CHAPTER II

CONCERNING THE FURTHER PERFECTION OF TELESCOPES FOLLOWING THE USE OF A SINGLE KIND OF GLASS.

PROBLEM 1

228. *If a new lens may be put in place of the image itself between the objective lens and the eyepiece, to inquire into the suitability by which the telescope may be improved by its aid.*

SOLUTION

Therefore since we have the case of three lenses, the magnification m will be produced at once $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$ where, since there must be an interval between the first and second lens $=\alpha$, there becomes $b=0$ and thus $\beta=Bb=0$, unless perhaps $B=\infty$. But so that we may be able to define the value of B, we shall introduce the focal length of the second lens into the calculation, which shall be $=q$, thus so that now we shall have $q=\frac{b\beta}{b+\beta}$, from which equation we will deduce $\beta=\frac{bq}{b-q}=0$, from which we attend to with the value of the letter B, evidently $B=\frac{\beta}{b}=-1$ and hence $\mathfrak{B}=\infty$. Therefore since both b as well as $\beta=0$, thus yet, so that there shall be $\frac{\beta}{b}=-1$, there will be $m=\frac{\alpha}{c}$; and thus $c=\frac{\alpha}{m}$, where c denotes the focal length of the eyepiece lens.

With these observed the radius of confusion will be

$$\frac{\mu m x^3}{4p^3}\left(\lambda+0+\frac{\lambda'}{m}\right),$$

[See Book I, Ch. I, § 41 onwards for Euler's initial treatment of defects produced by a lens, etc., & likewise Ch. II for the defects produced by several lenses.]

thus so that the middle lens clearly may add nothing to this confusion and likewise it shall be the case that any figure may be attributed to this lens. Then for the apparent [*i.e.* object] field of view we will have its radius :

$$\Phi=\frac{\pi-\pi'}{m+1},$$

where the value of π is defined by this formula :

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b};$$

from which so that likewise it may be concluded, in place of b we may introduce the focal length of the second lens q, and since there shall be $q=\mathfrak{B}b$, there will be $b=\frac{q}{\mathfrak{B}}$, which value will provide us with this equation :

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha\mathfrak{B}}{q}$$

or, on account of $\mathfrak{B}=\infty$,

$$\frac{\pi}{\Phi}=\frac{\alpha}{q} \quad \text{or} \quad \pi=\frac{\alpha\Phi}{q},$$

where finally it is required to note that the value of π must not exceed $\frac{1}{4}$. But with this value π allowed for the apparent field there will be :

$$\Phi=\frac{-\pi' q}{(m+1)q-\alpha}$$

and hence

$$\pi=\frac{-\alpha\pi'}{(m+1)q-\alpha};$$

whereby, if we may put $-\pi'=\frac{1}{4}$, also

$$\pi=\frac{+\frac{1}{4}\alpha}{(m+1)q-\alpha}$$

cannot be greater than $\frac{1}{4}$; if therefore also we may put $\pi=\frac{1}{4}$, we come upon this new determination :

$$1=\frac{\alpha}{(m+1)q-\alpha}$$

or

$$(m+1)q=2\alpha \quad \text{and} \quad q=\frac{2\alpha}{m+1}.$$

But if in the formula $\pi=\frac{-\alpha\pi'}{(m+1)q-\alpha}$ the fraction $\frac{\alpha}{(m+1)q-\alpha}$ were greater than one, then the value for $-\pi'$ will be required to be written less than $\frac{1}{4}$, so that $\pi=\frac{1}{4}$ may be produced ; but then a smaller field of view is going to arise, as if also $-\pi'$ may be $\frac{1}{4}$. From which we conclude, ether the fraction $\frac{\alpha}{(m+1)q-\alpha}$ shall be greater or less than one, in each case there shall become

$$\Phi<\frac{\frac{1}{4}+\frac{1}{4}}{m+1}$$

and in the single case $\frac{\alpha}{(m+1)q-\alpha}=1$ to be able to become

$$\Phi = \frac{1}{2(m+1)},$$

which value is twice as great as the case of two simple lenses. Yet meanwhile at this stage we may define nothing with regard to the quantity q , but rather we may see, whether the colored margin may be able to be destroyed, which will happen, if there were

$$0 = \frac{\pi b}{\Phi p} - \frac{\pi'}{m\Phi} \text{ or } 0 = 0 + \frac{(m+1)q-\alpha}{mq},$$

from which there follows $q = \frac{\alpha}{m+1}$, from which it is apparent the quantity q certainly can be assumed thus, so that the colored margin may be completely destroyed, which determination is required to be preferred by far.

[Thus, Euler still believed that he could remove a physical phenomenon by a mathematical transformation, even though it had been established how this was to be done experimentally, using an achromatic doublet with lenses of differing refractive indices, as designed by Dollond and others; see for example, A.C. King, *The History of the Telescope* (Dover).]

Therefore with $q = \frac{\alpha}{m+1}$ put for the apparent field, there shall become $\pi = \infty \cdot \pi'$ or $\pi' = \frac{\pi}{\infty}$; whereby, since π shall not be taken greater than $\frac{1}{4}$, there will become $\pi' = 0$, thus so that in this case clearly the eyepiece lens may add nothing to the field of view, certainly which will depends especially on the middle lens, and there will be

$$\Phi = \frac{1}{4(m+1)} \text{ or } \Phi = \frac{859}{m+1} \text{ minutes;}$$

then truly the distance for the position of the eye from the eyepiece lens will be produced

$$O = \frac{-\pi' r}{m\Phi} = 0,$$

or the eye will be required to be applied immediately to the eyepiece. Therefore the construction of this kind of telescope will itself thus be had.

Thus initially the focal length α is required to be defined, so that there shall be

$$\alpha = kmy\sqrt[3]{\mu(\lambda m + \lambda'')}$$

evidently on taking $x = my$, and λ is defined from the form of the objective lens, whether it were simple or multiple, as has been established in the preceding chapter. But about the second lens it is required to be considered, since the whole field of view depends on the fact that each side must be formed equally convex, so that it may be able for $\pi = \frac{1}{4}$ to be established; whereby, since for that there shall be $q = \frac{\alpha}{m+1}$, the radius of each face will be $= 1,10 \cdot \frac{\alpha}{m+1}$; but for the third eyepiece lens, since its aperture clearly is not present in the calculation, likewise there is, since it may be attributed to the figure itself, provided that a minimum aperture may be able to be taken, which at least shall be equal to the pupil. Therefore it may be agreed to put in place $\lambda''=1$, so that the distance α may be able to be taken smaller, and its figure will be able to be elaborated on following that prescribed given above.

COROLLARY 1

229. It will be seen to be wonderful, how the middle lens placed at the position of the image clearly may add nothing to the confusion, since still it will change the nature of the telescope so much, so that the eye therefore may be required to be applied to the eyepiece lens and with its help the colored fringe may be able to be destroyed. So that more is to be wondered at there, since this lens clearly will change neither the position of the image nor its size .

COROLLARY 2

230. Therefore with this same middle lens the diaphragm mentioned before will have to be put in place, of which the opening is taken equal to the aperture of this lens ; also why not indeed on this same lens the thinnest micrometer scale will be able to be put in place with its lines drawn, as it were, on the surface of this lens.

COROLLARY 3

231. Again we see this middle lens must be so much smaller than the eyepiece lens, since its focal length shall be $q=\frac{\alpha}{m+1}$, of this truly the focal length $=\frac{\alpha}{m}$, and nevertheless the apparent field of view to remain the same, as if we may use with a simple eyepiece lens as before.

SCHOLIUM 1

232. The introduction of this lens put in place at the position of the image thus has the greatest effect, since it may serve to remove the colored fringe. But the use of this kind of lens has been known by astronomers for some time now for another reason, since in this way they will increase the apparent field of view; but likewise they observe the inconvenience of a large lens present in there, since as if with a lens of this kind may itself disturb the image, all the smallest inequalities of the glass either from bubbles or striations left from the smoothing may be joined with the image itself and may be represented in the eyepiece magnified in the same ratio ; certainly which inconvenience there is to be avoided more, since scarcely can pieces of glass of this kind be allowed to be found, which shall be subject to no imperfections. Yet meanwhile it will not be at all difficult to distinguish these inequalities of the glass from the object itself being converted by the telescope itself in some way; then indeed it will soon be apparent, what may pertain to the object and what to the lens. But this

same inconvenience is found only, when the lens may be located at the position of the image; and as soon as this thence is moved a little, that emerges at once unchanged. Moreover I have began this investigation with this central case, so that the lens at the location of the image may constitute as it were the boundary of a lens, which will be arranged either to be closer to the objective or to the eyepiece; where thus it is agreed, that these closer to the objective to be distinguished from these referred to truly as closer to the eyepiece, just as with these lenses also, however many there were, accustomed to be attributed under the common name of the eyepiece lens, it will certainly be agreed at any rate which name to be used for these lenses closer to the objective.

SCHOLIUM 2

233. If we may not wish so much to change the colored fringe, as nevertheless we may wish to reject the increase in the apparent field of view, the case mentioned in the solution deserves all attention. Therefore we may put, as we have mentioned there, $q = \frac{2\alpha}{m+1}$, so that there may be established $\pi = -\pi' = \frac{1}{4}$, and the radius of the apparent field of view will be

$$\Phi = \frac{1}{2(m+1)} \text{ or } \Phi = \frac{1718}{m+1} \text{ minutes of arc,}$$

and both the second lens as well as the third will be required to have each side equally convex ; with this put in place the equation for the colored margin requiring to be removed will become

$$0 = \frac{m+1}{2m},$$

which with a double lens shall be less than that, which must be reduced to zero from the preceding chapter, here we arrive at this same gain, while it cannot be removed completely, yet it may become twice as small, thus so that may scarcely be observed; so that therefore if we may use common glass, for which $n = 1,55$, the limit of the focal length of the objective lens will be

$$a > kmy\sqrt[3]{0,9381(\lambda m + 1,6299)},$$

and for the position of the eye the distance is found

$$O = \frac{-\pi'}{m\Phi},$$

which on account of

$$\frac{-\pi'}{\Phi} = \frac{(m+1)q-\alpha}{q} = \frac{m+1}{2} \text{ and } r = \frac{\alpha}{m},$$

will become

$$O = \frac{m+1}{2m} \cdot \frac{\alpha}{m},$$

thus so that now the eye must be moved twice as close to the eyepiece lens as in the case of the previous chapter. But the distance of this lens from the objective so that there it is $=\frac{m+1}{m}\cdot\alpha$. From which the following construction arises.

Construction of a telescope composed from three lenses
formed from the same glass, for which $n=1,55$.

I. The objective lens may be selected as it pleases either to be simple with $\lambda=1$, or double for which $\lambda=0,1918$, or treble for which $\lambda=0,0422$, or finally quadruple for which $\lambda=-0,0102$, and thus so that α may be determined as in the preceding chapter from the focal length.

The radius of the aperture of this lens shall be $x=my$; the separation as far as to the second lens $=\alpha$.

II. The radius of each face of the second lens shall be $=1,10\cdot\frac{2\alpha}{m+1}$.

The radius of its aperture $=\frac{\alpha}{2(m+1)}$.

The distance to the eyepiece lens $\frac{\alpha}{m}$.

III. The radius of each face of the eyepiece lens $=1,10\cdot\frac{\alpha}{m+1}$

The radius of its aperture $=\frac{1}{4}\cdot\frac{\alpha}{m}$

For the position of the eye its distance from the eyepiece $O=\frac{m+1}{2m}\cdot\frac{\alpha}{m}$.

Truly the radius of the field of view $=\frac{1718}{m+1}$ minutes,

and, as now advised, the quantity α thus is required to be defined, so that there shall become

$$a>kmy\sqrt[3]{0,9381(\lambda m+1,6299)},$$

unless perhaps here a smaller value may be produced, as then as the prescribed aperture may be used; in which case the focal length from the aperture must be defined always, as we have done so far.

PROBLEM 2

234. *To establish a lens of this kind between the objective lens and the real image, by which all the confusion arising from the aperture of the lens may be removed and likewise the colored fringe may be removed, if it may be possible.*

SOLUTION

Since here again three lenses shall be required to be led into the calculation, the formula for the magnification will give $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$, where , since no real image may be given between the first and second lens, but for that which may fall between the second and third, the fraction $\frac{\alpha}{b}$ will be negative, but the fraction $\frac{\beta}{c}$ will be positive. Therefore we may put

$$\frac{\alpha}{b}=-k$$

and there will become $\frac{\beta}{c}=\frac{m}{k}$, from which we may gather

$$b=-\frac{\alpha}{k},\ \ \beta=Bb=\frac{-B\alpha}{k}\ \text{ and }\ c=\frac{k\beta}{m}=\frac{-B\alpha}{m}.$$

But the intervals, which must be positive will be $\alpha+b=\ \frac{k-1}{k}\alpha$, thus so that $(k-1)\alpha$ must be positive, and $\beta+c=-B\alpha(\frac{1}{k}+\frac{1}{m})$, and thus $B\alpha$ must be negative and hence also $\frac{B}{k-1}<0$.

From these observed we will consider the general formula

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=-k\ \text{ and thus }\ \pi=\frac{(1-k)\Phi}{\mathfrak{B}}.$$

Then truly [the object-field angle] is $\Phi=\frac{\pi-\pi'}{m+1}$; from which it is apparent, that some value π may be added to increase the field of view, there must be $\pi>0$ or $\frac{1-k}{\mathfrak{B}}>0$; but since $\frac{B}{k-1}<0,$ there will be $-\frac{B}{\mathfrak{B}}<0$ and thus $\frac{\mathfrak{B}}{B}>0$; this is evidently required, if we wish to increase the field of view. Now we will consider the equation for removing the colored fringe:

$$0=\frac{\pi b}{\Phi p}-\frac{\pi'}{m\Phi},$$

which on account of

$$p=\alpha,\ \ b=-\frac{\alpha}{k},\ \ \frac{\pi}{\Phi}=\frac{1-k}{\mathfrak{B}}\ \text{ and }\ \frac{\pi'}{\Phi}=\frac{1-k}{\mathfrak{B}}-m-1$$

will be changed into this :

$$0=\frac{(1-k)\Phi}{\mathfrak{B}}\left(\frac{1}{k}+\frac{1}{m}\right)+\frac{(m+1)}{m},$$

from which we find :

$$\mathfrak{B}=\frac{(1-k)(m+k)}{k(m+1)} \text{ and thus } B=\frac{(1-k)(m+k)}{2km-m+k^2}.$$

But there becomes from these values

$$\frac{B}{\mathfrak{B}}=\frac{k(m+1)}{2km-m+k^2} ;$$

from which it is apparent, so that also the second lens field of view may be increased, there must be $2km-m+k^2>0$, from which it is required, that there shall be $k>\sqrt{(m^2+m)}-m$ or $k>\frac{1}{2}$; therefore since $(k-1)\alpha$ must be positive, here two cases are required to be established:

I. where $\alpha>0$; then there must be $k>1$, from which there becomes

$$\mathfrak{B}=\frac{-(k-1)(m+k)}{k(m+1)} \text{ et } B=\frac{-(k-1)(m+k)}{2km-m+k^2}.$$

Now there will be

$$\pi=\frac{-(k-1)\Phi}{\mathfrak{B}}=\frac{+k(m+1)\Phi}{m+k} \text{ and } \pi'=\frac{-m(m+1)\Phi}{m+k}$$

and

$$\frac{\pi}{\pi'}=\frac{-k}{m},$$

from which it is apparent, if there may be put $-\pi'=\frac{1}{4}$, to become

$$\pi=+\frac{1}{4}\cdot\frac{k}{m}$$

Therefore both the fractions π and π' will be able to be taken unequal, unless there shall be $k=m$, with which in place there may be put $\pi=\frac{1}{4}$ and $-\pi'=\frac{1}{4}$, thus so that the field of view becomes a maximum.

But then there will become

$$b=-\frac{\alpha}{m} \text{ and } \beta=c=\frac{2(m-1)\alpha}{m(3m-1)}$$

on account of

$$\mathfrak{B}=-\frac{2(m-1)}{m+1} \text{ and } B=-\frac{2(m-1)}{3m-1},$$

thus so that now the focal length of the second lens shall be

$$\mathfrak{B}b=\frac{2(m-1)\alpha}{m(m+1)}$$

and of the third lens

$$c = \frac{2(m-1)\alpha}{m(3m-1)}$$

II. But if there shall be $\alpha < 0$, there must be $k < 1$ and also $k > \frac{1}{2}$, and the letters $\mathfrak{B}$ and B become positive. Hence there will be found

$$\frac{\pi}{\Phi} = \frac{k(m+1)}{m+k} \text{ et } \frac{\pi'}{\Phi} = \frac{k(m+1)}{m+k} - m - 1 = \frac{-m(m+1)}{m+k}$$

and thus

$$\frac{\pi}{\pi'} = \frac{-k}{m},$$

thus so that on account of $k < 1$ the letter π shall be much less than $-\pi'$; and thus the apparent field of view in this case may accept scarcely any increase.

Now finally we may consider that, on which the heart of the matter may depend, evidently the formula for the radius of confusion, which is :

$$\frac{\mu m x^3}{4\alpha^3}\left(\lambda - \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{\lambda''}{B^3 m}\right);$$

which, so that it may be reduced to zero, it is necessary that $\mathfrak{B}$ shall be a positive quantity, so that the first case mentioned above cannot occur, from which it is necessary, that there shall be $k < 1$ and thus also $\alpha < 0$ and $B > 0$; from which it follows there must be taken $k > \frac{1}{2}$, thus so that k may be contained between the limits $\frac{1}{2}$ and 1 ; whereby, since in this case there shall be $\frac{B}{\mathfrak{B}} > 0$, a certain increase in the field of view also will be taken, since therefore there shall be $\pi : -\pi' = k : m$, but which scarcely will be noticeable. Therefore if the radius of confusion may be put $= 0$, there will be obtained

$$\lambda = \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{\lambda''}{B^3 m},$$

where it is to be observed the letters λ and λ'' are unable to be less than one.

So that the resolution of this equation may be seen more clearly, in the first place I have noted that there cannot be taken $k = 1$, since then the first two lenses will be in contact, while truly there will be produced $\mathfrak{B} = 0$ and $B = 0$; but if there may be put $k = \frac{1}{2}$, indeed there may become

$$\mathfrak{B} = -\frac{2m+1}{2(m+1)} \text{ and } B = 2m+1;$$

from which our distances will be

$$b = -2\alpha,\ \beta = -2(2m+1)\alpha,\ c = \frac{-2(2m+1)\alpha}{m}$$

and thus the intervals

$$\alpha + b = -\alpha \text{ and } \beta l + c = -(2m+1)\alpha\left(2 + \tfrac{1}{m}\right) = -\tfrac{(2m+1)^2}{m}\cdot\alpha;$$

which latter increases greatly in length, unless $-\alpha$ may be taken exceedingly small, but which cannot happen, since the aperture of the first lens is itself defined on account of the clarity; from which it is evident the number *k* must be taken between the limits 1 and $\frac{1}{2}$.

COROLLARY 1

235. Therefore in this manner a twofold perfection may be able to be acquired, the one, where the colored margin may be destroyed completely, the other truly, where the confusion arising from the aperture is reduced to zero. Nor truly can any perceived increase be able to be added to the apparent field of view.

COROLLARY 2

236. So that it may be extended to the apertures of these lenses, for the first indeed the radius will be $x = my$; moreover for the second $\pi q \pm \frac{qx}{\mathfrak{B}p}$ (§ 23), or

$$\frac{-(1-k)(m+k)x\alpha}{k^2(m+1)} + \frac{x}{k};$$

but since there is $\pi = -\frac{k}{m}\cdot\pi'$, there can be taken $\pi' = -\frac{1}{4}$, since both sides of the eyepiece lens may become equally convex, there will be $\pi = \frac{k}{4m}$, and thus the radius of the second lens

$$= \frac{-(1-k)(m+k)\alpha}{4mk(m+1)} + \frac{x}{k}$$

as if the first part of which may vanish before the latter part, thus so that it may suffice to be putting this radius in place $= \frac{x}{k}$, which certainly is greater than *x*, on account of
$k < 1$.

But each side of the eyepiece lens must be equally convex, from which, since its focal length shall be

$$c = \frac{-(1-k)(m+k)\alpha}{m(2km-m+k^2)},$$

its fourth part will give the radius of the aperture.

COROLLARY 3

237. Moreover, since it may extend past the eyepiece lens to the position of the eye, of which the distance is found

$$O = \frac{-\pi' r}{m\Phi};$$

but since there shall be

$$\frac{-\pi'}{\Phi} = \frac{m(m+1)}{m+k} \quad \text{and} \quad r = \frac{-(1-k)(m+k)\alpha}{m(2km-m+k^2)},$$

there will be

$$O = \frac{-(1-k)(m+1)\alpha}{m(2km-m+k^2)},$$

which is a positive quantity.

SCHOLION 1

238. Certainly the effort is especially laborious, if we may wish to find these values for B and $\mathfrak{B}$ going to be substituted into the final equation, and thence to find the numbers λ and λ', and thus we may be required to undertake the calculation again for any magnification, for which an inconvenient remedy is required to be asked. Therefore we may consider these so complicated values for $\mathfrak{B}$ and B required to be elicited from the equation for removing the fringe, as clearly that equation shall be satisfied with the greatest rigor; but since it is superfluous that this equation may be fulfilled most perfectly, therefore so that the place of the eye may not be apparent ad the middle distance and the colored margin by its change may appear to be exceedingly small, if which perhaps is observed, it will be avoided most easily, it will suffice for that to be satisfied approximately ; whereby, since m may denote a large enough number, but k shall be less than unity, k will be able to be ignored besides m and m may be seen as if infinite ; from which we arrive at these values

$$\mathfrak{B} = \frac{(1-k)}{k}, \qquad B = \frac{1-k}{2k-1},$$

and thus with which we may make use in the setting out of our problem ; moreover from these our simpler elements are expressed thus:

$$b = -\frac{\alpha}{k}, \ \beta = \frac{-(1-k)\alpha}{k(2k-1)}, \ \text{and} \ c = \frac{-(1-k)\alpha}{m(2k-1)};$$

hence the interval

$$\alpha + b = \frac{-(1-k)\alpha}{k} \ \text{and} \ \beta + c = \frac{-(1-k)}{2k-1}\left(\frac{1}{k} + \frac{1}{m}\right)\alpha = \frac{-(1-k)(k+m)\alpha}{(2k-1)km}$$

and for the position of the eye:

$$O=\frac{-(m+1)(1-k)}{m(2k-1)}\cdot\frac{\alpha}{m}.$$

Moreover the focal lengths of these lenses will be

$$p=\alpha,\quad q=\frac{(1-k)\alpha}{k^2},\quad r=c=\frac{-(1-k)\alpha}{m(2k-1)}$$

and the radii of these apertures will be

$$\text{first } x=my,\quad \text{second } \tfrac{x}{k}=\tfrac{my}{k},\quad \text{third}=\tfrac{1}{4}r=\frac{-(1-k)\alpha}{4m(2k-1)}.$$

Finally the radius of the apparent field of view will be

$$\Phi=\frac{\frac{1}{4}\left(1+\frac{k}{m}\right)}{m+1}\quad\text{or}\quad \Phi=859\left(\frac{m+k}{m(m+1)}\right)\text{ minutes of arc.}$$

But now the equation requiring to be resolved at this stage will be

$$\lambda=\frac{1}{1-k}\left(\frac{\lambda' k^2}{(1-k)^2}+\frac{\nu(2k-1)}{1-k}\right)+\frac{\lambda''(2k-1)^3}{(1-k)^3 m}$$

or,

$$\lambda=\frac{1}{(1-k)^3}\left(\lambda' k^2+\nu(1-k)(2k-1)+\frac{\lambda''(2k-1)^3}{m}\right).$$

Therefore nothing else remains, except that we may resolve this equation for certain values of k, where it is required to observe λ'' must be put $=1,6299$, if indeed we wish to use common glass, for which $n=1,55$; in which case also $\nu=0,2326$.

EXAMPLE 1

239. We may put $k=\frac{3}{4}$, so that it may be held half way between its limits 1 and $\frac{1}{2}$, and our equation requiring to be resolved will lead to this form :

$$\lambda=64\left(\tfrac{9\lambda'}{16}+\tfrac{1}{8}\nu+\tfrac{\lambda''}{8m}\right)\text{ or }\lambda=36\lambda'+8\nu+\frac{8\lambda''}{m},$$
$$\lambda=36\lambda'+1,8608+\frac{13,0392}{m};$$

since now λ' cannot be less than one, we may put $\lambda'=1$ and there will become

$$\lambda=37,8608+\frac{13,0392}{m},$$

which value since it shall be so large, at no time can it be hoped any craftsman be able to prepare a lens of this kind ; from which this kind of telescope may be agreed to be set aside.

EXAMPLE 2

240. So that we may avoid such large number, we may assume $k=\frac{3}{5}$, so that there shall become $1-k=\frac{2}{5}$ and $2k-1=\frac{1}{5}$, and our equation will become

$$\lambda=\frac{125}{8}\left(\frac{9\lambda'}{25}+\frac{2}{25}\nu+\frac{\lambda''}{125m}\right),\ \lambda=\frac{45}{8}\lambda'+\frac{5}{4}\nu+\frac{\lambda''}{8m};$$

therefore on taking $\lambda'=1$ there will be

$$\lambda=5{,}9157+\frac{0{,}2037}{m},$$

which value even if still large in practice will be able to be tolerated. Meanwhile it will be agreed to define conveniently for this particular value $k=\frac{3}{5}$:

$$b=-\frac{5\alpha}{3},\quad \beta=-\frac{10\alpha}{3},\quad c=-\frac{2\alpha}{m}.$$

Hence the intervals $\alpha+b=-\frac{2\alpha}{3}$ and $\beta+c=-\alpha\left(\frac{10}{3}+\frac{2}{m}\right)$ and for the radius of the aperture of the first lens $=x$, of the second $=\frac{5}{3}x$, and of the third $=-\frac{\alpha}{2m}$, and the distance of the eye past the lens

$$O=-\frac{2(m+1)}{m}\cdot\frac{\alpha}{m}.$$

SCHOLIUM 2

241. If we may wish to establish cases of this kind for various magnifications, from the above it is understood only two cases suffice, so that thence the general formulas for whatever magnification may be able to be deduced, while evidently for one we may assume for m some moderately large number such as 20 , truly the other number as if infinite ; which investigation may be seen to be worthy of all attention, that we may establish in the following problem.

PROBLEM 3

242. *In the case of the preceding problem, if there may be taken* $k=\frac{5}{9}$, *to construct a telescope with some magnification greater than m, in which not only the colored fringe may vanish, but also the confusion arising from the aperture may be reduced to zero.*

SOLUTION

Since here there shall be $k=\frac{5}{9}$, there will be [$\mathfrak{B}=\frac{-(k-1)(m+k)}{k(m+1)}$ and $B=\frac{-(k-1)(m+k)}{2km-m+k^2}$.]

$$\mathfrak{B}=\frac{4(9m+5)\cdot 9}{9\cdot 9\cdot 5(m+1)}=\frac{4(9m+5)}{45(m+1)},\quad B=\frac{4(9m+5)\cdot 9^2}{9\cdot 9(9m+25)}=\frac{4(9m+5)}{9m+25}.$$

Now therefore we may establish two cases, in the former of which there shall be $m=20$, in the latter there shall be truly $m=\infty$.

