CHAPTER III

CONSIDERING THE FURTHER IMPROVEMENT OF TELESCOPES OF THE SECOND KIND BY USING DIFFERENT KINDS OF GLASS

PROBLEM 1

273. *IF the telescope shall be required to be composed from three lenses, to find the most important issues for making it perfect.*

SOLUTION

Therefore it is required to begin from the two fractions, by which method established before there may be put $\frac{\alpha}{b}=-P$ and $\frac{\beta}{c}=-Q$, thus so that of the letters *P* and *Q*, one shall be positive and the other negative, thus so that there shall become $PQ=-m$. Then therefore there will become

the determinable distances $b=-\frac{\alpha}{P}$, $\beta=-\frac{B\alpha}{P}$, $c=\frac{B\alpha}{PQ}$,

the focal lengths $p=\alpha$, $q=-\frac{\mathfrak{B}\alpha}{P}$, $r=\frac{B\alpha}{PQ}=-\frac{B\alpha}{m}$,

and the two intervals: $\alpha+b=\alpha(1-\frac{1}{P})$, $\beta+c=-\frac{B\alpha}{P}\left(1-\frac{1}{Q}\right)$.

Then, since for the field angle, $\Phi=\frac{\pi-\pi'}{m+1}$, we may put in place the small middle lens introduced, [see Ch. 2, § 265, for maximizing the field angle] both

$$\pi=\omega\xi \text{ and } \pi'=-\xi ,$$

so that there may become

$$\Phi=\frac{\omega+1}{m+1}\cdot\xi ,$$

from which there will be found, for the distance of the eye :

$$O=-\frac{\pi'}{\Phi}\cdot\frac{r}{m}=-\frac{(m+1)}{\omega+1}\cdot\frac{B\alpha}{mm}=-\frac{B\alpha}{m}\cdot\frac{m+1}{m(1+\omega)},$$

thus so that now $-B\alpha$ must be positive; or there will become from these three conditions requiring to be satisfied [for the intervals] :

$$1.\,\alpha\left(1-\tfrac{1}{P}\right)>0,\quad 2.\ -\tfrac{B\alpha}{P}\left(1-\tfrac{1}{Q}\right)>0,\quad 3.-B\alpha>0$$

and hence

$$\tfrac{1}{P}\left(1-\tfrac{1}{Q}\right)>0.$$

But again there will become

$$\mathfrak{B}\omega=(1-P)M,\ \ B=\tfrac{(1-P)M}{\omega-(1-P)M}$$

with there being,

$$M=\tfrac{\omega+1}{m+1}.$$

Now for the removal of the colored fringe , if indeed we may put the letters N, N', N'' in place of the fractions $\frac{dn}{n-1}$, $\frac{dn'}{n'-1}$, and $\frac{dn''}{n''-1}$
the equation is [see § 49]:
$0=N'\omega\cdot\frac{1}{P}+N''\cdot\frac{1}{PQ}$ or $0=N'\omega+N''\cdot\frac{1}{Q}$, [*i.e.* an early formula for the optical dispersion for each refractive index, where n is the average *r.i.* and dn its difference from the ends of the visible spectrum, to give the combined chromatic and spherical aberration], from which there becomes:

$$Q=-\tfrac{N''}{N'\omega}.$$

Moreover in order that this confusion may be removed completely, it is required [§ 53], that there shall become

$$0=N\alpha-\tfrac{N'\alpha}{\mathfrak{B}P}+\tfrac{N''\alpha}{BPQ},$$

with which value Q substituted in place there becomes

$$0=N-\tfrac{N'}{P}\left(\tfrac{1}{\mathfrak{B}}+\tfrac{\omega}{B}\right);$$

truly there is

$$\omega=\tfrac{1-P}{(m+1)\mathfrak{B}+P-1},$$

into which, if there may be put this value of ω, there will become

$$0=N-\tfrac{N'}{P}\left(\tfrac{m+P}{(m+1)\mathfrak{B}+P-1}\right);$$

then the equation for Q found on account of $PQ=-m$ will give also

$$m=\frac{N''P\left((m+1)\mathfrak{B}+P-1\right)}{N'(1-P)},$$

from which, if in the preceding equation for $\left(m+1\right)\mathfrak{B}+P-1$ there may be expressed the value of $\frac{N'm(1-P)}{N''P}$, this equation will arise :

$$0=NP\Big(\left(m+1\right)\mathfrak{B}+P-1\Big)-N'\left(m+P\right),\quad 0=NN'\left(1-P\right)-N'\left(m+P\right)$$

and thence again:

$$P=\frac{m\left(N-N''\right)}{Nm+N''};$$

and thus, since there shall be

$$\frac{N'm(1-P)}{N''P}=\left(m+1\right)\mathfrak{B}+P-1,$$

there is deduced

$$\mathfrak{B}=\frac{N'}{N-N''}+\frac{N'}{Nm+N''}\quad\text{or}\quad\mathfrak{B}=\frac{NN'm+NN''+N'N''-N''N''}{\left(N-N''\right)\left(Nm+N''\right)},$$

hence,

$$1-\mathfrak{B}=\frac{NNm-NN'm-NN''m-N'N''}{\left(N-N''\right)\left(Nm+N''\right)},$$

and thus,

$$B=\frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}$$

Now we may see, in what manner these determinations may be able to cease with the above conditions, and since there must be
$\frac{1}{P}(1-\frac{1}{Q})>0,$ or on account of $Q=-\frac{m}{P},\ \ \frac{1}{P}+\frac{1}{Q}>0$, there will become
$\frac{N}{N-N''}>0$ and $N>N''.$

But first the interval $\alpha(1-\frac{1}{P})>0$ will change into this $\frac{-N''(m+1)}{m\left(N-N''\right)}\cdot\alpha$ and thus $\frac{-N''\alpha}{N-N''}>0,$ and because the denominator now found is positive, it remains, so that there shall be $-N''\alpha>0.$ Truly the condition $-B\alpha>0$ now will give $B>0$; which since it shall be impossible, it is also impossible, that with the aid of three lenses these two suitable lenses may be obtained by which the confusion is removed completely.

[but from $B=\frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}=-\frac{NN'm+N'N''+N''\left(N-N''\right)}{mN\left(N'+N''-N\right)+N'N''}$ on account of $N>N''$ and $N'+N''-N>0$ there appears $B<0$; here there is an inconsistency that appears to render the original derivation by Euler to be in error (this corrected version given by *E. Ch.* editor of the *O.O*. version.)]

SCHOLIUM

274. Therefore this problem cannot be resolved, if indeed we may wish to remove the latter confusion completely. But with the final equation omitted the solution would have been simple, but then we would not be able to proceed further than in the preceding chapter, where we have assumed a single kind of glass to be used. Therefore since now it is not convenient to use two kinds of glass for making telescopes, which can be obtained equally well from a single kind of glass, we will not linger to investigate this, but we will try to produce telescopes only in a medium of this kind, which besides being of superior quality also may be freed from all confusion which may have remained there. But the reason why that investigation did not succeed here, therefore is evident, because the number of undefined letters was exceedingly small, if indeed for three equations to be fulfilled only three letters were present. Whereby if we may put several lenses in place, also we will have more letters of this kind, from which not only will it be required to satisfy the three equations, but also the remaining conditions.

PROBLEM 2

275. *If a telescope may be composed from four lenses, to determine the principle features required for its perfection.*

SOLUTION

Here the three fractions required to be considered may be put in place :

$$\frac{\alpha}{b} = -P, \quad -\frac{\beta}{c} = -Q, \quad \frac{\gamma}{d} = -R,$$

thus so that one of these letters *P, Q, R* one shall be negative and $m = -PQR$, from which the determinable distances will be

$$b = -\frac{\alpha}{P}, \; \beta = -\frac{B\alpha}{P}, \; c = \frac{B\alpha}{PQ}, \; \gamma = \frac{BC\alpha}{PQ}, \; d = \frac{BC\alpha}{PQR},$$

but the focal lengths

$$p = \alpha, \; q = \frac{-\mathfrak{B}\alpha}{P}, \; r = \frac{B\mathfrak{C}\alpha}{PQ}, \; s = \frac{BC\alpha}{PQR}$$

and the separations of the lenses

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right), \; \beta + c = -\frac{B\alpha}{P}\left(1 - \frac{1}{Q}\right), \; \gamma + d = \frac{BC\alpha}{PQ}\left(1 - \frac{1}{R}\right).$$

For the apparent field :

$$\Phi = \frac{\pi - \pi' + \pi''}{m+1}$$

there may be put

$$\pi=\omega\xi,\quad \pi'=-i\xi \text{ and } \pi''=\xi,$$

with the maximum value ξ, which these letters can take, evidently to be $\frac{1}{4}$; thus so that there shall be

$$\Phi=\frac{\omega+i+1}{m+1}\cdot\xi$$

or

$$\Phi=M\xi,$$

on putting

$$M=\frac{\omega+i+1}{m+1}.$$

Therefore from these we will obtain

$$\mathfrak{B}\omega=(1-P)M,\quad \mathfrak{C}i=(1-PQ)M-\omega.$$

Moreover for the location of the eye there will be

$$O=\frac{\pi''}{\Phi}\cdot\frac{s}{m}=\frac{m+1}{m^2}\cdot\frac{BC\alpha}{\omega+i+1}=\frac{BC\alpha}{m^2M};$$

so in order that it may be positive, there must be $BC\alpha>0$.
With these in place for the following three equations to be satisfied there must be [§ 49, 53, 42]:

$$\text{I.}\quad 0=\frac{N'\omega}{P}+\frac{N''i}{PQ}+\frac{N'''}{PQR},$$

$$\text{II.}\quad 0=N-\frac{N'}{\mathfrak{B}P}+\frac{N''}{B\mathfrak{C}PQ}-\frac{N'''}{BCPQR},$$

$$\text{III.}\quad 0=\mu\lambda-\frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''\lambda'''}{B^3C^3PQR};$$

which resolution so that it may be more easily put in place, we will consider in the first case, where the two first lenses are at in immediate contact, so that we have assumed for the two lenses above.

The first case, where $\alpha+b=0$ and thus $P=1$ and $\omega=0$; then the letter remains indeterminate and hence also *B*; with which done the resolution will be easy to put in place.

Indeed the first equation gives

$$0=N''i+\frac{N'''}{R},$$

from which it follows:

$$R=-\frac{N'''}{N''i},$$

thus so that *B* shall be a negative quantity, if indeed *i* shall be positive, just as the reasoning of the field demands. Hence therefore, since there shall be $P=1$, there will become

$$PQR = \frac{-QN'''}{N''i} = -m$$

and thus

$$Q = \frac{N''mi}{N'''}.$$

Moreover the second equation, in which now the two last terms emerge very small, if indeed the magnification m shall be great, gives at once

$$0 = N - \frac{N'}{\mathfrak{B}} \text{ and thus } \mathfrak{B} = \frac{N'}{N}$$

and hence there becomes

$$B = \frac{N'}{N-N'},$$

from which, if it should please, the value of $\mathfrak{B}$ will be able to be defined more carefully ; certainly there will become

$$\frac{1}{\mathfrak{B}} = \frac{N}{N'} + \frac{N'''(N-N')}{N'N'\mathfrak{C}mi} + \frac{N'''(N-N')}{N'N'Cm};$$

and generally, moreover it will suffice to be defined close enough by these two equations.

But the third equation must be resolved with more care ; of which the second term since it shall be negative, with the remaining being positive, as we will see soon, that must be equal to the remaining terms; for there will become:

$$\mathfrak{C}i = \frac{N'''-N''mi}{N''} \cdot \frac{i+1}{m+1},$$

and thus $\mathfrak{C}$ is negative and likewise also C, on which the conditions mentioned above depend.

But the first interval by hypothesis is $= 0$.

The second becomes $\beta + c = \frac{-N'(N''mi-N''')}{(N-N')N''mi} \cdot \alpha$, for which, if α shall be positive, $\frac{-N'}{N-N'}$ must be positive or $N < N'$; but on the other hand, if α shall be negative, there must become $N > N'$.

Again the third interval is $\frac{N'C\alpha}{(N-N')Q}\left(1+\frac{N''i}{N'''}\right)$; since here Q is positive, C is required to be negative, so that $-\frac{N'\alpha}{N-N'}$ shall be positive to be used for the second interval ; thus so that, if the second interval were positive, the third at once becomes positive.

Finally the formula for the location of the eye $O = \frac{BC\alpha}{m^2M}$ also becomes positive under the same conditions. From which it follows, if the first lens shall be made from crown glass, the second truly from crystal, or if $N' > N$, then α must be positive or $p > 0$, $q < 0$, $r > 0$, $s > 0$.

But if we may make the first lens from crystal glass, the second truly from crown glass, thus so that there shall be $N' < N$, a must become negative and thus $p < 0,\ q > 0,\ r > 0,\ s > 0$, whereby for each case the third equation will be easy to resolve.

With these conditions considered carefully, which now also have a place, provided ω shall be the least possible fraction, we may put the first interval $\alpha\left(1-\frac{1}{P}\right) = \eta\alpha$ with η being the minimum fraction, either positive, if $\alpha > 0$, or negative, if $\alpha < 0$, and there will become $P = \frac{1}{1-\eta}$; then $\mathfrak{B}$ may remain indefinite at this stage and ω may be sought and there will be

$$\mathfrak{B}\omega = \frac{-\eta}{1-\eta}\cdot\frac{(\omega+i+1)}{(m+1)};$$

in which the latter factor with ω is omitted with care, thus so that hence there shall be

$$\omega = \frac{-\eta}{1-\eta}\cdot\frac{i+1}{(m+1)\mathfrak{B}},$$

which value is diminished on account of a twofold reason : for in the first place η is very small, then that is divided by $m+1$, a big enough number; again truly both $\mathfrak{B}$ as well as $i+1$ differ little from unity; on account of which cause the value ω rightly will be considered as vanishing ; from which our first equation gives us as before :

$$0 = N''i + \frac{N''}{R} \quad \text{and} \quad R = \frac{-N'''}{N''i};$$

which value if it may be desired at this stage with more care, will be

$$-\frac{1}{R} = \frac{N''\omega Q}{N'''} + \frac{N''i}{N'''},$$

thus so that now P and R shall be known quantities; as far as in the first term it suffices to know Q at least minimally, which thus is allowed to be chosen from the preceding case, since ω now has been defined and $\mathfrak{B}$ will be defined soon. Hence therefore $Q = \frac{-m}{PR}$. The second equation in turn as in the preceding case will give as before $\mathfrak{B} = \frac{N'}{N}$; if for which indeed that may be desired more exact, this equation will be required to be used for that:

$$\frac{N'}{\mathfrak{B}} = NP + \frac{N''}{B\mathfrak{C}Q} - \frac{N'''}{BCQR}$$

where truly it will suffice to know a nearby value for B , surely $B = \frac{N}{N-N'}$. Then truly there will be had

$$\mathfrak{C}i = \left(1+\tfrac{m}{R}\right)\cdot\tfrac{(\omega+i+1)}{(m+1)} - \omega,$$

which value evidently is to be the actual negative value of R and thus also C. On which account the prescribed conditions will be fulfilled in the same cases as in the preceding, where $\omega = 0$, thus so that now it remains only to resolve the third equation, but only if we may remember on account of $\pi'' = \xi$, the fourth lens must become equally convex, which form also must be attributed to the third lens, if there shall be $i = 1$. Truly if there may be assumed $i = 1$, so that this lens will become equally convex, on account of $\mathfrak{C} = -2$ almost, there must be put in place for this lens

$$\lambda'' = 1 + N^2\left(1-2\mathfrak{C}\right)^2 = 1 + N^2\cdot 25,$$

where, as we have assumed before, there is $N = \frac{\sigma-\rho}{2\tau}$, [Clearly not the same as defined above for refractive indices!] and thus the number λ'' will obtain a large enough value; so that we will avoid this inconvenience by requiring to assume $i < 1$ and generally it will suffice to put in place $i = \frac{1}{2}$.

COROLLARY

276. Therefore this difference of the refraction of the glass comes into consideration only in the first two lenses and thus it will suffice for only a single lens to be prepared from crystal glass and all the rest from crown glass ; and thus only two cases are required to be established, the one, where the first lens is prepared from crystal glass, the other, where the second is so prepared.

CASE 1

277. Therefore the first lens shall be made from crystal glass, all the remaining made from crown glass; there will be $n = 1{,}58$ and $n' = n'' = n''' = 1{,}53$, then truly following Dollond's experiments : $N = 10$, $N' = N'' = N''' = 7$. With these in place and by assuming $i = \frac{1}{2}$, α will be negative and thus likewise also η. Moreover on assuming $\eta = -0{,}03$; therefore hence we will have $P = \frac{1}{1{,}03} = \frac{100}{103}$, and because there will be approximately $\mathfrak{B} = \frac{7}{10}$ and $B = \frac{7}{3}$, we find $\omega = \frac{45}{721(m+1)}$.

Then, since there shall be approximately $R = -2$ and thus $Q = \frac{103m}{200}$, with more care we will have

$$-\frac{1}{R} = \frac{1}{2} + \frac{927m}{28840(m+1)} \quad\text{or}\quad R = \frac{-28840(m+1)}{15347m+14420},$$

[Euler had found initially $-\frac{1}{R} = \frac{1}{2} + \frac{927m}{36050(m+1)}$ or $R = \frac{-36050(m+1)}{18952m+18025}$, corrected by E.Ch. in the *O.O.* edition, but Euler's values were maintained in the following calculations, as the error in R was small.] which is the correct value of $R,$ from which also more carefully Q will be able to be defined, evidently $Q=\frac{-m}{PR}$.

So that now also $\mathfrak{B}$ may be defined with more care, and there will be

$$\frac{1}{\mathfrak{B}}=\frac{10P}{7}+\frac{3\cdot 200}{7\cdot 103m\mathfrak{C}}+\frac{3\cdot 200}{7\cdot 2\cdot 103mC}.$$

Truly there is

$$\mathfrak{C}=\left(1+\frac{m}{R}\right)\left(\frac{3+2\omega}{m+1}\right)-2\omega \text{ and } C=\frac{\mathfrak{C}}{1-\mathfrak{C}};$$

from which B can be defined also.

Therefore with these principal values defined the third equation must be resolved by putting in place $\lambda''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2$, so that the fourth lens may be produced equally convex on both sides.

But for the third lens it may be seen to be able to put $\lambda''=1$.

Finally with the individual letters λ found also the construction of the individual lenses will be had. But now we will use the method now used a number of times, as it were for some case required to be determined, for example $m=25$, then for the case $m=\infty$ we may put in place the procedure and hence we may derive the construction for any magnification.

EXAMPLE 1

278. If the first lens shall be prepared from crystal glass, the three following shall be required to be prepared from crown glass, to investigate the construction of the telescope for a magnification $m=25$.

