CHAPTER III

CONCERNING OTHER KINDS OF TELESCOPES OF THE THIRD KIND, AND THE PRINCIPAL PERFECTIONS OF THESE.

DEFINITION

349. *We refer these telescopes to that other kind, which have been explained above in § 310, and especially in the appended corollary 2, § 314, in which evidently at this stage a second [diverging] lens may be put in place before the first real image, then truly a third lens after this image in that place, where the image of the first objective lens of the object considered may be projected by the second lens; which place since it may fall before the second image, a fourth ocular lens ought to be put in place here. Moreover in particular, if the focal length of the first lens were put* $=\alpha$, *the second lens thus is put in place, so that there shall be* $b=-\frac{\alpha}{\sqrt{m}}$; *or the distance between the first and second lens shall be*

$$=\alpha\left(1-\frac{1}{\sqrt{m}}\right).$$

COROLLARY 1

350. Therefore since these telescopes depend on four lenses, the terms for these will be had thus :

$$b=-\frac{\alpha}{\sqrt{m}},\ \ \beta=\frac{\sqrt{m}-1}{2m}\cdot\alpha,\ \ c=\frac{\sqrt{m}-1}{2m}\cdot\alpha,\ \ \gamma=\frac{\sqrt{m}-1}{2m}\cdot C\alpha,\ \ d=\frac{\sqrt{m}-1}{2m\sqrt{m}}\cdot C\alpha,$$

thus so that there shall be

$$B=\frac{1-\sqrt{m}}{2\sqrt{m}},\ \ \mathfrak{B}=\frac{1-\sqrt{m}}{1+\sqrt{m}}$$

$$2^{nd}\text{ lens}: \frac{1}{u}+\frac{1}{v}=\frac{1}{f};\ \ B=\frac{\beta}{b};\ \ \mathfrak{B}=\frac{B}{B+1}=\frac{\beta}{b}\cdot\frac{1}{\frac{\beta}{b}+1}=\frac{\beta}{\beta+b}=\frac{1}{b}\cdot\frac{1}{\frac{1}{\beta}+\frac{1}{b}}=\frac{q}{b}\ ;$$

[Recall that

$B=\frac{\beta}{b}$ and $\frac{B}{B+1}=\mathfrak{B}$, where $\mathfrak{B}b=q$, the focal length of the second lens, etc.]

and C may be left to our choice.

COROLLARY 2

351. From these terms, the focal lengths of the lenses will be

$$p=\alpha,\quad q=\frac{\sqrt{m}-1}{\left(1+\sqrt{m}\right)\sqrt{m}}\cdot\alpha,\quad r=\frac{\sqrt{m}-1}{2m}\cdot\mathfrak{C}\alpha \text{ and } s=\frac{\sqrt{m}-1}{2m\sqrt{m}}\cdot C\alpha$$

and the intervals between the lenses:

$$\alpha+B=\left(1-\frac{1}{\sqrt{m}}\right)\alpha,\quad \beta+c=\frac{\sqrt{m}-1}{m}\cdot\alpha,\quad \gamma+d=\frac{m-1}{2m\sqrt{m}}\cdot C\alpha$$

and the distance to the eye

$$O=\frac{m-1}{2mm}\cdot\alpha,$$

thus so that the total distance shall become

$$=\frac{m-1}{m}\left(1+\frac{1+\sqrt{m}\cdot C}{2m}\right)\cdot\alpha,$$

while the only point of concern is that a positive number must be taken for *C*.

COROLLARY 3

352. But the capital letters *P, Q, R* for this kind will become

$$P=\sqrt{m},\ Q=-1 \text{ and } R=-\sqrt{m},$$

thus so that hence there may be produced $PQR=m$, as the nature of the matter demands.

SCHOLIUM

353. Moreover here especially it will be necessary to render an account of the conditions mentioned in the definition, where as we have said the third lens is required to be located there, where the image of the object considered by the first lens to be projected by the second lens falls. Indeed, since the focal length of the second lens shall be

$$q=\frac{\sqrt{m}-1}{\left(1+\sqrt{m}\right)\sqrt{m}}\cdot\alpha,$$

moreover, its distance from the first lens will be $=\left(1-\frac{1}{\sqrt{m}}\right)\alpha$, which may be called *y,* if the first lens may be considered as the object, its image after the second lens falls at the distance

$$\zeta=\frac{yq}{y-q};$$

and truly there is

$$y-q=\left(\frac{\sqrt{m}-1}{\sqrt{m}+1}\right)\cdot\alpha \text{, and in addition } \zeta=\frac{\sqrt{m}-1}{m}\cdot\alpha,$$

to which the distance of the third lens from the second is precisely equal. But thus we have introduced this condition in the definition, since with its help we may assign the position of the third lens easily in practice. Moreover above [in § 314] we have observed now the radius of the apparent field of view to become $\Phi=\frac{859}{m+\sqrt{m}}$ min., which certainly it lacks from an increase, since we will try to perfect these lenses. Finally also in the same place it is shown the radius of the aperture of the third lens must be put in place $=\frac{\sqrt{m}}{50}$ in.

But for the second lens, since we have put

$$\pi=\omega\xi \text{ and } \omega=-\xi=-\frac{1}{\sqrt{m}},$$

the radius of its aperture must be

$$=\frac{q}{4\sqrt{m}}=\frac{\sqrt{m}-1}{4m\left(1+\sqrt{m}\right)}\cdot\alpha.$$

PROBLEM 1

354. *To insert a new lens between the last two lenses of a telescope of this kind, by which the field of view may be made bigger.*

SOLUTION

Therefore since here five lenses may occur, we may put our four fractions in place

$$\frac{\alpha}{b}=-P,\quad \frac{\beta}{c}=-Q,\quad \frac{\gamma}{d}=-R,\quad \frac{\delta}{e}=-S;$$

of which two of the letters must be negative, the first of which must be Q and we may put $Q=-k$, truly the other will be either R or S; but it may be convenient to have another negative quantity in its place, that we have not yet defined.
Hence our elements will become therefore

$$b=\frac{-\alpha}{P},\quad c=\frac{-B\alpha}{Pk},\quad d=\frac{BC\alpha}{PkR},\quad e=\frac{BCD\alpha}{PkRS},$$

$$\beta=\frac{-B\alpha}{P},\quad \gamma=\frac{-BC\alpha}{Pk},\quad \delta=\frac{-BCD\alpha}{PkR},$$

moreover the focal lengths:

$$p=\alpha,\ \ q=\frac{-\mathfrak{B}\alpha}{P},\ \ r=\frac{-B\mathfrak{C}\alpha}{Pk},\ \ s=\frac{BC\mathfrak{D}\alpha}{PkR},\ \ t=\frac{-BCD\alpha}{PkRS}$$

and hence the intervals of the lenses:

$$\alpha+b=\alpha\left(1-\tfrac{1}{P}\right),\qquad \beta+c=\frac{-B\alpha}{P}\left(1+\tfrac{1}{k}\right),$$

$$\gamma+d=\frac{-BC\alpha}{Pk}\left(1-\tfrac{1}{R}\right),\qquad \delta+e=\frac{+BCD\alpha}{PkR}\left(1-\tfrac{1}{S}\right);$$

which since they must be positive, and now α shall be positive, it is necessary that there shall become :

1. $P>1$, 2. $B<0$; 3. So that it will be agreed to distinguish between the two remaining intervals;

First case, where $R>0$ and $S=-k'$. And in this case there must become :

$$C\left(1-\tfrac{1}{R}\right)>0 \text{ and } CD<0,$$

so that also e itself shall become positive.

The latter case, where $R<0$ or $R=-k'$ and $S>0$. Therefore in this case there must become $C>0$ and thus also

$$\mathfrak{C}>0 \text{ or } <1 \text{ and } D(1-\tfrac{1}{S})>0.$$

So that also if there may become $e>0$, there must become $D<0$ and thus $S<1$.

Therefore now we will consider the apparent field, the radius of which [measured in degrees or minutes] is

$$\Phi=\frac{-\pi+\pi'-\pi''+\pi'''}{m-1},$$

and we may put in place so that at this point $\pi=-\omega\xi$, $\pi'=0$ from the nature of this kind, $\pi''=-\xi$ and $\pi'''=\xi$, so that there may become

$$\Phi=\frac{\omega+2}{m-1}\cdot\xi=\mathrm{M}\xi,\text{ with } M \text{ being } =\frac{\omega+2}{m-1},$$

and hence now at once there will be produced for the position of the eye :

$$O=\frac{e}{Mm}=\frac{(m-1)e}{m(\omega+2)}.$$

Again the fundamental equations will be:

1. $\frac{\mathfrak{B}\pi}{\Phi} = 1-P$ or $\mathfrak{B}\omega = -(1-P)M$
2. $0 = -(1+Pk)M - \omega$
3. $\mathfrak{D} = -\left(1+PkR\right)M - \omega;$

where since from the first there shall be

$$\omega = \frac{-(1-P)M}{\mathfrak{B}},$$

this value substituted into the second gives $0 = (1+Pk)\mathfrak{B} + P - 1,$ from which it follows:

$$\mathfrak{B} = -\frac{(P-1)}{1+Pk},$$

thus so that $\mathfrak{B}$ and therefore also B shall be a negative number ; moreover there becomes

$$B = \frac{-(P-1)}{P(1+k)} \text{ and } \omega = -(1+Pk)M;$$

then truly from the third there will be

$$\mathfrak{D} = Pk\left(1-R\right)M;$$

truly the letters C and $\mathfrak{C}$ remain left to our choice. Therefore for the two cases mentioned there will be :

For the first, where $S = -k'$, $\mathfrak{D} = Pk\left(1-R\right)M$. Therefore if there were $R > 1$, there must be $C > 0$ and $D < 0$; but since there may become $\mathfrak{D} < 0$, at once that condition $D < 0$ is fulfilled. But if there shall be $R < 1$, there will be $\mathfrak{D} > 0$, moreover there must become $C < 0$ and $D > 0$, consequently $\mathfrak{D} < 1$ and thus $Pk(1-R)M < 1$.

For the latter case, where $R = -k'$, there will be $\mathfrak{D} = Pk(1+k')M$ and thus $\mathfrak{D} > 0$; but before we have seen in this case there must be $C > 0$ and thus $\mathfrak{C} > 0$ and $\mathfrak{C} < 1$. Then truly $D(1-\frac{1}{S}) > 0$. Whereby, since there must be $S < 1$, there will be $D < 0$, from which on account of $\mathfrak{D} > 0$ it is deduced that $\mathfrak{D} > 1$.

Now this equation will be had for removing the colored margin :

$$0 = \frac{\omega}{P} - \frac{1}{PkR} - PkRS,$$

from which there is deduced:

$$0 = \omega kRS - S - 1 \text{ or } 0 = kRS\left(1+Pk\right)M + S + 1,$$

where therefore our two cases will be required to be distinguished.

I. If $S = -k'$, there will be had $0 = kk'R(1+Pk)M - k' + 1$, from which there becomes

$$R = \frac{1-k'}{kk'(1+Pk)M},$$

from which it is apparent to be $k' < 1$, from which, if there may be produced $R > 1$, there must become $C > 0$ and $D < 0$. But if there may be produced $R < 1$, there must become $D > 0$, $C < 0$, $\mathfrak{D} > 0$ and $\mathfrak{D} < 1$ and thus $Pk(1-R)M < 1$.

II. If $R = -k'$, there will become $0 = -kk'S(1+Pk)M + S + 1$, from which there is deduced

$$k' = \frac{S+1}{kS(1+Pk)M},$$

which expression itself is positive. But in this case we have seen above there must be $C > 0$, and thus both $\mathfrak{C} > 0$, $\mathfrak{C} < 1$, and $D < 0$, thus so that in this case it shall be required to be accepted that $S < 1$.

Here finally it will be required to be remembered that $PkRS = -m$, which condition must be considered according to the two cases.

In the first case, where $S = -k'$, on account of $R = \frac{m}{Pkk'}$ our equation gives

$$0 = -\tfrac{m}{P}(1+Pk)M - k' + 1,$$

from which it is gathered:

$$k' = 1 - \tfrac{m}{P}(1+Pk)M,$$

thus so that there must become $m(1+Pk)M < P$; where it may be observed, if there may be produced $R > 1$, there must become $C > 0$ and $D < 0$, but if there may arise $R < 1$, there must become $C < 0$ and $D > 0$, $\mathfrak{D} > 0$ and $\mathfrak{D} < 1$.

For the other case, if $R = -k'$, so that there shall become $m = Pkk'S$, our equation gives

$$0 = -\tfrac{m}{P}(1+Pk)M + S + 1,$$

from which it is deduced:

$$S = \tfrac{m}{P}(1+Pk)M - 1,$$

thus so that there must become $m(1+Pk)M > P$. But since there must be $S < 1$, there also must become $m(1+Pk)M < 2P$; besides we may recall there must be $C>0$ and thus $\mathfrak{C} > 0$, $\mathfrak{C} < 1$, and $D < 0$.

Finally, concerning these formulas, it is required to be observed properly on account of the value ω found, the letter M can be conveniently expressed by the remaining terms. Indeed since there shall be $\omega = -(1+Pk)M$, the equation

$$\frac{\omega+2}{m-1}=M$$

will give

$$M=\frac{2}{m+Pk}$$

and

$$\omega=\frac{-2(1+Pk)}{m+Pk},$$

thus so that for the apparent field there may be produced

$$\Phi=\frac{2}{m+Pk}\cdot\xi \text{ or } \Phi=\frac{1718}{m+Pk}\text{ min.}$$

Then truly also for the place of the eye

$$O=\frac{e(m+Pk)}{2m}.$$

From which observations we may establish the two cases separately.

I. ESTABLISHING THE CASE WHERE $S=-k'$

355. Therefore in this case our elements themselves will be had thus :

$$b=-\frac{\alpha}{P},\ c=\frac{-B\alpha}{Pk},\ d=\frac{BC\alpha}{PkR},\ e=\frac{BCD\alpha}{m},$$

$$\beta=\frac{-B\alpha}{P},\ \gamma=\frac{-BC\alpha}{Pk},\ \delta=\frac{BCD\alpha}{PkR}$$

and hence the intervals

$$\alpha+b=\alpha\left(1-\frac{1}{P}\right),\ \beta+c=\frac{-B\alpha}{P}\left(1+\frac{1}{k}\right),$$

$$\gamma+d=-\frac{BC\alpha}{Pk}\left(1-\frac{1}{R}\right),\ \delta+e=\frac{BCD\alpha}{PkR}\left(1+\frac{1}{k'}\right),$$

$$\beta=\frac{-B\alpha}{P},\ \gamma=\frac{-BC\alpha}{Pk},\ \delta=\frac{BCD\alpha}{PkR}$$

where therefore there must become:

$$P>1 \text{ and } \mathfrak{B}=\frac{-(P-1)}{1+Pk}, \text{ and hence } B=\frac{-(P-1)}{P(1+k)}.$$

Truly the third interval gives this condition $C\left(1-\frac{1}{R}\right)>0$ and finally $CD<0$; moreover there is :

$$\mathfrak{D}=Pk(1-R)M=\frac{2Pk(1-R)}{m+Pk} \quad\text{and}\quad D=\frac{2Pk(1-R)}{m-Pk+2PkR}.$$

But the destruction of the colored margin demands, that there shall be

$$k' = 1 - \tfrac{m}{P}\left(1 + Pk\right)M = 1 - \frac{2m(1+Pk)}{P(m+Pk)}$$

and

$$R = \frac{m(m+Pk)}{k\left(P(m+Pk) - 2m(1+Pk)\right)};$$

on account of which there must become

$$P\left(m + Pk\right) > 2m(1 + Pk)$$

and thus

$$k < \frac{m(P-2)}{P(2m-P)};$$

whereby, since this quantity shall be greater than *k,* on account of $2m > P$ there must be $P > 2$, from which condition it is evident also there must be $R > 1$, $C > 0$ and $D < 0$, as is evident from the value of D. From which we may put $Pk = \sqrt{m}$ in place, which with these conditions may satisfy the formulas and which may emerge simpler, so that there may become

$$M = \frac{2}{m+\sqrt{m}}$$

and thus

$$\Phi = \frac{2}{m+\sqrt{m}} \cdot \xi = \frac{1718}{m+\sqrt{m}} \text{ min.},$$

which value is twice as great as before [§ 314]. Then truly there will be

$$\omega = \frac{-2\left(1+\sqrt{m}\right)}{m+\sqrt{m}};$$

again if there may be put $P = 4\sqrt{m}$, there is produced $k = \frac{1}{4}$, $R = 2\sqrt{m}$ and $k' = \frac{1}{2}$, and hence

$$\mathfrak{D} = \frac{2\left(1-2\sqrt{m}\right)}{1+\sqrt{m}} \text{ and } D = \frac{2\left(1-2\sqrt{m}\right)}{5\sqrt{m}-1}.$$

Besides truly,

$$\mathfrak{B} = -\frac{\left(4\sqrt{m}-1\right)}{1+\sqrt{m}} \text{ and } B = -\frac{\left(4\sqrt{m}-1\right)}{5\sqrt{m}},$$

from which all the intervals will be produced positive, provided a positive quantity may be taken for C .

II. ESTABLISHING THE CASE WHERE $R = -k'$

356. Therefore for this case, the removal of the colored margin provides the equation:

$$0 = -\frac{2m(1+Pk)}{P(m+Pk)} + S + 1,$$

from which it is concluded

$$S = \frac{2m(1+Pk)}{P(m+Pk)} - 1,$$

thus, so that there must become

$$2m(1+Pk) > P(m+Pk);$$

then truly, on account of $S < 1$, there must become

$$2m(1+Pk) < 2P(m+Pk).$$

Now we may assume as before $Pk = \sqrt{m}$ and there will become $S = \frac{2\sqrt{m}}{P} - 1$, thus so that now there must be taken $P < 2\sqrt{m}$ and $P > \sqrt{m}$; moreover the letter *k* lies between the limits 1 and $\frac{1}{2}$.

Then truly, on account of $S = \frac{2\sqrt{m}}{P} - 1$, there will become

$$k' = \frac{m}{S\sqrt{m}} = \frac{P\sqrt{m}}{2\sqrt{m}-P}.$$

Moreover, from the definition *P* there will become :

$$\mathfrak{B} = -\frac{(P-1)}{1+\sqrt{m}} \quad \text{and} \quad B = -\frac{(P-1)}{P+\sqrt{m}}$$

and

$$\mathfrak{D} = \frac{2(1+k')}{1+\sqrt{m}} \quad \text{and} \quad D = \frac{2(1+k')}{\sqrt{m}+2k'-1}$$

or with

$$\mathfrak{D} = \frac{2\left(2\sqrt{m}-P+P\sqrt{m}\right)}{\left(1+\sqrt{m}\right)\left(2\sqrt{m}-P\right)};$$

which value since it shall be positive and greater than unity, the letter *D* at once becomes negative, just as the conditions demand, provided *C* may be taken positive. But where all the conditions may be determined fully, we may put in addition $P=\frac{3}{2}\sqrt{m}$ and there will become

$$k=\tfrac{2}{3},\quad k'=3\sqrt{m}\quad\text{and}\quad S=\tfrac{1}{3},$$

$$\mathfrak{B}=-\frac{(3\sqrt{m}-2)}{2(1+\sqrt{m})}\quad\text{and}\quad B=-\frac{(3\sqrt{m}-2)}{5\sqrt{m}},$$

$$\mathfrak{D}=\frac{2(3\sqrt{m}+1)}{\sqrt{m}+1}\quad\text{and}\quad D=-\frac{2(3\sqrt{m}+1)}{5\sqrt{m}+1},$$

with which values, it is satisfied by all the conditions.