I. On account of $m=20$ there will be $\mathfrak{B}=\frac{148}{189}$ and $B=\frac{148}{41}$; from which our equation, which is

$$\lambda=\frac{9\lambda'}{5\mathfrak{B}^3}+\frac{9v}{5\mathfrak{B}B}+\frac{\lambda''}{B^3 m},$$

will adopt the following form, if all the lenses shall be from the common glass, for which $n=1,55$ and $v=0,2326$, moreover we assume each side of the eyepiece lens to be equally convex, so that there shall be $\lambda''=1,6299$; with the aid of the named logarithms

$$\text{Log.}\mathfrak{B}=9,8937999,\quad \text{Log.}B=0,5574778$$

and hence

$$\text{Log.}\tfrac{1}{\mathfrak{B}}=0,1062000,\quad \text{Log.}\tfrac{1}{B}=9,4425221$$

and

$$\text{Log.}\tfrac{9}{5}=0,2552725:$$

$$\lambda=3,74861\lambda'+0,14812+0,00173;$$
$$\text{Log. }3,7486\lambda'=0,5738728+\text{Log.}\lambda'.$$

Here with regard to the number λ' it will help to have noted, so that, since the second lens may have a maximum aperture, of which evidently the radius shall be $\frac{9}{5}x=\frac{9}{5}\cdot my$, it may extricate this lens to render each side of this lens equally convex ; on account of this reason it is required to put in place

$$\sqrt{(\lambda'-1)}=\frac{\sigma-\rho}{2\tau}\cdot\frac{\beta-b}{\beta+b}=\frac{\sigma-\rho}{2\tau}\cdot\frac{B-1}{B+1}$$

and hence

$$\lambda'=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\cdot\left(\frac{B-1}{B+1}\right)^2.$$

But since there may be agreed to be $\left(\frac{\sigma-\rho}{2\tau}\right)^2=0,6299$, there will be

$$\lambda' = 1 + 0,6299 \cdot \left(\tfrac{107}{189}\right)^2 \quad \text{or} \quad \lambda' = 1,20189$$

and hence

$$\lambda = 4,50544 + 0,14812 + 0,00173, \quad \lambda = 4,65529.$$

From which there becomes:

$$\lambda - 1 = 3,65529 \quad \text{and} \quad \tau\sqrt{\lambda - 1} = 1,7304.$$

Therefore for the formation of this lens we will have

$$F = \tfrac{\alpha}{\sigma \pm 1,7304} = \tfrac{\alpha}{-0,1030} = -9,7087\alpha,$$
$$G = \tfrac{\alpha}{\sigma \mp 1,7304} = \tfrac{\alpha}{+1,9211} = +0,52053\alpha,$$

of which the radius of the aperture of the lens must be $x = my$. But the distance of the second lens from this is

$$\alpha + b = -0,8\alpha\ .$$

But for the second lens, since its focal length shall be

$$q = \mathfrak{B}b = -\tfrac{9}{5}\mathfrak{B}\alpha = -1,4095\alpha,$$

the radius of the face of each side will be $= 1,10q = -1,5504\alpha$, the radius of which aperture $= \tfrac{9}{5}x = \tfrac{9}{5} \cdot my$. Moreover from this lens to the third the distance is

$$\beta + c = -B\alpha\left(\tfrac{m+k}{mk}\right) = -6,4975\alpha - 36097 \cdot \tfrac{\alpha}{m} = -6,6780\alpha.$$

For the third lens, of which the focal length is

$$c = -\tfrac{148}{41} \cdot \tfrac{\alpha}{m} = -3,6097 \cdot \tfrac{\alpha}{m},$$

the radius of each face $= 1,1c = -\ 3,9707 \cdot \tfrac{\alpha}{m}$; and hence the distance as far as to the eye

$$O = -\tfrac{(m+1)B\alpha}{(m+k)m} = -\tfrac{4 \cdot 9 \cdot 21}{5 \cdot 41} \cdot \tfrac{\alpha}{m} = -3,6878 \cdot \tfrac{\alpha}{m}.$$

II. Now there shall become $m = \infty$, there will be $\mathfrak{B} = \tfrac{4}{5}, B = -4$, from which our equation adopts this form :

$$\lambda = \tfrac{225}{64}\lambda' + \tfrac{9}{16}\nu.$$

Here again we may render the following lens equally convex and on account of $\beta = Bb = 4b$ there will become:

$$\sqrt{(\lambda' - 1)} = \tfrac{\sigma-\rho}{2\tau} \cdot \tfrac{\beta-b}{\beta+b} = \tfrac{\sigma-\rho}{2\tau} \cdot \tfrac{3}{5}$$

Hence $\lambda' = 1 + 0,6299 \cdot \tfrac{9}{25}$; $\lambda' = 1,2267$, from which we deduce

$$\lambda = 4,3126 + 0,1308 = 4,4434$$

and hence

$$\lambda - 1 = 3,4434 \text{ and } \tau\sqrt{(\lambda - 1)} = 1,6796;$$

whereby the following construction is had:

I. For the first lens:

$$F = \tfrac{\alpha}{\sigma \pm 1{:}6796} = \tfrac{\alpha}{-0,0522} = -19,1570\alpha,$$
$$G = \tfrac{\alpha}{\sigma \mp 1,6796} = \tfrac{\alpha}{+1,8703} = +\ \ 0,5346\alpha.$$

The aperture is as before and equally the distance to the second lens.

II. For the second lens,

along with its focal length $= -\tfrac{9}{5}\mathfrak{B}\alpha = -\tfrac{36}{25} \cdot \alpha = -1,44\alpha$,

the radius of each face will become $= -1,584\alpha$, and the radius of its aperture $= \tfrac{9}{5}x$;

but the distance to the third lens $= -7,2\alpha - 4 \cdot \tfrac{\alpha}{m}$.

III. For the third lens,

of which the focal length $= -4 \cdot \tfrac{\alpha}{m}$,

the radius of each face $= -4,4 \cdot \tfrac{\alpha}{m}$,

and the radius of its aperture $= -1,1 \cdot \tfrac{\alpha}{m}$.

The distance from this lens as far as to the eye will be $O = -4 \cdot \tfrac{\alpha}{m}$.

Thus from these two cases established we may add on the solution of our general question for some multiple *m* greater than 20 :

I. For the first lens,

we may put in place the radius of the $\begin{cases}\text{anterior face} & = -\left(19,1570+\frac{f}{m}\right)\alpha \\ \text{posterior face} & = +\left(\ \ 0,5346+\frac{g}{m}\right)\alpha\end{cases}$

and with the application made to the case $m = 20$ there will be found:

$$19,1570+\tfrac{f}{20} = 9,7087, \text{ from which there becomes } f = -188,966;$$

again,

$$0,5346+\tfrac{g}{20} = 0,5205, \text{ hence } g = -0,2820.$$

II. For the second lens,

the radius of each face may be put in place $= -\left(1,584+\frac{h}{m}\right)\alpha$, and since there may become $1,584+\frac{h}{20} = 1,5504$, there will become $h = -0,6720$.

The focal length of this being

$$= -\left(1,440-\tfrac{0,6100}{m}\right)\alpha,$$

and the radius of the aperture $= \frac{9}{5}x$.

We will find for the distance to the third lens

$$-\left(7,2-\tfrac{14,0500}{m}\right)\alpha-\left(4,00-\tfrac{7,8060}{m}\right)\tfrac{\alpha}{m}$$

either

$$-7,200\alpha+10,0500\cdot\tfrac{\alpha}{m}+\tfrac{7,8060}{m^2}\alpha,$$

or

$$-\left(7,200-\tfrac{10,0500}{m}-\tfrac{7,8060}{mm}\right)\alpha.$$

III. For the third lens,

of which the focal length is found $= -\left(4,00-\frac{7,8060}{m}\right)\frac{\alpha}{m}$,

the radius of each face must be taken $= -\left(4,400-\frac{8,5860}{m}\right)\frac{\alpha}{m}$,

of which the fourth part can be put equal to the radius of the aperture.

Finally the distance of the eye from this lens is found :

$$O = -\left(4-\tfrac{6,2440}{m}\right)\tfrac{\alpha}{m}.$$

Truly the radius of the apparent field will be $\frac{859}{m+1}$ minutes of arc.

COROLLARY 1

243. Since the interval between the first and second lens shall be $=-0,8\alpha$, the whole length of the telescope from the objective as far as to the eye will be produced:

$$-\left(8-\frac{6,0500}{m}-\frac{14,0500}{mm}\right)\alpha,$$

thus so that this length shall be almost eight times greater than the focal distance α, by which circumstances these telescopes do not merit to be recommended highly.

COROLLARY 2

244. Since the radius of the aperture of the first lens must be $x=my=\frac{m}{50}$ in., but which cannot be greater than the fourth part of the lesser radius, which is

$$-0,1336\alpha=-\tfrac{1}{8}\alpha \text{ roughly},$$

it is apparent there must be taken $-\alpha>\frac{16m}{100}$ or $-\alpha>0,16m$.

But since the radius of the aperture of the second lens must be

$$=\tfrac{9}{5}x=\tfrac{9m}{250} \text{ in.},$$

this also must be smaller than the quarter part of the radius, which is $-0,396\alpha$; from which there must become $-\alpha>0,0909m$; which limit since it shall be less than the preceding, that will be required to be observed.

SCHOLIUM

245. Therefore since $-\alpha$ must be greater than $0,16m$, we may put $-\alpha=\frac{2}{10}m$ or $\alpha=-0,2m$ and we will obtain the following construction for telescopes of this kind,

Construction of telescopes for any magnification m, with the lenses made from common glass

I. For the objective lens in inches:

$$\text{Radius of the}\begin{cases}\text{anterior face} &=+3,8314m-37,7932\\ \text{posterior face} &=-0,1069m+0,0564.\end{cases}$$

The radius of its aperture $=\frac{m}{50}$.

The interval to the second lens $=0,16m$.

II. For the second lens in inches :

Focal length $=+0,2880m-0,12.$

Radius of each face $=+\ 0,3168m-0,13.$

Radius of its aperture $=\frac{9}{250}m=0,036m$.

Interval to the third lens $=+1,4400m-2,01-\frac{1,56}{m}.$

III. For the third lens in inches

Focal length $=+0,800-\frac{1,56}{m}.$

Radius of each face $=+\ 0,88-\frac{1,72}{m},$

of which the fourth part $=\frac{1}{5}$ dig. gives the radius of the aperture.

Hence the distance as far as to the eye will become

$$O=0,8-\frac{1,25}{m}.$$

The radius of the apparent field $=\frac{859}{m+1}$ minutes of arc.

Moreover the length of the whole telescope will be

$$=(-1,21+1,6m-\frac{2,81}{m})\text{ in.}$$

Thus for example for $m=100$ the length will be $=158\frac{3}{4}$ in. or 13 ft. $2\frac{3}{4}$ in.

Therefore since with the above tube scarcely exceeding one foot we will have produced almost as great a magnification, this kind of telescope now indeed will have to be rejected, even if it may be considered especially with respect of the common astronomical telescopes, since it may produce no colored fringe, while truly also because the confusion arising from the aperture may be removed completely. Whereby it will be agreed by us to inquire, whether with two lenses arranged between the object and the image, this inconvenience may be able to be avoided.

PROBLEM 4

246. *To insert two lenses of such a kind between the object and the image, so that not only the colored fringe, but also the confusion arising on account of the aperture, may be removed completely.*

SOLUTION

Since here four lenses shall be required to be considered, the magnification will give this formula $m=\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, of which three fractions the first two must be negative, truly the third positive. Therefore we may put in place

$$\frac{\alpha}{b}=-k,\quad \frac{\beta}{c}=-k',\ \text{ and there will become }\ \frac{\gamma}{d}=\frac{m}{kk'}.$$

From which there will become:

$$b=-\frac{\alpha}{k},\quad c=-\frac{\beta}{k'},\quad d=\frac{\gamma kk'}{m};$$

besides truly there is $\beta=Bb,\ \gamma=Cc$, from which all these elements thus may be defined from α :

$$b=-\frac{\alpha}{k},\quad \beta=-\frac{B\alpha}{k},\quad c=\frac{B\alpha}{kk'},\quad \gamma=\frac{BC\alpha}{kk'},\quad d=\frac{BC\alpha}{m};$$

hence the intervals between the lenses will become :

$$1.\ \alpha+b=\frac{k-1}{k}\cdot\alpha$$

$$2\cdot\ \beta+c=\frac{B\alpha}{k}\left(\frac{1-k'}{k'}\right)$$

$$3.\ \gamma+d=BC\alpha\left(\frac{1}{kk'}+\frac{1}{m}\right),$$

from which, since $k,\ k'$ and m are themselves positive numbers, these conditions follow:

$$1.\ \alpha(k-1)>0,\quad 2.\ B\alpha(1-k')>0,\quad 3.\ BC\alpha>0,$$

which with α removed are reduced to these two:

$$4.\ \frac{B(1-k')}{k-1}>0,\quad 5.\ \frac{C}{1-k'}>0\quad \text{or}\quad C(1-k')>0.$$

Now from the above we see that the colored fringe cannot be removed, unless the fractions $\pi,\ \pi'$ and π'' may be defined; so that in the end the following equations must be considered :

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=-k,\quad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{B\alpha}{c}=kk'\quad \text{and}\quad \Phi=\frac{\pi-\pi'+\pi''}{m+1},$$

from which there follows:

$$\frac{\pi}{\Phi}=\frac{1-k}{\mathfrak{B}},\quad \frac{\pi'}{\Phi}=\frac{1-k}{\mathfrak{B}\mathfrak{C}}+\frac{kk'-1}{\mathfrak{C}},\qquad \frac{\pi''}{\Phi}=\frac{\pi'}{\Phi}-\frac{\pi}{\Phi}+m+1=\frac{1-k}{\mathfrak{B}C}+\frac{kk'-1}{\mathfrak{C}}+m+1;$$

with which values substitutes, this equation is required to be made for the colored fringe to be removed with the division by $\frac{dn}{n-1}$:

$$0=-\frac{\pi}{\Phi}\cdot\frac{1}{k}+\frac{\pi'}{\Phi}\cdot\frac{1}{kk'}+\frac{\pi''}{\Phi}\cdot\frac{1}{m},$$

or

$$0=\frac{k-1}{\mathfrak{B}k}+\frac{1-k}{\mathfrak{B}\mathfrak{C}kk'}+\frac{kk'-1}{\mathfrak{C}kk'}+\frac{1-k}{\mathfrak{B}Cm}+\frac{kk'-1}{\mathfrak{C}m}+\frac{m+1}{m},$$

from which equation either $\mathfrak{B}$ or $\mathfrak{C}$ can be defined; then truly, so that the radius of confusion may be reduced to zero, there must become

$$0=\lambda-\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{1}{\mathfrak{B}^3\mathfrak{C}kk'}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)+\frac{\lambda'''}{B^3C^3m};$$

which must be resolved, so that the letter $\mathfrak{B}$ must become positive, or if $\mathfrak{B}$ were negative, on account of B likewise being negative, the letter $\mathfrak{C}$ must be positive.

COROLLARY 1

247. The equation for removing the colored margin is reduced to this form:

$$0=\frac{1-k}{\mathfrak{B}\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1-k}{\mathfrak{B}}\left(\frac{1}{k}+\frac{1}{m}\right)+\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}$$

or to this:

$$0=\frac{1-k}{\mathfrak{B}}\left(\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1}{k}-\frac{1}{m}\right)+\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m},$$

from which there is found:

$$\frac{k-1}{\mathfrak{B}}=\frac{\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}}{\left(\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1}{k}-\frac{1}{m}\right)}$$

or

$$\frac{k-1}{\mathfrak{B}}=\frac{(kk'-1)(m+kk')+(m+1)kk'\mathfrak{C}}{m+kk'-k'm\mathfrak{C}-kk'\mathfrak{C}},$$
$$\frac{k-1}{\mathfrak{B}}=\frac{(kk'-1)(m+kk')+\mathfrak{C}kk'(m+1)}{m+kk'-\mathfrak{C}k'(m+k)},$$
$$\frac{k-1}{\mathfrak{B}}=kk'-1+\frac{\mathfrak{C}k'\left(m(k-1)+kk'(k+m)\right)}{m+kk'-\mathfrak{C}k'(m+k)}.$$

COROLLARY 2

248. If this equation may be freed at once from fractions, there will be obtained

$$0=(1-k)(m+kk')-\mathfrak{C}(1-k)k'(m+k)+\mathfrak{B}(kk'-1)(m+kk')+\mathfrak{B}\mathfrak{C}kk'(m+1),$$

from which there is found

$$\mathfrak{C}=\frac{(m+kk')(k-1+\mathfrak{B}(1-kk'))}{\mathfrak{B}kk'(m+1)+(k-1)k'(m+k)}.$$

[Thus, one might reduce the spherical aberration for white light, or so-called image confusion by Euler, by adjusting the aperture sizes of the lenses successively which have parts of spheres as surfaces, but chromatic aberration would only be partially dealt with in this manner, by transmitting only rays close to the optical axis, for which the chromatic aberration would be less ; at the time, though Euler suspected light behaved in an analogous manner to sound in having a wave nature, with associated frequencies and wavelengths, but this had not yet been established experimentally: to be fulfilled later by Thomas Young's double slit experiment; however, an experimental method of producing almost achromatic lenses had already been established by John Dollon and others, which has not been referred to at this stage. The best approach would presumably involve application of both methods. See King's *History of the Telescope* for the details.]

SCHOLIUM

249. But initially the case is noteworthy, where the number B becomes infinite and the number $O=0$, which we have set out above now for another situation [§183]; which operation may be seen to be more difficult with the above, now may be set out more plainly in the following manner. Clearly we will consider the number B as huge and may put $B=\frac{1}{\omega}$, with ω specifying the minimum fraction, so that ω may be introduced in place of B in the calculation. Therefore then there will be now $\mathfrak{B}=\frac{1}{1+\omega}$, lest the second interval $\beta+c$ may increase exceedingly, there may be put

$$\beta+c=\frac{\eta\alpha}{k}$$

and there will become

$$c=\frac{\eta\alpha}{k}-\beta,\quad \frac{\beta}{c}=-k'=\frac{k\beta}{\eta\alpha-k\beta},$$

and since there is

$$\beta k=-B\alpha=-\frac{\alpha}{\omega},$$

there will become

$$+k'=\frac{1}{\eta\omega+1}\quad\text{and}\quad 1-k'=\frac{\eta\omega}{1+\eta\omega},$$

thus so that now the letter η may be introduced into the calculation in place of the letter k' ; finally, lest the third interval may become exceedingly great on account of

$B=\frac{1}{\omega}$, we may put $C=\theta\omega$, so that there may become $BC=\theta$; thus so that here in place of the letter C [*i.e.* the magnification produced by this lens], θ may be introduced into the calculation. Hence there will become $\mathfrak{C}=\frac{\theta\omega}{1+\theta\omega}$ [*i.e.* the radius of the ingoing aperture for this lens] and hence again [§ 246] :

$$\frac{\pi}{\Phi}=\left[\frac{1-k}{\mathfrak{B}}=\frac{1-k}{B/B+1}=\frac{1-k}{\frac{1}{\omega}/\frac{1}{\omega}+1}=\omega\left(1-k\right)\left(\frac{1}{\omega}+1\right)\right]=(1-k)+\omega(1-k),$$

$$\frac{\pi'}{\Phi}=\left[\frac{1-k}{\mathfrak{BC}}+\frac{kk'-1}{\mathfrak{C}}\right]=\frac{1+\theta\omega}{(1+\eta\omega)\theta}\left(1-k-\eta k+\eta\omega(1-k)\right)$$

and hence on putting $\omega=0$ there will become

$$\frac{\pi}{\Phi}=(1-k),\quad \frac{\pi'}{\Phi}=\frac{1}{\theta}(1-k-\eta k)\ \text{ and hence }\ \frac{\pi''}{\Phi}=\frac{1-k-\eta k}{\theta}+k+m$$

Whereby, since for the colored margin requiring to be removed this equation shall be found:

$$0=-\frac{\pi}{\Phi}\cdot\frac{1}{k}+\frac{\pi'}{\Phi}\cdot\frac{1}{kk'}+\frac{\pi''}{\Phi}\cdot\frac{1}{m},$$

on account of $k'=1$, if these values may be substituted, there will be produced,

$$0=\frac{k-1}{k}+\frac{1-k-\eta k}{k\theta}+\frac{1-k-\eta k}{m\theta}+\frac{k+m}{m}$$

or

$$0=m\theta\left(k-1\right)+m\left(1-k-\eta k\right)+k\left(1-k-\eta k\right)+k\theta\left(k+m\right),$$

$$0=\theta\left(k^2+\left(2k-1\right)m\right)+\left(k+m\right)\left(1-k-\eta k\right)$$

and hence there is found

$$\theta=\frac{+\left(k+m\right)\left(k+\eta k-1\right)}{\left(2k-1\right)m+k^2};$$

hence moreover our elements will become

$$b=-\frac{\alpha}{k},\quad \beta=\infty,\quad c=\infty,\quad \gamma=\frac{\theta\alpha}{k},\quad d=\frac{\theta\alpha}{m}$$

and the intervals

$$\alpha+b=\frac{k-1}{k}\cdot\alpha,\qquad \beta+c=\frac{\eta\alpha}{k},\qquad \gamma+d=\theta\alpha\left(\frac{1}{m}+\frac{1}{k}\right),$$

which must be positive; and thus

$$\eta(k-1)>0, \quad \theta(k-1)>0 \text{ and } \theta\eta>0.$$

But for the position of the eye we will have

$$O=\frac{\pi''}{m\Phi}\cdot d=\frac{1-k-\eta k}{mm}\cdot\alpha+\frac{(k+m)\theta\alpha}{m^2}$$

and with the value for θ substituted

$$O=\frac{1-k-\eta k}{m^2}\cdot\alpha+\frac{(k+m)^2(k+\eta k-1)\alpha}{m^2\left((2k-1)m+k^2\right)},$$
$$O=\frac{m+1}{m}\cdot\frac{k+\eta k-1}{(2k-1)m+k^2}\cdot\alpha;$$

finally from these observed this equation remains to be resolved :

$$0=\lambda-\frac{\lambda'}{k}+\frac{\lambda''}{\theta^3 k}+\frac{\lambda'''}{\theta^3 m},$$

which, since the following member by itself shall be negative, it will be able easily to be resolved, which we will show in the following problem.

PROBLEM 5

250. *In the case of the preceding problem if the two first lenses were prepared thus, so that the rays may emerge parallel again, to set out the construction of telescopes of this kind.*

SOLUTION

Since in this case there may become $B=\infty$ and $C=0$, in the preceding scholium the elements are defined thus, so that here it would be superfluous to repeat this ; but so that we may establish the solution more clearly, two cases are required to be considered; the one where the distance α is positive, the other, where that is negative.

I. Therefore there shall be $\alpha>0$ and there must become $k>1$, $\eta>0$ and $\theta>0$, which final condition is satisfied at once ; and O also shall be positive, then truly there will become

$$\frac{\pi}{\Phi}<0 \text{ and } \frac{\pi'}{\Phi}<0,$$

truly,

$$\frac{\pi'}{\Phi}=\frac{1-k-\eta k}{\theta}=\frac{-(2k-1)-k^2}{k+m}.$$

Therefore from the final formula the radius of the field of view may be deduced

$$\Phi = \frac{\pi''\theta}{1-k-\eta k+(k+m)\theta} \quad \text{or} \quad \Phi = \frac{(m+k))\pi''}{m(m+1)},$$

evidently with the value θ substituted, but only if the preceding formulas may not produce a smaller field. In order that the values π and π' required to be judged may be compared with π'' and on account of $\frac{\pi''}{\Phi} = \frac{m(m+1)}{m+k}$ there will be

$$\frac{\pi}{\pi''} = \frac{(1-k)(m+k)}{m(m+1)} \quad \text{and} \quad \frac{\pi'}{\pi''} = \frac{-(2k-1)m-k^2}{m(m+1)}$$

and hence

$$\frac{\pi-\pi'}{\pi''} = \frac{k}{m};$$

but from these formulas it is apparent both π as well as π' to be less than π'', provided k shall be less than $\frac{5}{12}m$, and since there shall be $\Phi = \frac{\pi-\pi'+\pi''}{m+1}$, on account of $\frac{\pi-\pi'}{\pi''} > 0$ the apparent field of view may be taken a little greater and there will be $\Phi = \frac{k+m}{m(m+1)}\pi''$, which certainly is greater than the simple case, evidently $\Phi = \frac{\pi''}{m+1}$, and that in the ratio $m+k:m$.

Now again the equation is required to be resolved as before.

II. But if α shall be negative, there must become $k<1$, $\eta<0$, $\theta<0$, for which it is necessary, that there shall be $k>\frac{1}{2}$.

Since now we had for the previous case $\frac{\pi-\pi'}{\pi''} = \frac{k}{m}$, hence the apparent field of view takes a much smaller increase in this case than in that, which indeed is scarcely perceptible, and the distance O for the place of the eye also in this case becomes positive; as for this reason the prior case is seen to be preferred to this of the latter.

But even for the prior case the apparent field of view has been found to be increased notably, while clearly k is increased to the value $\frac{5}{12}m$, yet does not permit the resolution of our equation, since the number λ' will have to be taken exceedingly large, on account of which the letter k scarcely can give an increase to the sum of two or three, as may be shown more clearly in the adjoining examples, which we have derived for the first case, since it is easy to foresee in the second case there the fault to be of the work involved, so that the length of the telescope may increase exceedingly.

EXAMPLE

251. We may put $k=2$ and the magnification $m=50$, since this is performed by astronomical telescopes, and there shall be

$$\theta=\frac{52(1+2\eta)}{154}=\frac{26}{77}(1+2\eta\ );$$

which value lest it may become exceedingly small, since then the term $\frac{\lambda''}{\theta^3 k}$ in our equation may not become exceedingly large, thus so that λ' may arrive at an enormous value, in addition we may put $\eta=1$, so that there may become $\theta=\frac{78}{77}$, and hence our elements may thus themselves be found:

$$b=-\frac{\alpha}{2},\quad \beta=\infty,\quad c=-\infty,\quad \beta+c=\frac{\alpha}{2},\quad \gamma=\frac{39}{77}\alpha,\quad d=\frac{39\alpha}{25\cdot 77};$$

now truly the equation requiring to be resolved is prepared thus :

$$0=\lambda-\frac{\lambda'}{2}+\frac{77^3\lambda''}{78^3\cdot 2}+\frac{77^3\lambda'''}{78^3\cdot 50}$$

and hence

$$\lambda'=2\lambda+\frac{77^3\lambda''}{78^3}+\frac{77^3\lambda'''}{78^3\cdot 25}.$$

Now in order that both the first lens as well as the final one may allow the maximum aperture, we may put $\lambda=\lambda'''=1,6299$, while clearly all the lenses are assumed to be made from the common glass $n=1,55$, but λ'' shall be $=1$; with which in place we will deduce

$$\lambda'=3,2598+0,9620+0,0628,\quad \lambda'=4,2846,$$

hence therefore

$$\lambda'-1=3,2846 \text{ and } \tau\sqrt{(\lambda'-1)}=1,64031;$$

whereby the construction of the individual lenses thus themselves will be had:

I. For the first lens

which since each face shall be equally convex and its focal length $=\alpha$, the radius of each face will be $=1,10\alpha$, then truly the radius of its aperture $x=my=1$ in. on account of $m=50$ and $y=\frac{1}{50}$in. and the interval from this lens to the second $=\frac{1}{2}\alpha$.

II. For the second lens

on account of $\beta=\infty$ there will become

$$F' = \frac{b}{\rho \pm 1,64031} = \frac{b}{1,8310},\ G' = \frac{b}{\sigma \mp 1,64031} = \frac{b}{-0,0129},$$

hence

$$F' = -0,2730\alpha,\ G' = +38,7597\alpha;$$

then truly the radius of its aperture $= \frac{1}{2}$ in. from § 23 and the distance to the following lens $= \frac{1}{2}\alpha$.

III. For the third lens

on account of $c = -\infty$ and $\lambda'' = 1$ there will become

$$F'' = \frac{\gamma}{\sigma} = 0,31123\alpha, \quad G' = \frac{\gamma}{\rho} = 2,6559\alpha,$$

then truly the radius of the aperture $= \frac{1}{2}$ in. and the distance to the following lens $\frac{78\cdot 13}{77\cdot 26}\alpha$ or $= \frac{1}{2}\alpha$ approx.

IV. For the fourth lens

the radius of each face $= 1,10d$ with $d = \frac{39}{25\cdot 77}\alpha$ being present, of which the fourth part gives the radius of the aperture; hence finally the distance as far as to the eye will be $\frac{3\cdot 51}{50\cdot 154}\alpha = \frac{1}{50}\alpha$ approx.

If we may consider the first lens alone, so that it may be restricted to the distance α, since that allows an aperture, of which the radius $= \frac{1}{4}\alpha$, there may be able to take $\alpha = 4$ in.; with regard to the second lens, of which the lesser radius $= \frac{1}{4}\alpha$, of which the fourth part $= \frac{1}{16}\alpha$, the radius of the aperture put equal to $\frac{1}{2}$ in. will give $\alpha = 8$ in., it will be required to retained also as a measure ; from which the length of the telescope will exceed 12 in. The reason for this being that we have assumed each face of the first lens to be equally convex.

Therefore in addition we may apply another solution by assuming $\lambda = 1$; from which there becomes

$$\lambda' = 2 + 0,9620 + 0,0628, \quad \lambda' = 3,0248,$$
$$\lambda' - 1 = 2,0248 \quad \text{and} \quad \tau\sqrt{(\lambda' - 1)} = 1,2879;$$

from which this construction of the lenses follows:

I. For the first lens

$$F = \frac{\alpha}{\sigma} = 0,6145\alpha, \quad G = \frac{\alpha}{\rho} = 5,2439\alpha.$$

II. For the second lens

$$F' = \frac{b}{0,1907 \pm 1,2879} = \frac{-0,5\alpha}{1,4786}, \quad G' = \frac{b}{1,6274 \mp 1,2879} = \frac{-0,5\alpha}{0,3395},$$

$$F' = -0,3382\alpha, \quad G' = -1,4723\alpha.$$

The rest remain as before. Therefore here it is apparent at once the second lens can receive the given aperture $\frac{1}{2}x$, if the first may be granted the aperture x. But the first smaller lens, since it shall be approximately $\frac{6}{10}\alpha$, of which the fourth part $\frac{3}{20}\alpha$ equated to $x = 1$ in. gives $\alpha = \frac{20}{3}$ in. $= 6\frac{2}{3}$ in.; why not also may the third lens require that there shall become $\frac{3}{40}\alpha = \frac{1}{2}$ in., from which again α becomes $6\frac{2}{3}$ in., and thus the whole length of the telescope scarcely will exceed 10 in.