Therefore on assuming $\eta=-0,03$, so that the interval between the first two lenses may become $-\frac{3\alpha}{100}$ on account of α being negative, we will have at once

$$P=\frac{100}{103}=0,97087,\ \text{Log.}P=9,9871628,\ \text{hence } \omega=0,00240,$$

then

$$R=\frac{-36050\cdot 26}{18\ 952\cdot 25+18025} \text{ or } R=\frac{36050\cdot 26}{491825}=-1,90576,$$

$$\text{Log.}R=\ 0,2800680\ (-)$$

and hence

$$Q=13,51160, \qquad \text{Log.}Q=1,1307092;$$

now for $\mathfrak{C}$ requiring to be found there may be observed to be

$$PQ=13,11810,\ \text{Log.}PQ=\ 1,1178720$$

and hence there will be

$$\mathfrak{C} = -\ 12{,}1181 \cdot \frac{(3{,}00480)}{26} - 0{,}00480 \text{ or } \mathfrak{C} = -1{,}40528,$$

$$\text{Log.}\mathfrak{C} = 0{,}1477628\ (-)$$

and hence

$$C = -0{,}58425, \quad \text{Log.}C = 9{,}7665972\ (-).$$

Now finally for finding B we need not use an approximation, since on account of

$\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ we have with care

$$1 - \tfrac{1}{\mathfrak{C}Q} + \tfrac{1}{CQR} = \left(\tfrac{10}{7} - P - 1\right)B$$

or

$$1 + 0{,}05266 + 0{,}06647 = (1{,}38696 - 1)B$$

and thus

$$1{,}11913 = 0{,}38696B$$

therefore

$$B = 2{,}89210,\ \text{Log. } B = 0{,}4612145,$$

consequently

$$\mathfrak{B} = \tfrac{2{,}89210}{3\ 89210} = 0{,}74307, \quad \text{Log.}\mathfrak{B} = 9{,}8710305.$$

We may seek our elements from these values found and we will find initially

$$b = -1{,}03000\alpha$$

$$\begin{aligned}
&\text{Log.}\tfrac{b}{\alpha} = 0{,}0128372\ (-) \\
&\text{Log.}B = 0{,}4612145 \\
&\text{Log.}\tfrac{\beta}{\alpha} = 0{,}4740517\ (-) && \beta = -2{,}97887\alpha \\
&\text{Log.}Q = 1{,}1307092 \\
&\text{Log}\,\tfrac{c}{\alpha} = 9{,}3433425 && c = +0{,}22046\alpha \\
&\text{Log.}C = 9{,}7665972\ (-) \\
&\text{Log.}\tfrac{\gamma}{\alpha} = 9{,}1099397\ (-) && \gamma = -0{,}12880\alpha \\
&\text{Log.}R = 0{,}2800680\ (-) \\
&\text{Log.}\tfrac{d}{\alpha} = 8{,}8298717\ (-) && d = -0{,}06759\alpha.
\end{aligned}$$

Hence again also the focal lengths will be

$$p=\alpha, \quad q=-0,76536\alpha, \qquad \text{Log.}\tfrac{q}{\alpha}=9,8838677\ (-)$$
$$r=-0,30982\alpha, \qquad \text{Log.}\tfrac{r}{\alpha}=9,4911053\ (-)$$
$$s=-0,06759\alpha, \qquad \text{Log.}\tfrac{s}{\alpha}=8,8298717\ (-).$$

Again the separation of the lenses will become

$$\alpha+b=-0,03000\alpha, \quad \beta+c=-2,75841\alpha, \quad \gamma+d=-0,19639\alpha.$$

Finally the distance of the eye from the final lens will be

$$O=-1,12480\cdot\tfrac{m+1}{mm}\cdot\alpha$$

and thus the distance between the first and last lens $=-2,98480\alpha$.

Now we will consider the third equation, which resolved and divided by μ for this case will give

$$0=\lambda-\frac{\mu'}{\mu}\cdot\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\mu'}{\mu}\cdot\frac{\lambda''}{B^3\mathfrak{C}^3P}-\frac{\mu'}{\mu}\cdot\frac{\nu'}{B\mathfrak{B}P}+\frac{\mu'}{\mu}\cdot\frac{\nu'}{B^3C\mathfrak{C}^3PQ}-\frac{\mu'}{\mu}\cdot\frac{\lambda'''}{B^3C^3PQR},$$

where $\mu=0,8724$, $\mu'=0,9875$ and $\nu'=0,2196$, from which this equation will be obtained for the reduced numbers

$$0=\lambda-2,84162\lambda'-0,00128\lambda''$$
$$(\text{Log. } 7,1090175)$$
$$-0,00938\lambda'''-0,11913$$
$$(\text{Log. } 7,9724463)+0,00095.$$

Now in order that we may resolve this equation, initially it is required to be observed the fourth lens to be equally convex on both sides and thus $\lambda'''=1,60006$, from which, since its focal length shall be $s=-0,06759\alpha$, the radius of each face will be

$$=1,06s=-0,07164\alpha.$$

But for the third lens, its focal length is $r=\mathfrak{C}c$, since the radius of its aperture must be $=\frac{1}{8}\mathfrak{C}c$, to which therefore the fourth part of the smaller radius must be equal, thence the lesser radius must be $\frac{1}{2}\mathfrak{C}c$, from which λ'' is required to be defined. Finally we may now define this lens. Moreover the radius of its anterior face is

$$=\frac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}=\frac{Cc}{\rho C+\sigma\mp\tau(1+C)\sqrt{(\lambda''-1)}}$$

and the radius of its posterior face

$$\frac{Cc}{\sigma C+\rho\pm\tau(1+C)\sqrt{(\lambda''-1)}},$$

thus so that we will have

$$\text{radius of the}\begin{cases}\text{anterior face} &=\frac{Cc}{1,5277\mp 0,41575\tau\sqrt{\lambda''-1}},\\ \text{posterior face} &=\frac{Cc}{-0,7432\pm 0,41575\tau\sqrt{\lambda''-1}},\end{cases}$$

where the upper signs must prevail; then truly the first radius is the smaller and thus there may be put $\frac{1}{2}\mathfrak{C}c$; from which it is deduced

$$\frac{C}{1,5277-0,41575\tau\sqrt{\lambda''-1}}=\frac{1}{2}\mathfrak{C}$$

and thus there will become

$$0,41575\tau\sqrt{\lambda''-1}=0,6961$$

and thus

$$\text{radius of the}\begin{cases}\text{anterior face} &=\frac{\gamma}{0,8316}=-0,15489\alpha,\\ \text{posterior face} &=\frac{\gamma}{-0,0471}=+2,73475\alpha,\end{cases}$$

of which lens therefore the radius of the aperture will be $=0,03872\alpha$; but from the value of x we will deduce $\sqrt{\lambda''-1}=1,80969$ and hence $\lambda''=4,27497$.

Now we may revert to our equation, which was

$$0=\lambda-2,84162\lambda'-0,00549-0,01501-0,11818;$$

since now λ' cannot be less than one, we may put $\lambda'=1$ and there will become

$$\lambda=2,98030,\quad \lambda-1=1,98030 \text{ and } \tau\sqrt{\lambda-1}=1,23484;$$

from which the first lens thus will be required to be made:

$$F=\frac{\alpha}{\sigma\pm 1,23484}=\frac{\alpha}{0,3479}=2,87439\alpha,$$

$$G=\frac{\alpha}{\rho\mp 1,23484}=\frac{\alpha}{1,3762}=0,72664\alpha.$$

Finally for the second lens we have

$$F = \frac{b\beta}{\rho\beta+\sigma b} = \frac{Bb}{2,3157} = -1,28638\alpha,$$

$$G = \frac{b\beta}{\sigma\beta+\rho b} = \frac{Bb}{5,0279} = -0,59247\alpha;$$

of which the fourth part of the smaller radius or $0,14812\alpha$ gives the radius of the aperture both for the first lens as well as for the second lens. Whereby hence the following construction of the telescope is deduced for the magnification $m = 25$.

I. For the first lens, Flint Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} = 2,87439\alpha, \\ \text{posterior face} = 0,72664\alpha. \end{cases}$$

Radius of the aperture $= 0,14812\alpha$.
Distance to the second lens $= -0,03\alpha$.

II. For the second lens, Crown Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} = -1,28638\alpha, \\ \text{posterior face} = -0,59247\alpha. \end{cases}$$

Radius of the aperture as before.
Distance $= -2,75841\alpha$.

III. For the third lens, Crown Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} = -0,15489\alpha, \\ \text{posterior face} = +\ 2,73475\alpha. \end{cases}$$

Radius of the aperture $= 0,03872\alpha$
Distance $= -0,19639\alpha$.

IV. For the fourth lens, Crown Glass,
radius of each face $= -0,07164\alpha$.
Radius of the aperture $= 0,01791\alpha$.
Distance to the eye

$$= -11248 \cdot \tfrac{m+1}{m^2}\alpha, \quad \text{or} \quad O = -1,1248\ \left(1+\tfrac{1}{m}\right)\cdot\tfrac{\alpha}{m}, \quad O = -0,04679\alpha,$$

where it is required to observe that α is to be negative, and if we assume the radius of the aperture $=\frac{m}{50}$ in. $=\frac{1}{2}$ in., there shall become approx. $\alpha=-\frac{7}{2}$ in. or more, and the length of the telescope $=10\frac{1}{2}$ in. Finally the radius of the field will be approx. $\Phi=49\frac{1}{2}$ minutes.

EXAMPLE 2

279. If the first lens shall be prepared from crystal glass, and the rest from crown glass, to investigate the construction of the telescope for the maximum magnification.

Again there shall be $\eta=-0,03$; we will have as before

$$P=0,97087,\ \text{Log.}P=9,9871628;$$

then truly $\omega=0$ and

$$R=-\tfrac{28840}{15347}\ \text{ or }\ R=-1,87919,\ \text{Log.}R=0,2739707\ (-)$$

$$Q=0,54148m,\quad \text{Log. }\tfrac{Q}{m}=9,7335868.$$

Again,

$$\mathfrak{C}=\tfrac{3}{R}=-1,57714,\ \text{Log. }\mathfrak{C}=0,1978710\ (-)$$

and

$$C=-0,61197,\quad \text{Log.}C=9,7867330\ (-).$$

But this equation will be had for finding B

$$1-\tfrac{1}{\mathfrak{C}Q}+\tfrac{1}{CQR}=\left(\tfrac{10}{7}P-1\right)B,$$

which, since the terms divided by Q vanish, it will be changed into this

$$1=0,3869B,\ \text{ hence }\ B=2,58464,\ \text{Log.}B=0,4124012$$

and

$$\mathfrak{B}=0,72103,\ \ \text{Log.}\mathfrak{B}=9,8579557.$$

Hence we will have the determinable distances

$$b=-1,03000\alpha,\qquad \text{Log.}\tfrac{b}{\alpha}=0,0128371(-)$$
$$\beta=-2,66218\alpha,\qquad \text{Log.}\tfrac{\beta}{\alpha}=0,4252383(-)$$
$$c=+4,91645\tfrac{\alpha}{m},\qquad \text{Log.}c\cdot\tfrac{m}{\alpha}=0,6916515$$

$$\gamma = -3,00874\tfrac{\alpha}{m}, \qquad \text{Log.}\gamma \cdot \tfrac{m}{\alpha} = 0,4783845(-)$$
$$d = -1,58173\tfrac{a}{m}, \qquad \text{Log.}d \cdot \tfrac{m}{\alpha} = 0,1991342(-)$$

and the separation of the lenses

$$a + b = -0,03000\alpha,$$
$$\beta + c = -2,66218\alpha + 4,91645 \cdot \tfrac{\alpha}{m},$$
$$\gamma + d = -4,59047 \cdot \tfrac{\alpha}{m}$$

and the distance of the eye past the final lens

$$O = -1,05449 \cdot \tfrac{\alpha}{m},$$

then truly the focal lengths :

$$p = \alpha, \;\; q = -0,74267\alpha, \;\; r = -7,75394 \cdot \tfrac{\alpha}{m}, \;\; s = -1,58173 \cdot \tfrac{\alpha}{m}.$$

Now therefore we will consider the third equation :

$$0 \;= \lambda - \tfrac{\mu'}{\mu} \cdot \tfrac{\lambda'}{\mathfrak{B}^3 P} - \tfrac{\mu' \nu'}{\mu \mathfrak{B} BP},$$

$$0 = \lambda - 3,11022\lambda' - 0,13738,$$

which with $\lambda' = 1$ assumed as before gives

$$\lambda = 3,24760, \text{ hence } \lambda - 1 = 2,24760 \text{ and } \tau\sqrt{\lambda - 1} = 1,31555,$$

from which the construction of the lenses may be set out in the following manner :

I. For the first lens from crystal glass

$$F = \tfrac{\alpha}{\sigma - 1,31555} = \tfrac{\alpha}{0,2672} = 3,74251\alpha,$$

$$G = \tfrac{\alpha}{\rho + 1,731555} = \tfrac{\alpha}{1,4569} = 0,68639\alpha.$$

II. For the second lens from crown glass

$$F=\frac{b\beta}{\rho\beta+\sigma b}=\frac{\beta}{2{,}2461}=-1{,}18525\alpha,$$

$$G=\frac{b\beta}{\sigma\beta+\rho b}=\frac{\beta}{4{,}5175}=-0{,}58931\alpha.$$

III. For the third lens from crown glass

$$F=\frac{Cc}{C\rho+\sigma\mp x},\quad G=\frac{Cc}{C\sigma+\rho\pm x},$$

$$F=\frac{\gamma}{1{,}5214\mp x},\quad G=\frac{\gamma}{-0{,}7892\pm x}.$$

Now since as before the radius of the anterior face shall be made smaller, this may be put equal to half of the focal length $=\frac{1}{2}\mathfrak{C}c$ and there will become

$$1{,}5214-x=\frac{2C}{\mathfrak{C}}=2(C+1) \text{ or } x=0{,}7454.$$

From which there is had at once

$$F=\frac{\gamma}{0{,}7760}=-3{,}87697\cdot\frac{\alpha}{m},\quad G=\frac{\gamma}{-0{,}1438}=+\,68{,}69269\cdot\frac{\alpha}{m}.$$

IV. Finally for the fourth lens, the focal length of which $s=-1{,}58173\cdot\frac{\alpha}{m}$; the radius of each face will be $=1{,}06s=-1{,}67663\cdot\frac{\alpha}{m}$; and thus all the main features for this case have been defined.

EXAMPLE 3

280. If the first lens shall be prepared from crystal glass, and the rest from crown glass, to present the construction of the telescope for any magnification.

SOLUTION

We will be able to define the main features easily from a comparison of the two preceding examples by the method presented above. Initially for the determinable distances there will be $b=-1{,}0300\alpha$; moreover for the remaining there may be put in place :

$$\beta=-2{,}6622\alpha-\beta'\cdot\frac{\alpha}{m},$$

$$c=+4{,}9164\cdot\frac{\alpha}{m}+\frac{c'}{m}\cdot\frac{\alpha}{m},$$

$$\gamma=-3{,}0087\cdot\frac{\alpha}{m}-\frac{\gamma'}{m}\cdot\frac{\alpha}{m},$$

$$d=-1,5817\cdot\frac{\alpha}{m}-\frac{d'}{m}\cdot\frac{\alpha}{m}$$

and there will become :

$$\beta'=7,9150,\quad c'=14,8775,\quad \gamma'=5,283,\quad d'=2,701;$$

in a similar manner there is for the focal lengths $p=\alpha$; for the rest there may be put

$$q=-07427\alpha-\frac{q'}{m}\cdot\alpha,$$

$$r=-7\ 7539\cdot\frac{\alpha}{m}-\frac{r'}{m}\cdot\frac{\alpha}{m},$$

$$s=-1,5817\cdot\frac{\alpha}{m}-\frac{s'}{m}\cdot\frac{\alpha}{m}$$

and there will become:

$$q'=0,5665,\quad r'=-0,2100,\quad s'=2,701;$$

from which the separations of the lenses are :

$$\alpha+b=-0,0300\alpha,$$

$$\beta+c=-2\ 6622\alpha-\frac{2,998}{m}\cdot\alpha+\frac{14,88}{m}\cdot\frac{\alpha}{m},$$

$$\gamma+d=-4,5904\cdot\frac{\alpha}{m}-\frac{7,984}{m}\cdot\frac{\alpha}{m}.$$

For the location of the eye there may be put :

$$O=-1,05449\cdot\frac{\alpha}{m}-\frac{O'}{m}\cdot\frac{\alpha}{m};\ \text{there will be}\ O'=2,8815.$$

1. For the first lens

$$\text{the radius}\begin{cases}\text{of the anterior face} = 3,7425\alpha+F'\cdot\frac{\alpha}{m}\\ \text{of the posterior face} = 0,6864\alpha+G'\cdot\frac{\alpha}{m};\end{cases}$$

there will become $F'=-21,70,\quad G'=1,005.$

II. For the second lens

$$\text{the radius}\begin{cases}\text{of the anterior face} = -1,1853\alpha - F'\cdot\frac{\alpha}{m}\\ \text{of the posterior face} = -0,5893\alpha - G'\cdot\frac{\alpha}{m};\end{cases}$$

there will become $F' = 2,52, \quad G' = 0,08.$

III. For the third lens

$$\text{the radius}\begin{cases}\text{of the anterior face} = -3,8769\cdot\frac{\alpha}{m} - F'\cdot\frac{\alpha}{m}\\ \text{of the posterior face} = +68,6929\cdot\frac{\alpha}{m} + G'\cdot\frac{\alpha}{m};\end{cases}$$

there will become $F' = -0,114, \quad G' = -8,098$.

IV. Finally for the fourth lens the radius of each face $= -1,6766\cdot\frac{\alpha}{m} - \frac{H}{m}\cdot\frac{\alpha}{m}$;
there will be $H = +2,862$, from which the construction of the telescope is made follows for any magnification *m*.

I. For the first lens, crystal glass,

$$\text{the radius}\begin{cases}\text{of the anterior face} = 3,7425\cdot\frac{\alpha}{m} - 21,70\cdot\frac{\alpha}{m}\\ \text{of the posterior face} = 0,6864\cdot\frac{\alpha}{m} + 1,005\cdot\frac{\alpha}{m}.\end{cases}$$

The focal length of this $= \alpha$.
The radius of the aperture $= \frac{m}{50}$ in.
The distance to the second lens $= -0,03\alpha$.

II. For the second lens, crown glass,

$$\text{the radius}\begin{cases}\text{of the anterior face} = -1,1853\alpha - 2,52\cdot\frac{\alpha}{m}\\ \text{of the posterior face} = -0,5893\alpha - 0,08\cdot\frac{\alpha}{m}.\end{cases}$$

The focal length $= -0,7427\alpha - \frac{0,5665\alpha}{m}$.
The radius of the aperture also $= \frac{m}{50}$ in.

Distance to the third lens $= -2{,}6622\alpha - 2\,998 \cdot \frac{\alpha}{m} + \frac{14{,}88}{m} \cdot \frac{\alpha}{m}$.

III. For the third lens, crown glass,

$$\text{the radius}\begin{cases}\text{of the anterior face} = \quad -3{,}8769 \cdot \frac{\alpha}{m} - 0{,}114 \cdot \frac{\alpha}{m} \\ \text{of the posterior face} = +68{,}6929 \cdot \frac{\alpha}{m} - \ 8{,}098 \cdot \frac{\alpha}{m}.\end{cases}$$

The focal length $= -7{,}7539 \cdot \frac{\alpha}{m} + 0{,}210 \cdot \frac{\alpha}{mm}$, of which the eighth part gives the radius of the aperture.

The distance to the fourth $= -4{,}5904 \cdot \frac{\alpha}{m} - 7{,}984 \cdot \frac{\alpha}{mm}$.

But the distance of the real image after this lens

$$= \gamma = 3{,}0087 \cdot \frac{\alpha}{m} - 5{,}283 \cdot \frac{\alpha}{mm}.$$

IV. For the fourth lens, crown glass,

$$\text{the radius of each face} = -1{,}6766 \cdot \frac{\alpha}{m} - \frac{2{,}862\alpha}{mm}.$$

The focal length of this $= -1{,}5817 \cdot \frac{\alpha}{m} - \frac{2{,}70\alpha}{mm}$, of which the fourth part gives the radius of the aperture.

And the distance to the eye

$$O = -1{,}0545 \cdot \frac{\alpha}{m} - 2{,}8815 \cdot \frac{\alpha}{mm}.$$

Hence the length of the whole telescope will be

$$\begin{aligned} &= -0{,}03\alpha \quad -2{,}998\frac{\alpha}{m} + 14{,}88\frac{\alpha}{mm} \\ &\quad -2{,}6622\alpha - 4{,}590\frac{\alpha}{m} - \ 7{,}98\frac{\alpha}{mm} \\ &\qquad\qquad\quad -1{,}054\frac{\alpha}{m} - \ 2{,}88\frac{\alpha}{mm} \\ \hline &= -2{,}6922\alpha - 8{,}642\frac{\alpha}{m} + \ 4{,}02\frac{\alpha}{mm}. \end{aligned}$$

Truly the radius of the apparent field of view will be $= \frac{1289}{m+1}$ minutes.

