 240$ dig., $m = 89$ and $y = \frac{1}{73}$, and by taking $p = \frac{9}{10}$ and $\lambda = 1$ and with the other term rejected in the formula of the roots these values substituted into our formula of the roots will give

$$240 = 89 \cdot \tfrac{1}{73} \cdot k \cdot \sqrt[3]{\tfrac{9}{10} \cdot 89}, \text{ therefore } k = \tfrac{73 \cdot 240}{89 \cdot \sqrt[3]{80}},$$

which fraction evaluated gives $k = 45$. Whereby, since we will have supposed everywhere k = 50, we will have reached a greater order of distinctness than hence arises. Therefore while ky occurs in our formula, following Huygens it will suffice to put in place $ky = \frac{45}{73} = \frac{5}{8}$ approximately, from which it will be apparent, if we may put $ky = 1$, a greater order to be obtained, not only of the clarity [defined by $y = mx$] but also of the distinctness, likewise truly a greater length of telescope is going to be produced, than if we put $ky = \frac{5}{8}$.

SCHOLIUM 3

201. Since crystal glass is indicated to be inappropriate for this kind of telescope, if indeed it may increase the colored margin, here we may use two kinds of glass for the constructions, the one where $n = 1,53$, the other where $n = 1,55$.

Construction of a telescope of this kind with each lens prepared from crown glass $n = 1,53$

I. For the objective lens.

$$\text{Radius of the } \begin{cases} \text{anterior face} = 0,6023\alpha \\ \text{posterior face} = 4,4131\alpha. \end{cases}$$

$$\text{Interval} = \tfrac{m+1}{m} \cdot \alpha.$$

II. For the eyepiece lens

$$\text{Radius of each face } = 1,06 \cdot \tfrac{\alpha}{m}$$

Distance of the eye from this lens $= \frac{m+1}{m} \cdot \frac{\alpha}{m}$.

Radius of the aperture of the objective lens $= my$, of the eyepiece lens $= \frac{1}{4} \cdot \frac{\alpha}{m}$.

Radius of the field of view $\Phi = \frac{859}{m+1}$ minutes on taking

$$\alpha = kmy\sqrt[3]{0,9875(m+1,60006)}.$$

Construction of a telescope of this kind with each lens prepared from common glass with $n = 1,55$

I. For the objective lens

$$\text{Radius of the } \begin{cases} \text{anterior face} & = 0,6145\alpha \\ \text{posterior face} & = 5,2438\alpha. \end{cases}$$

Radius of the aperture $= my$. Interval $= \frac{m+1}{m} \cdot \alpha$.

II. For the eyepiece lens

$$\text{Radius of each face } = 1,10 \cdot \frac{\alpha}{m}.$$

Radius of the aperture $= \frac{1}{4} \cdot \frac{\alpha}{m}$. Distance of the eye $= \frac{m+1}{m} \cdot \frac{\alpha}{m}$.
Radius of the apparent field of view $\Phi = \frac{859}{m+1}$ minutes on taking

$$\alpha = kmy\sqrt[3]{0,9381(m + 1,62991)},$$

where any degree of clarity and distinction can be assumed as wished.

PROBLEM 2

202. *If the objective lens were two of that kind, as we have described in* § 65*, to describe the construction of the telescope.*

SOLUTION

Here everything remains the same, which we have defined in the previous problem with regard to the magnification, the apparent field of view, and to the place of the eye; only in the expression found for the radius of confusion is a number λ arrived at smaller than a value diminished beyond the fifth part of unity ; from which the focal length of the objective lens also will be able to have a smaller value; that which without doubt is required to be regarded as a significant gain, since in this manner the length of the instrument may be contracted considerably. But here initially it is required to be attending to the kind of glass from which the lenses may be prepared, however great the number λ is defined by that, from which with crystal glass excluded on account of the reasons raised previously against the construction of telescopes of this kind for the two remaining kinds we may show here.

Construction of a telescope of this kind, with each lens prepared from crown glass for which $n = 1,53$.

I. For the duplicate objective lens

$$\text{radius of the first lens} \begin{cases} \text{anterior face} = +1,2047\alpha \\ \text{posterior face} = +\ 8,8222\alpha \end{cases}$$

$$\text{radius of the second lens} \begin{cases} \text{anterior face} = +0,6464\alpha \\ \text{posterior face} = -\ 1,6570\alpha. \end{cases}$$

Radius of the aperture $x = my$. Interval as far as the eyepiece lens $= \frac{m+1}{m} \cdot \alpha.$

II. For the eyepiece lens

Radius of each face $= 1,06 \cdot \frac{\alpha}{m}.$

Radius of the aperture $= \frac{1}{4} \cdot \frac{\alpha}{m}.$

The distance of the eye past this lens is given by $O = \frac{m+1}{m} \cdot \frac{\alpha}{m}.$ Here clearly that double objective lens may be considered as simple, the focal length of which shall be $= \alpha$, which now thus must be taken, so that there may become,

$$\alpha \geq\ kmy\sqrt[3]{0,9875\left(0,1951m + 1,60006\right)}.$$

Truly the radius of the apparent field of view is as before $\Phi = \frac{859}{m+1}$ minutes.

Construction of this kind of telescope

with each lens prepared from common glass with $n = 1,55$

I. For the double objective lens

$$\text{radius of the first lens} \begin{cases} \text{anterior face} = +\ 1,2289\alpha \\ \text{posterior face} = +10,4876\alpha \end{cases}$$

$$\text{radius of the second lens} \begin{cases} \text{anterior face} = +\ 0,6527\alpha \\ \text{posterior face} = -\ 1,6053\alpha. \end{cases}$$

Radius of the aperture $x = my$. Distance as far as to the eyepiece lens $= \frac{m+1}{m} \cdot \alpha.$

II. For the eyepiece lens

Radius of each face $= 1,01 \cdot \frac{\alpha}{m}.$

Radius of the aperture $= \frac{1}{4} \cdot \frac{\alpha}{m}.$

Distance of the eye past this lens $O=\frac{m+1}{m}\cdot\frac{\alpha}{m}$, where the field of view will be determined, of which the radius $=\frac{859}{m+1}$ minutes.

But the focal length of the double objective lens itself must be taken thus, so that

$$\alpha \geq kmy\sqrt[3]{0,9381(0,1918m+1,6299)}.$$

COROLLARY 1

203. Therefore if the magnification shall be so great, so that in the value of α itself the latter term may vanish before the other, in this case the distance α will be less than in the preceding, in the approximate ratio $\sqrt[3]{\frac{1}{5}}:1$ or $1:\sqrt[3]{5}$, that is almost as 10 : 17.

COROLLARY 2

204. Since these lenses shall be deduced from the minimum principle, therefore in practice they are adapted to be greater, as there shall be no need to fear, that they will be disturbed by small errors introduced by being brought together, which may be feared the most, if the remaining composite lenses, which indeed have been called perfect, we may wish to be substituted in place of the objective.

SCHOLIUM

205. So that it may appear clearer, how much there may be expected to gain, we may adapt both cases to a given magnification, for example $m=100$, where we will consider only a certain common glass.

If therefore I.) we may use a simple objective lens, the focal length α thus must be taken, so that there shall be

$$\alpha=100ky\sqrt[3]{0,9381\cdot 101,6299}, \qquad \alpha=100ky\cdot 4,5684;$$

from which , if with Huygens there may be taken $ky=\frac{5}{8}$ dig., there becomes $\alpha=285\frac{1}{2}$ in. $=$ 23 ft. $9\frac{1}{2}$ in.

But if II.) we may use the double objective lens, we will have :

$$\alpha=100ky\sqrt[3]{0,9381\cdot 20,8099}, \qquad \alpha=100ky\cdot 2,6926;$$

and again on taking $ky=\frac{5}{8}$ in. there will be

$$a = 168\tfrac{1}{3}\text{ in.} = 14\text{ ft.}\tfrac{1}{3}\text{ in.};$$

certainly this shortening of the previous maximum may be seen to be of the greatest moment; moreover now, since we may wish for much shorter telescopes at this stage, it will be seen not to be very noteworthy, since it may arise also in the case of the following problems, where we will make a triple objective lens.

PROBLEM 3

206. *If the objective lens were triple, as we have described in* § 66 *, to describe the construction of the telescope.*

SOLUTION

Everything remains as before, except that for the triple lens there shall be going to become $\lambda = \frac{3-8\nu}{27}$; from which only the common glass being required to be considered, for which $n = 1,55$, the focal length α of this lens thus must be determined, so that there shall be :

$$\alpha = kmy\sqrt[3]{0,9381(0,0422m + 1,6299)}\ ;$$

hence therefore telescopes will require to be constructed in the following manner.

Construction of this kind of telescope with each lens made from common glass
for which $n = 1,55$

I. For the triple objective lens

Radius of the first lens $\begin{cases}\text{anterior face} & = +15,7315\alpha\\ \text{posterior face} & = +\ \ 0,9791\alpha\end{cases}$

Radius of the second lens $\begin{cases}\text{anterior face} & = +\ \ 0,9791\alpha\\ \text{posterior face} & = -\ \ 2,4077\alpha\end{cases}$

Radius of the third lens $\begin{cases}\text{anterior face} & = +\ \ 0,6665\alpha\\ \text{posterior face} & = -\ \ 1,1183\alpha.\end{cases}$

Radius of its aperture $x = my$.

Interval as far as to the eyepiece lens $= \frac{m+1}{m}\cdot\alpha$.

II. For the eyepiece lens:

Radius of each face $=1,10\cdot\frac{\alpha}{m}$.

Radius of its aperture $=\frac{1}{4}\cdot\frac{\alpha}{m}$.

Distance of the eye $=\frac{m+1}{m}\cdot\frac{\alpha}{m}$ and the apparent radius of the field of view $\Phi=\frac{859}{m+1}$ minutes.

But so great a focal length of the objective lens must be taken, so that there shall be

$$\alpha=kmy\sqrt[3]{0,9381(0,0422m+1,6299)}.$$

COROLLARY 1

207. Therefore if a magnification must be put in place $m=100$, there will be taken

$$\alpha=100ky\sqrt[3]{0,9381\cdot 5,8499}$$

or $\alpha=100ky\cdot 1,7639$ and on assuming $ky=\frac{5}{8}$ in.

$$\alpha=110\tfrac{1}{4}\text{ in.}=9\text{ ft. }2\tfrac{1}{4}\text{ in.};$$

and thus the whole length of the telescope as far as the eyepiece will be produced :

$$\left(\frac{m+1}{m}\right)^2\cdot\alpha=112\tfrac{1}{2}\text{ in.}=9\text{ ft. }4\tfrac{1}{2}\text{ in.}$$

COROLLARY 2

208. So that it may reach a degree of clarity *y*, since here, through which the transmission of the rays is required through several lenses, and a greater loss from these is to be feared, even if we may not require a greater clarity than Huygens, yet if y may be granted a greater value than $\frac{1}{73}$, whereby with the value of k retained the length of the telescope produced will be greater.