Whereby the following construction of a telescope with a magnification of fifty times will deserve a mention with the lenses prepared from common glass.

I. For the first lens

$$\text{the radius of the} \begin{cases} \text{anterior face} = 4,10 \text{ in.} \\ \text{posterior face} = 34,96 \text{ in.} \end{cases}$$

Radius of the aperture $= 1$ in.
Interval to the second lens $= 3\frac{1}{3}$ in.

II. For the second lens

$$\text{The radius of the} \begin{cases} \text{anterior face} = -2,25 \text{ in.} \\ \text{posterior face} = -9,82 \text{ in.} \end{cases}$$

Radius of the aperture $= \frac{1}{2}$ in.
Interval to the third lens $= 3\frac{1}{3}$ in.

III. For the third lens

$$\text{The radius of the} \begin{cases} \text{anterior face} = 2,07 \text{ in.} \\ \text{posterior face} = 17,71 \text{ in.} \end{cases}$$

Radius of the aperture $= \frac{1}{2}$ in.
Interval to the third lens $= 3\frac{1}{3}$ in.

IV. For the fourth lens

Radius of each face $= 0,15$ in.
Radius of the aperture $= \frac{3}{80}$ in.

and distance of the eye $=\frac{2}{15}$ in. approx.,

from which the total length $=10\frac{2}{15}$ in.

Truly the radius of the field of view as thus far $\frac{859}{51}$ min. $=16$ min. 51 sec.

SCHOLION

252. Here I do not undertake the calculation of greater magnifications, since there is also a greater field of view than is commonly accustomed to be expected from telescopes of this kind. On account of which we may pursue our investigation for increasing the apparent field of view and with that suitably retained, which the three first lenses given to us generously. Hence we shall be able to use the values assumed here, evidently $k=2,\ \eta=1$ and $\theta=1$, but since in this manner the two first intervals become large enough, evidently $\frac{1}{2}\alpha$, with which agreed on the whole length may not be increased quite a lot, it is seen these two intervals to produce much less, thus only so that they may not vanish and neither the lenses may be able to touch each other. In the end a number scarcely greater than unity will be assumed for k, from which likewise we obtain this advantage, so that for λ' a number scarcely in excess of two may be found. Therefore we may put in place

$$k=1+\omega,$$

with ω denoting a small fraction, thus so that there shall become

$$b=-\tfrac{\alpha}{1+\omega}=-(1-\omega)\alpha,\quad \alpha+b=\omega\alpha;$$

on account of which by the same reasoning there may be established also $\eta=\omega$, so that the second interval also may become $\omega\alpha$. So that then it may pertain to the letter θ, which here has been defined from the color margin, from these hypothetical facts it may be going to be produced much smaller than unity, evidently $\theta=2\omega$, which might be subject to the maximum inconvenience ; indeed in the first place the elements γ and d will vanish, unless α may be increased greatly, then also the value of λ' itself will be huge. But here it is required to observe properly this hypothesis is not to be treated for the case where a single eyepiece lens is allowed to be resolved, but to be proposed by us to be used in the following, where two or more eyepiece lenses may be introduced ; from which, since new letters may be introduced into the calculation, no longer will there be a need to define this letter θ from the equation for removing the colored margin, but we will be able to consider that as arbitrary, thus so that now nothing stands in the way why there may not be put $\theta=1$. But because I have chosen this value, there are two provisos : the one is because, since the distance γ here shall be $\theta\alpha$, if θ were increased beyond unity, the length of the telescope shall be going to be produced longer; but the other proposes, lest θ may be taken less than unity, since then λ' soon may be going to obtain an enormous value ; therefore if the reckoning may be put in place

$$1.\ k = 1+\omega,\quad 2.\ \eta = \omega,\quad 3.\ \theta = 1;$$

so that, however many lenses may be used, for the first three there will be always

$$b = -(1-\omega)\alpha,\ \beta = \infty,\ c = -\infty,\ \beta + c = \omega\alpha,\ \gamma = \alpha.$$

Then for the letters π and π' there will be always also

$$\tfrac{\pi}{\Phi} = -\omega,\quad \tfrac{\pi'}{\Phi} = -2\omega$$

moreover there may be noted to be

$$B = \infty,\ \mathfrak{B} = 1,\ \ C = \mathfrak{C} = 0 \text{ and } BC = 1$$

and hence the equation for removing the margin will begin always from these two terms $+\omega - 2\omega$; thus so that these two terms always may merge into $-\omega$. Finally also the equation for the confusion being removed always begins from these three terms

$$0 = \lambda - \tfrac{\lambda'}{1+\omega} + \tfrac{\lambda''}{1+\omega} \cdots\cdots,$$

from which the calculation will follow for any number of lenses, where chiefly for us will be to magnify the proposed field of view, and that as far as it will have pleased.

PROBLEM 6

253. *With the first three lenses disposed thus before a real image, as indicated in the previous paragraph, if after the image two lenses may be put in place, in order to effect that the apparent field of view may emerge a maximum.*

SOLUTION

Since here there may be had five lenses, the formula for the magnification will be

$$m = -\tfrac{\alpha}{b} \cdot \tfrac{\beta}{c} \cdot \tfrac{\gamma}{d} \cdot \tfrac{\delta}{e},$$

of which the fraction $\tfrac{\gamma}{d}$ will be positive, the rest negative. Since there shall be

$$\tfrac{\alpha}{b}=-1-\omega, \quad \tfrac{\beta}{c}=-1,$$

there may be put

$$\tfrac{\gamma}{d}=i \text{ and } \tfrac{\delta}{e}=-l,$$

and we will have the following elements:

$$b=-\tfrac{\alpha}{1+\omega}, \ \beta=-\infty, \ c=\infty, \ \gamma=\alpha, \ d=\tfrac{\alpha}{i}, \ \delta=\tfrac{D\alpha}{i}, \ e=-\tfrac{D\alpha}{il},$$

with m being $=(1+\omega)il$.

The focal lengths will be

$$p=\alpha, \ q=b, \ r=\gamma, \ s=\mathfrak{D}d \text{ and } t=e.$$

Again the separation of the lenses

$$\alpha+b=\omega\alpha, \ \beta+c=\omega\alpha, \ \gamma+d=\tfrac{1+i}{i}\alpha, \ \delta+e=D\left(\tfrac{l-1}{il}\right)\alpha,$$

of which the first three by themselves shall be positive, and it remains only that $D(l-1)$ shall be positive.

We will now have for the fractions $\pi, \ \pi'$ etc.

$$\tfrac{\pi}{\Phi}=-\omega, \quad \tfrac{\pi'}{\Phi}=-2\omega$$

and thus

$$\tfrac{\pi-\pi'}{\Phi}=\omega.$$

Truly we will have these equations for the two remaining :

$$\tfrac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\tfrac{\alpha}{d}=i,$$
$$\tfrac{\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\tfrac{D\alpha}{e}=-il=-m.$$

From which there is deduced:

$$\tfrac{\pi''}{\Phi}=\tfrac{1+i-\omega}{\mathfrak{D}}=\tfrac{1+i}{\mathfrak{D}},$$
$$\tfrac{\pi'''}{\Phi}=\tfrac{1+i}{\mathfrak{D}}+\omega-1-m.$$

From which for the position of the eye we will have

$$O=-\tfrac{\pi'''}{m\Phi}\cdot t=\left(\tfrac{1+i}{\mathfrak{D}}+\omega-1-m\right)\tfrac{D\alpha}{m^2},$$
$$O=\left(\tfrac{1+i}{\mathfrak{D}}-1-m\right)\tfrac{D\alpha}{m^2};$$

which in order that it may become positive, it is necessary, that D shall be negative, and thus on account of $D(l-\lambda)>0$ there will be also $l<1$. Whereby, since the distance O shall be made positive, this equation will be had for the colored margin being removed :

$$0=-\omega+\frac{\pi''}{\Phi}\cdot\frac{1}{i}-\frac{\pi'''}{\Phi}\cdot\frac{1}{m}$$

or

$$0=-\omega+\frac{1+i}{\mathfrak{D}i}-\frac{(1+i)}{\mathfrak{D}m}+\frac{1+m}{m}$$

or with ω removed

$$0=\frac{1+i}{\mathfrak{D}}\cdot\frac{l-1}{il}+\frac{1+m}{m}$$

or

$$0=\frac{(1+i)(l-1)}{\mathfrak{D}}+1+m$$

hence

$$\mathfrak{D}=\frac{-(1+i)(l-1)}{m+1}=\frac{(1+i)(1-l)}{m+1}$$

and

$$D=\frac{(1+i)(1-l)}{2m-i+l},$$

which value must always be negative on account of $l<1$ and hence there will be required to be:

$$l<\frac{i}{2i+1} \text{ or } l<\frac{1}{2},$$

thus so that hence there must be $i>2m$ and

$$D=\frac{-(1+i)(1-l)}{i(1-2l)-l}.$$

With these defined concerning the values D and l we may examine the apparent field of view, the radius Φ of which is expressed in two ways :

$$1.\ \Phi=\frac{\mathfrak{D}\pi''}{1+i}=\frac{(1-l)\pi''}{m+1},\quad 2.\ \Phi=\frac{\mathfrak{D}\pi'''}{1+i-\mathfrak{D}(m+1)}=\frac{(1-l)\pi'''}{(m+1)l},$$

of which the smaller only has a place, if indeed π'' and π''' may obtain a maximum value, which is around $\frac{1}{4}$. But since there shall be $\pi'':\pi'''=1:l$, there can be assumed $\pi'=\pi''=\frac{1}{4}$: and there will become $\pi'''=\frac{1}{4}$ and hence the field produce

$$\Phi = \frac{1}{4} \cdot \frac{1-l}{m+1}$$

and thus the smaller, as if we may use for the simple ocular lens, against our tradition, thus, so that this problem may be unable to be resolved according to our investigation.

THE SAME PRECEDING PROBLEM

254. *Where all the rest remain the same only a fourth lens is located before the real image.*

SOLUTION

Therefore in the solution also all will remain the same as before, except that the signs of the two quantities i and l shall be changed. Therefore initially the elements will be

$$b = -\frac{\alpha}{1+\omega}, \ \beta = -\infty, \ c = \infty, \ \gamma = \alpha, \ d = -\frac{\alpha}{i}, \ \delta = -\frac{D\alpha}{i}, \ e = -\frac{D\alpha}{il}.$$

The focal lengths

$$p = \alpha, \ q = -\alpha, \ r = \gamma = \alpha, \ s = -\frac{\mathfrak{D}\alpha}{i}, \ t = -\frac{\mathfrak{D}\alpha}{il}.$$

Truly the separations of the lenses

$$\alpha + b = \omega\alpha, \ \beta + c = \omega\alpha, \ \gamma + d = +\frac{(1-i)}{i}\alpha, \ \delta + e = \frac{-(l+1)}{il} \cdot D\alpha;$$

from which it is apparent D must be negative, but $i > 1$; then truly there must be $m = il$.
Then we will find

$$\frac{\pi}{\Phi} = -\omega, \quad \frac{\pi'}{\Phi} = -2\omega, \quad \frac{\pi''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}}, \quad \frac{\pi'''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}} - 1 - m$$

and hence for the position of the eye

$$O = \left(-\frac{(1-i)}{\mathfrak{D}} - 1 - m\right)\frac{D\alpha}{mm},$$

which value therefore is positive on account of $D < 0$. Whereby, so that the colored margin may vanish, there must be

$$\mathfrak{D} = \frac{-(1-i)(1+l)}{m+1}, \quad D = \frac{-(i-1)(1+l)}{2m+i-l},$$

which value, since it shall be negative, satisfies the preceding conditions, only if there shall be $i>1$, and with these values substituted there will be

$$b=-\alpha,\ \ \beta=-\infty,\ \ c=\infty,\ \gamma=\alpha,\ d=-\tfrac{\alpha}{i},$$
$$\delta=\frac{(i-1)(1+l)\alpha}{(2m+i-l)i},\ \ e=\frac{(i-1)(1+l)\alpha}{(2m+1-l)il},$$

and hence

$$p=\alpha,\ \ q=-\alpha,\ \ r=\alpha,\ s=\frac{(i-1)(1+l)\alpha}{(m+1)i},\ t=\frac{(i-1)(1+l)\alpha}{(2m+i-l)il},$$
$$\alpha+b=\omega\alpha,\ \ \beta+c=\omega\alpha,\ \ \gamma+d=\tfrac{i-1}{i}\alpha,\ \ \delta+e=\frac{(i-1)(1+l)^2\alpha}{(2m+i-l)il},$$
$$\tfrac{\pi}{\Phi}=-\omega,\ \ \tfrac{\pi'}{\Phi}=-2\omega,\ \ \tfrac{\pi''}{\Phi}=+\tfrac{m+1}{l+1},\ \ \tfrac{\pi'''}{\Phi}=+\tfrac{m+1}{l+1}-1-m=\frac{l(m+1)}{1+l}$$

and hence

$$O=\frac{l(i-1)(m+1)}{2m+i-l}\cdot\frac{\alpha}{m^2}.$$

Therefore since there shall be $\pi'':\pi'''=1:-l$, two cases are required to be considered for the field of view.

I. If $l>1$, then there will be taken $\pi'''=-\frac{1}{4}$, so that there may become $\pi''=\frac{1}{4l}<\frac{1}{4}$ and hence the radius of the field of view

$$\Phi=\frac{1}{4}\cdot\frac{1+\frac{1}{l}}{m+1}.$$

II. If $l<1$, there will be taken $\pi''=\frac{1}{4}$, so that there may become $\pi'''=-\frac{l}{4}>-\frac{1}{4}$ and hence

$$\Phi=\frac{1}{4}\cdot\frac{1+l}{m+1}.$$

Therefore in each case the greater field of view will be, as if a singular eyepiece lens were present, as we have found in the case

$$\Phi=\frac{1}{4}\cdot\frac{1}{m+1}.$$

Therefore the maximum field will be obtained, if there may be taken $l=1$, in which case on account of $il=m$ there becomes $i=m$, then truly

$$\Phi=\frac{1}{2(m+1)}=\frac{1718}{m+1}\ \text{min. of arc},$$

which is twice as great. Therefore it will be agreed to take $l=1$, only if the resolution of the second equation may permit that, which is

$$0=\lambda-\frac{\lambda'}{1+\omega}+\frac{\lambda''}{1+\omega}-\frac{1}{\mathfrak{D}i}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu}{D}\right)-\frac{\lambda''''}{\mathfrak{D}^3m},$$

where, if there may be taken $l = 1$, so that there shall become $i = m$, there becomes

$$\mathfrak{D} = -2 \cdot \tfrac{m-1}{m+1}, \quad D = -2 \cdot \tfrac{m-1}{3m-1};$$

whereby, if m shall be a very large number, there will become $\mathfrak{D} = -2$, $D = -\tfrac{2}{3}$, from which it is evident in this manner the resolution of these equations far from being impeded is also to be advanced, thus so that by putting this $l = 1$ may agree especially with our aim.
Hence therefore we follow with

$$\lambda' = (1+\omega)\lambda + \lambda'' - \tfrac{\lambda'''}{\mathfrak{D}^3 m} - \tfrac{\lambda''''}{D^3 m} - \tfrac{\nu}{\mathfrak{D} D m},$$

for resolving this it may be observed initially, since the final two lenses require a maximum aperture, these must be taken equally convex on both sides ; from which for the final lens there must be taken $\lambda'''' = 1,6299$, for the last but one truly there is obtained

$$\sqrt{(\lambda''' - 1)} = \tfrac{\sigma-\rho}{2\tau} \cdot \tfrac{\delta-d}{\delta+d} = \tfrac{\sigma-\rho}{2\tau} \cdot \tfrac{-5m+3}{m+1}.$$

Now since there shall be

$$\left(\tfrac{\sigma-\rho}{2\tau}\right)^2 = 0,6299,$$

there will become

$$\lambda''' = 1 + 0,6299 \cdot \left(\tfrac{-5m+3}{m+1}\right)^2;$$

then truly we may assume $\lambda = 1$ and $\lambda'' = 1$, but for ω it is seen convenient to assume $\omega = \tfrac{1}{m}$, since in this way the distances between the first lenses do not become very small, so that in practice they may be able to have a position.

COROLLARY 1

255. Therefore so that if we may put $l = 1$, so that there shall become $i = m$, then truly $\omega = \tfrac{1}{m}$, our elements themselves thus will be obtained :

$$b = -\tfrac{m\alpha}{m+1}, \quad \beta = -\infty, \quad c = \infty, \quad \gamma = \alpha, \quad d = -\tfrac{\alpha}{m}, \quad \delta = \tfrac{2(m-1)\alpha}{m(3m-1)}, \quad e = \tfrac{2(m-1)\alpha}{m(3m-1)},$$

thus so that there shall be $\delta = e$ and a real image may be placed between the two final lenses.

Moreover the focal lengths will be

$$p=\alpha,\ \ q=-\tfrac{m\alpha}{m+1},\ \ r=\alpha,\ \ s=\frac{2(m-1)\alpha}{m(m+1)},\ \ t=\frac{2(m-1)\alpha}{m(3m-1)},$$

truly the lens separations

$$\alpha+b=\tfrac{\alpha}{m},\ \ \beta+c=\tfrac{\alpha}{m},\ \gamma+d=\tfrac{m-1}{m}\alpha,\ \ \delta+e=\frac{4(m-1)\alpha}{m(3m-1)}$$

and

$$O=\tfrac{mm-1}{3m-1}\cdot\tfrac{\alpha}{mm}.$$

COROLLARY 2

256. Therefore with a single lens added we have gained a conspicuous improvement, since the magnitude of the field of view shall be made twice as great, as if we may use a single lens for the eyepiece, where it is to be observed properly, that this new lens may not be added after the real image, but must be put in place before that.

SCHOLIUM 1

257. So that we may adapt this, which we have found, most conveniently in practice, we may now make use of the method treated above and we will investigate in the first place the construction of a telescope for a certain small magnification such as $m=25$, from that truly for $m=\infty$; from the comparison of these cases it will be allowed to deduce without trouble the construction for any magnification in between.

EXAMPLE 1

258. To show the construction of the telescope for $m=25$.
Since here there shall be $m=25$, there will be

$$\mathfrak{D}=-\tfrac{24}{13}=-1{,}84615,\quad D=-\tfrac{24}{37}=-0{,}64865,$$

hence there will be

$$\tfrac{1-D}{1+D}=\tfrac{1{,}64865}{0{,}35134}\quad\text{hence}\quad \text{Log.}\left(\tfrac{1-D}{1+D}\right)^2=1{,}3428018,$$

from which there is deduced:

$$\lambda'''=14{,}8699.$$

Now since there shall be

$$\text{Log.}(-\mathfrak{D})=0{,}2662669,\quad \text{Log.}(-D)=9{,}8120104,$$

we will find

$$\lambda'=1{,}04+1+0{,}094529+0{,}23888-0{,}00777,$$

$$\lambda'=2{,}3656,\quad \lambda'-1=1{,}3656\ \ \text{et}\ \ \tau\sqrt{(\lambda'-1)}=1{,}0577.$$

The construction of the lenses themselves thus will be had:

I. For the first lens

$$\text{Radius of the}\begin{cases}\text{anterior face} &= 0,6145\alpha\\ \text{posterior face} &= 5,2439\alpha.\end{cases}$$

The radius of the aperture $=\frac{25}{50}\text{in.}=\frac{1}{2}\text{in.}$

The distance to the following lens $=\frac{1}{25}\alpha=0,04a.$

II. For the second lens

the calculation will be had:

$$F=\frac{b}{\rho\pm1,0577}=\frac{-0,96\alpha}{1,2484},\quad G=\frac{b}{\sigma\mp1,0577}=\frac{-0,96\alpha}{0,5697}$$

or

$$F=-0,7690\alpha,\ G=-1,6851\alpha.$$

The distance to the following as before $=0,04\alpha$.

III. For the third lens

Since its focal length shall be actually

$$\gamma=\frac{\alpha}{1+\omega}=(1-\omega)\alpha \quad\text{and}\quad \lambda''=1,$$

from the first lens this is defined thus:

$$\text{Radius of the}\begin{cases}\text{anterior face} &= 0,5900\alpha\\ \text{posterior face} &= 5,0342\alpha.\end{cases}$$

The distance to the fourth lens $=\alpha-\frac{2\alpha}{m}=0,92\alpha.$

IV. For the fourth lens

of which the focal length is $1,84615\cdot\frac{\alpha}{m}$, since each side must be equally convex, the radius of each face $=2,03076\cdot\frac{\alpha}{m}$.

Radius of the aperture $=0,50769\cdot\frac{\alpha}{m}$.

Distance to the following lens $=1,29730\cdot\frac{\alpha}{m}$.

V. For the fifth lens

of which the focal length $=0,64865\frac{\alpha}{m}$,

the radius of each face will be $=0,71851\cdot\frac{\alpha}{m}$.

The radius of the aperture $=0,17838\cdot\frac{\alpha}{m}$.

Hence the distance as far as to the eye will be $=0,3373\cdot\frac{\alpha}{m}$ with m being $=25$ and the radius of the apparent field of view will be

$$\Phi=\frac{1718}{26}\text{min.}=66\text{ min. of arc.}$$

and the whole length of the instrument will be

$$=\alpha+1,6346\cdot\frac{\alpha}{m}=1,06538\alpha.$$

EXAMPLE 2

259. If $m=\infty$, to describe the construction of the telescope.
Therefore there will be

$$\mathfrak{D}=-2,\ \ D=-\frac{2}{3}\quad\text{and}\quad\omega=0,\ \ \frac{1-D}{1+\mathrm{D}}=5;$$

from which there becomes

$$\lambda'''=1+0,6299\cdot25=16,75.$$

And hence there is deduced

$$\lambda'=1+1=2,\ \ \lambda'-1=1\text{ and thus }\sqrt{(\lambda'-1)}=\tau=-0,9051.$$

Therefore the construction of the lenses thus will be obtained:

1. For the first lens

$$\text{Radius of the}\begin{cases}\text{anterior face} &= 0,6145\alpha\\ \text{posterior face} &= 5,2439\alpha.\end{cases}$$

Radius of the aperture $=\frac{m}{50}$ in.

Distance to the following lens $=\frac{\alpha}{m}$.

II. For the second lens

of which the focal length $b=-\alpha$, we will have

$$F = \frac{b}{\rho \pm 0{,}9051} = \frac{-\alpha}{1{,}0958}, \quad G = \frac{b}{\sigma \mp 0{,}9051} = \frac{-\alpha}{0{,}7223}.$$

Hence

$$F = -0{,}91257\alpha, \quad G = -1{,}38446\alpha.$$

Distance to the following lens $= \frac{\alpha}{m}$.

III. For the third lens

$$\text{Radius of the} \begin{cases} \text{anterior face} & = 0{,}6145\alpha \\ \text{posterior face} & = 5{,}2439\alpha. \end{cases}$$

Distance to the following lens $= \alpha - \frac{2\alpha}{m}$.

IV. For the fourth lens

of which the focal length is $\mathfrak{D}d = +\frac{2\alpha}{m}$,

the radius of each face will be $= 2{,}2 \cdot \frac{\alpha}{m}$.

The distance $= \frac{4}{3} \cdot \frac{\alpha}{m} = 1{,}333 \cdot \frac{\alpha}{m}$.

V. For the fifth lens

of which the focal length $= 0{,}666 \cdot \frac{\alpha}{m}$,

the radius of each face will be $= 0{,}7333 \cdot \frac{\alpha}{m}$.

The distance to the eye $= \frac{1}{3} \cdot \frac{\alpha}{m}$.

EXAMPLE 3

260. To describe a telescope of this kind For any magnification m .

Here to be assumed all the lenses to be prepared from the same kind of glass, for which there is $n = 1{,}55$.

Moreover the following construction will be prepared from the preceding:

I. For the first lens

there will be as thus far

$$\text{Radius of the} \begin{cases} \text{anterior face} & = 0{,}6145\alpha \\ \text{posterior face} & = 5{,}2439\alpha. \end{cases}$$

Radius of the aperture $x = \frac{m}{50}$ in.

Distance to the following lens $= \frac{\alpha}{m}$.

II. For the second lens

there may be put
$$F=-\left(0,91257+\tfrac{f}{m}\right)\alpha, \qquad G=-\left(1,38446+\tfrac{g}{m}\right)\alpha;$$
moreover there will be
$$0,91257\;+\tfrac{f}{25}=0,7690, \qquad 1,38446+\tfrac{g}{25}=1,6851;$$
from which
$$f=-3,59, \qquad g=+7,52.$$
Distance $=\frac{\alpha}{m}$.

III. For the third lens

$$\text{Radius of the}\begin{cases}\text{anterior face} &= 0,6145\left(1-\tfrac{1}{m}\right)\alpha\\ \text{posterior face} &= 5,2439\left(1-\tfrac{1}{m}\right)\alpha.\end{cases}$$

Distance to the following lens $=\alpha-\frac{2\alpha}{m}$.

IV. For the fourth lens

the radius of each face may be put $=\left(2,2+\frac{h}{m}\right)\frac{\alpha}{m}$
and there will become
$$2,2+\tfrac{h}{25}=2,03076;\ \text{hence there is deduced } h=-\,4,231.$$
Distance to the following lens
$$=\left(1,333+\tfrac{k}{m}\right)\tfrac{\alpha}{m},\quad \text{hence } k=-0,90;$$
and thus the distance will be
$$=(1,333-\tfrac{0,90}{m})\tfrac{\alpha}{m}.$$

V. For the fifth lens

of which the focal length $=\left(0,666-\frac{0,434}{m}\right)\frac{\alpha}{m}$,
the radius of each face $=\left(0,7333-\frac{0,495}{m}\right)\frac{\alpha}{m}$.

Distance to the eye $=\left(\frac{1}{3}+\frac{l}{m}\right)\frac{\alpha}{m}$,
from which $l=0,097$, and thus this distance will be
$$\left(0,333+\tfrac{0,097}{m}\right)\tfrac{\alpha}{m}$$
and the whole length of the telescope

$$=\alpha+\left(1,666-\tfrac{0,803}{m}\right)\tfrac{\alpha}{m} \text{ or } \alpha+1,666\cdot\tfrac{\alpha}{m}-0,803\cdot\tfrac{\alpha}{m^2}.$$

Now we may consider, how large a value of α it may be appropriate to use, and since the first three lenses may be able to be considered as a triple lens, the smallest radius is 0,6145α, of which the fourth part $\frac{3}{20}\alpha$ put equal to $x=\frac{m}{50}$ gives

$$\alpha=\tfrac{2m}{15}\text{ in.}=\tfrac{4m}{30}\text{ in.}$$

Therefore we may put $\alpha=\frac{4}{30}m$ in. and the following construction of telescopes of this kind follows for any magnification *m*, with the lenses made from glass with $n=1,55$.

Concerning the diaphragms required to be inserted with these telescopes the following may be seen following Scholium 3.

I. For the first lens

$$\text{Radius of the }\begin{cases}\text{anterior face} = 0,08193m \text{ in.}\\ \text{posterior face} = 0,69918m \text{ in.}\end{cases}$$

The radius of the aperture $x=\frac{m}{50}$ in. The distance $=\frac{2}{15}$ in.

II. For the second lens

$$\text{Radius of the }\begin{cases}\text{anterior face} = (-\,0,1217m+0,478)\text{ dig.}\\ \text{posterior face} = (-\,0,18459m-1,003)\text{ dig.}\end{cases}$$

The distance $=\frac{2}{15}$ in.

III. For the third lens

$$\text{Radius of the }\begin{cases}\text{anterior face} = (-\,0,08193m-0,0819)\text{ in.}\\ \text{posterior face} = (-\,0,69918m-0,699)\text{ in.}\end{cases}$$

The distance $=\left(\frac{2}{15}m-\frac{4}{15}\right)$ in.

IV. For the fourth lens

The radius of each face $=\left(0,2933-\frac{0,5641}{m}\right)$ in.

The distance $=(0{,}1777-\frac{0{,}12}{m})$ in.

V. For the fifth lens

Radius of each face $=\left(0,098-\frac{0{,}066}{m}\right)$ in.

Hence the distance to the eye $=\left(0,044-\frac{0{,}013}{m}\right)$ in.

The total length $=\left(\frac{2}{15}m+0,222-\frac{0{,}107}{m}\right)$ dig.,

thus so that for the case $m=100$ this length shall be $13\frac{1}{2}$ in., finally the radius of the apparent field of view $=\frac{1718}{m+1}$ minutes of arc., or, since also the first lenses contribute a little to increasing the field of view, $\Phi=\frac{1718}{m}$ min., thus so that for $m=100$ there may become $\Phi=17$ minutes 11 sec.