SCHOLIUM

357. I. Behold therefore two cases of telescopes of this kind completely determined by the given magnification *m,* the effect of which in practice must be the same. But since in the latter case the length of the instrument may emerge smaller than the former, we will prefer from that merit; on account of which it will be worth the effort to inquire into the construction of these telescopes with more. Therefore with the values of the particular letters noted, evidently

$$P=\tfrac{3}{2}\sqrt{m},\quad k=\tfrac{2}{3},\quad k'=3\sqrt{m},\quad S=\tfrac{1}{3},$$

$$\mathfrak{B}=-\frac{(3\sqrt{m}-2)}{2(1+\sqrt{m})},\qquad B=-\frac{(3\sqrt{m}-2)}{5\sqrt{m}},$$

$$\mathfrak{D}=\frac{2(3\sqrt{m}+1)}{\sqrt{m}+1},\qquad D=-\frac{2(3\sqrt{m}+1)}{5\sqrt{m}+1},$$

and since C must be positive, there may be put

$$C=\theta,\quad\text{so that there shall become}\quad \mathfrak{C}=\frac{\theta}{1+\theta},$$

and our elements will be expressed thus :

$$b=-\frac{2\alpha}{3\sqrt{m}},\quad \beta=\frac{2(3\sqrt{m}-2)}{15m}\cdot\alpha,\quad c=\frac{3\sqrt{m}-2}{5m}\cdot\alpha,\quad \gamma=\frac{\theta(3\sqrt{m}-2)}{5m}\cdot\alpha,$$

$$d=\frac{\theta(3\sqrt{m}-2)}{15m\sqrt{m}}\cdot\alpha,\quad \delta=\frac{-2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{15(5\sqrt{m}+1)m\sqrt{m}}\cdot\alpha,\quad e=\frac{2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{5(5\sqrt{m}+1)m\sqrt{m}}\cdot\alpha;$$

hence the focal lengths:

$$p=\alpha,\quad q=\frac{3\sqrt{m}-2}{3\left(1+\sqrt{m}\right)\sqrt{m}}\cdot\alpha,\quad r=\frac{\theta}{1+\theta}\cdot\frac{3\sqrt{m}-2}{5m}\cdot\alpha,$$

$$s=\frac{2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15m\left(\sqrt{m}+1\right)\sqrt{m}}\cdot\alpha,\quad \text{and}\quad t=\frac{2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{5\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha,$$

and the separations of the lenses:

$$\alpha+b=\alpha\left(1-\frac{2}{3\sqrt{m}}\right),\quad \beta+c=\frac{3\sqrt{m}-2}{3m}\cdot\alpha,$$

$$\gamma+d=\frac{\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15m\sqrt{m}}\cdot\alpha,\quad \delta+e=\frac{4\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha,$$

and the distance of the eye:

$$O=\frac{e\left(1+\sqrt{m}\right)}{2\sqrt{m}}=\frac{\theta\left(1+\sqrt{m}\right)\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{5m^2\left(5\sqrt{m}+1\right)}\cdot\alpha;$$

from which the total length of the telescope arises

$$=\left(\frac{\left(3\sqrt{m}-2\right)\left(1+\sqrt{m}\right)}{3m}+\frac{\theta\left(\sqrt{m}+1\right)\left(3\sqrt{m}+1\right)\left(3\sqrt{m}-2\right)\left(5\sqrt{m}+3\right)}{15m^2\left(5\sqrt{m}+1\right)}\right)\alpha,$$

thus so that, if m may be a very large number, this length becomes

$$\left(1+\frac{1}{3\sqrt{m}}+\frac{3\theta}{5\sqrt{m}}\right)\alpha,$$

and since in this case here becomes $e=\frac{18\theta\alpha}{25m}$, if it may be allowed to take $\alpha=\frac{m}{7}$ in., it will be convenient to put in put in place $\theta=5$, so that the focal length of the final lens will become approximately $\frac{1}{2}$ in.; but when α maintains a much greater value, it will be easy to take $\theta=1$.

II. Also we must inquire more carefully, how great an aperture may be needed to be attributed to each lens, and certainly for the first lens always it is customary to assume the radius of the aperture to be $x=\frac{m}{50}$ in.; for the remaining lenses, the radius is deduced from the formulas established above:

The radius of the second aperture

$$=\pi q\pm\frac{qx}{\mathfrak{B}p}=\frac{1}{4}\omega q+\frac{qx}{\mathfrak{B}\alpha}=\frac{1}{4}q\left(\frac{2}{\sqrt{m}}+\frac{8\left(1+\sqrt{m}\right)}{3\sqrt{m}-2}\cdot\frac{x}{\alpha}\right),$$

the radius of the third aperture

$$=\frac{rx}{B\mathfrak{C}p}=\frac{x}{\sqrt{m}}=\frac{\sqrt{m}}{50}\text{ in.}$$

But the fourth and fifth lens must take the maximum aperture ; from which these will be required to be made convex on both sides.

III. So that it may pertain now to the letters λ, for the first lens it is always convenient to take $\lambda=1$, which value also may be seen able to be taken for the second lens also, if indeed the number *m* may not be exceedingly small, but where in any case it will be required to be considered separately. Indeed for the third lens there is no doubt, on account of the minimum aperture, why it may not be considered to take $\lambda''=1$. Truly since the fourth lens must be with both sides equally convex, for that there must be taken

$$\lambda'''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\left(1-2\mathfrak{D}\right)^2=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\left(\frac{11\sqrt{m}+3}{\sqrt{m}+1}\right)^2.$$

Moreover, for the fifth lens there will be $\lambda''''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2$.

IV. Therefore with these values for $\lambda,\ \lambda',\ldots$ established, the quantity α must be defined from the following formula :

$$\alpha=kx\sqrt[3]{\mu m}\left\{\begin{array}{l}\lambda-\frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{v}{B}\right)-\frac{1}{B^3\mathfrak{C}Pk}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{v}{C}\right)\\ -\frac{1}{B^3C^3\mathfrak{D}Pkk'}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{v}{D}\right)+\frac{\lambda''''}{B^3C^3D^3m}\end{array}\right\}$$

where it will help to remember to be accustomed to take $x=\frac{m}{50}$, and $k=50$, so that there shall be $kx=m$. Yet meanwhile, of we wish for either a clearer or more distinct order to be held, $\frac{1}{2}m$ will be able to be taken for kx. Accordingly it is hence evident on account of that huge value of λ''', which evidently will include $\left(2\mathfrak{D}-1\right)^2$, hence this term arising again will be able to become small enough, since it shall be divided by $\mathfrak{D}^3$ except that its denominator on account of $Pkk'=3m$ by itself may be large enough. Finally at this point it must be observed the number λ'' to be multiplied by a notably large enough number, since $-\frac{1}{B^3}$ to be almost $\frac{125}{27}$, and $\frac{1}{\mathfrak{C}^3}>1$, and thus $-\frac{1}{B^3\mathfrak{C}^3}$ may be greater than 5 and thus as far as to 40, if there may be taken $\theta = 1$, thus in order that this term $Pk = \sqrt{m}$ in the denominator may be able to be diminished to become scarcely greater than one. A remedy may be introduced for this inconvenience by doubling this lens following the precepts in Book I with regard to treating composite lenses. But this will not be necessary, when thus we will duplicate the objective lens itself, so that the confusion may be removed arising from all the remaining lenses.

EXAMPLE

358. With $m = 25$ assumed, to describe the construction of a telescope of this kind.

I. Since there shall be $m = 25$, there will be $\sqrt{m} = 5$ and thence

$$P = \tfrac{15}{2}, \quad k = \tfrac{2}{3}, \quad k' = 15, \quad S = \tfrac{1}{3},$$
$$\mathfrak{B} = \tfrac{13}{12}, \quad B = -\tfrac{13}{25}, \quad \mathfrak{D} = \tfrac{16}{3}, \quad D = -\tfrac{16}{3};$$

from which our elements will be

$$b = -\tfrac{2\alpha}{15}, \quad \beta = \tfrac{26\alpha}{375}, \quad c = \tfrac{13\alpha}{125}, \quad \gamma = \tfrac{13\theta\alpha}{125},$$

$$d = \tfrac{13\theta\alpha}{1875}, \quad \delta = -\tfrac{16\theta}{1875}\alpha, \quad e = \tfrac{16\theta\alpha}{625}$$

and the focal lengths

$$p = \alpha, \quad q = \tfrac{13}{90}\alpha, \quad r = \tfrac{\theta}{1+\theta}\cdot\tfrac{13}{125}\alpha, \quad s = \tfrac{208}{5625}\alpha \text{ and } t = \tfrac{16\theta}{625}\alpha$$

and the separations of the lenses

$$\alpha + b = \tfrac{13}{15}\alpha, \quad \beta + c = \tfrac{13}{75}\alpha, \quad \gamma + d = \tfrac{208\theta}{1875}\alpha, \quad \delta + e = \tfrac{32\theta}{1875}\alpha$$

and the distance of the eye

$$O = \tfrac{48\theta}{3125}\alpha,$$

thus so that the total length shall be going to become $\alpha\left(\tfrac{26}{25} + \tfrac{448\theta}{3125}\right)$. Moreover the apparent field of view will be

$$\tfrac{1718}{30}\,\text{min.} = 57'\ 16''.$$

II. The radius of the aperture of the first lens $=\frac{1}{2}$ in.

The radius of the aperture of the second lens $=\frac{1}{4}q\left(\frac{2}{5}+\frac{48}{13}\cdot\frac{x}{\alpha}\right)$, from which it may be able to be deduced half the aperture to suffice for this lens.

The radius of the aperture of the third lens $=\frac{1}{10}$ in.

III. Thence again there will become $\lambda=1,\;\lambda'=1$ perhaps, $\lambda''=1,\;\lambda'''=1+\frac{841}{9}\left(\frac{\sigma-\rho}{2\tau}\right)^2$, were it is required to be observed, if common glass may be used, for which $n=1,55$, to become

$$\lambda''=1+0,6299\cdot\frac{841}{9}=59,861 \text{ et } \lambda'''=1,6299.$$

From the equation for α it will be able to be observed the number held under the sign of the root to increase to a little more than 2 μm, from which we are able to write 64 in its place without harm , and thus we will obtain $\alpha=100$ in. $=8\frac{1}{3}$ ft.

But for greater magnifications this quantity will increase in the ratio $m\sqrt[3]{m}$ nor will the great length be able to be diminished, unless we may be able to reduce the formula for the radius of the confusion to zero, since that, as will be apparent from the above, will be easily established, if at this stage we may put in front a concave lens being prepared either from the same glass or from crystal glass.

PROBLEM 2

359. *This kind of telescope thus is to be completed by placing a concave lens before the first lens, so that the confusion may be removed completely and thus the telescopes may be rendered the shortest with the field preserved found before.*

SOLUTION

Therefore now since we shall have six lenses, the five letters *P, Q, R, S, T* will be required to be considered related to the intervals between the lenses, the first of which *P* ought to give the minimum interval, which we may establish to become $=-\frac{1}{50}\alpha$ on account of negative α, so that there may become $P=\frac{50}{51}$. Then since the following letters may correspond to the letters *Q, R, S, T,* which before were *P, Q, R, S,* now we may put $R=-k$ and $S=-k'$ and the elements will become

$$b=-\tfrac{\alpha}{P},\quad c=\tfrac{B\alpha}{PQ},\quad d=\tfrac{BC\alpha}{PQk},\quad e=\tfrac{BCD\alpha}{PQkk'}$$

and

$$f=\tfrac{-BCDE\alpha}{PQkk'T}=\tfrac{-BCDE\alpha}{m}$$

$$\beta=-\tfrac{B\alpha}{P},\quad \gamma=\tfrac{BC\alpha}{PQ},\quad \delta=\tfrac{BCD\alpha}{PQk},\quad \varepsilon=\tfrac{BCDE\alpha}{PQkk'},$$

from which the intervals are deduced

1. $\alpha+b=\alpha\left(1-\tfrac{1}{P}\right)$; which happens on assuming $P=\tfrac{50}{51}$.
2. $\alpha+c=-\tfrac{B\alpha}{P}\left(1-\tfrac{1}{Q}\right)$; from which, since Q must be taken >1, B must be positive and thus $\mathfrak{B}>0$ and <1.
3. $\gamma+d=\tfrac{BC\alpha}{PQ}\left(1+\tfrac{1}{k}\right)$; from which C must be negative.
4. $\delta+e=\tfrac{BCD\alpha}{PQk}\left(1+\tfrac{1}{k'}\right)$; from which D must be positive and thus $\mathfrak{D}>0$ and $\mathfrak{D}<1$.
5. $\varepsilon+f=\tfrac{BCDE\alpha}{PQkk'}\left(1-\tfrac{1}{T}\right)$; from which $E\left(1-\tfrac{1}{T}\right)$ must be positive, but since f must be greater than zero, E must be negative, and therefore $T<1$.

Now for the apparent field of view we may put

$$\pi=-\nu\xi,\quad \pi'=\omega\xi,\quad \pi''=0,\quad \pi'''=\xi \text{ and } \pi''''=-\xi,$$

so that there may become

$$\Phi=\tfrac{\nu+\omega+2}{m-1}\cdot\xi=M\xi$$

with there being

$$M=\tfrac{\nu+\omega+2}{m-1};$$

from which there becomes for the position of the eye

$$O=\tfrac{f}{Mm}.$$

Moreover from these, the following fundamental equations may be formed :

1. $\mathfrak{B}\nu=-(1-P)M$
2. $\mathfrak{C}\omega=-(1-PQ)M-\nu$
3. $\mathfrak{D}\cdot 0=-(1+PQk)M-\nu-\omega$
4. $\mathfrak{E}=-(1-PQkk')M-\nu-\omega.$

From the third of which we have at once

$$v+\omega=-\left(1+PQk\right)M;$$

truly there is also

$$v+\omega=\left(m-1\right)M-2;$$

from which

$$M=\frac{2}{m+PQk}$$

and thus in turn

$$v+\omega=\frac{-2\left(1+PQk\right)}{m+PQk}$$

Now since the first equation gives

$$v=\frac{-2\left(1-P\right)}{\mathfrak{B}\left(m+PQk\right)},$$

the second provides

$$\mathfrak{C}\omega=\frac{-2\left(1-PQ\right)}{m+PQk}+\frac{2\left(1-P\right)}{\mathfrak{B}\left(m+PQk\right)};$$

whereby now there may become

$$v+\omega=\frac{2\left(1-P\right)}{\mathfrak{B}C\left(m+PQk\right)}-\frac{2\left(1-PQ\right)}{\mathfrak{C}\left(m+PQk\right)}=\frac{-2\left(1+PQk\right)}{m+PQk},$$

which equation reduced will give

$$(1-\mathfrak{B})(1-\mathfrak{C})-(1-\mathfrak{C})P+\mathfrak{B}PQ+\mathfrak{B}\mathfrak{C}PQk=0,$$

which is reduced to this form:

$$\frac{1-P}{BC}-\frac{P(1-Q)}{C}+PQ\left(1+k\right)=0,$$

which equation takes care of the relation requiring to be defined between the letters *B* and *C*. But the letter *D* remains left to our choice, provided it may be taken positive. Finally truly the fourth equation gives

$$\mathfrak{E}=-\frac{2(1-PQkk')}{m+PQk}+\frac{2(1+PQk)}{m+PQk}=\frac{2PQk\left(1+k'\right)}{m+PQk},$$

which value since it shall be positive, must become

$$2PQk(1+k')>m+PQk \text{ or } PQk(1+2k')>m.$$

Finally the removal of the colored margin demands this equation:

$$0=\frac{v}{P}+\frac{\omega}{PQ}-\frac{0}{PQk}+\frac{1}{PQkk'}+\frac{1}{PQkk'T'},$$

which with the values substituted for v and ω will be changed into this :

$$0=\frac{-2(1-P)}{\mathfrak{B}(m+PQk)}-\frac{2(1-PQ)}{\mathfrak{C}(m+PQk)Q}+\frac{2(1-P)}{\mathfrak{BC}(m+PQk)Q}+\frac{1}{Qkk'}+\frac{1}{Qkk'T}$$

or

$$0=\frac{2}{Q(m+PQk)}\left(\frac{(1-P)(1-Q)}{\mathfrak{B}}-1-PQk\right)+\frac{1}{Qkk'}+\frac{1}{Qkk'T}.$$

So that we may satisfy this equation most conveniently, in the first place we may reject the terms affected by the factor $(1-P)$ on account of the small sum, since there is no need that we may aim for greatest rigor in this resolution, and we will have

$$\frac{2(1+PQk)}{m+PQk}=\frac{1}{kk'}\left(1+\frac{1}{T}\right),$$

where at once according to the nature of telescopes of this kind established above we may put in place $PQk=\sqrt{m}$ and $T=\frac{1}{2}$; from which there will become $\frac{2}{\sqrt{m}}=\frac{3}{kk'}$, hence $kk'=\frac{3\sqrt{m}}{2}$. Now since there will become $kk'T=\frac{3\sqrt{m}}{4}=\frac{m}{PQ}$, so that thus there shall be $PQ=\frac{4}{3}\sqrt{m}$, on account of P being given, Q also will be defined. Again since there is $PQk=\sqrt{m}$, there will be $k=\frac{3}{4}$ and hence $k'=2\sqrt{m}$ and thus the values of these letters will themselves be had thus:

$$P=\frac{50}{51},\quad PQ=\frac{4}{3}\sqrt{m},\quad k=\frac{3}{4},\quad k'=2\sqrt{m}\ \text{ and }\ T=\frac{1}{2}$$

and hence

$$PQk=\sqrt{m},\quad PQkk'=2m\quad\text{and}\quad PQkk'T=m.$$

So that now it may be extended to the remaining letters B, C ..., the equation given above will give, if also the $1-P$ may be removed:

$$\frac{-1+PQ}{C}+PQ(1+k)=0,$$

from which there is found:

$$C=\frac{1-PQ}{PQ(1+k)}=\frac{3-4\sqrt{m}}{7\sqrt{m}}\quad\text{and}\quad\mathfrak{C}=\frac{3-4\sqrt{m}}{3(1+\sqrt{m})}.$$

But the letters B and $\mathfrak{B}$ are permitted according to our choice, thus so that, if the first concave lens may be prepared from crystal glass, as we have seen above [§ 342], it may be agreed to put $\mathfrak{B}=\frac{5}{7}$; again truly the letters $\mathfrak{D}$ and D hence clearly cannot be determined, unless each shall be required to be positive, from which we may establish $D=\theta$ and hence $\mathfrak{D}=\frac{\theta}{1+\theta}$; finally there will be

$$\mathfrak{E}=\frac{2(1+2\sqrt{m})}{1+\sqrt{m}} \text{ and hence } E=\frac{-2(1+2\sqrt{m})}{1+3\sqrt{m}};$$

which values may be shown to be viewed together:

$$\mathfrak{B}=\frac{5}{7},\quad \mathfrak{C}=\frac{3-4\sqrt{m}}{3(1+\sqrt{m})},\quad \mathfrak{D}=\frac{\theta}{1+\theta} \text{ and } \mathfrak{E}=\frac{2(1+2\sqrt{m})}{1+\sqrt{m}},$$

$$B=\frac{5}{2},\quad C=\frac{3-4\sqrt{m}}{7\sqrt{m}},\quad D=\mathfrak{D} \text{ and } E=\frac{-2(1+2\sqrt{m})}{1+3\sqrt{m}},$$

and hence

$$BC=\frac{5(3-4\sqrt{m})}{14\sqrt{m}},\quad BCD=\frac{5\theta(3-4\sqrt{m})}{14\sqrt{m}},\quad BCDE=\frac{5\theta(4\sqrt{m}-3)(1+2\sqrt{m})}{7\sqrt{m}(1+3\sqrt{m})};$$

$$\mathfrak{E}=\frac{2(1+2\sqrt{m})}{1+\sqrt{m}} \text{ and hence } E=\frac{-2(1+2\sqrt{m})}{1+3\sqrt{m}};$$

from which our elements are determined completely.