SCHOLIUM

209. This final caution is of the greatest importance and is required always to be investigated properly, whenever we may use a greater number of lenses, and I may make use of this opportunity concerning this matter to mention the case of telescopes brought from England, which are called nocturnal ; concerning which I will observe in the first

place there can be no use of those in complete darkness, but these are accustomed to be used only for viewing objects not too far away at twilight or by moonlight. But the whole mystery, and which most have asked about these telescopes, comes to this, that with these a higher order of clarity may be agreed on, or that the radius of the pupil may be attributed to the letter y, or there may be put approximately $y = \frac{1}{12}$ in., if indeed the clarity viewed may appear to be 36 times greater, than if there were taken $y = \frac{1}{73}$. Whereby, lest these telescopes may become exceedingly long, it will be required for us to be content with a much smaller magnification. But for this a magnification aim of $m = 10$ is accustomed to be more than sufficient. For if by night far away objects may appear to be ten times closer and these will be allowed to be seen by us with the same degree of clarity than by the naked eye, certainly more cannot be desired.

PROBLEM 4

210. *Finally if the objective lens were quadruple, to describe the construction of the telescope, constructed according to the instructions treated in the above book in* §154.

SOLUTION

Here from the beginning everything remains as before ; but since here in the first place it is worth noting, because in the formula for the resulting focal length α, as it were

$$\alpha = kmy\sqrt[3]{\mu\left(\tfrac{1-5\nu}{16}\cdot m + \lambda''\right)},$$

the value of the number λ may be produced negative and thus in a certain case the whole confusion may be able to vanish ; which case especially deserves merit, so that it may be set out with great care. Therefore we may take all these lenses to be made from a common glass, for which $n = 1,55$, and the eyepiece lens as up to this stage to be formed equally convex, and we will have

$$\lambda = \tfrac{1-5\nu}{16} = -0,010216 \text{ and } \lambda'' = 1,6299,$$

from which it is understood the radius of confusion evidently may go to zero, if there may be taken

$$m = \frac{1,6999}{0,010216} = 159\tfrac{1}{2}.$$

Therefore for this case α no longer will be determined from this but uniquely from the aperture, which the degree of clarity demands; if indeed for the order of clarity in general we may assume to be y, the radius of the aperture must be $= my$, from which the distance α must be taken just as large as the aperture may be able to receive for the rays of the individual faces. Whereby, if we may now consider α also as an indefinite quantity, the construction of the telescope thus will be had.

Construction of this kind of telescope with lenses prepared from a common glass

I. For the quadruple objective lens

$$\text{radius of first lens}\begin{cases}\text{anterior face} & =+\ 2,4580\alpha\\ \text{posterior face} & =+20,9754\alpha\end{cases}$$

$$\text{radius of second lens}\begin{cases}\text{anterior face} & =+1,3054\alpha\\ \text{posterior face} & =-3,2103\alpha\end{cases}$$

$$\text{radius of third lens}\begin{cases}\text{anterior face} & =+0,8887\alpha\\ \text{posterior face} & =-\ 1,4910\alpha.\end{cases}$$

$$\text{radius of fourth lens}\begin{cases}\text{anterior face} & =+0,6710\alpha\\ \text{posterior face} & =-0,9710\alpha.\end{cases}$$

Radius of the aperture $x = my$.

Interval as far as to the eyepiece lens $= \frac{m+1}{m}\cdot\alpha$.

II. For the eyepiece lens

Radius of each face $=1,10\cdot\frac{\alpha}{m}$.

Radius of the aperture $=\frac{1}{4}\cdot\frac{\alpha}{m}$.

Distance of the eye $=\frac{m+1}{m}\cdot\frac{\alpha}{m}$

Radius of the field of view observed $=\frac{859}{m+1}$ minutes.

But here in general there must be taken

$$\alpha \geq kmy\sqrt[3]{0,9381(-0,010216m+1,6299)},$$

unless the value hence being produced were smaller, so that then the prescribed aperture $x = my$ may be used, which chiefly may arise for $m = 159\frac{1}{2}$.

For which the radii of the faces must depend in the manner shown, between which since the minimum shall be $0,6710\alpha$, of which the fourth part $0,1678\alpha$ or almost $\frac{1}{6}\alpha$ will determine the radius of the aperture; which, as on account of $m = 159\frac{1}{2}$, shall become $159\frac{1}{2}y$, there will have to be taken $a > 6\cdot159\frac{1}{2}y$ or $\alpha > 957y$; therefore if we may assume $y=\frac{1}{50}$ in., then there will have to be taken $\alpha > 19\frac{7}{50}$ dig., on account of which we may put in place $\alpha = 20$ in. And with the magnification taken to be $m = 160$ we will have this most special construction.

Construction of the telescope for the magnification $m = 160$ with the lenses made from the common glass with $n = 1,55$

Radius of the first lens $\begin{cases}\text{anterior face} = +\ 49,16 \text{ in.}\\ \text{posterior face} = +419,50 \text{ in.}\end{cases}$

Radius of the second lens $\begin{cases}\text{anterior face} = +\ 26,10 \text{ in.}\\ \text{posterior face} = -\ 64,21 \text{ in.}\end{cases}$

Radius of the third lens $\begin{cases}\text{anterior face} = +\ 17,77 \text{ in.}\\ \text{posterior face} = -\ 29,82 \text{ in.}\end{cases}$

Radius of the fourth lens $\begin{cases}\text{anterior face} = +\ 13,42 \text{ in.}\\ \text{posterior face} = -\ 19,42 \text{ in.}\end{cases}$

Radius of its aperture $x = my = 3,2$ in.
Interval as far as to the eyepiece lens $= 20\frac{1}{8}$ in.

II. For the eyepiece lens

Radius of each face $= 0,1375$ in.
and the radius of its aperture $= \frac{1}{32}$ in.
Distance of the eye $= 0,1258$ in.,
thus so that the total length of the telescope shall be $= 20\frac{1}{4}$ in.
and the [angular] radius of the field of view seen corresponds to $5' \, 20''$.

COROLLARY 1

211. If we may have put a smaller magnification in place, a greater length of the telescope may have been produced. For if we may put $m = 50$ and we may give the value 50 to the letter k, there will be produced $\alpha = 50\sqrt[3]{0,9381 \cdot 1,1189}$ or $\alpha > 50,81$ and thus more than twice as much greater than the case $m = 160$, which certainly is a remarkable paradox.

COROLLARY 2

212. If the craftsman may err a little in the construction of the objective lens, its error will increase the value of the number λ only by a little amount, since that value $\lambda = -0,010216$ generally is a minimum ; indeed if on account of these errors λ may be increased by small amount $\frac{1}{1000}$, it produces $\lambda = -\ 0,010154$, [from the formula $\frac{1,001-5v}{16}$, for $v = 0,232692$, correcting the original value dervied for λ] thus so that this same quadruple lens shall be suitable for producing a greater magnification; which the above paradox does not allow.

SCHOLlUM 1

213. Neither here nor in the preceding have we defined a measure of any kind of inches which we may understand, whether they shall be French, English, or Rhenish, etc. Truly the decision is rather that this measure be left undefined. So that indeed if we may have reason to doubt lenses to be taking the trouble to follow the prescribed rules accurately, especially according to that it will be the greater measure to use for the inches. But if moreover plainly we may be certain about the execution, we will be able to use the measure with the lesser of the inches without risk. But it will always be advised in practice to use the greater measure of the inches; and thus the account itself which led us to the measure of the inches has led to this ; for this account has arisen for us from the aperture of the pupil, as we have expressed in parts of an inch. Therefore since the pupil itself shall be so changeable, so that clearly nothing certain may be able to be put in place concerning that, it is evident so much to be lacking, so that a certain reliable measure shall be prescribed by us, so that it shall be free to be changed perceptibly by us either by increasing or decreasing.

SCHOLIUM 2

214. Si far we have shown, just as we have shown by using composite lenses in place of the objective they may be able to shorten the length of this telescope considerably. Truly in this way plainly no increase is induced in the apparent field of view. Now formerly moreover it has been observed also the apparent field of view can be increased significantly, if also the eyepiece lens may be thus doubled or tripled. For since in the first place the apparent field shall depend on the aperture of the eyepiece lens, as for this reason we have attributed equal shapes on both sides to this lens, so that it may be rendered capable of a greater aperture, it is evident, if this lens may be allowed to be constructed thus, so that at this stage it may be able to receive a larger aperture, the apparent field of view is going to be increased in the same ratio. So that this may become more apparent, we may put the focal length of the eyepiece lens to be one inch, thus so that it may allow an aperture of which the radius $=\frac{1}{4}$ in. Now it is clear enough, if in its place two lenses joined together may be substituted, the focal lengths of each of which $=2$ in., then the focal length of this lens also to be on one inch, but this composite lens is going to be allowed twice as great an aperture, if indeed it may be established with both faces equal to each other and thus an aperture is allowed, of which the radius shall be half an inch, and in this way the apparent field of view will be doubled. In a similar manner, if in place of the simple eyepiece lens three lenses may be substituted, of which the individual focal lengths shall be a third of an inch, the same effect will be obtained in the account of the magnification, but since they allow an aperture three times greater, the field of view will be tripled. Moreover these generally are consider worthwhile, so that they may be established from our principles with more care, and we will determine especially the influx of lenses of this kind of composition, where they may affect the confusion.

PROBLEM 5

215. *If the eyepiece lens may be doubled, so that the* [*angular*] *radius of the apparent field of view of the eyepiece may obtain twice as large a value, to describe the construction of this kind of telescope.*

SOLUTION

Since this telescope actually is made from three lenses, of which the two posterior ones are joined to each other, this investigation is returned from the case of three lenses. Therefore in the first place for the magnification we will have $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$, where, since there must be the interval $\beta+c=0$, there will be $c=-\beta$ and thus $\frac{\beta}{c}=-1$, from which there becomes so that at this point $m=\frac{\alpha}{b}$ or $b=\frac{\alpha}{m}$; then truly we may put $\beta=Bb=\frac{B\alpha}{m}$ and thus also $c=-\frac{B\alpha}{m}$; which since the focal length of the posterior lens on account of $\gamma=\infty$, if the following lens joined to it may have an equal focal length, there shall become $\frac{b\beta}{b+\beta}=c$ or $\frac{B\alpha}{m(1+B)}=\frac{-B\alpha}{m}$, and hence $B=-2$; but it is better to deduce this from our principles; for since the radius of the apparent field of view now is $\varPhi=\frac{\pi-\pi'}{m+1}$, so that this may become twofold greater than the preceding case, there must become $-\pi=\pi'$, so that there may become $\varPhi=\frac{2\pi}{m+1}$. But from the above principles we may deduce $\frac{\mathfrak{B}\pi-\varPhi}{\varPhi}=\frac{\alpha}{b}=m,$ from which there becomes $\mathfrak{B}\pi=(m+1)\varPhi$ and thus $\mathfrak{B}=2$ and hence $B=-2$, thus so that the latter lenses may become equal to each other. But we will have found this value for the radius of confusion

$$\frac{mx^3}{4p^3}\mu\left(\lambda+\frac{q}{\mathfrak{B}^2p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)-\frac{\lambda''}{B^3m}\right),$$

where there is $p=\alpha,\ \ q=\mathfrak{B}b=\frac{2\alpha}{m}$, thus so that this expression may be changed into that same one

$$\frac{\mu mx^3}{4\alpha^3}\left(\lambda+\frac{1}{2m}\left(\frac{\lambda'}{4}-\frac{\nu'}{2}\right)+\frac{\lambda''}{8m}\right).$$

