CHAPTER II

CONCERNING COMMON TERRESTRIAL TELESCOPES AND THEIR PERFECTION

DEFINITION

325. *The characteristic of telescopes of this kind is consistent with this, so that rays transmitted through the first two lenses must become parallel to each other again, thus so that these telescopes may be made from two astronomical tubes.*

COROLLARY 1

326. Since these telescopes consist of four lenses, of which both the first two as well as the latter two shall themselves be connected according to the reasoning of astronomical tubes, the magnification of the telescope is on that account composed from the ratio of both the magnifications, which both these astronomical tubes may produce.

COROLLARY 2

327. Evidently if the focal length of the first lens may be put $= p$, of the second $= q$, of the third $= r$ and of the fourth $= s$, the two first lenses arranged for the interval $= p+q$ provide the magnification $= \frac{p}{q}$, truly the two latter lenses arranged for the interval $= r+s$ provide the magnification $= \frac{r}{s}$; the composite telescope will provide the magnification $= \frac{pr}{qs}$.

SCHOLIUM 1

328. Immediately from the start the two posterior lenses are made equal to each other and of the same certain focal length as the second lens, which three lenses are accustomed to be called the eyepieces, thus so that then the latter tube plainly may produce no magnification on account of $r = s = q$. But by how great a distance these two tubes, or the second and third lenses must be moved back from each other, the increases are not well defined ; but generally they appoint this distance to be $= 2q$, thus so that there shall become $r = s = q$, the whole length is going to become $= p + 5q$. But then the craftsmen have observed these telescopes to be made more effectively, if the three posterior lenses may be diminished continually in a certain ratio, which agrees outstandingly well with these, which we have above with regard to telescopes generally, whereby not only have we acquired a much greater field of view from these, whereby they supply a common construction, but also that we have effected especially, that the

colored fringe may be removed completely. Concerning which it will be agreed to propose here the precepts for the construction found before.

Construction of terrestrial telescopes composed from four lenses
for some magnification *m.*

Hence we will define, how great a focal length of the objective lens must be put in place, when we may have designated the following numbers $\lambda, \ \lambda', \ \lambda'', \ \lambda'''$ for the individual lenses.
[See Ch. 1 Book II for definitions of these quantities.]

1. Therefore if $p=\alpha$ may denote the focal length of the objective lens, its shape certainly may be agreed to be taken with the number $\lambda=1$, thus so that, if the ratio of the refraction shall be $n=1,55$, there may be had:

$$\text{Radius} \begin{cases} \text{of the anterior face } = \frac{p}{\sigma} = 0,6145p \\ \text{of the posterior face} = \frac{p}{\rho} = 5,2438p; \end{cases}$$

where so far $\frac{m}{50}$ in. has been put in place for the radius of its aperture. But if either greater or lesser clarity may be desired, either a greater or lesser value will be able to be taken.

The distance of the second lens from the first must be $=p+q$, where the value of q will be indicated soon.

II. For the second lens, if its focal length may be put $=q$, we have seen in the above chapter to be agreed to take $q=\frac{p}{k}$, with k being $=-m+\sqrt{2m(m-1)}$; and since there must be for its aperture $\omega=\frac{-(1+k)(m+k)}{m(m-1)}$, which value for the greater magnification will be $\omega=-\frac{3}{5}$, from which this aperture will not become the maximum, also there is no need, that this lens may be either equally convex, but it will suffice, so that there may be taken $\lambda'=1$ for that, from which the construction of this lens will be:

$$\text{Radius} \begin{cases} \text{of the anterior face} = \frac{q}{\rho} = 5,2438q \\ \text{of ther posterior face} = \frac{q}{\sigma} = 0,6145q; \end{cases}$$

And the radius of the aperture will satisfy the prescribed condition if it may be taken $=\frac{1}{4}\cdot\frac{q}{\sigma}$.

Moreover the distance of the third lens from the second, which above has been put $=\eta\alpha$, has been defined

$$\eta = \frac{k+1}{k^2} + \frac{\theta\sqrt{m(m-1)}}{k^2\sqrt{2}},$$

where the number θ is left to our choice, which moreover may be agreed to be taken neither much greater nor less than unity.

III. For the third lens, since that must receive the maximum aperture on account of $i=1$ and thus must be made equally convex on both sides, there will be $\lambda' = 1,6299$, and since its focal length shall be $r = \frac{\theta p}{k}$, the radius of each face will be $=1,10r$, of which the quarter part will give the radius of the aperture.
From this lens the distance to the fourth lens is

$$= r + s = \theta\alpha(\tfrac{1}{k} + \tfrac{1}{m}).$$

IV. Since the fourth lens also admits the maximum aperture and thus also each side must be equally convex, for that also there will be $\lambda''' = 1,6299$; from which, since its focal length shall be $s = \frac{\theta\alpha}{m}$, the radius of each face $=1,10s$ and $\frac{1}{4}s$ will give the radius of its aperture ; then truly the distance from this lens to the eye will be

$$= \frac{s}{Mm} = \frac{s(m-1)}{\sqrt{2m(m-1)}} = \frac{s\sqrt{(m-1)}}{\sqrt{2m}}.$$

V. And this will show the field of view of a telescope, of which the radius is

$$\Phi == \frac{\sqrt{2}}{\sqrt{m(m-1)}}.\xi$$

or in the measure of minutes of arc

$$\Phi = \frac{1215}{\sqrt{m(m-1)}}$$

VI. Moreover the total length of this instrument as far as to the eye will be

$$= \left(\frac{(k+1)^2}{k^2} + \frac{\theta(m-1)\sqrt{2(m-1)}}{k^2\sqrt{m}} \right) p.$$

VII. But for the focal length *p*, if clarity $y = \frac{1}{50}$ in. may be wished and for the degree of distinction $k = 50$ [this symbol has been used already in § 321 in the formula for α], so that there shall be $kx = m$, on account of the letter μ being a little less than one there will need to be taken in inches :

$$p = m\sqrt[3]{m\left((1+\tfrac{1}{k}+\tfrac{1{,}6299}{\theta^3}(\tfrac{1}{k}+\tfrac{1}{m})\right)};$$

and if a little less clarity may be desired, consider $y=\frac{1}{70}$, and with a lesser degree of distinction, consider $k=35$, we may wish to agree, this same value of p will be able to be reduced to half an inch.

EXAMPLE

329. If a telescope of this kind may magnify only by nine times, so that there shall be $m=9$, there shall be found $k=3$ and hence

$$q=\frac{p}{3},\ r=\frac{\theta p}{3} \text{ and } s=\frac{\theta p}{9},$$

from which the whole length of the telescope $=\left(\frac{16}{9}+\frac{32}{27}\theta\right)p$, and the radius of the field of view $=2°23'$.

Then truly the focal length p thus assumed will have to become

$$p = 9\sqrt[3]{9\left(1+\tfrac{1}{3}+\tfrac{4}{9}\cdot\tfrac{1{,}6299}{\theta^3}\right)};$$

therefore with $\theta=1$ taken, so that the length may become $=\frac{80}{27}p$ or approximately $=3p$, there will be deduced $p=9\sqrt[3]{18{,}5196}$ or approx. $p=24$ in., from which the whole length $=72$ in. $=6$ ft. ; which length, as we have noticed, will be able to be reduced to half.

SCHOLIUM 2

330. Truly also a length of three feet for so small a magnification will be seen to be enormous, especially with common telescopes of this kind being carried around much shorter and with greater magnification. But the particular cause of this length is present in the apparent field of view, which we have tried to produce a maximum ; which without doubt is much greater, than is taken in common instruments of this kind. Yet meanwhile the removal of the colored margin confers quite a lot to the length and likewise a conspicuous amount to the clarity and order of distinction, proposed by us, from which the instruments prepared according to this precept generally surpass these, which are carried around commonly and which generally labour under so many faults, that in practice they may scarcely be tolerated. But the length of these may be able to be diminished considerably, if in place of the objective either a double, or even triple lens may be substituted, such as have been found above from the minimum principle, since

then the value of λ will be reduced from $\frac{1}{5}$ in the first case, truly to $\frac{1}{24}$ for the latter; thus, if in our example λ were $=\frac{1}{5}$, we may have found $p = 20$ in. and the length of the telescope at this stage may have exceeded 5 ft. But with the triple objective lens we may have used, so that it may have become $\lambda = \frac{1}{24}$, there would be produced $p = 19\frac{1}{3}$ in.; from which it appears from these double or triple lenses, such as have been described above, and thus for perfect lenses, where there would be $\lambda = 0$, hardly any significant decrease in the length can be expected, perhaps for the smaller magnifications, where after the cube root sign the terms following λ are certainly notable; but for the greater magnifications there may be going to be a greater gain, so that it may yet be able to be reduced to a half. Whereby it is required to be especially incumbent for this kind of telescopes, that the objective lens shall be either doubled or tripled, so that not only the confusion arising from that itself may be reduced to zero, but also from that which arises from all the following lenses ; since then it will not be necessary for a greater distance p, than the aperture demands on account of the required clarity ; which case we have thus presented in the following problem, so that we may allow an extremely small distance between the first lenses.

[Thus, even at this stage before the wave nature of light was demonstrated by Young's double slit experiment, whereby the resolution of the telescope could be defined in terms of the separation of the max. min. of adjacent diffraction patterns from nearby angular sources, such as binary stars, it had been found experimentally that a wider aperture aided the resolution; however, other difficulties regarding aberrations might be increased.]

PROBLEM 1

331. *In this kind of telescopes in place of the objective lens two lenses of this kind to be substituted prepared from the same kind of glass, so that all the confusion also arising from the remaining lenses may be reduced to nothing and thus the minimal length may be acquired by these telescopes.*

SOLUTION

Here since five lenses shall be had, we may establish our fractions

$$\frac{\alpha}{b} = -P, \quad \frac{\beta}{c} = -Q, \quad \frac{\gamma}{d} = -R \text{ and } \frac{\delta}{e} = -S.$$

Of which letters the first P will be roughly $=1$; the second Q will be negative $=-k$; the third R also will be $=1$, but thus yet, so that the third interval $\gamma + d$ may become a finite quantity, evidently $\eta\alpha$; finally truly there shall be $S = -k'$, thus so that our terms shall be going to become

$$b=-\tfrac{\alpha}{P},\ \ c=-\tfrac{B\alpha}{Pk},\ d=\tfrac{BC\alpha}{PkR}=\infty,\ \ e=\tfrac{BCD\alpha}{PkRk'}=\tfrac{BCD\alpha}{m},$$

$$\beta=-\tfrac{B\alpha}{P},\ \ \gamma=-\tfrac{BC\alpha}{Pk}=\infty,\ \ \delta=\tfrac{BCD\alpha}{PkR}.$$

Hence the intervals:

$$1.\ \ \alpha+b=\alpha\left(1-\tfrac{1}{P}\right)=\tfrac{\alpha}{50},$$

as now we have assumed above [§ 292], thus so that there shall be $P=1\tfrac{1}{49}$,

$$2.\ \ \beta+c=-\tfrac{B\alpha}{P}\left(1+\tfrac{1}{P}\right),$$

$$3.\ \ \gamma+d=-\tfrac{BC\alpha}{Pk}\left(1-\tfrac{1}{R}\right)=\eta\alpha,$$

where evidently there is $C=\infty$ and hence $\tfrac{1}{R}=1+\tfrac{\eta Pk}{BC}$.

4. Since there were $C=\infty$ then D must be infinitely small, thus so that there shall be $CD=-\theta$, and there will be this interval

$$\delta+e=-\tfrac{B\theta\alpha}{PkR}\left(1+\tfrac{1}{k'}\right)$$

with the magnification being $m=PkRk'$, or $m=kk'$ approximately .

But because it may be able to happen, that the distance α may be arranged to be taken negative, we may place the first interval $\alpha+b=\zeta\alpha$ and there will become $P=\tfrac{1}{1-\zeta}$, where it is required to be observes, if α may be a negative quantity, then both ζ as well as η must be taken negative ; but it is necessary always, so that there shall be $-B\alpha>0$ or $B\alpha<0$ and $\theta>0$, as we have now assumed initially, where we have put $CD=-\theta$.

Now since for the apparent field there shall be

$$\Phi=\tfrac{-\pi+\pi'-\pi''+\pi'''}{m-1},$$

we may place

$$\pi=-\nu\xi,\ \ \pi'=\omega\xi,\ \ \pi''=-\xi\ \text{ and }\ \pi'''=\xi,$$

so that there shall be

$$\Phi=\tfrac{\nu+\omega+2}{m-1}\xi=M\xi$$

with there becoming

$$M=\tfrac{\nu+\omega+2}{m-1};$$

from which for the place of the eye we may deduce

$$O=\tfrac{e}{Mm},$$

with there being :

$$e=\tfrac{-B\theta\alpha}{m}.$$

Now we may consider our fundamental formulas :

$$1.\ \mathfrak{B}v = (P-1)M,$$
$$2.\ \mathfrak{C}\omega = -(1+Pk)M - v,$$
$$3.\ \mathfrak{D} = -(1+PkR)M - v - \omega,$$

from which it will be required to be observed the first to become $\mathfrak{B}v = \frac{\zeta}{1-\zeta}M$ *;* and thus the value *v* on account of the twofold reason will become a minimum quantity, thus so that also *mv* according to this shall be very small. Moreover for the second, since there is $C = \infty$, there will be $\mathfrak{C} = \frac{C}{C+1} = 1 - \frac{1}{C}$; but for the third, since there is $D = 0$ or rather $D = -\frac{\theta}{C}$, there will become $\mathfrak{D} = \frac{-\theta}{C-\theta} = \frac{-\theta}{C}$; then also it is necessary to note here to be

$$R = \frac{BC}{BC+\eta Pk} = 1 - \frac{\eta Pk}{BC};$$

since from the second equation on account of $\mathfrak{C} = \frac{C}{C+1}$ therefore there is

$$\omega = -(1+Pk)M\left(1+\tfrac{1}{C}\right) - v\left(1+\tfrac{1}{C}\right),$$

if this value may be substituted into the third equation, there will be

$$-\frac{\theta}{C} = -(1+Pk)M + \frac{\eta P^2k^2M}{BC} - v + (1+Pk)M\left(1+\tfrac{1}{C}\right) + v\left(1+\tfrac{1}{C}\right),$$

where, since the finite terms cancel each other mutually, from the infinitely small there may be concluded to become

$$\theta = -\frac{\eta P^2k^2M}{B} - (1+Pk)M - v,$$

from which there becomes

$$\eta = -\frac{(1+Pk)B}{P^2k^2} - \frac{B\theta}{P^2k^2M} - \frac{Bv}{P^2k^2M},$$

where the final term can be omitted with care.