Therefore nothing other remains, except that the radius of the confusion may be reduced to zero, which is done by the following equation:

$$\lambda=\frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)-\frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)-\frac{1}{B^3C^3PQk}\left(\frac{\lambda'''}{\mathfrak{D}^3}+\frac{\nu}{D\mathfrak{D}}\right)$$

$$-\frac{1}{B^3C^3D^3PQkk'}\left(\frac{\lambda''''}{\mathfrak{E}^3}+\frac{\nu}{E\mathfrak{E}}\right)+\frac{\lambda'''''}{B^3C^3D^3E^3m},$$

evidently if all the lenses shall be made from the same kind of glass. But if the first lens shall be crystal glass, truly the rest crown glass, the value of λ hence found must be multiplied by $\frac{9875}{8724}\left[=\frac{\mu'}{\mu}\right]$, which fraction is almost $\frac{17}{15}$ or rather truly closer $\frac{163}{144}$.

Truly with regard to this equation it is required to observe there must be taken $\lambda'=1,\ \lambda''=1,\ \lambda'''=1$. But for the fifth lens, so that each side may become equally convex, there must be taken

$$\lambda'''' = 1 + 0,60006(1-2\mathfrak{C})^2 = 1 + \frac{0,60006(3+7\sqrt{m})^2}{(1+\sqrt{m})^2}.$$

Truly for the sixth $\lambda''''' = 1,60006.$

COROLLARY 1

360. Therefore for these telescopes since there shall become $M = \frac{2}{m+\sqrt{m}}$, the radius of the apparent field of view $\Phi = \frac{1718}{m+\sqrt{m}}$ min.

COROLLARY 2

361. Moreover the radii of the apertures of the individual lenses thus will be defined from § 23:

For the first $= x,$
for the second $= \frac{x}{P},$
for the third $= \frac{r}{2\sqrt{m}} \pm \frac{x}{PQ},$
for the fourth $= 0s \pm \frac{x}{PQk},$
for the fifth $= \frac{t}{4} \pm \frac{x}{PQkk'},$
for the sixth $= \frac{u}{4} \pm \frac{x}{PQkk'T} = \frac{u}{4} \pm \frac{x}{m}.$

COROLLARY 3

362. If in place of real images we may wish to put in place diaphragms, there is found [§224-227]:
For the radius of the first aperture $= \frac{2BC}{m+\sqrt{m}} \cdot \frac{\alpha}{4}.$
For the second $= \frac{2BCD}{m+\sqrt{m}} \cdot \frac{\alpha}{4}.$

SCHOLIUM

363. Behold therefore the twofold perfection of this kind of telescopes ; on the one hand clearly it may be seen for the apparent field, that we have rendered almost twice as great ; the other truly consists in the destruction of the confusion, from which it is effected, that there shall be no need for the quantity α to be taken greater, than the aperture of the objective lens demands for the required clarity, and thus the length of the telescope may be shortened so much, certainly as much as it may be allowed to be done. Since here two lenses shall be found after the final image, by which the field of view has

been made twice as great, thus, if we wish to use three or more lenses, the field will be able to be enlarged as far as we will have wished. Which since it scarcely demands a greater calculation than the preceding problem, it will certainly be worth the effort to extend this investigation generally for any number of lenses.

PROBLEM 3

364. *Thus so that the apparent field of view may be increased as much as it may be desired, to be set out as before with a concave lens put in place before several lenses put in place after the final real image.*

SOLUTION

Here just as everything remains as in the preceding problem, which clearly extends to the focal lengths and to the distances between the lenses, only with this distinction, that both the series of letters *B, C, D* etc. and *P, Q, k, k', T* etc. must be continued further. Then the letter *M,* from which the apparent field is defined, will obtain another value from the number of lenses required to be inserted after the final image. Therefore the number of lenses shall be $=i$ and

$$M=\frac{v+\omega+i}{m-1}.$$

Then truly the fundamental equations will themselves be had as before, except that they may progress further; moreover after the third we may define any with the help of the third, as follows:

$$1.\ \mathfrak{B}v=-(1-P)M$$

$$2.\ \mathfrak{C}\omega=-(1-PQ)M-v$$

$$3.\ 0=-(1+PQk)M-v-\omega \text{ or } v+\omega=-(1+PQk)M,$$

from which

$$M(m-1)=-(1+PQk)M+i$$

and

$$M=\frac{i}{m+PQk},$$

$$4.\ \mathfrak{E}=PQk(1+k')M$$

$$5.\ \mathfrak{F}=PQk(1+k'T)M-1$$

$$6.\ \mathfrak{G}=PQk(1+k'TU)M-2$$

$$7.\ \mathfrak{H}=PQk(1+k'TUV)M-3$$

etc.

But from the first formulas there is deduced as before :

$$\frac{1-P}{BC}-\frac{P(1-Q)}{C}+PQ(1+k)=0,$$

from which, since $P=1$ approx. and thus v can be taken as zero, will be exact enough

$$\omega = -\,(1+PQk)M = -\frac{(1-PQ)M}{\mathfrak{C}},$$

from which we gather

$$\mathfrak{C} = \frac{1-PQ}{1+PQk} \text{ and } C = \frac{1-PQ}{PQ(1+k)}.$$

But here it suffices for this value to be defined approximately, since the aperture of the lens, on which the letters *v,* ω etc. will depend, rejects a precise sum. With which also it may prevail in the equation, where with the colored fringe removed, it will be obtained with the value substituted in place of M :

$$\frac{i(1+PQk)}{m+PQk} = \frac{1}{kk'}\left(1+\frac{1}{T}+\frac{1}{TU}+\frac{1}{TUV} \text{ etc.}\right);$$

since the number of terms of which shall be *i* and the individual letters *T, U, V* etc. to be smaller than unity, we may put in place the following both for the sake of neatness, as well as so that the latter lenses may be almost equally distant from each other :

$$T=\tfrac{1}{2},\quad U=\tfrac{2}{3},\quad V=\tfrac{3}{4},\quad \mathrm{W}=\tfrac{4}{5} \text{ etc.},$$

so that the multiplier of $\frac{1}{kk'}$ may become

$$1+2+3+4\cdots+i = \frac{(1+i)i}{2};$$

then also as before we may put $PQk=\sqrt{m}$, so that this equation may be produced :

$$\frac{i}{\sqrt{m}} = \frac{1}{kk'}\cdot\frac{i(1+i)}{2},$$

from which there is elicited

$$kk' = \frac{(1+i)\sqrt{m}}{2}.$$

Truly the product of the remaining letters

$$TUV\cdots = \frac{1}{i};$$

there will become

$$kk'TUV\cdots = \frac{(1+i)\sqrt{m}}{2i} = \frac{m}{PQ}.$$

and hence therefore there is deduced

$$PQ = \frac{2i\sqrt{m}}{1+i},$$

and since P itself is given, hence Q will be defined. Finally on account of $PQk=\sqrt{m}$ there is elicited

$$k=\tfrac{(1+i)}{2i} \quad \text{and} \quad k'=i\sqrt{m}.$$

Hence therefore all the values will be had in the following manner :

$$PQ=\tfrac{2i\sqrt{m}}{1+i},\quad k=\tfrac{1+i}{2i},\quad k'=i\sqrt{m},$$

$$T=\tfrac{1}{2},\quad U=\tfrac{2}{3},\quad V=\tfrac{3}{4},\quad W=\tfrac{4}{5}\ \text{etc.},$$

$$PQk=\sqrt{m},\quad PQkk'=im,\quad PQkk'T=\tfrac{im}{2},$$

$$PQkk'TU=\tfrac{im}{3}\quad \text{and}\quad PQkk'TUV\ldots=\tfrac{im}{i}=m.$$

Concerning the letters B, C, D etc. the first B with the third D hence is not defined; now truly we have shown to be

$$C=\tfrac{1-PQ}{PQ(1+k)}=\tfrac{1+i-2i\sqrt{m}}{(1+3i)\sqrt{m}}$$

and

$$\mathfrak{C}=\tfrac{1-PQ}{1+kPQ}=\tfrac{1+i-2i\sqrt{m}}{(1+i)(1+\sqrt{m})}$$

Therefore we may put as before

$$D=\theta \quad \text{and} \quad \mathfrak{D}=\tfrac{\theta}{1+\theta};$$

truly the following will become

$$\mathfrak{E}=\frac{i(1+i\sqrt{m})}{1+\sqrt{m}},$$
$$\mathfrak{F}=\frac{i(2+i\sqrt{m})}{2(1+\sqrt{m})}-1,$$
$$\mathfrak{G}=\frac{i(3+i\sqrt{m})}{3(1+\sqrt{m})}-2,$$
$$\mathfrak{H}=\frac{i(4+i\sqrt{m})}{4(1+\sqrt{m})}-3,$$

of which the penultimate of the letters will be

$$\frac{2(i-1)+(3i-2)\sqrt{m}}{(i-1)(1+\sqrt{m})}$$

and the final $=1$.

Therefore we may express these letters here to be viewed together:

$$\mathfrak{B}=\tfrac{5}{7}\ \text{approx.}\qquad B=\tfrac{5}{2}\ \text{approx.}$$

$$\mathfrak{C}=\frac{\left(2i\sqrt{m}-i-1\right)}{(1+i)\left(1+\sqrt{m}\right)}\qquad C=\frac{-\left(2i\sqrt{m}-i-1\right)}{(1+3i)\sqrt{m}}$$

$$\mathfrak{D}=\frac{\theta}{1+\theta}\qquad D=\theta$$

$$\mathfrak{E}=\frac{i+ii\sqrt{m}}{1+\sqrt{m}}\qquad E=\frac{-\left(i+ii\sqrt{m}\right)}{(i-1)\left(1+(i+1)\sqrt{m}\right)}$$

$$\mathfrak{F}=\frac{2(i-1)+(ii-1\cdot 2)\sqrt{m}}{2\left(1+\sqrt{m}\right)},\qquad F=\frac{-\left(2(i-1)+(ii-1\cdot 2)\sqrt{m}\right)}{(i-2)\left(2+(i+2)\sqrt{m}\right)}$$

$$\mathfrak{G}=\frac{3(i-1)+(ii-2\cdot 3)\sqrt{m}}{3\left(1+\sqrt{m}\right)},\qquad G=\frac{-\left(3(i-1)+(ii-2\cdot 3)\sqrt{m}\right)}{(i-3)\left(3+(i+3)\sqrt{m}\right)}$$

$$\mathfrak{H}=\frac{4(i-3)+(ii-3\cdot 4)\sqrt{m}}{4\left(1+\sqrt{m}\right)},\qquad G=\frac{-\left(4(i-1)+(ii-3\cdot 4)\sqrt{m}\right)}{(i-4)\left(4+(i+4)\sqrt{m}\right)}$$

etc.,

from which values all the elements according to the well-known formulas can be defined. Then truly so that all the confusion may be removed, this equation will be required to be fulfilled:

$$\lambda=\frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{v}{B\mathfrak{B}}\right)-\frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{v}{C\mathfrak{C}}\right)-\frac{1}{B^3C^3PQk}\left(\frac{\lambda'''}{\mathfrak{D}^3}+\frac{v}{D\mathfrak{D}}\right)$$

$$-\frac{1}{B^3C^3D^3PQkk'}\left(\frac{\lambda''''}{\mathfrak{E}^3}+\frac{v}{E\mathfrak{E}}\right)+\frac{1}{B^3C^3D^3E^3PQkk'T}\left(\frac{\lambda'''''}{\mathfrak{F}^3}+\frac{v}{F\mathfrak{F}}\right)$$

$$-\frac{1}{B^3C^3D^3E^3F^3PQkk'TU}\left(\frac{\lambda''''''}{\mathfrak{G}^3}+\frac{v}{G\mathfrak{G}}\right)+\text{etc.},$$

where as before it is required to be observed, if the first concave lens may be prepared from crystal glass, but all the rest from crown glass, then hence the value for λ found above must be multiplied by the fraction $\frac{9875}{8724}$; in which case, if indeed there may be put $\mathfrak{B}=\frac{5}{7}$, also all the confusion arising from the different refrangibility of the rays must be removed, clearly according to Dolland's experiments. Moreover, as we have now advised, unity will be able to be put in place for the letters λ', λ'' et λ'''. Truly for the following lenses, which all must be equally convex on both sides, there must be put in place :

$$\lambda''''=1+0,60006\left(2\mathfrak{E}-1\right)^2,$$

$$\lambda'''''=1+0,60006\left(2\mathfrak{F}-1\right)^2,$$

$$\lambda''''''=1+0,60006\left(2\mathfrak{G}-1\right)^2,\ \text{etc.}$$

COROLLARY 1

365. Therefore in this manner the radius of the apparent field of view will be

$$\Phi=\frac{i\xi}{m+\sqrt{m}} \quad \text{or} \quad \Phi=\frac{859i}{m+\sqrt{m}} \text{ minut.},$$

and if for the final lens the focal length were $=\zeta$, we will have for the position of the eye

$$O=\frac{\xi}{Mm}=\frac{\zeta\left(m+\sqrt{m}\right)}{im}=\frac{\zeta\left(m+\sqrt{m}\right)}{i\sqrt{m}},$$

from which, if the magnification were very large, there will become $O=\frac{\zeta}{i}$.

COROLLARY 2

366. The radii of the individual apertures of the lenses may be defined thus :

For the first $=x$, for the fifth $=\frac{t}{4}\pm\frac{1}{im}x$,
for the second $=\frac{x}{P}$, for the sixth $=\frac{u}{4}\pm\frac{2}{im}x$,
for the third $=\frac{i}{\sqrt{m}}\cdot\frac{r}{4}\pm\frac{x(1+i)}{2i\sqrt{m}}$, for the seventh $=\frac{v}{4}\pm\frac{3}{im}x$.
for the fourth $=0\frac{s}{4}\pm\frac{x}{\sqrt{m}}$, etc.

COROLLARY 3

367. Concerning the diaphragms the reasoning is the same as in the previous problem ; clearly for the diaphragm located at the position of the first image it must be

$$=\frac{iBC}{m+\sqrt{m}}\cdot\frac{\alpha}{4},$$

but for the next diaphragm so that it may become $=\frac{iBCD}{m+\sqrt{m}}\cdot\frac{\alpha}{4}$, from which it is apparent this opening thus must become greater, so that the field may be enlarged.

SCHOLIUM

368. Therefore with this problem we have completed this whole treatment of telescopes, since all the precepts for the construction of these have been established well

enough, nor may general constructions be able to be shown here conveniently, therefore since here not only as before two kinds of quantities will occur, whereby clearly they may be divided either by absolute numbers or by the magnification *m*, but indeed thus with three kinds of quantities, clearly besides absolute numbers in the first place there are number required to be introduced into the calculation divided by $\sqrt{m}$ or also by *m*, thus so that from the comparison of the two cases no general conclusion may be able to be deduced. Therefore nothing other remains here, except for any magnification that someone may demand, and also from the beginning a calculation may be put in place for the magnitude of the apparent field, or the value of the number *i*, which for any case offered to be undertaken on account of being a worthwhile matter, without doubt it is worthy of the effort. In which investigation also a certain letter θ must be determined, which we are allowed to choose at this stage, as equal to unity conveniently or allowed to taken equal to some greater number. But it is seen to be most convenient to put $\theta = 2$, from which the latter spacing of the lenses of the instrument may not be increased very much, likewise truly the value for λ is produced notably less, as if there were $\theta = 1$. Moreover, so that this whole calculation may be able to be undertaken and resolved more easily, we may adjoin some examples here.

EXAMPLE

369. There shall be $m = 49$, so that there becomes $\sqrt{m} = 7$, and $i = 2$ for the apparent field, thus so that the telescope shall be going to be prepared from six lenses, and besides there may be taken $\theta = 2$.

I. Initially the letters *P, Q,* etc. may be deduced, so that there follows,

$$P = \tfrac{50}{51}, \quad PQ = \tfrac{28}{3}, \quad k = \tfrac{3}{4}, \quad k' = 14, \quad T = \tfrac{1}{2}$$

$$\text{Log.}\tfrac{1}{P} = 0,0086002 \qquad \text{Log.}\tfrac{1}{PQ} = 9,0299632$$

$$\text{Log.}\tfrac{1}{PQk} = 9,1549019 \qquad \text{Log.}\tfrac{1}{PQkk'} = 8,0087738$$

$$\text{Log.}\tfrac{1}{PQkk'T} = 8,3098038$$

$$\mathfrak{B} = \tfrac{5}{7}, \quad l.\mathfrak{B} = 9,8538719 \qquad B = \tfrac{5}{2}, \quad l.B = 0,3979399$$

$$\mathfrak{C} = -\tfrac{25}{24}, \quad l.\mathfrak{C} = 0,0177287(-) \qquad C = -\tfrac{25}{49}, \quad l.C = 9,7077438(-)$$

$$\mathfrak{D} = \tfrac{15}{4}, \quad l.\mathfrak{D} = 9,8239086 \qquad D = 2, \quad l.D = 0,3010300$$

$$\mathfrak{D} = \tfrac{15}{4}, \quad l.\mathfrak{D} = 0,5740313 \qquad E = -\tfrac{15}{11}, \quad l.E = 0,1346984(-)$$

From these logarithms the following may be formed:

$$l.BC = 0{,}1056837(-) \qquad l.\,BCD = 0{,}4067137(-)$$
$$l.BCDE = 0{,}5414121(+) \qquad l.B\mathfrak{B} = 0{,}2518118(+)$$
$$l.C\mathfrak{C} = 9{,}7254725(+) \qquad l.\,D\mathfrak{D} = 0{,}1249386(+)$$
$$l.E\mathfrak{E} = 0{,}7087297(-).$$

II. We may deduce our elements from this as if our first effort made, which thus themselves will be had:

$$b = -1{,}02\alpha \qquad \beta = -2{,}55\alpha \qquad q = -\ 0{,}72857\alpha$$

$$\text{Log.}\tfrac{q}{\alpha} = 9{,}8624713(-)$$
$$c = +0{,}26785\alpha \qquad \gamma = -0{,}13666\alpha \qquad r = -0{,}27901\alpha$$

$$\text{Log.}\tfrac{r}{\alpha} = 9{,}4456318(-)$$

$$d = -0{,}18221\alpha \qquad \delta = -0{,}36443\alpha \qquad s = -0{,}12148\alpha$$
$$e = -0{,}02603\alpha \qquad \varepsilon = +0{,}03549\alpha \qquad \text{Log.}\tfrac{s}{\alpha} = 9{,}0844942(-)$$
$$f = -0{,}07099\alpha \qquad t = -0{,}09762\alpha$$

$$\text{Log.}\tfrac{t}{\alpha} = 8{,}9895188(-)$$
$$u = -0{,}07099\alpha$$

Moreover, for the position of the eye there will become $O = \frac{4u}{7} = -0{,}04057\alpha$.