But now it is required to note properly that these two posterior lenses assumed aperture, so that there may become $\pi=\frac{1}{4}$, cannot be allowed, unless each may be rendered equal on both sides. From which condition, if indeed we may make use of common glass, for which $n=1,55$, there will become for the third lens, as we have seen, $\lambda''=1,6299$. But since the value of the number λ' will to be obtained, which we will be able to define from that established above, since there shall be

$$\sqrt{\lambda'-1} = \frac{\sigma-\rho}{2\tau} \cdot \frac{b-\beta}{b+\beta} = -\frac{3(\sigma-\rho)}{2\tau},$$

from which there becomes

$$\lambda' = 1 + \frac{(\sigma-\rho)^2 \cdot 9}{4\tau^2};$$

whereby, since there was

$$\lambda'' = 1 + \frac{(\sigma-\rho)^2}{4\tau^2} = 1,6299,$$

there will become

$$\frac{(\sigma-\rho)^2}{4\tau^2} = 0,6299, \text{ and thus } \lambda' = 6,6691,$$

from which we obtain $\frac{\lambda'}{4} - \frac{\nu'}{2} = 1,5509$, and hence the part of the confusion arising from the second lens becomes $\frac{0,7754}{m}$, then the part arising from the third lens is $\frac{0,2037}{m}$; and thus our whole duplicate eyepiece lens will produce in the expression of the confusion a part $= \frac{0,9791}{m}$. Therefore with that radius in place $= \frac{1}{4k^2}$ we may deduce the focal length of the objective lens

$$\alpha = kmy\sqrt[3]{0,9381(\lambda m + 0,9791)}$$

on account of $x = my$, where I leave λ undefined, so that also the objective lens may be able to be assumed as it pleases either simple, double, treble, or even quadruple. But the two posterior lenses may become equal to each other and equally on both sides, with the radius of convexity being $= \frac{2,20\alpha}{m}$. Truly the distance of the eye after the last lens is found $O = \frac{-\pi' r}{m\Phi} = \frac{\pi r}{m\Phi}$; since now there is $\gamma = \frac{2\alpha}{m}$ and $\frac{\pi}{\Phi} = \frac{m+1}{2}$, there will be

$$O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$$

just as before ; then moreover the angular radius of the apparent field of view will be $= \frac{1718}{m+1}$ minutes.

COROLLARY 1

216. Hence therefore it will be apparent, if the eyepiece lens in this account may be duplicated, its effect on the confusion arising is going to be smaller, than if this lens were simple.

COROLLARY 2

217. In this case it will be worth the effort to examine the colored margin, for which with the division made by $\frac{dn}{n-1}$ in the above will result in this equation :

$$0=\frac{\pi b}{\Phi p}-\frac{\pi'}{m\Phi}=\frac{\pi}{\Phi}\cdot\frac{2}{m};$$

since now there shall become $\frac{\pi}{\Phi}=\frac{m+1}{2}$, this quantity, which must vanish, shall become $\frac{m+1}{m}$, just as before we have found for the simple eyepiece lens, thus so that hence nothing more need be feared about the colored margin.

COROLLARY 3

218. Therefore all the above formulae proposed for the construction of telescopes, whether the objective lens will have been simple or multiple, can be found here, only if a duplicate lens of this kind may be substituted in place of the eyepiece lens, although twice as many individual faces of this have been put in place following the radius ; then truly also in the value of the distance α after the root sign in place of the number 1,6299 this number 0,9791 may be written, and then the radius of the field of view may emerge twice as great. But scarcely is there a need in the formula for α to make this same correction, since it is only according to an agreed limit, within which α cannot be taken.

SCHOLIUM

219. But here in the first place the case deserves to be considered, where the objective lens is quadruple or $\lambda=-0,010216$ and the magnification is taken so great, that the confusion may vanish completely, which happens, if there were $m=\frac{0,9791}{0,010216}=95+\frac{4}{5}$; whereby there can be taken $m=96$, and if for this order of clarity there may be taken $y=\frac{1}{48}$ in., the radius of the aperture of the objective lens will have to be $=my=2$, from which α is defined easily; indeed above we have seen this lens does not allow a greater quadruple aperture, than the radius of which shall be $\frac{1}{6}\alpha$, from which on putting $\frac{1}{6}\alpha=2$ in. there will become $\alpha=12$ in., from which the following construction will be had.

Construction of a telescope for the magnification $m=96$
with the lenses made from common glass for which $n=1,55$.

I. For the quadruple objective lens

Radius of the first lens $\begin{cases}\text{anterior face} & = +\ \ 29{,}50 \text{ in.}\\ \text{posterior face} & = +251{,}70 \text{ in.}\end{cases}$

Radius of the second lens $\begin{cases}\text{anterior face} & = +\ \ 15{,}66 \text{ in.}\\ \text{posterior face} & = -\ \ 38{,}53 \text{ in.}\end{cases}$

Radius of the third lens $\begin{cases}\text{anterior face} & = +\ \ 10{,}66 \text{ in.}\\ \text{posterior face} & = -\ \ 17{,}90 \text{ in.}\end{cases}$

Radius of the fourth lens $\begin{cases}\text{anterior face} & = +\ \ 8{,}05 \text{ in.}\\ \text{posterior face} & = -\ \ 11{,}65 \text{ in.}\end{cases}$

Radius of its aperture $= 2$ in.
Interval as far as to the eyepiece lens $= 12\frac{1}{8}$ in.

II. For the double eyepiece lens

Radius of each lens and of each face $= 0{,}275$ in .
Radius of its aperture $= \frac{1}{16}$ in .
Distance of the eye $= 0{,}126$ in ,
thus so that the total length shall be $= 12{,}251$ in ,
but the radius of the apparent field of view $= \frac{1718}{97}$ min. $=$ 17 min. 43 sec.

PROBLEM 6

220. *If the eyepiece lens were triplicate, so that the radius of the field of view were returned greater by threefold, to describe the construction of the telescope.*

SOLUTION

Since here four lenses are required to be considered, the formula for the magnification will be $m = \frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, and since the three latter lenses are joined together, there will become $\beta + c = 0$ and $\gamma + d = 0$, from which the following determinations arise

$$b = \frac{\alpha}{m},\quad \beta = Bb = \frac{B\alpha}{m},\quad c = -\frac{B\alpha}{m},\quad \gamma = Cc = -\frac{BC\alpha}{m} \text{ and } d = \frac{BC\alpha}{m};$$

moreover the formula for the apparent field of view is :

$$\Phi = \frac{\pi - \pi' + \pi''}{m+1}\ ;$$

which so that it may become greater by three fold as above, it is required to put $\pi' = -\pi$ and $\pi'' = \pi$; for then there will be

$$\Phi = \frac{3\pi}{m+1},$$

thus so that there shall be

$$\pi = -\pi' = \pi'' = \frac{m+1}{3}\cdot\Phi.$$

Moreover from these letters our formulas are :

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi} = \frac{\alpha}{b} = m, \qquad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi} = \frac{B\alpha}{c} = -m,$$

where with the values of π and π' themselves substituted there will be had

$$\frac{\mathfrak{B}(m+1)}{3} - 1 = m \text{ and } \mathfrak{B} = 3 \text{ and hence } B = -\frac{3}{2}.$$

Then

$$+\frac{1}{3}\mathfrak{C} + \frac{1}{3} = 1 \text{ and } \mathfrak{C} = 2 \text{ and hence } C = -2$$

and thus the focal lengths of the last three lenses will be

of the second, $\mathfrak{B}b = \frac{3\alpha}{m}$, third $\mathfrak{C}c = \frac{3\alpha}{m}$, fourth $d = \frac{3\alpha}{m}$

thus so that these three lenses may become equal to each other; therefore the determinable distances will be

$$b = \frac{\alpha}{m}, \ \beta = -\frac{3}{2}b = -\frac{3\alpha}{2m}, \ c = \frac{3\alpha}{2m}, \ \gamma = -2c = -\frac{3\alpha}{m}, \ d = \frac{3\alpha}{m}.$$

We may substitute these values into the formula for the confusion radius, which shall become

$$\frac{\mu m x^3}{4p^3}\left(\lambda + \frac{q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{r}{\mathfrak{B}^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) + \frac{\lambda'''}{B^3C^3m}\right),$$

which on account of $p = \alpha$, $q = \frac{3\alpha}{m} = r$ will go into this form:

$$\frac{\mu m x^3}{4\alpha^3}\left(\lambda + \frac{1}{3m}\left(\frac{\lambda'}{9} - \frac{3\nu}{3}\right) + \frac{4}{27m}\left(\frac{\lambda''}{4} - \frac{\nu}{2}\right) + \frac{\lambda'''}{27m}\right),$$

where suitable numbers are required to be found for λ', λ'', λ'''. But since we wish, so that any of these lenses may be allowed the maximum aperture, which happens, if the values of the letters π, π', π'' shall be given $=\frac{1}{4}$, it is necessary, that any of these shall be equally convex on both sides, which may happen, if there may be put

$$\sqrt{\lambda'''-1}=\tfrac{\sigma-\rho}{2\tau},$$

$$\sqrt{\lambda''-1}=\tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{c-\gamma}{c+\gamma} \text{ and } \sqrt{\lambda'-1}=\tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{b-\beta}{b+\beta}$$

and thus

$$\sqrt{\lambda''-1}=-\tfrac{3}{1}\cdot\tfrac{\sigma-\rho}{2\tau} \text{ and } \sqrt{\lambda'-1}=-\tfrac{5}{1}\cdot\tfrac{\sigma-\rho}{2\tau}.$$

Therefore since there shall be, as has been shown above,

$$\lambda'''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2=1,6299,$$

there will be $\left(\tfrac{\sigma-\rho}{2\tau}\right)^2=0,6299$, from which we gather the number

$$\lambda''=1+9\cdot(0,6299)=6,6691$$

$$\text{and } \lambda'=1+25\cdot(0,6299) \text{ or } \lambda'=16,7475.$$

Now since for the kind of glass proposed, for which $n=1,55$, there shall be $\mu=0,9381$ and $\nu=0,2326$, now we will be able to designate the part, which this triple eyepiece lens may bring into the formula for the confusion, clearly which is the main point concerning the matter.
Moreover there will be found

$$\tfrac{\lambda'}{9}-\tfrac{2\nu}{3}=1,7058 \text{ and } \tfrac{1}{3m}\left(\tfrac{\lambda'}{9}-\tfrac{2\nu}{3}\right)=\tfrac{0,5686}{m}$$

and

$$\tfrac{\lambda''}{4}-\tfrac{\nu}{2}=1,5509 \text{ and the whole term } =\tfrac{0,2297}{m},$$

and thus

$$\tfrac{\lambda'''}{27m}=\tfrac{0,0603}{m},$$

from which the part arising for the whole eyepiece lens will be $=\tfrac{0,8586}{m}$, thus so that the whole expression $\tfrac{\mu m x^3}{4\alpha^3}\left(\lambda+\tfrac{1}{3m}\left(\tfrac{\lambda'}{9}-\tfrac{3\nu}{3}\right)+\tfrac{4}{27m}\left(\tfrac{\lambda''}{4}-\tfrac{\nu}{2}\right)+\tfrac{\lambda'''}{27m}\right)$

$$=\tfrac{\mu x^3}{4\alpha^3}(\lambda m+0,8586);$$

therefore by taking $x=my$ and with this formula $=\tfrac{1}{4k^3}$ put in place, we will determine the focal length of the objective lens $=\alpha$, so that there shall become

$$\alpha=kmy\sqrt[3]{0,9881(\lambda m+0,8586)},$$

or more.