SCHOLIUM 2

261. These telescopes in their kind thus are seen with all unrestricted numbers, so that scarcely anything more perfect may be able to be desired, unless we wish to use diverse kinds of glass. For not only have they been freed from the confusion arising from the apertures as well as from the colored margin, but also they reveal an apparent field of view twice as great as the simple one, and besides as they are shorter, so that indeed anything shorter may not be hoped for. Then also in the execution thence a conspicuous convenience can be obtained, since a little variation can be obtained between the first three lenses ; indeed if perhaps it may arise, that on account of a smallest error in practice bringing these lenses together they may not be adapted most exactly to the intervals prescribed here, it can easily arise, so that from these smallest changes they may be going to be placed together to produce an outstanding effect. Yet meanwhile it will be agreed always the following concave lens to be taken pains over many times following the same measures ; since indeed always some distinction may arise, between several lenses of the same kind and the best will be able to be selected easily. Truly nothing less may be agreed on but to pursue our investigation further, and to examine in this manner telescopes of this kind, for which a field of view shall be going to be produced three or four times greater.

SCHOLIUM 3

262. So that these telescopes may be effected to excel better, it is necessary, so that in place of the true image a diaphragm or hole, just as now has been described above [§ 224,225], since it may be put in place with a hole of a due magnitude. But this image on account of $\delta=e$ falls precisely in the middle of the interval between the

fourth and fifth lenses and thus at a distance $=\left(0,0888-\frac{0,06}{m}\right)$ in. Then since the hole must be equal to the magnitude of this image and the radius of the image shall be in general

$$=\alpha\Phi BCD=\alpha\Phi D=-2\tfrac{m-1}{3m-1}\cdot\alpha\Phi,$$

the radius of the hole must be $=-2\frac{m-1}{3m-1}\cdot\alpha\Phi$. Now since in our example set out there shall be $\alpha=\frac{2}{15}m$ and $\Phi=\frac{1}{2m}$, this same radius of the hole is deduced

$$=\tfrac{1}{15}\cdot\tfrac{2(m-1)}{3m-1}\text{ in.}=\left(\tfrac{2}{45}-\tfrac{4}{135m}\right)\text{ in.}$$

Moreover even if we have acquired a much greater order of clarity with these telescopes, than generally is accustomed to be done, while we have assumed $y=\frac{1}{50}$ in., from Huygens' rules moreover there follows $y=\frac{1}{73}$ in., yet, if from which it may be feared, lest here on account of the number of lenses the clarity may be allowed to be reduced, measures given may effect a remedy for this inconvenience by augmenting some part of itself so much, or what returns the same, by increasing the measure of one inch as it pleases, which until now we have left indefinite.
[*i.e.* make the unit of measurement greater by some amount.]

PROBLEM 7

268. *If besides the three earlier lenses, as have been set up in the preceding problem, hitherto a single lens may be arranged before the location of the image, after that two lenses in addition may be put in place, so that the maximum field may be obtained.*

SOLUTION

Since here six lenses are present, there will be

$$m=\tfrac{\alpha}{b}\cdot\tfrac{\beta}{c}\cdot\tfrac{\gamma}{d}\cdot\tfrac{\delta}{e}\cdot\tfrac{\varepsilon}{f},$$

of which fraction the three first are negative, the fourth positive and the fifth again negative. Now we have assumed for the prior ones to be $\frac{\alpha}{b}=-1-\omega$, $\frac{\beta}{c}=-1$. For the posterior ones truly we have put in place $\frac{\gamma}{d}=-k$, $\frac{\delta}{e}=i$ and $\frac{\varepsilon}{f}=-l$, so that there shall be $m=(1+\omega)\,kli$; from which on account of $B=\infty$, $C=0$ and $BC=1$ our elements will become :

$$b=-\tfrac{\alpha}{1+\omega},\ \beta=\infty,\ c=-\infty,\ \gamma=\tfrac{\alpha}{1+\omega},$$
$$d=-\tfrac{\alpha}{k},\ \delta=-\tfrac{D\alpha}{k},\ e=-\tfrac{D\alpha}{ki},\ \varepsilon=-\tfrac{DE\alpha}{ki},\ f=\tfrac{DE\alpha}{kil}.$$

And hence the focal lengths will be found

$$p=\alpha,\ q=-(1-\omega)\alpha,\ r=\tfrac{\alpha}{1+\omega},$$
$$s=-\tfrac{\mathfrak{D}\alpha}{k},\ t=-\tfrac{\mathfrak{E}D\alpha}{ki},\ u=\tfrac{DE\alpha}{kil}.$$

Then truly the separations of the lenses :

$$\alpha+b=\omega\alpha,\ \beta+c=\omega\alpha,\ \gamma+d=\tfrac{k-1}{k}\cdot\alpha,$$
$$\delta+e=-D\alpha\cdot\tfrac{i+1}{ki},\quad \varepsilon+f=DE\alpha\tfrac{1-l}{ki};$$

which so that they may be produced positive, there must be

$$1.\ k>1,\qquad 2.\ D<0,\qquad 3.\ +E(l-1)>0.$$

The letters π, π' etc. will be defined in the following manner:

$$\tfrac{\pi}{\Phi}=-\omega,\qquad \tfrac{\pi'}{\Phi}=-2\omega,$$

and the rest must be determined from the following formulas:

$$\tfrac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\tfrac{BC\alpha}{d}=-k,\qquad \tfrac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\tfrac{BCD\alpha}{e}=-ki,$$
$$\tfrac{\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}{\Phi}=m;$$

Hence therefore we deduce :

$$\tfrac{\pi''}{\Phi}=\tfrac{1-k}{\mathfrak{D}},\qquad \tfrac{\pi'''}{\Phi}=\tfrac{\pi''}{\mathfrak{E}\Phi}-\left(\tfrac{1+ki}{\mathfrak{E}}\right)\quad\text{and}\quad \tfrac{\pi''''}{\Phi}=\tfrac{\pi'''}{\Phi}-\tfrac{\pi''}{\Phi}+m+1.$$

Now since for the apparent field of view there shall be

$$\Phi=\tfrac{\pi-\pi'+\pi''-\pi'''+\pi''''}{m+1},$$

this may become a maximum, if there may be taken $\pi''=\tfrac{1}{4}$, $\pi'''=-\tfrac{1}{4}$, $\pi''''=\tfrac{1}{4}$;
thence there will become $\Phi=\tfrac{3}{4}\cdot\tfrac{1}{m+1}$, from which these equations will give

$$\frac{m+1}{3} = \frac{1-k}{\mathfrak{D}}, \qquad \frac{-m-1}{3} = \frac{m+1}{3\mathfrak{E}} - \left(\frac{1+ki}{\mathfrak{E}}\right)$$

and

$$\frac{m+1}{3} = \frac{-m-1}{3} + m + 1$$

which is an identity, as must be the case.

From which it follows, for the position of the eye,

$$O = \frac{m+1}{3} \cdot \frac{u}{m} = \frac{m+1}{3} \cdot \frac{DE\alpha}{kilm};$$

which distance, in order that it may become positive, on account of $D < 0$, there must also be $E < 0$ and thus $l < 1$, if indeed we may assume α positive. But it is apparent, if we may take $l = 1$, the two latter lenses to be joined to each other at once and to produce the case of the double eyepiece lens now considered above [§ 215]. Moreover we may see, before we may contemplate the equation for the colored margin, whatever values the letters $\mathfrak{D}$ and $\mathfrak{E}$ may obtain from the two above equations, and from the former certainly there is found $\mathfrak{D} = -\frac{3(k+1)}{m+1}$; which since it shall be negative, also D becomes negative, as required ; and from the other there becomes $\mathfrak{E} = \frac{3ki-m+2}{m+1}$ and hence $E = \frac{3ki-m+2}{2m-3ki-1}$; which value since it must be negative, the two cases are required to be considered.

I. If the numerator were negative and the denominator positive, there will become $3ki + 2 < m$ and $m > \frac{3ki+1}{2}$ or more simply, $m > 3ki + 2$.

II. But if the numerator shall be positive and the denominator negative, there will become $m < 3ki + 2$ and $m < \frac{3ki+1}{2}$ or more simply $m < \frac{3ki+1}{2}$.

But since there shall become $m = kil$, on account of $l < 1$ there will be $m < ki$, from which it is apparent the first case cannot happen, but the second alone, thus so that there shall become $m < ki$, with which agreed all the conditions are fulfilled. Thus nothing stand in the way, whereby we may not undertake the resolution of the equations for removing the colored fringes, which beyond expectation may emerge so very easily, so that no difficulties, such as before were being met, will disturb the process. Moreover this equation with the first two terms missing as being minimal thus will be had :

$$0 = \frac{\pi''}{\Phi} \cdot \frac{d}{p} + \frac{\pi'''}{\Phi} \cdot \frac{e}{Dp} + \frac{\pi''''}{\Phi} \cdot \frac{f}{DEp},$$

which on account of $\pi'' = -\pi''' = +\pi''''$ will change into this :

$$0 = \frac{d}{p} - \frac{e}{Dp} + \frac{f}{Dep};$$

since now there shall become

$$p=\alpha \quad \text{et} \quad \tfrac{d}{\alpha}=-\tfrac{1}{k}, \quad \tfrac{e}{\alpha}=-\tfrac{D}{ki}, \quad \tfrac{f}{\alpha}=\tfrac{DE}{kil},$$

our equation will become

$$0=-\tfrac{1}{k}+\tfrac{1}{ki}+\tfrac{1}{kil}$$

or on multiplying by k

$$0=-1+\tfrac{1}{i}+\tfrac{1}{il}, \quad 0=-il+l+1;$$

and hence $i=\frac{l+1}{2}$, where there must be $l<1$; hence $il=1+l$ and $m=(1+l)k$, thus so that there shall become $k=\frac{m}{1+l}$. Then there will become

$$\mathfrak{D}=\frac{-3(k-1)}{m+1}, \quad \mathfrak{E}=\frac{3ki-m+2}{m+1}$$

and thus

$$D=\frac{-3(k-1)}{3k+m-2}, \quad E=\frac{3ki-m+2}{2m-3ki-1}.$$

From these found the equation for the removal of the confusion will be

$$0=\lambda-\frac{\lambda'}{1+\omega}+\frac{\lambda''}{1+\omega}-\frac{1}{\mathfrak{D}k}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu}{D}\right)-\frac{1}{D^3\mathfrak{E}ki}\left(\frac{\lambda''''}{\mathfrak{E}^2}+\frac{\nu}{E}\right)+\frac{\lambda'''''}{D^3E^3m},$$

which, as until now has been done, is resolved easily, evidently by seeking the value of λ'.

SCHOLIUM

264. The solution of this problem provides us with the maximum help for resolving the following investigations, since this new method supplies us with the equation needed for removing the colored fringe required to be resolved most readily, which was involved with significant difficulties above. Moreover the strength of this method consists in that we may attribute at once the determined values of the letters π, π', π'' etc., which indeed shall be prepared thus, so that they may produce the maximum apparent field of view. For with this done these letters may be removed at once from the equation mentioned and in place of the letters *d, e, f* by substituting the values found before also the capital letters vanish at once from the calculation, thus so that the whole equation may involve no further other elements besides the letters *k, i, l;* of which one is defined without any difficulty; then truly from the other values assumed for the letters π the letters $\mathfrak{D}$, $\mathfrak{E}$ etc. may be determined easily and thence also *D, E* etc., of which the values transcribed into the final equation make the whole operation simple; this method also can be called into use for the first lenses, but where it is required to be observed, since these lenses concur as if to constituting the objective lens, from these letters π and π' nothing or very little can be added to the field requiring to be increased. On account of which from these letters, not the value

$\frac{1}{4}$ as from the following , but rather so that a minimum may be attributed, as $\frac{1}{4}\omega$ and $\frac{1}{4}\omega'$, evidently with ω and ω' denoting minimum fractions. Whereby, so that this new method may be seen more clearly, we will use that here for the following general problem requiring to be solved.

PROBLEM 8

265. *To construct a telescope of this kind from six lenses, the first three of which serve to remove all the confusion, moreover the final three by tripling the field, while evidently we may assign a simple field to the simple eyepiece lens.*

SOLUTION

Here therefore the five following fractions arise requiring to be considered:

$$\frac{\alpha}{b},\ \frac{\beta}{c},\ \frac{\gamma}{d},\ \frac{\delta}{e},\ \frac{\varepsilon}{f},$$

of which all except the single one must become negative. Whereby, if we may put in place :

$$\frac{\alpha}{b}=-P,\ \ \frac{\beta}{c}=-Q,\ \ \frac{\gamma}{d}=-R,\ \ \frac{\delta}{e}=-S,\ \ \frac{\varepsilon}{f}=-T,$$

it is evident of these five letters *P, Q, R, S, T* one to become negative and with the rest present positive. But which shall be negative, there is no need yet to define here. With this in place our elements may be represented as well as equally the focal lengths with the distances between the lenses to be viewed in the following manner :

Determinable distances		Focal lengths	Intervals between lenses
		$p=\alpha$	
$b=\frac{-\alpha}{P}$	$\beta=\frac{-B\alpha}{P}$	$q=\mathfrak{B}b$	$\alpha+b=\alpha\left(1-\frac{1}{P}\right)>0$
$c=\frac{B\alpha}{PQ}$	$\gamma=\frac{BC\alpha}{PQ}$	$r=\mathfrak{C}c$	$\beta+c=\frac{-B\alpha}{P}\left(1-\frac{1}{Q}\right)>0$
$d=\frac{-BC\alpha}{PQR}$	$\delta=\frac{-BCD\alpha}{PQR}$	$s=\mathfrak{D}d$	$\gamma+d=\frac{BC\alpha}{PQ}\left(1-\frac{1}{R}\right)>0$
$e=\frac{BCD\alpha}{PQRS}$	$e=\frac{BCDE\alpha}{PQRS}$	$t=\mathfrak{E}e$	$\delta+d=\frac{-BCD\alpha}{PQR}\left(1-\frac{1}{S}\right)>0$
$f=\frac{-BCDE\alpha}{PQRST}$		$u=f$	$\varepsilon+f=\frac{BCDE\alpha}{PQRS}\left(1-\frac{1}{T}\right)>0$

where, since the product *PQRST* shall be negative, for the magnification there will become

$$m=-PQRST.$$

Then, since there may be obtained for the apparent field of view

$$\Phi = \frac{\pi-\pi'+\pi''-\pi'''+\pi''''}{m+1},$$

the maximum value shall be ξ, which these letters π, π' etc. are able to receive, and we may put in place

$$\pi = \omega\xi, \quad \pi' = -\omega'\xi, \quad \pi'' = \xi, \quad \pi''' = -\xi, \quad \pi'''' = \xi,$$

so that there shall become

$$\Phi = \frac{\omega+\omega'+3}{m+1}\xi.$$

Therefore since hence there shall be :

$$\frac{\pi''''}{\Phi} = \frac{m+1}{\omega+\omega'+3},$$

we will have for the distance of the eye

$$O = \frac{\pi''''}{\Phi}\cdot\frac{u}{m} = \frac{m+1}{\omega+\omega'+3}\cdot\frac{-BCDE\alpha}{PQRSTm}$$

or

$$O = \frac{m+1}{\omega+\omega'+3}\cdot\frac{BCDE\alpha}{mm}.$$

Therefore so that O may become positive, since $\frac{\pi''''}{\Phi}$ is positive, there must become $u > 0$ and thus the final lens is convex, which condition in addition is required to be observed properly.

Therefore now the condition for the colored fringe being removed thus will itself be had:

$$0 = \omega\cdot\frac{b}{\alpha} - \omega'\cdot\frac{c}{B\alpha} + \frac{d}{BC\alpha} - \frac{e}{BCD\alpha} + \frac{f}{BCDE\alpha},$$

which is reduced to this form:

$$0 = +\omega\cdot\frac{1}{P} + \omega'\cdot\frac{1}{PQ} + \frac{1}{PQR} + \frac{1}{PQRS} + \frac{1}{PQRST},$$

in which the two first terms on account of being insignificant may be ignored, thus so that according to this there shall become

$$0 = 1 + \frac{1}{S} + \frac{1}{ST},$$

which equation is resolved easily, while either of the letters S and T shall be negative; from which it is apparent the three first letters P, Q, R by necessity must be positive.

Now the order demands, that also we will determine the letters $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc. from the fundamental equations:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=-P, \quad \frac{1}{2}\ \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=PQ,$$

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=-PQR, \quad \frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=PQRS,$$

from which, if for the sake of brevity we may put

$$\frac{\omega+\omega'+3}{m+1}=M,$$

we gather

$$\begin{array}{l|l} \mathfrak{B}=\frac{(1-P)M}{\omega} & B=\frac{\mathfrak{B}}{1-\mathfrak{B}} \\ \mathfrak{C}=\frac{(1-PQ)M-\omega}{\omega'} & C=\frac{\mathfrak{C}}{1-\mathfrak{C}} \\ \mathfrak{D}=(1-PQR)M-\omega'-\omega & D=\frac{\mathfrak{D}}{1-\mathfrak{D}} \\ \mathfrak{E}=(1-PQRS)M-\omega'-\omega-1 & E=\frac{\mathfrak{E}}{1-\mathfrak{E}} \end{array}$$

and thus

$$B=\frac{(1-P)M}{\omega-(1-P)M}, \qquad C=\frac{(1-PQ)M-\omega}{\omega'+\omega-(1-PQ)M},$$
$$D=\frac{(1-PQR)M-\omega'-\omega}{1+\omega+\omega'-(1-PQR)M}, \qquad E=\frac{(1-PQRS)M-\omega'-\omega-1}{2+\omega+\omega'-(1-PQRS)M}.$$

Now finally in order for the confusion of the aperture to be removed, which is

$$0=\lambda-\frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{1}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)-\frac{1}{B^3C^3\mathfrak{D}PQR}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu}{D}\right)$$
$$+\frac{1}{B^3C^3D^3\mathfrak{E}PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^2}+\frac{\nu}{E}\right)-\frac{1}{B^3C^3D^3E^3\mathfrak{F}PQRST}\cdot\lambda'''''; $$

for this equation in order that it may be satisfied, noting the term following after the first three to become exceedingly small. For since there shall be $PQRST=-m$, this is a huge number, but the first factors P and Q scarcely will differ from unity, it is necessary so that the product RST may produce almost the whole number m ; then since we have found the equation $0=1+\frac{1}{S}+\frac{1}{ST}$ between S and T, it is apparent the number m to be contained neither in S nor in T and therefore the factor R includes the greatest part of the number m . On account of which the fourth member of this equation and the following as if with the first three were to vanish, thus so that the first three may be able to just about cancel each other out, from which there will be established approximately

$$0=\lambda-\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3\mathfrak{C}^3PQ}$$

and thus

$$\lambda'=\mathfrak{B}^3P\lambda+\frac{\mathfrak{B}^3\lambda''}{B^3\mathfrak{C}^3Q},$$

where for a pleasing resolution it shall be required to choose, that there will be produced $\lambda=1$, $\lambda'=1$ and $\lambda''=1$, since then the lightest errors brought forth in practice are of the least trouble. Whereby, since the following small terms shall be positive, it will be necessary, so that there shall be $1>\mathfrak{B}^3P+\frac{\mathfrak{B}^3}{B^3\mathfrak{C}^3Q}$; from which, if there shall be $P=1$ and $Q=1$ and as above $B\mathfrak{C}=1$, there must become $1>2\mathfrak{B}^3$ or $\mathfrak{B}<\sqrt[3]{\frac{1}{2}}$, or if $\mathfrak{B}<\frac{4}{5}$; which deserves to be mentioned in the application.

COROLLARY 1

266. Therefore since now it is certain all five letters *P, Q, R, S, T* to be positive besides *S* or *T*, if we may assume the distance α always to be positive, from the first interval we conclude $P>1$, and since this interval may be set to be a minimum, *P* will be only a little greater than unity, and because also the second interval is taken a minimum, the letter *Q* also will differ only a little from unity. Then since also *f* must be a positive quantity and thus the product *BCDE* positive, on account of $\varepsilon=-Tf$ the final interval becomes $(1-T)f$ and this at once presents the $T<1$, from which, if *T* shall be positive, the required condition shall be so that $T<1$; but if *T* shall be negative, there is no need for a restriction.

COROLLARY 2

267. Because in the final equation all the members are positive except the second, thus so that the second alone must cancel out all the remaining, it is necessary that $\mathfrak{B}$ shall be positive and as we have noted, in order that it shall have a value just a little less than unity. Whereby, since there shall be found $\mathfrak{B}=\frac{(1-P)M}{\omega}$, the letter M moreover always shall be, but $1-P$ negative, it follows the small part ω always shall be negative.

COROLLARY 3

268. Therefore if we may set the first interval to be $=\eta\alpha$ and equally the second $=\eta\alpha$, for the minimal fraction η, since here we wish to avoid only that case, where

these lenses as it were must coalesce into a single lens, it is agreed to assume η to be so small, that it will be allowed to be carried out; for which it will suffice to be seen, if there shall be $\eta\alpha = 0,03$; hence therefore there will be $1-\frac{1}{P}=\eta$ and thus $P=1-\frac{1}{\eta}$, and now we will be able to define ω closer, evidently $\omega=\frac{-\eta M}{(1-\eta)\mathfrak{B}}$, and since $M=\frac{3}{m+1}$, there will become $\omega=\frac{-3\eta}{(m+1)\mathfrak{B}}$ and thus the letter $\mathfrak{B}$ is left to our choice at this point.

COROLLARY 4

269. Since $\mathfrak{B}>0$ and a little less than unity, B will be positive ; from which we will have for the second interval $Q=\frac{B}{B+\eta P}$ and thus $Q<1$, or $Q=\frac{(1-\eta)B}{(1-\eta)B+\eta}$ or approximately, $Q=1-\frac{\eta}{B}$; and hence it will be possible to define ω', evidently

$$\omega'=\frac{(1-PQ)M-\omega}{\mathfrak{C}} \text{ or } \omega'=\frac{6\eta}{(m+1)\mathfrak{B}\mathfrak{C}},$$

thus so that $\mathfrak{C}$ also may be left to our choice.

SCHOLIUM 1

270. Hence the above case treated, where there was $B=\infty$ and $\mathfrak{C}=0$, is readily deduced; for since there will be $Q=1$ with there remaining

$$P=1+\eta, \quad \omega=\frac{-3\eta}{m+1}, \quad \omega'=\frac{6\eta}{(m+1)\mathfrak{B}\mathfrak{C}},$$

and since both $\beta=\infty$ as well as $c=-\infty$, γ will become the focal length of the third lens $r=\frac{B\mathfrak{C}\alpha}{PQ}$, where $B\mathfrak{C}$ will be defined thus, so that the third lens may emerge perfectly equal to the first lens, which in practice is a great convenience ; namely, there may be put

$$B\mathfrak{C}=PQ=1+\eta .$$

But since then there shall be $\mathfrak{B}=1$, the latter condition [§ 265] is unable to be satisfied ; which since it shall be of greater concern than the preceding, we may put rather $\mathfrak{B}=\frac{4}{5}$, so that there shall be $B=4$, and on assuming $P=1,03$ there must be assumed to find $\mathfrak{C}>0,256$, whereby there is taken $\mathfrak{C}=0,257$ and there will become

$$C=\frac{0,257}{0,743}=0,34589,$$

so that it may suffice to be observed by those, who may wish to use such a resolution.

COROLLARY 5

271. Now since both *BC* as well as *PQ* are positive numbers, so that the third interval also may become positive, it is necessary, that there shall become $R > 1$, which condition is fulfilled at once, since *R* will be a much greater number at this stage. It is necessary the fourth interval $-\frac{BCD}{PQR}\left(1-\frac{1}{S}\right)\alpha$, that $-D\left(1-\frac{1}{S}\right)\alpha$ shall be positive; but from the preceding it is apparent that $\mathfrak{D}$ and thus *D* must be negative ; from which there must become $1-\frac{1}{S} > 0$, which will happens, if S were either negative or $S > 1$, if it shall be positive. Concerning the fifth interval we have now considered above.

SCHOLIUM 2

272. Here therefore two cases are required to be considered, the one, where *S* is a negative number, the other truly, where T is negative.

I. Let $S < 0$ and there may be put $S = -K$, and this equation will be obtained :

$$0 = 1 - \frac{1}{K} - \frac{1}{KT} \text{ and there will be } K = 1 + \frac{1}{T};$$

truly here there is $T < 1$, as we have seen above, from which there will be $K > 2$, and *KT* is contained within the limits 1 and 2; from which, since there shall be $RKT = m$, *R* will be contained within the limits m and $\frac{1}{2}m$; again in this case there will be

$$\mathfrak{E} = (1 + RK)M - 1,$$

which value on account of $RK > m$ and $M = \frac{3}{m+1}$ will become

$$\mathfrak{E} > \frac{3(m+1)}{m+1} - 1 > 2,$$

and thus *E* always to be negative, just as the remaining condition demands, clearly so that *BCDE* shall be positive.

II. Now there shall become $T < 0$, and there may be put $T = -K$, and there will be $0 = 1 + \frac{1}{S} - \frac{1}{SK}$ and thus $S = \frac{1}{K} - 1$, where certainly the account of the final interval *K* may be taken as it pleases, now truly it is required, that there shall be $K < 1$. Therefore since there shall be $RSK = m$, there will be $RS = \frac{m}{K}$ and thus $RS > m$ and the letter $\mathfrak{E}$ clearly shall be negative and thus also *E*; with which noted the setting out of the examples plainly will labour under no difficulty ; only that here it has be seen to apply, thus so that the three latter lenses may take the maximum aperture, these must be prepared equally convex on each side ; with which condition the numbers λ''', λ'''', λ''''' may be determined in the following manner, as indeed now has been shown above, evidently, if for the nature of the glass there may be put $\frac{\sigma-\rho}{2\tau}=N$, there will be found

$$\lambda'''=1+N^2\left(\frac{1-D}{1+D}\right)^2,$$

where the form on account of $D=\frac{\mathfrak{D}}{1-\mathfrak{D}}$ will become

$$\lambda'''=1+N^2\left(1-2\mathfrak{D}\right)^2,,$$

and in a similar manner,

$$\lambda''''=1+N^2\left(1-2\mathfrak{E}\right)^2,\quad \lambda'''''=1+N^2.$$

But I consider it to be superfluous to illustrate this example by an example, since for the case of five lenses several examples now have been presented, then truly, in the first place if someone will have wished for a greater field , he will consider rather to use two kinds of glass, so that also the final kinds of confusion may be removed completely. Which argument it remains to be explained further in the following chapter.

CAPUT II

DE ULTERIORI HORUM TELESCOPIORUM PERFECTIONE

QUAM QUIDEM UNICAM VITRI SPECIEM ADHIBENDO

ASSEQUI LICET

PROBLEMA 1

228. *Si inter lentem obiectivam et ocularem in ipso loco imaginis nova lens constituatur, inquirere in commoda, quae eius ope telescopio conciliare licet.*

SOLUTIO

Quia igitur casum trium lentium habemus, multiplicatio *m* statim praebet $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$ ubi, cum esse debeat intervallum inter lentem primam et secundam $=\alpha$, fit $b=0$ ideoque $\beta=Bb=0$, nisi forte $B=\infty$. Quo autem hinc valorem ipsius *B* definire queamus, distantiam focalem secundae lentis in computum introducamus, quae sit $=q$, ita ut iam habeamus $q=\frac{b\beta}{b+\beta}$, ex qua aequatione colligemus $\beta=\frac{bq}{b-q}=0$, unde valorem litterae *B* consequimur, scilicet $B=\frac{\beta}{b}=-1$ hincque $\mathfrak{B}=\infty$. Quoniam igitur tam *b* quam $\beta=0$, ita tamen, ut sit $\frac{\beta}{b}=-1$, erit $m=\frac{\alpha}{c}$; ideoque $c=\frac{\alpha}{m}$, ubi *c* denotat distantiam focalem lentis ocularis.