Again the removal of the colored margin demands this equation:

$$0 = \frac{v}{P} + \frac{\omega}{PQ} + \frac{1}{PQR} + \frac{1}{PQRS},$$

which for our case becomes

$$0 = v - \frac{\omega}{k} - \frac{1}{k} + \frac{1}{kk'},$$

from which with the first term ignored there is deduced

$$k' = \frac{1}{\omega+1},$$

and on account of $m = Pkk'$ there will be

$$m = \tfrac{Pk}{\omega+1}.$$

But since there shall be

$$\omega = -(1+Pk)M \text{ and } M = \tfrac{\omega+2}{m-1},$$

with the term v ignored there will become

$$(m-1)\omega = -(1+Pk)(\omega+2)$$

and hence

$$\omega = \tfrac{-2(1+Pk)}{m+Pk} \text{ and } M = \tfrac{2}{m+Pk}.$$

Whereby, since there shall be $m = \tfrac{Pk}{\omega+1}$, with the value of ω substituted we will obtain

$$m - \tfrac{2m(1+Pk)}{m+Pk} = Pk,$$

and hence

$$mm - 2m = P^2k^2 + 2Pkm,$$

to which with m^2 added to each side provides $2m(m-1) = (Pk+m)^2$ and thus

$$Pk = -m + \sqrt{2m(m-1)}.$$

Therefore with this value for Pk assumed we will gain the maximum value for the apparent field, which will be

$$\Phi = \tfrac{2}{\sqrt{2m(m-1)}} \cdot \xi,$$

and in terms of the angles measured on account of $\xi = \tfrac{1}{4}$, there will become

$$\Phi = \tfrac{1718}{\sqrt{2m(m-1)}} \text{ min}.$$

But now a particular need remains standing there, as the two first lenses must be defined thus, so that the formula for the radius of the confusion found may vanish completely, from which the following equation will be required to be resolved:

$$0 = \lambda - \tfrac{1}{\mathfrak{B}P}\left(\tfrac{\lambda'}{\mathfrak{B}^2} + \tfrac{v}{B}\right) - \tfrac{\lambda''}{\mathfrak{B}^3 Pk} - \tfrac{\lambda'''}{\mathfrak{B}^3\theta^3 Pk} - \tfrac{\lambda''''}{\mathfrak{B}^3\theta^3 m}$$

or

$$0 = \lambda - \tfrac{1}{\mathfrak{B}P} - \tfrac{\lambda''}{\mathfrak{B}^3 Pk} - \tfrac{\lambda'''}{\mathfrak{B}^3\theta^3 Pk} - \tfrac{\lambda''''}{\mathfrak{B}^3\theta^3 m} - \tfrac{v}{\mathfrak{B}BP},$$

in which equation, as we have seen before now, there must be taken $\lambda''=1$, and since the two latter lenses must be equally convex on both sides, there will be for the common glass $\lambda'''=\lambda''''=1,6299$. Truly from this equation either λ or λ' must be defined, just as the coefficient of λ' is greater or less than one. Likewise it is required to be observed all the quantities here besides the letters B and $\mathfrak{B}$ to be determined well enough, thus so that in this matter only the letters B and $\mathfrak{B}$ may be allowed to be chosen by us; in which two cases are to be considered, the one, where $\mathfrak{B}$ is a fraction greater than unity, for example $\frac{1+i}{i}$, the other truly, which is less than unity, for example $=\frac{i}{1+i}$.

For the first case, if there shall be $\mathfrak{B}=\frac{1+i}{i}$, then there will $B=-1-i$ and thus a negative number ; for which case therefore α must be positive or the first lens to be convex, the second truly to be concave, for which the value λ' must be determined, and indeed from this equation :

$$\lambda'=\frac{(1+i)^3P\lambda}{i^3}+\frac{\lambda''}{i^3k}+\frac{\lambda'''}{i^3\theta^3k}+\frac{\lambda''''}{i^3\theta^3m}+\frac{(1+i)v}{i^3},$$

where by taking $\lambda=1$ it is evident λ' to become greater than one.

But the second, if there shall be $\mathfrak{B}=\frac{i}{1+i}$ will be $B=i$ and thus positive ; from which the distance α will become negative or the first lens shall be concave, the second truly convex, in which case the number λ will be required to be defined by this equation :

$$\lambda=\frac{(1+i)^3\lambda'}{i^3P}+\frac{\lambda''}{i^3Pk}+\frac{\lambda'''}{i^3\theta^3Pk}+\frac{\lambda''''}{i^3\theta^3m}+\frac{(1+i)v}{i^3P},$$

and here there will be able to be taken $\lambda'=1$; λ truly will become greater than one.

Therefore it is evident in an almost similar manner, where in the former case λ' is defined, in the second case λ to be defined, therefore so that $P=1$ approximately, wince we have found $P=\frac{1}{1-\zeta}$; where it may be observed in the first case, where α is positive, to be able to take $\zeta=\frac{1}{50}$, so that there shall become $P=\frac{50}{49}$; and in the same manner η also will be positive, just as also our formula declares on putting $B=-1-i$, evidently

$$\eta=\frac{(1+Pk)(1+i)}{P^2k^2}+\frac{(1+i)\theta}{P^2k^2M}.$$

But for the other case, where α is a negative quantity, there must be taken $\zeta=-\frac{1}{50}$, so that there shall be $P=\frac{50}{51}$; and on account of the same reason η also will become, clearly

$$\eta = \frac{i(1+Pk)}{P^2k^2} + \frac{\theta i}{P^2k^2M}.$$

COROLLARY 1

332. Since by removing the colored margin we will have come upon this equation:

$$Pk = -m + \sqrt{2m(m-1)},$$

from which on account of P given the value of k is determined, hence we will have

$$M = \frac{2}{\sqrt{2m(m-1)}} \text{ and } \omega = \frac{-2(1+Pk)}{\sqrt{2m(m-1)}}$$

and hence

$$\eta = \frac{-(1+Pk)B}{P^2k^2} - \frac{B\theta\sqrt{2m(m-1)}}{2P^2k^2}.$$

COROLLARY 2

333. Since $C = \infty$, $D = 0$ and $CD = -\theta$, our elements will become

$$b = \frac{-\alpha}{P},\ \ c = \frac{-B\alpha}{Pk},\ \ d = -\infty,\ \ e = \frac{-B\theta\alpha}{m},$$

$$\beta = \frac{-B\alpha}{P},\ \ \gamma = \infty,\ \ \delta = \frac{-B\theta\alpha}{Pk}$$

and hence the focal lengths

$$p = \alpha,\ \ q = \frac{-B\alpha}{P},\ \ r = \frac{-B\alpha}{Pk},\ \ s = \delta = \frac{-B\theta\alpha}{Pk},\ \ t = \frac{-B\theta\alpha}{m};$$

then truly the separations of the lenses

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right) = \zeta\alpha,\ \ \beta + c = -\frac{B\alpha}{P}\left(1 + \frac{1}{k}\right),$$

$$\gamma + d = \eta\alpha,\ \ \delta + e = -B\theta\alpha\left(\frac{1}{Pk} + \frac{1}{m}\right)$$

and finally the distance of the eye

$$O=\frac{-B\theta\alpha\sqrt{2m(m-1)}}{2m^2},$$

where it will be observed the letter θ to be allowed by our choice, where it will be able with precautions taken, lest the final lenses may become exceedingly small.

COROLLARY 3

334. From these it is clear, so that the greater the letter B may be taken*,* there the greater the following interval to be produced, with the following and hence the length of the telescope there to be increased more; but the letter B may emerge greater, so that the letter $\mathfrak{B}$ approaches closer to one ; indeed whether there shall be $\mathfrak{B}=\frac{1+i}{i}$ or $\mathfrak{B}=\frac{i}{1+i}$, with the number i increased, the number B will be increased *;* whereby, since the letter $\mathfrak{B}$ now also may be allowed by our choice, by no means may that be arranged to be placed exceedingly close to unity nor yet also will it be agreed for the number i be assumed to be very small such as a half or a smaller fraction at this stage; for then an exceedingly large number λ or λ' may be produced from the final equation, evidently thus greater than 27. From which it is concluded the number i must take a minimum value greater than unity.

COROLLARY 4

335. Here therefore convenience is balanced with inconvenience ; indeed if i may be taken less than one, we will obtain the convenience of a short tube, truly on the other hand an exceedingly large value of the number λ or λ' would be a conspicuous inconvenience; but if we may take the number i much greater than one, so that λ or λ' may exceed one by too little, truly on the other hand the tube will become exceedingly long.

COROLLARY 5

336. But if an option may be given between the assumed values $\frac{1+i}{i}$ and $\frac{i}{1+i}$ for $\mathfrak{B}$ with i retaining the same value in each, then λ or λ' may obtain almost the same value. Truly in the first case since there may become $B=-1-i$, the length of the tube will be made longer, than in the other case, where there may be $B=i$*;* on account of which it is always more prudent to select the latter, where the first lens is concave and the second convex, than the former, where in turn the first lens shall be convex, the second truly concave.

SCHOLIUM 1

337. So that which may appear clearer, we may put $i=2$ and $\mathfrak{B}=\frac{2}{3}$, so that there may become $B=2$; then truly therefore there will be $P=\frac{50}{51}$ and our terms will be had in the following manner with the quantity α negative :

$$b=\tfrac{-51}{50}\alpha,\quad c=\tfrac{-2\alpha}{Pk},\quad d=-\infty,\quad e=\tfrac{-2\theta\alpha}{m},$$

$$\beta=\tfrac{-2\alpha}{P}=\tfrac{-51\alpha}{25},\quad \gamma=\infty,\quad \delta=\tfrac{-2\theta\alpha}{Pk}$$

with Pk being $=-m+\sqrt{2m(m-1)}$; then truly the focal lengths will be

$$p=\alpha,\quad q=\tfrac{-51\alpha}{75},\quad r=\tfrac{-2\alpha}{Pk},\quad s=\tfrac{-2\theta\alpha}{Pk},\ \text{et}\ t=\tfrac{-2\theta\alpha}{m},$$

and the lens spacing

$$\alpha+b=-\tfrac{1}{50}\alpha,\quad \beta+c=-\tfrac{51}{25}\alpha-\tfrac{2\alpha}{Pk},$$

$$\gamma+d=\eta\alpha=\frac{-2(1+Pk)\alpha}{P^2k^2}-\frac{\theta\sqrt{2m(m-1)}}{P^2k^2}\alpha,$$

$$\delta+e=\tfrac{-2\theta\alpha}{m}-\tfrac{2\theta\alpha}{Pk}=\tfrac{-2\theta\alpha}{Pkm}\sqrt{2m(m-1)}$$

and the distance of the eye

$$O=\frac{-\theta\alpha\sqrt{2m(m-1)}}{mm},$$

with which performed the radius of the field will be

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\quad \text{min}.$$

But for the radius of the third lens it is required to be observed $\omega=\frac{-2(1+Pk)}{\sqrt{2m(m-1)}}$, thus so that, if m shall be a large enough number, there may become $\omega=-\frac{10}{17}$; from which, since this lens may not be required to have the maximum aperture, but smaller, which shall be to the maximum as 10:17, it will suffice for this lens to have taken $\lambda''=1$; whereby, and if there shall be $\lambda'=1$, but $\lambda'''=\lambda''''=1,6299$, we will find for the objective lens

$$\lambda=\tfrac{27\cdot 51}{8\cdot 0}+\tfrac{1}{8Pk}+\tfrac{1,6299}{8\theta^3Pk}+\tfrac{1,6299}{8\theta^3m}+\tfrac{153v}{200},$$

with v being $=0,2326$, evidently for the refraction $n=1,55$.

Hence moreover with the number λ found the first objective lens thus must be constructed so that there may become,

$$\text{the radius of the }\begin{cases}\text{anterior face} = \frac{\alpha}{\sigma-\tau\sqrt{\lambda-1}}\\ \\ \text{posterior face} = \frac{\alpha}{\sigma+\tau\sqrt{\lambda-1}},\end{cases}$$

with there being $\rho = 0{,}1907, \quad \sigma = 1{,}6274, \quad \tau = 0{,}9051.$

But for the second lens there must be taken,

$$\text{the radius of the }\begin{cases}\text{anterior face} = \frac{2b}{2\rho+\sigma}\\ \\ \text{posterior face truly} = \frac{2b}{2\sigma+\rho},\end{cases}$$

with there being $b = -\frac{51}{50}\alpha.$

For the third lens there will become:

$$\text{the radius of the }\begin{cases}\text{anterior face} = \frac{c}{\rho}\\ \\ \text{posterior face} = \frac{c}{\sigma}\end{cases}$$

with there being $c = \frac{-2\alpha}{Pk}$.

Truly for the fourth lens,

$$\text{the radius of each face } = 1{,}10s,$$

and for the fifth lens,

$$\text{the radius of each face } = 1{,}10t.$$

Truly for the absolute measures requiring to be found the radius of the first lens and the minimum radius of the second will be considered, which shall be $= \mathfrak{m}\alpha$, the fourth part of which $\frac{1}{4}\mathfrak{m}\alpha$ will be equal to the radius of the aperture on account of the clarity sought, which shall be $\frac{m}{50}$ in., and hence there becomes $\alpha = \frac{-2m}{25\mathfrak{m}}$ in., if perhaps it may give the final lenses exceedingly small measures, as above it happens in use, it may be attributed the value of the letter θ greater only by one as it pleases, since hence the length of the telescope may be hardly increased. Moreover this total length as far as to the eye is deduced to be

$$= -\alpha\left(2\frac{3}{50} + \frac{2(1+2Pk)}{P^2k^2} + \frac{(m+Pk)^3\theta}{m^2P^2k^2}\right).$$

EXAMPLE 1

338. If there were $m=9,$ there will be $Pk=3$ and $k=\frac{153}{50}$ on account of $P=\frac{50}{51}$; from which the terms of the telescope will be

$$b=-\tfrac{51}{50}\alpha,\ \ \beta=-\tfrac{51}{25}\alpha,\ \ c=-\tfrac{2\alpha}{3},\ \ \gamma=\infty,$$

$$d=-\infty,\quad \delta=-\tfrac{2\theta\alpha}{3},\quad e=-\tfrac{2\theta\alpha}{9}$$

and the focal lengths

$$p=\alpha,\quad q=-\tfrac{17}{25}\alpha,\quad r=-\tfrac{2}{3}\alpha,\quad s=-\tfrac{2\theta\alpha}{3},\quad t=-\tfrac{2\theta\alpha}{9},$$

and the intervals

$$\alpha+b=-\tfrac{1}{50}\alpha,\quad \beta+c=-\tfrac{203}{75}\alpha,\quad \gamma+d=-\tfrac{(8+12\theta)}{9}\alpha,\quad \delta+e=-\tfrac{8\theta\alpha}{9}$$

and the distance of the eye

$$O=-\tfrac{4\theta\alpha}{27}.$$

Then truly the radius of the apparent field of view

$$\Phi=143\text{ min.}=\ 2°23'.$$

Now truly we will have

$$\begin{aligned}\lambda&=3,4425+0,04166+\tfrac{0,09055}{\theta^3}\\ &\ \ +\ 0,1779\\ \hline &\ \ \ 3,6204\\ &\ \ \ 0,0416\\ \hline \lambda&=3,6620+\tfrac{0,09055}{\theta^3}.\end{aligned}$$

Now we may assume $\theta=1$, so that there may become

$$\lambda=3,75255,\quad \lambda-1=2,75255\ \text{ and }\ \tau\sqrt{\lambda-1}=1,50162.$$

Whereby the construction of the first lens thus will itself be had :

$$\text{radius of the}\begin{cases}\text{anterior face } =\tfrac{\alpha}{0,1258}=7,9491\alpha\\ \\ \text{posterior face}=\tfrac{\alpha}{1,6923}=0,5909\alpha.\end{cases}$$

But for the second lens there will be

$$\text{radius of the}\begin{cases}\text{anterior face} &= \frac{2b}{2,0088} = -1,0155\alpha\\ \text{posterior face} &= \frac{2b}{3,4455} = -0,5921\alpha.\end{cases}$$

Moreover, for the third lens there will be

$$\text{radius of the}\begin{cases}\text{anterior face} &= \frac{c}{0,1907} = -3,4959\alpha\\ \text{posterior face} &= \frac{2b}{1,6274} = -0,4097\alpha.\end{cases}$$

For the fourth lens,

$$\text{the radius of each face } = -0,7333\alpha.$$

For the fifth and final lens

$$\text{the radius of each face} = -0,2444\alpha\,.$$

Now in the two first lenses the minimal radius occurs $= 0,5909\alpha$, so that there shall be $\mathfrak{m} = 0,5909$ and thus

$$\alpha = -\frac{72}{59,09} \text{ in. or } \alpha = -1\tfrac{1}{4} \text{ in.}$$

From which the following construction of this telescope for a magnification $m = 9$ will be produced, with the lenses made from common glass.