Again the interval between the objective lens and the eyepiece is [for the final image at infinity]

$$\alpha + b = \tfrac{m+1}{m} \cdot \alpha .$$

And since the three lenses constituting the eyepiece shall be equal to each other and equally convex on both sides on account of which the focal length $= \tfrac{3\alpha}{m}$, the radius of the individual faces will be $= 3,30 \tfrac{\alpha}{m}$, the focal length of this triple lens itself being $= \tfrac{\alpha}{m}$ and the radius of the aperture $= \tfrac{3}{4} \cdot \tfrac{\alpha}{m}$. For the distance of the eye past the eyepiece lens there is found $O = \tfrac{\pi'' s}{m\Phi}$, which on account of $\pi'' = \tfrac{m+1}{3} \cdot \Phi$ and $s = \tfrac{3\alpha}{m}$ will become

$$O = \tfrac{m+1}{m} \cdot \tfrac{\alpha}{m}$$

just as before; but the apparent field of view will be

$$\Phi = \tfrac{3 \cdot 859}{m+1} \text{ minutes} = \tfrac{2577}{m+1} \text{ minutes.}$$

COROLLARY 1

221. Here we have defined nothing about the objective lens and that for argument's sake can be put in place to be either simper, double, treble, or even quadruple, and the rules of the construction remain the same as before, provided that the quantity α may be defined from this given formula.

COROLLARY 2

222. It can be shown in the same way as before these telescopes to be no more freed from the troublesome colored fringes than the preceding ones; nor indeed can the colored fringe be removed by two lenses, to which defect it is allowed to refer all these telescopes.

SCHOLIUM

223. In a similar way the eyepiece lens also may be quadruple, thus so that the radius of the field of view will be rendered four times greater; but I will not pursue this investigation further, because, if we wish to use more lenses, these above can lead to other convenient telescopes, just as we will instruct in the following. Evidently here we have considered only the most simple kind of these telescopes, which only with two lenses, the one the objective, the other the eyepiece, is agreed to be considered, even if for each composite lenses may be allowed to be used; since also we have assumed both these lenses to be made from the same kind of glass and also in the following chapter we will use a single kind of glass, so that it may be understood, to which order of perfection

these telescopes may be able to be raised, before which we may call in the aid of diverse kinds of glass. For these degrees of perfection are required to be distinguished properly, which can be obtained from a single kind of glass, from these, which demand diverse kinds; with which agreed on this treatment may be rendered more transparent. But here at this point it will be required to remember, by which account these instruments may be agreed to be freed from another conspicuous inconvenience, which consists in that, repeatedly stray rays, clearly which have not been produced by the object are being seen, may enter into the tube and disturb the vision considerably. We will show in the following problem, how such rays must be kept away.

PROBLEM 7

224. *With these lenses constructed and placed in a tube stray rays, which may enter the tube by the objective lens, are to be kept away, lest they may be incident in the eye and disturb the vision.*

SOLUTION

This tube is accustomed to be set diverging a little from the objective lens at the end, so that rays advancing from the sides may be intercepted ; likewise truly this divergence must be so great, so that none of the rays emitted by the object towards the objective lens may be excluded ; which happens, if the divergence may be equal to the angular radius of the field of view. Yet meanwhile in this way not all the stray rays are prevented from entering the objective; whereby, lest they illuminate the inner walls of the tube, it is necessary, that the internal surface of the tube must be covered over by the color black, as also is required to be understood regarding the front part of the tube. Nor yet does this suffice completely, since also the blackest color shall be capable of some light, and on that account diaphragms or screens are accustomed to be inserted into these tubes, bored through with holes, which must not be greater, than the passage of the rays to the vision by necessity demands, which will happen most conveniently at the place of the image $F\zeta$, where all these rays are repelled in the smallest space. Therefore at this place a diaphragm or circular orb equally most black may be put in place, the hole of which shall be precisely equal to the magnitude of the image, as may be allowed to be discerned by the eye, and in this way general access of all stray rays to the eyepiece lens will be closed, and if by chance they may reach there, they will not be refracted, so that they may be able to pass into the eye.

COROLLARY 1

225. For the magnitude of this opening requiring to be defined we may consider the radius of the apparent field of view Φ, and since the radius of the image $F\zeta$ shall be $\alpha\Phi$, this likewise may be taken for the radius of the opening.

COROLLARY 2

226. So that the greater were the apparent field of view, there is need for a greater hole, where the diaphragm may be bored through. Thus in the final example given § 219, since there shall be $\alpha = 12$ in. and $\Phi = 17$ min. 43 sec. or in parts of a radius $\Phi = \frac{1}{194}$, the radius of this opening must be $\frac{6}{97}$ in. or approx. $\frac{1}{16}$ in., thus so that its diameter may be made equal to $\frac{1}{8}$ in.

SCHOLIUM

227. In astronomical tubes referred to according to this kind, this diaphragm itself is accustomed to be constructed with a small measure, or with the thinnest wires set out in this space, which, since they may be extended into the place of the image, since these as if placed together, and may be equally distinct to the eye, will be represented in the image itself; from which the true magnitude of an astronomical object and the separation of the parts are accustomed to be judged.

LIBRI SECUNDI,

DE CONSTRUCTIONE TELESCOPIORUM SECTIO SECUNDA.

DE TELESCOPIIS SECUNDI GENERIS, QUAE LENTE OCULARI CONVEXA INSTRUCTA,

OBIECTA SITU INVERSO REPRAESENTANT.

CAPUT I

DE TELESCOPIIS SIMPLICIORIBUS SECUNDI GENERIS

EX UNICA VITRI SPECIE PARATIS

PRAECEPTUM GENERALE

192. Cum in hac sectione obiectorum repraesentatio semper futura sit inversa, hic ante omnia monendum est in omnibus formulis generalibus supra traditis litteram *m,* qua multiplicatio indicatur, ubique negative capi debere, ita ut in illis formulis, quoties *m* occurrit, eius loco scribi $-m$ oporteat.

PROBLEMA 1

193. *Simplicissimum huius generis telescopium ex duabus lentibus eademque vitri specie construere, quod obiecta secundum datam multiplicationem m aucta situque inverso repraesentet.*

SOLUTIO

Proposita multiplicatione *m* formulae nostrae generales statim praebent hanc determinationem $m=\frac{\alpha}{b}$, ubi manifestum est α exprimere distantiam focalem lentis obiectivae, *b* vero ocularis ob $\beta=\infty$. Cum igitur fractio $\frac{\alpha}{b}$

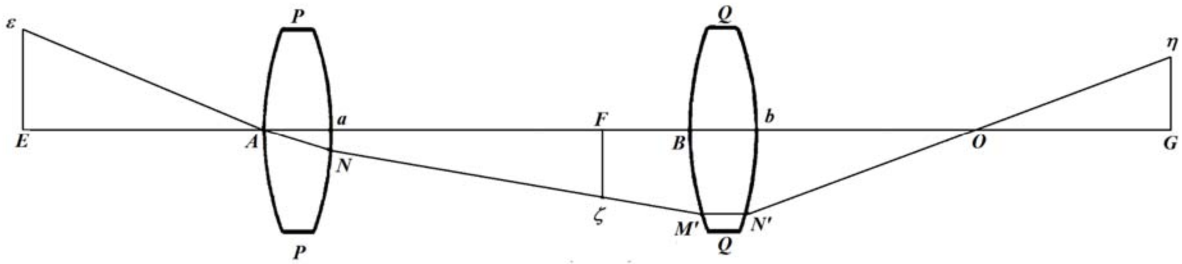

Fig. 13 (iterata).

hic sit positiva simulque harum lentium distantia $\alpha + b$, utramque distantiam a et b positivam esse oportet, ita ut ambae lentes futurae sint convexae et imago realis in puncto F (Fig. 13, Lib. 1, pag. 146) repraesentetur, quod simul est focus communis utriusque lentis. Tum vero campi apparentis semidiameter erit $\Phi = \frac{\pi}{m+1}$, qui autem non conspicietur nisi paulo in certo loco constituto, cuius distantia post lentem ocularem est $O = \frac{\pi q}{m\Phi}$ denotante q distantiam focalem lentis ocularis, quam vidimus esse $= b$. Cum igitur sit $\pi = (m+1)\Phi$, erit haec distantia $O = \frac{m+1}{m} \cdot q$ ideoque tantillo maior quam q. Ut iam obiecta dato claritatis gradu adpareant, quem vocavimus $= y$, ita ut y sit mensura semidiametro pupillae minor, ostensum est aperturam lentis obiectivae tantam esse debere, ut eius semidiameter sit $x = my$, unde iam intelligitur eius distantiam focalem p vel α certe minorem statui non posse quam $4x$. Videamus nunc etiam, quomodo hoc telescopium ratione marginis colorati futurum sit comparatum. Cum is prorsus tolli non possit, quia fieri nequit, ut sit $0 = \frac{dn'}{n'-1} \cdot \frac{\pi}{m\Phi} = \frac{dn'}{n'-1} \cdot \frac{m+1}{m}$ multo minus heac confusionio penitus destrui potest, cum esse deberet $0 = \frac{dn'}{n'-1} \cdot (p+q)$, quia $p+q$ est distantia lentium. Eo magis autem in id est incumbendum, ut confusio primae speciei ab apertura pendens insensibilis reddatur seu ut semidiameter huius confusionis certum quendam limitem, quem littera k indicavimus, non superet. Quare ex superioribus [§ 42, 44] colligetur haec conditio:

$$+\frac{m\mu x^3}{4p^3}\left(\lambda + \frac{\lambda'}{m}\right) < \frac{1}{4k^3} \quad \text{seu} \quad \frac{x^3}{p^3}\left(\mu\lambda m + \mu\lambda'\right) < \frac{1}{k^3},$$

unde pro distantia focali lentis obiectivae $p = \alpha$ hanc obtinemus conditionem :

$$p > kx\sqrt[3]{(\mu\lambda m + \mu\lambda')},$$

et ob $x = my$ erit

$$p > kky\sqrt[3]{(\mu\lambda m + \mu\lambda')}$$

seu ad minimum p huic formulae aequalis capi poterit.

COROLLARIUM 1

194. Hinc ergo statim apparet, quo maior requiratur multiplicatio, eo maiorem esse debere lentis obiectivae distantiam focalem ideoque etiam longitudinem telescopii neque id in ratione tantum simplici, sed fere in ratione sesquitriplicata multiplicationis, scilicet ut $m^{\frac{4}{3}}$; hincque ista longitudo mox tanta evadit, ut neutiquam sit verendum, ne quantitas p minor fiat quam $4my$.

COROLLARIUM 2

195. Numerus μ ab indole vitri pendet, unde sequitur, quo minor is fuerit, eo magis longitudinem p imminui. Vidimus autem supra crescente ratione refractionis n istum numerum μ diminui; sed quia tum formula $\frac{dn}{n-1}$ crescit ideoque margo coloratus augetur, praestabit vitro uti communi.

COROLLARIUM 3

196. Hinc etiam intelligimus, quo maior gradus claritatis y desideretur, eo magis quantitatem p augeri debere, quod etiam usu venit, si maior distinctio requiratur, quia tum litterae k maior valor tribui deberet.

COROLLARIUM 4

197. Ad longitudinem autem horum instrumentorum Contrahendam plurimum interest lentem obiectivam ita conficere, ut fiat $\lambda = 1$, quippe qui huius litterae minimus est valor. Quare huic lenti eam formam tribui conveniet, quam supra in capite de lentibus obiectivis descripsimus.