His notatis semidiameter confusionis erit

$$\frac{\mu m x^3}{4p^3}\left(\lambda+0+\frac{\lambda'}{m}\right),$$

ita ut lens media nihil plane ad hanc confusionem conferat perindeque sit, quaecunque figura huic lenti tribuatur. Deinde pro campo apparente habebimus eius semidiametrum

$$\Phi=\frac{\pi-\pi'}{m+1},$$

ubi valor ipsius π per hanc formulam definitur

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b};$$

ex qua ut aliquid concludi possit, loco b introducamus distantiam focalem secundae lentis q, et cum sit $q=\mathfrak{B}b$, erit $b=\frac{q}{\mathfrak{B}}$, qui valor nobis praebet hanc aequationem:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha\mathfrak{B}}{q}$$

sive ob $\mathfrak{B}=\infty$

$$\frac{\pi}{\Phi}=\frac{\alpha}{q} \quad \text{seu} \quad \pi=\frac{\alpha\Phi}{q},$$

ubi tantum est animadvertendum valorem π quadrantem unitatis superare non debere. Hoc autem valore π admisso pro campo apparente erit

$$\Phi=\frac{-\pi' q}{(m+1)q-\alpha}$$

hincque

$$\pi=\frac{-\alpha\pi'}{(m+1)q-\alpha};$$

quare, si ponamus $-\pi'=\frac{1}{4}$, etiam

$$\pi=\frac{+\frac{1}{4}\alpha}{(m+1)q-\alpha}$$

maior quam $\frac{1}{4}$ esse nequit; si igitur quoque sumamus $\pi=\frac{1}{4}$, novam hanc nanciscimur determinationem

$$1=\frac{\alpha}{(m+1)q-\alpha}$$

sive

$$(m+1)q=2\alpha \quad \text{et} \quad q=\frac{2\alpha}{m+1}.$$

Sin autem in formula $\pi=\frac{-\alpha\pi'}{(m+1)q-\alpha}$ fractio $\frac{\alpha}{(m+1)q-\alpha}$ maior esset unitate, tum pro $-\pi'$ minorem valorem quam $\frac{1}{4}$ scribi oporteret, ut prodiret $\pi=\frac{1}{4}$; tum autem campus apparens minor esset prodituris, quam si etiam $-\pi'$ esset $\frac{1}{4}$. Unde concludimus, sive haec fractio $\frac{\alpha}{(m+1)q-\alpha}$ maior sit unitate sive minor, utroque casu fore

$$\Phi<\frac{\frac{1}{4}+\frac{1}{4}}{m+1}$$

ac solo casu $\frac{\alpha}{(m+1)q-\alpha}=1$ fieri posse

$$\Phi=\frac{1}{2(m+1)},$$

qui valor duplo maior est quam casu duarum lentium simplicium. Interim tamen de quantitate q nihil adhuc definiamus, sed potius videamus, num hoc modo margo coloratus destrui possit, quod eveniet, si fuerit

$$0=\frac{\pi b}{\Phi p}-\frac{\pi'}{m\Phi} \text{ sive } 0=0+\frac{(m+1)q-\alpha}{mq},$$

ex qua sequitur $q=\frac{\alpha}{m+1}$, unde patet quantitatem q utique ita assumi posse, ut margo coloratus penitus destruatur, quae determinatio praecedenti longe est anteferenda. Posito igitur $q=\frac{\alpha}{m+1}$ pro campo apparente foret $\pi=\infty\cdot\pi'$ seu $\pi'=\frac{\pi}{\infty}$; quare, cum π maius quam $\frac{1}{4}$ capi non possit, fiet $\pi'=0$, ita ut hoc casu lens ocularis nihil plane ad campum conferat, quippe qui unice a lente media pendebit, eritque

$$\Phi=\frac{1}{4(m+1)} \text{ seu } \Phi=\frac{859}{m+1} \text{ minut.;}$$

tum vero pro loco oculi prodibit eius distantia a lente oculari

$$O=\frac{-\pi' r}{m\Phi}=0$$

seu oculum lenti oculari immediate adplicari oportet. Constructio ergo huius modi telescopii ita se habebit.

Primo distantia focalis α ita est definienda, ut sit

$$\alpha= kmy\sqrt[3]{\mu\left(\lambda m +\lambda''\right)}$$

sumto scilicet $x=my$, et λ ex forma lentis obiectivae, quaecunque fuerit sive simplex sive multiplicata, definitur, ut in capite praecedente est expositum. Circa lentem autem secundam tenendum est, quia ab ea totus campus pendet, eam utrinque aeque convexam formari debere, ut statui possit $\pi=\frac{1}{4}$; quare, cum pro ea sit $q=\frac{\alpha}{m+1}$, semidiameter utriusque faciei erit $=1{,}10\cdot\frac{\alpha}{m+1}$; pro tertia autem lente oculari, quoniam eius apertura plane non in calculum ingreditur, perinde est, quaenam ipsi figura tribuatur, dummodo minimam aperturam recipere possit, quae saltim pupillae sit aequalis. Conveniet igitur statui $\lambda''=1$, ut distantia α minor capi possit, eiusque figura secundum praecepta supra data elaborari poterit.

COROLLARIUM 1

229. Mirum videbitur, quod media lens in ipso loco imaginis constituta nihil plane ad confusionem conferat, cum tamen naturam telescopii tantopere immutet, ut oculum adeo lenti oculari immediate adplicari oporteat eiusque ope margo coloratus destrui possit. Quod eo magis adhuc est mirandum, quod haec lens nihil plane in imagine neque in eius loco vel quantitate immutet.

COROLLARIUM 2

230. In ipsa igitur hac lente media diaphragma ante memoratum constitui debebit, cuius foramen ipsi huius lentis aperturae aequale est capiendum; quin etiam super hac

ipsa lente micrometrum statui poterit tenuissimis scilicet lineis super eius superficie ducendis.

COROLLARIUM 3

231. Videmus porro hanc lentem mediam tantillo minorem esse debere quam lentem ocularem, cum eius distantia focalis sit $q=\frac{\alpha}{m+1}$, huius vero $=\frac{\alpha}{m}$, nihiloque minus campum apparentem manere eundem, ac si simplici lente oculari ut ante uteremur.

SCHOLION 1

232. Introductio huius lentis in ipso loco imaginis collocandae ideo est maximi momenti, quod margini colorato penitus tollendo inserviat. Usus autem huiusmodi lentis astronomis ob aliam rationem iam dudum innotuit, siquidem hoc modo campum apparentem auxerunt; simul autem ingens huius lentis incommodum observarunt in eo constans, quod, cum lentis huius quasi substantia se cum imagine permisceat, omnes vel minimae inaequalitates vitri veluti bullulae vel striae a politura relictae cum imagine ipsa uniantur oculoque in pari ratione multiplicatae repraesententur ; quod certe incommodum eo magis est vitandum, quod vix eiusmodi vitri frusta reperire liceat, quae nullis plane inaequalitatibus sint obnoxia. Interim tamen haud difficile erit
has vitri inaequalitates ab ipso obiecto distinguera tubum quodammodo convertendo; tum enim mox apparebit, quid ad obiectum pertineat quidve ad lentem. Istud autem incommodum tantum locum habet, quando lens in ipso imaginis loco collocatur; simulatque ea tantillum inde removetur, illud mox insensibile evadit. Ceterum hanc investigationem ab hoc casu sum exorsus, quod lens in loco imaginis constituta terminum quasi constituat lentium, quae vel propius ad obiectivam vel ad ocularem collocabuntur; quas ideo distingui convenit, quod illae magis ad obiectivam, hae vero magis ad ocularem sint referendae, quemadmodum etiam his, quotquot fuerint, commune nomen lentium ocularium tribui solet, quae appellatio illis lentibus, quae obiectivae sunt propiores, utiquam certe conveniet.

SCHOLION 2

233. Si marginem coloratum non tantopere reformidemus, ut velimus tam insigne campi apparentis augmentum repudiare, casus in solutione memoratus omnem attentionem meretur. Ponamus igitur, ut ibi animadvertimus, $q=\frac{2\alpha}{m+1}$, ut statui possit $\pi=-\pi'=\frac{1}{4}$, et campi apparentis semidiameter erit

$$\Phi=\frac{1}{2(m+1)} \quad \text{sive} \quad \Phi=\frac{1718}{m+1} \text{ minut.},$$

atque tam lentem secundam quam tertiam utrinque fieri oportebit aeque convexam; hac facta positione pro margine colorato tollendo aequatio fiet

$$0=\frac{m+1}{2m},$$

quae cum duplo sit minor quam ea, quae capite praecedente debebat ad nihilum redigi, hic istud lucrum adipiscimur, ut margo coloratus, dum penitus tolli nequit, duplo tamen minor fiat, ita ut vix sensibilis evadat; quod si ergo vitro communi, pro quo $n=1,55$, utamur, limes distantiae focalis lentis obiectivae erit

$$a>kmy\sqrt[3]{0,9381(\lambda m+1,6299)},$$

et pro loco oculi reperitur distantia

$$O=\frac{-\pi'}{m\Phi},$$

quae ob

$$\frac{-\pi'}{\Phi}=\frac{(m+1)q-\alpha}{q}=\frac{m+1}{2} \quad \text{et} \quad r=\frac{\alpha}{m}$$

abit in hanc

$$O=\frac{m+1}{2m}\cdot\frac{\alpha}{m},$$

ita ut iam oculus duplo propius lenti oculari admoveri debeat quam casu praecedentis capitis. Distantia autem huius lentis ab obiectiva est ut ibi $=\frac{m+1}{m}\cdot\alpha$. Unde sequens oritur constructio.

Constructio telescopii ex tribus lentibus compositi
ex eadem vitri specie formatis
pro qua $n=1,55$

I. Lens obiectiva pro lubitu sive simplex existante $\lambda=1$, sive duplicata pro $\lambda=0,1918$, sive triplicata pro $\lambda=0,0422$, sive denique quadruplicata pro $\lambda=-0,0102$ eligatur et ita ut in capite praecedente ex distantia focali α determinetur.

Istius lentis semidiameter aperturae esto $x=my$; intervallum usque ad secundam lentem $=\alpha$.

II. Lentis secundae semidiameter utriusque faciei $=1,10\cdot\frac{2\alpha}{m+1}$.

Eius aperturae semidiameter $=\frac{\alpha}{2(m+1)}$.

Intervallum ad lentem ocularem $\frac{\alpha}{m}$.

III. Lentis ocularis semidiameter faciei utriusque $=1,10\cdot\frac{\alpha}{m+1}$

Eius aperturae semidiameter $=\frac{1}{4}\cdot\frac{\alpha}{m}$

Pro loco oculi eius distantia ab oculari $O=\frac{m+1}{2m}\cdot\frac{\alpha}{m}$.

Campi vero visi semidiameter $=\frac{1718}{m+1}$ minut.

et, ut iam monitum, quantitas α ita est definienda, ut sit

$$a > kmy\sqrt[3]{0,9381\left(\lambda m+1,6299\right)},$$

nisi forte hic valor minor prodeat, quam ut apertura praescripta locum habere possit; quo casu semper distantia focalis ex apertura definiri debet, ut hactenus fecimus.

PROBLEMA 2

234. *Inter lentem obiectivam et imaginem realem eiusmodi lentem constituere, qua omnis confusio ab apertura lentium oriunda destruatur simulque margo coloratus, si fieri queat, tollatur.*

SOLUTIO

Cum hic iterum tres lentes in computum sint ducendae, formula pro multiplicatione dabit $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}$, ubi, cum inter primam et secundam lentem non detur imago realis, sed ea inter secundam et tertiam cadat, fractio $\frac{\alpha}{b}$ erit negativa, at fractio $\frac{\beta}{c}$ erit positiva. Ponamus ergo

$$\frac{\alpha}{b}=-k$$

eritque $\frac{\beta}{c}=\frac{m}{k}$, unde colligimus

$$b=-\frac{\alpha}{k},\quad \beta=Bb=\frac{-B\alpha}{k}\quad\text{et}\quad c=\frac{k\beta}{m}=\frac{-B\alpha}{m}.$$

Intervalla autem, quae debent esse positiva, erunt $\alpha+b=\frac{k-1}{k}\alpha$, ita ut $(k-1)\alpha$ debeat esse positivum, et $\beta+c=-B\alpha(\frac{1}{k}+\frac{1}{m})$, sicque $B\alpha$ debet esse negativum hincque etiam $\frac{B}{k-1}<0$.

His notatis consideremus formulas generales

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=-k\quad\text{ideoque}\quad \pi=\frac{(1-k)\Phi}{\mathfrak{B}}.$$

Tum vero est $\Phi=\frac{\pi-\pi'}{m+1}$; unde patet, ut valor π aliquid conferat ad campum augendum, debere esse $\pi>0$ seu $\frac{1-k}{\mathfrak{B}}>0$; at quia $\frac{B}{k-1}<0$, erit $-\frac{B}{\mathfrak{B}}<0$ ideoque $\frac{\mathfrak{B}}{B}>0$; hoc scilicet requiritur, si campum augere velimus. Nunc consideremus aequationem pro margine colorato tollendo

$$0=\frac{\pi b}{\Phi p}-\frac{\pi'}{m\Phi},$$

quae ob

$$p=\alpha,\;\; b=-\tfrac{\alpha}{k},\;\; \tfrac{\pi}{\Phi}=\tfrac{1-k}{\mathfrak{B}}\;\text{ et }\;\tfrac{\pi'}{\Phi}=\tfrac{1-k}{\mathfrak{B}}-m-1$$

abit in hanc

$$0=\tfrac{(1-k)\Phi}{\mathfrak{B}}\left(\tfrac{1}{k}+\tfrac{1}{m}\right)+\tfrac{(m+1)}{m},$$

unde invenimus

$$\mathfrak{B}=\tfrac{(1-k)(m+k)}{k(m+1)}\;\text{ ideoque }\;B=\tfrac{(1-k)(m+k)}{2km-m+k^2}.$$

Ex his autem valoribus fit

$$\tfrac{B}{\mathfrak{B}}=\tfrac{k(m+1)}{2km-m+k^2}\;;$$

unde patet, ut etiam secunda lens campum augeat, esse debere $2km-m+k^2>0$, ad quod requiritur, ut sit $k>\sqrt{\left(m^2+m\right)}-m$ sive $k>\frac{1}{2}$; cum igitur esse debeat $(k-1)\alpha$ positivum, duo hic casus sunt constituendi:

I. quo $\alpha>0$; tum esse debet $k>1$, unde fit

$$\mathfrak{B}=\tfrac{-(k-1)(m+k)}{k(m+1)}\;\text{ et }\;B=\tfrac{-(k-1)(m+k)}{2km-m+k^2}.$$

Nunc igitur erit

$$\pi=\tfrac{-(k-1)\Phi}{\mathfrak{B}}=\tfrac{+k(m+1)\Phi}{m+k}\;\text{ et }\;\pi'=\tfrac{-m(m+1)\Phi}{m+k}$$

et

$$\tfrac{\pi}{\pi'}=\tfrac{-k}{m},$$

unde patet, si ponatur $-\pi'=\frac{1}{4}$, fore

$$\pi=+\tfrac{1}{4}\cdot\tfrac{k}{m}$$

Ambae ergo fractiones π et π' non aequales sumi poterunt, nisi sit $k=m$, quo casu statui poterit $\pi=\frac{1}{4}$ et $-\pi'=\frac{1}{4}$, ita ut campus fiat maximus.

Tum autem erit

$$b=-\tfrac{\alpha}{m}\;\text{ et }\;\beta=c=\tfrac{2(m-1)\alpha}{m(3m-1)}$$

ob

$$\mathfrak{B}=-\tfrac{2(m-1)}{m+1}\;\text{ et }\;B=-\tfrac{2(m-1)}{3m-1},$$

ita ut nunc sit distantia focalis lentis secundae

$$\mathfrak{B}b=\tfrac{2(m-1)\alpha}{m(m+1)}$$

et lentis tertiae

$$c=\frac{2(m-1)\alpha}{m(3m-1)}$$

II. Sin autem sit $\alpha<0$, debet esse $k<1$ et tamen $k>\frac{1}{2}$, et litterae $\mathfrak{B}$ et B fiunt positivae. Hincque habebitur

$$\frac{\pi}{\Phi}=\frac{k(m+1)}{m+k} \text{ et } \frac{\pi'}{\Phi}=\frac{k(m+1)}{m+k}-m-1=\frac{-m(m+1)}{m+k}$$

ideoque

$$\frac{\pi}{\pi'}=\frac{-k}{m},$$

ita ut ob $k<1$ littera π multo minor sit quam $-\pi'$; ideoque campus apparens hoc casu vix ullum accipiat augmentum.

Nunc denique id, in quo cardo rei versatur, perpendamus, formulam scilicet pro semidiametro confusionis, quae est:

$$\frac{\mu m x^3}{4\alpha^3}\left(\lambda-\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)-\frac{\lambda''}{B^3 m}\right);$$

quae ut ad nihilum redigi queat, necesse est, ut $\mathfrak{B}$ sit quantitas positiva, unde casus prior ante memoratus locum habere nequit, ex quo necesse est, ut sit $k<1$ ideoque etiam $\alpha<0$ et $B>0$; ex quo sequitur capi debere $k>\frac{1}{2}$, ita ut k intra limites $\frac{1}{2}$ et 1 contineri debeat; quare, cum hoc casu sit $\frac{B}{\mathfrak{B}}>0$, campus quoque augmentum quoddam accipiet, propterea quod sit $\pi:-\pi'=k:m$, quod autem vix erit sensibile. Si itaque statuatur semidiameter confusionis $=0$, habebitur

$$\lambda=\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{\lambda''}{B^3 m},$$

ubi notandum est litteras λ et λ'' unitate minores esse non posse.

Quo resolutio huius aequationis clarius perspiciatur, primum observo sumi non posse $k=1$, tum quia duae priores lentes fierent contiguae, tum vero quod prodiret $\mathfrak{B}=0$ et $B=0$; sin autem poneretur $k=\frac{1}{2}$, fieret quidem

$$\mathfrak{B}=-\frac{2m+1}{2(m+1)} \text{ et } B=2m+1;$$

unde nostrae distantiae erunt

$$b=-2\alpha,\ \beta=-2(2m+1)\alpha,\ c=\frac{-2(2m+1)\alpha}{m}$$

ideoque intervalla

$$\alpha + b = -\alpha \text{ et } \beta l + c = -(2m+1)\alpha\left(2+\tfrac{1}{m}\right) = -\tfrac{(2m+1)^2}{m}\cdot\alpha\,;$$

quod posterius in enormem longitudinem excresceret, nisi $-\alpha$ perexiguum caperetur, quod autem fieri nequit, quia primae lentis apertura ob claritatem per se definitur; ex quo manifestum est numerum k intra limites 1 et $\frac{1}{2}$ accipi debere.

COROLLARIUM 1

235. Hoc ergo modo duplicem perfectionem his telescopiis conciliare licet, alteram, qua margo coloratus prorsus destruitur, alteram vero, qua confusio ab apertura oriunda ad nihilum redigitur. Neque vero campo apparenti ullum augmentum sensibile addi potest.

COROLLARIUM 2

236. Quod ad lentium harum aperturas attinet, pro prima quidam erit semidiameter $x = my$; pro secunda autem $\pi q \pm \frac{qx}{\mathfrak{B}p}$; (§ 23) sive

$$\frac{-(1-k)(m+k)x\alpha}{k^2(m+1)} + \frac{x}{k}\,;$$

quia autem est $\pi = -\frac{k}{m}\cdot\pi'$, capique potest $\pi' = -\frac{1}{4}$, siquidem lens ocularis fiat utrinque aequaliter convexa, erit $\pi = \frac{k}{4m}$, ideoque semidiameter aperturae secundae lentis

$$\frac{-(1-k)(m+k)\alpha}{4mk(m+1)} + \frac{x}{k}$$

cuius pars prior prae posteriore quasi evanescit, ita ut sufficiat hanc semidiametrum statuisse $= \frac{x}{k}$, quae utique maior est quam x ob $k < 1$.

Lens autem ocularis utrinque aequaliter convexa esse debet, unde, cum eius distantia focalis sit

$$c = \frac{-(1-k)(m+k)\alpha}{m(2km-m+k^2)},$$

eius pars quarta dabit semidiametrum aperturae.

COROLLARIUM 3

237. Quod autem ad locum oculi attinet post lentem ocularem, eius distantia reperitur

$$O = \frac{-\pi' r}{m\Phi}\,;$$

quia autem est

$$\frac{-\pi'}{\Phi}=\frac{m(m+1)}{m+k} \quad \text{et} \quad r=\frac{-(1-k)(m+k)\alpha}{m(2km-m+k^2)},$$

erit

$$O=\frac{-(1-k)(m+1)\alpha}{m\left(2km-m+k^2\right)},$$

quae est quantitas positiva.

SCHOLION 1

238. Labor certe esset maxime operosus, si hos valores pro B et $\mathfrak{B}$ inventos vellemus in ultima aequatione substituera indeque numeros λ et λ' investigare atque adeo coacti essemus pro quavis multiplicatione calculum de novo suscipere, cui incommodo medela est quaerenda. Perpendamus igitur istos tam complicatos valores pro $\mathfrak{B}$ et B ex aequatione pro margine tollendo esse erutos, ut scilicet illi aequationi summo rigore satisfieret; quoniam autem superfluum est hanc aequationem perfectissime adimplere, propterea quod locus oculi ob aperturam pupillae haud mediocrem latitudinem patitur exiguaque eius mutatione margo coloratus, si quis forte observatur, facillime evitabitur, sufficiet ei quam proxima satisfecisse; quare, cum semper m denotet numerum satis magnum, k autem sit unitate minor, prae m facile licebit k negligera et m quasi infinitum spectare; unde nanciscemur hos valores

$$\mathfrak{B}=\frac{(1-k)}{k}, \qquad B=\frac{1-k}{2k-1},$$

quibus itaque in evolutione nostri problematis utemur; ex iis autem nostra elementa ita simplicius exprimentur:

$$b=-\frac{\alpha}{k}, \quad \beta=\frac{-(1-k)\alpha}{k(2k-1)}, \quad \text{et} \quad c=\frac{-(1-k)\alpha}{m(2k-1)};$$

hinc intervalla

$$\alpha+b=\frac{-(1-k)\alpha}{k} \quad \text{et} \quad \beta+c=\frac{-(1-k)}{2k-1}\left(\frac{1}{k}+\frac{1}{m}\right)\alpha=\frac{-(1-k)(k+m)\alpha}{(2k-1)km}$$

et pro oculi loco

$$O=\frac{-(m+1)(1-k)}{m(2k-1)}\cdot\frac{\alpha}{m}.$$

Lentium autem harum distantiae focales erunt

$$p=\alpha, \quad q=\frac{(1-k)\alpha}{k^2}, \quad r=c=\frac{-(1-k)\alpha}{m(2k-1)}$$

earumque aperturae semidiametri

primae $x = my$, secundae $\frac{x}{k} = \frac{my}{k}$, tertiae $= \frac{1}{4} r = \frac{-(1-k)\alpha}{4m(2k-1)}$.

Campi denique apparentis semidiameter erit

$$\Phi = \frac{\frac{1}{4}\left(1+\frac{k}{m}\right)}{m+1} \quad \text{sive} \quad \Phi = 859\left(\frac{m+k}{m(m+1)}\right) \text{ minut.}$$

Nunc autem aequatio adhuc resolvenda erit

$$\lambda = \frac{1}{1-k}\left(\frac{\lambda' k^2}{(1-k)^2} + \frac{\nu(2k-1)}{1-k}\right) + \frac{\lambda''(2k-1)^3}{(1-k)^3 m}$$

seu

$$\lambda = \frac{1}{(1-k)^3}\left(\lambda' k^2 + \nu(1-k)(2k-1) + \frac{\lambda''(2k-1)^3}{m}\right).$$

Nihil aliud igitur superest, nisi ut pro quibusdam valoribus ipsius k hanc aequationem resolvamus, ubi notandum est λ'' poni debere $=1,6299$, siquidem vitro communi, pro quo est $n = 1,55$, uti velimus; quo casu etiam est $\nu = 0,2326$.

EXEMPLUM 1

239. Statuamus $k = \frac{3}{4}$, ut intra limites suos 1 et $\frac{1}{2}$ medium teneat, et aequatio nostra resolvenda induet hanc formam:

$$\lambda = 64\left(\frac{9\lambda'}{16} + \frac{1}{8}\nu + \frac{\lambda''}{8m}\right) \text{ sive } \lambda = 36\lambda' + 8\nu + \frac{8\lambda''}{m},$$
$$\lambda = 36\lambda' + 1,8608 + \frac{13,0392}{m};$$

quia nunc λ' unitate minus esse nequit, statuamus $\lambda' = 1$ fietque

$$\lambda = 37,8608 + \frac{13,0392}{m},$$

qui valor cum tam sit enormis, nunquam sperandum est ullum artificem huiusmodi lentem parare posse; unde hanc telescopiorum speciem praetermitti conveniet.

EXEMPLUM 2

240. Ut tantos numeros evitemus, sumamus $k = \frac{3}{5}$, ut fiat $1-k = \frac{2}{5}$ et $2k-1 = \frac{1}{5}$, et aequatio nostra fiet

$$\lambda = \frac{125}{8}\left(\frac{9\lambda'}{25} + \frac{2}{25}\nu + \frac{\lambda''}{125m}\right),\ \lambda = \frac{45}{8}\lambda' + \frac{5}{4}\nu + \frac{\lambda''}{8m};$$

sumto igitur $\lambda' = 1$ erit

$$\lambda = 5{,}9157 + \frac{0{,}2037}{m},$$

qui valor etsi satis magnus tamen in praxi tolerari poterit. Interim conveniet singula huic valori $k = \frac{3}{5}$ convenienter definire:

$$b = -\frac{5\alpha}{3}, \quad \beta = -\frac{10\alpha}{3}, \quad c = -\frac{2\alpha}{m}.$$

Hinc intervalla

$$\alpha + b = -\frac{2\alpha}{3} \quad \text{et} \quad \beta + c = -\alpha\left(\frac{10}{3} + \frac{2}{m}\right)$$

et pro aperturis lentium semidiameter primae $= x$, secundae $= \frac{5}{3}x$ et tertiae $= -\frac{\alpha}{2m}$ oculique post lentem distantia

$$O = -\frac{2(m+1)}{m} \cdot \frac{\alpha}{m}.$$

SCHOLION 2

241. Si huiusmodi casus pro variis multiplicationibus evolvere vellemus, ex superioribus intelligitur duos tantum casus sufficere posse, ut inde formulae generales pro quavis multiplicatione elici queant, dum scilicet altero pro m numerus modice magnus veluti 20 assumatur, altero vero numerus quasi infinitus; quae investigatio cum omni attentione digna videatur, eam in sequente problemate instituamus.

PROBLEMA 3

242. *In casu praecedentis problematis si capiatur* $k = \frac{5}{9}$ *, pro quacunque multiplicatione maiore m telescopium construere, in quo non solum margo coloratus prorsus evanescat, sed etiam confusio ex apertura oriunda ad nihilum redigatur.*

SOLUTIO

Cum hic sit $k = \frac{5}{9}$, erit

$$\mathfrak{B} = \frac{4(9m+5)\cdot 9}{9\cdot 9\cdot 5(m+1)} = \frac{4(9m+5)}{45(m+1)}, \quad B = \frac{4(9m+5)\cdot 9^2}{9\cdot 9(9m+25)} = \frac{4(9m+5)}{9m+25}.$$

Nunc igitur duos casus evolvamus, in quorum priore sit $m = 20$, in posteriore vero $m = \infty$.