III. Hence now the intervals of the lenses will become known :

1. $\alpha + b = -0{,}02000\alpha$
2. $\beta + c = -2{,}28215\alpha$
3. $\gamma + d = -0{,}31887\alpha$
4. $\delta + e = -0{,}39046\alpha$
5. $\varepsilon + f = -0{,}03550\alpha$
6. $O = -0{,}04057\alpha$

Total length $= 3{,}08755\alpha$.

From that the diaphragms thus are defined:
The former is placed at the distance $\gamma = -\ 0{,}13666\alpha$ after the third lens.
The radius of its opening $= -0{,}0114\alpha$.
The latter is put after the fourth lens at the distance $\delta = -0{,}36443\alpha$.
The radius of its opening $= -0{,}0228\alpha$.

Truly again the radius of its apparent field of view will be $30\frac{2}{3}$ minutes.

IV. Now it may be appropriate to consider the individual lenses, of which not only the construction, but also the degree of the confusion is required to be defined, which may be attributed to whatever value λ, where indeed the first lens will have to be discussed in the final place, evidently after the value λ were found. Therefore since all the following lenses are assumed to be made from the same crown glass, the values pertaining to that will be:

$$\begin{aligned} \nu &= 0,2196, \quad \text{Log.}\nu = 9,3416323, \\ \sigma &= 1,6601, \\ \rho &= 0,2267, \\ \hline \sigma-\rho &= 1,4334, \quad \text{Log.}(\sigma-\rho) = 0,1568674, \\ \tau &= 0,9252. \end{aligned}$$

Therefore now we may run through the individual lenses after the first in order.

For the second lens

$$1. \quad \text{Radius}\begin{cases} \text{anterior} = \dfrac{1}{\sigma-\mathfrak{B}(\sigma-\rho)+\tau\sqrt{(\lambda'-1)}} \\ \text{posterior} = \dfrac{1}{\sigma+\mathfrak{B}(\sigma-\rho)-\tau\sqrt{(\lambda'-1)}}, \end{cases}$$

which formulas are elicited easily from the above. Here truly there is $\lambda'=1$ and the calculation may be put in place thus:

$l.(\sigma-\rho)=0,1563674$ $l.\mathfrak{B}=9,8538719$ $0,0102393$ $\mathfrak{B}(\sigma-\rho)=1,02386$	$\sigma=1,6601$ subtr. $1,0239$ $0,6362$ denom. of anterior radius. $\rho=0,2267$ add. $1,0239$ $1,2506$ denom. posterior radius.
$\log.\frac{q}{\alpha}=9,8624713(-)$ $\log.\text{ denom.}=9,8035937$	$9,8624713(-)$ $0,0971184$
$0,0588776(-)$ anterior radius $=-1,14519\alpha$	$9,7653529(-)$ posterior radius $=-0,58257\alpha$.

2. Radius of the required aperture

$$=\frac{51}{50}x=\frac{51}{50}\cdot\frac{m}{50}\text{ dig.}$$

3. Calculation for the moment of the confusion:

$l.\frac{1}{P} = -\ 0,0086002$	$l.\lambda' = \quad 0,0000000$	$l.v = 9,3416323$
	$l.\mathfrak{B}^2 = -\ 9,5616157$	$l.B\mathfrak{B} = 0,2518118$
	$0,4383843$	$9,0898205$
	add log. coeffic. $= 0,0086002$	$0,0086002$
	$0,00469845$	$9,0984207$

Therefore first part $= 2,79888$
posterior part $= 0,12543$
Moment of the confusion $= 2,92431$.

For the third lens

$$1.\ \text{Radius}\begin{cases}\text{anterior} = \frac{1}{\sigma - \mathfrak{C}(\sigma-\rho) + \tau\sqrt{(\lambda''-1)}}\\ \text{posterior} = \frac{1}{\sigma + \mathfrak{C}(\sigma-\rho) - \tau\sqrt{(\lambda''-1)}},\end{cases}$$

where it is to be observed $\lambda'' = 1$.

$l.(\sigma-\rho) = 0,1563674$	$\sigma = 1,6601$	$\rho = 0,2267$
$l.(-\mathfrak{C}) = 0,0177287$	$+\ 1,4931$	$-1,4931$
$0,174096$	$3,1532$	$-1,2664$
$\mathfrak{C}(\sigma-\rho) = -1,49313$	denom. anter.	denom. poster.

$\text{Log.}\frac{r}{\alpha} = 9,4456318(-)$	$9,4456318\ (-)$
Log. denom. $= 0,4987515(+)$	$0,1025709(-)$
$8,9468803(-)$	$9,3430609(\ +)$

Therefore

$$\text{radius}\begin{cases}\text{anterior} = -0,08848\alpha\\ \text{posterior} = +\ 0,22032\alpha.\end{cases}$$

2. Radius of the required aperture

$$= \tfrac{2}{7}\cdot\tfrac{r}{4} + \tfrac{3}{28}x \ \text{ or } = 0,02\alpha + \tfrac{3}{28}x,$$

as the aperture this lens certainly can support.

3. Calculation for the moment of the confusion:

		$l.v = 9,3416323$
	$l.\lambda'' = 0,0000000$	$l.C\mathfrak{C} = 9,7254725$
$l.\frac{1}{PQ} = 9,0299632$	$3l.\ \mathfrak{C} = 0,0531861(-)$	$9,6161598$
$3l.\ B = 1,1938197$	$9,9468139$	$7,8361435$
$7,8361435$	$7,8361435$	$7,4523033$
	$7,7829574(-)$	

Therefore the first part $= +\ 0,00606$
latter part $= -\ 0,00283$
Momentum of the confusion $= 0,00323.$

For the fourth lens

$$1.\ \text{Radius}\begin{cases}\text{anterior} = \dfrac{1}{\sigma-\mathfrak{D}(\sigma-\rho)+\tau\sqrt{(\lambda'''-1)}}\\ \text{posterior} = \dfrac{1}{\sigma+\mathfrak{D}(\sigma-\rho)-\tau\sqrt{(\lambda'''-1)}},\end{cases}$$

where again there may be taken $\lambda''' = 1$.

$l.\ (\sigma-\rho) = 0,1563674$	$\sigma = 1,6601$	$\rho = 0,2267$
$l.\ \mathfrak{D} = 9,8239086$	$0,9556$	$0,9556$
$l.\mathfrak{D}(\sigma-\rho) = 9,9802760$	$0,7045$	$1,1823$
$\mathfrak{D}(\sigma-\rho) = 0,95560$	denom. anter.	denom. poster.

$\log.\ \frac{s}{\alpha} = 9,0844942(-)$	$9,0844942\ (-)$
$\log.\ \text{denom.} = 9,8478810$	$0,0727277$
$= 9,2366132\ (-)$	$9,0117665\ (-)$

$$\text{radius}\begin{cases}\text{anterior} = -0,17243\alpha\\ \text{posterior} = -0,10273\alpha.\end{cases}$$

2. Required radius of the aperture $= \frac{1}{7}x$, which the lens aperture will support conveniently; indeed if the smaller radius of the second lens, which is $0,58257\alpha$, sustains the aperture x, here the smaller radius, which is $0,10273\alpha$, will sustain the aperture $\frac{1}{7}x$ conveniently.

3. Calculation for the moment of confusion:

$l.\frac{1}{PQk} = 9{,}1549019$	$l.\lambda''' = 0{,}0000000$	$l.v = 9{,}3416323$
$3l.BC = \frac{0{,}3170511(-)}{8{,}8378508}$	$3l.\mathfrak{D} = 9{,}4717258$	$l.\mathfrak{D}D = 0{,}1249386$
	$0{,}5282742$	$9{,}2166937$
	$8{,}8378508$	$8{,}8378508$
	$9{,}3661250$	$8{,}0545445$

Therefore the first part $= 0{,}23234$
the posterior part $= 0{,}01133$
The momentum of confusion $= 0{,}24367.$

For the fifth lens

1. Since this lens must be equally convex on both sides, on account of its focal length

$$t = -0{,}09762\alpha$$

the radius of each face will be $= 1{,}06t = -0{,}10348\alpha$;
now truly there will be $\lambda''' = 1 + 0{,}60006(2\mathfrak{E}-1)^2$; but there is $2\mathfrak{E}-1 = 6{,}5$, therefore

$$\log.(2\mathfrak{E}-1) = 0{,}8129134$$

and

$$\log.(2\mathfrak{E}-1)^2 = 1{,}6258268$$
$$\log.\ 0{,}60006 = 9{,}7781947$$
$$1{,}4040215$$

and thus $\lambda'''' = 26{,}352$.

2. Here the radius of the aperture by hypothesis is $\frac{1}{4}t = -0{,}02440\alpha$; indeed the other part that this lens easily allows, is $\frac{1}{98}x$.

3. The calculation for the moment of confusion :

$l.\frac{1}{PQkk'} = 8{,}0087738$	$l.\lambda'''' = 1{,}4208136$	$l.v = 9{,}3416323$
$3l.BCD = 1{,}2201411$	$3l.\mathfrak{E} = 1{,}7220939$	$l.E\mathfrak{E} = 0{,}7087297$
$6{,}7886327$	$9{,}6987197$	$8{,}6329026$
	$6{,}7886327$	$6{,}7886327$
	$6{,}4873524$	$5{,}4215353$

$$\begin{aligned} \text{Therefore the foremost part} &= \;\;0,00031 \\ \text{and the posterior} &= \underline{-0,00002} \\ \text{Momentum of confusion} &= \;\;0,00029. \end{aligned}$$

For the sixth lens

1. Since by hypothesis this lens must be equally convex on both sides, on account of its focal length

$$u = -0,07099\alpha$$

there will be

the radius of each face $1,06u = -0,07525\alpha$;
then truly there will become $\lambda'''''=1,60006$.

2. Radius of the aperture $= \frac{1}{4}u = -0,01775\alpha$.
3. Calculation for the moment of the confusion:

$$\begin{array}{r|r} l.\frac{1}{PQkk'T} = 8,3098038 & l.\lambda''''' = 0,2041363 \\ 3l.BCDE = 1,6242363 & \underline{6,6855675} \\ \hline 6,6855675 & 6,8897038 \end{array}$$

Therefore the confusion moment $= 0,00077$.

V. With these found all the confusion moments may be gathered together into one sum, which will be 3,17227. But now two cases are required to be considered, just as the first concave lens to be prepared either from crown glass or crystal glass, which will require to be presented separately.

1. For the first concave lens requiring to be prepared from crown glass:
Therefore for this lens there will be

$$\lambda = 3,17227, \text{ from which } \lambda - 1 = 2,17227;$$

and hence there calculation arises:

$$\begin{array}{r|l} \text{Log.}(\lambda-1) = 0,3369138 & \\ \text{Log.}\sqrt{(\lambda-1)} = 0,1684569 & \\ \text{Log.}\tau = 9,9662356 & \text{thererfore} \\ \hline 0,1346925 & \tau\sqrt{(\lambda-1)} = 1,3636. \end{array}$$

Now since for this lens there shall be

$$\text{radius}\begin{cases}\text{anterior} &= \frac{1}{\sigma-\tau\sqrt{(\lambda-1)}}\\ \text{posterior} &= \frac{1}{\sigma+\tau\sqrt{(\lambda-1)}},\end{cases}$$

this calculation thus will be had :

$$\begin{array}{rr} \sigma = 1,6601 & \rho = 0,2267 \\ \tau\sqrt{(\lambda-1)} = 1,3636 & 1,3636 \\ \hline 0,2965 & 1,5903 \end{array}$$

$$\begin{array}{l|l} l.0,2965 = 9,4720247 & l.1,5903 = 0,2014791 \\ \text{complement} = 0,5279753 & \text{complement} = 9,7985208 \end{array}$$

and

$$\text{radius}\begin{cases}\text{anterior} = 3,37268\alpha\\ \text{posterior} = 0,62881\alpha\end{cases}$$

with the radius of the aperture proving to be $x = \frac{m}{50}\text{in.} = 1$ in.

2. For the first concave lens requiring to be prepared from crystal glass :
Therefore for this lens there will be

$$\lambda = \tfrac{9875}{8724}\cdot 3,17227 \text{ or } \lambda = 3,59080,$$

and since for the following crystal there is

$$\rho = 0,1414, \quad \sigma = 1,5827, \quad \tau = 0,8775,$$

the calculation thus will be had:

$$\begin{array}{l|r} \text{Log. }(\lambda-1) = 0,4134339 & \\ \text{Log.}\sqrt{(\lambda-1)} = 0,2067169 & \\ \text{Log.}\tau = 9,9432471 & \text{therefore} \\ 0,1499640 & \tau\sqrt{(\lambda-1)} = 1,41242 \\ \sigma = 1,5827 & \rho = 0,1414 \\ \text{subtr. } 1,4124 & \text{add. } 1,4124 \\ \hline 0,1703 & 1,5538 \\ \text{log. } 9,2312146 & \text{log. } 0,1913951 \\ \text{complement } 0,7687854 & \text{complement } 9,8086049 \end{array}$$

and thus there becomes

$$\text{radius}\begin{cases}\text{anterior} &= 5{,}87199\alpha\\ \text{posterior} &= 0{,}64358\alpha\end{cases}$$

with the radius of the aperture proving to be $x = \frac{m}{50}\text{dig.} = 1\text{ dig.}$

VI. Since the two first lenses jointly constitute the objective lens, of which the radius of the aperture $= 1$ in., the minimum radius of these may be put in place, which is $-0{,}58257\alpha, \ > 4$ in., and hence it may be concluded there must be taken $-\alpha > \frac{4}{0{,}58257}$ in., that is, $-\alpha > 7$ in. or at least not smaller, thus so that, if the best outcome may be able to be hoped for, it will be allowed to take $-\alpha = 7$ in. But if nevertheless some aberration shall be required to be avoided, there will be the need to increase the measurement by as much as one inch. But for the sake of convenience we may suppose $\alpha = -10$ in.; from which the following is produced for the construction of this telescope for the magnification $m = 49$.

1. For the objective lens
as far as it may be prepared from crown glass

$$\text{Radius of the}\begin{cases}\text{anterior face} &= -33{,}73\text{ in.}\\ \text{posterior face} &= -\ \ 6{,}29\text{ in.}\end{cases}\text{Crown Glass.}$$

(1). For the objective lens
provided it may be prepared from crystal glass

$$\text{Radius of the}\begin{cases}\text{anterior face} &= -58{,}72\text{ in.}\\ \text{posterior face} &= -\ \ 6{,}44\text{ in.}\end{cases}\text{Flint Glass.}$$

The focal length of which for each case $= -10$ in.
The radius of the aperture $= 1$ in.
Distance to the second lens $= 0{,}2 = \frac{1}{5}$ in.

2. For the second lens

$$\text{Radius of the}\begin{cases}\text{anterior face} &= 11{,}45\text{ in.}\\ \text{posterior face} &= \ \ 5{,}83\text{ in.}\end{cases}\text{Crown Glass.}$$

Of which the focal length $= 7{,}28$ in.
Radius of the aperture $= 1$ in.
Distance to the third lens $= 22{,}82$ in.

3. For the third lens

$$\text{Radius of the }\left\{\begin{array}{ll}\text{anterior face} & = \quad 0,884 \text{ in.}\\ \text{posterior face} & = -\,2,20 \text{ in.}\end{array}\right\}\text{Crown Glass.}$$

Of which the focal length $= 2,79$ dig.
Radius of the aperture $= 0,3$ dig.
Distance to the fourth lens $= 3,19$ dig.

4. For the fourth lens

$$\text{Radius of the }\left\{\begin{array}{ll}\text{anterior face} & = 1,72 \text{ in.}\\ \text{posterior face} & = 1,03 \text{ in.}\end{array}\right\}\text{Crown Glass.}$$

Of which the focal length $= 1,21$ in.
Radius of the aperture $= \frac{1}{7}$ in.
Distance to the fifth lens $= 3,90$ in.

5. For the fifth lens

Radius of each face $= 1,03$ in. Crown Glass.

Of which the focal length is 0,97 in.
Radius of the aperture $= \frac{1}{4}$ in.
Distance to the sixth lens $= 0,36$ in.

6. For the sixth lens

Radius of each face $= 0,75$ in. Crown Glass.
Of which the focal length $= 0,71$ in.
Radius of the aperture $= 0,18 = \frac{1}{6}$ in.
Distance as far as to the eye $= 0,40$ in.
Therefore of which the total length of this telescope will become $= 30,87$ in. $= 2\frac{1}{2}$ ft.
and the radius of the field of apparent field of view $= 30\frac{2}{3}$ min.

CAPUT III

DE ALTERA TERTII GENERIS TELESCOPIORUM SPECIE PRINCIPALI EORUMQUE PERFECTIONE

DEFINITIO

349. *Ad alteram hanc speciem referimus ea telescopia, quae supra § 310 et quidem speciatim in subnexo corollario 2, § 314, sunt explicata, in quibus scilicet lens secunda adhuc ante primam imaginem realem collocatur, tertia vero lens post hanc imaginem in eo loco, ubi lentis primae instar obiecti consideratae imago per secundam lentem proiiceretur; qui locus cum ante imaginem secundam cadat, lens quarta ocularis in debito loco constituitur. Speciatim autem, si primae lentis distantia focalis ponatur* $=\alpha$, *secunda lens ita statuitur, ut sit* $b=-\frac{\alpha}{\sqrt{m}}$; *sive intervallum primae et secundae lentis*

$$=\alpha\left(1-\frac{1}{\sqrt{m}}\right).$$

COROLLARIUM 1

350. Cum igitur haec telescopia quatuor constent lentibus, pro iis elementa ita se habebunt:

$$b=-\frac{\alpha}{\sqrt{m}},\quad \beta=\frac{\sqrt{m}-1}{2m}\cdot\alpha,\quad c=\frac{\sqrt{m}-1}{2m}\cdot\alpha,\quad \gamma=\frac{\sqrt{m}-1}{2m}\cdot C\alpha,\quad d=\frac{\sqrt{m}-1}{2m\sqrt{m}}\cdot C\alpha,$$

ita ut sit

$$B=\frac{1-\sqrt{m}}{2\sqrt{m}},\quad \mathfrak{B}=\frac{1-\sqrt{m}}{1+\sqrt{m}}$$

et C arbitrio nostro relinquatur.