I. For the first lens

$$\text{radius of the}\begin{cases}\text{anterior face} &= -9,93 \text{ in.}\\ \text{posterior face} &= -0,73 \text{ in.}\end{cases}$$

Of which the focal length $= 1\frac{1}{4}$ in..

Radius of the aperture $= 0,18$ in.

Distance to the second lens $= 0,025$ dig.

II. For the second lens

$$\text{radius of the}\begin{cases}\text{anterior face} = 1,27 \text{ dig.}\\ \\ \text{posterior face} = 0,74 \text{ dig.}\end{cases}$$

Of which the focal length $= 0,85$ in.
Radius of the aperture as before $= 0,18$ in.
Distance to the third lens $= 3,38$ in.

III. For the third lens

$$\text{radius of the}\begin{cases}\text{anterior face} = 4,37 \text{ in.}\\ \\ \text{posterior face} = 0,51 \text{ in.}\end{cases}$$

Of which the focal length $= 0,83$ in.
Radius of the aperture $= 0,13$ in.
Distance to the fourth lens $= 2,78$ in.

IV. For the fourth lens

Radius of each face $= 0,92$ in.
Of which the focal length $= 0,83$ in.
Radius of the aperture $= 0,23$ in.
Distance to the fifth lens $= 1,11$ in.

V. For the fifth lens

Radius of each face $= 0,30$ in.
Of which the focal length $= 0,28$ in.
Radius of the aperture $= 0,07$ in.,
and the distance to the eye $= 0,19$ in.
and thus the total length of the instrument $= 7,49$ in.
and the radius of the field of view $= 2°23'$.

Therefore with this perfection of the telescope used, which before was 6 ft., has been reduced to $7\frac{1}{2}$ in.

EXAMPLE 2

339. If the magnification shall be $m = 50,$ there will be $Pk = 20$ and $k = \frac{102}{5}$; from which our terms will be :

$$b = -\tfrac{51}{50}\alpha, \quad \beta = -\tfrac{51}{25}\alpha, \quad c = -\tfrac{\alpha}{10}, \ \gamma = \infty,$$

$$d = -\infty, \quad \delta = -\tfrac{\theta\alpha}{10}, \quad e = -\tfrac{\theta\alpha}{25}$$

and the focal lengths:

$$p = \alpha, \quad q = -\tfrac{17}{25}\alpha, \quad r = -\tfrac{\alpha}{10}, \quad s = -\tfrac{\theta\alpha}{10}, \ \text{ and } \ t = -\tfrac{\theta\alpha}{25}$$

and the separations of the lenses:

$$\alpha + b = -\tfrac{1}{50}\alpha, \quad \beta + c = -\tfrac{107}{50}\alpha, \ \gamma + d = \tfrac{-(21+35\theta)\alpha}{200}, \quad \delta + e = \tfrac{-7\theta\alpha}{50}$$

and the distance of the eye

$$O = -\tfrac{7\theta\alpha}{250},$$

and the radius of the apparent field of view will be $= 24\frac{1}{2}$ min.
Now truly there will be produced

$$\begin{array}{l} \lambda = 3,4425 + \frac{0,0143}{\theta^3} \\ \quad + 0,0063 \\ \quad\ \ 0,1779 \\ \hline \lambda = 3,6267 + \frac{0,0143}{\theta^3} \end{array}$$

Now there may be taken $\theta = 2$ and there will become

$$\lambda = 3,6285, \quad \lambda - 1 = 2,6285 \ \text{ and } \ \tau\sqrt{(\lambda - 1)} = 1,4674,$$

from which there becomes :

I. For the first lens

$$\text{radius of the} \begin{cases} \text{anterior face } = \frac{\alpha}{0,1600} = 6,2500\alpha \\ \text{posterior face} = \frac{\alpha}{1,0581} = 0,6031\alpha. \end{cases}$$

II. For the second lens

$$\text{As before the radius of the} \begin{cases} \text{anterior face } = \frac{2b}{2,0088} = -1,0155\alpha \\ \text{posterior face } = \frac{2b}{3,4455} = -0,5921\alpha. \end{cases}$$

III. For the third lens

$$\text{radius of the}\begin{cases}\text{anterior face} = \frac{c}{0,1907} = -0,5244\alpha \\ \text{posterior face} = \frac{c}{1,6274} = -0,0615\alpha.\end{cases}$$

IV. For the fourth lens

Radius of each face $= -0,2200\alpha$.

V. For the fifth lens

Radius of each face $= -0,0880\alpha$.

Now since in the first two lenses the minimum radius shall be $0,5921\alpha$, there will become $\mathfrak{m} = 0,5921$ and thus $\alpha = -\frac{400}{59,91}$ in., thus so that it may be taken $= -7$ in.

From which the following construction of this telescope for the magnification $m = 50$ will be produced.

I. For the first lens

$$\text{radius of the}\begin{cases}\text{anterior face} = -43,75 \text{ in.} \\ \text{posterior face} = -4,22 \text{ in.}\end{cases}$$

Of which the focal length $= -7$ in.
The radius of the aperture $= 1,05$ in.
The distance to the second lens $= 0,14$ in.

II. For the second lens

$$\text{radius of the}\begin{cases}\text{anterior face} = 7,11 \text{ dig.} \\ \text{posterior face} = 4,14 \text{ dig.}\end{cases}$$

Of which the focal length is $= 4,76$ in.

The radius of the aperture as before $=1,05$ in.
The distance to the third lens $=14,98$ in.

III. For the third lens

$$\text{radius of the} \begin{cases} \text{anterior face} = 3,67 \text{ dig.} \\ \text{posterior face} = 0,43 \text{ dig.} \end{cases}$$

Distance of the focus is $=0,7$ in.
Radius of the aperture $=0,11$ in.
Interval to the fourth lens $=3,18$ in.

IV. For the fourth lens

Radius of each face $=1,54$ in.
Of which the focal length is $=1,40$ in.
Radius of the aperture $=0,38$ in.
Interval to the fifth lens $=1,96$ in.

V. For the fifth lens

Radius of each face $=0,61$ dig.
Of which the focal length is $=0,56$ in.
Radius of the aperture $=0,15$ in.
Distance to the eye $=0,39$ in.
And thus the total length $=20\frac{2}{3}$ in. approximately,
and the radius of the field of view $=24\frac{1}{2}$ min.

SCHOLION 2

340. Therefore this latter telescope thus can be prepared easily by tubes sliding into each other, so that it may be able to be carried around easily by anyone, since without that concave lens this telescope may have increased beyond twenty feet. But this has to be observed concerning the sliding tube, while it may be drawn to accommodate the eye, the eyepiece lens alone must be able to be moved, truly all the other lenses always must stand in assigned places , which is required to be maintained always for all the telescopes which are treated here ; besides there is no need, that it may be perfected, as the various

kinds of glass are provided, that we may attribute to the special principle, as we have done so far, as the solution of the preceding problems with a few changes will be able to accommodate this aim, as we will show in the following problem.

PROBLEM 3

341. *If the first concave objective lens may be prepared from crystal glass, while the rest are prepared from crown glass, to describe the construction of the telescope, in which not only the colored margin, but also the whole confusion arising from the different refrangibility of the rays may be removed completely.*

SOLUTION

If we wish to resolve this problem, as we have done at this stage, from the principles established above, clearly everything may be able to be going to be used in the same manner in the preceding problem up to that point, where we have removed the colored margin, and also this equation itself may not differ from that, which we have treated in the previous problem, since in that the first lens does not enter into the calculation, thus so that hence also these same determinations may prevail and thus far hence also the letters $\mathfrak{B}$ and B may obtain the same determinations and now also are going to remain indeterminate;
[See § 53: recall that $0=\frac{dn}{n-1}\cdot\alpha+\frac{dn'}{n'-1}\cdot\frac{b}{\mathfrak{B}}+\frac{dn''}{n''-1}\cdot\frac{c}{\mathfrak{C}B^2}+\frac{dn'''}{n'''-1}\cdot\frac{d}{\mathfrak{D}B^2C^2}+\frac{dn''''}{n''''-1}\cdot\frac{e}{\mathfrak{E}B^2C^2D^2}+\text{etc.}$;
where the dispersions of the red and blue colors may be cancelled by using lenses with slightly differing refractive indices and positive and negative lenses, leading to the achromatic doublet for two lenses close together, which has a long focal length useful for the telescope objective.]

but now at last an account will be required to be had of the final equation, by which the confusion is removed completely from the central axis, and the equation pertaining to that will become, if for the first lens we may denote the differential formula $\frac{dn}{n-1}$ by the letter N, but for the following lenses we may denote by the letters N', and we may divide this equation,

$$0=\frac{N}{N'}-\frac{1}{\mathfrak{B}P}-\frac{1}{B\mathfrak{C}^2Pk}-\frac{1}{BP\theta k}-\frac{1}{B\theta m},$$

in which equation the third term with the following are very small in comparison with the first two terms, so that they may be able to be ignored without error, especially since, as we have often noted, the nature of the investigation may not allow, that this equation may be resolved carefully nor that also that our aim demands. Whereby with the first two terms taken only, we may deduce

$$\mathfrak{B}=\frac{N'}{NP},$$

evidently on account of this condition, with the first lens required to be prepared from crystal glass the whole distinction in the resolution consists in this only, so that now it may be defined, since the letter $\mathfrak{B}$ may be left to our choice; on account of which, since from Dolland's experiments we have $N:N'=10:7$ and besides there shall be $P=\frac{50}{51}$, now we follow on with $\mathfrak{B}=\frac{357}{500}$, which value is reduced approximately to this, $\mathfrak{B}=\frac{5}{7}$, or also even to $\mathfrak{B}=\frac{2}{3}$, which is the value we have attributed to $\mathfrak{B}$ itself in the preceding examples; but any value may be attributed to $\mathfrak{B}$ in the final equation, from which the number λ is defined, a certain small distinction may be introduced; since indeed now the first term is required to be multiplied by μ, truly the following by μ', with division made by μ' this equation becomes :

$$\frac{\mu}{\mu'}\lambda=\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3Pk}+\frac{\lambda'''}{B^3\theta^3Pk}+\frac{\lambda''''}{B^3\theta^3P}+\frac{\nu'}{\mathfrak{B}BP},$$

where as before there can be taken $\lambda'=1$ and $\lambda''=1$; but since the latter lenses are made from crown glass, so that $n=1,53$, for the two last lenses, which must be equally convex, there will be $\lambda'''=\lambda''''=1,60006$, but there the related letters will be

$$\mu'=0,9875,\quad \nu'=0,2196,\quad \rho'=0,2267 \text{ and } \sigma'=1,6601,\ \tau'=0,9252.$$

For the first lens there will be

$$\mu=0,8724,\quad \nu=0,2529,\quad \rho=0,1414,\quad \sigma=1,5827 \text{ and } \tau=0,8775.$$

COROLLARY 1

342. Now at last it is understood, why the first lens established to be prepared from crystal glass rather than the second ; for if the first is crystal, there becomes $\mathfrak{B}=\frac{5}{7}$ and $B=\frac{5}{2}$. But if we shall make the second lens from crystal, there will become $\mathfrak{B}=\frac{7}{5}$ and $B=-\frac{7}{2}$. Whereby, since all the following distances shall be multiplied by *B,* and these therefore shall produce the total length of the tube in the latter case than in the former case and that in the ratio $7:5$.

COROLLARY 2

343. If the difference of the dispersion of both kinds of glass were less, than we have assumed here following Dollond's experiments, then the fraction required to be assumed for $\mathfrak{B}$ may approach closer to one and thence *B* may approach a greater value and thus the instrument may emerge longer ; from which in practice generally it will come about, so that the two kinds of glass may be chosen with the greatest ratio of dispersion between them, if indeed in this manner the telescopes may be rendered much shorter.

SCHOLIUM 1

344. Therefore since here the first lens to be made from crystal glass, the rest we have assumed to be made from crown glass, with regard to Dollond's experiments we may put in place $\mathfrak{B}=\frac{5}{7}$ so that there shall be $B=\frac{5}{2}$, and putting $\theta=2$, lest the eyepiece lens may become exceedingly small, our terms will be had in the following manner:

$$b=-\tfrac{51}{50}\alpha,\quad c=-\tfrac{5\alpha}{2Pk},\ d=-\infty,\ e=-\tfrac{5\alpha}{m},$$

$$\beta=-\tfrac{51}{20}\alpha,\quad \gamma=\infty,\ \delta=-\tfrac{5\alpha}{Pk}$$

and the focal lengths

$$p=\alpha,\ q=-\tfrac{51}{70}\alpha,\quad r=-\tfrac{5\alpha}{2Pk},\ s=-\tfrac{5\alpha}{Pk},\ t=-\tfrac{5\alpha}{m}$$

and hence the intervals

$$\alpha+b=-\tfrac{1}{50}\alpha,\quad \beta+c=-\tfrac{51}{20}\alpha-\tfrac{5\alpha}{2Pk},$$

$$\gamma+d=\eta\alpha=-\frac{5(1+Pk)\alpha}{2P^2k^2}-\frac{5\sqrt{2m(m-1)}\alpha}{2P^2k^2},\ e=-\tfrac{5\alpha}{m},$$

$$\delta+e=-\frac{5\sqrt{2m(m-1)}}{mPk}\alpha$$

and the distance of the eye

$$O=-\frac{5\sqrt{2m(m-1)}}{2m^2}\alpha$$

with there being

$$Pk=-m+\sqrt{2m(m-1)};$$

then moreover the radius of the field

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\text{ min.}$$

Hence therefore so that we may investigate the construction for any magnification *m*, the method now more often being used for the first case we have set out, where $m=25$, then truly the case, where $m=\infty$

EXAMPLE 1

345. The magnification shall be $m = 25$ and there will be found

$$\sqrt{2m(m-1)} = 34,64101 \text{ and hence } Pk = 9,64101,$$

from which the intervals themselves will be had :

$$\alpha + b = -0,02\alpha, \quad \beta + c = -\ 2,80930\alpha, \quad \gamma + d = -1,21770\alpha,$$

$$\delta + e = -0,71860\alpha$$

and the distance of the eye $= -0,13844\alpha$.