Here truly it is required to observe that α to be negative, the value of which is defined from the latter radius of the second lens, of which the fourth part $-0,1478\alpha$ or $=-\frac{1}{7}\alpha$ approx. put equal to $\frac{m}{50}$ will give $\alpha=-\frac{7m}{50}$; therefore there will be put in place $\alpha=-\frac{m}{7}$.

COROLLARY

281. Therefore if we may put $\alpha=-\frac{m}{7}$, the following construction will be had requiring to determine the magnification of the telescope in the ratio $m:1$.

I. For the first lens, flint glass,

$$\text{radius in inches}\begin{cases}\text{of the anterior face} = -\ 0,5346m+3,10\\ \text{of the posterior face} = -0,0981m-0,14.\end{cases}$$

Focal length $=-\frac{m}{7}$.

Radius of the aperture $=\frac{m}{50}$.

Distance $=0,0043m$.

II. For the second lens, crown glass,

$$\text{radius}\begin{cases}\text{of the anterior face} = +\ 0,1693m+0,36\\ \text{of the posterior face} = +\ 0,0842m+0,01.\end{cases}$$

Focal length $=+0,1061m+0,081$.

Radius of the aperture $=\frac{m}{50}$.

Distance $=+0,3803m+0,43-\frac{2,12}{m}$.

III. For the third lens, crown glass,

$$\text{radius}\begin{cases}\text{of the anterior face} = +\ 0,55-\frac{0,02}{m}\\ \text{of the posterior face} = -9,81+\frac{1,16}{m}.\end{cases}$$

Focal length $=+1,11-\frac{0,03}{m}$.

Distance $= 0,66 + \frac{1,14}{m}$.

Distance of the real image after this lens $= 0,43 + \frac{0,75}{m}$.

IV. For the fourth lens, crown glass,

$$\text{radius of each face} = +0,24 + \frac{0,41}{m}$$

Focal length $= 0,23 + \frac{0,4}{m}$.

Radius of the aperture $= 0,06 + \frac{0,1}{m}$.

Distance as far as to the eye $= 0,15 + \frac{0,4}{m}$,

and total length of the telescope

$$= 0,3846m + 1,23 - \frac{06}{m}$$

and the radius of the field of view $= \frac{1289}{m+1}$ minutes of arc.

SCHOLIUM

282. Therefore these telescopes become much longer than these, which were found in the previous chapter, for each with the same magnification. Indeed here for the magnification $m = 100$, a length $= 40$ in. is produced, while in the preceding chapter a length $= 13\frac{1}{2}$ in. was sufficient, that is three times smaller. Therefore this deserves to be sought, whether the quality, by which also the diffusion length arising from the different refraction of the rays is reduced to zero, shall be required to be of such a size, that the length of the telescope may be tripled, which question will only be able to be judged in practice , but which therefore will be more difficult, where the execution of these precepts with less than the greatest accuracy may be allowed to be expected. Whereby unless this prescribed condition may succeed by the greatest luck, the telescopes of the preceding chapter always will be preferred, thus so that we may be able without twofold kinds of glass. Moreover we will not dwell on the establishment of the second case of this kind by setting it out numerically, since which method of treating such calculations has been explained well enough, then truly because the proposed need only to be attached as an appendix to the book, in which the precepts alone may be gathered together as a favour to the skilled lens makers for use in practice.

[See E446: *Detailed instructions for carrying lenses of all kinds to the highest degree of perfection…… according to theory of M. Euler…. by Nicolaus Fuss.* St. P. 1774.]

CASE 2

283. Now the second lens shall be made from crystal glass, truly the rest from crown glass, thus so that there shall be $n' = 1,58$ and $n = n'' = n''' = 1,53$ and thence also $N = N'' = N'''$, and there may be put $\frac{N'}{N} = \zeta$ so that there shall become, following Dollond, $\zeta = \frac{10}{7}$. With these in place and on taking $i = \frac{1}{2}$, α will be positive and thus also η. Therefore there may be taken $\eta = 0,03$, from which we will have $P = \frac{1}{1-\eta}$. Since now, as we have seen, there becomes $\mathfrak{B} = \zeta$ approx., there will become

$$\omega = \frac{-\eta}{1-\eta} \cdot \frac{i+1}{(m+1)\zeta} = \frac{-9}{194\zeta(m+1)}.$$

Then, since there shall be approximately $R = -2$ and thus $Q = \frac{m}{2P}$, where $P = +\frac{100}{97}$, for we will have with more care from the first equation

$$0 = \zeta Q\omega + i + \frac{1}{R},$$

which will change into this:

$$-\frac{1}{R} = \frac{1}{2} - \frac{9m}{388P(m+1)} \text{ or } -\frac{1}{R} = \frac{1}{2} - \frac{9m}{400P(m+1)},$$

from which also Q can be defined with more care, since there shall be $Q = \frac{-m}{PR}$.

Now so that $\mathfrak{B}$ also may be defined with more care, we deduce from the second equation [§ 275]

$$0 = 1 - \frac{\zeta}{\mathfrak{B}P} + \frac{1}{B\mathfrak{C}PQ} - \frac{1}{BCPQR},$$

which on account of $\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ will give

$$-B(\zeta - P) = \zeta - \frac{1}{\mathfrak{C}Q} + \frac{1}{CQR},$$

But there was approximately $B = \frac{-\zeta}{\zeta-1}$ and thus negative. Again from the value of B there will be with care $\mathfrak{B} = \frac{B}{B+1}$.

Therefore with these defined the third equation will become

$$0 = \lambda - \frac{\mu'\lambda'}{\mu\mathfrak{B}^3P} + \frac{\lambda''}{B^3\mathfrak{C}^3PQ} - \frac{\lambda'''}{B^3C^3PQR} - \frac{\mu'\nu'}{\mu B\mathfrak{B}P} + \frac{\nu}{B^3C\mathfrak{C}PQ}.$$

Here the two first terms taken as very large will give approximately $\lambda' = \frac{\mu}{\mu'}\mathfrak{B}^3 P\lambda$, from which it may be understood for the concave lens its value λ to be produced greater than in the first case, thus so that the first case may be judged more suitable in practice.

So that now also we shall be able to judge from the length of the telescope, since in the preceding case that was produced exceedingly great, a part of this must be considered especially, which is the letter β, the value of which before was $=-2,6622\alpha$ approx., but which now on account of $\beta = \frac{-B\alpha}{P}$ and $B = \frac{-\zeta}{\zeta-1}$ approx. or $B = -\frac{10}{3}$ and $P = 1$ there is found $\beta = \frac{10}{3}\alpha = 3,33\alpha$, thus so that hence telescopes designed at this stage may become longer, which here therefore will not be worth the effort of setting out further.

SCHOLIUM

284. Since the length of the telescope, which hence may result to become so great, may stand in the way of perfection, it is required to enquire, whether or not some remedy may be applied to remove this inconvenience, but since the hypothesis made here by no means gives us hope, whereby the objective lens has been assumed to be made from two lenses. On which account it will be convenient to have assumed the objective lens as if consisting of three lenses, of which either one or two shall be concave and formed from crystal glass. Nevertheless lest we should be urged to undertake a new inquiry about a greater field of view, here at once we may introduce also as if three ocular lenses, so that in this way, if the arrangement may be successful, not only may shorter telescopes be obtained, but also a notable increase in the apparent field of view may be taken.

PROBLEM 3

285. *If the three first lenses may be referred to as constituting the objective, then truly three lenses may be added as if the eyepiece, to define the principal features necessary for the construction of the telescope.*

SOLUTION

Therefore since here six lenses occur, we may put

$$\frac{\alpha}{b} = -P, \quad \frac{\beta}{c} = -Q, \quad \frac{\gamma}{d} = -R, \quad \frac{\delta}{e} = -S, \quad \frac{\varepsilon}{f} = -T,$$

so that our elements shall become:

$$b=-\tfrac{\alpha}{P},\ \ c=\tfrac{B\alpha}{PQ},\ \ d=\tfrac{-BC\alpha}{PQR},\ \ e=\tfrac{BCD\alpha}{PQRS}\ \text{ and }\ f=\tfrac{-BCDE\alpha}{PQRST},$$

$$\beta=\tfrac{-B\alpha}{P},\quad \gamma=\tfrac{BC\alpha}{PQ},\quad \delta=\tfrac{-BCD\alpha}{PQR},\quad \varepsilon=\tfrac{BCDE\alpha}{PQRS}$$

and thus the focal lengths:

$$p=\alpha,\ \ q=\tfrac{-\mathfrak{B}\alpha}{P},\ \ r=\tfrac{B\mathfrak{C}\alpha}{PQ},\ \ s=\tfrac{-BC\mathfrak{D}\alpha}{PQR},\ \ t=\tfrac{BCD\mathfrak{E}\alpha}{PQRS},\ \ u=\tfrac{-BCDE\alpha}{PQRST},$$

with there being $m=-PQRST$.

Now of these numbers *P, Q, R, S, T* one must be negative; moreover the separations of the lenses may be expressed thus

$$\alpha+b=\alpha\left(1-\tfrac{1}{P}\right),\ \ \beta+c=\tfrac{-B\alpha}{P}\left(1-\tfrac{1}{Q}\right),\ \ \gamma+d=\tfrac{BC\alpha}{PQ}\left(1-\tfrac{1}{R}\right),$$

$$\delta+e=\tfrac{-BCD\alpha}{PQR}\left(1-\tfrac{1}{S}\right)\ \text{and}\ \ \varepsilon+f=\tfrac{BCDE\alpha}{PQRS}\left(1-\tfrac{1}{T}\right),$$

so that the final interval on account of $\varepsilon=-Tf$ may be shown also thus $\varepsilon+f=f(1-T)$; from which, since f must be a positive quantity, it as apparent at once there must become $T<1$.

Now since the three first lenses must be separated from each other by exceedingly small distances, we may establish each interval $=\eta\alpha$ and hence we will have

$$P=\tfrac{1}{1-\eta}\ \text{ and }\ Q=\tfrac{B}{B+\eta P}.$$

Truly now, since the radius of the field shall become

$$\Phi=\tfrac{\pi-\pi'+\pi''-\pi'''+\pi''''}{m+1}$$

we may put

$$\pi=\omega\xi,\ \ \pi'=-\omega'\xi,\ \ \pi''=i\xi,\ \ \pi'''=-\xi,\ \text{ and }\ \pi''''=\xi,$$

thus so that there shall be

$$\Phi=\tfrac{\omega+\omega'+i+2}{m+1}\xi=M\xi,$$

with there being

$$M=\tfrac{\omega+\omega'+i+2}{m+1}\ \text{ or }\ M=\tfrac{i+2}{m+1}\ \text{approx.}$$

on account of the minimal fractions ω and ω', as we will see soon. And hence on account of $\frac{\pi''''}{\Phi}=\frac{1}{M}$ the distance of the eye from the eyepiece arises at once :

$$O=\frac{1}{M}\cdot\frac{u}{m}=\frac{m+1}{m(i+2)}\cdot u.$$

Again it will be required to consider the following equations :

$$\mathfrak{B}\omega=(1-P)M,$$
$$\mathfrak{C}\omega'=(1-PQ)M-\omega,$$
$$\mathfrak{D}i=(1-PQR)M-\omega'-\omega,$$
$$\mathfrak{E}\ =\ (1-PQRS)M-\omega'-\omega-i.$$

The small parts ω and ω' are sought from the first two of these equations , evidently

$$\omega=\frac{(1-P)M}{\mathfrak{B}}=\frac{-\eta M}{(1-\eta)\mathfrak{B}},$$

$$\omega'=\frac{(1-PQ)M-\omega}{\mathfrak{C}}$$

or

$$\omega'=\frac{\eta(1-B)M}{(B+(1-B)\eta)\mathfrak{C}}+\frac{\eta M}{(1-\eta)\mathfrak{B}\mathfrak{C}},$$

$$\omega'=\frac{\eta M}{\mathfrak{C}}\left(\frac{2B+\eta(1-B)}{(B+\eta(1-B))(1-\eta)B}\right);$$

whereby, since η shall be a very small fraction, there will be approximately

$$\omega=\frac{-\eta M}{\mathfrak{B}},\quad \omega'=\frac{2\eta M}{B\mathfrak{C}}$$

and even now the letters $\mathfrak{B}$ and $\mathfrak{C}$ remain indeterminate, while the following $\mathfrak{D}$ and $\mathfrak{E}$ may be determined by the formulas provided here.

Therefore now the three equations may be examined, which are required to fulfill:

$$\text{I.}\quad 0=+\tfrac{N'\omega}{P}+\tfrac{N''\omega'}{PQ}+\tfrac{N'''i}{PQR}+\tfrac{N''''}{PQRS}+\tfrac{N'''''}{PQRST}$$

$$\text{II.}\quad 0=N-\tfrac{N'}{\mathfrak{B}P}+\tfrac{N''}{B\mathfrak{C}PQ}-\tfrac{N'''}{BC\mathfrak{D}PQR}+\tfrac{N''''}{BCD\mathfrak{E}PQRS}-\tfrac{N'''''}{BCDE\mathfrak{F}PQRST}$$

$$\text{III.}\quad 0=\mu\lambda-\tfrac{\mu'}{\mathfrak{B}P}\left(\tfrac{\lambda'}{\mathfrak{B}^2}+\tfrac{\nu'}{B}\right)+\tfrac{\mu''}{B^3\mathfrak{C}PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^2}+\tfrac{\nu''}{C}\right)-\tfrac{\mu'''}{B^3C^3\mathfrak{D}PQR}\left(\tfrac{\lambda'''}{\mathfrak{D}^2}+\tfrac{\nu'''}{D}\right)$$

$$+\tfrac{\mu''''}{B^3C^3D^3\mathfrak{E}PQRS}\left(\tfrac{\lambda''''}{\mathfrak{E}^2}+\tfrac{\nu''''}{E}\right)-\tfrac{\mu'''''\lambda'''''}{B^3C^3D^3E^3PQRST}.$$

Moreover in the first equation evidently the two first members are very small besides the following, while also with the following multiplied by *m* at this stage remain much smaller. With these omitted there will be had

$$0=N'''i+\tfrac{N''''}{S}+\tfrac{N'''''}{ST},$$

where, since it is agreed the three last lenses to be made from the same kind of glass, there will become

$$0=iST+T+1;$$

from which it is apparent either S or T must be a negative quantity. On this account there may be put $S=-K$, so that a single one of these lenses may lie before the real image, from which there becomes $K=\tfrac{T+1}{iT}$; but before now we have seen to be $T<1$ and thus $K>2$ if $i<1$, and if $i=\tfrac{1}{2}$, thus there will become $K>4$; from which there will become

$$KT=\tfrac{1+T}{i}=-ST.$$

Now since P and Q will be approximately equal to unity $RST=-m$ and thus $R=\tfrac{im}{1+T}$ and thus a large number. But from these approximate values it will be easy to deduce the accurate values from the same first equation.

Now in the second equation it is evident enough the three first terms to be much greater than the following. Therefore with these omitted there will be had

$$0=N-\tfrac{N'}{\mathfrak{B}P}+\tfrac{N''}{B\mathfrak{C}PQ},$$

from which on account of P and $Q=1$ approx. there is deduced to be approximately

$$0=N-\tfrac{N'}{\mathfrak{B}}+\tfrac{N''}{B\mathfrak{C}},$$

from which there is gathered

$$\mathfrak{C} = \frac{N''\mathfrak{B}}{B(N'-B\mathfrak{B})} = \frac{N''(1-\mathfrak{B})}{N'-N\mathfrak{B}}$$

and hence

$$C = \frac{N''(1-\mathfrak{B})}{N'-N\mathfrak{B}-N''+N''\mathfrak{B}}, \quad C = \frac{N''(1-\mathfrak{B})}{N-N''+\mathfrak{B}(N''-N)},$$

where the letter $\mathfrak{B}$ is allowed by us at this stage to be chosen as we wish.

But for $\mathfrak{C}$ the following cases are noteworthy:

1. If $\mathfrak{B} > 1$ and $\mathfrak{B} < \frac{N'}{N}$; then $\mathfrak{C}$ will be negative and thus also C negative.
2. If $\mathfrak{B} > 1$ and $\mathfrak{B}$ likewise $> \frac{N'}{N}$; then $\mathfrak{C}$ will be positive.
3. If there were $\mathfrak{B} < 1$ and $\mathfrak{B} < \frac{N'}{N}$ then $\mathfrak{C}$ will be positive.
4. If there were $\mathfrak{B} < 1$ and $\mathfrak{B} > \frac{N'}{N}$; then $\mathfrak{C}$ will be negative and thus C negative.

But C will be negative for the second case, if N' [and N''] $> N$; but C will be positive if there were N' [and N''] $< N$. For the third case C will be positive, if there shall be $N' > N$ [and $N'' < N$]; but if there shall be $N' < N$ [and $N'' > N$], the C will be negative.

It is required to observe concerning these two remaining letters $\mathfrak{D}$ and $\mathfrak{E}$, $\mathfrak{D}$ and thus D to be negative, while truly $\mathfrak{E}$ to be positive; for there shall become approximately

$$\mathfrak{E} = \frac{i+2}{T} - i$$

and thus

$$E = \frac{\frac{i+2}{T} - i}{1+i-\frac{i+2}{T}}$$

and therefore E is negative.

Now we will examine the separations of the lenses, and indeed in the first place, since there shall be $d < \gamma$, it is necessary that γ shall be positive, and thus there must become $BC\alpha > 0$. Truly, there is

$$BC = \frac{N''\mathfrak{B}}{N'-N''+\mathfrak{B}(N''-N)}.$$

Therefore since the distance γ may contain the greater part of the whole length, $\mathfrak{B}$ must be taken thus, so that the quantity BC will scarcely exceed unity, as before this the coefficient only was in excess of two, and then truly of three.

But at once also γ is returned a positive quantity, and d will be negative and evidently $\gamma + d > 0$.

Again truly δ becomes positive , truly $\delta = Dd$, and also e is positive, thus so that $\delta + e$ shall be positive.

Finally $\varepsilon = Ee$ and thus negative and f positive; also there will be $\varepsilon + f > 0$ on account of $T < 1$, as now we have noted before; from which also the distance of the eye becomes positive, and thus all the conditions have been fulfilled.

Finally it is evident also in the third equation the three first terms to enter mainly into the calculation, as they are much greater besides the rest ; from which there becomes

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu''\lambda''}{B^3\mathfrak{C}^3},$$

where it is required to be effected also, that none of the letters λ, λ', λ'' may exceed unity to any extend. Therefore so that all these may be put in place most conveniently, it will be required to set out the different cases, so that among the first three lenses either one or two crystal lenses may be present and in which place.

CASE 1

286. Where the first lens is crystal, the second and third truly has been prepared from crown glass. Therefore there will become $N = 10$, $N' = 7$ and $N'' = 7$ and thus in the first place

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ and } C = \frac{-7(1-\mathfrak{B})}{3\mathfrak{B}}$$

and thus

$$B\mathfrak{C} = \frac{7\mathfrak{B}}{7-10\mathfrak{B}} \text{ and } BC = -\frac{7}{3};$$

hence therefore $\gamma = -\frac{7}{3}\alpha$, from which it is evident α must be taken negative; which value since it shall not be smaller than in the preceding problem, we do not pursue this case further.