COROLLARIUM 2

351. Ex his elementis erunt lentium distantiae focales

$$p=\alpha,\quad q=\frac{\sqrt{m}-1}{\left(1+\sqrt{m}\right)\sqrt{m}}\cdot\alpha,\quad r=\frac{\sqrt{m}-1}{2m}\cdot\mathfrak{C}\alpha \quad\text{et}\quad s=\frac{\sqrt{m}-1}{2m\sqrt{m}}\cdot C\alpha$$

et lentium intervalla

$$\alpha+B=\left(1-\frac{1}{\sqrt{m}}\right)\alpha,\quad \beta+c=\frac{\sqrt{m}-1}{m}\cdot\alpha,\quad \gamma+d=\frac{m-1}{2m\sqrt{m}}\cdot C\alpha$$

et distantia oculi

$$O=\frac{m-1}{2mm}\cdot\alpha,$$

ita ut tota longitudo futura sit

$$=\frac{m-1}{m}\left(1+\frac{1+\sqrt{m}\cdot C}{2m}\right)\cdot\alpha,$$

ubi tantum monendum est pro C numerum positivum accipi debere.

COROLLARIUM 3

352. Litterae autem maiusculae P, Q, R pro hac specie fient

$$P=\sqrt{m},\ Q=-1 \text{ et } R=-\sqrt{m},$$

ita ut hinc prodeat $PQR=m$, uti rei natura postulat.

SCHOLION

353. Hic autem inprimis rationem reddere oportet conditionis in definitione commemoratae, qua diximus lentem tertiam ibi esse collocandam, ubi primae lentis instar obiecti consideratae imago per secundam lentem proiecta esset casura. Cum enim secundae lentis distantia focalis sit

$$q=\frac{\sqrt{m}-1}{(1+\sqrt{m})\sqrt{m}}\cdot\alpha,$$

eius autem distantia a prima lente $=\left(1-\frac{1}{\sqrt{m}}\right)\alpha$ quae vocetur y, si prima lens uti obiectum consideretur, eius imago post secundam lentem cadet ad distantiam

$$\xi=\frac{yq}{y-q};$$

est vero

$$y-q=\left(\frac{\sqrt{m}-1}{\sqrt{m}+1}\right)\cdot\alpha \text{ hincque } \xi=\frac{\sqrt{m}-1}{m}\cdot\alpha,$$

cui praecise distantia tertiae lentis a secunda aequatur. Hanc autem conditionem ideo in definitionem introduximus, quoniam eius ope locus tertiae lentis facillime per praxin assignatur. Ceterum supra [§ 314] iam notavimus semidiametrum campi apparentis fore $\Phi=\frac{859}{m+\sqrt{m}}$ min., quae utique augmentatione indiget, cum has lentes perficere conabimur. Denique ibidem quoque est ostensum semidiametrum aperturae tertiae lentis statui debere $=\frac{\sqrt{m}}{50}$ dig.

Pro secunda autem lente, quia posuimus

$$\pi=\omega\xi \text{ et } \omega=-\xi=-\tfrac{1}{\sqrt{m}},$$

semidiameter eius aperturae esse debet

$$=\tfrac{q}{4\sqrt{m}}=\tfrac{\sqrt{m}-1}{4m\left(1+\sqrt{m}\right)}\cdot\alpha.$$

PROBLEMA 1

354. *Inter binas postremas lentes huius telescopiorum speciei novam lentem inserere, qua campus apparens magis amplificetur.*

SOLUTIO

Cum igitur hic occurrant quinque lentes, statuantur nostrae quaternae fractiones

$$\tfrac{\alpha}{b}=-P,\quad \tfrac{\beta}{c}=-Q,\quad \tfrac{\gamma}{d}=-R,\quad \tfrac{\delta}{e}=-S;$$

quarum litterarum duae debent esse negativae, quarum prior erit Q statuaturque $Q=-k$,

[Notandum est littera ξ in hac paragrapho duas plane differentes quantitates designari, scilicet $\tfrac{yq}{y-q}$ et $\tfrac{1}{\sqrt{m}}$ E. Ch.]

altera vero erit R vel S; utram autem negativam statui conveniat, nondum definiamus. Hinc igitur elementa nostra erunt

$$b=\tfrac{-\alpha}{P},\quad c=\tfrac{-B\alpha}{Pk},\quad d=\tfrac{BC\alpha}{PkR},\quad e=\tfrac{BCD\alpha}{PkRS},$$

$$\beta=\tfrac{-B\alpha}{P},\quad \gamma=\tfrac{-BC\alpha}{Pk},\quad \delta=\tfrac{-BCD\alpha}{PkR},$$

distantiae autem focales

$$p=\alpha,\quad q=\tfrac{-\mathfrak{B}\alpha}{P},\quad r=\tfrac{-B\mathfrak{C}\alpha}{Pk},\quad s=\tfrac{BC\mathfrak{D}\alpha}{PkR},\quad t=\tfrac{-BCD\alpha}{PkRS}$$

hincque lentium intervalla

$$\alpha+b=\alpha\left(1-\tfrac{1}{P}\right),\qquad \beta+c=\tfrac{-B\alpha}{P}\left(1+\tfrac{1}{k}\right),$$

$$\gamma+d=\tfrac{-BC\alpha}{Pk}\left(1-\tfrac{1}{R}\right),\qquad \delta+e=\tfrac{+BCD\alpha}{PkR}\left(1-\tfrac{1}{S}\right);$$

quae cum esse debeant positiva et α iam sit positivum, necesse est, ut sit 1. $P>1$, 2. $B<0$; 3. quod ad bina reliqua intervalla attinet, duos casus distingui convenit.

Casus prior, quo $R > 0$ et $S = -k'$. Hocque casu debet esse

$$C\left(1-\tfrac{1}{R}\right) > 0 \text{ et } CD < 0,$$

quo ipso etiam fit e positivum.

Casus posterior, quo $R < 0$ seu $R = -k'$ et $S > 0$. Hoc ergo casu esse debet $C > 0$ ideoque etiam

$$\mathfrak{C} > 0 \text{ at } < 1 \text{ et } D(1-\tfrac{1}{S}) > 0.$$

Ut autem etiam fiat $e > 0$, debet esse $D < 0$ ideoque $S < 1$.

Nunc igitur consideremus campum apparentem, cuius semidiameter est

$$\Phi = \tfrac{-\pi+\pi'-\pi''+\pi'''}{m-1},$$

ac statuamus ut hactenus $\pi = -\omega\xi$, $\pi' = 0$ ex natura huius speciei, $\pi'' = -\xi$ et $\pi''' = \xi$, ut fiat

$$\Phi = \tfrac{\omega+2}{m-1}\cdot\xi = \mathrm{M}\xi \text{ existente } M = \tfrac{\omega+2}{m-1},$$

atque hinc iam statim pro loco oculi prodit

$$O = \tfrac{e}{Mm} = \tfrac{(m-1)e}{m(\omega+2)}.$$

Aequationes porro fundamentales erunt:

$$\begin{aligned} &1.\ \tfrac{\mathfrak{B}\pi}{\Phi} = 1-P \text{ seu } \mathfrak{B}\omega = -(1-P)M \\ &2.\ 0 = -(1+Pk)M - \omega \\ &3.\ \mathfrak{D} = -\left(1+PkR\right)M - \omega; \end{aligned}$$

ubi cum ex prima sit

$$\omega = \tfrac{-(1-P)M}{\mathfrak{B}},$$

hic valor in secunda substitutus dat $0 = (1+Pk)\mathfrak{B} + P - 1$, unde sequitur

$$\mathfrak{B} = -\tfrac{(P-1)}{1+Pk},$$

ita ut $\mathfrak{B}$ ac proinde etiam B sit numerus negativus; fit autem

$$B = \tfrac{-(P-1)}{P(1+k)} \text{ et } \omega = -(1+Pk)M;$$

tum vero ex tertia erit

$$\mathfrak{D} = Pk\left(1-R\right)M;$$

litterae vero C et $\mathfrak{C}$ arbitrio nostro manent relictae. Pro binis ergo casibus memoratis erit:

Pro priore, quo $S=-k'$, $\mathfrak{D}=Pk(1-R)M$. Si ergo fuerit $R>1$, debet esse $C>0$ et $D<0$; at cum fiat $\mathfrak{D}<0$, sponte illa conditio $D<0$ impletur. Sin autem sit $R<1$, erit $\mathfrak{D}>0$, debet autem esse $C<0$ et $D>0$, consequenter $\mathfrak{D}<1$ ideoque $Pk(1-R)M<1$.

Pro posteriore casu, quo $R=-k'$, erit $\mathfrak{D}=Pk(1+k')M$ ideoque $\mathfrak{D}>0$; ante autem vidimus hoc casu esse debere $C>0$ adeoque $\mathfrak{C}>0$ et $\mathfrak{C}<1$. Tum vero $D(1-\frac{1}{S})>0$. Quare, cum esse debeat $S<1$, erit $D<0$, unde ob $\mathfrak{D}>0$ colligitur $\mathfrak{D}>1$.

Nunc pro tollendo margine colorato habebitur haec aequatio:

$$0=\frac{\omega}{P}-\frac{1}{PkR}-PkRS,$$

ex qua colligitur

$$0=\omega kRS-S-1 \text{ seu } 0=kRS(1+Pk)M+S+1,$$

ubi ergo binos nostros casus distingui oportet.

I. Si $S=-k'$, habebitur $0=kk'R(1+Pk)M-k'+1$, unde fit

$$R=\frac{1-k'}{kk'(1+Pk)M},$$

unde patet esse debere $k'<1$, unde, si prodeat $R>1$, debet esse $C>0$ et $D<0$. Sin autem prodeat $R<1$, debet esse $D>0$, $C<0$, $\mathfrak{D}>0$ et $\mathfrak{D}<1$ adeoque $Pk(1-R)M<1$.

II. Si $R=-k'$, erit $0=-kk'S(1+Pk)M+S+1$, unde colligitur

$$k'=\frac{S+1}{kS(1+Pk)M},$$

quae expressio per se est positiva. Hoc autem casu supra vidimus esse debere $C>0$ adeoque $\mathfrak{C}>0$ et $\mathfrak{C}<1$ et $D<0$, ita ut hoc casu sumendum sit $S<1$.

Denique hic meminisse oportet esse $PkRS=-m$, quae conditio secundum binos casus considerari debet.

Primo casu, quo $S=-k'$, ob $R=\frac{m}{Pkk'}$ nostra aequatio dat

$$0=-\tfrac{m}{P}(1+Pk)M-k'+1,$$

unde colligitur

$$k'=1-\tfrac{m}{P}(1+Pk)M,$$

ita ut esse debeat $m(1+Pk)M<P$; ubi notetur, si prodeat $R>1$, esse debere $C>0$ et $D<0$, sin autem prodeat $R<1$, debere esse $C<0$ et $D>0$, $\mathfrak{D}>0$ et $\mathfrak{D}<1$.

Altero casu, si $R=-k'$, ut sit $m=Pkk'S$, nostra aequatio dat

$$0=-\tfrac{m}{P}\left(1+Pk\right)M+S+1,$$

unde colligitur

$$S=\tfrac{m}{P}\left(1+Pk\right)M-1,$$

ita ut esse debeat $m(1+Pk)M>P$. Cum autem debeat esse $S<1$, etiam esse debet $m(1+Pk)M<2P$; praeterea recordemur esse debere $C>0$ adeoque $\mathfrak{C}>0$ et $\mathfrak{C}<1$ et $D<0$.

Tandem circa has formulas probe observandum est ob valorem ω inventum litteram M per reliqua elementa commode exprimi posse. Cum enim sit $\omega=-(1+Pk)M$, aequatio

$$\tfrac{\omega+2}{m-1}=M$$

dabit

$$M=\tfrac{2}{m+Pk}$$

et

$$\omega=\tfrac{-2(1+Pk)}{m+Pk},$$

ita ut pro campo apparente prodeat

$$\Phi=\tfrac{2}{m+Pk}\cdot\xi \quad\text{seu}\quad \Phi=\tfrac{1718}{m+Pk}\ \text{min.}$$

Tum vero etiam pro loco oculi

$$O=\tfrac{e\left(m+Pk\right)}{2m}.$$

Quibus observatis binos casus seorsim evolvamus.

I. EVOLUTIO CASUS QUO $S=-k'$

355. Hoc ergo casu elementa nostra ita se habebunt:

$$b=-\frac{\alpha}{P},\quad c=\frac{-B\alpha}{Pk},\quad d=\frac{BC\alpha}{PkR},\quad e=\frac{BCD\alpha}{m},$$

$$\beta=\frac{-B\alpha}{P},\quad \gamma=\frac{-BC\alpha}{Pk},\quad \delta=\frac{BCD\alpha}{PkR}$$

hincque intervalla

$$\alpha+b=\alpha\left(1-\frac{1}{P}\right),\quad \beta+c=\frac{-B\alpha}{P}\left(1+\frac{1}{k}\right),$$

$$\gamma+d=-\frac{BC\alpha}{Pk}\left(1-\frac{1}{R}\right),\quad \delta+e=\frac{BCD\alpha}{PkR}\left(1+\frac{1}{k'}\right),$$

$$\beta=\frac{-B\alpha}{P},\quad \gamma=\frac{-BC\alpha}{Pk},\quad \delta=\frac{BCD\alpha}{PkR}$$

ubi ergo esse debet

$$P>1 \text{ et } \mathfrak{B}=\frac{-(P-1)}{1+Pk}, \text{ hincque } B=\frac{-(P-1)}{P(1+k)}.$$

Tertium vero intervallum dat hanc conditionem $C\left(1-\frac{1}{R}\right)>0$ et ultimum $CD<0$; est autem

$$\mathfrak{D}=Pk(1-R)M=\frac{2Pk(1-R)}{m+Pk} \quad \text{et} \quad D=\frac{2Pk(1-R)}{m-Pk+2PkR}.$$

Destructio autem marginis colorati postulat, ut sit

$$k'=1-\frac{m}{P}(1+Pk)M=1-\frac{2m(1+Pk)}{P(m+Pk)}$$

et

$$R=\frac{m(m+Pk)}{k\big(P(m+Pk)-2m(1+Pk)\big)};$$

quamobrem debet esse

$$P(m+Pk)>2m(1+Pk)$$

ideoque

$$k<\frac{m(P-2)}{P(2m-P)};$$

quare, cum illa quantitas maior debeat esse quam *k*, ob $2m>P$ debet esse $P>2$, ex qua etiam conditione patet semper esse debere $R>1$ adeoque $C>0$ et $D<0$, uti ex valore ipsius *D* manifestum est. Quo his conditionibus satisfiat formulaeque evadant simpliciores, statuamus $Pk=\sqrt{m}$, ut fiat

$$M=\frac{2}{m+\sqrt{m}}$$

ideoque

$$\Phi=\frac{2}{m+\sqrt{m}}\cdot\xi=\frac{1718}{m+\sqrt{m}}\text{ min.},$$

qui valor duplo maior est quam ante [§ 314]. Tum vero erit

$$\omega = \frac{-2\left(1+\sqrt{m}\right)}{m+\sqrt{m}};$$

porro si capiatur $P = 4\sqrt{m}$, prodit $k = \frac{1}{4}$, $R = 2\sqrt{m}$ et $k' = \frac{1}{2}$, hincque

$$\mathfrak{D} = \frac{2\left(1-2\sqrt{m}\right)}{1+\sqrt{m}} \text{ et } D = \frac{2\left(1-2\sqrt{m}\right)}{5\sqrt{m}-1}.$$

Praeterea vero

$$\mathfrak{B} = -\frac{\left(4\sqrt{m}-1\right)}{1+\sqrt{m}} \text{ et } B = -\frac{\left(4\sqrt{m}-1\right)}{5\sqrt{m}},$$

unde omnia intervalla prodibunt positiva, dummodo pro C sumatur quantitas positiva.

II. EVOLUTIO CASUS QUO $R = -k'$

356. Pro hoc ergo casu destructio marginis colorati praebet

$$0 = -\frac{2m(1+Pk)}{P(m+Pk)} + S + 1,$$

unde concluditur

$$S = \frac{2m(1+Pk)}{P(m+Pk)} - 1,$$

ita ut esse debeat

$$2m(1+Pk) > P\left(m+Pk\right);$$

tum vero ob $S < 1$ debet esse

$$2m(1+Pk) < 2P\left(m+Pk\right).$$

Statuamus nunc iterum ut ante $Pk = \sqrt{m}$ fietque $S = \frac{2\sqrt{m}}{P} - 1$, ita ut nunc capi debeat $P < 2\sqrt{m}$ et $P > \sqrt{m}$; littera autem k cadet intra limites 1 et $\frac{1}{2}$.
Tum vero ob $S = \frac{2\sqrt{m}}{P} - 1$ erit

$$k' = \frac{m}{S\sqrt{m}} = \frac{P\sqrt{m}}{2\sqrt{m}-P}.$$

Definito autem P erit

$$\mathfrak{B} = -\frac{(P-1)}{1+\sqrt{m}} \quad \text{et} \quad B = -\frac{(P-1)}{P+\sqrt{m}}$$

et

$$\mathfrak{D} = \frac{2(1+k')}{1+\sqrt{m}} \quad \text{et} \quad D = \frac{2(1+k')}{\sqrt{m}+2k'-1}$$

sive

$$\mathfrak{D} = \frac{2\left(2\sqrt{m}-P+P\sqrt{m}\right)}{\left(1+\sqrt{m}\right)\left(2\sqrt{m}-P\right)};$$

qui valor cum sit positivus et unitate maior, littera D sponte fit negativa, quemadmodum conditiones postulant, dummodo C capiatur positivum. Quo autem omnia plene determinentur, statuamus insuper $P = \frac{3}{2}\sqrt{m}$ ac fiet

$$k = \frac{2}{3}, \quad k' = 3\sqrt{m} \quad \text{et} \quad S = \frac{1}{3},$$

$$\mathfrak{B} = -\frac{\left(3\sqrt{m}-2\right)}{2\left(1+\sqrt{m}\right)} \quad \text{et} \quad B = -\frac{\left(3\sqrt{m}-2\right)}{5\sqrt{m}},$$

$$\mathfrak{D} = \frac{2\left(3\sqrt{m}+1\right)}{\sqrt{m}+1} \quad \text{et} \quad D = -\frac{2\left(3\sqrt{m}+1\right)}{5\sqrt{m}+1},$$

quibus valoribus omnibus conditionibus satisfit.