From these premises λ may be sought from the equation given above [§ 341], and there will be found :

$$\lambda = 3,16815 + 0,007514 + 0,001502 + 0,000579 + 0,14198,$$

or

$$\lambda = 3,31972,$$

from which there becomes:

$$\tau\sqrt{(\lambda - 1)} = 1,33648.$$

Hence therefore, if *F* and *G* may denote the anterior and posterior radius of the face, we will have

I. For the first crystal lens

$$F = \frac{\alpha}{\sigma - 1,3365} = \frac{\alpha}{0,2462} = 4,0617\alpha, \quad G = \frac{\alpha}{\rho + 1,3365} = \frac{\alpha}{1,4779} = 0,6766\alpha.$$

II. For the second but crown glass lens there will be

$$F = \frac{5b}{5\rho' + 2\sigma'} = \frac{5b}{4,4537} = -1,14537\alpha, \quad G = \frac{5b}{5\sigma' + 2\rho'} = \frac{5b}{8,7539} = -0,5826\alpha,$$

which construction prevails for any magnification.

III. For the third crown glass lens we will have

$$F = \frac{c}{\rho'} = \frac{c}{0,2267} = -\frac{11,0278\alpha}{Pk} = -1,1438\alpha,$$

$$G = \frac{c}{\sigma'} = \frac{c}{1,6601} = -\frac{1,5059\alpha}{Pk} = -0,1562\alpha,$$

were the penultimate values for any magnification prevail.

IV. Likewise for the fourth crown glass lens, the focal length of which $=s=-\frac{5\alpha}{Pk}$, will be

$$F=G=1,06s=-\frac{5,30\alpha}{Pk}=-0,5497\alpha,$$

where the penultimate value for any magnification prevails.

V. For the fifth lens also crown glass, the focal length of which is $t=-\frac{5\alpha}{m}$, there will be

$$F=G=1,06t=-\frac{5,3\alpha}{m}=-0,212\alpha,$$

where again the penultimate form prevails for any magnification.

EXAMPLE 2

346. If the magnification m becomes infinite or extremely large, there will become $\sqrt{2m(m-1)}=m\sqrt{2}=1,41421m$ and hence Pk= 0,41421 m, from which the intervals will become

$$\alpha+b=-\ 0,02\alpha,\quad \beta+c=-2,55\alpha-6,0356\cdot\frac{\alpha}{m},$$
$$\gamma+d=-\ 26,6425\cdot\frac{\alpha}{m},\quad \delta+e=-17,0712\cdot\frac{\alpha}{m},$$

and the distance of the eye

$$O=-3,5355\cdot\frac{\alpha}{m}.$$

From these premises λ is sought from the given equation, and there will be had

$$\lambda=3,16815+0,14198=3,31013,$$

from which there becomes

$$\tau\sqrt{(\lambda-1)}=1,3337;$$

whereby there will be had :

I. For the first lens

$$F=\frac{\alpha}{\sigma-1,3337}=\frac{\alpha}{0,2490}=4,0160\alpha,\quad G=\frac{\alpha}{\rho+1,3337}=\frac{\alpha}{1,4751}=0,6779\alpha.$$

II. The second lens agrees with the preceding example.

III. For the third lens there will be

$$F=\frac{-11{,}0278\alpha}{Pk}=-26{,}6237\cdot\frac{\alpha}{m},\quad G=\frac{-1{,}5059\alpha}{Pk}=-3{,}6357\cdot\frac{\alpha}{m}.$$

IV. For the fourth lens there will be

$$F=G=\frac{-5{,}3\alpha}{Pk}=-12{,}7955\frac{\alpha}{m}.$$

V. Finally for the fifth lens

$$F=G=-5{,}3\cdot\frac{\alpha}{m}.$$

Moreover, the terms themselves will be had in the following manner:

$$b=-1{,}02\alpha,\quad c=-6{,}0355\cdot\frac{\alpha}{m},\quad d=-\infty,\quad e=-5\cdot\frac{\alpha}{m},$$

$$\beta=-2{,}55\alpha,\quad \gamma=\infty,\quad \delta=-12{,}0710\cdot\frac{\alpha}{m}$$

and hence the focal lengths

$$p=\alpha,\ \ q=-0{,}72857\alpha,\quad \gamma=-\ 6{,}0355\cdot\frac{\alpha}{m},$$

$$s=-12{,}0710\cdot\frac{\alpha}{m},\quad t=-5\cdot\frac{\alpha}{m}.$$

EXAMPLE 3

347. From the collation of the preceding examples to describe the construction of telescopes of this kind for any greater magnification *m*.

Initially the elements will be required to be expressed in the following manner:

$$b=-1{,}02\alpha,\quad \beta=-2{,}55\alpha,\quad c=-\left(6{,}0355+\frac{11{,}1750}{m}\right)\frac{\alpha}{m},\ \ \gamma=\infty,$$

$$d=-\infty,\quad \delta=-\left(12{,}0710+\frac{22{,}3500}{m}\right)\frac{\alpha}{m},\quad e=-5\cdot\frac{\alpha}{m}$$

and hence the focal lengths

$$p=\alpha,\quad q=-0{,}72857\alpha,\quad r=-\left(\ 6{,}0355+\frac{11{,}1750}{m}\right)\frac{\alpha}{m},$$
$$s=\left(12{,}0710+\frac{22{,}3500}{m}\right)\frac{\alpha}{m},\quad t=-5\cdot\frac{\alpha}{m}$$

and the separation of the lenses

$$\alpha+b=-\ 0{,}02\alpha,\quad \beta+c=-2{,}55\alpha-\left(6{,}0355+\tfrac{11{,}1750}{m}\right)\tfrac{\alpha}{m}$$

$$\gamma+d=-\left(26{,}6425+\tfrac{m}{95}\ \right)\tfrac{\alpha}{m},\quad \delta+e=-\left(17{,}0710+\tfrac{22{,}3500}{m}\right)\tfrac{\alpha}{m}$$

and the distance of the eye

$$O=-\left(3{,}5355-\tfrac{1{,}8625}{m}\right)\tfrac{\alpha}{m}.$$

and finally the radius of the field always is

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\ \text{min.}$$

Truly the construction of the lens thus itself will be had :

I. For the first crystal lens

$$\text{Radius of the}\begin{cases}\text{anterior face } =\left(4{,}0160+\tfrac{1{,}14}{m}\right)\alpha\\ \text{posterior face}=\left(\ 0{,}6779-\tfrac{0{,}0325}{m}\right)\alpha.\end{cases}$$

II. For the second crown glass lens

$$\text{Radius of the }\begin{cases}\text{anterior face } =-1{,}1451\alpha\\ \text{posterior face}=-\ 0{,}5826\alpha.\end{cases}$$

III. For the third crown glass lens

$$\text{Radius of the }\begin{cases}\text{anterior face } =-\left(26{,}6237+\tfrac{49{,}28}{m}\right)\tfrac{\alpha}{m}\\ \text{posterior face } =-\left(\ 3{,}6357-\tfrac{6{,}73}{m}\right)\tfrac{\alpha}{m}.\end{cases}$$

IV. For the fourth crown glass lens

Radius of each face $=-\left(12{,}7953+\tfrac{23{,}68}{m}\right)\tfrac{\alpha}{m}.$

V. For the fifth crown glass lens

$$\text{Radius of each face} = -5{,}30\cdot\tfrac{\alpha}{m}.$$

Now finally it remains to be required to judge, how great a value may be agreed to be attributed to α . In the end this minimum radius of the two first lenses may be considered, which is $-0{,}5826\alpha$, of which the fourth part $-0{,}1456\alpha$ may be put equal to the aperture $\frac{m}{50}$, and thence there will be found $\alpha = -\frac{m}{7{,}28}$, from which value indeed the quantity α must not accept a smaller value ; whereby there may be taken $\alpha = -\frac{m}{7}$ and the following construction of telescopes of this kind will be obtained for any magnification *m.*

Therefore with the focal length being put $\alpha = -\frac{m}{7}$ in. we will obtain the following measures for the construction sought :

I. For the first crystal lens

$$\text{Radius of the}\begin{cases}\text{anterior face} = (-0{,}5737m - 0{,}16)\,\text{in.}\\ \text{posterior face} = (-0{,}0968m + 0{,}004)\,\text{in.}\end{cases}$$

Of which the focal length $= -\frac{m}{7}$ in.

Radius of the aperture $= \frac{m}{50}$ in.

Distance to the second lens $= 0{,}00286m$ in.

II. For the second crown glass lens

$$\text{Radius of the}\begin{cases}\text{anterior face} = 0{,}1636m\ \text{in.}\\ \text{posterior face} = 0{,}0832m\ \text{in.}\end{cases}$$

Of which the focal length is $= 0{,}10408m$ in.

Radius of the aperture $= \frac{m}{50}$ in.

Distance to the third lens $= \left(0{,}3643m + 0{,}86 + \frac{1{,}6}{m}\right)$ in.

III. For the third crown glass lens

$$\text{Radius of the}\begin{cases}\text{anterior face} = \left(3,80+\frac{7,04}{m}\right)\\ \text{posterior face} = \left(0,52+\frac{0,9}{m}\right)\end{cases}$$

Of which the focal length is $=\left(0,86+\frac{1,6}{m}\right)$ in.

Radius of the aperture $=0,13$ in.

Distance to the fourth $=\left(3,80+\frac{14}{m}\right)$ in.

IV. For the fourth crown glass lens

Radius of each face $=(1,82+\frac{3,4}{m})$ in.

Of which the focal length is $=\left(1,72+\frac{3,2}{m}\right)$ in.

Radius of the aperture $=0,45$ in.

Distance to the fifth lens $=(2,44+\frac{3,2}{m})$ in.

V. For the fifth crown glass lens

Radius of each face $=0,76$ in.

Of which the focal length $=0,71$ in.

Radius of the aperture $=0,19$ in.

Distance to the eye $=\left(0,50-\frac{0,2}{m}\right)$ in.

Therefore these gathered together thence is the length of the whole telescope:

$$\left(0,3672m+7,60+\frac{18,6}{m}\right) \text{ in.,}$$

from which it is clear, if $m=100$, the length of the instrument is not going to exceed $44\frac{1}{2}$ in.

Finally the radius of the apparent field will be

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}} \text{ minutes,}$$

which therefore for $m=100$ will become 12 minutes.

SCHOLIUM 2

348. Therefore these telescopes at this stage will be short enough, only if in practice it will be permitted to elaborate lenses that follow the most exact measurements prescribed, and if also each kind of glass may be permitted to have precisely the same refractive index, as we have supposed here; but always it is required to maintain, if the refraction of the glass may differ from that, clearly which we have assumed with regard to the formation of the lenses, then the whole calculation is required to be established anew ; then truly this rule is required to be observed properly, so that, where we may expect a less happy outcome from the artificer, the measures here prescribed to be increased and thus must be doubled or trebled; that which may happen most conveniently, if we may accept a measure of many digits. But always, even if the artificer may have used the greatest diligence, scarcely at any time will it be required to hope, that the first telescope produced shall correspond to the instrument intended; but rather instead it will be necessary, so that with the lenses the first of the concave lenses especially may be worked through several stages, so that from these by trial and error the best may be selected ; [Thus, Euler persisted with separate lenses rather than the achromatic doublet, although it was well established at the time, and had even been patented by Dolland.]
for even if the same measures may be retained, yet always it shall arise in use, as with more examples all will disagree a little among themselves. Also, since quite often an agreement will be reached with the same measure for the construction of this lens changed a little, yet still, so that it may maintain the same focal length, and for whatever the measure some examples may be made ; evidently if from the theory the radii of the anterior and posterior faces of this lens were found to be F and G, thus this figure often may be agreed to be changed, so that the radius of the anterior face $=F\mp F^2\omega$, truly the posterior $=G\pm G^2\omega$, for some small assumed fraction ω, which at this stage may be perceived in practice ; since then nothing will be changed in the focal length. Finally also certain advise is required to be given concerning the use of diaphragms in telescopes of this kind; since indeed in these two real images are found, also in each place a diaphragm will be able to be put in place, of which the aperture must capture that same image. But the radius of the first image is

$$=\alpha\Phi B=B\alpha M\xi=\tfrac{1}{4}MB\alpha;$$

truly in our case $M=\frac{2}{\sqrt{2m(m-1)}}$ and $B=\frac{5}{2}$, and thus that same radius will be $=\frac{5\alpha}{4\sqrt{2m(m-1)}}$ on taking $\alpha=\frac{m}{7}$ so that the prior radius will be

$$=\frac{5m}{28\sqrt{2m(m-1)}}=\frac{5}{28\sqrt{2}}=\frac{1}{8}\text{ in.},$$

unless m shall be a small number. But the radius of the second image will be

$$=\alpha\Phi BCD=\alpha\Phi B\theta;$$

whereby, since we will have assumed $\theta = 2$, the latter diaphragm must have an aperture, of which the radius shall be twice as great as the preceding, clearly $\frac{1}{4}$ in., from which truly no use can be expected, since the latter lenses themselves shall require a much smaller aperture, thus so that only the first diaphragm may be able to have a use, to which also, if it were desired, a micrometer will be able to be attached.

CAPUT II

DE TELESCOPIIS TERRESTRIBUS COMMUNIBUS

EORUMQUE PERFECTIONE

DEFINITIO

325. *Character huiusmodi telescopiorum in hoc consistit, quod radii per duas priores lentes transmissi iterum inter se fiant paralleli, ita ut haec telescopia ex duobus tubis astronomicis sint composita.*

COROLLARIUM 1

326. Cum haec telescopia ex quatuor lentibus constant, quarum tam binae priores quam binae posteriores secundum rationem tuborum astronomicorum sibi sunt iunctae, multiplicatio telescopii est in ratione composita ambarum multiplicationum, quas ambo isti tubi astronomici producerent.