COROLLARIUM 5

198. Circa lentem autem ocularem parum lucraremur, si et $\lambda' = 1$ capere vellemus, quoniam in maioribus multiplicationibus hic terminus prae primo evanescit; quin potius huic lenti eiusmodi figuram tribui necesse est, quae maximae aperturae sit capax, quoniam ab ea campus apparens potissimum pendet; quare haec sanciatur regula, ut lens ocularis utrinque aequaliter convexa conficiatur, quoniam tum demum littera π valorem $\frac{1}{4}$ vel etiam maiorem accipere potest. Tum vero erit

I. Pro vitro coronario seu $n = 1,53$:

$$\lambda' = 1,60006 .$$

II. Pro vitro communi seu $n = 1,55$:

$$\lambda' = 1,62991 .$$

III. Pro vitro denique crystallino seu $n = 1,58$:

$\lambda' = 1,67445$.

SCHOLION 1

199. Hugenius partim theoriae satis incompletae partim experimentis innixus distantiam focalem lentis obiectivae quadrato multiplicationis proportionalem statuit, cui tantum abest, ut adversari velim, ut potius in praxi eius praesertim temporis assentiar; nostra enim determinatio innititur huic rationi, quod facies lentium ad figuram sphaericam perfecte sint formatae, quam si artifex exacte efficere posset, nullum est dubium, quin nostra formula veritati sit consentanea, quod quidem nunc summorum artificum industriae concedendum videtur; sed quando figura lentium a sphaerica figura tantillum aberrat, notum est vitium eo magis esse sensibile, quo maior fuerit distantia focalis lentis, cui propterea aliter occurri nequit nisi distantiam focalem maiorem reddendo quam secundum nostram regulam. Num autem praecise ratio duplicata inde exsurgat, neutiquam affirmare licet, sed prout quaeque lens feliciori successu fuerit elaborata, eo minor distantia focalis sufficit eidem multiplicationi producendae, seu potius eadem lens maiori multiplicationi producendae erit apta; quod etsi perpetuo est observandum, tamen hic assumo lentibus non solum sphaericas figuras, sed etiam secundum datos radios tribui posse.

SCHOLION 2

200. His autem Hugenii observationibus praecipue utemur ad gradus tam claritatis quam distinctionis definiendos, quibus astronomi contenti esse solent, etiamsi cuique liberum relinquatur sive maiorem sive minorem gradum eligere. Quod igitur primo ad gradum claritatis attinet, Hugenius lenti obiectivae, cuius distantia focalis $= 20$ ped. sive 240 digit., assignat aperturam, cuius semidiameter $= 1,225$ digit., eamque ad multiplicationem $m = 89$ aptam iudicat; quam rationem etiam in reliquis lentibus obiectivis observat; quare, cum hic sit $x = 1,225$ dig. et $m = 89$, ob $x = my$ hinc colligimus $y = \frac{x}{m} = \frac{1,225}{89} = \frac{1}{73}$; quare, cum supra passim assumserimus $y = \frac{1}{50}$, multo maiorem claritatis gradum illis instrumentis conciliavimus cumque adeo duplo maiorem.

Quod dein ad gradum distinctionis attinet littera k contentum, in allegato Hugenii exemplo perpendamus esse $p = 240$ dig., $m = 89$ et $y = \frac{1}{73}$, sumtoque $p = \frac{9}{10}$ et $\lambda = 1$ reiectoque altero termino in formula radicali hi valores in nostra formula substituti dabunt

$$240 = 89 \cdot \frac{1}{73} \cdot k \cdot \sqrt[3]{\frac{9}{10} \cdot 89}, \text{ ergo } k = \frac{73 \cdot 240}{89 \cdot \sqrt[3]{80}},$$

quae fractio evoluta dat $k = 45$. Quare, cum supra passim sumserimus $k = 50$, maiorem distinctionis gradum, quam hinc oritur, sumus complexi. Dum igitur in nostra formula ky occurrat, secundum Hugenium sufficeret statuere $ky = \frac{45}{73} = \frac{5}{8}$ circiter, ex quo patet, si statuamus $ky = 1$, non solum claritatis, sed et distinctionis maiorem gradum obtineri, simul vero longitudinem telescopii multo maiorem esse proditurum, quam si poneremus $ky = \frac{5}{8}$.

SCHOLION 3

201. Quoniam vitrum crystallinum ad huiusmodi telescopia ineptum est indicandum, siquidem margo coloratus augeretur, pro duabus vitri speciebus, altera qua $n = 1,53$, altera qua $n = 1,55$, constructiones hic apponamus.

Constructio huiusmodi telescopii utraque lente ex vitro coronario $n = 1,53$ parata

I. Pro lente obiectiva.

$$\text{Radius faciei} \begin{cases} \text{anterioris} & = 0,6023\alpha \\ \text{posterioris} & = 4,4131\alpha. \end{cases}$$

$$\text{Intervallum} = \frac{m+1}{m} \cdot \alpha.$$

II. Pro lente oculari

$$\text{Radius utriusque faciei} = 1,06 \cdot \frac{\alpha}{m}$$

Distantia oculi ab hac lente $= \frac{m+1}{m} \cdot \frac{\alpha}{m}$.

Semidiameter aperturae lentis obiectivae $= my$, lentis ocularis $= \frac{1}{4} \cdot \frac{\alpha}{m}$.

Semidiameter campi $\Phi = \frac{859}{m+1}$ minut. sumendo

$$\alpha = kmy\sqrt[3]{0,9875(m + 1,60006)}.$$

Constructio huiusmodi telescopii

utraque lente ex vitro communi $n = 1,55$ parata

I. Pro lente obiectiva

$$\text{Radius faciei} \begin{cases} \text{anterioris} & = 0,6145\alpha \\ \text{posterioris} & = 5,2438\alpha. \end{cases}$$

Semidiameter aperturae $= my$. Intervallum $= \frac{m+1}{m} \cdot \alpha$.

II. Pro lente oculari

Radius utriusque faciei $=1,10\cdot\frac{\alpha}{m}$.

Semidiameter aperturae $=\frac{1}{4}\cdot\frac{\alpha}{m}$. Distantia oculi $=\frac{m+1}{m}\cdot\frac{\alpha}{m}$.

Semidiameter campi apparentis $\Phi=\frac{859}{m+1}$ minut. sumendo

$$\alpha=kmy\sqrt[3]{0,9381(m+1,62991)},$$

ubi quilibet gradum claritatis et distinctionis pro lubitu assumere potest.

PROBLEMA 2

202. *Si lens obiectiva fuerit duplicata eius generis, quod descripsimus § 65, constructionem telescopii describere.*

SOLUTIO

Hic omnia, quae in praecedente problemate de multiplicatione, campo apparente et loco oculi definivimus, manent eadem; tantum in expressione pro semidiametro confusionis inventa numerus λ minorem adipiscitur valorem ultra partem quintam unitatis imminutum; unde distantia focalis lentis obiectivae etiam minorem valorem habere poterit; id quod sine dubio tanquam insigne lucrum est spectandum, cum hoc modo longitudo instrumenti haud mediocriter contrahatur. Hic autem ad vitri speciem, ex quo lentes parantur, inprimis est attendendum, quandoquidem numerus λ per eam definitur, unde excluso vitro crystallino ob rationes ante allegatas constructiones huiusmodi telescopiorum pro binis reliquis speciebus hic exhibeamus.

Constructio huiusmodi telescopii

utraque lente ex vitro coronario pro quo $n=1,53$ parata

I. Pro lente obiectiva duplicata

$$\text{Lentis prioris radius faciei}\begin{cases}\text{anterioris} &=+1,2047\alpha\\ \text{posterioris} &=+\ 8,8222\alpha\end{cases}$$

$$\text{Lentis posterioris radius faciei}\begin{cases}\text{anterioris} &=+0,6464\alpha\\ \text{posterioris} &=-\ 1,6570\alpha.\end{cases}$$

Semidiameter aperturae $x=my$. Intervallum usque ad lentem ocularem $=\frac{m+1}{m}\cdot\alpha$.

II. Pro lente oculari

Radius faciei utriusque $= 1,06 \cdot \frac{\alpha}{m}$.

Semidiameter aperturae $= \frac{1}{4} \cdot \frac{\alpha}{m}$.

Distantia oculi post hanc lentem $O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$. Hic scilicet ipsa lens obiectiva duplicata ut simplex spectatur, cuius distantia focalis sit $= \alpha$, quae iam ita capi debet, ut fiat

$$\alpha \geq kmy\sqrt[3]{0,9875(0,1951m + 1,60006)}.$$

Semidiameter vero campi apparentis est ut ante $\Phi = \frac{859}{m+1}$ minut.

Constructio huiusmodi telescopii

utraque lente ex vitro communi $n = 1,55$ parata
I. Pro lente obiectiva duplicata

Lentis prioris radius faciei $\begin{cases} \text{anterioris} = +\ 1,2289\alpha \\ \text{posterioris} = +\ 10,4876\alpha \end{cases}$

Lentis posterioris radius faciei $\begin{cases} \text{anterioris} = +0,6527\alpha \\ \text{posterioris} = -\ 1,6053\alpha. \end{cases}$

Semidiameter aperturae $x = my$. Intervallum usque ad lentem ocularem $= \frac{m+1}{m} \cdot \alpha$.

II. Pro lente oculari

Radius faciei utriusque $= 1,01 \cdot \frac{\alpha}{m}$.

Semidiameter aperturae $= \frac{1}{4} \cdot \frac{\alpha}{m}$.

Distantia oculi post hanc lentem $O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$, ubi cernetur campus, cuius semidiameter $= \frac{859}{m+1}$ minut.

At distantia focalis ipsius lentis obiectivae duplicatae ita capi debet, ut sit

$$\alpha \geq kmy\sqrt[3]{0,9381(0,1918m + 1,6299)}.$$

COROLLARIUM 1

203. Si ergo multiplicatio tanta sit, ut in valore ipsius α postremus terminus prae altero evanescat, hoc casu distantia α minor erit quam praecedente, in ratione circiter $\sqrt[3]{\frac{1}{5}}:1$ vel $1:\sqrt[3]{5}$, hoc est fere ut 10 : 17.

COROLLARIUM 2

204. Cum istae lentes duplicatae ex principio minimi sint deductae, eo magis sunt ad praxin accommodatae, cum metuendum non sit, ut exigui errores ab artifice commissi effectum perturbent, quod maxime esset metuendum, si reliquas lentes compositas, quae quidem perfectae sunt vocatae, loco obiectivae substituera vellemus.

SCHOLION

205. Quo clarius appareat, quantum lucrum hinc sit exspectandum, accommodemus ambos casus ad datam multiplicationem, puta $m = 100$, ubi quidam solum vitrum commune consideremus.