CASE 2

287. Where the second lens is crystal, truly the first and third are made from crown glass; there will be $N = 7$, $N' = 10$, $N'' = 7$ and hence

$$C = \frac{7(1-\mathfrak{B})}{3} \text{ and thus } BC = \frac{7}{3}\mathfrak{B}$$

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ and } B\mathfrak{C} = \frac{7\mathfrak{B}}{10-7\mathfrak{B}},$$

from which there becomes $\gamma = \frac{7}{3}\mathfrak{B}\alpha$. Therefore so that the telescope may be shortened, there must be taken $\mathfrak{B} < 1$; then truly *B* will be positive; but from the third equation we will have

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu\lambda''(10-7\mathfrak{B})^3}{7^3\mathfrak{B}^3},$$

from which there becomes

$$\frac{\mu'\lambda'}{\mu} = \mathfrak{B}^3\lambda + \frac{(10-7\mathfrak{B})^3\lambda''}{7^3};$$

therefore here we may observe, either the letters λ, λ', and λ'' may be reduced to unity, for which it is required that there shall be

$$\frac{\mu'}{\mu} = \mathfrak{B}^3 + \left(\tfrac{10}{7} - \mathfrak{B}\right)^3;$$

of which the expansion on account of $\frac{\mu'}{\mu}$ being almost $=1$ gives

$$\left(\tfrac{10}{7}\right)^3 - 3\left(\tfrac{10}{7}\right)^2\mathfrak{B} + 3\left(\tfrac{10}{7}\right)\mathfrak{B}^2 = 1,$$

of which the roots are first $\mathfrak{B} = 0,97$, by which moreover here we gain nothing to the length, truly the other $\mathfrak{B} = 0,46$ or $\mathfrak{B} = \frac{5}{11}$ and in this manner there becomes $\gamma = \frac{7}{6}\alpha$, which is a significant gain ; which case therefore deserves special attention, so that it may be set out more carefully.

CASE 3

288. Now the third lens shall be made from crystal glass, the first and second from crown glass, so that there shall become $N = 7$, $N' = 7$, but $N'' = 10$. Hence therefore it follows that

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-7\mathfrak{B}} = \frac{10}{7}, \quad C = -\frac{10}{3}$$

and thence

$$B\mathfrak{C} = \tfrac{10}{7}B \text{ and } BC = -\tfrac{10}{3}B,$$

from which there becomes $\gamma = -\frac{10}{3}\alpha$. Therefore there must be $-B < 1$ or $B > -1$, hence $\mathfrak{B} < 1$. Hence therefore the third equation with unity written in place of any λ will be

$$0 = 1 - \frac{1}{\mathfrak{B}^3} + \frac{343(1-\mathfrak{B})^3}{1000\mathfrak{B}^3},$$

or

$$0 = \mathfrak{B}^3 - 1 + \tfrac{343}{1000}(1-\mathfrak{B})^3,$$

or

$$0 = -657 - 1029\mathfrak{B} + 1029\mathfrak{B}^2 + 657\mathfrak{B}^3,$$

which case therefore also is seen worth evaluating.

CASE 4

289. If the first and second lens shall be crystal and the third crown, there will be $N' = N = 10$ and $N'' = 7$; hence

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10(1-\mathfrak{B})} = \frac{7}{10} \text{ and } C = \frac{7}{3}$$

and hence

$$BC = \frac{7B}{3}.$$

Now, since α is negative, but γ positive, there must become $\gamma = +\frac{7}{3}B\alpha$. Now we may put $\frac{7}{3}B = -1$ and now there will become $B = -\frac{3}{7}$, $\mathfrak{B} = -\frac{3}{4}$, and $B\mathfrak{C} = -\frac{3}{10}$, from which the third equation becomes

$$0 = \lambda + \frac{\lambda' 4^3}{3^3} - \frac{\mu''}{\mu} \cdot \frac{\lambda'' 10^3}{3^3},$$

thus so that there may become:

$$\lambda + \frac{64\lambda'}{27} = \frac{\mu''}{\mu} \cdot \frac{1000\lambda''}{27} + \text{etc.},$$

so that since it may not happen, unless large numbers may be taken for λ and λ', this case in no manner can be admitted in practice.

CASE 5

290. The first and third lens shall be crystal, the second from crown glass; there will be $N = N'' = 10$ and $N' = 7$, and thus

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ and } C = -\frac{10(1-\mathfrak{B})}{3},$$

hence

$$BC = -\frac{10}{3}\mathfrak{B} \text{ and } \gamma = -\frac{10}{3}\mathfrak{B}\alpha.$$

Therefore there may be put $\frac{10}{3}\mathfrak{B} = 1$ and there will become $\mathfrak{B} = \frac{3}{10}$, $B = \frac{3}{7}$ and $B\mathfrak{C} = \frac{3}{4}$; from which the third equation will give

$$0 = \lambda - \frac{\mu''}{\mu} \cdot \frac{\lambda' 1000}{27} + \frac{64\lambda''}{27} \text{ or } \lambda + \frac{64\lambda''}{27} = \frac{\mu''}{\mu} \cdot \frac{\lambda' 1000}{27} + \text{etc.},$$

and equally small is suitable in practice.

CASE 6

291. The second and third lens shall be crystal, the first made from crown glass; there will be $N = 7$ and $N' = N'' = 10$, from which it is gathered

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ and } C = -\frac{10(1-\mathfrak{B})}{3\mathfrak{B}}$$

and hence

$$BC = \frac{10}{3} \text{ and } \gamma = \frac{10}{3}\alpha,$$

which case now follows at once.

FURTHER EVALUATION OF THE SECOND CASE

292. So that it may be extended to the value of the letter η, for any magnification *m,* which the aperture of the objective lens demands, its radius shall be approx. $\frac{m}{50}$ in., and we may assume to take $\alpha = \frac{m}{6}$ in., since the lens almost generally is plano-convex, and the thickness of this lens around $\frac{1}{64}\alpha$; whereby, if we may put the separation of the two first lenses to be $\frac{1}{50}\alpha$, there is nothing to be feared, lest the two lenses may touch each other, but enough space may be left over, so that also they will be able to be moved in some manner. Therefore we may put $\eta = \pm\frac{1}{50} = \pm 0,02.$

Now since the first lens is made from crown glass and therefore convex, α will be positive and $\eta = +\frac{1}{50} = +0,02.$ Hence we find at once

$$P = \frac{50}{49} \text{ and } Q = \frac{49B}{49B+1},$$

as we will see from *B* below. Here it may suffice to be observing $Q = 1$ and $P = 1$ approx.

From these premises with the fraction assumed $i = \frac{1}{2}$ and $T = \frac{1}{2}$, since there must be $T < 1$, our first equation will give $K = 6 = -S$ and because $PQ = 1$ approx., $R = \frac{1}{3}m$; nor is there a need, that this value may be elicited more accurately.

Moreover the second equation, for which also it will suffice equally to be satisfied approximately only, since in this case there is $N = 7$, $N' = 10$, $N'' = N''' = N'''' = N''''' = 7$, gives us

$$0 = 7 - \frac{10}{\mathfrak{B}P} + \frac{7}{B\mathfrak{C}PQ},$$

which on taking $P = 1$ and $PQ = 1$ gives

$$\mathfrak{C}=\frac{-7(1-\mathfrak{B})}{(7\mathfrak{B}-10)}=\frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}}$$

and thence

$$C=\frac{7(1-\mathfrak{B})}{3} \text{ and } BC=\frac{7\mathfrak{B}}{3}.$$

Since from the first elements henceforth there shall be $\gamma=\frac{BC\alpha}{PQ}$, which distance contains a particular part of the whole length, we may make $\gamma=\alpha$ or perhaps approx. $=\alpha$ and thus $BC=1$; from which it follows that $\frac{7}{3}\mathfrak{B}=1$ and $\mathfrak{B}=\frac{3}{7}$, from which again

$$B=\frac{3}{4},\quad \mathfrak{C}=\frac{4}{7} \text{ and } C=\frac{4}{3},$$

and thus $B\mathfrak{C}=\frac{3}{7}$. Now from the value B found we will have besides $P=\frac{50}{49}$ also $Q=\frac{147}{151}$ and $PQ=\frac{150}{151}$ and thus now there will be accurately $R=\frac{151m}{3\cdot 150}$.

Therefore now we may progress to the third equation:

$$0=\lambda-\frac{\mu'}{\mu}\cdot\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3\mathfrak{C}^3PQ}-\frac{\mu'\nu'}{\mu}\cdot\frac{\lambda'}{B\mathfrak{B}P}+\frac{\nu}{B^3C\mathfrak{B}PQ}-\frac{\lambda'''}{B^3C^3\mathfrak{D}^3PQR}+\frac{\lambda''''}{B^3C^3D^3\mathfrak{E}^3PQRS}$$
$$-\frac{\nu}{B^3C^3D\mathfrak{D}PQR}+\frac{\nu}{B^3C^3D^3E\mathfrak{E}PQRS}+\frac{\lambda'''''}{B^3C^3D^3E^3m};$$

which so that it may be able to be set out in numbers, it is necessary first to investigate the values of the letters D and E, which are found from the above formulas:

$$\tfrac{1}{2}\mathfrak{D}=(1-PQR)M \quad\text{or}\quad \mathfrak{D}=\left(1-\tfrac{1}{3}m\right)\frac{5}{m+1}=-\frac{5}{3}$$

in order that m be assumed rather large, especially since $\mathfrak{D}$ may occur itself only in small numbers ; and thus $D=-\frac{5}{8}$. In a similar manner there will be

$$\mathfrak{E}=\frac{5(1+2m)}{2(m+1)}-\frac{1}{2}=\frac{9}{2} \quad\text{et}\quad E=\frac{-9}{7}.$$

Therefore with these values substituted we will have

$$\begin{array}{l} 0=\lambda-\underset{(1,0413348)}{10,9985\lambda'}+\underset{(1,1068156)}{12,7884\lambda''}-\underset{(9,8332690)}{0,68119}+\underset{(9,8374334)}{0,68775} \\ +\underset{(9,8115752)}{\frac{0,64800\lambda'''}{m}}+\underset{(8,3516924)}{\frac{0,02247\lambda''''}{m}}-\underset{(9,8010248)}{\frac{0,63245}{m}}-\underset{(8,8906053)}{\frac{0,07773}{m}} \\ +\underset{(0,2849264)}{\frac{1,92720\lambda'''''}{m}} \end{array}$$

from which equation, if there may be taken $\lambda = 1$ and $\lambda' = 1$, there may be deduced

$$\begin{aligned}\lambda' = 0,09092 &+ 0,05891\lambda''' : m - \tfrac{0,06451}{m}\\ +1,16275 &+ 0,0020\lambda'''' : m\\ +\ 0,00060 &+ 0,1752\lambda''''' : m\\ \overline{1,25427} &\end{aligned}$$

It is required to observe with regard to the letters λ''', λ'''', λ''''', since the two latter must allow a full aperture, there must be

$$\lambda''''' = 1,60006 \text{ and } \lambda'''' = 1 + 0,60006(1 - 2\mathfrak{C})^2 = 1 + 0,60006 \cdot 64,$$

from which these two lenses can be computed at once.

But for the fourth lens the value of the number λ''' may be defined from the rays themselves. Then truly the first and the third lens too may be determined by calculation. With which done the value of λ' may be sought; which since it may involve *m* also, first for a determined value of *m,* e.g. $m = 25$, then for $m = \infty$ radii of the faces of this lens will be found from these and in turn the values of these may be inferred for any magnification, as now has been done several times above.

Moreover the separations of the lenses with the focal lengths themselves may be found in the following manner :

$$\begin{aligned}&b = -0,98\alpha, &&\beta = -0,73500\alpha, &&\log.(-\beta) = 9,8662874,\\ &c = 0,75500\alpha, &&\gamma = 1,00667\alpha, &&\log.\gamma = 0,0028856,\\ &d = -\tfrac{3\alpha}{m}, &&\delta = +\tfrac{1,875\alpha}{m},\\ &e = +\tfrac{0,3125\alpha}{m}, &&\varepsilon = -\tfrac{0,40178\alpha}{m}, &&f = +\tfrac{0,80867\alpha}{m}\end{aligned}$$

and the focal lengths

$$\begin{aligned}&p = \alpha, &&q = -0,42000\alpha, &&r = 0,43143\alpha,\\ &s = \tfrac{5\alpha}{m} &&t = +\tfrac{1,40625\alpha}{m} &&u = \tfrac{0,80367\alpha}{m}.\end{aligned}$$

Hence the separation of the lenses :

$$\alpha + b = 0,02\alpha,\ \beta + c = 0,0200\alpha,\ \gamma + d = 1,00667\alpha - \tfrac{3\alpha}{m},$$
$$\delta + e = \tfrac{2,1876\alpha}{m},\quad \varepsilon + f = \tfrac{0,40179\alpha}{m},$$

and for the position of the eye $O = 0,3214 \cdot \tfrac{\alpha}{m}$.

CONSTRUCTION OF THE LENSES

Initially we will investigate the construction for individual lenses prepared from crown glass and with these in place for some lens

$$\text{with the radius}\begin{cases}\text{of the anterior face } = F\\ \text{of the posterior face} = G.\end{cases}$$

This determination will itself be had in the following manner:

I. For the first lens on account of $\lambda = 1$ there will be found:

$$F = \frac{\alpha}{\sigma} = \frac{\alpha}{1{,}6601} = 0{,}60237\alpha, \quad G = \frac{\alpha}{\rho} = \frac{\alpha}{0{,}2267} = 4{,}41111\alpha.$$

II. For the third lens on account of $\lambda'' = 1$ there will become

$$F = \frac{c\gamma}{\gamma\rho + c\sigma} = \frac{Cc}{C\rho+\sigma} = \frac{\gamma}{C\rho+\sigma}, \quad G = \frac{c\gamma}{\gamma\rho + c\rho} = \frac{Cc}{C\sigma+\rho} = \frac{\gamma}{C\sigma+\rho},$$

$$F = \frac{\gamma}{0{,}9624} = 0{,}51298\alpha, \quad G = \frac{\gamma}{2{,}4401} = 0{,}41255\alpha.$$

III. For the fourth lens on account of λ''' even now unknown, there may be put for brevity

$$\tau(1+D)\sqrt{(\lambda''' - 1)} = x$$

and there will become

$$F = \frac{\delta}{D\rho+\sigma\pm x}, \qquad G = \frac{\delta}{\rho+D\sigma\mp x}$$

and thus

$$F = \frac{\delta}{1{,}5184\pm x}, \qquad G = \frac{\delta}{-0{,}8109\mp x}$$

Now in order that this lens may allow the aperture $\frac{1}{2}\xi$, this may eventuate, if the posterior face were made plane or the denominator $= 0$; therefore the lower signs may prevail and there may be put $x = 0{,}8109$, from which there will become

$$G = \infty \quad \text{and} \quad F = \frac{\delta}{0{,}7075} \quad \text{or} \quad F = \frac{2{,}650\alpha}{m},$$

as there must become, since $F = (n-1)5\frac{\alpha}{m}$.

Therefore since there shall be

$$\tau(1+D)\sqrt{(\lambda''' - 1)} = 0{,}8109,$$

there will become

$$\sqrt{(\lambda''' - 1)} = \tfrac{0,8109}{0,3469} \text{ and hence } \lambda''' = 6,4642.$$

IV. For the fifth lens there is $\lambda'''' = 39,40384$, and because this lens is equally convex on both sides, the radius of each face will be

$$= 1,06t = 1,4906 \cdot \tfrac{\alpha}{m}.$$

V. For the sixth lens, as we have seen, $\lambda''''' = 1,60006$ and thus the radius of each face

$$= 1,06u = 0,8518 \cdot \tfrac{\alpha}{m}.$$

VI. For the second lens there will now be found initially

$$\lambda' = 1,25427 + \tfrac{0,6894}{m}.$$

We may now put to be $m = 25$ and there will become $\lambda' = 1,28184$. Whereby, since for the second lens there shall be

$$F = \frac{\beta}{B\rho' + \sigma' \pm \tau'(1+B)\sqrt{(\lambda'-1)}}, \quad G = \frac{\beta}{B\sigma' + \rho' \mp \tau'(1+B)\sqrt{(\lambda'-1)}},$$

there will be $\tau'(1+B)\sqrt{(\lambda'-1)} = 0,81524$, from which there is deduced

$$F = \frac{\beta}{1,6888 \pm 0,81524} = \frac{\beta}{0,8736}, \quad G = \frac{\beta}{1,3284 \mp 0,81524} = -2,1436,$$

hence

$$F = -0,84134\alpha, \quad G = -0,34286\alpha.$$

Now there shall become $m = \infty$, there will become

$$\lambda' = 1,25427 \text{ and } \tau'(1+B)\sqrt{(\lambda'-1)} = 0,77434,$$

from which the radii of the faces

$$F = \frac{\beta}{1,6888 \pm 0,7743} = \frac{\beta}{0,9145}, \quad G = \frac{\beta}{1,3284 \mp 0,7743} = \frac{\beta}{2,1027},$$

hence

$$F = -0,80373\alpha, \quad G = -0,34955\alpha.$$

From these two cases we conclude for any magnification :

$$F = -0,80373\alpha - \frac{f}{m}, \qquad F = -0,80373\alpha - 0,940 \cdot \frac{\alpha}{m}$$

and

$$G = -0,34955\alpha - \frac{g}{m}, \qquad G = -0,34955\alpha + 0,167 \cdot \frac{\alpha}{m}.$$

Finally the radius of the field of view will be $\Phi = \frac{2148}{m+1}$ minutes.

SCHOLIUM

293. Because the three first lenses demand a common aperture, of which the radius shall be $\frac{m}{50}$ in., this must be with respect to the minimum radius of these lenses, which is $0,343\alpha$, of which the fourth part $0,086\alpha$, that is around $\frac{1}{12}\alpha$, itself put equal to $\frac{m}{50}$ in. will give $\alpha = \frac{6}{25}m$ or $\alpha = \frac{1}{4}m$, since before it may have been allowed to set $\alpha = \frac{1}{7}m$, nor therefore have we made use of our avowed arrangements, while we have tried to shorten the length of the telescope ; indeed even if here the length of the telescope may hold a shorter ratio to α, yet the quantity α itself has emerged here almost by so much more. From which it is understood, if we should wish everything to be clearly perfect, in short it is necessary to allow a greater length. Yet meanwhile the length hence resulting is a little less than found above ; but here in this case it has depended properly on the elaboration of the lenses to be liable to much greater difficulties than before, thus so that the skilled practitioner may only reach the goal after many attempts. On account of which we shall not linger further over these investigations, since the construction of telescopes of this kind shall be easy from the calculations brought into the use of the skilled craftsman.