SCHOLION

357. I. En ergo duos casus huiusmodi telescopiorum penitus determinatos pro data multiplicatione *m,* quorum effectus in praxi idem esse debet. Cum autem posteriore casu longitudo instrumenti minor evadat quam priore, eum merito hic praeferimus; quamobrem operae pretium erit in constructionem istorum telescopiorum adcuratius inquirere. Notatis igitur praecipuarum litterarum valoribus, scilicet

$$P = \frac{3}{2}\sqrt{m}, \quad k = \frac{2}{3}, \quad k' = 3\sqrt{m}, \quad S = \frac{1}{3},$$

$$\mathfrak{B} = -\frac{\left(3\sqrt{m}-2\right)}{2\left(1+\sqrt{m}\right)}, \quad B = -\frac{\left(3\sqrt{m}-2\right)}{5\sqrt{m}},$$

$$\mathfrak{D} = \frac{2\left(3\sqrt{m}+1\right)}{\sqrt{m}+1}, \quad D = -\frac{2\left(3\sqrt{m}+1\right)}{5\sqrt{m}+1},$$

et quia C debet esse positivum, ponatur

$$C=\theta,\ \text{ ut sit }\ \mathfrak{C}=\tfrac{\theta}{1+\theta},$$

et elementa nostra ita erunt expressa:

$$b=-\tfrac{2\alpha}{3\sqrt{m}},\quad \beta=\frac{2\left(3\sqrt{m}-2\right)}{15m}\cdot\alpha,\quad c=\tfrac{3\sqrt{m}-2}{5m}\cdot\alpha,\quad \gamma=\frac{\theta\left(3\sqrt{m}-2\right)}{5m}\cdot\alpha,$$

$$d=\frac{\theta\left(3\sqrt{m}-2\right)}{15m\sqrt{m}}\cdot\alpha,\quad \delta=\frac{-2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha,\quad e=\frac{2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{5\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha;$$

hinc distantiae focales

$$p=\alpha,\quad q=\frac{3\sqrt{m}-2}{3\left(1+\sqrt{m}\right)\sqrt{m}}\cdot\alpha,\quad r=\tfrac{\theta}{1+\theta}\cdot\tfrac{3\sqrt{m}-2}{5m}\cdot\alpha,$$

$$s=\frac{2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15m\left(\sqrt{m}+1\right)\sqrt{m}}\cdot\alpha,\quad \text{et}\quad t=\frac{2\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{5\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha,$$

et lentium intervalla

$$\alpha+b=\alpha\left(1-\tfrac{2}{3\sqrt{m}}\right),\quad \beta+c=\tfrac{3\sqrt{m}-2}{3m}\cdot\alpha,$$

$$\gamma+d=\frac{\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15m\sqrt{m}}\cdot\alpha,\quad \delta+e=\frac{4\theta\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{15\left(5\sqrt{m}+1\right)m\sqrt{m}}\cdot\alpha,$$

et distantia oculi

$$O=\frac{e\left(1+\sqrt{m}\right)}{2\sqrt{m}}=\frac{\theta\left(1+\sqrt{m}\right)\left(3\sqrt{m}-2\right)\left(3\sqrt{m}+1\right)}{5m^2\left(5\sqrt{m}+1\right)}\cdot\alpha;$$

unde tota oritur longitudo telescopii

$$=\left(\frac{\left(3\sqrt{m}-2\right)\left(1+\sqrt{m}\right)}{3m}+\frac{\theta\left(\sqrt{m}+1\right)\left(3\sqrt{m}+1\right)\left(3\sqrt{m}-2\right)\left(5\sqrt{m}+3\right)}{15m^2\left(5\sqrt{m}+1\right)}\right)\alpha,$$

ita ut, si *m* sit numerus praemagnus, haec longitudo fiat

$$\left(1+\tfrac{1}{3\sqrt{m}}+\tfrac{3\theta}{5\sqrt{m}}\right)\alpha,$$

et quia hoc casu fit $e=\frac{18\theta\alpha}{25m}$, si liceret capere $\alpha=\frac{m}{7}$ dig., statui conveneret $\theta=5$, ut ultimae lentis distantia focalis fieret circiter $\frac{1}{2}$ dig.; quando autem α multo maiorem obtinet valorem, facile capi poterit $\theta=1$.

II. Adcuratius etiam inquirere debemus, quantam aperturam cuique lenti tribui oporteat, ac pro prima quidem lente semper sumi solet semidiameter aperturae $x=\frac{m}{50}$ dig.; pro reliquis lentibus ex formulis supra expositis colligitur:

Semidiameter aperturae secundae lentis

$$=\pi q\pm\frac{qx}{\mathfrak{B}p}=\frac{1}{4}\omega q+\frac{qx}{\mathfrak{B}\alpha}=\frac{1}{4}q\left(\frac{2}{\sqrt{m}}+\frac{8\left(1+\sqrt{m}\right)}{3\sqrt{m}-2}\cdot\frac{x}{\alpha}\right),$$

semidiameter aperturae tertiae lentis

$$=\frac{rx}{B\mathfrak{C}p}=\frac{x}{\sqrt{m}}=\frac{\sqrt{m}}{50}\ \text{dig.}$$

Quarta autem et quinta lens maximam aperturam capere debent; unde eas utrinque convexas effici oportet.

III. Quod nunc ad litteras λ attinet, pro prima lente semper sumi convenit $\lambda=1$, qui valor etiam pro secunda lente sumi posse videtur, siquidem numerus m non sit admodum parvus, de quo autem quovis casu seorsim erit dispiciendum. Pro tertia enim lente ob minimam aperturam nullum est dubium, quin sumi possit $\lambda''=1$. Quoniam vero quarta lens debet esse utrinque aequaliter convexa, pro ea sumi debet

$$\lambda'''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\left(1-2\mathfrak{D}\right)^2=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\left(\frac{11\sqrt{m}+3}{\sqrt{m}+1}\right)^2.$$

Pro quinta autem lente erit $\lambda''''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2$.

IV. His igitur valoribus pro λ, λ',..... stabilitis quantitas α ex sequente formula definiri debet:

$$\alpha=kx\sqrt[3]{\mu m}\left\{\begin{array}{l}\lambda-\frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)-\frac{1}{B^3\mathfrak{C}Pk}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)\\ -\frac{1}{B^3C^3\mathfrak{D}Pkk'}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu}{D}\right)+\frac{\lambda''''}{B^3C^3D^3m}\end{array}\right\}$$

ubi meminisse iuvabit sumi solere $x=\frac{m}{50}$, et $k=50$, ut sit $kx=m$. Interim tamen, si minore vel claritatis vel distinctionis gradu contenti esse velimus, pro kx sumi poterit $\frac{1}{2}m$. Deinde etiam hinc evidens est ob illum praegrandem valorem ipsius λ''', qui scilicet quadratum $(2\mathfrak{D}-1)^2$ involvebat, terminum inde hic oriundum iterum satis fieri parvum, cum is divisus sit per $\mathfrak{D}^3$ praeterquam quod eius denominator ob $Pkk'=3m$ per se sit satis magnus. Denique adhuc notari debet numerum λ'' multiplicari per quantitatem satis notabilem, cum sit $-\frac{1}{B^3}$ propemodum $\frac{125}{27}$ et $\frac{1}{\mathfrak{C}^3}>1$ ideoque $-\frac{1}{B^3\mathfrak{C}^3}$ ultra 5 assurgat atque adeo ad 40 usque, si sumeretur $\theta=1$, ita ut $Pk=\sqrt{m}$ in denominatore hunc terminum vix infra unitatem diminuere possit. Cui incommodo remedium afferri posset hanc lentem secundum praecepta in Libro I de lentibus compositis tradita duplicando. Hoc autem necesse non erit, quando ipsam lentem obiectivam ita duplicabimus, ut omnis confusio a reliquis etiam lentibus oriunda tollatur.

EXEMPLUM

358. Sumto $m=25$ constructionem huiusmodi telescopii describere.

I. Cum sit $m=25$, erit $\sqrt{m}=5$ indeque

$$P=\frac{15}{2},\quad k=\frac{2}{3},\quad k'=15,\quad S=\frac{1}{3},$$
$$\mathfrak{B}=\frac{13}{12},\quad B=-\frac{13}{25},\quad \mathfrak{D}=\frac{16}{3},\quad D=-\frac{16}{3};$$

unde elementa nostra erunt

$$b=-\frac{2\alpha}{15},\quad \beta=\frac{26\alpha}{375},\quad c=\frac{13\alpha}{125},\quad \gamma=\frac{13\theta\alpha}{125},$$

$$d=\frac{13\theta\alpha}{1875},\quad \delta=-\frac{16\theta}{1875}\alpha,\quad e=\frac{16\theta\alpha}{625}$$

et distantiae focales

$$p=\alpha,\quad q=\frac{13}{90}\alpha,\quad r=\frac{\theta}{1+\theta}\cdot\frac{13}{125}\alpha,\quad s=\frac{208}{5625}\alpha \text{ and } t=\frac{16\theta}{625}\alpha$$

et intervalla lentium

$$\alpha+b=\frac{13}{15}\alpha,\quad \beta+c=\frac{13}{75}\alpha,\quad \gamma+d=\frac{208\theta}{1875}\alpha,\quad \delta+e=\frac{32\theta}{1875}\alpha$$

ac distantia oculi

$$O=\frac{48\theta}{3125}\alpha,$$

ita ut tota longitudo futura sit $\alpha\left(\frac{26}{25}+\frac{448\theta}{3125}\right)$. Campi autem apparentis semidiameter erit

$$\tfrac{1718}{30}\,\text{min.} = 57'\ 16''.$$

II. Semidiameter aperturae lentis primae $= \frac{1}{2}$ in.

Semidiameter aperturae lentis secundae $= \frac{1}{4}q\left(\frac{2}{5} + \frac{48}{13}\cdot\frac{x}{\alpha}\right)$, unde colligera licet pro hac lente dimidiam aperturam sufficere.

Semidiameter aperturae lentis tertiae $= \frac{1}{10}$ dig.

III. Deinde porro erit $\lambda = 1$, $\lambda' = 1$ fortasse, $\lambda'' = 1$, $\lambda''' = 1 + \frac{841}{9}\left(\frac{\sigma-\rho}{2\tau}\right)^2$,
ubi notandum, si vitrum commune adhibeatur, quo $n = 1,55$, fore

$$\lambda'' = 1 + 0,6299\cdot\tfrac{841}{9} = 59,861 \ \text{ et } \ \lambda''' = 1,6299.$$

Ex aequatione pro α colligere licet numerum sub signo radicali contentum circiter ultra 2 μm excrescere, unde eius loco tuto scribere possumus 64, sicque obtinebimus
$\alpha = 100$ dig. $= 8\frac{1}{3}$ ped.

Pro maioribus autem multiplicationibus haec quantitas in ratione $m\sqrt[3]{m}$ crescet neque haec longitudo satis magna imminui poterit, nisi formulam pro semidiametro confusionis ad nihilum redigamus, id quod, uti ex superioribus liquet, facile praestabitur, si his quinque lentibus adhuc lentem concavam praefigamus sive ex eodem sive ex vitro crystallino parandam.

PROBLEMA 2

359. *Hanc telescopiorum speciem ante primam lentem praefigendo lentem concavam ita perficere, ut confusio penitus tollatur sicque haec telescopia brevissima reddantur servato campo ante invento.*

SOLUTIO

Cum igitur nunc sex habeamus lentes, quinque litterae erunt considerandae *P, Q, R, S, T* ad lentium intervalla relatae, quarum prima *P* debet dare intervallum minimum, quod ob α negativum statuamus $= -\frac{1}{50}\alpha$, ut fiat $P = \frac{50}{51}$. Deinde cum sequentia intervalla respondeant litteris *Q, R, S, T,* quae ante erant *P, Q, R, S,* nunc ponamus $R = -k$ et $S = -k'$ eruntque elementa

$$b = -\tfrac{\alpha}{P},\ \ c = \tfrac{B\alpha}{PQ},\ \ d = \tfrac{BC\alpha}{PQk},\ \ e = \tfrac{BCD\alpha}{PQkk'}$$

et

$$f = \tfrac{-BCDE\alpha}{PQkk'T} = \tfrac{-BCDE\alpha}{m}$$

$$\beta = -\tfrac{B\alpha}{P},\quad \gamma = \tfrac{BC\alpha}{PQ},\quad \delta = \tfrac{BCD\alpha}{PQk},\quad \varepsilon = \tfrac{BCDE\alpha}{PQkk'},$$

unde intervalla colliguntur

1. $\alpha + b = \alpha\left(1-\tfrac{1}{P}\right)$; quod fit sumto $P = \tfrac{50}{51}$.
2. $\alpha + c = -\tfrac{B\alpha}{P}\left(1-\tfrac{1}{Q}\right)$; unde, cum Q capi debeat > 1, debet esse B positivum ideoque $\mathfrak{B} > 0$ et < 1.
3. $\gamma + d = \tfrac{BC\alpha}{PQ}\left(1+\tfrac{1}{k}\right)$; unde C debet esse negativum.
4. $\delta + e = \tfrac{BCD\alpha}{PQk}\left(1+\tfrac{1}{k'}\right)$; unde D debet esse positivum ideoque $\mathfrak{D} > 0$ et $\mathfrak{D} < 1$.
5. $\varepsilon + f = \tfrac{BCDE\alpha}{PQkk'}\left(1-\tfrac{1}{T}\right)$; unde debet esse $E\left(1-\tfrac{1}{T}\right)$ positivum, sed cum et f debeat esse maius nihilo, debet esse E negativum, ergo $T < 1$.

Iam pro campo apparente ponamus

$$\pi = -\nu\xi,\ \ \pi' = \omega\xi,\ \ \pi'' = 0,\ \ \pi''' = \xi \ \text{ et } \ \pi'''' = -\xi,$$

ut fiat

$$\Phi = \tfrac{\nu+\omega+2}{m-1}\cdot\xi = M\xi$$

existante

$$M = \tfrac{\nu+\omega+2}{m-1};$$

unde pro loco oculi fit

$$O = \tfrac{f}{Mm}.$$

Ex his autem formabuntur sequentes aequationes fundamentales:

$$\begin{aligned}
&1.\ \mathfrak{B}\nu = -(1-P)M\\
&2.\ \mathfrak{C}\omega = -(1-PQ)M - \nu\\
&3.\ \mathfrak{D}\cdot 0 = -(1+PQk)M - \nu - \omega\\
&4.\ \ \mathfrak{E} = -(1-PQkk')M - \nu - \omega.
\end{aligned}$$

Ex quarum tertia statim habemus

$$\nu + \omega = -\left(1+PQk\right)M;$$

est vero etiam

$$v+\omega=(m-1)M-2;$$

unde

$$M=\tfrac{2}{m+PQk}$$

sicque vicissim

$$v+\omega=\tfrac{-2(1+PQk)}{m+PQk}$$

Quia nunc prima aequatio dat

$$v=\tfrac{-2(1-P)}{\mathfrak{B}(m+PQk)},$$

secunda praebebit

$$\mathfrak{C}\omega=\tfrac{-2(1-PQ)}{m+PQk}+\tfrac{2(1-P)}{\mathfrak{B}(m+PQk)};$$

quare nunc fiet

$$v+\omega=\tfrac{2(1-P)}{\mathfrak{B}C(m+PQk)}-\tfrac{2(1-PQ)}{\mathfrak{C}(m+PQk)}=\tfrac{-2(1+PQk)}{m+PQk},$$

quae aequatio reducta dabit

$$(1-\mathfrak{B})(1-\mathfrak{C})-(1-\mathfrak{C})P+\mathfrak{B}PQ+\mathfrak{B}\mathfrak{C}PQk=0,$$

quae ad formam hanc reducitur:

$$\tfrac{1-P}{BC}-\tfrac{P(1-Q)}{C}+PQ(1+k)=0,$$

quae aequatio inservit relationi inter litteras B et C definiendae. Littera autem D arbitrio nostro manet relicta, dummodo capiatur positiva. Tandem vero quarta aequatio dat

$$\mathfrak{E}=-\tfrac{2(1-PQkk')}{m+PQk}+\ \tfrac{2(1+PQk)}{m+PQk}=\tfrac{2PQk(1+k')}{m+PQk}$$

qui valor cum sit positivus, debet esse

$$2PQk(1+k')>m+PQk \ \text{ sive } \ PQk(1+2k')>m.$$

Denique destructio marginis colorati postulat hanc aequationem:

$$0=\tfrac{v}{P}+\tfrac{\omega}{PQ}-\tfrac{0}{PQk}+\tfrac{1}{PQkk'}+\tfrac{1}{PQkk'T'},$$

quae substitutis pro v et ω valoribus abit in hanc:

$$0=\frac{-2(1-P)}{\mathfrak{B}(m+PQk)}-\frac{2(1-PQ)}{\mathfrak{C}(m+PQk)Q}+\frac{2(1-P)}{\mathfrak{BC}(m+PQk)Q}+\frac{1}{Qkk'}+\frac{1}{Qkk'T}$$

sive

$$0=\frac{2}{Q(m+PQk)}\left(\frac{(1-P)(1-Q)}{\mathfrak{B}}-1-PQk\right)+\frac{1}{Qkk'}+\frac{1}{Qkk'T}.$$

Ut huic aequationi commodissime satisfaciamus, primo terminos factore $(1-P)$ adfectos ob summam parvitatem reiiciamus, quandoquidem non opus est, ut in hac resolutione summum rigorem sequamur, et habebimus

$$\frac{2(1+PQk)}{m+PQk}=\frac{1}{kk'}\left(1+\frac{1}{T}\right),$$

ubi statim secundum naturam huius speciei telescopiorum supra stabilitam statuamus $PQk=\sqrt{m}$ et $T=\frac{1}{2}$; unde fiet $\frac{2}{\sqrt{m}}=\frac{3}{kk'}$, hinc $kk'=\frac{3\sqrt{m}}{2}$. Quia nunc erit $kk'T=\frac{3\sqrt{m}}{4}=\frac{m}{PQ}$, ita ut sit $PQ=\frac{4}{3}\sqrt{m}$, ob P datum etiam Q definietur. Quia porro est $PQk=\sqrt{m}$, erit $k=\frac{3}{4}$ hincque $k'=2\sqrt{m}$ sicque valores harum litterarum ita se habebunt:

$$P=\frac{50}{51},\quad PQ=\frac{4}{3}\sqrt{m},\quad k=\frac{3}{4},\quad k'=2\sqrt{m}\quad \text{et}\quad T=\frac{1}{2}$$

hincque

$$PQk=\sqrt{m},\quad PQkk'=2m\quad \text{et}\quad PQkk'T=m.$$

Quod nunc ad reliquas litteras *B, C* ... attinet, aequatio supra data, si etiam factor $1-P$ reiiciatur, dabit:

$$\frac{-1+PQ}{C}+PQ(1+k)=0,$$

unde invenitur

$$C=\frac{1-PQ}{PQ(1+k)}=\frac{3-4\sqrt{m}}{7\sqrt{m}}\quad \text{et}\quad \mathfrak{C}=\frac{3-4\sqrt{m}}{3(1+\sqrt{m})}.$$

Litterae autem B et $\mathfrak{B}$ arbitrio nostro permittuntur, ita ut, si prima lens concava ex vitro crystallino paretur, ut supra [§ 342] vidimus, poni conveniat $\mathfrak{B}=\frac{5}{7}$; porro vero litterae $\mathfrak{D}$ et D hinc plane non determinantur, nisi quod utramque positivam esse oportet, ex quo statuamus $D=\theta$ hincque $\mathfrak{D}=\frac{\theta}{1+\theta}$; denique vero erit

$$\mathfrak{B} = \frac{5}{7}, \quad \mathfrak{C} = \frac{3-4\sqrt{m}}{3\left(1+\sqrt{m}\right)}, \quad \mathfrak{D} = \frac{\theta}{1+\theta} \quad \text{et} \quad \mathfrak{E} = \frac{2\left(1+2\sqrt{m}\right)}{1+\sqrt{m}},$$

$$B = \frac{5}{2}, \quad C = \frac{3-4\sqrt{m}}{7\sqrt{m}}, \quad D = \mathfrak{D} \quad \text{et} \quad E = \frac{-2\left(1+2\sqrt{m}\right)}{1+3\sqrt{m}},$$

hincque

$$BC = \frac{5\left(3-4\sqrt{m}\right)}{14\sqrt{m}}, \quad BCD = \frac{5\theta\left(3-4\sqrt{m}\right)}{14\sqrt{m}}, \quad BCDE = \frac{5\theta\left(4\sqrt{m}-3\right)\left(1+2\sqrt{m}\right)}{7\sqrt{m}\left(1+3\sqrt{m}\right)};$$

qui valores uni conspectui ita repraesentantur:

$$\mathfrak{E} = \frac{2\left(1+2\sqrt{m}\right)}{1+\sqrt{m}} \quad \text{hincque} \quad E = \frac{-2\left(1+2\sqrt{m}\right)}{1+3\sqrt{m}};$$

ex quibus elementa nostra penitus determinantur.