Si igitur I.) lente obiectiva simplici utamur, distantia focalis α ita accipi debet, ut sit

$$\alpha = 100ky\sqrt[3]{0,9381\cdot 101,6299}, \qquad \alpha = 100ky\cdot 4,5684;$$

unde, si cum Hugenio capiatur $ky = \frac{5}{8}$ dig., prodit

$$\alpha = 285\tfrac{1}{2} \text{ dig.} = 23 \text{ ped. } 9\tfrac{1}{2} \text{ dig.}$$

Sin autem II.) utamur lente obiectiva duplicata, habebimus

$$\alpha = 100ky\sqrt[3]{0,9381\cdot 20,8099}, \qquad \alpha = 100ky\cdot 2,6926;$$

sumtoque iterum $ky = \frac{5}{8}$ dig. erit

$$a = 168\tfrac{1}{3} \text{ dig.} = 14 \text{ ped.} \tfrac{1}{3} \text{ dig.};$$

haec certe contractio antehac maximi momenti foret visa; nunc autem, cum multo adhuc breviora telescopia desideremus, non admodum notatu digna videbitur, quod etiam eveniet in casu sequentis problematis, ubi lentem obiectivam triplicatam faciemus.

PROBLEMA 3

206. *Si lens obiectiva, fuerit triplicata, quam* § 66 *descripsimus, telescopii constructionem describere.*

SOLUTIO

Omnia manent ut ante, nisi quod pro hac lente triplicata futurum sit $\lambda = \frac{3-8\nu}{27}$; unde considerando tantum vitrum commune, pro quo $n = 1,55$, lentis huius obiectivae distantia focalis α ita definiri debet, ut sit

$$\alpha = kmy\sqrt[3]{0,9381(0,0422m + 1,6299)} ;$$

hinc igitur sequenti modo talia telescopia erunt construenda.

Constructio huiusmodi telescopii
utraque lente ex vitro communi pro quo $n = 1,55$ parata

I. Pro lente obiectiva triplicata

Lentis primae radius faciei $\begin{cases} \text{anterioris} & = +15,7315\alpha \\ \text{posterioris} & = +\ 0,9791\alpha \end{cases}$

lentis secundae radius faciei $\begin{cases} \text{anterioris} & = +0,9791\alpha \\ \text{posterioris} & = -2,4077\alpha \end{cases}$

lentis tertiae radius faciei $\begin{cases} \text{anterioris} & = +0,6665\alpha \\ \text{posterioris} & = -\ 1,1183\alpha. \end{cases}$

Eius aperturae semidiameter $x = my$.

Intervallum usque ad lentem ocularem $= \frac{m+1}{m} \cdot \alpha$.

II. Pro lente oculari

Radius faciei utriusque $= 1,10 \cdot \frac{\alpha}{m}$.

Eius aperturae semidiameter $= \frac{1}{4} \cdot \frac{\alpha}{m}$.

Distantia oculi $= \frac{m+1}{m} \cdot \frac{\alpha}{m}$ campique apparentis semidiameter $\Phi = \frac{859}{m+1}$ minut.

Ipsa autem lentis obiectivae distantia focalis tanta accipi debet, ut sit

$$\alpha = kmy\sqrt[3]{0,9381(0,0422m + 1,6299)}.$$

COROLLARIUM 1

207. Si ergo multiplicatio statuatur $m = 100$, capi poterit

$$\alpha = 100ky\sqrt[3]{0,9381 \cdot 5,8499}$$

sive $\alpha = 100ky \cdot 1,7639$ sumtoque $ky = \frac{5}{8}$ dig.

$$\alpha = 110\tfrac{1}{4} \text{ dig.} = 9 \text{ ped. } 2\tfrac{1}{4} \text{ dig.};$$

sicque longitudo totius telescopii usque ad oculum prodibit

$$\left(\frac{m+1}{m}\right)^2 \cdot \alpha = 112\tfrac{1}{2} \text{ dig.} = 9 \text{ ped. } 4\tfrac{1}{2} \text{ dig.}$$

COROLLARIUM 2

208. Quod ad gradum claritatis y attinet, quoniam hic plures sunt lentes, per quas radiis est transeundum, eorumque ideo maior iactura metuenda, etiamsi maiorem claritatem non requiramus quam Hugenius, tamen ipsi y maior valor tribui debet quam $\frac{1}{73}$; quare retento valore k longitudo telescopii maior prodibit.

SCHOLION

209. Haec ultima cautela maximi est momenti et semper probe observanda, quoties maiore lentium numero utemur, atque hac occasione haud abs re erit eorum telescopiorum ex Anglia allatorum, quae nocturna sunt appellata, mentionem facere; circa quae primum observo eorum usum in summis tenebris plane fore nullum, sed tantum tempore crepusculi vel lucente luna ea adhiberi solere ad obiecta non nimis longinqua spectanda. Totum autem mysterium, quod in his telescopiis plerique quaesiverunt, huc redit, ut iis summus claritatis gradus concilietur, seu ut litterae y semidiameter ipsius pupillae tribuatur sive circiter statuatur $y = \frac{1}{12}$ dig., siquidem tum claritas visa tricies sexies maior sentietur, quam si sumeretur $y = \frac{1}{73}$. Quare, ne haec telescopia nimis fiant longa, multo minori multiplicatione nos contentos esse oportet. Ad hunc autem scopum multiplicatio $m = 10$ plus quam sufficiens esse solet. Si enim noctu obiecta longinqua quasi nobis decuplo essent propiora eaque eodem claritatis gradu aspicere licebit atque nudis oculis, plus certe desiderari non poterit.

PROBLEMA 4

210. *Si denique lens obiectiva fuerit quadruplicata, secundum principia § 154 libro superiore tradita constructa, telescopii constructionem describere.*

SOLUTIO

Hic denuo omnia manent ut ante; sed quod hic inprimis notatu dignum occurrit, est, quod in formula pro distantia focali α resultante, scilicet

$$\alpha = kmy\sqrt[3]{\mu\left(\tfrac{1-5v}{16}\cdot m+\lambda''\right)},$$

valor numeri λ prodeat negativus ideoque certo casu tota confusio evanescere queat; qui casus maxime meretur, ut omni diligentia evolvatur. Sumamus igitur omnes istas lentes ex vitro communi, pro quo $n=1,55$, esse confectas lentemque ocularem utrinque ut hactenus aeque convexam formari, atque habebimus

$$\lambda=\tfrac{1-5v}{16}=-0,010216 \text{ et } \lambda''=1,6299,$$

unde intelligitur semidiametrum confusionis prorsus in nihilum abire, si capiatur

$$m=\tfrac{1,6999}{0,010216}=159\tfrac{1}{2}.$$

Pro hoc ergo casu quantitas α non amplius ex hac formula sed unice ex apertum, quam gradus claritatis postulat, determinabitur; si enim pro gradu claritatis in genere sumamus *y*, semidiameter aperturae debet esse $=my$, unde distantia α tanta accipi debet, ut pro radiis singularum facierum tantam aperturam recipere possit. Quare, si α etiam nunc ut quantitatem indefinitam spectemus, constructio telescopii ita se habebit.

Constructio huiusmodi telescopii
lentibus ex vitro communi paratis

I. Pro lente obiectiva quadruplicata

Lentis primae radius faciei $\begin{cases}\text{anterioris} & =+\ 2,4580\alpha\\ \text{posterioris} & =+20,9754\alpha\end{cases}$

lentis secundae radius faciei $\begin{cases}\text{anterioris} & =+1,3054\alpha\\ \text{posterioris} & =-3,2103\alpha\end{cases}$

lentis tertiae radius faciei $\begin{cases}\text{anterioris} & =+0,8887\alpha\\ \text{posterioris} & =-\ 1,4910\alpha.\end{cases}$

lentis quartae radius faciei $\begin{cases}\text{anterioris} & =+0,6710\alpha\\ \text{posterioris} & =-0,9710\alpha.\end{cases}$

Semidiameter aperturae $x=my$.

Intervallum usque ad lentem ocularem $=\frac{m+1}{m}\cdot\alpha$.

II. Pro lente oculari

Radius utriusque faciei $=1,10\cdot\frac{\alpha}{m}$.

Semidiameter aperturae $=\frac{1}{4}\cdot\frac{\alpha}{m}$.

Distantia oculi $=\frac{m+1}{m}\cdot\frac{\alpha}{m}$

campique visi semidiameter $= \frac{859}{m+1}$ minut.
Hic autem in genere capi deberet

$$\alpha \geq kmy\sqrt[3]{0,9381(-0,010216m+1,6299)},$$

nisi valor hinc prodiens minor fuerit, quam ut praescripta apertura $x = my$ locum habere possit, id quod potissimum pro $m = 159\frac{1}{2}$ eveniet.

Pro quo radii facierum modo exhibiti perpendi debent, inter quos minimus cum sit $0,6710\alpha$, huius pars quarta $0,1678\alpha$ seu fere $\frac{1}{6}\alpha$ determinabit semidiametrum aperturae; quae, cum ob $m = 159\frac{1}{2}$ sit $159\frac{1}{2}y$, capi debebit $a > 6.159\frac{1}{2}y$ seu $\alpha > 957y$; si igitur sumamus $y = \frac{1}{50}$ dig., capi debebit $\alpha > 19\frac{7}{50}$ dig., quocirca statuamus $\alpha = 20$ dig. Sumtaque multiplicatione $m = 160$ habebimus hanc specialissimam constructionem.

Constructio telescopii pro multiplicatione $m = 160$ lentibus
e vitro communi $n = 1,55$ confectis

Lentis primae radius faciei	anterioris	$= +\ 49,16$	dig.
	posterioris	$= +419,50$	dig.
lentis secundae radius faciei	anterioris	$= +\ 26,10$	dig.
	posterioris	$= -\ 64,21$	dig.
lentis tertiae radius faciei	anterioris	$= +\ 17,77$	dig.
	posterioris	$= -\ 29,82$	dig.
lentis quartae radius faciei	anterioris	$= +13,42$	dig.
	posterioris	$= -19,42$	dig.

Eius aperturae semidiameter $x = my = 3,2$ dig.
Intervallum usque ad lentem ocularem $= 20\frac{1}{8}$ dig.

II. Pro lente oculari

Radius utriusque faciei $= 0,1375$ dig.
eiusque semidiameter aperturae $= \frac{1}{32}$ dig.
Distantia oculi $= 0,1258$ dig.,
ita ut sit tota telescopii longitudo $= 20\frac{1}{4}$ dig.
campique visi semidiameter $= 5'\ 20''$.

COROLLARIUM 1

211. Si multiplicationem minorem statuissemus, longitudo telescopii maior prodiisset. Si enim statuamus $m = 50$ litteraeque k etiam valorem 50 tribuarous, prodiret $\alpha = 50\sqrt[3]{0,9381 \cdot 1,1189}$ seu $\alpha > 50,81$ ideoque plus quam duplo maior quam casu $m = 160$, quod certe ingens est paradoxon.

COROLLARIUM 2

212. Si artifex in constructione lentis obiectivae tantillum aberret, eius error valorem numeri λ tantum paulisper augebit, quia ille valor $\lambda = -0,010216$ omnium est minimus; si enim ob hos errores λ particula $\frac{1}{1000}$ augeatur, prodit $\lambda = -\ 0,010154$, ita ut tum ista lens quadruplicata ad maiorem multiplicationem producendam sit apta; quod paradoxon priori non cedit.

SCHOLlON 1

213. Neque hic neque in praecedentibus definivimus, cuiusmodi mensuram digitorum intelligamus, an sint Parisini an Londinenses an Rhenani etc. Verum consultum potius est hanc mensuram prorsus indeterminatam relinquere. Quodsi enim causam dubitandi habeamus lentes secundum regulas praescriptas accurate esse elaboratas, maxime e re erit maiorem mensuram pro digitis adhibere. Sin autem de exsecutione plane simus certi, mensura digitorum minore tuto uti poterimus. Semper autem praxi consulendo utile erit maiorem digitorum mensuram adhibere; atque adeo ipsa ratio, quae nos ad digitorum mensuram perduxit, hoc suadet; haec enim ratio ex apertura pupillae nobis est nata, quam in partibus digiti expressimus. Cum igitur ipsa pupilla tantopere sit mutabilis, ut nihil plane certi de ea statui possit, manifestum est tantum abesse, ut nobis certa quaedam mensura sit praescripta, ut nobis potius liberum sit eam sive augendo sive minuendo notabiliter immutare.