I. Ob $m = 20$ erit $\mathfrak{B} = \frac{148}{189}$ et $B = \frac{148}{41}$; unde nostra aequatio, quae est

$$\lambda = \frac{9\lambda'}{5\mathfrak{B}^3} + \frac{9\nu}{5\mathfrak{B}B} + \frac{\lambda''}{B^3 m},$$

si omnes lentes ex vitro communi, pro quo $n=1,55$ et $v=0,2326$, lentem autem ocularem utrinque aeque convexam assumamus, ut sit $\lambda''=1,6299$, sequentem induet formam; in subsidium vocatis logarithmis

$$\text{Log.}\mathfrak{B}=9,8937999, \quad \text{Log.}B=0,5574778$$

hincque

$$\text{Log.}\tfrac{1}{\mathfrak{B}}=0,1062000, \quad \text{Log.}\tfrac{1}{B}=\ 9,4425221$$

et

$$\text{Log.}\tfrac{9}{5}=0,2552725:$$

$$\lambda=3,74861\lambda'+0,14812+0,00173;$$
$$\text{Log. } 3,7486\lambda'=0,5738728+\text{Log.}\lambda'.$$

Hic circa numerum λ' observasse iuvabit, quod, cum lens secunda maximam aperturam habere debeat, cuius scilicet semidiameter sit $\frac{9}{5}x=\frac{9}{5}\cdot my$, expediat hanc lentem utrinque aeque convexam reddere; quam ob causam statui oportet

$$\sqrt{(\lambda'-1)}=\tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{\beta-b}{\beta+b}=\tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{B-1}{B+1}$$

hincque

$$\lambda'=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2\cdot\left(\tfrac{B-1}{B+1}\right)^2.$$

Cum autem constet esse $\left(\frac{\sigma-\rho}{2\tau}\right)^2=0,6299$, erit

$$\lambda'=1+0,6299\cdot\left(\tfrac{107}{189}\right)^2 \quad \text{seu} \quad \lambda'=1,20189$$

hincque

$$\lambda=\ 4,50544+0,14812+0,00173, \quad \lambda=4,65529.$$

Unde fit

$$\lambda-1=3,65529 \quad \text{et} \quad \tau\sqrt{\lambda-1}=1,7304.$$

Pro formatione igitur primae lentis habebimus

$$F=\tfrac{\alpha}{\sigma\pm1,7304}=\tfrac{\alpha}{-0,1030}=-9,7087\alpha,$$
$$G=\tfrac{\alpha}{\sigma\mp1,7304}=\tfrac{\alpha}{+1,9211}=+0,52053\alpha,$$

cuius lentis aperturae semidiameter debet esse $x=my$. At intervallum secundae lentis ab hac est

$$\alpha+b=-0,8\alpha .$$

Pro secunda autem lente, cum sit eius distantia focalis

$$q = \mathfrak{B}b = -\tfrac{9}{5}\mathfrak{B}\alpha = -1,4095\alpha,$$

erit semidiameter utriusque faciei $= 1,10q = -1,5504\alpha$, eius aperturae semidiameter $= \tfrac{9}{5}x = \tfrac{9}{5}\cdot my$. Ab hac autem lente ad tertiam intervallum est

$$\beta + c = -B\alpha(\tfrac{m+k}{mk}) = -6,4975\alpha - 36097\cdot\tfrac{\alpha}{m} = -6,6780\alpha.$$

Pro lente tertia, cuius distantia focalis est

$$c = -\tfrac{148}{41}\cdot\tfrac{\alpha}{m} = -3,6097\cdot\tfrac{\alpha}{m},$$

semidiameter faciei utriusque $= 1,1c = -\ 3,9707\cdot\tfrac{\alpha}{m}$; hincque ad oculum usque erit distantia

$$O = -\frac{(m+1)B\alpha}{(m+k)m} = -\tfrac{4\cdot 9\cdot 21}{5\cdot 41}\cdot\tfrac{\alpha}{m} = -3,6878\cdot\tfrac{\alpha}{m}.$$

II. Sit nunc $m = \infty$, erit $\mathfrak{B} = \tfrac{4}{5}, B = -4$, unde nostra aequatio induet hanc formam:

$$\lambda = \tfrac{225}{64}\lambda' + \tfrac{9}{16}v.$$

Hic iterum lentem secundam aequaliter convexam reddamus et ob $\beta = Bb = 4b$ erit

$$\sqrt{(\lambda' - 1)} = \tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{\beta-b}{\beta+b} = \tfrac{\sigma-\rho}{2\tau}\cdot\tfrac{3}{5}$$

Hincque $\lambda' = 1 + 0,6299\cdot\tfrac{9}{25}$; $\lambda' = 1,2267$, ex quo colligimus

$$\lambda = 4,3126 + 0,1308 = 4,4434$$

hincque

$$\lambda - 1 = 3,4434 \text{ et } \tau\sqrt{(\lambda - 1)} = 1,6796;$$

quare sequens habetur constructio:

I. Pro prima lente

$$F = \tfrac{\alpha}{\sigma\pm 1:6796} = \tfrac{\alpha}{-0,0522} = -19,1570\alpha,$$
$$G = \tfrac{\alpha}{\sigma\mp 1,6796} = \tfrac{\alpha}{+1,8703} = +\ \ 0,5346\alpha.$$

Apertura est ut ante aeque ac distantia ad secundam lentem.

II. Pro secunda lente

Quum eius distantia focalis $= -\frac{9}{5}\mathfrak{B}\alpha = -\frac{36}{25}\cdot\alpha = -1,44\alpha$,
fiat semidiameter utriusque faciei $= -1,584\alpha$, eiusque aperturae semidiameter $= \frac{9}{5}x$;
at distantia ad lentem tertiam $= -7,2\alpha - 4\cdot\frac{\alpha}{m}$.

III. Pro tertia lente

cuius distantia focalis $= -4\cdot\frac{\alpha}{m}$,
semidiameter utriusque faciei $= -4,4\cdot\frac{\alpha}{m}$,
eiusque aperturae semidiameter $= -1,1\cdot\frac{\alpha}{m}$.
Ab hac lente ad oculum usque erit distantia $O = -4\cdot\frac{\alpha}{m}$.

His duobus casibus evolutis solutionem quaestionis nostrae generalis pro multiplicatione quacunque m maiore quam 20 ita adstruamus:

I. Pro prima lente

$$\text{statuamus radium faciei}\begin{cases}\text{anterioris} & = -\left(19,1570 + \frac{f}{m}\right)\alpha \\ \text{posterioris} & = +\left(\quad 0,5346 + \frac{g}{m}\right)\alpha\end{cases}$$

et adplicatione ad casum $m = 20$ facta reperietur

$$19,1570 + \frac{f}{20} = 9,7087, \text{ unde fit } f = -188,966;$$

porro

$$0,5346 + \frac{g}{20} = 0,5205, \text{ hinc } g = -0,2820.$$

II. Pro secunda lente

statuatur semidiameter utriusque faciei $= -\left(1,584 + \frac{h}{m}\right)\alpha$,
cumque esse debeat $1,584 + \frac{h}{20} = 1,5504$, erit $h = -0,6720$.
Eius distantia focali existante

$$= -\left(1,440 - \frac{0,6100}{m}\right)\alpha,$$

et aperturae semidiametro $= \frac{9}{5}x$.
Pro distantia ad tertiam lentem inveniemus

$$-\left(7,2-\tfrac{14,0500}{m}\right)\alpha-\left(4,00-\tfrac{7,8060}{m}\right)\tfrac{\alpha}{m}$$

sive

$$-7,200\alpha+10,0500\cdot\tfrac{\alpha}{m}+\tfrac{7,8060}{m^2}\alpha$$

sive

$$-\left(7,200-\tfrac{10,0500}{m}-\tfrac{7,8060}{mm}\right)\alpha.$$

III. Pro tertia lente

cuius distantia focalis reperitur $=-\left(4,00-\tfrac{7,8060}{m}\right)\tfrac{\alpha}{m}$,

sumi debet semidiameter utriusque faciei $=-\left(4,400-\tfrac{8,5860}{m}\right)\tfrac{\alpha}{m}$,

cuius parti quartae semidiameter aperturae aequalis statui potest.

Distantia denique oculi ab hae lente reperitur

$$O=-\left(4-\tfrac{6,2440}{m}\right)\tfrac{\alpha}{m}.$$

Campi vero apparentis semidiameter erit $\tfrac{859}{m+1}$ minut.

COROLLARIUM 1

243. Cum intervallum primae lentis et secundae sit $=-0,8\alpha$, prodibit tota telescopii longitudo ab obiectiva usque ad oculum

$$-\left(8-\tfrac{6,0500}{m}-\tfrac{14,0500}{mm}\right)\alpha,$$

ita ut haec longitudo fere sit octuplo maior quam distantia focalis α, qua circumstantia haec telescopia non admodum commendari merentur.

COROLLARIUM 2

244. Cum primae lentis semidiameter aperturae debeat esse $x=my=\tfrac{m}{50}$, dig., quae autem maior esse nequit parte quarta radii minoris, quae est

$$-0,1336\alpha=-\tfrac{1}{8}\alpha \text{ circiter},$$

patet capi debere $-\alpha>\tfrac{16m}{100}$ vel $-\alpha>0,16m.$

Quia autem lentis secundae semidiameter aperturae esse debet

$$=\tfrac{9}{5}x=\tfrac{9m}{250} \text{ dig.},$$

haec quoque minor esse debet parte quarta radii, quae est $-0,396\alpha$; unde esse debet $-\alpha > 0,0909m$; qui limes cum minor sit praecedente, illum observari oportet.

SCHOLION

245. Cum igitur $-\alpha$ maius esse debeat quam $0,16m$, statuamus $-\alpha = \frac{2}{10}m$ sive $\alpha = -0,2m$ atque sequentem constructionem pro telescopiis huius speciei obtinebimus,

Constructio telescopiorum pro quacunque multiplicatione m lentibus ex vitro communi confectis

I. Pro lente obiectiva in digitis

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= +\,3,8314m - 37,7932\\ \text{posterioris} &= -\,0,1069m + \ \ 0,0564.\end{cases}$$

Eius aperturae semidiameter $= \frac{m}{50}$.

Intervallum ad lentem secundam $= 0,16m$.

II. Pro lente secunda in digitis

Distantia focalis $= +0,2880m - 0,12$.

Semidiameter faciei utriusque $= +\ 0,3168m - 0,13$.

Eius aperturae semidiameter $= \frac{9}{250}m = 0,036m$.

Intervallum ad lentem tertiam $= +1,4400m - 2,01 - \frac{1,56}{m}$.

III. Pro tertia lente in digitis

Distantia focalis $= +0,800 - \frac{1,56}{m}$.

Semidiameter utriusque faciei $= +\ 0,88 - \frac{1,72}{m}$,

cuius pars quarta $= \frac{1}{5}$ dig. dat semidiametrum aperturae.

Hinc ad oculum usque distantia erit

$$O = 0,8 - \frac{1,25}{m}.$$

Campi apparentis semidiameter $= \frac{859}{m+1}$ minut.

Tota autem telescopii longitudo erit

$$= (-1,21 + 1,6m - \tfrac{2,81}{m}) \text{ dig.}$$

Ita v. gr. pro $m=100$ erit longitudo $=158\frac{3}{4}$ dig. sive 13 ped. $2\frac{3}{4}$ dig.

Cum igitur supra tubo unum pedem vix superante fere tantam multiplicationem produxerimus, haec telescopiorum species nunc quidem erit repudianda, etsi respectu vulgarium tuborum astronomicorum maxime foret aestimanda, cum quod nullum marginem coloratum praebeat, tum vero etiam quia confusio ab apertura oriunda prorsus sit sublata. Quamobrem nobis inquiri conveniet, num duabus lentibus inter obiectivam et imaginem collocandis hoc incommodum evitari possit.

PROBLEMA 4

246. *Inter lentem obiectivam et imaginem eiusmodi duas lentes interponere, ut non solum margo coloratus, sed etiam confusio ob apertura oriunda penitus destruatur.*

SOLUTIO

Cum hic quatuor lentes sint considerandae, multiplicatio dabit hanc formulam $m=\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, quarum trium fractionum binae priores negativae, tertia vero affirmativa esse debebit. Statuatur ergo

$$\frac{\alpha}{b}=-k,\quad \frac{\beta}{c}=-k',\quad \text{eritque}\quad \frac{\gamma}{d}=\frac{m}{kk'}.$$

Unde erit

$$b=-\frac{\alpha}{k},\quad c=-\frac{\beta}{k'},\quad d=\frac{\gamma kk'}{m};$$

praeterea vero est $\beta=Bb,\ \gamma=Cc$, unde omnia haec elementa ex α ita definientur:

$$b=-\frac{\alpha}{k},\quad \beta=-\frac{B\alpha}{k},\quad c=\frac{B\alpha}{kk'},\quad \gamma=\frac{BC\alpha}{kk'},\quad d=\frac{BC\alpha}{m};$$

hinc intervalla lentium fient

$$1.\quad \alpha+b=\frac{k-1}{k}\cdot\alpha$$

$$2\cdot\quad \beta+c=\frac{B\alpha}{k}\left(\frac{1-k'}{k'}\right)$$

$$3.\quad \gamma+d=BC\alpha\left(\frac{1}{kk'}+\frac{1}{m}\right),$$

unde, quia $k,\ k'$ et m sunt per se numeri positivi, hae sequuntur conditiones:

$$1.\quad \alpha(k-1)>0,\quad 2.\quad B\alpha(1-k')>0,\quad 3.\quad BC\alpha>0,$$

quae eliso α reducuntur ad has duas

$$4.\ \frac{B(1-k')}{k-1}>0,\quad 5.\ \frac{C}{1-k'}>0 \quad \text{seu} \quad C(1-k')>0.$$

Iam ex superioribus vidimus marginem coloratum destrui non posse, nisi ante fractiones π, π' et π'' definiantur; quem in finem sequentes aequationes considerari debent:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=-k,\quad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{B\alpha}{c}=kk' \quad \text{et} \quad \Phi=\frac{\pi-\pi'+\pi''}{m+1},$$

ex quibus assequimur

$$\frac{\pi}{\Phi}=\frac{1-k}{\mathfrak{B}},\quad \frac{\pi'}{\Phi}=\frac{1-k}{\mathfrak{B}\mathfrak{C}}+\frac{kk'-1}{\mathfrak{C}},\qquad \frac{\pi''}{\Phi}=\frac{\pi'}{\Phi}-\frac{\pi}{\Phi}+m+1=\frac{1-k}{\mathfrak{B}C}+\frac{kk'-1}{\mathfrak{C}}+m+1;$$

quibus valoribus substitutis ad marginem coloratum tollendum requiritur haec aequatio divisione per $\frac{dn}{n-1}$ facta:

$$0=-\frac{\pi}{\Phi}\cdot\frac{1}{k}+\frac{\pi'}{\Phi}\cdot\frac{1}{kk'}+\frac{\pi''}{\Phi}\cdot\frac{1}{m}$$

sive

$$0=\frac{k-1}{\mathfrak{B}k}+\frac{1-k}{\mathfrak{B}\mathfrak{C}kk'}+\frac{kk'-1}{\mathfrak{C}kk'}+\frac{1-k}{\mathfrak{B}Cm}+\frac{kk'-1}{\mathfrak{C}m}+\frac{m+1}{m},$$

ex qua aequatione vel $\mathfrak{B}$ vel $\mathfrak{C}$ definiri potest; tum vero, ut semidiameter confusionis ad nihilum redigatur, debet esse

$$0=\lambda-\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{1}{\mathfrak{B}^3\mathfrak{C}kk'}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)+\frac{\lambda'''}{B^3C^3m};$$

quae ut resolvi possit, littera $\mathfrak{B}$ debet esse positiva, vel si $\mathfrak{B}$ esset negativum, ob *B* quoque negativum littera $\mathfrak{C}$ debet esse positiva.

COROLLARIUM 1

247. Aequatio pro margine colorato tollendo ad hanc formam reducitur:

$$0=\frac{1-k}{\mathfrak{B}\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1-k}{\mathfrak{B}}\left(\frac{1}{k}+\frac{1}{m}\right)+\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}$$

seu ad hanc:

$$0=\frac{1-k}{\mathfrak{B}}\left(\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1}{k}-\frac{1}{m}\right)+\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}$$

unde reperitur

$$\frac{k-1}{\mathfrak{B}}=\frac{\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}}{\left(\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1}{k}-\frac{1}{m}\right)}$$

sive

$$\frac{k-1}{\mathfrak{B}}=\frac{(kk'-1)(m+kk')+(m+1)kk'\mathfrak{C}}{m+kk'-k'm\mathfrak{C}-kk'\mathfrak{C}},$$
$$\frac{k-1}{\mathfrak{B}}=\frac{(kk'-1)(m+kk')+\mathfrak{C}kk'(m+1)}{m+kk'-\mathfrak{C}k'(m+k)},$$
$$\frac{k-1}{\mathfrak{B}}=kk'-1+\frac{\mathfrak{C}k'(m(k-1)+kk'(k+m))}{m+kk'-\mathfrak{C}k'(m+k)}.$$

COROLLARIUM 2

248. Si haec aequatio statim a fractionibus liberetur, habebitur

$$0=(1-k)(m+kk')-\mathfrak{C}(1-k)k'(m+k)+\mathfrak{B}(kk'-1)(m+kk')+\mathfrak{B}\mathfrak{C}kk'(m+1),$$

unde reperitur

$$\mathfrak{C}=\frac{(m+kk')(k-1+\mathfrak{B}(1-kk'))}{\mathfrak{B}kk'(m+1)+(k-1)k'(m+k)}.$$

SCHOLION

249. Inprimis autem notatu dignus est casus, quo numerus B fit infinitus et numerus $O=0$, quem supra iam alia occasione evolvimus [§ 183]; quae operatio cum supra difficilior sit visa, nunc sequenti modo planiore expediatur. Considerabimus scilicet numerum B ut praegrandem sitque $B=\frac{1}{\omega}$, denotante ω fractionem minimam, ita ut w loco B in calculum introducatur. Tum igitur erit $\mathfrak{B}=\frac{1}{1+\omega}$ co iam, ne secundum intervallum $\beta+c$ nimis excrescat, statuatur

$$\beta+c=\frac{\eta\alpha}{k}$$

eritque

$$c=\frac{\eta\alpha}{k}-\beta,\quad \frac{\beta}{c}=-k'=\frac{k\beta}{\eta\alpha-k\beta},$$

et quia est

$$\beta k=-B\alpha=-\frac{\alpha}{\omega},$$

erit

$$+k'=\frac{1}{\eta\omega+1}\ \text{ et }\ 1-k'=\frac{\eta\omega}{1+\eta\omega},$$

ita ut nunc loco litterae k' in calculum introducatur littera η; denique, ne tertium intervallum nimis excrescat ob $B=\frac{1}{\omega}$, statuamus $C=\theta\omega$, ut fiat $BC=\theta$; ita ut hic loco litterae C θ in calculum ingrediatur. Hinc erit $\mathfrak{C}=\frac{\theta\omega}{1+\theta\omega}$ atque hinc porro

$$\frac{\pi}{\Phi}=(1-k)+\omega(1-k),$$

$$\frac{\pi'}{\Phi}=\frac{1+\theta\omega}{(1+\eta\omega)\theta}\big(1-k-\eta k+\eta\omega(1-k)\big)$$

hincque posito $\omega=0$ erit

$$\frac{\pi}{\Phi}=(1-k),\quad \frac{\pi'}{\Phi}=\frac{1}{\theta}(1-k-\eta k)\ \text{ hincque }\ \frac{\pi''}{\Phi}=\frac{1-k-\eta k}{\theta}+k+m$$

Quare, cum pro margine tollendo inventa sit haec aequatio:

$$0=-\frac{\pi}{\Phi}\cdot\frac{1}{k}+\frac{\pi'}{\Phi}\cdot\frac{1}{kk'}+\frac{\pi''}{\Phi}\cdot\frac{1}{m},$$

ob $k'=1$, si illi valores substituantur, prodibit

$$0=\frac{k-1}{k}+\frac{1-k-\eta k}{k\theta}+\frac{1-k-\eta k}{m\theta}+\frac{k+m}{m}$$

sive

$$0=m\theta\big(k-1\big)+m\big(1-k-\eta k\big)+k\big(1-k-\eta k\big)+k\theta\big(k+m\big),$$

$$0=\theta\Big(k^2+\big(2k-1\big)m\Big)+\big(k+m\big)\big(1-k-\eta k\big)$$

hincque invenietur

$$\theta=\frac{+\big(k+m\big)\big(k+\eta k-1\big)}{\big(2k-1\big)m+k^2};$$

hinc autem nostra elementa erunt

$$b=-\frac{\alpha}{k},\quad \beta=\infty,\quad c=\infty,\quad \gamma=\frac{\theta\alpha}{k},\quad d=\frac{\theta\alpha}{m}$$

et intervalla

$$\alpha+b=\frac{k-1}{k}\cdot\alpha,\qquad \beta+c=\frac{\eta\alpha}{k},\qquad \gamma+d=\theta\alpha\Big(\frac{1}{m}+\frac{1}{k}\Big),$$

quae debent esse positiva; ideoque

$$\eta\big(k-1\big)>0,\qquad \theta(k-1)>0\quad\text{et}\quad \theta\eta>0.$$

Pro loco oculi autem habebimus

$$O=\frac{\pi''}{m\Phi}\cdot d=\frac{1-k-\eta k}{mm}\cdot\alpha+\frac{\big(k+m\big)\theta\alpha}{m^2}$$

et valore pro θ substituto

$$O=\frac{1-k-\eta k}{m^2}\cdot\alpha+\frac{(k+m)^2(k+\eta k-1)\alpha}{m^2((2k-1)m+k^2)},$$
$$O=\frac{m+1}{m}\cdot\frac{k+\eta k-1}{(2k-1)m+k^2}\cdot\alpha;$$

his denique observatis resolvenda restat haec aequatio:

$$0=\lambda-\frac{\lambda'}{k}+\frac{\lambda''}{\theta^3 k}+\frac{\lambda'''}{\theta^3 m},$$

quae, cum secundum membrum per se sit negativum, facile resolvi poterit, id quod in sequente problemate ostendemus.

PROBLEMA 5

250. *In casu praecedentis problematis si binae priores lentes ita fuerint comparatae, ut radii iterum paralleli evadant, constructionem huiusmodi telescopiorum exponere.*

SOLUTIO

Cum hoc casu fiat $B=\infty$ et $C=0$, in scholio praecedente elementa iam sunt definita, unde ea hic repetera superfluum foret; quo autem clarius solutionem evolvamus, duo sunt casus perpendendi, alter, quo distantia α est positiva, alter, quo ea est negativa.

I. Sit igitur $\alpha>0$ debebitque esse $k>1$, $\eta>0$ et $\theta>0$, quae ultima conditio sponte impletur; sitque etiam O positivum, tum vero fiet

$$\frac{\pi}{\Phi}<0 \text{ et } \frac{\pi'}{\Phi}<0,$$

nempe

$$\frac{\pi'}{\Phi}=\frac{1-k-\eta k}{\theta}=\frac{-(2k-1)-k^2}{k+m}.$$

Ex ultima igitur formula colligetur semidiameter campi visi

$$\Phi=\frac{\pi''\theta}{1-k-\eta k+(k+m)\theta} \quad \text{seu} \quad \Phi=\frac{(m+k))\pi''}{m(m+1)},$$

substituto scilicet valore θ, si modo praecedentes formulae non praebeant campum minorem Ad quod diiudicandum comparentur valores π et π' cum π'' et ob $\frac{\pi''}{\Phi}=\frac{m(m+1)}{m+k}$ erit

$$\frac{\pi}{\pi''}=\frac{(1-k)(m+k)}{m(m+1)} \quad \text{et} \quad \frac{\pi'}{\pi''}=\frac{-(2k-1)m-k^2}{m(m+1)}$$

hincque

$$\frac{\pi-\pi'}{\pi''}=\frac{k}{m};$$

at ex illis formulis patet tam π quam π' minores esse quam π'', dummodo sit k minus quam, $\frac{5}{12}m$, et cum sit $\Phi=\frac{\pi-\pi'+\pi''}{m+1}$, ob $\frac{\pi-\pi'}{\pi''}>0$ campus apparens hinc aliquod augmentum accipiet eritque $\Phi=\frac{k+m}{m(m+1)}\pi''$, qui utique maior est quam simplex, scilicet $\Phi=\frac{\pi''}{m+1}$, idque in ratione $m+k:m$.

Iam porro aequatio resolvenda est ut ante.

II. Sin autem α sit negativum, fieri debet $k<1$, $\eta<0$, $\theta<0$, ad quod necessarium est, ut sit $k>\frac{1}{2}$.

Quia nunc pro casu praecedente habuimus $\frac{\pi-\pi'}{\pi''}=\frac{k}{m}$, hinc campus apparens multo minus augmentum accipit in hoc casu quam in illo, quod adeo vix erit sensibile, et pro loco oculi distantia O etiam hoc casu fit positiva; quam ob causam casus prior huic posteriori anteferendus videtur.

Etsi autem priori casu campus apparens notabiliter augeri posse est inventus, dum scilicet k usque ad valorem $\frac{5}{12}m$ augetur, tamen resolutio nostrae aequationis hoc non permittit, quoniam numerus λ' nimis magnus accipi deberet, quocirca littera k vix ultra binarium vel ternarium ad summum crescere potest, uti in subiunctis exemplis magis fiat manifestum, quae ex casu priore derivabimus, quoniam facile est praevidere posteriorem casum eo etiam vitio esse laboraturum, quod longitudo telescopii nimis excrescat.

EXEMPLUM

251. Statuamus $k=2$ et multiplicationem $m=50$, quandoquidem hic de tubis astronomicis agitur, eritque

$$\theta=\frac{52(1+2\eta)}{154}=\frac{26}{77}(1+2\eta);$$

qui valor ne fiat nimis parvus, quia tum in nostra aequatione terminus $\frac{\lambda''}{\theta^3 k}$ fieret nimis magnus, ita ut λ' enormem adipisceretur valorem, statuamus insuper $\eta=1$, ut fiat $\theta=\frac{78}{77}$, hincque elementa nostra ita se habebunt:

$$b = -\frac{\alpha}{2}, \quad \beta = \infty, \quad c = -\infty, \quad \beta + c = \frac{\alpha}{2}, \quad \gamma = \frac{39}{77}\alpha, \quad d = \frac{39\alpha}{25\cdot 77};$$

iam vero aequatio resolvenda ita est comparata:

$$0 = \lambda - \frac{\lambda'}{2} + \frac{77^3\lambda''}{78^3\cdot 2} + \frac{77^3\lambda'''}{78^3\cdot 50}$$

hincque

$$\lambda' = 2\lambda + \frac{77^3\lambda''}{78^3} + \frac{77^3\lambda'''}{78^3\cdot 25}.$$

Iam ut tam prima lens quam ultima maximam admittat aperturam, ponamus $\lambda = \lambda''' = 1,6299$, dum scilicet omnes lentes ex vitro communi $n = 1,55$ factae assumuntur, at λ'' sit $= 1$; quibus positis colligemus

$$\lambda' = 3,2598 + 0,9620 + 0,0628, \quad \lambda' = 4,2846,$$

hinc ergo

$$\lambda' - 1 = 3,2846 \text{ et } \tau\sqrt{(\lambda' - 1)} = 1,64031;$$

quare constructio singularum lentium ita se habebit:

I. Pro prima lente

quae cum sit aeque utrinque convexa eiusque distantia focalis $= \alpha$, erit semidiameter utriusque faciei $= 1,10\alpha$, tum vero eius semidiameter aperturae $x = my = 1$ dig. ob m = 50 et $y = \frac{1}{50}$ dig. et intervallum ab hac lente ad secundam $= \frac{1}{2}\alpha$.

II. Pro secunda lente

ob $\beta = \infty$ erit

$$F' = \frac{b}{\rho \pm 1,64031} = \frac{b}{1,8310}, \; G' = \frac{b}{\sigma \mp 1,64031} = \frac{b}{-0,0129},$$

hinc

$$F' = -0,2730\alpha, \; G' = +38,7597\alpha;$$

tum vero semidiameter eius aperturae $= \frac{1}{2}$ dig. ex § 23 et intervallum ad lentem sequentem $= \frac{1}{2}\alpha$.

III, Pro tertia lente

ob $c = -\infty$ et $\lambda'' = 1$ erit

$$F'' = \frac{\gamma}{\sigma} = 0,31123\alpha, \quad G' = \frac{\gamma}{\rho} = 2,6559\alpha,$$

tum vero aperturae semidiameter $= \frac{1}{2}$ dig. et intervallum ad sequentem lentem $\frac{78\cdot 13}{77\cdot 26}\alpha$ seu $= \frac{1}{2}\alpha$ proxime.

IV. Pro quarta lente

semidiameter utriusque faciei $=1{,}10d$ existante $d=\frac{39}{25\cdot 77}\alpha$, cuius pars quarta dat semidiameter aperturae; hinc denique intervallum usque ad oculum erit $\frac{3\cdot 51}{50\cdot 154}\alpha=\frac{1}{50}\alpha$ proxime.

Quod ad distantiam α attinet, si ad solam primam lentem respiceremus, quia ea aperturam admittit, cuius semidiameter $=\frac{1}{4}\alpha$, sumi posset $\alpha=4$ dig.; sed ad secundam lentem respiciendo, cuius minor semidiameter est $=\frac{1}{4}\alpha$, huius pars quarta $=\frac{1}{16}\alpha$, semidiametro aperturae $\frac{1}{2}$ dig. posita dabit $\alpha=8$ dig., quam mensuram etiam retinere oportet; unde longitudo telescopii excederet 12 dig. Huius rei causa est, quod primam lentem utrinque aeque convexam assumsimus.

Adiungamus igitur aliam insuper solutionem sumendo $\lambda=1$; unde fit

$$\lambda'=2+0{,}9620+0{,}0628,\quad \lambda'=3{,}0248,$$
$$\lambda'-1=2{,}0248 \quad \text{et} \quad \tau\sqrt{(\lambda'-1)}=1{,}2879;$$

unde haec sequitur lentium constructio:

I. Pro prima lente

$$F=\frac{\alpha}{\sigma}=0{,}6145\alpha,\quad G=\frac{\alpha}{\rho}=5{,}2439\alpha.$$

II. Pro secunda lente

$$F'=\frac{b}{0{,}1907\pm 1{,}2879}=\frac{-0{,}5\alpha}{1{,}4786},\quad G'=\frac{b}{1{,}6274\mp 1{,}2879}=\frac{-0{,}5\alpha}{0{,}3395},$$

$$F'=-0{,}3382\alpha,\quad G'=-1{,}4723\alpha.$$

Reliqua manent ut ante. Hic igitur statim patet secundam lentem debitam aperturam $\frac{1}{2}x$ recipere posse, si prima patiatur aperturam x. Primae autem semidiameter minor, cum sit circiter $\frac{6}{10}\alpha$, eius pars quarta $\frac{3}{20}\alpha$ ipsi $x=1$ dig. aequata dat $\alpha=\frac{20}{3}$ dig. $=6\frac{2}{3}$ dig.; quin etiam tertia lens postulat, ut sit $\frac{3}{40}\alpha=\frac{1}{2}$ dig., unde α iterum $6\frac{2}{3}$ dig., sicque tota telescopii longitudo vix superabit 10 dig.

Quocirca notari merebitur sequens constructio telescopii quinquagies multiplicantis lentibus ex vitro communi paratis.