CAPUT III

DE ULTERIORI TELESCOPIORUM SECUNDI GENERIS PERFECTIONE DIVERSAS VITRI SPECIES ADHIBENDO

PROBLEMA 1

273. *Si telescopium ex tribus lentibus sit componendum, invenire momenta ad eius perfectionem facientia.*

SOLUTIO

Incipiendum igitur est a duabus fractionibus, quae methodo ante exposita ponantur $\frac{\alpha}{b}=-P$ et $\frac{\beta}{c}=-Q$, ita ut litterarum *P* et *Q* altera sit positiva, altera vero negativa, ita ut sit $PQ=-m$. Tum igitur erunt

$$\text{distantiae determinatrices}: b=-\frac{\alpha}{P}\ ,\ \ \beta=-\frac{B\alpha}{P},\ \ c=\frac{B\alpha}{PQ},$$

$$\text{distantiae focales}: p=\alpha,\ \ q=-\frac{\mathfrak{B}\alpha}{P},\ \ r=\frac{B\alpha}{PQ}=-\frac{B\alpha}{m},$$

$$\text{et bina intervalla}: \alpha+b=\alpha(1-\tfrac{1}{P}),\quad \beta+c=-\frac{B\alpha}{P}\left(1-\frac{1}{Q}\right).$$

Deinde, cum sit pro campo $\Phi=\frac{\pi-\pi'}{m+1}$ et media lens parum confert, statuamus

$$\pi=\omega\xi \ \text{ et } \ \pi'=-\xi,$$

ut fiat

$$\Phi=\frac{\omega+1}{m+1}\cdot\xi,$$

unde pro distantia oculi habetur

$$O=-\frac{\pi'}{\Phi}\cdot\frac{r}{m}=-\frac{(m+1)}{\omega+1}\cdot\frac{B\alpha}{mm}=-\frac{B\alpha}{m}\cdot\frac{m+1}{m(1+\omega)},$$

ita ut nunc $-B\alpha$ debeat esse positivum; seu his tribus conditionibus erit satisfaciendum:

$$1.\ \alpha\left(1-\frac{1}{P}\right)>0,\quad 2.\ -\frac{B\alpha}{P}\left(1-\frac{1}{Q}\right)>0,\quad 3.\ -B\alpha>0$$

hincque

$$\tfrac{1}{P}\left(1-\tfrac{1}{Q}\right)>0.$$

Porro autem fiet

$$\mathfrak{B}\omega=(1-P)M,\quad B=\tfrac{(1-P)M}{\omega-(1-P)M}$$

existente

$$M=\tfrac{\omega+1}{m+1}.$$

Iam pro margine colorato tollendo aequatio est [§ 49], siquidem pro fractionibus $\tfrac{dn}{n-1}$, $\tfrac{dn'}{n'-1}$, et $\tfrac{dn''}{n''-1}$ litteras N, N', N'' statuamus

$$0=N'\omega\cdot\tfrac{1}{P}+N''\cdot\tfrac{1}{PQ}\quad\text{seu}\quad 0=N'\omega+N''\cdot\tfrac{1}{Q},$$

unde fit

$$Q=-\tfrac{N''}{N'\omega}.$$

Ut autem haec confusio penitus tollatur, requiritur [§ 53], ut sit

$$0=N\alpha-\tfrac{N'\alpha}{\mathfrak{B}P}+\tfrac{N''\alpha}{BPQ},$$

quae loco Q valore substituto fit

$$0=N-\tfrac{N'}{P}\left(\tfrac{1}{\mathfrak{B}}+\tfrac{\omega}{B}\right);$$

est vero

$$\omega=\tfrac{1-P}{(m+1)\mathfrak{B}+P-1},$$

in qua si ponatur valor ipsius ω, erit

$$0=N-\tfrac{N'}{P}\left(\tfrac{m+P}{(m+1)\mathfrak{B}+P-1}\right);$$

deinde aequatio pro Q inventa ob $PQ=-m$ dabit quoque

$$m=\tfrac{N''P\left((m+1)\mathfrak{B}+P-1\right)}{N'(1-P)},$$

ex qua, si in praecedente aequatione pro $(m+1)\mathfrak{B}+P-1$ scribatur valor ipsius $\tfrac{N'm(1-P)}{N''P}$, orietur haec aequatio :

$$0=NP\left((m+1)\mathfrak{B}+P-1\right)-N'(m+P),\quad 0=NN'(1-P)-N'(m+P)$$

indeque porro

$$P=\frac{m(N-N'')}{Nm+N''};$$

ideoque, cum sit

$$\frac{N'm(1-P)}{N''P}=(m+1)\mathfrak{B}+P-1,$$

colligitur

$$\mathfrak{B}=\frac{N'}{N-N''}+\frac{N'}{Nm+N''} \quad \text{sive} \quad \mathfrak{B}=\frac{NN'm+NN''+N'N''-N''N''}{(N-N'')(Nm+N'')},$$

hinc

$$1-\mathfrak{B}=\frac{NNm-NN'm-NN''m-N'N''}{(N-N'')(Nm+N'')}$$

adeoque

$$B=\frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}$$

Iam videamus, quo modo hae determinationes cum superioribus conditionibus subsistere queant, et cum esse debeat $\frac{1}{P}(1-\frac{1}{Q})>0$ sive ob $Q=-\frac{m}{P}$, $\frac{1}{P}+\frac{1}{Q}>0$, erit $\frac{N}{N-N''}>0$ et $N>N''$.

Primum autem intervallum $\alpha(1-\frac{1}{P})>0$ abit in hoc $\frac{-N''(m+1)}{m(N-N'')}\cdot\alpha$ ideoque $\frac{-N''\alpha}{N-N''}>0$, et quia denominator iam inventus est positivus, restat, ut sit $-N''\alpha>0$. Conditio vero $-B\alpha>0$ dabit nunc $B>0$, [sed ex $B=\frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}=-\frac{NN'm+N'N''+N''(N-N'')}{mN(N'+N''-N)+N'N''}$ ob $N>N''$ et $N'+N''-N>0$ prodit $B<0$]; quod cum sit impossibile, etiam impossibile est, ut ope trium lentium haec duo commoda, quibus altera confusio penitus tollitur, obtineantur.

SCHOLION

274. Hoc ergo problema resolvi nequit, siquidem posteriorem confusionem penitus tollere velimus. Omissa autem ultima aequatione solutio facilis fuisset, sed tum plus non essemus consecuti quam in praecedente capite, ubi unica vitri specie sumus usi. Quoniam igitur non convenit duas vitri species adhibere ad telescopia conficienda, quae ex unica specie aeque felici successu obtineri possunt, huic investigationi non immorabimur, sed tantum eiusmodi in medium producere conabimur, quae praeter superiores qualitates etiam omni confusione, quae ibi erat relicta, destituantur. Causa autem, cur ista investigatio hic non successit, in eo manifesto consistit, quod numerus litterarum indefinitarum erat nimis parvus, siquidem ad tres aequationes adimplendas tantum tres litterae praesto erant. Quare si plures lentes constituamus, plures etiam habebimus eiusmodi litteras, quibus non solum his tribus aequationibus, sed reliquis etiam conditionibus satisfieri poterit.

PROBLEMA 2

275. *Si telescopium ex quatuor lentibus sit componendum, determinare momenta ad eius perfectionem facientia.*

SOLUTIO

Tres fractiones hic considerandae ponantur

$$\frac{\alpha}{b} = -P, \quad -\frac{\beta}{c} = -Q, \quad \frac{\gamma}{d} = -R,$$

ita ut harum litterarum *P, Q, R* una sit negativa et $m = -PQR$, unde erunt distantiae determinatrices

$$b = -\frac{\alpha}{P}, \ \beta = -\frac{B\alpha}{P}, \ c = \frac{B\alpha}{PQ}, \ \gamma = \frac{BC\alpha}{PQ}, \ d = \frac{BC\alpha}{PQR},$$

distantiae autem focales

$$p = \alpha, \ q = \frac{-\mathfrak{B}\alpha}{P}, \ r = \frac{B\mathfrak{C}\alpha}{PQ}, \ s = \frac{BC\alpha}{PQR}$$

et intervalla lentium

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right), \ \beta + c = -\frac{B\alpha}{P}\left(1 - \frac{1}{Q}\right), \ \gamma + d = \frac{BC\alpha}{PQ}\left(1 - \frac{1}{R}\right).$$

Pro campo autem apparente

$$\Phi = \frac{\pi - \pi' + \pi''}{m+1}$$

statuatur

$$\pi = \omega\xi, \ \ \pi' = -i\xi \ \text{ et } \ \pi'' = \xi,$$

existante ξ valore maximo, quem hae litterae accipere possunt, scilicet $\frac{1}{4}$;
ita ut sit

$$\Phi = \frac{\omega + i + 1}{m+1} \cdot \xi$$

sive

$$\Phi = M\xi,$$

posito

$$M = \frac{\omega + i + 1}{m+1}.$$

Ex his igitur obtinemus

$$\mathfrak{B}\omega = (1 - P)M, \ \ \mathfrak{C}i = (1 - PQ)M - \omega.$$

Pro loco autem oculi erit

$$O = \frac{\pi''}{\Phi} \cdot \frac{s}{m} = \frac{m+1}{m^2} \cdot \frac{BC\alpha}{\omega + i + 1} = \frac{BC\alpha}{m^2 M};$$

quod ut fiat positivum, debet esse $BC\alpha > 0$.
His positis tribus sequentibus aequationibus satisfieri debet [§ 49, 53, 42]:

$$\text{I.}\quad 0 = \frac{N'\omega}{P} + \frac{N''i}{PQ} + \frac{N'''}{PQR},$$

$$\text{II.}\quad 0 = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ} - \frac{N'''}{BCPQR},$$

$$\text{III.}\quad 0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right) - \frac{\mu'''\lambda'''}{B^3C^3PQR};$$

quae resolutio quo facilius institui possit, consideremus primo casum, quo duae priores lentes sibi immediate iunguntur, ut supra de lentibus duplicatis assumsimus.

Primus casus, quo $\alpha + b = 0$ ideoque $P = 1$ et $\omega = 0$; tum littera manet indeterminata hincque etiam *B*; quo facto resolutio facile institui poterit.

Prima enim aequatio dat

$$0 = N''i + \frac{N'''}{R},$$

unde sequitur

$$R = -\frac{N'''}{N''i},$$

ita ut *B* sit quantitas negativa, siquidem *i* sit positivum, id quod ratio campi postulat. Hinc ergo, cum sit $P = 1$, erit

$$PQR = \frac{-QN'''}{N''i} = -m$$

ideoque

$$Q = \frac{N''mi}{N'''}.$$

Secunda autem aequatio, in qua iam duo postremi termini evadunt valde parvi, siquidem multiplicatio *m* sit magna, statim dat

$$0 = N - \frac{N'}{\mathfrak{B}} \text{ adeoque } \mathfrak{B} = \frac{N'}{N}$$

et hinc fit

$$B = \frac{N'}{N-N'},$$

unde, si libuerit, valor ipsius $\mathfrak{B}$ adcuratius definiri poterit; habebitur nempe

$$\frac{1}{\mathfrak{B}} = \frac{N}{N'} + \frac{N'''(N-N')}{N'N'\mathfrak{C}mi} + \frac{N'''(N-N')}{N'N'Cm};$$

plerumque autem sufficit his duabus aequationibus proxima satisfecisse.

Tertia autem adcurate resolvi debet; cuius secundus terminus cum sit negativus, reliquis existentibus positivis, ut mox videbimus, ille reliquis aequalis esse debet; erit enim

$$\mathfrak{C}i = \frac{N''' - N''mi}{N''} \cdot \frac{i+1}{m+1},$$

ideoque $\mathfrak{C}$ est negativum simulque etiam C, unde conditiones supra memoratae sunt perpendendae.

Primum autem intervallum est per hypothesin $=0$.

Secundum fit $\beta+c=\frac{-N'(N''mi-N''')}{(N-N')N''mi}\cdot\alpha$, pro quo, si α sit positivum, debet esse $\frac{-N'}{N-N'}$ positivum seu $N<N'$; contra autem, si α sit negativum, debet esse $N>N'$.

Tertium porro intervallum est $\frac{N'C\alpha}{(N-N')Q}\left(1+\frac{N''i}{N'''}\right)$; quia hic Q est positivum, C negativum, requiritur, ut sit $-\frac{N'\alpha}{N-N'}$ positivum uti pro secundo intervallo; ita ut, si secundum intervallum fuerit positivum, tertium sponte evadat positivum.

Denique formula pro loco oculi $O=\frac{BC\alpha}{m^2M}$; etiam fit positiva sub conditionibus iisdem. Ex quibus sequitur, si lens prima sit ex vitro coronario, secunda vero ex crystallino sive $N'>N$, tunc debere esse α positivum seu $p>0$, $q<0$, $r>0$, $s>0$.

Sin autem primam lentem ex vitro crystallino, secundam vero ex coronario faciamus, ita ut sit $N'<N$, debebit esse a negativum ideoque $p<0$, $q>0$, $r>0$, $s>0$, quare pro utroque casu facile erit tertiam aequationem resolvere.

His conditionibus perpensis, quae etiam nunc locum habebunt, dummodo ω sit fractio quam minima, statuamus primum intervallum $\alpha\left(1-\frac{1}{P}\right)=\eta\alpha$ existante η fractione minima, sive positiva, si $\alpha>0$, sive negativa, si $\alpha<0$, eritque $P=\frac{1}{1-\eta}$; deinde maneat $\mathfrak{B}$ adhuc indefinita et quaeratur ω eritque

$$\mathfrak{B}\omega=\frac{-\eta}{1-\eta}\cdot\frac{(\omega+i+1)}{(m+1)};$$

in quo postremo factore ω tuto omittitur, ita ut hinc sit

$$\omega=\frac{-\eta}{1-\eta}\cdot\frac{i+1}{(m+1)\mathfrak{B}},$$

qui valor ob duplicem causam diminuitur: primo enim η est valde parvum, deinde ea dividitur per $m+1$, numerum satis magnum; porro vero tam $\mathfrak{B}$ quam $i+1$ ab unitate parum discrepant; quam ob causam valor ω recte pro evanescente haberi poterit; unde prima aequatio nobis dabit ut ante

$$0=N''i+\frac{N''}{R}\quad\text{and}\quad R=\frac{-N'''}{N''i};$$

quem valorem siquis adhuc adcuratius desideret, erit

$$-\frac{1}{R}=\frac{N''\omega Q}{N'''}+\frac{N''i}{N'''},$$

ita ut nunc P et R sint quantitates cognitae; in primo termino utpote minimo sufficit Q proxima nosse, quem adeo ex casu praecedente desumere licet, quia ω iam est definitum et $\mathfrak{B}$ mox definietur. Hinc igitur $Q=\frac{-m}{PR}$. Secunda aequatio iterum ut in casu praecedente proxime dabit ut ante $\mathfrak{B}=\frac{N'}{N}$; si quis eum vero exactius desideret, erit ei hac aequatione utendum:

$$\frac{N'}{\mathfrak{B}}=NP+\frac{N''}{B\mathfrak{C}Q}-\frac{N'''}{BCQR}$$

ubi pro B sufficit valorem prope verum nosse, nempe $B=\frac{N}{N-N'}$. Tum vero habebitur

$$\mathfrak{C}i=\left(1+\frac{m}{R}\right)\cdot\frac{(\omega+i+1)}{(m+1)}-\omega,$$

quem valorem manifestum est propter valorem ipsius R esse negativum ideoque etiam C. Quocirca conditiones praescriptae iisdem casibus implentur ut in praecedente, ubi $\omega=0$, ita ut nunc tantum supersit aequationem tertiam resolvere, si modo meminerimus ob $\pi''=\xi$ quartam lentem fieri debere aeque convexam, quae forma etiam tertiae lenti tribui deberet, si esset $i=1$. Verum si sumeretur $i=1$, unde haec lens fieret utrinque aeque convexa, ob $\mathfrak{C}=-2$ propemodum pro hac lente statui deberet

$$\lambda''=1+N^2\left(1-2\mathfrak{C}\right)^2=1+N^2\cdot 25,$$

ubi, ut ante sumsimus, est $N=\frac{\sigma-\rho}{2\tau}$, sicque numerus λ'' satis magnum obtineret valorem; quod incommodum evitabimus sumendo $i<1$ et plerumque sufficiet statuere $i=\frac{1}{2}$.

COROLLARIUM

276. Hic ergo differentia refractionis vitri tantum in duabus prioribus lentibus in considerationem venit ideoque sufficiet unicam tantum lentem ex vitro crystallino conficere et reliquas omnes ex vitro coronario; sicque duos tantum casus habebimus evolvendos, alterum, quo prima lens ex vitro crystallino conficitur, alterum, quo secunda.

CASUS 1

277. Sit igitur prima lens crystallina, reliquae omnes ex vitro coronario factae; erit $n=1,58$ et $n'=n''=n'''=1,53$, tum vero secundum Dollondi experimenta $N=10,\ \ N'=N''=N'''=7$. His positis et sumto $i=\frac{1}{2}$ erit α negativum ideoque etiam η. Sumatur autem $\eta=-0,03$; hinc ergo habebimus $P=\frac{1}{1,03}=\frac{100}{103}$, et quia erit proxime $\mathfrak{B}=\frac{7}{10}$ et $B=\frac{7}{3}$, invenimus $\omega=\frac{45}{721(m+1)}$.

Deinde, cum sit proxime $R=-2$ ideoque $Q=\frac{103m}{200}$, adcuratius habebimus

$$-\frac{1}{R}=\frac{1}{2}+\frac{927m}{28840(m+1)} \text{ seu } R=\frac{-28840(m+1)}{15347m+14420},$$

qui est valor correctus ipsius *R,* ex quo etiam adcuratius *Q* definiri poterit, scilicet $Q=\frac{-m}{PR}$.

Ut nunc etiam $\mathfrak{B}$ adcuratius definiatur, erit

$$\frac{1}{\mathfrak{B}}=\frac{10P}{7}+\frac{3\cdot 200}{7\cdot 103m\mathfrak{C}}+\frac{3\cdot 200}{7\cdot 2\cdot 103mC}.$$

Est vero

$$\mathfrak{C}=\left(1+\frac{m}{R}\right)\left(\frac{3+2\omega}{m+1}\right)-2\omega \text{ et } C=\frac{\mathfrak{C}}{1-\mathfrak{C}};$$

unde etiam *B* definiri potest.

His igitur valoribus definitis tertia aequatio principalis resolvi debet statuendo $\lambda''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2$, ut quarta lens aeque convexa utrinque reddatur.
Pro tertia autem lente videtur statui posse $\lambda''=1$.

Denique inventis singulis litteris λ etiam singularum lentium constructio habebitur. Utemur autem methodo iam aliquoties usitata, scilicet pro casu quodam determinato, puta $m=25$, deinde pro casu $m=\infty$ evolutionem instituamus indeque constructionem pro quacunque multiplicatione derivemus.

EXEMPLUM 1

278. Si prima lens ex vitro crystallino, tres sequentes autem ex coronario sint parandae, pro multiplicatione $m=25$ constructionem telescopii investigare.
Sumto igitur $\eta=-0,03$, ut intervallum duarum priorum lentium fiat $-\frac{3\alpha}{100}$ ob α negativum, habemus statim

$$P=\frac{100}{103}=0,97087,\ \text{Log.}P=9,9871628,\ \text{hinc } \omega=0,00240,$$

deinde

$$R=\frac{-36050\cdot 26}{18\,952\cdot 25+18025} \text{ seu } R=\frac{36050\cdot 26}{491825}=-1,90576,$$

$$\text{Log.}R=\ 0,2800680\ (-)$$

hincque

$$Q=13,51160, \qquad \text{Log.}Q=1,1307092;$$

nunc pro $\mathfrak{C}$ inveniendo notetur esse

$$PQ = 13{,}11810,\ \text{Log.}PQ = \ 1{,}1178720$$

hincque erit

$$\mathfrak{C} = -\ 12{,}1181\tfrac{(3{,}00480)}{26} - 0{,}00480 \text{ seu } \mathfrak{C} = -1{,}40528,$$

$$\text{Log.}\mathfrak{C} = 0{,}1477628\ (-)$$

hincque

$$C = -0{,}58425, \quad \text{Log.}C = 9{,}7665972\ (-).$$

Nunc denique pro B inveniendo nulla approximatione utamur, quia ob

$\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ adcurate habemus

$$1 - \tfrac{1}{\mathfrak{C}Q} + \tfrac{1}{CQR} = \left(\tfrac{10}{7} - P - 1\right)B$$

sive

$$1 + 0{,}05266 + 0{,}06647 = \left(1{,}38696 - 1\right)B$$

adeoque

$$1{,}11913 = 0{,}38696B$$

ideoque

$$B = 2{,}89210,\ \text{Log. } B = 0{,}4612145,$$

consequenter

$$\mathfrak{B} = \tfrac{2{,}89210}{3\ 89210} = 0{,}74307, \quad \text{Log.}\mathfrak{B} = 9{,}8710305.$$

His valoribus definitis primo quaeramus nostra elementa ac reperiemus

$$b = -1{,}03000\alpha$$

$\text{Log.}\frac{b}{\alpha} = 0{,}0128372\ (-)$

$\text{Log.}B = 0{,}4612145$

$\text{Log.}\frac{\beta}{\alpha} = 0{,}4740517\ (-)$ $\qquad \beta = -2{,}97887\alpha$

$\text{Log.}Q = 1{,}1307092$

$\text{Log}\frac{c}{\alpha} = 9{,}3433425$ $\qquad c = +0{,}22046\alpha$

$\text{Log.}C = 9{,}7665972\ (-)$

$\text{Log.}\frac{\gamma}{\alpha} = 9{,}1099397\ (-)$ $\qquad \gamma = -0{,}12880\alpha$

$\text{Log.}R = 0{,}2800680\ (-)$

$\text{Log.}\frac{d}{\alpha} = 8{,}8298717\ (-)$ $\qquad d = -0{,}06759\alpha.$

Hinc porro etiam distantias focales

$$p=\alpha\,,\quad q=-0,76536\alpha,\qquad \text{Log.}\tfrac{q}{\alpha}=9,8838677\ (-)$$
$$r=-0,30982\alpha,\qquad \text{Log.}\tfrac{r}{\alpha}=9,4911053\ (-)$$
$$s=-0,06759\alpha,\qquad \text{Log.}\tfrac{s}{\alpha}=8,8298717\ (-).$$

Porro intervalla lentium erunt

$$\alpha+b=-0,03000\alpha,\quad \beta+c=-2,75841\alpha,\quad \gamma+d=-0,19639\alpha.$$

Distantia denique oculi ab ultima lente

$$O=-1,12480\cdot\tfrac{m+1}{mm}\cdot\alpha$$

ideoque intervallum inter primam et ultimam lentem $=-2,98480\alpha$.