Nihil igitur aliud superest, nisi ut semidiameter confusionis ad nihilum redigatur, id quod fit sequente aequatione:

$$\lambda = \frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) - \frac{1}{B^3C^3PQk}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right)$$

$$- \frac{1}{B^3C^3D^3PQkk'}\left(\frac{\lambda''''}{\mathfrak{E}^3} + \frac{\nu}{E\mathfrak{E}}\right) + \frac{\lambda'''''}{B^3C^3D^3E^3m},$$

si scilicet omnes lentes ex eodem vitro sint factae. Sin autem prima lens sit crystallina, reliquae vero coronariae, valor ipsius λ hinc inventus insuper multiplicari debet per $\frac{9875}{8724}\left[=\frac{\mu'}{\mu}\right]$, quae fractio est fere $\frac{17}{15}$ propius vero $\frac{163}{144}$.

Circa hanc vero aequationem observandum est sumi debere $\lambda' = 1$, $\lambda'' = 1$, $\lambda''' = 1$. Pro quinta autem lente, ut utrinque fiat aeque convexa, sumi debet

$$\lambda'''' = 1 + 0,60006\left(1-2\mathfrak{E}\right)^2 = 1 + \frac{0,60006\left(3+7\sqrt{m}\right)^2}{(1+\sqrt{m})^2}.$$

Pro sexta vero $\lambda''''' = 1,60006$.

COROLLARIUM 1

360. Pro his igitur telescopiis cum fiat $M = \frac{2}{m+\sqrt{m}}$, erit semidiameter campi apparentis $\Phi = \frac{1718}{m+\sqrt{m}}$ min.

COROLLARIUM 2

361. Semidiametri autem aperturae singularum lentium ita definiuntur ex § 23:

Pro prima $= x$,
pro secunda $= \frac{x}{P}$,
pro tertia $= \frac{r}{2\sqrt{m}} \pm \frac{x}{PQ}$,
pro quarta $= 0s \pm \frac{x}{PQk}$,
pro quinta $= \frac{t}{4} \pm \frac{x}{PQkk'}$,
pro sexta $= \frac{u}{4} \pm \frac{x}{PQkk'T} = \frac{u}{4} \pm \frac{x}{m}$.

COROLLARIUM 3

362. Si in locis imaginum realium velimus diaphragmata constituera, reperitur [§224-227]:
Pro priori semidiameter aperturae $= \frac{2BC}{m+\sqrt{m}} \cdot \frac{\alpha}{4}$.
Pro posteriore vero $= \frac{2BCD}{m+\sqrt{m}} \cdot \frac{\alpha}{4}$.

SCHOLION

363. En ergo duplicem perfectionem huius generis telescopiorum; altera scilicet spectat ad campum apparentem, quem fere duplo maiorem reddidimus; altera vero consistit in destructione confusionis, qua efficitur, ut non opus sit quantitatem α maiorem accipere, quam apertura lentis obiectivae ad claritatem requisita postulat, sicque longitudo telescopii tantopere contrahatur, quantum quidem fieri licet. Cum hic duae lentes post ultimam imaginem reperiantur, quibus campus duplo maior est factus, ita, si tres pluresve lentes adhibere velimus, campum, quousque voluerimus, amplificare licebit. Quod cum vix maiorem calculum postulat quam praecedens problema, operae pretium utique erit hanc investigationem generatim ad quotcunque lentes extendere.

PROBLEMA 3

364. *Praefixa, ut ante lente concava plures lentes post ultimam imaginem realem ita disponere, ut campus apparens, quantum libuerit, amplificetur.*

SOLUTIO

Hic omnia prorsus manent ut in problemate antecedente, quod scilicet ad elements, distantias focales et intervalla lentium attinet, hoc tantum discrimine, ut ambae series litterarum *B, C, D* etc. et *P, Q, k, k', T* etc. ulterius continuari debeant. Deinde littera *M,* qua campus apparens definitur, alium nanciscetur valorem a numero lentium post ultimam imaginem inserendarum. Sit igitur harum lentium numerus $=i$ eritque

$$M=\frac{v+\omega+i}{m-1}.$$

Tum vero aequationes fundamentales se habebunt ut ante, nisi quod ulterius progrediantur; post tertiam autem quamlibet sequentium ope tertiae definiamus, uti sequitur:

$$1.\ \mathfrak{B}v=-(1-P)M$$
$$2.\ \mathfrak{C}\omega=-\left(1-PQ\right)M-v$$
$$3.\ 0=-(1+PQk)M-v-\omega \text{ sive } v+\omega=-(1+PQk)M,$$

unde

$$M\left(m-1\right)=-(1+PQk)M+i$$

et

$$M=\frac{i}{m+PQk},$$
$$4.\ \mathfrak{E}=PQk(1+k')M$$
$$5.\ \mathfrak{F}=PQk(1+k'T)M-1$$
$$6.\ \mathfrak{G}=PQk(1+k'TU)M-2$$
$$7.\ \mathfrak{H}=PQk(1+k'TUV)M-3$$
etc.

Ex primis autem formulis colligetur ut ante

$$\frac{1-P}{BC}-\frac{P(1-Q)}{C}+PQ(1+k)=0,$$

unde, quia *P* proxima $=1$ ideoque *v* pro nihilo haberi potest, erit satis exacte

$$\omega=-(1+PQk)M=-\frac{(1-PQ)M}{\mathfrak{C}},$$

unde colligimus

$$\mathfrak{C}=\frac{1-PQ}{1+PQk} \text{ et } C=\frac{1-PQ}{PQ(1+k)}.$$

Hic autem sufficit hunc valorem vero proxime definivisse, quia aperturae lentium, unde litterae *v,* ω etc. pendent, summam praecisionem respuunt. Quod cum etiam valeat in aequatione, qua margo coloratus destruitur, habebitur loco *M* substituto valore

$$\frac{i(1+PQk)}{m+PQk}=\frac{1}{kk'}\left(1+\frac{1}{T}+\frac{1}{TU}+\frac{1}{TUV}\text{ etc.}\right);$$

quorum terminorum numerus cum sit *i* et singulae litterae *T, U, V* etc. unitate debeant esse minores, statuamus tam conciunitatis gratia, quam ut lentes postremae aequis fere intervallis distent :

$$T=\tfrac{1}{2},\quad U=\tfrac{2}{3},\quad V=\tfrac{3}{4},\quad \mathrm{W}=\tfrac{4}{5}\ \text{etc.},$$

ut factor ipsius $\frac{1}{kk'}$ fiat

$$1+2+3+4\cdots+i=\tfrac{(1+i)i}{2};$$

deinde etiam ut ante ponamus $PQk=\sqrt{m}$, ut prodeat ista aequatio:

$$\tfrac{i}{\sqrt{m}}=\tfrac{1}{kk'}\cdot\tfrac{i(1+i)}{2},$$

unde elicitur

$$kk'=\tfrac{(1+i)\sqrt{m}}{2}.$$

Productum vero reliquarum litterarum

$$TUV\cdots=\tfrac{1}{i};$$

erit

$$kk'TUV\cdots=\tfrac{(1+i)\sqrt{m}}{2i}=\tfrac{m}{PQ}.$$

hincque ergo deducitur

$$PQ=\tfrac{2i\sqrt{m}}{1+i},$$

et quia *P* per se datur, hinc *Q* definietur. Denique ob $PQk=\sqrt{m}$ elicitur

$$k=\tfrac{(1+i)}{2i}\ \text{ et }\ k'=i\sqrt{m}.$$

Hinc ergo valores omnes sequenti modo se habent:

$$PQ=\tfrac{2i\sqrt{m}}{1+i},\quad k=\tfrac{1+i}{2i},\quad k'=i\sqrt{m},$$

$$T=\tfrac{1}{2},\quad U=\tfrac{2}{3},\quad V=\tfrac{3}{4},\quad W=\tfrac{4}{5}\ \text{etc.},$$

$$PQk=\sqrt{m},\quad PQkk'=im,\quad PQkk'T=\tfrac{im}{2},$$

$$PQkk'TU=\tfrac{im}{3}\quad\text{et}\quad PQkk'TUV\ldots=\tfrac{im}{i}=m.$$

Circa litteras *B, C, D* etc. prima *B* cum tertia *D* hinc non definitur; iam vero ostendimus esse

$$C=\frac{1-PQ}{PQ(1+k)}=\frac{1+i-2i\sqrt{m}}{(1+3i)\sqrt{m}}$$

et

$$\mathfrak{C} = \frac{1-PQ}{1+kPQ} = \frac{1+i-2i\sqrt{m}}{(1+i)\left(1+\sqrt{m}\right)}$$

Ponamus igitur ut ante

$$D = \theta \quad \text{et} \quad \mathfrak{D} = \frac{\theta}{1+\theta};$$

sequentes vero erunt

$$\mathfrak{E} = \frac{i\left(1+i\sqrt{m}\right)}{1+\sqrt{m}},$$

$$\mathfrak{F} = \frac{i\left(2+i\sqrt{m}\right)}{2\left(1+\sqrt{m}\right)} - 1,$$

$$\mathfrak{G} = \frac{i\left(3+i\sqrt{m}\right)}{3\left(1+\sqrt{m}\right)} - 2,$$

$$\mathfrak{H} = \frac{i\left(4+i\sqrt{m}\right)}{4\left(1+\sqrt{m}\right)} - 3,$$

quarum litterarum penultima erit

$$\frac{2(i-1)+(3i-2)\sqrt{m}}{(i-1)\left(1+\sqrt{m}\right)}$$

et ultima $=1$.

Has igitur quoque litteras hic coniunctim aspectui exponamus:

$$\mathfrak{B} = \tfrac{5}{7} \ \text{circiter} \qquad B = \tfrac{5}{2} \ \text{vel circiter}$$

$$\mathfrak{C} = \frac{\left(2i\sqrt{m}-i-1\right)}{(1+i)\left(1+\sqrt{m}\right)} \qquad C = \frac{-\left(2i\sqrt{m}-i-1\right)}{(1+3i)\sqrt{m}}$$

$$\mathfrak{D} = \frac{\theta}{1+\theta} \qquad D = \theta$$

$$\mathfrak{E} = \frac{i+ii\sqrt{m}}{1+\sqrt{m}} \qquad E = \frac{-\left(i+ii\sqrt{m}\right)}{(i-1)\left(1+(i+1)\sqrt{m}\right)}$$

$$\mathfrak{F} = \frac{2(i-1)+(ii-1\cdot 2)\sqrt{m}}{2\left(1+\sqrt{m}\right)}, \qquad F = \frac{-\left(2(i-1)+(ii-1\cdot 2)\sqrt{m}\right)}{(i-2)\left(2+(i+2)\sqrt{m}\right)}$$

$$\mathfrak{G} = \frac{3(i-1)+(ii-2\cdot 3)\sqrt{m}}{3\left(1+\sqrt{m}\right)}, \qquad G = \frac{-\left(3(i-1)+(ii-2\cdot 3)\sqrt{m}\right)}{(i-3)\left(3+(i+3)\sqrt{m}\right)}$$

$$\mathfrak{H} = \frac{4(i-3)+(ii-3\cdot 4)\sqrt{m}}{4\left(1+\sqrt{m}\right)}, \qquad G = \frac{-\left(4(i-1)+(ii-3\cdot 4)\sqrt{m}\right)}{(i-4)\left(4+(i+4)\sqrt{m}\right)}$$

etc.,

ex quibus valoribus omnia elementa secundum formulas satis cognitas definiri possunt. Deinde vero ut omnis confusio tollatur, haec aequatio erit adimplenda:

$$\lambda = \frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) - \frac{1}{B^3C^3PQk}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right)$$

$$-\frac{1}{B^3C^3D^3PQkk'}\left(\frac{\lambda''''}{\mathfrak{E}^3} + \frac{\nu}{E\mathfrak{E}}\right) + \frac{1}{B^3C^3D^3E^3PQkk'T}\left(\frac{\lambda'''''}{\mathfrak{F}^3} + \frac{\nu}{F\mathfrak{F}}\right)$$

$$-\frac{1}{B^3C^3D^3E^3F^3PQkk'TU}\left(\frac{\lambda''''''}{\mathfrak{G}^3} + \frac{\nu}{G\mathfrak{G}}\right) + \text{etc.},$$

ubi ut ante notandum est, si lens prima concava ex vitro crystallino paretur, reliquae autem omnes ex coronario, tum valorem hinc pro λ inventum insuper multiplicare debere per fractionem $\frac{9875}{8724}$; quo casu, siquidem statuatur $\mathfrak{B} = \frac{5}{7}$, etiam omnis confusio a diversa refrangibilitate radiorum oriunda tolli deberet, scilicet secundum Dollondi experimenta. Ceterum, ut iam monuimus, pro litteris λ', λ'' et λ''' unitas poni poterit. Pro sequentibus vero lentibus, quae omnes utrinque aeque convexae esse debent, statui debet

$$\lambda'''' = 1 + 0,60006\left(2\mathfrak{E} - 1\right)^2,$$
$$\lambda''''' = 1 + 0,60006\left(2\mathfrak{F} - 1\right)^2,$$
$$\lambda'''''' = 1 + 0,60006\left(2\mathfrak{G} - 1\right)^2, \text{ etc.}$$

COROLLARIUM 1

365. Hoc igitur modo campi apparentis semidiameter erit

$$\Phi = \frac{i\xi}{m+\sqrt{m}} \quad \text{sive} \quad \Phi = \frac{859i}{m+\sqrt{m}} \text{ minut.},$$

ac si pro lente ultima fuerit distantia focalis $= \zeta$ pro loco oculi habebimus

$$O = \frac{\xi}{Mm} = \frac{\zeta\left(m+\sqrt{m}\right)}{im} = \frac{\zeta\left(m+\sqrt{m}\right)}{i\sqrt{m}},$$

unde, si multiplicatio fuerit praemagna, erit $O = \frac{\zeta}{i}$.

COROLLARIUM 2

366. Semidiametri aperturae singularum lentium ita definientur:

Pro prima $= x$, pro quinta $= \frac{t}{4} \pm \frac{1}{im} x$,
pro secunda $= \frac{x}{P}$, pro sexta $= \frac{u}{4} \pm \frac{2}{im} x$,
pro tertia $= \frac{i}{\sqrt{m}} \cdot \frac{r}{4} \pm \frac{x(1+i)}{2i\sqrt{m}}$, pro septima $= \frac{v}{4} \pm \frac{3}{im} x$.
pro quarta $= 0\frac{s}{4} \pm \frac{x}{\sqrt{m}}$, etc.

COROLLARIUM 3

367. Circa diaphragmata eadem est ratio ut in problemate praecedente; scilicet pro diaphragmate in loco prioris imaginis collocando debet esse

$$= \frac{iBC}{m+\sqrt{m}} \cdot \frac{\alpha}{4},$$

pro altero autem diaphragmate unde patet $= \frac{iBCD}{m+\sqrt{m}} \cdot \frac{\alpha}{4}$, haec foramina eo maiora fieri debere, quo magis campus amplificetur.

SCHOLION

368. Hoc igitur problemate totum huncce de telescopiis tractatum finimus, quoniam cuncta praecepta pro illorum constructione satis sunt exposita neque hic constructiones generales commode exhiberi queant, propterea quod hic non solum quantitates duplicis generis ut ante, ubi scilicet vel numeri absoluti vel per multiplicationem *m* divisi occurrebant, sed triplicis adeo generis, scilicet praeter numeros absolutos quantitates primo per $\sqrt{m}$ vel etiam per *m* divisae, in computum sunt, ducendae, ita ut ex comparatione duorum casuum nulla conclusio generalis colligi queat. Nihil igitur aliud hic restat, nisi ut pro qualibet multiplicatione, quam quis postulat, atque etiam pro quantitate campi seu valore numeri *i* calculus ab initio instituatur, quem pro quovis casu oblato suscepisse ob rei dignitatem sine dubio operae erit pretium. In quo quidam negotio etiam littera θ, quae arbitrio nostro hactenus est permissa, determinari debet, quam commode unitati aequalem vel maiorem assumere licet. Videtur autem aptissime poni posse $\theta = 2$, unde posteriora instrumenti intervalla non nimis augentur, simul vero valor pro λ notabiliter minor prodit, quam si esset $\theta = 1$. Quo autem totus iste calculus facilius suscipi et absolvi queat, aliquot exempla hic subiungamus.

EXEMPLUM

369. Sit $m = 49$, ut sit $\sqrt{m} = 7$, et pro campo apparente $i = 2$, ita ut telescopium ex sex lentibus sit componendum, et sumatur praeterea $\theta = 2$.