I. Pro prima lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} = 4,10 \text{ dig.}\\ \text{posterioris} = 34,96 \text{ dig.}\end{cases}$$

Aperturae semidiameter $=1$ dig.

Intervallum ad secundam lentem $=3\frac{1}{3}$ dig.

II. Pro secunda lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} = -2,25 \text{ dig.}\\ \text{posterioris} = -9,82 \text{ dig.}\end{cases}$$

Semidiameter aperturae $=\frac{1}{2}$ dig.

Intervallum ad tertiam lentem $=3\frac{1}{3}$ dig.

III. Pro tertia lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} = 2,07 \text{ dig.}\\ \text{posterioris} = 17,71 \text{ dig.}\end{cases}$$

Semidiameter aperturae $=\frac{1}{2}$ dig.

Intervallum ad tertiam lentem $=3\frac{1}{3}$ dig.

IV. Pro quarta lente

Radius utriusque faciei $=0,15$ dig.

Semidiameter aperturae $=\frac{3}{80}$ dig.

et distantia oculi $=\frac{2}{15}$ dig. proxime,

unde tota longitudo $=10\frac{2}{15}$ dig.

Campi vero visi semidiameter ut hactenus $\frac{859}{51}$ minut. $=16$ minut. 51 sec.

SCHOLION

252. Maiores multiplicationes calculo hic non subiicio, quia ab huiusmodi telescopiis etiam maior campus quam vulgo exspectari solet. Quamobrem nostram investigationem ad campum apparentem augendum prosequamur idque retentis commodis, quae ternae lentes priores nobis sunt largitae. Hinc possemus valoribus hic assumtis uti, scilicet $k=2$, $\eta=1$ et $\theta=1$, sed

quia hoc modo duo priora intervalla satis fiunt magna, scilicet $\frac{1}{2}\alpha$, quo pacto tota longitudo non parum augetur, praestare videtur haec duo intervalla multo minora efficere, ita ut tantum non evanescant neque lentes se immediate contingere debeant. Hunc in finem pro k numerus unitatem vix superans assumi debebit, unde simul hoc lucrum nanciscimur, ut pro λ' numerus binarium vix superans reperiatur. Statuamus igitur

$$k = 1 + \omega,$$

denotante ω fractionem minimam, ita ut sit

$$b = -\tfrac{\alpha}{1+\omega} = -(1-\omega)\alpha, \quad \alpha + b = \omega\alpha;$$

ob eandemque rationem statuatur etiam $\eta = \omega$, ut secundum intervallum etiam fiat $\omega\alpha$. Quod deinde ad litteram θ attinet, quae hic ex margine colorato est definita, factis his hypothesibus multo minor unitate esset proditura, scilicet $\theta = 2\omega$, qui valor maximis incommodis foret obnoxius; primo enim elementa γ et d evanescerent, nisi α in immensum augeretur, deinde etiam
valor ipsius λ' fieret enormis. Sed probe hic notandum est has hypotheses non casui hic tractato, ubi unica lens ocularis admittitur, destinari, sed propositum nobis esse iis uti in sequentibus, ubi duae pluresve lentes oculares considerabuntur; quibus, cum novae litterae in calculum introducantur, non amplius opus erit ex aequatione marginem coloratum tollente hanc litteram θ
definire, sed eam poterimus ut arbitrariam contemplari, ita ut iam nihil obstet, quominus ponatur $\theta = 1$. Quod autem hunc valorem elegerim, duae sunt causae: altera est, quod, cum distantia γ hic sit $\theta\alpha$, si θ ultra unitatem augeretur, longitudo telescopii maior esset proditura; altera autem suadet, ne θ minus unitate capiatur, quia tum λ' mox enormem valorem esset obtenturum;
sit igitur ratum statuere

$$1.\ k = 1 + \omega, \quad 2.\ \eta = \omega, \quad 3.\ \theta = 1;$$

unde, quotcunque lentes adhibeantur, pro tribus prioribus semper erit

$$b = -(1-\omega)\alpha, \ \beta = \infty, \ 'c = -\infty, \ \beta + c = \omega\alpha, \ \gamma = \alpha.$$

Deinde pro litteris π et π' erit quoque semper

$$\tfrac{\pi}{\Phi} = -\omega, \quad \tfrac{\pi'}{\Phi} = -2\omega$$

ceterum notetur esse

$$B = \infty, \ \mathfrak{B} = 1, \ C = \mathfrak{C} = 0 \text{ et } BC = 1$$

atque hinc aequatio pro margine tollendo semper his duobus terminis exordietur $+\omega-2\omega$; ita ut hi duo termini semper coalescant in $-\omega$. Denique etiam aequatio pro confusione tollenda semper incipiet ab his tribus terminis

$$0=\lambda-\tfrac{\lambda'}{1+\omega}+\tfrac{\lambda''}{1+\omega}\cdots\cdots,$$

unde facile erit calculum pro quotvis lentibus ocularibus prosequi, ubi potissimum nobis erit propositum campum apparentem multiplicare, idque quousque libuerit.

PROBLEMA 6

253. *Tribus lentibus prioribus ita ante imaginem realem dispositis, uti paragrapho praecedente est indicatum, si post imaginem duae lentes constituantur, efficere, ut campus apparens evadat maximus.*

SOLUTIO

Cum hic habeantur quinque lentes, formula pro multiplicatione erit

$$m=-\tfrac{\alpha}{b}\cdot\tfrac{\beta}{c}\cdot\tfrac{\gamma}{d}\cdot\tfrac{\delta}{e},$$

quarum fractionum ista $\frac{\gamma}{d}$ erit positiva, reliquae negativae. Cum igitur sit

$\frac{\alpha}{b}=-1-\omega,\quad \frac{\beta}{c}=-1,$

statuatur

$$\tfrac{\gamma}{d}=i \text{ et } \tfrac{\delta}{e}=-l$$

habebimusque sequentia elementa:

$$b=-\tfrac{\alpha}{1+\omega},\ \beta=-\infty,\ c=\infty,\ \gamma=\alpha,\ d=\tfrac{\alpha}{i},\ \delta=\tfrac{D\alpha}{i},\ e=-\tfrac{D\alpha}{il},$$

existente $m=(1+\omega)il$.

Deinde distantiae focales

$$p=\alpha,\ q=b,\ r=\gamma,\ s=\mathfrak{D}d \text{ et } t=e.$$

Porro intervalla lentium

$$\alpha+b=\omega\alpha,\ \beta+c=\omega\alpha,\ \gamma+d=\tfrac{1+i}{i}\alpha,\ \delta+e=D\left(\tfrac{l-1}{il}\right)\alpha,$$

quorum trium priora cum per se sint positiva, tantum superest, ut sit $D(l-1)$ positivum.

Pro fractionibus π, π' etc. iam habemus

$$\tfrac{\pi}{\Phi}=-\omega,\quad \tfrac{\pi'}{\Phi}=-2\omega$$

ideoque

$$\frac{\pi-\pi'}{\Phi}=\omega.$$

Pro binis reliquis vero habentur hae aequationes:

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{\alpha}{d}=i,$$
$$\frac{\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{D\alpha}{e}=-il=-m.$$

Ex quibus elicitur

$$\frac{\pi''}{\Phi}=\frac{1+i-\omega}{\mathfrak{D}}=\frac{1+i}{\mathfrak{D}},$$
$$\frac{\pi'''}{\Phi}=\frac{1+i}{\mathfrak{D}}+\omega-1-m.$$

Unde pro loco oculi statim habemus

$$O=-\frac{\pi'''}{m\Phi}\cdot t=\left(\frac{1+i}{\mathfrak{D}}+\omega-1-m\right)\frac{D\alpha}{m^2},$$
$$O=\left(\frac{1+i}{\mathfrak{D}}-1-m\right)\frac{D\alpha}{m^2};$$

quae ut fiat positiva, necesse est, ut D sit negativum, adeoque ob $D(l-\lambda)>0$ erit quoque $l<1$. Quare, cum distantia O facta sit positiva, pro margine colorato tollendo habebitur haec aequatio:

$$0=-\omega+\frac{\pi''}{\Phi}\cdot\frac{1}{i}-\frac{\pi'''}{\Phi}\cdot\frac{1}{m}$$

seu

$$0=-\omega+\frac{1+i}{\mathfrak{D}i}-\frac{(1+i)}{\mathfrak{D}m}+\frac{1+m}{m}$$

seu reiectis ω

$$0=\frac{1+i}{\mathfrak{D}}\cdot\frac{l-1}{il}+\frac{1+m}{m}$$

seu

$$0=\frac{(1+i)(l-1)}{\mathfrak{D}}+1+m$$

hinc

$$\mathfrak{D}=\frac{-(1+i)(l-1)}{m+1}=\frac{(1+i)(1-l)}{m+1}$$

et

$$D=\frac{(1+i)(1-l)}{2m-i+l},$$

qui valor debet esse negativus ob $l<1$ hincque esse oporteret

$$l<\frac{i}{2i+1}\text{ sive }l<\frac{1}{2},$$

ita ut hinc esse debeat $i > 2m$ et

$$D = \frac{-(1+i)(1-l)}{i(1-2l)-l}.$$

His circa valores D et l definitis examinemus campum apparentem, cuius semidiameter Φ duplici modo exprimitur:

$$1.\ \ \Phi = \frac{\mathfrak{D}\pi''}{1+i} = \frac{(1-l)\pi''}{m+1}, \quad 2.\ \ \Phi = \frac{\mathfrak{D}\pi'''}{1+i-\mathfrak{D}(m+1)} = \frac{(1-l)\pi'''}{(m+1)l},$$

quorum minor tantum locum habet, siquidem π'' et π''' maximum valorem qui est circiter $\frac{1}{4}$, obtineant. Cum autem sit $\pi'' : \pi''' = 1 : l$, tantum sumi poterit $\pi' = \pi'' = \frac{1}{4}$:fietque $\pi''' = \frac{1}{4}$ hincque campus prodiret

$$\Phi = \frac{1}{4} \cdot \frac{1-l}{m+1}$$

ideoque minor, quam si lente oculari simplici uteremur, contra nostrum institutum, ita, ut hoc problema pro nostro scopo resolvi nequeat.

IDEM PROBLEMA PRAECEDENS

254. *Ubi ceteris manentibus omnibus tantum quarta lens ante imaginem realem collocatur.*

SOLUTIO

In solutione ergo etiam omnia manebunt ut ante, nisi quod binarum quantitatum i et l signa sint mutanda. Primo ergo erunt elementa

$$b = -\frac{\alpha}{1+\omega},\ \ \beta = -\infty,\ \ c = \infty,\ \ \gamma = \alpha,\ \ d = -\frac{\alpha}{i},\ \ \delta = -\frac{D\alpha}{i},\ \ e = -\frac{D\alpha}{il}.$$

Distantiae focales

$$p = \alpha,\ \ q = -\alpha,\ \ r = \gamma = \alpha,\ \ s = -\frac{\mathfrak{D}\alpha}{i},\ \ t = -\frac{\mathfrak{D}\alpha}{il}.$$

Lentium vero intervalla

$$\alpha + b = \omega\alpha,\ \ \beta + c = \omega\alpha,\ \ \gamma + d = +\frac{(1-i)}{i}\alpha,\ \ \delta + e = \frac{-(l+1)}{il}\cdot D\alpha;$$

e patet esse debere D negativum, at $i > 1$; tum vero notetur esse $m = il$.
Deinde inveniemus

$$\frac{\pi}{\Phi} = -\omega, \quad \frac{\pi'}{\Phi} = -2\omega, \quad \frac{\pi''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}}, \quad \frac{\pi'''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}} - 1 - m$$

hincque pro loco oculi

$$O=\left(-\frac{(1-i)}{\mathfrak{D}}-1-m\right)\frac{D\alpha}{mm},$$

qui ergo valor est positivus ob $D<0$. Quare, ut margo coloratus evanescat, debet esse

$$\mathfrak{D}=\frac{-(1-i)(1+l)}{m+1},\quad D=\frac{-(i-1)(1+l)}{2m+i-l},$$

qui valor, cum sit negativus, conditionibus praecedentibus satisfit, si modo sit $i>1$, atque his valoribus substitutis erit

$$b=-\alpha,\ \beta=-\infty,\ c=\infty,\ \gamma=\alpha,\ d=-\frac{\alpha}{i},$$
$$\delta=\frac{(i-1)(1+l)\alpha}{(2m+i-l)i},\ e=\frac{(i-1)(1+l)\alpha}{(2m+1-l)il},$$

hincque

$$p=\alpha,\ q=-\alpha,\ r=\alpha,\ s=\frac{(i-1)(1+l)\alpha}{(m+1)i},\ t=\frac{(i-1)(1+l)\alpha}{(2m+i-l)il},$$
$$\alpha+b=\omega\alpha,\ \beta+c=\omega\alpha,\ \gamma+d=\frac{i-1}{i}\alpha,\ \delta+e=\frac{(i-1)(1+l)^2\alpha}{(2m+i-l)il},$$
$$\frac{\pi}{\Phi}=-\omega,\ \frac{\pi'}{\Phi}=-2\omega,\ \frac{\pi''}{\Phi}=+\frac{m+1}{l+1},\ \frac{\pi'''}{\Phi}=+\frac{m+1}{l+1}-1-m=\frac{l(m+1)}{1+l}$$

hincque

$$O=\frac{l(i-1)(m+1)}{2m+i-l}\cdot\frac{\alpha}{m^2}.$$

Cum igitur sit $\pi'':\pi'''=1:-l$, pro campo duo casus sunt perpendendi.

I. Si $l>1$, tum poterit capi $\pi'''=-\frac{1}{4}$, ut fiat $\pi''=\frac{1}{4l}<\frac{1}{4}$ hincque semidiameter campi

$$\Phi=\frac{1}{4}\cdot\frac{1+\frac{1}{l}}{m+1}.$$

II. Si $l<1$, capi poterit $\pi''=\frac{1}{4}$, ut fiat $\pi'''=-\frac{l}{4}>-\frac{1}{4}$ hincque

$$\Phi=\frac{1}{4}\cdot\frac{1+l}{m+1}.$$

Utroque ergo casu campus maior erit, quam si unica adesset lens ocularis, quo casu invenimus

$$\Phi=\frac{1}{4}\cdot\frac{1}{m+1}.$$

Maximus igitur campus obtinebitur, si capiatur $l=1$, quo casu ob $il=m$ fit $i=m$, tum vero

$$\Phi = \frac{1}{2(m+1)} = \frac{1718}{m+1} \text{ minut.},$$

qui est duplo maior. Conveniet igitur sumi $l = 1$, si modo resolutio postremae aequationis id permittat, quae est

$$0 = \lambda - \frac{\lambda'}{1+\omega} + \frac{\lambda''}{1+\omega} - \frac{1}{\mathfrak{D}i}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu}{D}\right) - \frac{\lambda''''}{\mathfrak{D}^3 m},$$

ubi, si capiatur $l = 1$, ut sit $i = m$, fit

$$\mathfrak{D} = -2 \cdot \frac{m-1}{m+1}, \quad D = -2 \cdot \frac{m-1}{3m-1};$$

quare, si m sit numerus praemagnus, erit $\mathfrak{D} = -2$, $D = -\frac{2}{3}$, ex quo manifestum est resolutionem illius aequationis hoc modo non solum non impediri, sed et adiuvari, ita ut haec positio $l = 1$ nostro scopo maxime conveniat.
Hinc ergo consequimur

$$\lambda' = (1+\omega)\lambda + \lambda'' - \frac{\lambda'''}{\mathfrak{D}^3 m} - \frac{\lambda''''}{D^3 m} - \frac{\nu}{\mathfrak{D}Dm},$$

ad quam resolvendam primo notetur, quia duae postremae lentes maximam requirunt aperturam, eas utrinque aeque convexas capi debere; unde pro ultima lente sumi debebit $\lambda'''' = 1,6299$, pro penultima vero habetur

$$\sqrt{(\lambda''' - 1)} = \frac{\sigma-\rho}{2\tau} \cdot \frac{\delta-d}{\delta+d} = \frac{\sigma-\rho}{2\tau} \cdot \frac{-5m+3}{m+1}.$$

Cum nunc sit

$$\left(\frac{\sigma-\rho}{2\tau}\right)^2 = 0,6299,$$

erit

$$\lambda''' = 1 + 0,6299 \cdot \left(\frac{-5m+3}{m+1}\right)^2;$$

deinde vero sumamus $\lambda = 1$ et $\lambda'' = 1$, pro ω autem commode sumi posse videtur $\omega = \frac{1}{m}$, quoniam hoc modo intervalla lentium priorum non fiunt nimis parva, quam ut in praxi locum habere queant.

COROLLARIUM 1

255. Quodsi ergo statuamus $l = 1$, ut sit $i = m$, tum vero $\omega = \frac{1}{m}$, nostra elementa ita se habebunt:

$$b = -\frac{m\alpha}{m+1}, \ \beta = -\infty, \ c = \infty, \ \gamma = \alpha, \ d = -\frac{\alpha}{m}, \ \delta = \frac{2(m-1)\alpha}{m(3m-1)}, \ e = \frac{2(m-1)\alpha}{m(3m-1)},$$

ita ut sit $\delta = e$ et imago realis inter binas lentes postremas media interiaceat.

Distantiae autem focales erunt

$$p = \alpha, \quad q = -\frac{m\alpha}{m+1}, \quad r = \alpha, \quad s = \frac{2(m-1)\alpha}{m(m+1)}, \quad t = \frac{2(m-1)\alpha}{m(3m-1)},$$

intervalla vero lentium

$$\alpha + b = \frac{\alpha}{m}, \quad \beta + c = \frac{\alpha}{m}, \quad \gamma + d = \frac{m-1}{m}\alpha, \quad \delta + e = \frac{4(m-1)\alpha}{m(3m-1)}$$

et

$$O = \frac{mm-1}{3m-1} \cdot \frac{\alpha}{mm}.$$

COROLLARIUM 2

256. Adiecta igitur unica lente hoc insigne commodum feliciter sumus adepti, quod amplitudo campi duplo maior sit facta, quam si unica lente oculari uteremur, ubi probe notandum est, quod haec nova lens adiecta non post imaginem realem, sed ante eam debeat collocari.

SCHOLION 1

257. Quo haec, quae invenimus, commodissime ad praxin accommodemus, methodo iam supra tradita utamur ac primo constructionem telescopii pro multiplicatione quapiam modica veluti $m = 25$ investigemus, deinde vero pro $m = \infty$; ex quorum casuum comparatione non difficulter pro qualibet multiplicatione media constructionem colligera licebit.

EXEMPLUM 1

258. Pro $m = 25$ constructionem telescopii exhibere.

Cum hic sit $m = 25$, erit

$$\mathfrak{D} = -\tfrac{24}{13} = -1{,}84615, \quad D = -\tfrac{24}{37} = -0{,}64865,$$

hinc erit

$$\frac{1-D}{1+D} = \frac{1{,}64865}{0{,}35134} \quad \text{hinc} \quad \text{Log.}\left(\frac{1-D}{1+D}\right)^2 = 1{,}3428018,$$

unde colligitur

$$\lambda''' = 14{,}8699.$$

Iam cum sit

$$\text{Log.}(-\mathfrak{D}) = 0{,}2662669, \quad \text{Log.}(-D) = 9{,}8120104,$$

reperiemus

$$\lambda' = 1{,}04 + 1 + 0{,}094529 + 0{,}23888 - 0{,}00777,$$
$$\lambda' = 2{,}3656, \quad \lambda' - 1 = 1{,}3656 \text{ et } \tau\sqrt{(\lambda' - 1)} = 1{,}0577.$$

Constructio igitur lentium ita se habebit:

I. Pro prima lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,6145\alpha\\ \text{posterioris} &= 5,2439\alpha.\end{cases}$$

Semidiameter aperturae $=\frac{25}{50}\,\text{dig.}=\frac{1}{2}\,\text{dig.}$

Intervallum ad lentem sequentem $=\frac{1}{25}\alpha = 0,04a.$

II. Pro secunda lente

calculus ita se habebit:

$$F = \frac{b}{\rho\pm 1,0577} = \frac{-0,96\alpha}{1,2484},\quad G = \frac{b}{\sigma\mp 1,0577} = \frac{-0,96\alpha}{0,5697}$$

seu

$$F = -0,7690\alpha,\ G = -1,6851\alpha.$$

Intervallum ad sequentem ut ante $= 0,04\alpha$.

III. Pro tertia lente

Cum eius distantia focalis sit revera

$$\gamma = \frac{\alpha}{1+\omega} = (1-\omega)\alpha \quad \text{et} \quad \lambda'' = 1,$$

ex prima lente haec ita definitur:

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,5900\alpha\\ \text{posterioris} &= 5,0342\alpha.\end{cases}$$

Intervallum ad quartam $= \alpha - \frac{2\alpha}{m} = 0,92\alpha.$

IV. Pro quarta lente

cuius distantia focalis est $1,84615\cdot\frac{\alpha}{m}$, quia debet esse utrinque aeque convexa, erit utriusque faciei radius $= 2,03076\cdot\frac{\alpha}{m}$.

Semidiameter aperturae $= 0,50769\cdot\frac{\alpha}{m}$.

Intervallum ad sequentem $= 1,29730\cdot\frac{\alpha}{m}$.

V. Pro quinta lente

cuius distantia focalis $= 0,64865\frac{\alpha}{m}$,

erit semidiameter utriusque faciei $= 0,71851\cdot\frac{\alpha}{m}$.

Semidiameter aperturae $= 0,17838\cdot\frac{\alpha}{m}$.

Hinc intervallum ad oculum usque erit $=0,3373\cdot\frac{\alpha}{m}$ existante $m=25$ et campi apparentis semidiameter erit

$$\Phi=\tfrac{1718}{26}\text{minut.}=66\text{ minut.}$$

et longitudo totius instrumenti

$$=\alpha+1,6346\cdot\tfrac{\alpha}{m}=1,06538\alpha.$$

EXEMPLUM 2

259. Si $m=\infty$, constructionem telescopii describere.
Erit igitur

$$\mathfrak{D}=-2,\ \ D=-\tfrac{2}{3}\quad\text{et}\quad\omega=0,\ \ \tfrac{1-D}{1+\mathrm{D}}=5;$$

unde fit

$$\lambda'''=1+0,6299\cdot25=16,75.$$

Hincque colligitur

$$\lambda'=1+1=2,\ \ \lambda'-1=1\ \text{ideoque}\ \ \sqrt{(\lambda'-1)}=\tau=-0,9051.$$

Constructio igitur lentium ita se habebit:

1. Pro prima lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,6145\alpha\\ \text{posterioris} &= 5,2439\alpha.\end{cases}$$

Semidiameter aperturae $=\frac{m}{50}$ dig.

Intervallum ad lentem sequentem $=\frac{\alpha}{m}$.

II. Pro secunda lente

cuius distantia focalis $b=-\alpha$, habebimus

$$F=\ \frac{b}{\rho\pm0,9051}=\frac{-\alpha}{1,0958},\ \ G=\frac{b}{\sigma\mp0,9051}=\frac{-\alpha}{0,7223}.$$

Hinc

$$F=-0,91257\alpha,\quad G=-1,38446\alpha.$$

Intervallum ad sequentem $=\frac{\alpha}{m}$.

III. Pro tertia lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,6145\alpha\\ \text{posterioris} &= 5,2439\alpha.\end{cases}$$

Intervallum ad sequentem lentem $=\alpha-\frac{2\alpha}{m}$.

IV. Pro quarta lente

cuius distantia focalis est $\mathfrak{D}d=+\frac{2\alpha}{m}$,

erit radius utriusque faciei $=2,2\cdot\frac{\alpha}{m}$.

Intervallum $=\frac{4}{3}\cdot\frac{\alpha}{m}=1,333\cdot\frac{\alpha}{m}$.

V. Pro quinta lente

cuius distantia focalis $=0,666\cdot\frac{\alpha}{m}$,

erit semidiameter utriusque faeiei $=0,7333\cdot\frac{\alpha}{m}$.

Intervallum ad oculum $=\frac{1}{3}\cdot\frac{\alpha}{m}$.

EXEMPLUM 3

260. Pro multiplicatione quaconque *m* constructionem huiusmodi telescopii describere.

Hic assumsi omnes lentes ex ea vitri specie parari, pro qua est $n=1,55$.

Ex praecedentibus autem sequens constructio concinnabitur:

I. Pro prima lente

erit ut hactenus

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,6145\alpha\\ \text{posterioris} &= 5,2439\alpha.\end{cases}$$

Semidiameter aperturae $x=\frac{m}{50}$ dig.

Intervallum ad sequentem $=\frac{\alpha}{m}$.

II. Pro secunda lente

ponatur

$$F=-\left(0,91257+\frac{f}{m}\right)\alpha,\qquad G=-\left(1,38446+\frac{g}{m}\right)\alpha;$$

erit autem

$$0,91257 + \tfrac{f}{25} = 0,7690, \qquad 1,38446 + \tfrac{g}{25} = 1,6851;$$

unde

$$f = -3,59, \quad g = +7,52.$$

Intervallum $= \tfrac{\alpha}{m}$.

III. Pro tertia lente

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,6145\left(1-\tfrac{1}{m}\right)\alpha\\ \text{posterioris} &= 5,2439\left(1-\tfrac{1}{m}\right)\alpha.\end{cases}$$

Intervallum ad sequentem $= \alpha - \tfrac{2\alpha}{m}$.

IV. Pro quarta lente

statuatur semidiameter utriusque faciei $= \left(2,2 + \tfrac{h}{m}\right)\tfrac{\alpha}{m}$

eritque

$$2,2 + \tfrac{h}{25} = 2,03076; \text{ hinc colligitur } h = -4,231.$$

Intervallum ad sequentem

$$= \left(1,333 + \tfrac{k}{m}\right)\tfrac{\alpha}{m}, \quad \text{hinc } k = -0,90;$$

adeoque intervallum erit

$$= (1,333 - \tfrac{0,90}{m})\tfrac{\alpha}{m}.$$

V. Pro quinta lente

cuius distantia focalis $= \left(0,666 - \tfrac{0,434}{m}\right)\tfrac{\alpha}{m}$,

erit radius utriusque faciei $= \left(0,7333 - \tfrac{0,495}{m}\right)\tfrac{\alpha}{m}$.

Distantia ad oculum $= \left(\tfrac{1}{3} + \tfrac{l}{m}\right)\tfrac{\alpha}{m}$,

unde $l = 0,097$, adeoque haec distantia erit

$$\left(0,333 + \tfrac{0,097}{m}\right)\tfrac{\alpha}{m}$$

et tota telescopii longitudo

$$= \alpha + \left(1,666 - \tfrac{0,803}{m}\right)\tfrac{\alpha}{m} \quad \text{vel} \quad \alpha + 1,666 \cdot \tfrac{\alpha}{m} - 0,803 \cdot \tfrac{\alpha}{m^2}.$$

Perpendamus nunc, quantum valorem ipsi α tribui conveniat, et cum ternae priores lentes ut lens triplicata spectari queant, minimus semidiameter est $0,6145\alpha$, cuius pars quarta $\tfrac{3}{20}\alpha$ ipsi $x = \tfrac{m}{50}$ aequalis posita dat

$$\alpha = \tfrac{2m}{15}\ \text{dig.} = \tfrac{4m}{30}\ \text{dig.}$$

Ponamus igitur $\alpha = \frac{4}{30}m$ dig. et habebitur sequens constructio huiusmodi telescopiorum pro multiplicatione quacunque m, lentibus ex vitro $n = 1,55$ factis.

Circa diaphragma his telescopiis inserendum videatur sequens Scholion 3.

I. Pro prima lente

$$\text{Radius faciei} \begin{cases} \text{anterioris} = 0,08193m \\ \text{posterioris} = 0,69918m. \end{cases} \text{dig.}$$

Semidiameter aperturae $x = \frac{m}{50}$ dig. Intervallum $= \frac{2}{15}$ dig.

II. Pro secunda lente

$$\text{Radius faciei} \begin{cases} \text{anterioris} = \left(-\ 0,1217m + 0,478\right) \text{dig.} \\ \text{posterioris} = \left(-\ 0,18459m - 1,003\right) \text{dig.} \end{cases}$$

Intervallum $= \frac{2}{15}$ dig.

III. Pro tertia lente

$$\text{Radius faciei} \begin{cases} \text{anterioris} = \left(-\ 0,08193m - 0,0819\right) \text{dig.} \\ \text{posterioris} = \left(-\ 0,69918m - 0,699\right) \text{dig.} \end{cases}$$

Intervallum $= \left(\frac{2}{15}m - \frac{4}{15}\right)$ dig.

IV. Pro quarta lente

Semidiameter utrinsque faciei $= \left(0,2933 - \frac{0,5641}{m}\right)$ dig.

Intervallum $= (0,1777 - \frac{0,12}{m})$ dig.

V. Pro quinta lente

Radius utriusque faciei $= \left(0,098 - \frac{0,066}{m}\right)$ dig.