Tertiam iam consideremus aequationem, quae resoluta et per μ divisa pro hoc casu dabit

$$0=\lambda-\tfrac{\mu'}{\mu}\cdot\tfrac{\lambda'}{\mathfrak{B}^3P}+\tfrac{\mu'}{\mu}\cdot\tfrac{\lambda''}{B^3\mathfrak{C}^3P}-\tfrac{\mu'}{\mu}\cdot\tfrac{\nu'}{B\mathfrak{B}P}+\tfrac{\mu'}{\mu}\cdot\tfrac{\nu'}{B^3C\mathfrak{C}^3PQ}-\tfrac{\mu'}{\mu}\cdot\tfrac{\lambda'''}{B^3C^3PQR},$$

ubi $\mu=0,8724$ et $\mu'=0,9875$ et $\nu'=0,2196$, unde habebitur haec aequatio ad numeros reducta

$$0=\lambda-2,84162\lambda'-0,00128\lambda''$$
$$(\text{Log. } 7,1090175)$$
$$-0,00938\lambda'''-0,11913$$
$$(\text{Log. } 7,9724463)+0,00095.$$

Ut nunc hanc aequationem resolvamus, primo notandum est quartam lentem esse utrinque aeque convexam ideoque $\lambda'''=1,60006$, unde, cum eius distantia focalis sit $s=-0,06759\alpha$, erit radius utriusque faciei

$$=1,06s=-0,07164\alpha.$$

Pro tertia autem lente, cuius distantia focalis est $r=\mathfrak{C}c$, quia eius semidiameter aperturae esse debet $=\tfrac{1}{8}\mathfrak{C}c$, cui ergo quarta pars minoris radii aequalis esse debet, inde minor radius esse debet $\tfrac{1}{2}\mathfrak{C}c$, ex quo λ'' definiri oportet. Hunc in finem hanc lentem nunc definiamus. Eius autem radius anterioris faciei est

$$=\frac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}=\frac{Cc}{\rho C+\sigma\mp\tau(1+C)\sqrt{(\lambda''-1)}}$$

et radius faciei posterioris

$$\frac{Cc}{\sigma C+\rho\pm\tau(1+C)\sqrt{(\lambda''-1)}},$$

ita ut habeamus

$$\text{radium faciei}\begin{cases}\text{anterioris}=\frac{Cc}{1,5277\mp 0,41575\tau\sqrt{\lambda''-1}}\ ,\\ \text{posterioris}=\frac{Cc}{-0,7432\pm 0,41575\tau\sqrt{\lambda''-1}},\end{cases}$$

ubi signa superiora valere debent; tum vero prior radius est minor ideoque ponatur $\frac{1}{2}\mathfrak{C}c$; unde colligitur

$$\frac{C}{1,5277-0,41575\tau\sqrt{\lambda''-1}}=\frac{1}{2}\mathfrak{C}$$

sicque erit

$$0,41575\tau\sqrt{\lambda''-1}=0,6961$$

adeoque

$$\text{radium faciei}\begin{cases}\text{anterioris}=\frac{\gamma}{0,8316}=-0,15489\alpha\ ,\\ \text{posterioris}=\frac{\gamma}{-\ 0,0471}=+2,73475\alpha,\end{cases}$$

cuius ergo lentis semidiameter aperturae erit $=0,03872\alpha$; at ex valore ipsius x colligemus $\sqrt{\lambda''-1}=1,80969$ hincque $\lambda''=4,27497$.

Nunc revertamur ad nostram aequationem, quae erit

$$0=\lambda-2,84162\lambda'-0,00549-0,01501-0,11818;$$

quia nunc λ' unitate minus esse nequit, statuamus $\lambda'=1$ eritque

$$\lambda=2,98030,\quad \lambda-1=1,98030 \text{ et } \tau\sqrt{\lambda-1}=1,23484;$$

ex quo prima lens ita erit construenda:

$$F=\frac{\alpha}{\sigma\pm 1,23484}=\frac{\alpha}{0,3479}=2,87439\alpha,$$

$$G=\frac{\alpha}{\rho\mp 1,23484}=\frac{\alpha}{1,3762}=0,72664\alpha.$$

Tandem pro secunda lente habemus

$$F = \frac{b\beta}{\rho\beta+\sigma b} = \frac{Bb}{2,3157} = -1,28638\alpha,$$

$$G = \frac{b\beta}{\sigma\beta+\rho b} = \frac{Bb}{5,0279} = -0,59247\alpha;$$

cuius minoris radii pars quarta seu $0,14812\alpha$ dat semidiametrum aperturae tam pro lente prima quam pro lente secunda. Quare hinc deducitur sequens constructio telescopii pro multiplicatione $m = 25$.

I. Pro lente prima, Flint Glass,

$$\text{radium faciei} \begin{cases} \text{anterioris} = 2,87439\alpha, \\ \text{posterioris} = 0,72664\alpha. \end{cases}$$

Semidiameter aperturae $= 0,14812\alpha$.
Intervallum ad secundam $= -0,03\alpha$.

ll. Pro secunda lente, Crown Glass,

$$\text{radium faciei} \begin{cases} \text{anterioris} = -1,28638\alpha, \\ \text{posterioris} = -0,59247\alpha. \end{cases}$$

Semidiameter aperturae ut ante.
Intervallum $= -2,75841\alpha$.

III. Pro tertia lente, Crown Glass,

$$\text{radium faciei} \begin{cases} \text{anterioris} = -0,15489\alpha, \\ \text{posterioris} = +\ 2,73475\alpha. \end{cases}$$

Semidiameter aperturae $= 0,03872\alpha$
Intervallum $= -0,19639\alpha$.

IV. Pro quarta lente, Crown Glass,
radius utriusque faciei $= -0,07164\alpha$.
Semidiameter aperturae $= 0,01791\alpha$.
Distantia ad oculum
$= -11248 \cdot \frac{m+1}{m^2}\alpha$, seu $O = -1,1248\ \left(1+\frac{1}{m}\right)\cdot\frac{\alpha}{m}$, $O = -0,04679\alpha$,

ubi notandum est esse α negativum, ac si semidiametrum aperturae sumamus $= \frac{m}{50}$ dig. $= \frac{1}{2}$ dig., fiat $\alpha = -\frac{7}{2}$ dig. circiter vel maius et longitudo telescopii $= 10\frac{1}{2}$ dig. Denique semidiameter campi erit $\Phi = 49\frac{1}{2}$ minut. circiter.

EXEMPLUM 2

279. Si prima lens ex vitro crystallino, reliquae ex coronario sint parandae, pro multiplicatione maxima constructionem telescopii investigare.

Sit iterum $\eta = -0,03$; habebimus ut ante

$$P = 0,97087,\ \text{Log.}P = 9,9871628;$$

tum vero $\omega = 0$ et

$$R = -\tfrac{28840}{15347} \ \text{seu}\ R = -1,87919,\ \text{Log.}R = 0,2739707\ (-)$$

$$Q = 0,54148m, \quad \text{Log.}\ \tfrac{Q}{m} = 9,7335868.$$

Porro

$$\mathfrak{C} = \tfrac{3}{R} = -1,57714,\ \text{Log.}\ \mathfrak{C} = 0,1978710\ (-)$$

et

$$C = -0,61197, \quad \text{Log.}C = 9,7867330\ (-).$$

Pro B autem inveniendo habetur haec aequatio

$$1 - \tfrac{1}{\mathfrak{C}Q} + \tfrac{1}{CQR} = \left(\tfrac{10}{7}P - 1\right)B,$$

quae, quia termini per Q divisi evanescunt, abit in hanc

$$1 = 0,3869B,\ \text{hinc}\ B = 2,58464,\ \text{Log.}B = 0,4124012$$

et

$$\mathfrak{B} = 0,72103,\ \text{Log.}\mathfrak{B} = 9,8579557.$$

Hinc habebimus distantias determinatrices

$$\begin{aligned}
b &= -1,03000\alpha, & \text{Log.}\tfrac{b}{\alpha} &= 0,0128371(-)\\
\beta &= -2,66218\alpha, & \text{Log.}\tfrac{\beta}{\alpha} &= 0,4252383(-)\\
c &= +4,91645\tfrac{\alpha}{m}, & \text{Log.}c\cdot\tfrac{m}{\alpha} &= 0,6916515\\
\gamma &= -3,00874\tfrac{\alpha}{m}, & \text{Log.}\gamma\cdot\tfrac{m}{\alpha} &= 0,4783845(-)\\
d &= -1,58173\tfrac{a}{m}, & \text{Log.}d\cdot\tfrac{m}{\alpha} &= 0,1991342(-)
\end{aligned}$$

atque intervalla lentium

$$a+b=-0,03000\alpha,$$
$$\beta+c=-2,66218\alpha+4,91645\cdot\tfrac{\alpha}{m},$$
$$\gamma+d=-4,59047\cdot\tfrac{\alpha}{m}$$

et distantiam oculi post ultimam lentem

$$O=-1,05449\cdot\tfrac{\alpha}{m},$$

tum vero distantias focales

$$p=\alpha,\quad q=-0,74267\alpha,\quad r=-7,75394\cdot\tfrac{\alpha}{m},\quad s=-1,58173\cdot\tfrac{\alpha}{m}.$$

Nunc igitur consideremus aequationem tertiam

$$0=\lambda-\tfrac{\mu'}{\mu}\cdot\tfrac{\lambda'}{\mathfrak{B}^3P}-\tfrac{\mu'\nu'}{\mu\mathfrak{B}BP},$$

$$0=\lambda-3,11022\lambda'-0,13738,$$

quae sumto ut ante $\lambda'=1$ dat

$$\lambda=3,24760,\ \text{hinc}\ \lambda-1=2,24760\ \text{et}\ \tau\sqrt{\lambda-1}=1,31555,$$

unde lentium constructio sequenti modo expedietur:

I. Pro prima lente ex vitro crystallino

$$F=\tfrac{\alpha}{\sigma-1,31555}=\tfrac{\alpha}{0,2672}=3,74251\alpha,$$

$$G=\tfrac{\alpha}{\rho+1,731555}=\tfrac{\alpha}{1,4569}=0,68639\alpha.$$

II. Pro secunda lente ex vitro coronario

$$F=\tfrac{b\beta}{\rho\beta+\sigma b}=\tfrac{\beta}{2,2461}=-1,18525\alpha,$$

$$G=\tfrac{b\beta}{\sigma\beta+\rho b}=\tfrac{\beta}{4,5175}=-0,58931\alpha.$$

III. Pro tertia lente ex Crown Glass

$$F=\tfrac{Cc}{C\rho+\sigma\mp x},\quad G=\tfrac{Cc}{C\sigma+\rho\pm x},$$

$$F=\tfrac{\gamma}{1,5214\mp x},\quad G=\tfrac{\gamma}{-0,7892\pm x}.$$

Cum nunc ut ante radius faciei anterioris fiat minor, is semissi distantiae focalis $=\frac{1}{2}\mathfrak{C}c$ aequalis ponatur eritque

$$1,5214-x=\frac{2\mathrm{C}}{\mathfrak{C}}=2\left(C+1\right) \text{ seu } x=0,7454.$$

Unde statim habetur

$$F=\frac{\gamma}{0,7760}=-3,87697\cdot\frac{\alpha}{m},\quad G=\frac{\gamma}{-0,1438}=+\,68,69269\cdot\frac{\alpha}{m}.$$

IV. Denique pro quarta lente, cuius distantia focalis $s=-1,58173\cdot\frac{\alpha}{m}$; radius utriusque faciei erit

$$=1,06s=-1,67663\cdot\frac{\alpha}{m};$$

sicque omnia momenta pro hoc casu sunt definita.

EXEMPLUM 3

280. Si prima lens ex vitro crystallino, reliquae ex coronario sint parandae, pro multiplicatione quacunque constructionem telescopii exponere.

SOLUTIO

Ex comparatione duorum exemplorum praecedentium singula momenta methodo supra indicata facile definiemus. Primo pro distantiis determinatricibus erit $b=-1,0300\alpha$; pro reliquis autem statuatur

$$\beta=-2,6622\alpha-\beta'\cdot\frac{\alpha}{m},$$
$$c=+4,9164\cdot\frac{\alpha}{m}+\frac{c'}{m}\cdot\frac{\alpha}{m},$$
$$\gamma=-3,0087\cdot\frac{\alpha}{m}-\frac{\gamma'}{m}\cdot\frac{\alpha}{m},$$
$$d=-1,5817\cdot\frac{\alpha}{m}-\frac{d'}{m}\cdot\frac{\alpha}{m}$$

eritque

$$\beta'=7,9150,\quad c'=14,8775,\quad \gamma'=5,283,\quad d'=2,701;$$

simili modo pro distantiis focalibus est $p=\alpha$; pro reliquis statuatur

$$q = -07427\alpha - \tfrac{q'}{m} \cdot \alpha,$$

$$r = -7\ 7539 \cdot \tfrac{\alpha}{m} - \tfrac{r'}{m} \cdot \tfrac{\alpha}{m},$$

$$s = -1,5817 \cdot \tfrac{\alpha}{m} - \tfrac{s'}{m} \cdot \tfrac{\alpha}{m}$$

eritque

$$q' = 0,5665, \quad r' = -0,2100, \quad s' = 2,701;$$

unde lentium intervalla sunt

$$\alpha + b = -0,0300\alpha,$$

$$\beta + c = -2\ 6622\alpha - \frac{2,998}{m} \cdot \alpha + \frac{14,88}{m} \cdot \frac{\alpha}{m},$$

$$\gamma + d = -4,5904 \cdot \tfrac{\alpha}{m} - \frac{7,984}{m} \cdot \tfrac{\alpha}{m}.$$

Pro loco oculi statuatur

$$O = -1,05449 \cdot \tfrac{\alpha}{m} - \tfrac{O'}{m} \cdot \tfrac{\alpha}{m}; \text{ erit } O' = 2,8815.$$

1. Pro lente prima

$$\text{statuatur radius faciei} \begin{cases} \text{anterioris } = 3,7425\alpha + F' \cdot \tfrac{\alpha}{m} \\ \text{posterioris} = 0,6864\alpha + G' \cdot \tfrac{\alpha}{m}; \end{cases}$$

erit $F' = -21,70, \quad G' = 1,005.$

II. Pro lente secunda

$$\text{statuatur radius faciei} \begin{cases} \text{anterioris } = -1,1853\alpha - F' \cdot \tfrac{\alpha}{m} \\ \text{posterioris} = -0,5893\alpha - G' \cdot \tfrac{\alpha}{m}; \end{cases}$$

erit $F' = 2,52, \quad G' = 0,08.$

III. Pro lente tertia

statuatur radius faciei $\begin{cases} \text{anterioris} = -3,8769\cdot\frac{\alpha}{m} - F'\cdot\frac{\alpha}{m} \\ \text{posterioris} = +68,6929\cdot\frac{\alpha}{m} + G'\cdot\frac{\alpha}{m}; \end{cases}$

erit $F' = -0,114$, $G' = -8,098$.

IV. Denique pro quarta lente
radius utriusque faciei $= -1,6766\cdot\frac{\alpha}{m} - \frac{H}{m}\cdot\frac{\alpha}{m}$;
erit $H = +2,862$, ex quibus conficitur sequens constructio telescopii pro multiplicatione quacunque *m*.

I. Pro prima lente, Crystall Glass,

statuatur radius faciei $\begin{cases} \text{anterioris} = 3,7425\cdot\frac{\alpha}{m} - 21,70\cdot\frac{\alpha}{m} \\ \text{posterioris} = 0,6864\cdot\frac{\alpha}{m} + 1,005\cdot\frac{\alpha}{m}. \end{cases}$

Eius distantia focalis $= \alpha$.
Semidiameter aperturae $= \frac{m}{50}$ dig.
Intervallum ad lentem secundam $= -0,03\alpha$.

II. Pro secunda lente, Crown Glass,

statuatur radius faciei $\begin{cases} \text{anterioris} = -1,1853\alpha - 2,52\cdot\frac{\alpha}{m} \\ \text{posterioris} = -0,5893\alpha - 0,08\cdot\frac{\alpha}{m}. \end{cases}$

Distantia focalis $= -0,7427\alpha - \frac{0,5665\alpha}{m}$.
Semidiameter aperturae quoque $= \frac{m}{50}$ dig.
Intervallum ad tertiam $= -2,6622\alpha - 2\,998\cdot\frac{\alpha}{m} + \frac{14,88}{m}\cdot\frac{\alpha}{m}$.

III. Pro tertia lente, Crown Glass,

radius faciei $\begin{cases} \text{anterioris} = -3,8769\cdot\frac{\alpha}{m} - 0,114\cdot\frac{\alpha}{m} \\ \text{posterioris} = +68,6929\cdot\frac{\alpha}{m} - 8,098\cdot\frac{\alpha}{m}. \end{cases}$

Distantia focalis $= -7,7539 \cdot \frac{\alpha}{m} + 0,210 \cdot \frac{\alpha}{mm}$,
cuius pars octava dat semidiametrum aperturae.

Intervallum ad quartam $= -4,5904 \cdot \frac{\alpha}{m} - 7,984 \cdot \frac{\alpha}{mm}$.

At distantia imaginis realis post hanc lentem

$$= \gamma = 3,0087 \cdot \frac{\alpha}{m} - 5,283 \cdot \frac{\alpha}{mm}.$$

IV. Pro quarta lente, Crown Glass,

$$\text{radius utriusque faciei } = -1,6766 \cdot \frac{\alpha}{m} - \frac{2,862\alpha}{mm}.$$

Eius distantia focalis $= -1,5817 \cdot \frac{\alpha}{m} - \frac{2,70\alpha}{mm}$,
cuius pars quarta dat semidiametrum aperturae.

Et intervallum ad oculum

$$O = -1,0545 \cdot \frac{\alpha}{m} - 2,8815 \cdot \frac{\alpha}{mm}.$$

Hinc totius telescopii longitudo erit

$$\begin{aligned} &= -0,03\alpha \quad -2,998\frac{\alpha}{m} + 14,88\frac{\alpha}{mm} \\ &\quad -2,6622\alpha - 4,590\frac{\alpha}{m} - \ \ 7,98\frac{\alpha}{mm} \\ &\quad\quad\quad\quad\quad\quad -1,054\frac{\alpha}{m} - \ \ 2,88\frac{\alpha}{mm} \\ \hline &= -2,6922\alpha - 8,642\frac{\alpha}{m} + \ \ 4,02\frac{\alpha}{mm}. \end{aligned}$$

Semidiameter vero campi apparentis erit $= \frac{1289}{m+1}$ minut.