COROLLARIUM 2

327. Scilicet si lentis primae ponatur distantia focalis $= p$, secundae $= q$, tertiae $= r$ et quartae $= s$, binae priores lentes ad intervallum $= p + q$ dispositae multiplicationem praebent $= \frac{p}{q}$, binae posteriores vero ad intervallum $= r + s$ dispositae multiplicationem $= \frac{r}{s}$; telescopium compositum multiplicationem producet

$$= \frac{pr}{qs}.$$

SCHOLION 1

328. Statim ab initio binae lentes posteriores inter se factae sunt aequales et quidam eiusdem distantiae focalis ac lens secunda, quae tres lentes oculares vocari solent, ita ut

tum tubus posterior nullam plane multiplicationem producat ob $r = s = q$. Quanto autem intervallo hi duo tubi sive lentes secunda et tertia a se invicem debeant esse remotae, auctores non satis definiunt; plerumque autem hoc spatium fieri iubent $= 2q$, ita ut, cum etiam sit $r = s = q$, tota longitudo futura sit $= p + 5q$. Deinde autem artifices observarunt haec telescopia meliorem effectum producere, si tres lentes posteriores continuo certa ratione diminuantur, id quod egregie convenit cum iis, quae supra de hoc telescopiorum genere annotavimus, ubi non solum multo maiorem campum iis conciliavimus, quam vulgaris constructio suppeditat, sed etiam id inprimis effecimus, ut margo coloratus penitus evanesceret. Quocirca praecepta pro constructione ante inventa hic ordine proponi conveniet.

Constructio telescopiorum terrestrium
ex quatuor lentibus compositorum pro quavis multiplicatione m

Quanta statui debeat lentis obiectivae distantia focalis, deinceps definiemus, quando pro singulis lentibus sequentibus numeros λ, λ', λ'', λ''' assignaverimus.

1. Si igitur $p = \alpha$ denotet distantiam focalem lentis obiectivae, eius figuram utique ex numero $\lambda = 1$ peti conveniet, ita ut, si ratio refractionis sit $n = 1,55$, habeatur:

$$\text{Radius faciei} \begin{cases} \text{anterioris} = \frac{p}{\sigma} = 0,6145p \\ \text{posterioris} = \frac{p}{\rho} = 5,2438p; \end{cases}$$

pro eius apertura semidiameter hactenus posita est $\frac{m}{50}$ dig. Sin autem vel maior claritas desideretur vel minor sufficiat, loco 50 vel numerus maior vel minor assumi poterit.

Intervallum lentis secundae a prima debet esse $= p + q$, ubi valor ipsius q mox indicabitur.

II. Pro lente secunda, si eius distantia focalis ponatur $= q$, in superiore capite vidimus sumi convenire $q = \frac{p}{k}$ existante $k = -m + \sqrt{2m(m-1)}$; et quia pro eius apertura debet esse $\omega = \frac{-(1+k)(m+k)}{m(m-1)}$, qui valor pro maioribus multiplicationibus erit circiter $\omega = -\frac{3}{5}$, unde haec apertura non fit maxima, etiam non opus est, ut haec lens fiat utrinque aeque convexa, sed sufficiet, ut pro ea sumatur $\lambda' = 1$, unde huius lentis constructio erit:

$$\text{Radius faciei} \begin{cases} \text{anterioris} = \frac{q}{\rho} = 5,2438q \\ \text{posterioris} = \frac{q}{\sigma} = 0,6145q; \end{cases}$$

Et aperturae semidiameter si capiatur $= \frac{1}{4} \cdot \frac{q}{\sigma}$ conditioni praescriptae satisfaciet.

Distantia autem tertiae lentis a secunda, quae supra est posita $= \eta\alpha$, definita est

$$\eta = \frac{k+1}{k^2} + \frac{\theta\sqrt{m(m-1)}}{k^2\sqrt{2}},$$

ubi numerus θ arbitrio nostro relinquitur, quem autem neque multo maiorem neque minorem unitate sumi conveniet.

III. Pro tertia lente, quoniam ea maximam aperturam recipere debet ob $i = 1$ ideoque utrinque aeque convexa confici debet, erit $\lambda' = 1,6299$, et cum eius distantia focalis sit $r = \frac{\theta p}{k}$, erit radius utriusque faciei $= 1,10r$, cuiuspars quarta dabit semidiametrum aperturae.

Ab hac lente distantia ad quartam est

$$= r + s = \theta\alpha(\tfrac{1}{k} + \tfrac{1}{m}).$$

IV. Quia quarta lens etiam maximam aperturam admittere ideoque etiam utrinque aequaliter convexa esse debet, pro ea etiam erit $\lambda''' = 1,6299$; unde, cum eius distantia focalis sit $s = \frac{\theta\alpha}{m}$, erit radius utriusque faciei $= 1,10s$ et $\frac{1}{4}s$ dabit semidiametrum eius aperturae; tum vero distantia ab hac lente ad oculum erit

$$= \frac{s}{Mm} = \frac{s(m-1)}{\sqrt{2m(m-1)}} = \frac{s\sqrt{(m-1)}}{\sqrt{2m}}.$$

V. Hocque telescopium campum ostendet, cuius semidiameter est

$$\Phi == \frac{\sqrt{2}}{\sqrt{m(m-1)}}.\xi$$

seu in mensura

$$\Phi = \frac{1215}{\sqrt{m(m-1)}} \text{ min}.$$

VI. Tota autem huius instrumenti longitudo ad oculum usque erit

$$= \left(\frac{(k+1)^2}{k^2} + \frac{\theta(m-1)\sqrt{2(m-1)}}{k^2\sqrt{m}} \right) p.$$

VII. Pro distantia autem focali p, si desideretur claritas $y = \frac{1}{50}$ dig. et pro gradu distinctionis $k = 50$, ut sit $kx = m$, ob litteram μ parum ab unitate deficientem debebit sumi in digitis

$$p = m\sqrt[3]{m\left((1+\tfrac{1}{k}+\tfrac{1,6299}{\theta^3}(\tfrac{1}{k}+\tfrac{1}{m})\right)};$$

ac si tam minore claritate, puta $y=\frac{1}{70}$, et minore gradu distinctionis, puta $k=35$, acquiescere velimus, iste valor ipsius p ad semissem redigi poterit.

EXEMPLUM

329. Si huiusmodi telescopium tantum novies multiplicare debeat, ut sit $m=9$, reperietur $k=3$ hincque

$$q=\tfrac{p}{3},\ r=\tfrac{\theta p}{3} \text{ et } s=\tfrac{\theta p}{9},$$

unde erit totius telescopii longitudo $=\left(\frac{16}{9}+\frac{32}{27}\theta\right)p$ et semidiameter campi $=2°23'$.

Tum vero distantia focalis p ita assumi debebit

$$p = 9\sqrt[3]{9\left(1+\tfrac{1}{3}+\tfrac{4}{9}\cdot\tfrac{1,6299}{\theta^3}\right)};$$

sumto ergo $\theta=1$, ut longitudo fiat $=\frac{80}{27}p$ seu propemodum $=3p$, colligetur $p=9\sqrt[3]{18,5196}$ seu propemodum $p=24$ dig., unde longitudo tota $=72$ dig. $=6$ ped. ; quae longitudo, uti animadvertimus, ad semissem reduci posset.

SCHOLION 2

330. Verum etiam longitudo trium pedum pro tam exigua multiplicatione enormis videbitur, praecipue cum vulgo eiusmodi telescopia circumferantur multo breviora magisque amplificantia. At praecipua causa huius longitudinis in campo apparente est sita, quem maximum producere sumus conati; qui sine dubio multo maior est, quam in vulgaribus eiusmodi instrumentis deprehenditur. Interim tamen destructio marginis colorati non parum ad longitudinem confert perinde ac insignis claritatis et distinctionis gradus, qui nobis erat propositus, ex quo instrumenta secundum haec praecepta parata plurimum antecellent iis, quae vulgo circumferuntur et quae plerumque tot tantisque vitiis laborant, ut in praxi vix tolerari queant. Non mediocriter autem eorum longitudo diminui posset, si loco lentis obiectivae sive lens duplicata sive etiam triplicata, quales supra ex principio minimi sunt inventae, substituantur, siquidem tum valor ipsius λ priori casu ad $\frac{1}{5}$, posteriore vero ad $\frac{1}{24}$ reduceretur; ita, si in nostro exemplo λ fuisset $=\frac{1}{5}$, invenissemus $p=20$ dig. et telescopii longitudo adhuc ad 5 pedes excrevisset. Sin autem lente obiectiva triplicata usi essemus, ut fuisset $\lambda=\frac{1}{24}$, prodiisset $p=19\frac{1}{3}$ dig.; unde patet a lentibus illis duplicatis et triplicatis, quales supra sunt descriptae,

atque adeo a lentibus perfectis, ubi foret $\lambda = 0$, haud notabile decrementum longitudinis exspectari posse, saltem pro minoribus multiplicationibus, ubi post signum radicale cubicum termini λ sequentes admodum sunt notabiles; pro maioribus autem multiplicationibus maius lucrum esset futurum, quod vix tamen ad semissem redire posset. Quare pro hac specie telescopiorum praecipue in id est incumbendum, ut lens obiectiva ita duplicetur vel triplicetur, ut non solum confusio ab ipsa oriunda, sed et ea, quae a sequentibus lentibus omnibus nascitur, ad nihilum redigatur; tum enim distantiam *p* maiorem statui non erit necesse, quam apertura ob claritatem requisita postulet; quem casum in sequente problemate ita evolvamus, ut exiguum spatium intra lentes priores admittamus.

PROBLEMA 1

331. *In hac telescopiorum specie loco lentis obiectivae eiusmodi binas lentes ex eodem vitro parandas substituere, ut omnis confusio etiam a reliquis lentibus oriunda ad nihilum redigatur sicque his telescopiis minima longitudo concilietur.*

SOLUTIO

Cum igitur hic habeantur quinque lentes, statuamus nostras fractiones

$$\frac{\alpha}{b} = -P, \quad \frac{\beta}{c} = -Q, \quad \frac{\gamma}{d} = -R \text{ et } \frac{\delta}{\varepsilon} = -S.$$

Quarum litterarum prima *P* proxime erit $=1$; secunda *Q* erit negativa $=-k$; tertia *R* *etiam* erit $=1$, sed ita tamen, ut intervallum tertium $\gamma + d$ fiat quantitas finita, scilicet $\eta\alpha$; denique vero erit $S = -k'$, ita ut nostra elementa futura sint

$$b = -\frac{\alpha}{P}, \quad c = -\frac{B\alpha}{Pk}, \; d = \frac{BC\alpha}{PkR} = \infty, \quad e = \frac{BCD\alpha}{PkRk'} = \frac{BCD\alpha}{m},$$

$$\beta = -\frac{B\alpha}{P}, \quad \gamma = -\frac{BC\alpha}{Pk} = \infty, \quad \delta = \frac{BCD\alpha}{PkR}.$$

Hincque intervalla

$$1. \; \alpha + b = \alpha\left(1 - \frac{1}{P}\right) = \frac{\alpha}{50},$$

uti supra [§ 292] iam assumsimus, ita ut sit $P = 1\frac{1}{49}$,

$$2. \; \beta + c = -\frac{B\alpha}{P}\left(1 + \frac{1}{P}\right),$$

$$3. \; \gamma + d = -\frac{BC\alpha}{Pk}\left(1 - \frac{1}{R}\right) = \eta\alpha,$$

ubi scilicet est $C = \infty$ hincque $\frac{1}{R} = 1 + \frac{\eta Pk}{BC}$.

4. Quia erat $C = \infty$ debet esse *D* infinite parvum, ita ut sit $CD = -\theta$, eritque hoc intervallum

$$\delta + e = -\tfrac{B\theta\alpha}{PkR}\left(1+\tfrac{1}{k'}\right)$$

existante multiplicatione $m = PkRk'$ seu proxima $m = kk'$.

Quia autem fieri posset, ut distantiam α negativam capi expediret, statuamus primum intervallum $\alpha + b = \zeta\alpha$ fietque $P = \tfrac{1}{1-\zeta}$, ubi notandum, si α esset quantitas negativa, tam ζ quam η negative accipi debere; semper autem necesse erit, ut sit $-B\alpha > 0$ seu $B\alpha < 0$ et $\theta > 0$, uti initio iam assumsimus, ubi posuimus $CD = -\theta$.

Cum nunc pro campo apparente sit

$$\Phi = \tfrac{-\pi+\pi'-\pi''+\pi'''}{m-1},$$

statuamus

$$\pi = -v\xi, \quad \pi' = \omega\xi, \quad \pi'' = -\xi \quad \text{et} \quad \pi''' = \xi,$$

ut sit

$$\Phi = \tfrac{v+\omega+2}{m-1}\xi = M\xi$$

existente

$$M = \tfrac{v+\omega+2}{m-1};$$

ex quibus pro loco oculi colligimus

$$O = \tfrac{e}{Mm}$$

existante

$$e = \tfrac{-B\theta\alpha}{m}$$

Consideremus nunc nostras formulas fundamentales

1. $\mathfrak{B}v = (P-1)M,$
2. $\mathfrak{C}\omega = -(1+Pk)M - v,$
3. $\mathfrak{D} = -(1+PkR)M - v - \omega,$

de quibus observari oportet fore primam $\mathfrak{B}v = \tfrac{\zeta}{1-\zeta}M$ *;* sicque valor v ob duplicem causam fiet quantitas minima, ita ut etiam mv adhuc sit valde parvum. Pro secunda autem, quia est $C = \infty$, erit $\mathfrak{C} = \tfrac{C}{C+1} = 1 - \tfrac{1}{C}$; pro tertia autem, quia est $D = 0$ seu potius $D = -\tfrac{\theta}{C}$, erit $\mathfrak{D} = \tfrac{-\theta}{C-\theta} = \tfrac{-\theta}{C}$; deinde etiam hic recordari oportet esse

$$R = \tfrac{BC}{BC+\eta Pk} = 1 - \tfrac{\eta Pk}{BC};$$

quia igitur ex secunda aequatione ob $\mathfrak{C} = \tfrac{C}{C+1}$ est

$$\omega = -(1+Pk)M\left(1+\tfrac{1}{C}\right) - v\left(1+\tfrac{1}{C}\right),$$

si hic valor in tertia aequatione substituatur, erit

$$-\frac{\theta}{C}=-\left(1+Pk\right)M+\frac{\eta P^2k^2M}{BC}-v+\left(1+Pk\right)M\left(1+\frac{1}{C}\right)+v\left(1+\frac{1}{C}\right),$$

ubi, cum termini finiti se mutuo destruant, ex infinite parvis concluditur fore

$$\theta=-\frac{\eta P^2k^2M}{B}-\left(1+Pk\right)M-v,$$

unde fit

$$\eta=-\frac{(1+Pk)B}{P^2k^2}-\frac{B\theta}{P^2k^2M}-\frac{Bv}{P^2k^2M},$$

ubi terminus ultimus tuto omitti potest.