SCHOLION 2

214. Hactenus ostendimus, quemadmodum lentibus compositis loco obiectivae adhibendis haec telescopia non mediocriter contrahi queant. Verum hoc modo nullum plane augmentum campo apparenti inducitur. Iam dudum autem est observatum campum quoque apparentem non mediocriter augeri posse, si etiam lens ocularis sive duplicetur, sive adeo triplicetur. Cum enim campus apparens inprimis ab apertura lentis ocularis pendeat, quam ob causam etiam huic lenti figuram utrinque aequalem tribuimus, ut maioris aperturae capax redderetur, evidens est, si hanc lentem ita instruere liceret, ut adhuc maiorem aperturam recipere posset, campum apparentem in eadem ratione auctum iri. Quo hoc clarius perspiciatur, ponamus lentis ocularis distantiam focalem esse unius digiti, ita ut aperturam admittat, cuius semidiameter $= \frac{1}{4}$ dig. Iam satis manifestum est, si eius loco binae lentes inter se iunctae, quarum utriusque distantia focalis $= 2$ dig.,

substituantur, tum istius lentis compositae distantiam focalem quoque fore unius digiti, sed hanc lentem compositam duplo maiorem aperturam esse admissuram, siquidem utraque faciebus inter se aequalibus constet ideoque aperturam admittat, cuius semidiameter dimidii digiti, atque hoc modo campus apparens duplicabitur. Simili modo, si loco eius lentis ocularis simplicis substituantur ternae lentes, quarum singularum distantia focalis sit trium digitorum, idem effectua ratione multiplicationis obtinebitur, sed quia aperturam triplo maiorem admittunt, campus triplicabitur. Haec autem omnino digna sunt, ut adcuratius ex nostris principiis explicentur, atque inprimis influxum huiusmodi lentium compositarum, quo confusionem afficiunt, determinemus.

PROBLEMA 5

215. *Si lens ocularis duplicetur, ut semidiameter campi apparentis duplo maiorem valorem nanciscatur, constructionem huiusmodi telescopii describere.*

SOLUTIO

Cum hic telescopium revera tribus constat lentibus, quarum binae posteriores sibi immediate sunt iunctae, haec investigatio ex casu trium lentium est repetenda. Primo igitur pro multiplicatione habebimus $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$, ubi, cum esse debeat intervallum $\beta+c=0$, erit $c=-\beta$ ideoque $\frac{\beta}{c}=-1$, unde fit ut hactenus $m=\frac{\alpha}{b}$ seu $b=\frac{\alpha}{m}$; tum vero posuimus $\beta=Bb=\frac{B\alpha}{m}$ ideoque etiam $c=-\frac{B\alpha}{m}$; quae cum sit distantia focalis postremae lentis ob $\gamma=\infty$, si seconda lens ipsi iuncta parem haberet distantiam focalem, foret $\frac{b\beta}{b+\beta}=c$ sive $\frac{B\alpha}{m(1+B)}=\frac{-B\alpha}{m}$, hincque $B=-2$; sed praestat haec ex nostris principiis deducere; quia enim campi apparentis semidiameter nunc est $\Phi=\frac{\pi-\pi'}{m+1}$, ut hic duplo maior fiat quam casu praecedente, debet esse $-\pi=\pi'$, ut fiat $\Phi=\frac{2\pi}{m+1}$. Ex principiis autem superioribus colligimus $\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=m,$ unde fit $\mathfrak{B}\pi=(m+1)\Phi$ ideoque $\mathfrak{B}=2$ hincque $B=-2$, ita ut postremae lentes fiant inter se aequales. Hoc autem valore invento pro semidiametro confusionis habebimus

$$\frac{mx^3}{4p^3}\mu\left(\lambda+\frac{q}{\mathfrak{B}^2p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)-\frac{\lambda''}{B^3m}\right),$$

ubi est $p=\alpha,$ $q=\mathfrak{B}b=\frac{2\alpha}{m}$, ita ut haec expressio abeat in istam

$$\frac{\mu mx^3}{4\alpha^3}\left(\lambda+\frac{1}{2m}\left(\frac{\lambda'}{4}-\frac{\nu'}{2}\right)+\frac{\lambda''}{8m}\right).$$

Nunc autem probe notandum est has duas lentes posteriores assumtam aperturam, ut fiat $\pi = \frac{1}{4}$, admittere non posse, nisi utraque sibi utrinque reddatur aequalis. Ex qua conditione, si quidem vitro communi utamur, pro quo $n = 1,55$, pro lente tertia erit, uti vidimus, $\lambda'' = 1,6299$. Quemnam autem valorem numerus λ' sit habiturus, ex supra allatis definire poterimus, cum sit

$$\sqrt{\lambda' - 1} = \frac{\sigma - \rho}{2\tau} \cdot \frac{b - \beta}{b + \beta} = -\frac{3(\sigma - \rho)}{2\tau},$$

unde fit

$$\lambda' = 1 + \frac{(\sigma - \rho)^2 \cdot 9}{4\tau^2};$$

quare, cum fuerit

$$\lambda'' = 1 + \frac{(\sigma - \rho)^2}{4\tau^2} = 1,6299,$$

erit

$$\frac{(\sigma - \rho)^2}{4\tau^2} = 0,6299, \quad \text{ideoque} \quad \lambda' = 6,6691,$$

ex quo obtinemus $\frac{\lambda'}{4} - \frac{\nu'}{2} = 1,5509$, hincque confusionis pars ex secunda lente orta fit $\frac{0,7754}{m}$, dum pars ex tertia lente orta est $\frac{0,2037}{m}$; sicque tota nostra lens ocularis duplicata producet in expressione confusionis partem $= \frac{0,9791}{m}$. Posita igitur illa semidiametro $= \frac{1}{4k^2}$ colligemus distantiam focalem lentis obiectivae

$$\alpha = kmy\sqrt[3]{0,9381(\lambda m + 0,9791)}$$

ob $x = my$, ubi λ indefinitum relinquo, ut etiam lens obiectiva pro lubitu sive simplex sive duplicata sive triplicata sive etiam quadruplicata assumi queat. Binae autem lentes posteriores inter se aequales fient et utrinque aeque convexae, radio convexitatis existante $= \frac{2,20\alpha}{m}$. Oculi vero distantia post hanc lentem reperitur $O = \frac{-\pi' r}{m\Phi} = \frac{\pi r}{m\Phi}$; quia nunc est $\gamma = \frac{2\alpha}{m}$ et $\frac{\pi}{\Phi} = \frac{m+1}{2}$, erit

$$O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$$

prorsus ut ante ; tum autem campi apparentis semidiameter erit $= \frac{1718}{m+1}$ minut.

COROLLARIUM 1

216. Hinc ergo patet, si lens ocularis hac ratione duplicetur, eius effectum in confusione augenda minorem esse futurum, quam si haec lens esset simplex.

COROLLARIUM 2

217. Operae pretium erit pro hoc casu in marginem coloratum inquirere, pro quo divisione per $\frac{dn}{n-1}$ facta haec in superioribus occurrit aequatio

$$0=\frac{\pi b}{\Phi p}-\frac{\pi'}{m\Phi}=\frac{\pi}{\Phi}\cdot\frac{2}{m};$$

cum nunc sit $\frac{\pi}{\Phi}=\frac{m+1}{2}$, haec quantitas, quae evanescere deberet, fit $\frac{m+1}{m}$, prorsus ut ante invenimus pro lente oculari simplici, ita ut hinc pro margine colorato nihil amplius sit metuendum.

COROLLARIUM 3

218. Omnes igitur formulae supra allatae pro constructione telescopiorum, sive lens obiectiva fuerit simplex sive multiplicata, etiam hic locum obtinere possunt, si modo loco lentis ocularis simplicis huiusmodi lens duplicata substituatur, cuius singulae facies secundum radium duplo maiorem sunt elaborandae; tum vero etiam in valore distantiae α post signum radicale loco numeri 1,6299 scribatur hic numerus 0,9791, atque tum campi apparentis semidiameter duplo evadet maior. Vix autem opus est in formula pro α istam correctionem facere, quia tantum de limite sermo est, infra quem α accipi non oportet.

SCHOLION

219. Hic autem inprimis considerari meretur casus, quo lens obiectiva est quadruplicata sive $\lambda=-0,010216$ et multiplicatio tanta accipitur, ut confusio penitus evanescat, quod fit, si fuerit $m=\frac{0,9791}{0,010216}=95+\frac{4}{5}$; quare capi potest $m=96$, et si pro gradu claritatis capiatur potest $y=\frac{1}{48}$ dig., semidiameter aperturae lentis obiectivae debebit esse $=my=2$, unde α facile definitur; supra enim vidimus hanc lentem quadruplicatam maiorem aperturam non admittere, quam cuius semidiameter sit $\frac{1}{6}\alpha$, unde posito $\frac{1}{6}\alpha=2$ dig. fiet $\alpha=12$ dig., ex quo sequens habebitur constructio.

Constructio telescopii pro multiplicatione $m=96$
lentibus ex vitro communi pro quo $n=1,55$ confectis

I. Pro lente obiectiva quadruplicata

Lentis primae radius faciei $\begin{cases} \text{anterioris} & = + \ 29,50 \ \text{dig.} \\ \text{posterioris} & = +251,70 \ \text{dig.} \end{cases}$

lentis secundae radius faciei $\begin{cases} \text{anterioris} & = + \ 15,66 \ \text{dig.} \\ \text{posterioris} & = - \ 38,53 \ \text{dig.} \end{cases}$

lentis tertiae radius faciei $\begin{cases} \text{anterioris} & = + \ 10,66 \ \text{dig.} \\ \text{posterioris} & = - \ 17,90 \ \text{dig.} \end{cases}$

lentis quartae radius faciei $\begin{cases} \text{anterioris} & = + \ 8,05 \ \text{dig.} \\ \text{posterioris} & = - \ 11,65 \ \text{dig.} \end{cases}$

Eius aperturae semidiameter $= 2$ dig.
Intervallum usque ad lentem ocularem $= 12\frac{1}{8}$ dig.

II. Pro oculari duplicata

Lentis utriusque radius faciei utriusque $= 0,275$ dig.
Eius aperturae semidiameter $= \frac{1}{16}$ dig.
Distantia oculi $= 0,126$ dig.,
ita ut sit longitudo tota $= 12,251$ dig.,
campi autem apparent semidiameter $= \frac{1718}{97}$ minut. $= 17$ minut. 43 sec.