Hic vero notandum est a esse negativum, cuius valor definietur ex radio posteriore lentis secundae, cuius pars quarta $-0,1478\alpha$ seu circiter $= -\frac{1}{7}\alpha$ ipsi $\frac{m}{50}$ aequalis posita dabit $\alpha = -\frac{7m}{50}$; statui igitur poterit $\alpha = -\frac{m}{7}$.

COROLLARIUM

281. Si igitur statuamus $\alpha = -\frac{m}{7}$, habebitur sequens constructio determinata telescopii in ratione $m:1$ multiplicantis.

I. Pro prima lente, Flint Glass,

$$\text{radius faciei in digitis} \begin{cases} \text{anterioris} \ = - \ 0,5346m + 3,10 \\ \text{posterioris} = -0,0981m - 0,14. \end{cases}$$

Distantia focalis $=-\frac{m}{7}$.
Semidiameter aperturae $=\frac{m}{50}$.
Intervallum $=0,0043m$.

II. Pro secunda lente, Crown Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= +\ 0,1693m+0,36\\ \text{posterioris} &= +\ 0,0842m+0,01.\end{cases}$$

Distantia focalis $=+0,1061m+0,081$.
Semidiameter aperturae $=\frac{m}{50}$.
Intervallum $=+0,3803m+0,43-\frac{2,12}{m}$.

III. Pro tertia lente, Crown Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= +\ 0,55-\frac{0,02}{m}\\ \text{posterioris} &= -9,81+\frac{1,16}{m}.\end{cases}$$

Distantia focalis $=+1,11-\frac{0,03}{m}$.

Intervallum $=0,66+\frac{1,14}{m}$.
Distantia imaginis realis post hanc lentem $=0,43+\frac{0,75}{m}$.

IV. Pro quarta lente, Crown Glass,

$$\text{radius utriusque faciei} = +0,24+\frac{0,41}{m}$$

Distantia focalis $=0,23+\frac{0,4}{m}$.
Semidiameter aperturae $=0,06+\frac{0,1}{m}$.
Intervallum usque ad oculum $=0,15+\frac{0,4}{m}$
et tota telescopii longitudo

$$=0,3846m+1,23-\frac{06}{m}$$

et semidiameter campi $=\frac{1289}{m+1}$ minut.

SCHOLION

282. Haec itaque telescopia multo fiunt longiora quam ea, quae praecedente capite sunt inventa, pro eadem utrinque multiplicatione. Hic enim pro multiplicatione $m=100$ prodit longitudo $=40$ dig., dum in praecedente capite sufficiebat longitudo $=13\frac{1}{2}$ dig., hoc est triplo minor. Hic igitur merito quaeritur, utrum qualitas, qua etiam spatium diffusionis a diversa radiorum refrangibilitate oriundae ad nihilum redigitur, tanti sit habenda, ut longitudo telescopii triplicetur, quae quaestio non nisi per praxin diiudicari poterit, quae autem eo erit difficilior, quo minus accuratissimam exsecutionem horum praeceptorum exspectare licet. Quocirca nisi haec conditio praescripta felicissime succedat, semper praestabit telescopia praecedentis capitis praeferre, ita ut duplici vitri specie carere possimus. Ceterum in evolutione casus secundi huiusmodi evolutionibus numericis non immorabimur, cum quia methodus tales calculos tractandi iam satis est explicata, tum vero quia propositum est huic operi singularem librum subiungere, in quo sola praecepta pro praxi dirigenda in gratiam artificum colligentur.

CASUS 2

283. Sit iam secunda lens ex vitro crystallino, reliquae vero ex vitro coronario, ita ut sit $n'=1,58$ et $n=n''=n'''=1,53$ indeque etiam $N=N''=N'''$, et ponatur $\frac{N'}{N}=\zeta$ ut sit secundum Dollondum $\zeta=\frac{10}{7}$. His positis et sumto $i=\frac{1}{2}$ erit α positivum ideoque etiam η. Sumatur igitur $\eta=0,03$, unde habebimus $P=\frac{1}{1-\eta}$. Quia nunc, ut vidimus, est $\mathfrak{B}=\zeta$ proxime, erit

$$\omega=\frac{-\eta}{1-\eta}\cdot\frac{i+1}{(m+1)\zeta}=\frac{-9}{194\zeta(m+1)}.$$

Deinde, cum sit proxime $R=-2$ ideoque $Q=\frac{m}{2P}$, ubi $P=+\frac{100}{97}$, ad curatius habebimus [Vide Opus 446 (indicis ENESTROEMIANI): *Instruction détaillée pour porter les lunettes de toutes les différentes especes au plus haut degré de perfection dont elles sont susceptibles tirée de la théorie dioptrique de M. Euler le pere et mise à la portee de tous les ouvriers en ce genre par Nicolaus Fuss.* St.-Pétersbourg 1774];
ex prima aequatione

$$0=\zeta Q\omega+i+\frac{1}{R},$$

quae abit in hanc

$$-\frac{1}{R}=\frac{1}{2}-\frac{9m}{388P(m+1)} \text{ seu } -\frac{1}{R}=\frac{1}{2}-\frac{9m}{400P(m+1)},$$

ex quo etiam Q adcuratius definiri potest, cum sit $Q=\frac{-m}{PR}$.

Ut nunc etiam $\mathfrak{B}$ adcuratius definiatur, colligimus ex aequatione secunda [§ 275]

$$0=1-\frac{\zeta}{\mathfrak{B}P}+\frac{1}{B\mathfrak{C}PQ}-\frac{1}{BCPQR},$$

quae ob $\frac{1}{\mathfrak{B}}=\frac{1}{B}+1$ dabit

$$-B(\zeta-P)=\zeta-\frac{1}{\mathfrak{C}Q}+\frac{1}{CQR},$$

Erat autem proxime $B=\frac{-\zeta}{\zeta-1}$ ideoque negativum. Ex valore porro adcurato ipsius B erit $\mathfrak{B}=\frac{B}{B+1}$.

His igitur definitis tertia aequatio erit

$$0=\lambda-\frac{\mu'\lambda'}{\mu\mathfrak{B}^3P}+\frac{\lambda''}{B^3\mathfrak{C}^3PQ}-\frac{\lambda'''}{B^3C^3PQR}-\frac{\mu'\nu'}{\mu B\mathfrak{B}P}+\frac{\nu}{B^3C\mathfrak{C}PQ}.$$

Hic duo primi termini utpote valde magni dabunt proxime $\lambda'=\frac{\mu}{\mu'}\mathfrak{B}^3P\lambda,$ unde intelligere licet pro lente concava eius valorem λ maiorem prodire quam casu primo, ita ut casus primus ad praxin sit aptior iudicandus.

Ut nunc etiam de longitudine telescopii iudicare possimus, quia praecedente casu ea nimis magna prodierat, considerari debet eius pars praecipua, quae est littera β, cuius valor ante fuerat $=-2,6622\alpha$ circiter, qui autem nunc ob $\beta=\frac{-B\alpha}{P}$ et $B=\frac{-\zeta}{\zeta-1}$ proxime seu $B=-\frac{10}{3}$ et $P=1$ reperitur $\beta=\frac{10}{3}\alpha=3,33\alpha,$ ita ut telescopia hinc nata adhuc fiant longiora, quae propterea hic fusius evolvere operae non erit pretium.

SCHOLION

284. Quia longitudo telescopii, quae hinc tanta resultat, perfectioni non parum obstat, disquirendum est, utrum huic incommodo non aliquod remedium adferri possit, quod autem ex hypothesi hic facta, qua lens obiectiva quasi ex duabus lentibus constare est assumta, neutiquam sperare licet. Quamobrem lentem obiectivam quasi ex tribus lentibus constantem assumi conveniet, quarum vel una vel duae sint concavae et ex vitro crystallino formatae. Ne autem novam investigationem circa maiorem campum suscipere cogamur, hic statim quoque tres lentes quasi oculares introducamus, ut hoc modo, si negotium successerit, non solum breviora telescopia obtineantur, sed etiam simul campus apparens notabile incrementum accipiat.

PROBLEMA 3

285. *Si tres lentes priores ad obiectivam constituendam referantur, tum vero tres quasi lentes oculares adiungantur, definire momenta ad telescopii constructionem necessaria.*

SOLUTIO

Cum igitur hic occurrant sex lentes, statuamus

$$\tfrac{\alpha}{b}=-P,\ \tfrac{\beta}{c}=-Q,\ \tfrac{\gamma}{d}=-R,\ \tfrac{\delta}{e}=-S,\ \tfrac{\varepsilon}{f}=-T,$$

ut sint nostra elementa

$$b=-\tfrac{\alpha}{P},\ c=\tfrac{B\alpha}{PQ},\ d=\tfrac{-BC\alpha}{PQR},\ e=\tfrac{BCD\alpha}{PQRS}\ \text{ et }\ f=\tfrac{-BCDE\alpha}{PQRST},$$

$$\beta=\tfrac{-B\alpha}{P},\ \gamma=\tfrac{BC\alpha}{PQ},\ \delta=\tfrac{-BCD\alpha}{PQR},\ \varepsilon=\tfrac{BCDE\alpha}{PQRS}$$

adeoque distantiae focales

$$p=\alpha,\ q=\tfrac{-\mathfrak{B}\alpha}{P},\ r=\tfrac{B\mathfrak{C}\alpha}{PQ},\ s=\tfrac{-BC\mathfrak{D}\alpha}{PQR},\ t=\tfrac{BCD\mathfrak{E}\alpha}{PQRS},\ u=\tfrac{-BCDE\alpha}{PQRST},$$

existante $m=-PQRST$.

Horum iam numerorum *P, Q, R, S, T* unicus debet esse negativus; intervalla autem lentium ita exprimentur

$$\alpha+b=\alpha\left(1-\tfrac{1}{P}\right),\ \beta+c=\tfrac{-B\alpha}{P}\left(1-\tfrac{1}{Q}\right),\ \gamma+d=\tfrac{BC\alpha}{PQ}\left(1-\tfrac{1}{R}\right),$$

$$\delta+e=\tfrac{-BCD\alpha}{PQR}\left(1-\tfrac{1}{S}\right)\ \text{ et }\ \varepsilon+f=\tfrac{BCDE\alpha}{PQRS}\left(1-\tfrac{1}{T}\right),$$

quod ultimum intervallum ob $\varepsilon=-Tf$ etiam ita exhibetur $\varepsilon+f=f(1-T)$; unde, quia f debet esse quantitas positiva, statim patet esse debere $T<1$.

Quia nunc tres priores lentes exiguis intervallis a se invicem distare debent, statuamus utrumque intervallum $=\eta\alpha$ hincque habebimus

$$P=\tfrac{1}{1-\eta}\ \text{ et }\ Q=\tfrac{B}{B+\eta P}.$$

Nunc vero, cum sit campi semidiameter

$$\Phi=\tfrac{\pi-\pi'+\pi''-\pi'''+\pi''''}{m+1}$$

statuamus

$$\pi=\omega\xi,\ \pi'=-\omega'\xi,\ \pi''=i\xi,\ \pi'''=-\xi,\ \text{ et }\ \pi''''=\xi,$$

ita ut sit

$$\Phi=\tfrac{\omega+\omega'+i+2}{m+1}\xi=M\xi,$$

existante

$$M=\tfrac{\omega+\omega'+i+2}{m+1}\ \text{ seu proxime }\ M=\tfrac{i+2}{m+1}$$

ob ω et ω' fractiones minimas, ut mox videbimus. Hincque ob $\frac{\pi''''}{\Phi} = \frac{1}{M}$ statim oritur distantia oculi

$$O = \frac{1}{M} \cdot \frac{u}{m} = \frac{m+1}{m(i+2)} \cdot u.$$

Porro considerari oportet sequentes aequationes:

$$\begin{aligned} &\mathfrak{B}\omega = (1-P)M, \\ &\mathfrak{C}\omega' = (1-PQ)M - \omega, \\ &\mathfrak{D}i = (1-PQR)M - \omega' - \omega, \\ &\mathfrak{E} \ = \ (1-PQRS)M - \omega' - \omega - i. \end{aligned}$$

Ex harum aequationum duabus primis quaerantur particulae ω et ω', scilicet

$$\omega = \frac{(1-P)M}{\mathfrak{B}} = \frac{-\eta M}{(1-\eta)\mathfrak{B}},$$

$$\omega' = \frac{(1-PQ)M - \omega}{\mathfrak{C}}$$

seu

$$\omega' = \frac{\eta(1-B)M}{(B+(1-B)\eta)\mathfrak{C}} + \frac{\eta M}{(1-\eta)\mathfrak{B}\mathfrak{C}},$$

$$\omega' = \frac{\eta M}{\mathfrak{C}}\left(\frac{2B+\eta(1-B)}{(B+\eta(1-B))(1-\eta)B}\right);$$

quare, cum η sit fractio valde parva, erit proxime

$$\omega = \frac{-\eta M}{\mathfrak{B}}, \quad \omega' = \frac{2\eta M}{B\mathfrak{C}}$$

atque litterae $\mathfrak{B}$ et $\mathfrak{C}$ etiamnum manent indeterminatae, dum sequentes $\mathfrak{D}$ et $\mathfrak{E}$ per formulas hic allatas determinantur.

Nunc igitur considerentur tres aequationes, quas adimpleri oportet

$$\text{I.}\quad 0=+\frac{N'\omega}{P}+\frac{N''\omega'}{PQ}+\frac{N'''i}{PQR}+\frac{N''''}{PQRS}+\frac{N'''''}{PQRST}$$

$$\text{II.}\quad 0=N-\frac{N'}{\mathfrak{B}P}+\frac{N''}{B\mathfrak{C}PQ}-\frac{N'''}{BC\mathfrak{D}PQR}+\frac{N''''}{BCD\mathfrak{E}PQRS}-\frac{N'''''}{BCDE\mathfrak{F}PQRST}$$

$$\text{III.}\quad 0=\mu\lambda-\frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''}{B^3C^3\mathfrak{D}PQR}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right)$$

$$+\frac{\mu''''}{B^3C^3D^3\mathfrak{E}PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^2}+\frac{\nu''''}{E}\right)-\frac{\mu'''''\lambda'''''}{B^3C^3D^3E^3PQRST}.$$

In prima autem aequatione duo membra priora prae sequentibus manifesto sunt valde parva, dum etiam per m multiplicata adhuc multo minora manent sequentibus. Iis omissis habebitur

$$0=N'''i+\frac{N''''}{S}+\frac{N'''''}{ST},$$

ubi, quia tres posteriores lentes ex eadem vitri specie fieri convenit, erit

$$0=iST+T+1;$$

unde patet vel S vel T esse debere quantitatem negativam. Hanc ob rem statuatur $S=-K$, ut unica harum lentium ante imaginem realem cadat, unde fiet $K=\frac{T+1}{iT}$; ante autem iam vidimus esse $T<1$ ideoque $K>2$ ob $i<1$, et si $i=\frac{1}{2}$, fiet adeo $K>4$; ex quo erit

$$KT=\frac{1+T}{i}=-ST.$$

Iam ob P et Q unitati proxime aequales erit quoque proxime $RST=-m$ ideoque $R=\frac{im}{1+T}$ adeoque numerus magnus. Ex his autem valoribus proximis facile erit valores adcuratos ex eadem aequatione prima deducere.

Iam in aequatione secunda satis evidens est tres terminas priores multo maiores esse sequentibus. His ergo omissis habebitur

$$0=N-\frac{N'}{\mathfrak{B}P}+\frac{N''}{B\mathfrak{C}PQ},$$

unde ob P et Q proxima $=1$ colligitur fore etiam proxima

$$0=N-\frac{N'}{\mathfrak{B}}+\frac{N''}{B\mathfrak{C}},$$

unde colligitur

$$\mathfrak{C}=\frac{N''\mathfrak{B}}{B(N'-B\mathfrak{B})}=\frac{N''(1-\mathfrak{B})}{N'-N\mathfrak{B}}$$

hincque

$$C=\frac{N''(1-\mathfrak{B})}{N'-N\mathfrak{B}-N''+N''\mathfrak{B}},\qquad C=\frac{N''(1-\mathfrak{B})}{N-N''+\mathfrak{B}(N''-N)},$$

ubi littera $\mathfrak{B}$ adhuc arbitrio nostro permittitur.

Pro $\mathfrak{C}$ autem sequentes casus sunt notandi:

1. Si $\mathfrak{B} > 1$ et $\mathfrak{B} < \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negativum adeoque etiam *C* negativum.
2. Si $\mathfrak{B} > 1$ et $\mathfrak{B}$ simul $> \frac{N'}{N}$; tunc erit $\mathfrak{C}$ positivum.
3. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} < \frac{N'}{N}$ tunc erit $\mathfrak{C}$ positivum.
4. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} > \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negativum adeoque et *C* negativum.

Pro secundo autem casu erit *C* negativum, si N' [et N''] $> N$; sin autem fuerit N'[et N''] $< N$, erit *C* positivum. Pro casu tertio erit *C* positivum, si sit $N' > N$[et $N'' < N$]; sin autem sit $N' < N$[et $N'' > N$], erit *C* negativum.

Circa duas autem reliquas litteras $\mathfrak{D}$ et $\mathfrak{E}$ notandum est fore $\mathfrak{D}$ negativum ideoque et *D*, tum vero $\mathfrak{E}$ esse positivum; fit enim proxime

$$\mathfrak{E} = \frac{i+2}{T} - i$$

ideoque

$$E = \frac{\frac{i+2}{T} - i}{1 + i - \frac{i+2}{T}}$$

ideoque *E* negativum.

Examinerons iam intervalla lentium, ac primo quidem, cum sit $d < \gamma$, necesse est, ut sit γ positivum, ideoque debet esse $BC\alpha > 0$. Est vero

$$BC = \frac{N''\mathfrak{B}}{N' - N'' + \mathfrak{B}(N'' - N)}.$$

Cum igitur distantia γ maximam partem totius longitudinis contineat, $\mathfrak{B}$ ita accipi debet, ut quantitas *BC* unitatem vix superet, quia ante hic coefficiens modo binarium, tum vero et ternarium superaverat.

Statim autem ac γ reddita est quantitas positiva, erit *d* negativum et manifesto $\gamma + d > 0$.

Porro vero fit δ positivum, nempe $\delta = Dd$, atque etiam *e* positivum, ita ut sit $\delta + e$ positivum.

Denique $\varepsilon = Ee$ adeoque negativum et *f* positivum; eritque etiam $\varepsilon + f > 0$ ob $T < 1$, ut iam ante notavimus; ex quo etiam distantia oculi fit positiva, sicque omnes conditiones sunt adimpletae.

Denique in tertia aequatione etiam manifestum est tres tantum terminos priores potissimum in computum venire, utpote prae reliquis multo maiores; unde fit

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\ \frac{\mu''\lambda''}{B^3\mathfrak{C}^3},$$

ubi efficiendum quoque est, ut nulla litterarum $\lambda,\ \ \lambda',\ \ \lambda''$ unitatem notabiliter superet. Quemadmodum igitur haec omnia commodissime praestentur, diversos casus evolvi oportet, prouti inter tres lentes priores vel una vel duae crystallinae occurrant et quo loco.

CASUS 1

286. Quo prima lens est crystallina, secunda et tertia vero ex vitro coronario est parata. Erit ergo $N=10,\ \ N'=7$ et $N''=7$ adeoque primo

$$\mathfrak{C}=\frac{7(1-\mathfrak{B})}{7-10\mathfrak{B}}\ \text{ et }\ C=\frac{-7(1-\mathfrak{B})}{3\mathfrak{B}}$$

ideoque

$$B\mathfrak{C}=\frac{7\mathfrak{B}}{7-10\mathfrak{B}}\ \text{ et }\ BC=-\frac{7}{3};$$

hinc ergo fit $\gamma=-\frac{7}{3}\alpha$, unde evidens est sumi debere α negativum; qui valor cum non minor sit quam in problemate praecedente, hunc casum ulterius non prosequimur.