Destructio porro marginis colorati postulet hanc aequationem:

$$0=\frac{v}{P}+\frac{\omega}{PQ}+\frac{1}{PQR}+\frac{1}{PQRS},$$

quae pro nostro casu fit

$$0=v-\frac{\omega}{k}-\frac{1}{k}+\frac{1}{kk'},$$

unde neglecto termino primo deducitur

$$k'=\frac{1}{\omega+1},$$

et ob $m=Pkk'$ erit

$$m=\frac{Pk}{\omega+1}.$$

Cum autem sit

$$\omega=-(1+Pk)M \quad \text{et} \quad M=\frac{\omega+2}{m-1},$$

neglecto termino *v* fiet

$$\left(m-1\right)\omega=-(1+Pk)\left(\omega+2\right)$$

hincque

$$\omega=\frac{-2(1+Pk)}{m+Pk} \quad \text{atque} \quad M=\frac{2}{m+Pk}.$$

Quare, cum sit $m=\frac{Pk}{\omega+1}$, substituto valore ipsius ω obtinemus

$$m-\frac{2m(1+Pk)}{m+Pk}=Pk,$$

hincque

$$mm-2m=P^2k^2+2Pkm,$$

quae m^2 utrinque addito praebet $2m(m-1)=\left(Pk+m\right)^2$ ideoque

$$Pk = -m + \sqrt{2m(m-1)}.$$

Hoc ergo valore pro Pk assumto pro campo apparente adipiscemur maximum valorem, qui erit

$$\Phi = \frac{2}{\sqrt{2m(m-1)}} \cdot \xi,$$

et in mensura angulorum ob $\xi = \frac{1}{4}$ erit

$$\Phi = \frac{1718}{\sqrt{2m(m-1)}} \quad \text{min}.$$

Nunc autem praecipuum opus superest in eo consistens, ut binae priores lentes ita definiantur, ut formula pro semidiametro confusionis inventa penitus evanescat, unde sequens aequatio erit resolvenda:

$$0 = \lambda - \frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{v}{B}\right) - \frac{\lambda''}{\mathfrak{B}^3 Pk} - \frac{\lambda'''}{\mathfrak{B}^3\theta^3 Pk} - \frac{\lambda''''}{\mathfrak{B}^3\theta^3 m}$$

seu

$$0 = \lambda - \frac{1}{\mathfrak{B}P} - \frac{\lambda''}{\mathfrak{B}^3 Pk} - \frac{\lambda'''}{\mathfrak{B}^3\theta^3 Pk} - \frac{\lambda''''}{\mathfrak{B}^3\theta^3 m} - \frac{v}{\mathfrak{B}BP},$$

in qua aequatione, ut ante iam vidimus, sumi potest $\lambda'' = 1$, et quia duae postremae lentes debent esse utrinque aequaliter convexae, erit pro vitro communi $\lambda''' = \lambda'''' = 1,6299$. Ex hac vero aequatione vel λ vel λ' definiri debet, prouti coefficiens ipsius λ' maior est unitate sive minor. Ceterum notandum est omnes quantitates hic praeter litteras B et $\mathfrak{B}$ satis esse determinatas, ita ut in hoc negotio tantum litterae B et $\mathfrak{B}$ arbitrio nostro permittantur; in quo duo casus sunt perpendendi, alter, quo $\mathfrak{B}$ est fractio unitate maior, puta $\frac{1+i}{i}$, alter vero, quo est unitate minor, puta $= \frac{i}{1+i}$.

Primo si sit $\mathfrak{B} = \frac{1+i}{i}$, erit $B = -1 - i$ ideoque numerus negativus; quo ergo casu α debet esse positivum seu prima lens convexa, secunda vero concava, pro qua valor λ' determinari debet et quidem ex hac aequatione:

$$\lambda' = \frac{(1+i)^3 P\lambda}{i^3} + \frac{\lambda''}{i^3 k} + \frac{\lambda'''}{i^3\theta^3 k} + \frac{\lambda''''}{i^3\theta^3 m} + \frac{(1+i)v}{i^3},$$

ubi sumto $\lambda = 1$ evidens est λ' fieri unitate maius.

At secundo si sit $\mathfrak{B} = \frac{i}{1+i}$ erit $B = i$ ideoque positivum ; unde distantia α fiet negativa sive prima lens concava, secunda vero convexa, quo casu numerus λ definiri oportet per hanc aequationem:

$$\lambda = \frac{(1+i)^3 \lambda'}{i^3 P} + \frac{\lambda''}{i^3 Pk} + \frac{\lambda'''}{i^3\theta^3 Pk} + \frac{\lambda''''}{i^3\theta^3 m} + \frac{(1+i)v}{i^3 P},$$

atque hic sumi poterit $\lambda' = 1$; λ vero unitate maius fiet.

Perspicuum igitur est simili fere modo, quo in priore casu λ' definitur, in secundo casu litteram λ definiri, propterea quod proxime est $P=1$, quandoquidem invenimus $P=\frac{1}{1-\zeta}$; ubi notetur priore casu, quo α est positivum, sumi posse $\zeta=\frac{1}{50}$, ut sit $P=\frac{50}{49}$; eodemque modo etiam η erit positivum, quemadmodum etiam nostra formula posito $B=-1-i$ declarat, scilicet

$$\eta=\frac{(1+Pk)(1+i)}{P^2k^2}+\frac{(1+i)\theta}{P^2k^2M}.$$

Pro altero autem casu, quo α est quantitas negativa, sumi debet $\zeta=-\frac{1}{50}$, ut sit $P=\frac{50}{51}$; ob eandemque rationem etiam η fiet negativum, scilicet

$$\eta=\frac{i(1+Pk)}{P^2k^2}+\frac{\theta i}{P^2k^2M}.$$

COROLLARIUM 1

332. Cum tollendo marginem coloratum pervenerimus ad hanc aequationem:

$$Pk=-m+\sqrt{2m(m-1)},$$

qua ob *P* datum valor ipsius *k* determinatur, hinc habebimus

$$M=\frac{2}{\sqrt{2m(m-1)}} \quad \text{et} \quad \omega=\frac{-2(1+Pk)}{\sqrt{2m(m-1)}}$$

atque hinc

$$\eta=\frac{-(1+Pk)B}{P^2k^2}-\frac{B\theta\sqrt{2m(m-1)}}{2P^2k^2}.$$

COROLLARIUM 2

333. Cum sit $C=\infty$, $D=0$ et $CD=-\theta$, fient nostra elementa

$$b=\frac{-\alpha}{P},\quad c=\frac{-B\alpha}{Pk},\quad d=-\infty,\quad e=\frac{-B\theta\alpha}{m},$$

$$\beta=\frac{-B\alpha}{P},\quad \gamma=\infty,\quad \delta=\frac{-B\theta\alpha}{Pk}$$

hincque distantiae focales

$$p=\alpha,\ \ q=\tfrac{-B\alpha}{P},\ \ r=\tfrac{-B\alpha}{Pk},\ \ s=\delta=\tfrac{-B\theta\alpha}{Pk},\ \ t=\tfrac{-B\theta\alpha}{m};$$

tum vero lentium intervalla

$$\alpha+b=\alpha\left(1-\tfrac{1}{P}\right)=\zeta\alpha,\ \ \beta+c=-\tfrac{B\alpha}{P}\left(1+\tfrac{1}{k}\right),$$

$$\gamma+d=\eta\alpha,\ \ \delta+e=-B\theta\alpha\left(\tfrac{1}{Pk}+\tfrac{1}{m}\right)$$

ac denique distantia oculi

$$O=-\frac{B\theta\alpha\sqrt{2m(m-1)}}{2m^2},$$

ubi notetur litteram θ arbitrio nostro permitti, quo caveri poterit, ne ultimae lentes fiant nimis parvae.

COROLLARIUM 3

334. Ex his perspicitur, quo maior capiatur littera *B,* eo maius prodire secundum intervallum cum sequentibus hincque longitudinem telescopii eo magis augeri; at littera *B* eo maior evadit, quo propius littera $\mathfrak{B}$ ad unitatem accedit ; sive enim sit $\mathfrak{B}=\frac{1+i}{i}$ sive $\mathfrak{B}=\frac{i}{1+i}$; aucto numero *i* augetur numerus *B;* quare, cum littera $\mathfrak{B}$ etiam nunc arbitrio nostro permittatur, neutiquam expediet eam unitati nimis propinquam statui neque tamen etiam conveniet pro *i* numerum valde parvum assumi, veluti dimidium vel fractionem adhuc minorem; tum enim ex ultima aequatione numerus vel λ vel λ' prodiret nimis magnus, scilicet adeo maior quam 27. Unde concluditur numerum *i* ad minimum unitate maiorem capi debere.

COROLLARIUM 4

335. Hic igitur commodum cum incommodo compensatur; si enim *i* unitate minus caperetur, obtineremus commodum brevitatis tubi, contra vero nimis magnus valor numeri λ vel λ' insigne esset incommodum; sin autem numerum *i* unitate multo maiorem sumeremus, obtineremus quidem commodum, ut λ vel λ' parum unitatem excederent, contra vero tubus fieret nimis longus.

COROLLARIUM 5

336. Sin autem optio detur inter valores $\frac{1+i}{i}$ et $\frac{i}{1+i}$ pro $\mathfrak{B}$ assumendos retinente *i* in utroque eundem valorem, tunc λ vel λ' eundem fere valorem nancisceretur. Verum priore casu cum fiat $B=-1-i$, longitudo tubi maior prodiret, quam altero casu, quo esset $B=i$; quam ob rem semper consultius est posteriorem casum eligere, quo lens prima est

concava et secunda convexa, quam priorem, ubi vicissim lens prima esset convexa, secunda vero concava.

SCHOLION 1

337. Quae quo clarius perspiciantur, ponamus $i=2$ et $\mathfrak{B}=\frac{2}{3}$, ut fiat $B=2$; tum igitur erit $P=\frac{50}{51}$ et elementa nostra sequenti modo se habebunt existente α quantitate negativa:

$$b=\frac{-51}{50}\alpha,\ \ c=\frac{-2\alpha}{Pk},\ \ d=-\infty,\ \ e=\frac{-2\theta\alpha}{m},$$

$$\beta=\frac{-2\alpha}{P}=\frac{-51\alpha}{25},\ \ \gamma=\infty,\ \ \ \delta=\frac{-2\theta\alpha}{Pk}$$

existente $Pk=-m+\sqrt{2m(m-1)}$; tum vero distantiae focales erunt

$$p=\alpha,\ \ q=\frac{-51\alpha}{75},\ \ r=\frac{-2\alpha}{Pk},\ \ s=\frac{-2\theta\alpha}{Pk},\ \text{et}\ \ t=\frac{-2\theta\alpha}{m},$$

at intervalla lentium

$$\alpha+b=-\frac{1}{50}\alpha,\ \ \beta+c=-\frac{51}{25}\alpha-\frac{2\alpha}{Pk},$$

$$\gamma+d=\eta\alpha=\frac{-2(1+Pk)\alpha}{P^2k^2}-\frac{\theta\sqrt{2m(m-1)}}{P^2k^2}\alpha,$$

$$\delta+e=\frac{-2\theta\alpha}{m}-\frac{2\theta\alpha}{Pk}=\frac{-2\theta\alpha}{Pkm}\sqrt{2m(m-1)}$$

et distantia oculi

$$O=\frac{-\theta\alpha\sqrt{2m(m-1)}}{mm},$$

quibus factis campi semidiameter erit

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\quad\text{min}.$$

Pro apertura autem tertiae lentis notandum est esse $\omega=\frac{-2(1+Pk)}{\sqrt{2m(m-1)}}$, ita ut, si m sit numerus satis magnus, fiat $\omega=-\frac{10}{17}$; unde, cum haec lens non maximam aperturam, sed minorem, quae sit ad maximam ut 10:17, requirat, sufficiet pro hac lente sumsisse $\lambda''=1$; quare, si et $\lambda'=1$, at $\lambda'''=\lambda''''=1,6299$, pro lente obiectiva inveniemus

$$\lambda=\frac{27\cdot51}{8\cdot0}+\frac{1}{8Pk}+\frac{1,6299}{8\theta^3Pk}+\frac{1,6299}{8\theta^3m}+\frac{153\nu}{200},$$

existente $v = 0,2326$ pro refractione scilicet $n = 1,55$.

Hinc autem invento numero λ prima lens obiectiva concava ita construi debet ut fiat

$$\text{radius faciei}\begin{cases}\text{anterioris} & = \frac{\alpha}{\sigma - \tau\sqrt{\lambda - 1}}\\ \text{posterioris vero} & = \frac{\alpha}{\sigma + \tau\sqrt{\lambda - 1}}\end{cases}$$

existente $\rho = 0,1907, \quad \sigma = 1,6274, \quad \tau = 0,9051.$

Pro secunda autem lente capi debebit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = \frac{2b}{2\rho + \sigma}\\ \text{posterioris vero} & = \frac{2b}{2\sigma + \rho}\end{cases}$$

existente $b = -\frac{51}{50}\alpha.$

Pro tertia lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = \frac{c}{\rho}\\ \text{posterioris vero} & = \frac{c}{\sigma}\end{cases}$$

existente $c = \frac{-2\alpha}{Pk}$.

Pro quarta vero lente

$$\text{radius utriusque faciei } = 1,10s,$$

et pro quinta lente

$$\text{radius faciei utriusque } = 1,10t.$$

Ad mensuras vero absolutas inveniendas consideretur in constructione lentium primae et secundae minimus radius, qui sit $= \mathfrak{m}\alpha$, cuius pars quarta $\frac{1}{4}\mathfrak{m}\alpha$ aequetur semidiametro aperturae ob claritatem requisitae, quae sit $\frac{\mathfrak{m}}{50}$ dig., hincque fit $\alpha = \frac{-2\mathfrak{m}}{25\mathfrak{m}}$ dig., quae mensura si forte det ultimas lentes nimis exiguas, ut supra usu venit, tantum litterae θ tribuatur valor unitate pro lubitu maior, cum hinc longitudo telescopii vix augeatur. Colligitur autem tota haec longitudo ad oculum usque

$$=-\alpha\left(2\tfrac{3}{50}+\tfrac{2(1+2Pk)}{P^2k^2}+\tfrac{(m+Pk)^3\theta}{m^2P^2k^2}\right).$$

EXEMPLUM 1

338. Si fuerit $m=9$, erit $Pk=3$ et $k=\frac{153}{50}$ ob $P=\frac{50}{51}$; unde elementa telescopii erunt

$$b=-\tfrac{51}{50}\alpha,\ \ \beta=-\tfrac{51}{25}\alpha,\ \ c=-\tfrac{2\alpha}{3},\ \ \gamma=\infty,$$

$$d=-\infty,\quad \delta=-\tfrac{2\theta\alpha}{3},\quad e=-\tfrac{2\theta\alpha}{9}$$

et distantiae focales

$$p=\alpha,\quad q=-\tfrac{17}{25}\alpha,\quad r=-\tfrac{2}{3}\alpha,\quad s=-\tfrac{2\theta\alpha}{3},\quad t=-\tfrac{2\theta\alpha}{9}$$

et intervalla

$$\alpha+b=-\tfrac{1}{50}\alpha,\quad \beta+c=-\tfrac{203}{75}\alpha,\quad \gamma+d=-\tfrac{(8+12\theta)}{9}\alpha,\quad \delta+e=-\tfrac{8\theta\alpha}{9}$$

et distantia oculi

$$O=-\tfrac{4\theta\alpha}{27}.$$

Tum vero campi apparentis semidiameter

$$\Phi=143\text{ min.}=\ 2°23'.$$

Nunc vero habebimus

$$\lambda=3,4425+0,04166+\tfrac{0,09055}{\theta^3}$$
$$\begin{array}{r}+\ 0,1779\\ \hline 3,6204\\ 0,0416\\ \hline \end{array}$$
$$\lambda=3,6620+\tfrac{0,09055}{\theta^3}.$$

Sumamus nunc $\theta=1$, ut fiat

$$\lambda=3,75255,\quad \lambda-1=2,75255\ \text{ et }\ \tau\sqrt{\lambda-1}=1,50162.$$

Quare constructio lentis primae ita se habebit:

$$\text{radius faciei}\begin{cases}\text{anterioris }=\frac{\alpha}{0,1258}=7,9491\alpha\\[2ex] \text{posterioris}=\frac{\alpha}{1,6923}=0,5909\alpha.\end{cases}$$

Pro secunda autem lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{2b}{2{,}0088} = -1{,}0155\alpha \\ \text{posterioris} = \frac{2b}{3{,}4455} = -0{,}5921\alpha.\end{cases}$$

Pro tertia autem lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{c}{0{,}1907} = -3{,}4959\alpha \\ \text{posterioris} = \frac{2b}{1{,}6274} = -0{,}4097\alpha.\end{cases}$$

Pro lente quarta

$$\text{radius faciei utriusque} = -0{,}7333\alpha.$$

Pro lente denique quinta

$$\text{radius faciei utriusque} = -0{,}2444\alpha .$$

Iam in duabus prioribus lentibus occurrit radius minimus $= 0{,}5909\alpha$, ut sit $\mathfrak{m} = 0{,}5909$ adeoque

$$\alpha = -\frac{72}{59{,}09} \text{ dig. seu } \alpha = -1\tfrac{1}{4}\text{dig.}$$

Unde , sequens prodibit constructio huius telescopii pro multiplicatione $m = 9$, lentibus ex vitro communi factis.