Hinc intervallum ad oculum $= \left(0,044 - \frac{0,013}{m}\right)$ dig.

Longitudo tota $= \left(\frac{2}{15}m + 0,222 - \frac{0,107}{m}\right)$ dig., ita ut pro casu $m=100$ haec longitudo sit $13\frac{1}{2}$ dig., campi denique apparentis semidiameter $=\frac{1718}{m+1}$ minut., seu, quia etiam lentes priores aliquantillum ad campum augendum conferunt, $\Phi=\frac{1718}{m}$ minut., ita ut pro $m=100$ fiat $\Phi=17$ minut. 11 sec.

SCHOLION 2

261. Telescopia haec in suo genere ita omnibus numeris absoluta videntur, ut perfectiora vix desiderari queant, nisi diversas vitri species adhibere velimus. Non solum enim confusionis ab apertura oriundae sunt expertia aeque ac marginis colorati, sed etiam campum apparentem duplo maiorem patefaciunt quam simplicia ac praeterea tam sunt brevia, ut breviora ne sperare quidem liceat. Deinde etiam in exsecutione insigne commodum inde obtineri potest, quod inter tres priores lentes intervalla aliquantillum variari possunt; si enim forte eveniat, ut ob tantillum errorem in praxi commissum hae lentes non exactissime ad intervalla hic praescripta sint accommodata, facile evenire potest, ut iis paulisper mutatis egregium effectum sint praestaturae. Interim tamen semper consultum erit secundam lentem concavam pluries elaborari secundum easdem mensuras; cum enim semper aliquod discrimen deprehendatur, inter plures eiusmodi lente & optima facile eligi poterit. Nihilo minus vero conveniet nostram investigationem ulterius prosequi et in eiusmodi huius generis telescopia inquirere, quorum campus adeo triplo vel quadruplo maior sit proditurus.

SCHOLION 3

262. Quo haec telescopia meliorem effectum praestent, necesse est, ut in loco imaginis verae diaphragma sive septum, quemadmodum supra [§ 224,225] iam est descriptum, cum foramine debitae magnitudinis constituatur. Cadit autem haec imago ob $\delta=e$ praecise in medium intervalli quartae et quintae lentis ideoque ad distantiam $=\left(0,0888-\frac{0,06}{m}\right)$ dig. Deinde cum foramen magnitudini huius imaginis debeat esse aequale et semidiameter imaginis sit in genere

$$=\alpha\Phi BCD=\alpha\Phi D=-2\tfrac{m-1}{3m-1}\cdot\alpha\Phi,$$

debet esse semidiameter foraminis $=-2\frac{m-1}{3m-1}\cdot\alpha\Phi$. Iam cum in nostro exemplo evoluto sit $\alpha=\frac{2}{15}m$ et $\Phi=\frac{1}{2m}$, colligitur ista foraminis semidiameter

$$=\tfrac{1}{15}\cdot\tfrac{2(m-1)}{3m-1}\text{ dig.}=\left(\tfrac{2}{45}-\tfrac{4}{135m}\right)\text{ dig.}=\tfrac{1}{15}\cdot\tfrac{2(m-1)}{3m-1}\text{ dig. }\left(\tfrac{2}{45}-\tfrac{4}{135m}\right)\text{ dig.}$$

Ceterum etsi his telescopiis multo maiorem claritatis gradum conciliavimus, quam vulgo fieri solet, dum sumsimus $y=\frac{1}{50}$ dig., ex Hugenii regulis autem sequitur $y=\frac{1}{73}$ dig., tamen, si quis vereatur, ne hic ob multitudinem lentium claritas notabilem iacturam patiatur, huic incommodo facile medela afferetur mensuras datas tantum quapiam sui parte augendo seu, quod eodem redit, mensuram unius digiti, quam hactenus indefinitam reliquimus, pro lubitu augendo.

PROBLEMA 7

268. *Si praeter tres lentes priores, ut in praecedente problemate sunt constitutae, adhuc una lens ante locum imaginis collocetur, post eam insuper duas lentes ita disponere, ut maximus campus obtineatur.*

SOLUTIO

Cum hic occurrant sex lentes, erit

$$m = \frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}\cdot\frac{\delta}{e}\cdot\frac{\varepsilon}{f},$$

quarum fractionum tres priores sunt negativae, quarta positiva et quinta denuo negativa. Pro prioribus iam sumsimus esse $\frac{\alpha}{b} = -1-\omega$, $\frac{\beta}{c} = -1$. Pro posterioribus vero statuamus $\frac{\gamma}{d} = -k$, $\frac{\delta}{e} = i$ et $\frac{\varepsilon}{f} = -l$, ut sit $m = (1+\omega)\, kli$; unde ob $B = \infty$, $C = 0$ et $BC = 1$ elementa nostra erunt

$$b = -\frac{\alpha}{1+\omega},\ \ \beta = \infty,\ \ c = -\infty,\ \ \gamma = \frac{\alpha}{1+\omega},$$
$$d = -\frac{\alpha}{k},\ \ \delta = -\frac{D\alpha}{k},\ \ e = -\frac{D\alpha}{ki},\ \ \varepsilon = -\frac{DE\alpha}{ki},\ \ f = \frac{DE\alpha}{kil}.$$

Atque hinc distantiae focales reperientur

$$p = \alpha,\ \ q = -(1-\omega)\alpha,\ \ r = \frac{\alpha}{1+\omega},$$
$$s = -\frac{\mathfrak{D}\alpha}{k},\ \ t = -\frac{\mathfrak{E}D\alpha}{ki},\ \ u = \frac{DE\alpha}{kil}.$$

Tum vero intervalla lentium

$$\alpha + b = \omega\alpha,\ \ \beta + c = \omega\alpha,\ \ \gamma + d = \frac{k-1}{k}\cdot\alpha,$$
$$\delta + e = -D\alpha\cdot\frac{i+1}{ki},\ \ \ \varepsilon + f = DE\alpha\frac{1-l}{ki};$$

quae ut prodeant positiva, debet esse

$$1.\ k > 1,\qquad 2. D < 0,\qquad 3.\ +E(l-1) > 0.$$

Litterae π, π' etc. sequenti modo definientur:

$$\frac{\pi}{\Phi} = -\omega,\qquad \frac{\pi'}{\Phi} = -2\omega,$$

et reliquae ex sequentibus formulis determinari debent:

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{BC\alpha}{d}=-k,\quad \frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{BCD\alpha}{e}=-ki,$$

$$\frac{\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}{\Phi}=m;$$

hinc ergo colligimus

$$\frac{\pi''}{\Phi}=\frac{1-k}{\mathfrak{D}},\quad \frac{\pi'''}{\Phi}=\frac{\pi''}{\mathfrak{E}\Phi}-\left(\frac{1+ki}{\mathfrak{E}}\right)\quad \text{et}\quad \frac{\pi''''}{\Phi}=\frac{\pi'''}{\Phi}-\frac{\pi''}{\Phi}+m+1.$$

Nunc cum pro campo apparente sit

$$\Phi=\frac{\pi-\pi'+\pi''-\pi'''+\pi''''}{m+1},$$

is fiet maximus, si sumatur $\pi''=\frac{1}{4}$, $\pi'''=-\frac{1}{4}$, $\pi''''=\frac{1}{4}$; inde enim fiet $\Phi=\frac{3}{4}\cdot\frac{1}{m+1}$, unde illae aequationes dabunt

$$\frac{m+1}{3}=\frac{1-k}{\mathfrak{D}},\quad \frac{-m-1}{3}=\frac{m+1}{3\mathfrak{E}}-\left(\frac{1+ki}{\mathfrak{E}}\right)$$

et

$$\frac{m+1}{3}=\frac{-m-1}{3}+m+1$$

quae est, uti debet, identica.

Unde pro loco oculi sequitur

$$O=\frac{m+1}{3}\cdot\frac{u}{m}=\frac{m+1}{3}\cdot\frac{DE\alpha}{kilm};$$

quae distantia ut fiat positiva, ob $D<0$ debet etiam esse $E<0$ adeoque $l<1$, siquidem assumamus α positivum. Patet autem, si caperemus $l=1$, binas postremas lentes sibi immediate iungi et prodire casum lentis ocularis duplicatae iam supra [§ 215] consideratum Videamus autem, ante quam aequationem pro margine colorato contemplemur, cuiusmodi valores litterae $\mathfrak{D}$ et $\mathfrak{E}$ ex binis aequationibus superioribus obtineant, et ex priori quidem invenitur $\mathfrak{D}=-\frac{3(k+1)}{m+1}$; qui cum sit negativus, etiam D fit negativum, uti oportet; et ex altera fit $\mathfrak{E}=\frac{3ki-m+2}{m+1}$ hincque $E=\frac{3ki-m+2}{2m-3ki-1}$; qui valor cum debeat esse negativus, duo casus sunt considerandi.

I. Si numerator negativus et denominator positivus, erit $3ki+2<m$ et $m>\frac{3ki+1}{2}$.

II. Sin autem numerator sit positivus et denominator negativus, erit $m<3ki+2$ et $m<\frac{3ki+1}{2}$ seu simpliciter $m<\frac{3ki+1}{2}$.

Cum autem sit $m = kil$, ob $l < 1$ erit $m < ki$, unde patet priorem casum locum habere non posse, sed solum secundum, ita ut sit $m < ki$, quo pacto omnes conditiones sunt adimpletae. Sicque nihil impedit, quominus resolutionem aequationis pro margine tollendo suscipiamus, quae praeter exspectationem tam facilis evadet, ut nullae difficultates, quales ante occurrebant, negotium turbent. Haec autem aequatio omissis duobus primis terminis utpote minimis ita se habebit:

$$0 = \frac{\pi''}{\Phi} \cdot \frac{d}{p} + \frac{\pi'''}{\Phi} \cdot \frac{e}{Dp} + \frac{\pi''''}{\Phi} \cdot \frac{f}{Dep},$$

quae ob $\pi'' = -\pi''' = +\pi''''$ abit in hanc:

$$0 = \frac{d}{p} - \frac{e}{Dp} + \frac{f}{Dep};$$

cum nunc sit

$$p = \alpha \quad \text{et} \quad \frac{d}{\alpha} = -\frac{1}{k}, \quad \frac{e}{\alpha} = -\frac{D}{ki}, \quad \frac{f}{\alpha} = \frac{DE}{kil},$$

erit nostra aequatio

$$0 = -\frac{1}{k} + \frac{1}{ki} + \frac{1}{kil}$$

seu per *k* multiplicando

$$0 = -1 + \frac{1}{i} + \frac{1}{il}, \quad 0 = -il + l + 1;$$

hincque $i = \frac{l+1}{2}$, ubi debet esse $l < 1$; hinc $il = 1 + l$ et $m = (1+l)k$, ita ut sit $k = \frac{m}{1+l}$. Deinde erit

$$\mathfrak{D} = \frac{-3(k-1)}{m+1}, \quad \mathfrak{E} = \frac{3ki-m+2}{m+1}$$

ideoque

$$D = \frac{-3(k-1)}{3k+m-2}, \quad E = \frac{3ki-m+2}{2m-3ki-1}.$$

His inventis aequatio pro confusione tollenda erit

$$0 = \lambda - \frac{\lambda'}{1+\omega} + \frac{\lambda''}{1+\omega} - \frac{1}{\mathfrak{D}k}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{v}{D}\right) - \frac{1}{D^3\mathfrak{E}ki}\left(\frac{\lambda''''}{\mathfrak{E}^2} + \frac{v}{E}\right) + \frac{\lambda'''''}{D^3E^3m},$$

quae, ut hactenus est factum, facile resolvitur, quaerendo scilicet valorem ipsius λ'.

SCHOLlON

264. Solutio huius problematis ad sequentes investigationes expediendas maximum adiumentum nobis affert, dum ea nobis novam methodum suppeditat aequationem pro margine colorato tollendo, quae supra insignibus difficultatibus erat involuta, expeditissime resolvendi. Huius methodi autem vis in eo consistit, ut litteris π, π', π'' etc. statim determinatos valores tribuamus, qui quidem ita sint comparati, ut maximum campum apparentem producant. Hoc enim facto istae litterae ex memorata aequatione statim tolluntur et loco litterarum *d, e, f* valores ante inventos substituendo etiam litterae maiusculae sponte ex calculo evanescunt, ita ut tota

aequatio nulla amplius alia elementa involvat praeter litteras *k, i, l;* quarum una inde sine ulla difficultate definitur; deinde vero ex illis valoribus pro litteris π assumtis facile determinantur litterae $\mathfrak{D}$, $\mathfrak{E}$ etc. indeque etiam *D, E* etc., quarum valores in ultimam aequationem translati totum negotium facile conficiunt; quin etiam haec methodus pro prioribus lentibus in usum vocari potest, ubi autem notandum est, quia hae lentes quasi ad obiectivam constituendam concurrunt, ex earum litteris π et π' nihil vel perparum ad campum amplificandum redundare posse. Quocirca his litteris non ut sequentibus valor $\frac{1}{4}$,
sed potius quam minimus, puta $\frac{1}{4}\omega$ et $\frac{1}{4}\omega'$, tribui debet, denotantibus scilicet ω et ω' fractiones quam minimas. Quare, quo haec nova methodus clarius perspiciatur, ea ad sequens problema generale huc spectans solvendum utemur.

PROBLEMA 8

265. *Telescopium huius generis ex sex lentibus construere, quarum tres priores inserviant omni confusioni tollendae, tres autem posteriores campo triplicando, dum scilicet lenti oculari simplici campum simplicem assignamus.*

SOLUTIO

Hic igitur quinque sequentes fractiones considerandae veniunt:
$$\frac{\alpha}{b},\ \frac{\beta}{c},\ \frac{\gamma}{d},\ \frac{\delta}{e},\ \frac{\varepsilon}{f},$$
quarum omnes praeter unicam debent esse negativae. Quare, si statuamus:
$$\frac{\alpha}{b}=-P,\ \frac{\beta}{c}=-Q,\ \frac{\gamma}{d}=-R,\ \frac{\delta}{e}=-S,\ \frac{\varepsilon}{f}=-T,$$
evidens est harum quinque litterarum *P, Q, R, S, T* unicam fore negativam reliquis existentibus positivis. Quaenam autem sit negativa, hic nondum opus est definire. Hoc posito nostra elementa aeque ac distantiae focales cum intervallis lentium sequenti modo conspectui repraesententur:

Distantiae determinatrices		Distantiae focales	Intervalla lentium
		$p=\alpha$	
$b=\frac{-\alpha}{P}$	$\beta=\frac{-B\alpha}{P}$	$q=\mathfrak{B}b$	$\alpha+b=\alpha\left(1-\frac{1}{P}\right)>0$
$c=\frac{B\alpha}{PQ}$	$\gamma=\frac{BC\alpha}{PQ}$	$r=\mathfrak{C}c$	$\beta+c=\frac{-B\alpha}{P}\left(1-\frac{1}{Q}\right)>0$
$d=\frac{-BC\alpha}{PQR}$	$\delta=\frac{-BCD\alpha}{PQR}$	$s=\mathfrak{D}d$	$\gamma+d=\frac{BC\alpha}{PQ}\left(1-\frac{1}{R}\right)>0$
$e=\frac{BCD\alpha}{PQRS}$	$e=\frac{BCDE\alpha}{PQRS}$	$t=\mathfrak{E}e$	$\delta+d=\frac{-BCD\alpha}{PQR}\left(1-\frac{1}{S}\right)>0$
$f=\frac{-BCDE\alpha}{PQRST}$		$u=f$	$\varepsilon+f=\frac{BCDE\alpha}{PQRS}\left(1-\frac{1}{T}\right)>0$

ubi, cum productum *PQRST* sit negativum, pro multiplicatione erit

$$m = -PQRST.$$

Deinde, cum pro campo apparente habeatur

$$\Phi = \frac{\pi - \pi' + \pi'' - \pi''' + \pi''''}{m+1},$$

sit ξ maximus valor, quem hae litterae π, π′ etc. recipere possunt, et statuamus

$$\pi = \omega\xi, \quad \pi' = -\omega'\xi, \quad \pi'' = \xi, \quad \pi''' = -\xi, \quad \pi'''' = \xi,$$

ut sit

$$\Phi = \frac{\omega + \omega' + 3}{m+1}\xi.$$

Cum igitur hinc sit

$$\frac{\pi''''}{\Phi} = \frac{m+1}{\omega + \omega' + 3},$$

pro distantia oculi habebimus

$$O = \frac{\pi''''}{\Phi} \cdot \frac{u}{m} = \frac{m+1}{\omega + \omega' + 3} \cdot \frac{-BCDE\alpha}{PQRSTm}$$

seu

$$O = \frac{m+1}{\omega + \omega' + 3} \cdot \frac{BCDE\alpha}{mm}.$$

Ut igitur *O* fiat positivum, quia $\frac{\pi''''}{\Phi}$, est positivum, debet esse $u > 0$ ideoque ultima lens convexa, quae conditio insuper est probe observanda.

Nunc igitur aequatio pro margine colorato tollendo ita se habebit:

$$0 = \omega \cdot \frac{b}{\alpha} - \omega' \cdot \frac{c}{B\alpha} + \frac{d}{BC\alpha} - \frac{e}{BCD\alpha} + \frac{f}{BCDE\alpha},$$

quae reducitur ad hanc formam:

$$0 = +\omega \cdot \frac{1}{P} + \omega' \cdot \frac{1}{PQ} + \frac{1}{PQR} + \frac{1}{PQRS} + \frac{1}{PQRST},$$

in qua duos priores terminos ob parvitatem negligera licet, ita ut adhuc sit

$$0 = 1 + \frac{1}{S} + \frac{1}{\mathrm{ST}},$$

quae aequatio facile resolvitur, dummodo litterarum *S* et *T* altera sit negativa; unde patet tres priores litteras *P, Q, R* necessario esse positivas.

Nunc ordo postulat, ut etiam litteras $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc. ex aequationibus fundamentalibus determinerons:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=-P,\quad \frac{1}{2}\ \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=PQ,$$

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=-PQR,\quad \frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=PQRS,$$

ex quibus, si brevitatis gratia ponamus

$$\frac{\omega+\omega'+3}{m+1}=M,$$

colligimus

$$\begin{array}{l|l} \mathfrak{B}=\frac{(1-P)M}{\omega} & B=\frac{\mathfrak{B}}{1-\mathfrak{B}} \\ \mathfrak{C}=\frac{(1-PQ)M-\omega}{\omega'} & C=\frac{\mathfrak{C}}{1-\mathfrak{C}} \\ \mathfrak{D}=(1-PQR)M-\omega'-\omega & D=\frac{\mathfrak{D}}{1-\mathfrak{D}} \\ \mathfrak{E}=(1-PQRS)M-\omega'-\omega-1 & E=\frac{\mathfrak{E}}{1-\mathfrak{E}} \end{array}$$

adeoque

$$B=\frac{(1-P)M}{\omega-(1-P)M},\qquad C=\frac{(1-PQ)M-\omega}{\omega'+\omega-(1-PQ)M},$$
$$D=\frac{(1-PQR)M-\omega'-\omega}{1+\omega+\omega'-(1-PQR)M},\quad E=\frac{(1-PQRS)M-\omega'-\omega-1}{2+\omega+\omega'-(1-PQRS)M}.$$

Nunc denique aequatio pro confusione aperturae tollenda considerari debet, quae est

$$0=\lambda-\frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{1}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)-\frac{1}{B^3C^3\mathfrak{D}PQR}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu}{D}\right)$$
$$+\frac{1}{B^3C^3D^3\mathfrak{E}PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^2}+\frac{\nu}{E}\right)-\frac{1}{B^3C^3D^3E^3\mathfrak{F}PQRST}\cdot\lambda'''''; $$

cui aequationi ut satisfieri queat, notandum est terminos post tres priores sequentes admodum fieri parvos. Cum enim sit $PQRST=-m$, hoc est numero ingenti, primi autem factores P et Q vix ab unitate discrepent, necesse est, ut productum RST numerum m fere totum producat; deinde quia inter S et T invenimus aequationem $0=1+\frac{1}{S}+\frac{1}{ST}$, patet numerum m neque in S neque in T contineri ideoque factorem R maximam partem numerum m complecti. Quocirca huius aequationis membra quartum et sequentia prae tribus prioribus quasi evanescent, ita ut tria priora se mutuo propemodum destruere debeant, unde proxima statuendum erit

$$0=\lambda-\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3\mathfrak{C}^3PQ}$$

adeoque

$$\lambda' = \mathfrak{B}^3 P\lambda + \frac{\mathfrak{B}^3\lambda''}{B^3\mathfrak{C}^3 Q},$$

ubi pro felici exsecutione optandum esset, ut prodiret $\lambda = 1$, $\lambda' = 1$ et $\lambda'' = 1$, quia tum leves errores in praxi commissi minimi sunt momenti. Quare, cum sequentes termini parvi sint positivi, necesse erit, ut sit $1 > \mathfrak{B}^3 P + \frac{\mathfrak{B}^3}{B^3\mathfrak{C}^3 Q}$; unde, si esset $P = 1$ et $Q = 1$ et ut supra $B\mathfrak{C} = 1$, deberet esse $1 > 2\mathfrak{B}^3$ seu $\mathfrak{B} < \sqrt[3]{\frac{1}{2}}$ sive $\mathfrak{B} < \frac{4}{5}$; quod praeceptum in adplicatione attendi meretur.

COROLLARIUM 1

266. Cum igitur nunc certum sit quinque litteras *P, Q, R, S, T* omnes esse positivas praeter *S* vel *T*, si sumamus distantiam a semper esse positivam, ex primo intervallo concludimus esse
$P > 1$, et quia hoc intervallum statuitur minimum, *P* parum tantum superabit unitatem, et quia etiam secundum intervallum sumitur minimum, littera *Q* parum quoque ab unitate discrepabit.
Deinde quia quoque *f* debet esse quantitas positiva ideoque productum *BCDE* positivum, ob $\varepsilon = -Tf$ ultimum intervallum fit $(1-T)f$ statimque hanc praebet conditionem $T < 1$, unde, si *T* sit positivum, conditio postulat, ut sit $T < 1$; sin autem *T* sit negativum, nulla restrictione opus est.

COROLLARIUM 2

267. Quia in ultima aequatione omnia membra praeter secundum sunt positiva, ita ut solum secundum omnia reliqua destruere debeat, necesse est, ut $\mathfrak{B}$ sit positivum et propemodum, uti notavimus, valorem habeat unitate aliquantillum minorem. Quare, cum inventum sit $\mathfrak{B} = \frac{(1-P)M}{\omega}$, littera autem M semper sit positiva, at $1-P$ negativum, sequitur fore particulam ω negativam

COROLLARIUM 3

268. Si igitur intervallum primum statuamus $= \eta\alpha$ pariterque secundum etiam $= \eta\alpha$, existente η fractione minima, quoniam tantum hic illum casum evitare volumus, quo hae lentes quasi in unam coalescere deberent, η tam parvum assumi convenit, quam exsecutio permittit; ad quod sufficere videtur, si sit $\eta\alpha = 0,03$; hinc igitur erit $1 - \frac{1}{P} = \eta$ adeoque $P = 1 - \frac{1}{\eta}$, et nunc ω propius definire poterimus, scilicet
$\omega = \frac{-\eta M}{(1-\eta)\mathfrak{B}}$, et quia $M = \frac{3}{m+1}$, erit
$\omega = \frac{-3\eta}{(m+1)\mathfrak{B}}$ sicque littera $\mathfrak{B}$ adhuc nostro arbitrio relinquitur.

COROLLARIUM 4

269. Quia $\mathfrak{B} > 0$ et parumper minus unitate, erit B positivum; unde pro secundo intervallo habebimus $Q = \frac{B}{B+\eta P}$ ideoque $Q < 1$ sive $Q = \frac{(1-\eta)B}{(1-\eta)B+\eta}$ seu proxima $Q = 1 - \frac{\eta}{B}$; hincque definire licet ω', scilicet

$$\omega' = \frac{(1-PQ)M - \omega}{\mathfrak{C}} \text{ sive } \omega' = \frac{6\eta}{(m+1)\mathfrak{BC}}$$

ita ut etiam $\mathfrak{C}$ nostro arbitrio relinquatur.

SCHOLION 1

270. Hinc casus supra tractatus, quo erat $B = \infty$ et $\mathfrak{C} = 0$, facile deducitur; tum enim erit $Q = 1$ manente

$$P = 1 + \eta, \quad \omega = \frac{-3\eta}{m+1}, \quad \omega' = \frac{6\eta}{(m+1)\mathfrak{BC}},$$

et quia tam $\beta = \infty$ quam $c = -\infty$, fiet γ distantia focalis tertiae lentis $r = \frac{B\mathfrak{C}\alpha}{PQ}$, ubi $B\mathfrak{C}$ ita definiri poterit, ut tertia lens primae perfecte evadat aequalis, quod ad praxin valde est conveniens; statuatur nempe

$$B\mathfrak{C} = PQ = 1 + \eta .$$

Quia autem tum fit $\mathfrak{B} = 1$, postremae conditioni [§ 265] satisfieri nequit; quae cum sit maioris momenti quam praecedens, statuamus potius $\mathfrak{B} = \frac{4}{5}$, ut sit $B = 4$, et sumto $P = 1{,}03$ reperitur sumi debere $\mathfrak{C} > 0{,}256$, quare sumatur $\mathfrak{C} = 0{,}257$ eritque

$$C = \frac{0{,}257}{0{,}743} = 0{,}34589,$$

quod notasse sufficiat pro iis, qui tali resolutione uti velint.

COROLLARIUM 5

271. Quia nunc tam BC quam PQ sunt numeri positivi, ut tertium quoque intervallum fiat positivum, necesse est, ut sit $R > 1$, quae conditio sponte impletur, quoniam R erit numerus multo adhuc maior. Pro quarto intervallo $-\frac{BCD}{PQR}\left(1 - \frac{1}{S}\right)\alpha$ necesse est, ut $-D\left(1 - \frac{1}{S}\right)\alpha$ sit positivum; ex praecedentibus autem patet $\mathfrak{D}$ ideoque et D esse negativum; unde fieri debet $1 - \frac{1}{S} > 0$, quod fit, si fuerit vel S negativum vel $S > 1$, si sit positivum. De quinto intervallo iam supra vidimus.

SCHOLION 2

272. Hic ergo duo casus sunt considerandi, alter, quo S est numerus negativus, alter vero, quo T est negativum

I. Sit $\mathrm{S}<0$ et ponatur $S=-K$ habebiturque ista aequatio

$$0=1-\tfrac{1}{K}-\tfrac{1}{KT} \quad \text{eritque} \quad K=1+\tfrac{1}{\mathrm{T}};$$

verum hic est $\mathrm{T}<1$, uti supra vidimus, unde erit $K>2$, et KT continetur intra limites 1 et 2; unde, cum sit $RKT=m$, continebitur R intra limites m et $\tfrac{1}{2}m$; hoc porro casu erit

$$\mathfrak{C}=\left(1+RK\right)M-1,$$

qui valor ob $RK>m$ et $M=\tfrac{3}{m+1}$ erit

$$\mathfrak{C}>\tfrac{3\left(m+1\right)}{m+1}-1>2$$

ideoque E semper negativum, uti reliqua conditio postulat, scilicet ut $BCDE$ sit positivum

II. Sit iam $\mathrm{T}<0$ et ponatur $T=-K$ eritque
$0=1+\tfrac{1}{S}-\tfrac{1}{SK}$ ideoque $S=\tfrac{1}{K}-1,$ ubi quidem ratione ultimi intervalli K pro lubitu accipi posset, nunc vero requiritur, ut sit $K<1$. Cum igitur sit $RSK=m$, erit $RS=\tfrac{m}{K}$ adeoque $RS>m$ et littera $\mathfrak{C}$ manifesto fit negativa ideoque etiam E; quibus notatis evolutio exemplorum nulla plane laborat difficultate; id tantum hic adhuc adiungere visum est, ut tres posteriores lentes maximam aperturam accipiant, eas utrinque aeque convexas confici debere; qua conditione numeri λ''', λ'''', λ''''' sequenti modo determinantur, uti quidem iam supra est ostensum, scilicet, si pro indole vitri ponatur $\tfrac{\sigma-\rho}{2\tau}=N$, reperitur

$$\lambda'''=1+N^2\left(\tfrac{1-D}{1+D}\right)^2,$$

quae forma ob $D=\tfrac{\mathfrak{D}}{1-\mathfrak{D}}$ erit

$$\lambda'''=1+N^2\left(1-2\mathfrak{D}\right)^2,,$$

similique modo

$$\lambda''''=1+N^2\left(1-2\mathfrak{C}\right)^2,\quad \lambda'''''=1+N^2.$$

Superfluum autem iudico hanc investigationem exemplo illustrare, cum quia pro casu quinque lentium plurima exempla iam sunt allata, tum vero, inprimis si quis campum maiorem desideraverit, consultum potius erit duas vitri species adhibere, ut etiam ultima confusionis species penitus removeatur. Quod argumentum in sequente adhuc capite fusius nobis explicandum restat.