CASUS 2

287. Quo lens secunda est crystallina, prima vero et tertia ex vitro coronario; erit $N=7,\ \ N'=10,\ \ N''=7$ atque hinc

$$C=\frac{7(1-\mathfrak{B})}{3}\ \text{ adeoque }\ BC=\frac{7}{3}\mathfrak{B}$$

$$\mathfrak{C}=\frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}}\ \text{ et }\ B\mathfrak{C}=\frac{7\mathfrak{B}}{10-7\mathfrak{B}},$$

unde fit $\gamma=\frac{7}{3}\mathfrak{B}\alpha$. Quo igitur telescopium contrahatur, sumi debet $\mathfrak{B}<1$; tum vero erit B positivum; at ex tertia aequatione habebimus

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\frac{\mu\lambda''(10-7\mathfrak{B})^3}{7^3\mathfrak{B}^3},$$

unde fit

$$\frac{\mu'\lambda'}{\mu}=\mathfrak{B}^3\lambda+\frac{(10-7\mathfrak{B})^3\lambda''}{7^3};$$

hic ergo videamus, an litterae λ et λ' et λ'' possint ad unitatem redigi; ad quod requiritur, ut sit

$$\frac{\mu'}{\mu} = \mathfrak{B}^3 + \left(\tfrac{10}{7} - \mathfrak{B}\right)^3;$$

cuius evolutio ob $\frac{\mu'}{\mu}$ propemodum $=1$ dat

$$\left(\tfrac{10}{7}\right)^3 - 3\left(\tfrac{10}{7}\right)^2 \mathfrak{B} + 3\left(\tfrac{10}{7}\right)\mathfrak{B}^2 = 1,$$

cuius radices sunt prior $\mathfrak{B} = 0,97$, qua autem hic nihil in longitudine lucramur, altera vero $\mathfrak{B} = 0,46$ sive $\mathfrak{B} = \frac{5}{11}$ hocque modo fiet $\gamma = \frac{7}{6}\alpha$, quod insigne lucrum est; qui ergo casus inprimis meretur, ut adcuratius evolvatur.

CASUS 3

288. Sit nunc tertia lens crystallina, prima et secunda ex vitro coronario, ut sit $N = 7$, $N' = 7$, at $N'' = 10$. Hinc igitur sequitur

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-7\mathfrak{B}} = \frac{10}{7}, \quad C = -\frac{10}{3}$$

indeque

$$B\mathfrak{C} = \frac{10}{7}B \text{ et } BC = -\frac{10}{3}B,$$

unde fit $\gamma = -\frac{10}{3}\alpha$. Deberet ergo esse $-B < 1$ vel $B > -1$, hinc $\mathfrak{B} < 1$.
Hinc ergo tertia aequatio unitate loco cuiuslibet λ scripta erit

$$0 = 1 - \frac{1}{\mathfrak{B}^3} + \frac{343(1-\mathfrak{B})^3}{1000\mathfrak{B}^3}$$

seu

$$0 = \mathfrak{B}^3 - 1 + \tfrac{343}{1000}(1-\mathfrak{B})^3$$

seu

$$0 = -657 - 1029\mathfrak{B} + 1029\mathfrak{B}^2 + 657\mathfrak{B}^3,$$

qui ergo casus etiam evolutione dignus videtur.

CASUS 4

289. Si prima et secunda lens sint crystallinae et tertia coronaria, erit $N' = N = 10$ et $N'' = 7$; hincque

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10(1-\mathfrak{B})} = \frac{7}{10} \text{ et } C = \frac{7}{3}$$

hincque

$$BC = \tfrac{7B}{3}.$$

Nunc, quia est α negativum, γ autem positivum, debet esse $\gamma = +\tfrac{7}{3}B\alpha$. Ponamus nunc $\tfrac{7}{3}B = -1$ erit $B = -\tfrac{3}{7}$ et $\mathfrak{B} = -\tfrac{3}{4}$ et $B\mathfrak{C} = -\tfrac{3}{10}$, unde tertia aequatio fiet

$$0 = \lambda + \tfrac{\lambda' 4^3}{3^3} - \tfrac{\mu''}{\mu} \cdot \tfrac{\lambda'' 10^3}{3^3},$$

ita ut esse debeat

$$\lambda + \tfrac{64\lambda'}{27} = \tfrac{\mu''}{\mu} \cdot \tfrac{1000\lambda''}{27} + \text{etc.},$$

quod cum fieri nequeat, nisi pro λ et λ' numeri maximi accipiantur, hic casus nullo modo in praxi admitti potest.

CASUS 5

290. Sint prima et tertia lens crystallinae, secunda ex vitro coronario; erit $N = N'' = 10$ et $N' = 7$ adeoque

$$\mathfrak{C} = \tfrac{10(1-\mathfrak{B})}{7-10\mathfrak{B}} \quad \text{et} \quad C = -\tfrac{10(1-\mathfrak{B})}{3},$$

hinc

$$BC = -\tfrac{10}{3}\mathfrak{B} \quad \text{et} \quad \gamma = -\tfrac{10}{3}\mathfrak{B}\alpha.$$

Ponatur ergo $\tfrac{10}{3}\mathfrak{B} = 1$ eritque $\mathfrak{B} = \tfrac{3}{10}$ et $B = \tfrac{3}{7}$ et $B\mathfrak{C} = \tfrac{3}{4}$; unde aequatio tertia dabit

$$0 = \lambda - \tfrac{\mu''}{\mu} \cdot \tfrac{\lambda' 1000}{27} + \tfrac{64\lambda''}{27} \quad \text{seu} \quad \lambda + \tfrac{64\lambda''}{27} = \tfrac{\mu''}{\mu} \cdot \tfrac{\lambda' 1000}{27} + \text{etc.},$$

quae aeque parum ad praxin est idonea.

CASUS 6

291. Sint secunda et tertia lens crystallinae, prima ex vitro coronario confecta; erit $N = 7$ et $N' = N'' = 10$, unde colligitur

$$\mathfrak{C} = \tfrac{10(1-\mathfrak{B})}{10-7\mathfrak{B}} \quad \text{et} \quad C = -\tfrac{10(1-\mathfrak{B})}{3\mathfrak{B}}$$

hincque

$$BC = \tfrac{10}{3} \quad \text{et} \quad \gamma = \tfrac{10}{3}\alpha,$$

qui ergo casus iam sponte cadit.

EVOLUTIO ULTERIOR CASUS SECUNDI

292. Quod ad valorem litterae η attinet, pro quavis multiplicatione *m,* quae lentis obiectivae aperturam postulat, cuius semidiameter sit circiter $\frac{m}{50}$ dig., sumamus accipi $\alpha=\frac{m}{6}$ dig., quia lens plerumque fere est plano-convexa, eritque huius lentis crassities circiter $\frac{1}{64}\alpha$; quare, si intervallum binarum lentium priorum statuamus $\frac{1}{50}\alpha$, metuendum non est, ne duae lentes se mutuo tangant, sed satis relinquetur spatii, ut etiam quodammodo moveri possint. Ponamus ergo $\eta=\pm\frac{1}{50}=\pm 0,02.$

Quia nunc prima lens est ex vitro coronario ideoque convexa, erit α positivum et $\eta=+\frac{1}{50}=+0,02.$ Hinc reperimus statim

$$P=\frac{50}{49} \text{ et } Q=\frac{49B}{49B+1},$$

ubi de *B* infra dispiciemus. Hic notasse sufficiat esse proxime $Q=1$ et $P=1.$

His praemissis sumta fractione $i=\frac{1}{2}$ et $T=\frac{1}{2},$ quandoquidem esse debet $T<1$, prima aequatio nostra dabit $K=6=-S$ et ob $PQ=1$ proxime $R=\frac{1}{3}m$*;* neque opus est, ut hic valor accuratius eruatur.

Secunda autem aequatio, cui pariter proxime tantum satisfecisse sufficit, quia hoc casu est $N=7,$ $N'=10,$ $N''=N'''=N''''=N'''''=7,$ nobis praebet

$$0=7-\frac{10}{\mathfrak{B}P}+\frac{7}{B\mathfrak{C}PQ},$$

quae sumto $P=1$ et $PQ=1$ dat

$$\mathfrak{C}=\frac{-7(1-\mathfrak{B})}{(7\mathfrak{B}-10)}=\frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}}$$

indeque

$$C=\frac{7(1-\mathfrak{B})}{3} \text{ et } BC=\frac{7\mathfrak{B}}{3}.$$

Cum dein ex primis elementis sit $\gamma=\frac{BC\alpha}{PQ}$, quae distantia praecipuam partem totius longitudinis continet, faciamus $\gamma=\alpha$ sive proxime saltem $=\alpha$ adeoque $BC=1$; unde sequitur $\frac{7}{3}\mathfrak{B}=1$ et $\mathfrak{B}=\frac{3}{7}$, unde porro

$$B=\frac{3}{4}, \quad \mathfrak{C}=\frac{4}{7} \text{ et } C=\frac{4}{3},$$

ideoque $B\mathfrak{C}=\frac{3}{7}.$ Nunc ex valore *B* invento habebimus praeter $P=\frac{50}{49}$ etiam $Q=\frac{147}{151}$ et $PQ=\frac{150}{151}$ sicque adcurate iam erit $R=\frac{151m}{3\cdot 150}$.

Nunc igitur ad aequationem tertiam progrediamur:

$$0=\lambda-\frac{\mu'}{\mu}\cdot\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3\mathfrak{C}^3PQ}-\frac{\mu'\nu'}{\mu}\cdot\frac{\lambda'}{B\mathfrak{B}P}+\frac{\nu}{B^3C\mathfrak{B}PQ}-\frac{\lambda'''}{B^3C^3\mathfrak{D}^3PQR}+\frac{\lambda''''}{B^3C^3D^3\mathfrak{E}^3PQRS}$$
$$-\frac{\nu}{B^3C^3D\mathfrak{D}PQR}+\frac{\nu}{B^3C^3D^3E\mathfrak{E}PQRS}+\frac{\lambda'''''}{B^3C^3D^3E^3m};$$

quae ut in numeros evolvi possit, ante necesse est valores litterarum D et E investigare, qui ex formulis supra datis inveniuntur:

$$\tfrac{1}{2}\mathfrak{D}=(1-PQR)M \quad \text{seu} \quad \mathfrak{D}=\left(1-\tfrac{1}{3}m\right)\tfrac{5}{m+1}=-\tfrac{5}{3}$$

ob m ut valde magnum assumtum, praecipue cum $\mathfrak{D}$ tantum occurrat in numeris per se minimis; adeoque $D=-\frac{5}{8}$. Simili modo erit

$$\mathfrak{E}=\frac{5(1+2m)}{2(m+1)}-\frac{1}{2}=\frac{9}{2} \quad \text{et} \quad E=\frac{-9}{7}.$$

His igitur valoribus substitutis habebimus

$$\begin{array}{l} 0=\lambda-\underset{(1,0413348)}{10,9985\lambda'}+\underset{(1,1068156)}{12,7884\lambda''}-\underset{(9,8332690)}{0,68119}+\underset{(9,8374334)}{0,68775} \\ +\underset{(9,8115752)}{\frac{0,64800\lambda'''}{m}}+\underset{(8,3516924)}{\frac{0,02247\lambda''''}{m}}-\underset{(9,8010248)}{\frac{0,63246}{m}}-\underset{(8,8906053)}{\frac{0,07773}{m}} \\ +\underset{(0,2849264)}{\frac{1,92720\lambda'''''}{m}} \end{array}$$

ex qua aequatione, si sumatur $\lambda=1$ et $\lambda'=1$, colligitur

$$\begin{array}{l} \lambda'=0,09092+0,05891\lambda''' :m-\frac{0,06451}{m} \\ \quad +1,16275 \ +0,0020\lambda'''' \ :m \\ \quad \underline{+\ 0,00060} \ +0,1752\lambda''''' \ :m \\ \quad\ 1,25427 \end{array}$$

Circa litteras λ''', λ'''', λ''''' observandum est, quia binae postremae plenam aperturam admittere debent, esse debere

$$\lambda'''''=1,60006 \ \text{et} \ \lambda''''=1+0,60006(1\ -2\mathfrak{E})^2=1+0,60006\cdot 64,$$

unde hae duae lentes statim computari possunt.

Pro quarta autem lente in ipso radiorum calculo valor numeri λ''' definiatur. Tum vero lens prima et tertia quoque per calculum determinantur.

Quo facto quaeratur valor ipsius λ'; qui cum etiam *m* involvat, primo pro valore determinato ipsius *m,* v. g. $m=25$, deinde pro $m=\infty$ radii facierum huius lentis investigentur ex iisque pro multiplicatione quacunque eorum valores concludantur, ut iam supra aliquoties est factum.

Intervalla autem lentium cum distantiis focalibus sequenti modo se habebunt:

$$b=-0,98\alpha,\qquad \beta=-0,73500\alpha,\qquad \log.(-\beta)=9,8662874,$$
$$c=0,75500\alpha,\qquad \gamma=1,00667\alpha,\qquad \log.\gamma=0,0028856,$$
$$d=-\tfrac{3\alpha}{m},\qquad \delta=+\tfrac{1,875\alpha}{m},$$
$$e=+\tfrac{0,3125\alpha}{m},\qquad \varepsilon=-\tfrac{0,40178\alpha}{m},\qquad f=+\tfrac{0,80867\alpha}{m}$$

et distantiae focales

$$p=\alpha,\qquad q=-0,42000\alpha,\qquad r=0,43143\alpha,$$
$$s=\tfrac{5\alpha}{m}\qquad t=+\tfrac{1,40625\alpha}{m}\qquad u=\tfrac{0,80367\alpha}{m}.$$

Hincque intervalla lentium

$$\alpha+b=0,02\alpha,\quad \beta+c=0,0200\alpha,\quad \gamma+d=1,00667\alpha-\tfrac{3\alpha}{m},$$
$$\delta+e=\tfrac{2,1876\alpha}{m},\qquad \varepsilon+f=\tfrac{0,40179\alpha}{m},$$

et pro loco oculi $O=0,3214\cdot\frac{\alpha}{m}.$

CONSTRUCTIO LENTIUM

Investigemus primo constructionem pro singulis lentibus ex vitro coronario parandis positisque pro quavis lente

$$\text{radiis faciei}\begin{cases}\text{anterioris} &= F\\ \text{posterioris} &= G.\end{cases}$$

Haec determinatio sequenti modo se habebit:

I. Pro prima lente ob $\lambda=1$ reperietur

$$F=\tfrac{\alpha}{\sigma}=\tfrac{\alpha}{1,6601}=0,60237\alpha,\qquad G=\tfrac{\alpha}{\rho}=\tfrac{\alpha}{0,2267}=4,41111\alpha.$$

II. Pro tertia lente ob $\lambda''=1$ erit

$$F=\frac{c\gamma}{\gamma\rho+c\sigma}=\frac{Cc}{C\rho+\sigma}=\frac{\gamma}{C\rho+\sigma},\quad G=\frac{c\gamma}{\gamma\rho+c\rho}=\frac{Cc}{C\sigma+\rho}=\frac{\gamma}{C\sigma+\rho},$$

$$F=\frac{\gamma}{0,9624}=0,51298\alpha,\quad G=\frac{\gamma}{2,4401}=0,41255\alpha.$$

III. Pro quarta lente ob λ''' etiamnunc incognitum ponatur brevitatis gratia

$$\tau(1+D)\sqrt{(\lambda'''-1)}=x$$

eritque

$$F=\frac{\delta}{D\rho+\sigma\pm x},\qquad G=\frac{\delta}{\rho+D\sigma\mp x}$$

adeoque

$$F=\frac{\delta}{1,5184\pm x},\qquad G=\frac{\delta}{-0,8109\mp x}$$

Ut nunc haec lens aperturam $\frac{1}{2}\xi$ admittat, hoc eveniet, si posterior facies fuerit plana seu denominator $=0$; valeant igitur signa inferiora et ponatur $x=0,8109$, unde fiet

$$G=\infty \quad \text{et} \quad F=\frac{\delta}{0,7075} \quad \text{seu} \quad F=\frac{2,650\alpha}{m},$$

uti debet esse, quia $F=(n-1)5\frac{\alpha}{m}$.
Cum igitur sit

$$\tau(1+D)\sqrt{(\lambda'''-1)}=0,8109,$$

erit

$$\sqrt{(\lambda'''-1)}=\frac{0,8109}{0,3469} \quad \text{hincque} \quad \lambda'''=6,4642.$$

IV. Pro quinta lente est $\lambda''''=39,40384$, et quia haec lens est utrinque aeque convexa, erit

$$\text{radius faciei utriusque } =1,06t=1,4906\cdot\frac{\alpha}{m}.$$

V. Pro sexta lente est, uti vidimus, $\lambda'''''=1,60006$ ideoque

$$\text{radius utriusque faciei } =1,06u=0,8518\cdot\frac{\alpha}{m}.$$

VI. Pro secunda lente reperietur nunc primo

$$\lambda'=1,25427+\frac{0,6894}{m}.$$

Statuamus nunc esse $m=25$ eritque $\lambda'=1,28184$. Quare, cum pro secunda lente sit

$$F=\frac{\beta}{B\rho'+\sigma'\pm\tau'(1+B)\sqrt{(\lambda'-1)}},\quad G=\frac{\beta}{B\sigma'+\rho'\mp\tau'(1+B)\sqrt{(\lambda'-1)}},$$

erit $\tau'(1+B)\sqrt{(\lambda'-1)}=0,81524,$ unde colligitur

$$F=\frac{\beta}{1,6888\pm0,81524}=\frac{\beta}{0,8736},\quad G=\frac{\beta}{1,3284\mp0,81524}=-2,1436,$$

hinc

$$F=-0,84134\alpha,\quad G=-0,34286\alpha.$$

Sit nunc $m=\infty$, erit

$$\lambda'=1,25427\quad\text{et}\quad\tau'(1+B)\sqrt{(\lambda'-1)}=0,77434,$$

unde radii facierum

$$F=\frac{\beta}{1,6888\pm0,7743}=\frac{\beta}{0,9145},\quad G=\frac{\beta}{1,3284\mp0,7743}=\frac{\beta}{2,1027},$$

hinc

$$F=-0,80373\alpha,\quad G=-0,34955\alpha.$$

Ex his igitur duobus casibus pro multiplicatione quacunque concludimus

$$F=-0,80373\alpha-\frac{f}{m},\qquad F=-0,80373\alpha-0,940\cdot\frac{\alpha}{m}$$

et

$$G=-0,34955\alpha-\frac{g}{m},\qquad G=-0,34955\alpha+0,167\cdot\frac{\alpha}{m}.$$

Denique semidiameter campi visi erit $\Phi=\frac{2148}{m+1}$ minut.

SCHOLION

293. Quia ternae lentes priores communem postulant aperturam, cuius semidiameter sit $\frac{m}{50}$ dig., hic ad radium minimum istarum lentium, qui est $0,343\alpha$, respici debet, cuius pars quarta $0,086\alpha$, hoc est circiter $\frac{1}{12}\alpha$, ipsi $\frac{m}{50}$ dig. aequalis posita dabit $\alpha=\frac{6}{25}m$ sive $\alpha=\frac{1}{4}m,$ cum ante licuisset statuere $\alpha=\frac{1}{7}m,$ neque ergo voti nostri compotes sumus facti, dum longitudinem telescopii contrahere sumus conati; etsi enim hic longitudo telescopii minorem tenet rationem ad α, tamen ipsa quantitas α fere tanto maior hic prodiit. Ex quo intelligitur, si omnes plane perfectiones desideremus, necesse prorsus esse maiorem longitudinem admittere. Interim tamen longitudo hinc resultans

aliquanto minor est quam supra inventa; sed probe hic est perpendendum hoc casu elaborationem lentium multo maioribus difficultatibus esse obnoxiam quam ante, ita ut artifex non nisi post plurima tentamina scopum attingere possit. Quocirca his investigationibus non ulterius immoror, cum ex calculis allatis facile sit huiusmodi telescopiorum constructionem in usum artificum depromere.