I. Pro prima lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = -9{,}93 \text{ dig.} \\ \text{posterioris} = -0{,}73 \text{ dig.}\end{cases}$$

Cuius distantia focalis $= 1\tfrac{1}{4}$ dig .
Semidiameter aperturae $= 0{,}18$ dig.
Distantia ad lentem secundam $= 0{,}025$ dig.

II. Pro secunda lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = 1{,}27 \text{ dig.} \\ \text{posterioris} = 0{,}74 \text{ dig.}\end{cases}$$

Cuius distantia focalis $= 0,85$ dig.
Semidiameter aperturae ut ante $= 0,18$ dig.
Distantia ad lentem tertiam $= 3,38$ dig.

III. Pro tertia lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = 4,37 \text{ dig.}\\ \text{posterioris} = 0,51 \text{ dig.}\end{cases}$$

Cuius distantia focalis $= 0,83$ dig.
Semidiameter aperturae $= 0,13$ dig.
Distantia ad quartam $= 2,78$ dig.

IV. Pro quarta lente

Radius utriusque faciei $= 0,92$ dig.
Cuius distantia focalis $= 0,83$ dig.
Semidiameter aperturae $= 0,23$ dig.
Intervallum ad quintam $= 1,11$ dig.

V. Pro quinta lente

Radius utriusque faciei $= 0,30$ dig.
Cuius distantia focalis $= 0,28$ dig.
Semidiameter aperturae $= 0,07$ dig.
et distantia ad oculum $= 0,19$ dig.
sicque tota instrumenti longitudo $= 7,49$ dig.
et semidiameter campi $= 2°23'$.

Hac ergo perfectione adhibita telescopium, quod ante erat 6 ped., reductum est ad $7\frac{1}{2}$ dig.

EXEMPLUM 2

339. Si multiplicatio sit $m = 50$, erit $Pk = 20$ et $k = \frac{102}{5}$; unde elementa nostra erunt

$$b=-\tfrac{51}{50}\alpha,\quad \beta=-\tfrac{51}{25}\alpha,\quad c=-\tfrac{\alpha}{10},\ \gamma=\infty,$$

$$d=-\infty,\quad \delta=-\tfrac{\theta\alpha}{10},\quad e=-\tfrac{\theta\alpha}{25}$$

et distantiae focales

$$p=\alpha,\quad q=-\tfrac{17}{25}\alpha,\quad r=-\tfrac{\alpha}{10},\quad s=-\tfrac{\theta\alpha}{10},\ \text{ et }\ t=-\tfrac{\theta\alpha}{25}$$

et intervalla lentium

$$\alpha+b=-\tfrac{1}{50}\alpha,\quad \beta+c=-\tfrac{107}{50}\alpha,\ \gamma+d=\frac{-(21+35\theta)\alpha}{200},\quad \delta+e=\frac{-7\theta\alpha}{50}$$

atque distantia oculi

$$O=-\tfrac{7\theta\alpha}{250},$$

et campi apparentis semidiameter erit $=24\tfrac{1}{2}$ min.

Nunc vero prodibit

$$\begin{array}{l}\lambda=3,4425+\frac{0,0143}{\theta^3}\\ \quad +0,0063\\ \quad\ \ 0,1779\\ \hline \lambda=3,6267+\frac{0,0143}{\theta^3}\end{array}$$

Sumatur nunc $\theta=2$ eritque

$$\lambda=3,6285,\quad \lambda-1=2,6285\quad \text{et}\quad \tau\sqrt{(\lambda-1)}=1,4674,$$

unde fiet:

I. Pro prima lente

$$\text{radius faciei}\begin{cases}\text{anterioris } =\frac{\alpha}{0,1600}=6,2500\alpha\\ \\ \text{posterioris}=\frac{\alpha}{1,0581}=0,6031\alpha.\end{cases}$$

II. Pro secunda lente

$$\text{Uti ante radius faciei}\begin{cases}\text{anterioris } =\frac{2b}{2,0088}=-1,0155\alpha\\ \\ \text{posterioris}=\frac{2b}{3,4455}=-0,5921\alpha.\end{cases}$$

III. Pro tertia lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{c}{0,1907} = -0,5244\alpha\\ \text{posterioris} = \frac{c}{1,6274} = -0,0615\alpha.\end{cases}$$

IV. Pro quarta lente

Radius faciei utriusque $= -0,2200\alpha$.

V. Pro quinta lente

Radius faciei utriusque $= -0,0880\alpha$.

Iam cum sit in duabus prioribus lentibus radius minimus $0,5921\alpha$, erit $\mathfrak{m} = 0,5921$ adeoque $\alpha = -\frac{400}{59,91}$ dig., ita ut capi posset $= -7$ dig.

Unde sequens prodibit constructio huius telescopii pro multiplicatione $m = 50$.

I. Pro prima lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = -43,75 \text{ dig.}\\ \text{posterioris} = -4,22 \text{ dig.}\end{cases}$$

Cuius distantia focalis $= -7$ dig.
Semidiameter aperturae $= 1,05$ dig.
Distantia ad lentem secundam $= 0,14$ dig.

II. Pro secunda lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = 7,11 \text{ dig.}\\ \text{posterioris} = 4,14 \text{ dig.}\end{cases}$$

Cuius distantia focalis est $= 4,76$ dig.
Semidiameter aperturae ut ante $= 1,05$ dig.
Intervallum ad tertiam lentem $= 14,98$ dig.

III. Pro tertia lente

$$\text{radius faciei}\begin{cases}\text{anterioris} & = 3,67 \text{ dig.}\\ \text{posterioris} & = 0,43 \text{ dig.}\end{cases}$$

Distantia focalis est $= 0,7$ dig.
Semidiameter aperturae $= 0,11$ dig.
Intervallum ad quartam $= 3,18$ dig.

IV. Pro quarta lente

Radius utriusque faciei $= 1,54$ dig.
Cuius distantia focalis est $= 1,40$ dig.
Semidiameter aperturae $= 0,38$ dig.
Intervallum ad quintam lentem $= 1,96$ dig.

V. Pro quinta lente

Radius utriusque faciei $= 0,61$ dig.
Cuius distantia focalis $= 0,56$ dig.
Semidiameter aperturae $= 0,15$ dig.
Distantia ad oculum $= 0,39$ dig.
Sicque longitudo tota $= 20\frac{2}{3}$ dig. propemodum
et semidiameter campi $= 24\frac{1}{2}$ min.

SCHOLION 2

340. Hoc ergo etiam postremum telescopium facile per tubos ductitios ita parari potest, ut commode quis secum id portare possit, cum lente illa concava omissa hoc telescopium ultra viginti pedes excrevisset. Circa tubos autem ductitios hic notari oportet, dum ductus ad oculum accommodatur, solam lentem ocularem mobilem esse debere, reliquas vero lentes omnes in locis hic assignatis perpetuo consistere debere, id quod in perpetuum de omnibus telescopiis, quae hic tractantur, est tenendum ; ceterum non opus est, ut perfectioni, quam variae vitri species largiuntur, caput peculiare tribuamus, ut hactenus fecimus, sed solutio praecedentis problematis paucis mutandis ad hune scopum accommodari potest, uti in problemate sequente ostendemus.

PROBLEMA 3

341. *Si prima lens obiectiva concava ex vitro crystallino paretur, dum reliquae ex vitro coronario conficiuntur, constructionem telescopii describere, in quo non margo solum coloratus, sed etiam tota confusio a diversa radiorum refrangibilitate oriunda penitus destruatur.*

SOLUTIO

Hoc problema, ut hactenus fecimus, ex principiis supra stabilitis si resolvere vellemus, omnia plane eodem modo se essent habitura uti in problemate praecedente usque ad eum locum, ubi marginem coloratum sustulimus, atque etiam haec ipsa aequatio non esset discrepatura ab ea, quam in praecedenti problemate tractavimus, quoniam in ea prima lens non in computum venit, ita ut hinc etiam eaedem determinationes obtinerentur atque hucusque litterae $\mathfrak{B}$ et B etiam nunc mansurae essent indeterminatae; iam autem demum ultimae aequationis, qua confusio penitus e medio tollitur, ratio erit habenda, et aequatio eo pertinens [§ 53], si pro prima lente formulam differentialem $\frac{dn}{n-1}$ littera N, pro sequentibus autem lentibus litteris N' denotemus per hasque aequationem dividamus, erit

$$0=\frac{N}{N'}-\frac{1}{\mathfrak{B}P}-\frac{1}{B\mathfrak{C}^2Pk}-\frac{1}{BP\theta k}-\frac{1}{B\theta m},$$

in qua aequatione terminus tertius cum sequentibus prae duobus primis tam sunt exigui, ut sine errore negligi queant, praecipue cum, uti iam saepius notavimus, natura rei non permittat, ut haec aequatio adcurate resolvatur neque id etiam scopus noster postulet. Quare sumtis tantum duobus terminis prioribus colligemus

$$\mathfrak{B}=\frac{N'}{NP},$$

scilicet ob hanc conditionem lentis primae e vitro crystallino parandae totum discrimen in resolutione in hoc tantum consistit, ut nunc, cum littera $\mathfrak{B}$ ante arbitrio nostro mansisset relicta, definiatur; quocirca, quia ex Dollondi experimentis habemus $N:N'=10:7$ ac praeterea sit $P=\frac{50}{51}$, consequimur nunc $\mathfrak{B}=\frac{357}{500}$, qui valor proxime reductur ad hunc $\mathfrak{B}=\frac{5}{7}$, sive etiam $\mathfrak{B}=\frac{2}{3}$, qui est ipse valor, quem in praecedentibus iam exemplis ipsi $\mathfrak{B}$ tribuimus; quicunque autem valor ipsi $\mathfrak{B}$ tribuatur, in aequationem ultimam, ex qua numerus λ definitur, leve quoddam discrimen ingreditur; cum enim nunc primus terminus per μ, sequentes vero per μ' sint multiplicandi, divisione per μ' facta haec aequatio fiet

$$\frac{\mu}{\mu'}\lambda=\frac{\lambda'}{\mathfrak{B}^3P}+\frac{\lambda''}{B^3Pk}+\frac{\lambda'''}{B^3\theta^3Pk}+\frac{\lambda''''}{B^3\theta^3P}+\frac{\nu'}{\mathfrak{B}BP},$$

ubi ut ante sumi potest $\lambda'=1$ et $\lambda''=1$; at quia lentes posteriores ex vitro coronario, quo $n=1,53$, conficiuntur, pro duabus postremis lentibus, quae utrinque aequaliter convexae esse debent, erit $\lambda'''=\lambda''''=1,60006$, litterae autem eo pertinentes erunt

$$\mu'=0,9875,\quad \nu'=0,2196,\quad \rho'=0,2267 \text{ et } \sigma'=1,6601,\quad \tau'=0,9252.$$

Pro prima autem lente crystallina erit

$$\mu=0,8724,\quad \nu=0,2529,\quad \rho=0,1414,\quad \sigma=1,5827 \text{ et } \tau=0,8775.$$

COROLLARIUM 1

342. Nunc igitur demum intelligitur, cur praestet primam lentem ex vitro crystallino parare quam secundam; si enim prima est crystallina, fit $\mathfrak{B}=\frac{5}{7}$ et $B=\frac{5}{2}$. Sin autem secundam crystallinam faceremus, foret $\mathfrak{B}=\frac{7}{5}$ et $B=-\frac{7}{2}$. Quare, cum omnes sequentes distantiae multiplicatae sint per *B,* eae ac propterea tota longitudo tubi prodiret posteriore casu maior quam primo idque in ratione $7:5$.

COROLLARIUM 2

343. Si discrimen dispersionis ambarum vitri specierum minus esset, quam hic secundum Dollondi experimenta assumimus, tunc fractio pro $\mathfrak{B}$ assumenda propius ad unitatem accederet indeque *B* maiorem nancisceretur valorem sicque instrumentum longius evaderet; ex quo ad praxin plurimum expedit, ut duae vitri species ratione dispersionis maxime inter se differentes eligantur, siquidem hoc modo telescopia multo breviora redderentur.