I. Primo colligantur litterae *P, Q* etc., ut sequitur,

$$P=\tfrac{50}{51},\quad PQ=\tfrac{28}{3},\quad k=\tfrac{3}{4},\quad k'=14,\quad T=\tfrac{1}{2}$$

$\text{Log.}\frac{1}{P}=0,0086002$	$\text{Log.}\frac{1}{PQ}=9,0299632$
$\text{Log.}\frac{1}{PQk}=9,1549019$	$\text{Log.}\frac{1}{PQkk'}=8,0087738$
$\text{Log.}\frac{1}{PQkk'T}=8,3098038$	
$\mathfrak{B}=\ \frac{5}{7},\quad l.\mathfrak{B}=9,8538719$	$B=\ \frac{5}{2},\quad l.B=0,3979399$
$\mathfrak{C}=-\frac{25}{24},\quad l.\mathfrak{C}=0,0177287(-)$	$C=-\frac{25}{49},\quad l.C=9,7077438(-)$
$\mathfrak{D}=\ \frac{15}{4},\quad l.\mathfrak{D}=9,8239086$	$D=\ 2,\quad l.D=0,3010300$
$\mathfrak{D}=\ \frac{15}{4},\quad l.\mathfrak{D}=0,5740313$	$E=-\frac{15}{11},\quad l.E=0,1346984(-)$

Ex his logarithmis formantur sequentes:

$$\begin{array}{ll} l.BC \;\;=0,1056837(-) & l.\,BCD=0,4067137(-) \\ l.BCDE=0,5414121(+) & l.B\mathfrak{B}\;\;=0,2518118(+) \\ l.C\mathfrak{C}\;\;=9,7254725(+) & l.\,D\mathfrak{D}\;\;=0,1249386(+) \\ l.E\mathfrak{E}\;\;=0,7087297(-). & \end{array}$$

II. Hoc quasi primo labore confecto colligamus nostra elementa, quae ita se habebunt:

$b=-1,02\alpha$	$\beta=-2,55\alpha$	$q=-\ 0,72857\alpha$
		$\text{Log.}\frac{q}{\alpha}=9,8624713(-)$
$c=+0,26785\alpha$	$\gamma=-0,13666\alpha$	$r=-0,27901\alpha$
		$\text{Log.}\frac{r}{\alpha}=9,4456318(-)$
$d=-0,18221\alpha$	$\delta=-0,36443\alpha$	$s=-0,12148\alpha$
$e=-0,02603\alpha$	$\varepsilon=+0,03549\alpha$	$\text{Log.}\frac{s}{\alpha}=9,0844942(-)$
$f=-0,07099\alpha$		$t=-0,09762\alpha$
		$\text{Log.}\frac{t}{\alpha}=8,9895188(-)$
		$u=-0,07099\alpha$

Pro oculo autem erit $O=\frac{4u}{7}=-0,04057\alpha$.

III. Hinc iam lentium intervalla cognoscuntur:

$$\begin{array}{ll} 1. & \alpha + b = -0,02000\alpha \\ 2. & \beta + c = -2,28215\alpha \\ 3. & \gamma + d = -0,31887\alpha \\ 4. & \delta + e = -0,39046\alpha \\ 5. & \varepsilon + f = -0,03550\alpha \\ 6. & O = -0,04057\alpha \\ \hline & \text{Tota longitudo} = 3,08755\alpha. \end{array}$$

Deinde etiam diaphragmata ita definiuntur:
Prius post lentem tertiam ad distantiam $\gamma = -\ 0,13666\alpha$ ponitur.
Eius semidiameter foraminis $= -0,0114\alpha$.
Posterius ponitur post quartam lentem ad distantiam $\delta = -0,36443\alpha$.
Eius semidiameter foraminis $= -0,0228\alpha$.
Porro vero semidiameter campi apparentis erit $30\frac{2}{3}$ minut.

IV. Nunc singulas lentes examinari conveniet, quarum non solum constructio, sed etiam momentum confusionis, quod quaelibet ad valorem λ confert, est definiendum, ubi quidem prima lens ultimo loco, postquam scilicet valor λ fuerit inventus, tractari debebit. Quoniam igitur sequentes lentes omnes ex vitro coronario fieri sumuntur, valores eo pertinentes erunt:

$$\begin{array}{rl} \nu = 0,2196, & \text{Log.}\nu = 9,3416323, \\ \sigma = 1,6601, & \\ \rho = 0,2267, & \\ \hline \sigma - \rho = 1,4334, & \text{Log.}(\sigma - \rho) = 0,1568674, \\ \tau = 0,9252. & \end{array}$$

Nunc igitur singulas lentes post primam ordine percurramus.

Pro lente secunda

$$1.\quad \text{Radius} \begin{cases} \text{anterior} = \dfrac{1}{\sigma - \mathfrak{B}(\sigma-\rho) + \tau\sqrt{(\lambda'-1)}} \\ \text{posterior} = \dfrac{1}{\sigma + \mathfrak{B}(\sigma-\rho) - \tau\sqrt{(\lambda'-1)}}, \end{cases}$$

quae formulae ex superioribus facile eliciuntur. Hic vero est $\lambda' = 1$ et calculus ita instituatur:

$l.(\sigma-\rho)=0{,}1563674$ $l.\mathfrak{B}=9{,}8538719$ $0{,}0102393$ $\mathfrak{B}(\sigma-\rho)=1{,}02386$	$\sigma=1{,}6601$ subtr. $1{,}0239$ $0{,}6362$ denom. radii anter. $\rho=0{,}2267$ add. $1{,}0239$ $1{,}2506$ denom. radii poster.
$\log.\frac{q}{\alpha}=9{,}8624713(-)$ log. denom. $=9{,}8035937$	$9{,}8624713(-)$ $0{,}0971184$
$0{,}0588776(-)$ radius anterior $=-1{,}14519\alpha$	$9{,}7653529(-)$ radius posterior $=-0{,}58257\alpha.$

2. Semidiameter aperturae requiritur

$$=\frac{51}{50}x=\frac{51}{50}\cdot\frac{m}{50}\ \text{dig.}$$

3. Calculus pro momento confusionis:

$l.\frac{1}{P}=-\ 0{,}0086002$	$l.\lambda'=\ \ 0{,}0000000$ $l.\mathfrak{B}^2=-\ 9{,}5616157$ $0{,}4383843$ adde log. coeffic. $=0{,}0086002$ $0{,}00469845$	$l.v=9{,}3416323$ $l.B\mathfrak{B}=0{,}2518118$ $9{,}0898205$ $0{,}0086002$ $9{,}0984207$

Ergo pars prior $=2{,}79888$
posterior $=0{,}12543$
Momentum confusionis $=2{,}92431.$

Pro lente tertia

$$1.\ \text{Radius}\begin{cases}\text{anterior} =\dfrac{1}{\sigma-\mathfrak{C}(\sigma-\rho)+\tau\sqrt{(\lambda''-1)}}\\[2ex] \text{posterior}=\dfrac{1}{\sigma+\mathfrak{C}(\sigma-\rho)-\tau\sqrt{(\lambda''-1)}},\end{cases}$$

ubi notetur esse $\lambda''=1$.

$l.(\sigma-\rho)=0,1563674$	$\sigma=1,6601$	$\rho=0,2267$
$l.(-\mathfrak{C})=0,0177287$	$+\ 1,4931$	$-1,4931$
$0,174096$	$3,1532$	$-1,2664$
$\mathfrak{C}(\sigma-\rho)=-1,49313$	denom. anter.	denom. poster.

$\text{Log.}\frac{r}{\alpha}=9,4456318(-)$	$9,4456318\ (-)$
$\text{Log. denom.}=0,4987515(+)$	$0,1025709(-)$
$8,9468803(-)$	$9,3430609(\ +)$

Ergo

$$\text{radius}\begin{cases}\text{anterior} &= -0,08848\alpha\\ \text{posterior} &= +\ 0,22032\alpha.\end{cases}$$

2. Semidiameter aperturae requisita

$$=\frac{2}{7}\cdot\frac{r}{4}+\frac{3}{28}x \ \text{ sive } \ =0,02\alpha+\frac{3}{28}x,$$

quam aperturam haec lens utique sustinere potest.

3. Calculus pro momento confusionis:

		$l.v=9,3416323$
	$l.\lambda''=0,0000000$	$l.C\mathfrak{C}=9,7254725$
$l.\frac{1}{PQ}=9,0299632$	$3l.\ \mathfrak{C}=0,0531861(-)$	$9,6161598$
$3l.\ B=1,1938197$	$9,9468139$	$7,8361435$
$7,8361435$	$7,8361435$	$7,4523033$
	$7,7829574(-)$	

Ergo pars prior $=+\ 0,00606$
posterior $=-\ 0,00283$
Momentum confusionis $=0,00323.$

Pro lente quarta

$$1.\ \text{Radius}\begin{cases}\text{anterior} &= \frac{1}{\sigma-\mathfrak{D}(\sigma-\rho)+\tau\sqrt{(\lambda'''-1)}}\\ \text{posterior} &= \frac{1}{\sigma+\mathfrak{D}(\sigma-\rho)-\tau\sqrt{(\lambda'''-1)}},\end{cases}$$

ubi iterum sumatur $\lambda'''=1.$

$l.\,(\sigma-\rho)=0{,}1563674$	$\sigma=1{,}6601$	$\rho=0{,}2267$
$l.\,\mathfrak{D}\;=\;9{,}8239086$	$0{,}9556$	$0{,}9556$
$l.\mathfrak{D}(\sigma-\rho)\;=\;9{,}9802760$	$0{,}7045$	$1{,}1823$
$\mathfrak{D}(\sigma-\rho)\;=\;0{,}95560$	denom. anter.	denom. poster.

$\log.\ \frac{s}{\alpha}=9{,}0844942(-)$	$9{,}0844942\ (-)$
$\log.\ \text{denom.}=9{,}8478810$	$0{,}0727277$
$=9{,}2366132\ (-)$	$9{,}0117665\ (-)$

$$\text{radius}\begin{cases}\text{anterior} & =-0{,}17243\alpha\\ \text{posterior} & =-0{,}10273\alpha.\end{cases}$$

2. Semidiameter aperturae requisita $=\frac{1}{7}x$, quam aperturam lens commode sustinebit; si enim minor radius lentis secundae, qui est $0{,}58257\alpha$, sustinet aperturam x, hic radius minor, qui est $0{,}10273\alpha$, commode sustinebit aperturam $\frac{1}{7}x$.

3. Calculus pro momento confusionis:

	$l.\lambda'''=0{,}0000000$	$l.v\;=9{,}3416323$
$l.\ \frac{1}{PQk}=\ 9{,}1549019$	$3l.\ \mathfrak{D}=9{,}4717258$	$l.\mathfrak{D}D=0{,}1249386$
$3l.BC=\frac{0{,}3170511(-)}{8{,}8378508}$	$0{,}5282742$	$9{,}2166937$
	$8{,}8378508$	$8{,}8378508$
	$9{,}3661250$	$8{,}0545445$

$$\begin{aligned}\text{Ergo pars prior} &= 0{,}23234\\ \text{posterior} &= 0{,}01133\\ \text{Momentum confusionis} &= 0{,}24367.\end{aligned}$$

Pro lente quinta

1. Quia haec lens utrinque debet esse aeque convexa, ob eius distantiam focalem

$$t=-0{,}09762\alpha$$

erit

$$\text{radius utriusque faciei } =1{,}06t=-0{,}10348\alpha\,;$$

nunc vero erit $\lambda'''=1+0{,}60006\left(2\mathfrak{C}-1\right)^2$; at est $2\mathfrak{C}-1=6{,}5$, ergo

$$\log.\left(2\mathfrak{C}-1\right)=0{,}8129134$$

et

$$\begin{array}{r l}
\log.(2\mathfrak{C}-1)^2 = & 1,6258268 \\
\log.\ 0,60006 = & \underline{9,7781947} \\
 & 1,4040215
\end{array}$$

adeoque $\lambda'''' = 26,352$.

2. Semidiameter aperturae hic per hypothesin est $\frac{1}{4}t = -0,02440\alpha$; altera enim pars $\frac{1}{98}x$, quam haec lens facillime patitur.

3. Calculus pro momento confusionis:

$$\left|\begin{array}{r}
l.\frac{1}{PQkk'} = 8,0087738 \\
3l.BCD = \underline{1,2201411} \\
6,7886327 \\
\\
\end{array}\right|\begin{array}{r}
l.\lambda'''' = 1,4208136 \\
3l.\mathfrak{C} = \underline{1,7220939} \\
9,6987197 \\
\underline{6,7886327} \\
6,4873524
\end{array}\left|\begin{array}{r}
l.v = 9,3416323 \\
l.E\mathfrak{C} = \underline{0,7087297} \\
8,6329026 \\
\underline{6,7886327} \\
5,4215353
\end{array}\right.$$

$$\begin{array}{r l}
\text{Ergo pars prior} = & 0,00031 \\
\text{posterior} = & \underline{-0,00002} \\
\text{Momentum confusionis} = & 0,00029.
\end{array}$$

Pro lente sexta

1. Quia per hypothesin haec lens utrinque debet esse aeque convexa, ob eius distantiam focalem

$$u = -0,07099\alpha$$

erit

$$\text{radius utriusque faciei } 1,06u = -0,07525\alpha;$$
$$\text{tum vero erit } \lambda''''' = 1,60006.$$

2. Semidiameter aperturae $= \frac{1}{4}u = -0,01775\alpha$.

3. Calculus pro momento confusionis:

$$\left|\begin{array}{r}
l.\frac{1}{PQkk'T} = 8,3098038 \\
3l.BCDE = \underline{1,6242363} \\
6,6855675
\end{array}\right|\begin{array}{r}
l.\lambda''''' = 0,2041363 \\
\underline{6,6855675} \\
6,8897038
\end{array}$$

Ergo momentum confusionis $= 0,00077$.

V. His inventis colligantur omnia momenta confusionis in unam summam, quae erit 3,17227. Nunc autem duo casus sunt considerandi, prout primam lentem concavam vel ex vitro coronario vel ex crystallino parare voluerimus, quos seorsim evolvi oportet.

1. Pro prima lente concava ex vitro coronario paranda
Pro hac ergo lente erit

$$\lambda = 3{,}17227, \text{ unde } \lambda - 1 = 2{,}17227;;$$

hincque fiet sequens calculus:

$$\begin{array}{r|l} \text{Log.}(\lambda-1) = 0{,}3369138 & \\ \text{Log.}\sqrt{(\lambda-1)} = 0{,}1684569 & \\ \text{Log.}\tau = 9{,}9662356 & \text{ergo} \\ 0{,}1346925 & \tau\sqrt{(\lambda-1)} = 1{,}3636. \end{array}$$

Nunc cum sit pro hac lente

$$\text{radius}\begin{cases} \text{anterior} = \frac{1}{\sigma - \tau\sqrt{(\lambda-1)}} \\ \text{posterior} = \frac{1}{\sigma + \tau\sqrt{(\lambda-1)}}, \end{cases}$$

calculus ita se habebit:

$$\begin{array}{rr} \sigma = 1{,}6601 & \rho = 0{,}2267 \\ \tau\sqrt{(\lambda-1)} = 1{,}3636 & 1{,}3636 \\ 0{,}2965 & 1{,}5903 \end{array}$$

$$\begin{array}{r|r} l.0{,}2965 = 9{,}4720247 & l.1{,}5903 = 0{,}2014791 \\ \text{complementum} = 0{,}5279753 & \text{complementum} = 9{,}7985208 \end{array}$$

sicque prodit

$$\text{radius}\begin{cases} \text{anterior} = 3{,}37268\alpha \\ \text{posterior} = 0{,}62881\alpha \end{cases}$$

semidiametro aperturae existante $x = \frac{m}{50}\text{dig.} = 1 \text{ dig.}$

2. Pro prima lente concava ex vitro crystallino paranda
Pro hac igitur lente erit

$$\lambda = \tfrac{9875}{8724} \cdot 3{,}17227 \text{ seu } \lambda = 3{,}59080,$$

et quia pro vitro crystallino est

$$\rho = 0{,}1414, \quad \sigma = 1{,}5827, \quad \tau = 0{,}8775,$$

calculus ita se habebit:

Log. $(\lambda-1) = 0,4134339$	
Log. $\sqrt{(\lambda-1)} = 0,2067169$	
Log. $\tau = 9,9432471$	ergo
$0,1499640$	$\tau\sqrt{(\lambda-1)} = 1,41242$
$\sigma = 1,5827$	$\rho = 0,1414$
subtr. $1,4124$	add. $1,4124$
$0,1703$	$1,5538$
log. $9,2312146$	log. $0,1913951$
complementum $0,7687854$	complementum $9,8086049$

sicque prodit

$$\text{radius}\begin{cases}\text{anterior} = 5,87199\alpha\\ \text{posterior} = 0,64358\alpha\end{cases}$$

semidiametro aperturae existente $x = \frac{m}{50}\text{dig.} = 1$ dig.

VI. Quia binae priores lentes coniunctim lentem obiectivam constituunt, cuius semidiameter aperturae $= 1$ dig., statuatur earum minimus radius, qui est $-\ 0,58257\alpha$, $>$ 4 dig. hincque concludetur sumi debere $-\alpha > \frac{4}{0,58257}$ dig., hoc est $-\alpha > 7$ dig. vel saltim non minus, ita ut, si optimus successus sperari posset, accipere liceret $-\alpha = 7$ dig. Sin autem aberratio quaedam sit pertimescenda, tantum opus erit mensuram unius digiti augere. Commoditatis autem gratia sumamus $\alpha = -10$ dig.; unde sequens prodit constructio huius telescopii determinata pro multiplicatione $m = 49$.

1. Pro lente obiectiva
quatenus ex vitro coronario paratur

$$\text{Radius faciei}\begin{cases}\text{anterioris} = -33,73 \text{ dig.}\\ \text{posterioris} = -\ \ 6,29 \text{ dig.}\end{cases}\text{Crown Glass.}$$

(1). Pro lente obicetiva
quatenus ex vitro crystallino paratur

$$\text{Radius faciei}\begin{Bmatrix}\text{anterioris} &= -58,72\ \text{dig.}\\ \text{posterioris} &= -\ \ 6,44\ \text{dig.}\end{Bmatrix}\text{Flint Glass.}$$

Cuius distantia focalis pro utroque casu $= -10$ dig.
Semidiameter aperturae $= 1$ dig.
Intervallum ad secundam $= 0,2 = \frac{1}{5}$ dig.

2. Pro lente secunda

$$\text{Radius faciei}\begin{Bmatrix}\text{anterioris} &= 11,45\ \text{dig.}\\ \text{posterioris} &= \ \ 5,83\ \text{dig.}\end{Bmatrix}\text{Crown Glass.}$$

Cuius distantia focalis $= 7,28$ dig.
Semidiameter aperturae $= 1$ dig.
Intervallum ad tertiam $= 22,82$ dig.

3. Pro lente tertia

$$\text{Radius faciei}\begin{Bmatrix}\text{anterioris} &= \ \ 0,884\ \text{dig.}\\ \text{posterioris} &= -\ 2,20\ \text{dig.}\end{Bmatrix}\text{Crown Glass.}$$

Cuius distantia focalis $= 2,79$ dig.
Semidiameter aperturae $= 0,3$ dig.
Intervallum ad quartam $= 3,19$ dig.

4. Pro lente quarta

$$\text{Radius faciei}\begin{Bmatrix}\text{anterioris} &= 1,72\ \text{dig.}\\ \text{posterioris} &= 1,03\ \text{dig.}\end{Bmatrix}\text{Crown Glass.}$$

Cuius distantia focalis $= 1,21$ dig.
Semidiameter aperturae $= \frac{1}{7}$ dig.
Intervallum ad quintam $= 3,90$ dig.

5. Pro lente quinta

Radius utriusque faciei $= 1,03$ dig. Crown Glass.

Cuius distantia focalis est 0,97 dig.
Semidiameter aperturae $= \frac{1}{4}$ dig.
lntervallum ad sextam $= 0,36$ dig.

6. Pro lente sexta
Radius faciei utriusque $= 0,75$ dig. Crown Glass.
Cuius distantia focalis $= 0,71$ dig.

Semidiameter aperturae $= 0,18 = \frac{1}{6}$ dig.

Distantia ad oculum usque $= 0,40$ dig.

Huius igitur telescopii longitudo tota fiet

$= 30,87$ dig. $= 2\frac{1}{2}$ ped.

et semidiameter campi apparentis $= 30\frac{2}{3}$ min.