PROBLEMA 6

220. *Si lens ocularis fuerit triplicata, ut semidiameter campi reddatur triplo maior, telescopii constructionem describere.*

SOLUTIO

Quia hic quatuor lentes sunt considerandae, formula pro multiplicatione erit $m = \frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$, et quia ternae posteriores sibi immediate iunguntur, fiet $\beta + c = 0$ et $\gamma + d = 0$, unde sequentes prodeunt determinationes

$$b = \frac{\alpha}{m}, \quad \beta = Bb = \frac{B\alpha}{m}, \quad c = -\frac{B\alpha}{m}, \quad \gamma = Cc = -\frac{BC\alpha}{m} \quad \text{et} \quad d = \frac{BC\alpha}{m};$$

formula autem pro campo apparente est

$$\Phi = \frac{\pi - \pi' + \pi''}{m+1} \ ;$$

qui ut triplo fiat maior quam supra, statui debet $\pi' = -\pi$ et $\pi'' = \pi$; tum enim erit

$$\Phi = \frac{3\pi}{m+1},$$

ita ut sit

$$\pi = -\pi' = \pi'' = \frac{m+1}{3}\cdot\Phi.$$

Pro his autem litteris formulae nostrae sunt

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi} = \frac{\alpha}{b} = m, \qquad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi} = \frac{B\alpha}{c} = -m,$$

ubi substitutis valoribus ipsius π et π' habebitur

$$\frac{\mathfrak{B}(m+1)}{3} - 1 = m \text{ et } \mathfrak{B} = 3 \text{ hincque } B = -\frac{3}{2}.$$

Deinde

$$+\frac{1}{3}\mathfrak{C} + \frac{1}{3} = 1 \text{ et } \mathfrak{C} = 2 \text{ hincque } C = -2$$

sicque trium lentium postremarum distantiae focales erunt

$$\text{secundae } \mathfrak{B}b = \frac{3\alpha}{m}, \text{ tertiae } \mathfrak{C}c = \frac{3\alpha}{m}, \text{ quartae } d = \frac{3\alpha}{m}$$

ita ut hae tres lentes fiant inter se aequales; distantiae vero determinatrices erunt

$$b = \frac{\alpha}{m}, \ \beta = -\frac{3}{2}b = -\frac{3\alpha}{2m}, \ c = \frac{3\alpha}{2m}, \ \gamma = -2c = -\frac{3\alpha}{m}, \ d = \frac{3\alpha}{m}.$$

Substituamus hos valores in formula pro semidiametro confusionis, quae fiet

$$\frac{\mu m x^3}{4p^3}\left(\lambda + \frac{q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{r}{\mathfrak{B}^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) + \frac{\lambda'''}{B^3C^3m}\right),$$

quae ob $p = \alpha,\ q = \frac{3\alpha}{m} = r$ abit in hanc formam:

$$\frac{\mu m x^3}{4\alpha^3}\left(\lambda + \frac{1}{3m}\left(\frac{\lambda'}{9} - \frac{3\nu}{3}\right) + \frac{4}{27m}\left(\frac{\lambda''}{4} - \frac{\nu}{2}\right) + \frac{\lambda'''}{27m}\right),$$

ubi pro $\lambda',\ \lambda'',\ \lambda'''$ numeri idonei sunt quaerendi. Quia autem volumus, ut quaevis harum lentium maximam admittat aperturam, quod fit, si litteris $\pi,\ \pi',\ \pi''$ valor $=\frac{1}{4}$ tribui possit, necesse est, ut quaelibet earum sit utrinque aequaliter convexa, id quod eveniet, si statuatur

$$\sqrt{\lambda''' - 1} = \tfrac{\sigma-\rho}{2\tau},$$

$$\sqrt{\lambda'' - 1} = \tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{c-\gamma}{c+\gamma} \text{ et } \sqrt{\lambda' - 1} = \tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{b-\beta}{b+\beta}$$

ideoque

$$\sqrt{\lambda'' - 1} = -\tfrac{3}{1}\cdot\tfrac{\sigma-\rho}{2\tau} \text{ et } \sqrt{\lambda' - 1} = -\tfrac{5}{1}\cdot\tfrac{\sigma-\rho}{2\tau}.$$

Cum igitur sit, ut supra est ostensum,

$$\lambda''' = 1 + \left(\tfrac{\sigma-\rho}{2\tau}\right)^2 = 1,6299,$$

erit $\left(\tfrac{\sigma-\rho}{2\tau}\right)^2 = 0,6299$, ex quo valore colligimus

$$\lambda'' = 1 + 9\cdot(0,6299) = 6,6691$$

$$\text{et } \lambda' = 1 + 25\cdot(0,6299) \text{ seu } \lambda' = 16,7475.$$

Cum iam pro vitri specie proposita, pro qua $n = 1,55,$ sit $\mu = 0,9381$ et $\nu = 0,2326$, nunc poterimus partem assignare, quam haec lens ocularis triplicata in formulam pro confusione infert, quippe in quo cardo rei versatur.
Reperietur autem

$$\tfrac{\lambda'}{9} - \tfrac{2\nu}{3} = 1,7058 \text{ et } \tfrac{1}{3m}\left(\tfrac{\lambda'}{9} - \tfrac{2\nu}{3}\right) = \tfrac{0,5686}{m}$$

et

$$\tfrac{\lambda''}{4} - \tfrac{\nu}{2} = 1,5509 \text{ et totus terminus } = \tfrac{0,2297}{m}$$

et

$$\tfrac{\lambda'''}{27m} = \tfrac{0,0603}{m},$$

unde pars a tota lente oculari orta erit $= \tfrac{0,8586}{m}$, ita ut sit tota expressio

$$= \tfrac{\mu x^3}{4\alpha^3}\left(\lambda m + 0,8586\right);$$

sumto igitur $x = my$ positaque hac formula $\tfrac{1}{4k^3}$ determinabimus lentis obiectivae distantiam focalem $= \alpha$, ut sit

$$\alpha = kmy\sqrt[3]{0,9881(\lambda m + 0,8586)}$$

sive maius.

Intervallum porro inter lentem obiectivam et ocularem est

$$\alpha + b = \tfrac{m+1}{m}\cdot\alpha.$$

Et cum tres lentes ocularem constituentes sint inter se aequales et utrinque aequaliter convexae ob cuiuslibet distantiam focalem $=\frac{3\alpha}{m}$, radius singularum facierum erit = $=3,30\frac{\alpha}{m}$, ipsius huius lentis triplicatae distantia focali existante $=\frac{\alpha}{m}$ et semidiametro aperturae $=\frac{3}{4}\cdot\frac{\alpha}{m}$. Pro distantia oculi autem post hanc lentem reperitur $O=\frac{\pi''s}{m\Phi}$, quae ob $\pi''=\frac{m+1}{3}\cdot\Phi$ et $s=\frac{3\alpha}{m}$ fiet

$$O=\frac{m+1}{m}\cdot\frac{\alpha}{m}$$

prorsus ut ante; at campi apparentis semidiameter erit

$$\Phi=\frac{3\cdot 859}{m+1}\text{ minut.}=\frac{2577}{m+1}\text{ minut.}$$

COROLLARIUM 1

221. Circa lentem obiectivam hic nihil definivimus et ea pro lubitu sive simplex sive duplicata sive triplicata sive etiam quadruplicata statui potest, atque etiam regulae constructionis manent eaedem ut ante, dummodo quantitas α ex formula hic data definiatur.

COROLLARIUM 2

222. Eodem etiam modo quo ante ostendi potest haec telescopia non magis margini colorato esse obnoxia quam praecedentia; neque enim duabus lentibus, ad quem casum omnia haec telescopia referre licet, margo coloratus tolli potest.

SCHOLION

223. Simili modo etiam lens ocularis quadruplicari posset, ita ut semidiameter campi quadrupla maior redderetur; at hanc investigationem non ulterius prosequor, quoniam, si plures lentes adhibere velimus, iis insuper alia commoda telescopiis induci possunt, quemadmodum in sequentibus docebimus. Hic scilicet tantum simplicissimam horum telescopiorum speciem sumus contemplati, quae non nisi duabus lentibus, altera obiectiva, altera oculari, constare est censenda, etiamsi pro utraque lentibus compositis uti liceat; quin etiam ambas has lentes ex eadem vitri specie factas assumsimus atque etiam in sequente capite unicam vitri speciem adhibebimus, ut intelligatur, ad quemnam perfectionis gradum haec telescopia evehi queant, ante quam vitri species diversas in subsidium vocemus. Probe enim distinguendae sunt eae perfectiones, quae unica vitri specie obtineri possunt, ab iis, quae diversas species postulant; quo pacto ista tractatio magis perspicua reddetur. Hic autem adhuc meminisse oportet, qua ratione haec instrumenta ab alio insigni incommodo liberare conveniat, quod in eo consistit, quod

saepenumero etiam radii peregrini, qui scilicet non ab obiecto spectando sunt profecti, in tubum intrent atque visionem non mediocriter perturbent. Quemadmodum igitur tales radii peregrini arceri debeant, in sequenti problemate ostendemus.

PROBLEMA 7

224. *Constructis his lentibus ac tubo insertis radios peregrinos, qui per lentem obiectivam in tubum ingrediuntur, arcere, ne in oculum incidant et visionem turbent.*

SOLUTIO

Hunc in finem quandoque solet tubus aliquantillum divergens lenti obiectivae praefigi, ut radii a lateribus advenientes intercipiantur; simul vero haec divergentia tanta esse debet, ut radiorum ab obiecto versus lentem obiectivam emissorum nulli excludantur; id quod fit, si divergentia semidiametro campi fiat aequalis. Interim tamen hoc modo non omnes radii alieni ab introitu in obiectivam arcentur; quare, ne iis parietes tubi intus illuminentur, necesse est, ut tubi interna superficies ubique colore nigro obducatur, quod etiam de tubo praefixo est intelligendum. Neque tamen hoc prorsus sufficit, cum etiam color nigerrimus cuiuspiam illuminationis sit capax, atque ob hanc causam diaphragmata seu septa his tubis inseri solent, pertusa foraminibus, quae maiora esse non debent, quam transitus radiorum ad visionem necessariorum postulat, id quod commodissime fiet in ipso loco imaginis $F\zeta$, ubi omnes isti radii in spatium arctissimum sunt redacti. In hoc ergo loco huiusmodi diaphragma seu orbis circularis pariter nigerrimus constituatur, cuius foramen praecise sit aequale magnitudini imaginis, quam oculo cernere licet, hocque modo radiis peregrinis omnis accessus ad lentem ocularem praecludetur, et si qui forte eo pertingant, non ita refringentur, ut in oculum ingredi possint.

COROLLARIUM 1

225. Ad quantitatem huius foraminis definiendam consideretur semidiameter campi apparentis Φ, et cum semidiameter imaginis $F\zeta$ sit $\alpha\Phi$, hic simul capiatur pro semidiametro foraminis.

COROLLARIUM 2

226. Quo maior ergo fuerit campus apparens, eo maiore opus erit foramine, quo diaphragma pertundatur. Ita in exemplo ultimo § 219 allato, cum sit $\alpha = 12$ dig. et $\Phi = 17$ min. 43 sec. seu in partibus radii $\Phi = \frac{1}{194}$, semidiameter istius foraminis debet esse $\frac{6}{97}$ dig. sive circiter $\frac{1}{16}$ dig., ita ut eius diameter adaequet $\frac{1}{8}$ dig.

SCHOLION

227. In tubis astronomicis ad hoc genus referendis hoc ipsum diaphragma etiam micrometro sive filis tenuissimis per hoc spatium dispositis instrui solet, quae, cum in ipso loco imaginis sint extensa, cum ea se quasi confundunt et oculo aeque distincte atque ipsa imago repraesentabuntur; unde astronomi veram quantitatem obiecti distantiamque eius partium diiudicare solent.