SCHOLION 1

344. Quoniam igitur hic primam lentem ex vitro crystallino, reliquas ex coronario fieri assumimus, experimentis Dollondianis innixi statuamus $\mathfrak{B}=\frac{5}{7}$ ut sit $B=\frac{5}{2}$, ac posito $\theta=2$, ne lens ocularis fiat nimis parva, elementa nostra sequenti modo se habebunt:

$$b=-\frac{51}{50}\alpha, \quad c=-\frac{5\alpha}{2Pk}, \quad d=-\infty, \quad e=-\frac{5\alpha}{m},$$

$$\beta=-\frac{51}{20}\alpha, \quad \gamma=\infty, \quad \delta=-\frac{5\alpha}{Pk}$$

et distantiae focales

$$p=\alpha, \quad q=-\frac{51}{70}\alpha, \quad r=-\frac{5\alpha}{2Pk}, \quad s=-\frac{5\alpha}{Pk}, \quad t=-\frac{5\alpha}{m}$$

hincque intervalla

$$\alpha+b=-\frac{1}{50}\alpha, \quad \beta+c=-\frac{51}{20}\alpha-\frac{5\alpha}{2Pk},$$

$$\gamma+d=\eta\alpha=-\frac{5(1+Pk)\alpha}{2P^2k^2}-\frac{5\sqrt{2m(m-1)\alpha}}{2P^2k^2}, \quad e=-\frac{5\alpha}{m},$$

$$\delta+e=-\frac{5\sqrt{2m(m-1)}}{mPk}\alpha$$

et distantia oculi

$$O=-\frac{5\sqrt{2m(m-1)}}{2m^2}\alpha$$

existante

$$Pk=-m+\sqrt{2m(m-1)};$$

tum autem semidiameter campi

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\text{ min.}$$

Ut igitur hinc constructionem pro quavis multiplicatione m investigemus, methodo iam saepius adhibita utentes primo evolvamus casum, quo $m=25$, tum vero casum, quo $m=\infty$

EXEMPLUM 1

345. Sit multiplicatio $m=25$ ac reperietur

$$\sqrt{2m(m-1)}=34,64101 \text{ hincque } Pk=9,64101,$$

unde intervalla ita se habebunt:

$$\alpha+b=-0,02\alpha,\quad \beta+c=-\ 2,80930\alpha,\quad \gamma+d=-1,21770\alpha,$$

$$\delta+e=-0,71860\alpha$$

et distantia oculi $=-0,13844\alpha$.

His praemissis quaeratur λ ex aequatione supra data et invenietur

$$\lambda=3,16815+0,007514+0,001502+0,000579+0,14198$$

seu

$$\lambda=3,31972,$$

unde fit

$$\tau\sqrt{(\lambda-1)}=1,33648.$$

Hinc igitur, si F et G denotent radios anterioris et posterioris faciei, habebimus

I. Pro prima lente crystallina

$$F=\frac{\alpha}{\sigma-1,3365}=\frac{\alpha}{0,2462}=4,0617\alpha,\quad G=\frac{\alpha}{\rho+1,3365}=\frac{\alpha}{1,4779}=0,6766\alpha.$$

II. Pro secunda autem lente coronaria erit

$$F=\frac{5b}{5\rho'+2\sigma'}=\frac{5b}{4,4537}=-1,14537\alpha,\quad G=\frac{5b}{5\sigma'+2\rho'}=\frac{5b}{8,7539}=-0,5826\alpha,$$

quae constructio pro omni multiplicatione valet.

III. Pro tertia lente coronaria habebimus

$$F = \frac{c}{\rho'} = \frac{c}{0,2267} = -\frac{11,0278\alpha}{Pk} = -1,1438\alpha,$$

$$G = \frac{c}{\sigma'} = \frac{c}{1,6601} = -\frac{1,5059\alpha}{Pk} = -0,1562\alpha,$$

ubi valores penultimi pro omni multiplicatione valent.

IV. Pro quarta lente itidem coronaria, cuius distantia focalis $= s = -\frac{5\alpha}{Pk}$, erit

$$F = G = 1,06s = -\frac{5,30\alpha}{Pk} = -0,5497\alpha,$$

ubi valor penultimus pro omni multiplicatione valet.

V. Pro quinta lente etiam coronaria, cuius distantia focalis est $t = -\frac{5\alpha}{m}$, erit

$$F = G = 1,06t = -\frac{5,3\alpha}{m} = -0,212\alpha,$$

ubi iterum forma penultima pro omni multiplicatione valet.

EXEMPLUM 2

346. Si fit multiplicatio *m* infinita seu praegrandis, erit $\sqrt{2m(m-1)} = m\sqrt{2} = 1,41421m$ hincque Pk= 0,41421 *m,* unde intervalla erunt

$$\alpha + b = -\ 0,02\alpha, \quad \beta + c = -2,55\alpha -\ 6,0356\cdot\frac{\alpha}{m},$$
$$\gamma + d = -\ 26,6425\cdot\frac{\alpha}{m}, \quad \delta + e = -17,0712\cdot\frac{\alpha}{m}$$

et distantia oculi

$$O = -3,5355\cdot\frac{\alpha}{m}.$$

His praemissis quaeratur λ ex aequatione data et habebitur

$$\lambda = 3,16815 + 0,14198 = 3,31013,$$

unde fit

$$\tau\sqrt{(\lambda - 1)} = 1,3337;$$

quare habebitur:

I. Pro prima lente

$$F=\frac{\alpha}{\sigma-1,3337}=\frac{\alpha}{0,2490}=4,0160\alpha,\quad G=\frac{\alpha}{\rho+1,3337}=\frac{\alpha}{1,4751}=0,6779\alpha.$$

II. Secunda lens convenit cum exemplo praecedente.
III. Pro tertia lente erit

$$F=\frac{-11,0278\alpha}{Pk}=-26,6237\cdot\frac{\alpha}{m},\quad G=\frac{-1,5059\alpha}{Pk}=-3,6357\cdot\frac{\alpha}{m}.$$

IV. Pro quarta lente erit

$$F=G=\frac{-5,3\alpha}{Pk}=-12,7955\frac{\alpha}{m}.$$

V. Pro quinta denique lente

$$F=G=-5,3\cdot\frac{\alpha}{m}.$$

Elementa autem sequenti modo se habebunt:

$$b=-1,02\alpha,\quad c=-6,0355\cdot\frac{\alpha}{m},\quad d=-\infty,\quad e=-5\cdot\frac{\alpha}{m},$$

$$\beta=-2,55\alpha,\quad \gamma=\infty,\quad \delta=-12,0710\cdot\frac{\alpha}{m}$$

hincque distantiae focales

$$p=\alpha,\ \ q=-0,72857\alpha,\quad \gamma=-\ 6,0355\cdot\frac{\alpha}{m},$$

$$s=-12,0710\cdot\frac{\alpha}{m},\quad t=-5\cdot\frac{\alpha}{m}.$$

EXEMPLUM 3

347. Ex collatione praecedentium exemplorum pro quavis multiplicatione maiore *m* constructionem huiusmodi telescopiorum describere.

Primo elementa sequenti modo expressa reperientur:

$$b=-1,02\alpha,\quad \beta=-2,55\alpha,\quad c=-\left(6,0355+\frac{11,1750}{m}\right)\frac{\alpha}{m},\ \ \gamma=\infty,$$

$$d=-\infty,\quad \delta=-\left(12,0710+\frac{22,3500}{m}\right)\frac{\alpha}{m},\quad e=-5\cdot\frac{\alpha}{m}$$

hincque distantiae focales

$$p=\alpha,\quad q=-0,72857\alpha,\quad r=-\left(\ 6,0355+\frac{11,1750}{m}\right)\frac{\alpha}{m},$$

$$s=\ \left(12,0710\ +\ \frac{22,3500}{m}\right)\frac{\alpha}{m},\quad t=-5\cdot\frac{\alpha}{m}$$

et intervalla lentium

$$\alpha + b = -\ 0,02\alpha, \quad \beta + c = -2,55\alpha - \left(6,0355 + \tfrac{11,1750}{m}\right)\tfrac{\alpha}{m}$$

$$\gamma + d = -\left(26,6425 + \tfrac{m}{95}\ \right)\tfrac{\alpha}{m}, \quad \delta + e = -\left(17,0710 + \tfrac{22,3500}{m}\right)\tfrac{\alpha}{m}$$

et distantia oculi

$$O = -\left(3,5355 - \tfrac{1,8625}{m}\right)\tfrac{\alpha}{m}.$$

et tandem semidiameter campi semper est

$$\Phi = \tfrac{1718}{\sqrt{2m(m-1)}}\ \text{min.}$$

Lentium vero constructio ipsa ita se habebit:

I. Pro prima lente crystallina

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= \left(4,0160 + \tfrac{1,14}{m}\right)\alpha \\ \text{posterioris} &= \left(\ 0,6779 - \tfrac{0,0325}{m}\right)\alpha.\end{cases}$$

II. Pro secunda lente coronaria

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= -1,1451\alpha \\ \text{posterioris} &= -\ 0,5826\alpha.\end{cases}$$

III. Pro tertia lente coronaria

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= -\left(26,6237 + \tfrac{49,28}{m}\right)\tfrac{\alpha}{m} \\ \text{posterioris} &= -\left(\ \ 3,6357 - \tfrac{6,73}{m}\right)\tfrac{\alpha}{m}.\end{cases}$$

IV. Pro quarta lente coronaria

Radius utriusque faciei $= -\left(12,7953 + \tfrac{23,68}{m}\right)\tfrac{\alpha}{m}.$

V. Pro quinta lente coronaria

Radius utriusque faciei $=-5{,}30\cdot\frac{\alpha}{m}$.

Nunc denique iudicandum restat, quantum valorem ipsi α tribui conveniat. Hunc in finem consideretur duarum priorum lentium radius minimus, qui est $-0,5826\alpha$, cuius pars quarta $-0,1456\alpha$ ponatur aequalis semidiametro aperturae $\frac{m}{50}$, indeque reperietur $\alpha=-\frac{m}{7{,}28}$, quo quidem valore quantitas α minor accipi non debet; quocirca sumatur $\alpha=-\frac{m}{7}$ atque obtinebitur sequens constructio huiusmodi telescopiorum pro quavis multiplicatione *m*.

Posita igitur distantia focali $\alpha=-\frac{m}{7}$ dig. impetrabimus pro constructione quaesita sequentes mensuras:

I. Pro prima lente crystallina

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= (-\,0,5737m-0,16)\,\text{dig.}\\ \text{posterioris} &= (-0,0968m+0,004)\,\text{dig.}\end{cases}$$

Cuius distantia focalis $=-\frac{m}{7}$ dig.

Semidiameter aperturae $=\frac{m}{50}$ dig.

Intervallum ad lentem secundam $=0,00286m$ dig.

II. Pro secunda lente coronaria

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= 0,1636m\ \text{dig.}\\ \text{posterioris} &= 0,0832m\ \text{dig.}\end{cases}$$

Cuius distantia focalis est $=0,10408m$ dig.

Semidiameter aperturae $=\frac{m}{50}$ dig.

Intervallum ad lentem tertiam $=\left(0,3643m+0,86+\frac{1{,}6}{m}\right)$ dig.

III. Pro tertia lente coronaria

$$\text{Radius faciei}\begin{cases}\text{anterioris} &= \left(3,80+\frac{7,04}{m}\right)\\ \text{posterioris} &= \left(\ 0,52+\frac{0,9}{m}\right)\end{cases}$$

Cuius distantia focalis est $=\left(0,86+\frac{1,6}{m}\right)$ dig.

Semidiameter aperturae $=0,13$ dig.

Intervallum ad quartam $=\left(3,80+\frac{14}{m}\right)$ dig.

IV. Pro quarta lente coronaria

$$\text{Radius faciei utriusque} = (1,82+\tfrac{3,4}{m})\text{ dig.}$$

Cuius distantia focalis est $=\left(1,72+\frac{3,2}{m}\right)$ dig.

Semidiameter aperturae $=0,45$ dig.

Intervallum ad quintam $=(2,44+\frac{3,2}{m})$ dig.

V. Pro quinta lente coronaria

$$\text{Radius utriusque faciei} = 0,76\text{ dig.}$$

Cuius distantia focalis $=0,71$ dig.

Semidiameter aperturae $=0,19$ dig.

Intervallum ad oculum $=\left(0,50-\frac{0,2}{m}\right)$ dig.

Tota ergo telescopii longitudo inde colligitur haec:

$$\left(0,3672m+7,60+\frac{18,6}{m}\right)\text{ dig.,}$$

unde patet, si $m=100$, longitudinem instrumenti non esse superaturam $44\frac{1}{2}$ dig. Semidiameter denique campi apparentis erit

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\text{ minut.}$$

quae ergo pro $m=100$ fiet 12 minut.

SCHOLION 2

348. Haec ergo telescopia adhuc satis brevia forent, si modo in praxi lentes quam exactissime secundum mensuras praescriptas liceret elaborare et si etiam utraque vitri species praecise eandem refractionem admitteret, quam hic supposuimus; perpetuo autem tenendum est, si vitri refractio discrepet ab ea, quam assumsimus, tunc totum calculum de novo esse instituendum, qui scilicet ad formationem lentium spectat; deinde vero etiam haec regula probe est observanda, ut, quo minus felicissimum successum ab artifice exspectare queamus, mensurae hic praescriptae augeri atque adeo duplicari vel triplicari debeant; id quod commodissime fiet, si digiti mensuram multo maiorem accipiamus. Semper autem, etiamsi artifex summam industriam adhibeat, vix unquam sperandum erit, ut primum statim, quod produxerit, instrumentum voto respondeat; quin potius semper necesse erit, ut lentis primae concavae praesertim plura exempla elaborentur, ut ex iis optimum per experientiam eligi possit; quamvis enim eaedem mensurae retineantur, tamen semper usu veniet, ut plura exempla omnia inter se aliquantillum discrepent. Quin etiam saepe consultum erit ipsam mensuram pro constructione huius lentis aliquantillum immutare, ita tamen, ut eadem distantia focalis conservetur, et pro quavis mensura aliquot exempla conficere; scilicet si ex theoria radii facierum anterioris et posterioris istius lentis inventi fuerint *F* et *G,* hanc figuram saepe ita immutari conveniet, ut capiatur radius faciei anterioris $=F\mp F^2\omega,$ posterioris vero $=G\pm G^2\omega,$ sumendo pro ω tantillam fractionem, quae adhuc in praxi sentiri queat; tum enim in distantia focali nihil mutabitur. Denique etiam quaedam monenda restant circa diaphragmata in huiusmodi telescopiis usurpanda; quia enim in iis duae imagines reales reperiuntur, in utriusque loco etiam diaphragma constitui poterit, cuius apertura ipsam illam imaginem capere debet. Primae autem imaginis semidiameter est

$$=\alpha\Phi B=B\alpha M\xi=\tfrac{1}{4}MB\alpha;$$

est vero *M* in nostro casu $=\frac{2}{\sqrt{2m(m-1)}}$ et $B=\frac{5}{2}$, adeoque ista semidiameter erit $=\frac{5\alpha}{4\sqrt{2m(m-1)}}$ sumtoque $\alpha=\frac{m}{7}$ ut ante semidiameter ista erit

$$=\frac{5m}{28\sqrt{2m(m-1)}}=\frac{5}{28\sqrt{2}}=\frac{1}{8}\text{ dig.,}$$

nisi *m* sit numerus parvus. Secundae autem imaginis semidiameter est

$$=\alpha\Phi BCD=\alpha\Phi B\theta;$$

quare, cum sumserimus $\theta=2$, posterius diaphragma aperturam habere debet, cuius semidiameter sit duplo maior quam antecedens, scilicet $\frac{1}{4}$ dig., a quo vero nullus usus exspectari poterit, cum postremae lentes ipsae multo minorem aperturam postulent, ita ut solum diaphragma prius utilitatem habere possit, cui etiam, si libuerit, micrometrum adplicari poterit.