CHAPTER III

CONCERNING SIMPLE MICROSCOPES,

CLEARLY FREED FROM ALL CONFUSION,

CONSTRUCTED FROM GLASS EITHER OF THE SAME KIND,

OR OF DIFFERENT KINDS.

76. *If the microscope may be constructed from two lenses with the first concave and the second convex placed together close to each other, to effect that the confusion arising from the aperture may be removed completely.*

SOLUTION

Since here two lenses only are present, the minimum separation of these $Aa(1-\frac{1}{P})$ may be put $=\eta a$ and hence there will become $P=\frac{A}{A-\eta}$; then since the focal lengths shall become

$$p=\mathfrak{A}a \text{ and } q=-\frac{A\mathfrak{B}}{P}\cdot a,$$

on account of the first concave lens $\mathfrak{A}$ must be negative and hence $A<0$; whereby the second lens certainly shall be convex on account of $\mathfrak{B}=1$. Again the magnification will be expressed thus, so that there shall be $m=P\cdot\frac{h}{a}$; from which the distance is deduced

$$a=P\cdot\frac{h}{m}=\frac{A}{A-\eta}\cdot\frac{h}{m},$$

thus so that the distance of the object also shall be required to be less than $\frac{h}{m}$. Now so that the confusion arising from the aperture may be reduced to zero, it will be required for this equation to be satisfied:

$$\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{\lambda'}{A^3P}=0,$$

if indeed both lenses may be constructed from the same kind of glass. But if they may be prepared from the different kinds of glass, for the second case in place of μ there may be written μ' and this equation will be had :

$$\frac{\mu\lambda}{\mathfrak{A}^3}+\frac{\mu\nu}{A\mathfrak{A}}-\frac{\mu'\lambda'}{A^3P}=0.$$

Which case we establish here, since the case scarcely becomes more complicated, and from this equation we will be able to define either λ or λ'; clearly there will become

$$\text{either} \quad \lambda = \frac{\mu'}{\mu} \cdot \frac{(1-\mathfrak{A})^3}{P} \cdot \lambda' - \nu \mathfrak{A}(1-\mathfrak{A})$$

$$\text{or} \quad \lambda' = \frac{\mu}{\mu'} \cdot \frac{P}{(1-\mathfrak{A})^3} \cdot \lambda + \frac{\mu\nu}{\mu'} \cdot \frac{P\mathfrak{A}}{(1-\mathfrak{A})^3},$$

thus so that at this point the letter $\mathfrak{A}$ may be left to our choice, provided it may be taken negative ; whereby this letter will be allowed to be defined thus, so that also the other confusion may be removed arising from the different refrangibility; which in the end if, as we have done above, there may be put $\frac{dn}{n-1} = N$ for the first lens and $\frac{dn'}{n'-1} = N'$ for the second, it will be required for this equation to be satisfied:

$$0 = N \cdot \frac{1}{\mathfrak{A}} - \frac{N'}{P} \cdot \frac{1}{A},$$

from which there is deduced:

$$\frac{\mathfrak{A}}{A} = \frac{N}{N'} \cdot P = 1 - \mathfrak{A},$$

thus, so that on account of $P = 1$, there may become approximately:

$$\mathfrak{A} = 1 - \frac{N}{N'} = \frac{N' - N}{N'},$$

which value, since it must be negative, it is necessary, that there shall be $N' < N$.

Therefore we may take the first lens concave made from crystal glass, and truly the latter made from crown glass, and on account of $N : N' = 10 : 7$ there will become $\mathfrak{A} = -\frac{3}{7}$, for which value we can be content. But if we may demand a more exact value, in place of P we may substitute its value and our equation will become :

$$0 = \frac{10}{\mathfrak{A}} - 7\frac{(\mathrm{A}-\eta)}{A^2} = 10A(A+1) - 7(A-\eta),$$

which by taking as before $\eta = \frac{1}{5}$ will give:

$$A = -\frac{3}{20} \mp \sqrt{\frac{9}{400} - \frac{7}{50}};$$

which value evidently is imaginary, thus so that it shall be unable to satisfy this condition, unless the distance ηa may be taken much smaller, clearly there must be taken $\eta < \frac{1}{31}$; but since such a small distance cannot be obtained with so very small lenses, also this condition will be unable to be satisfied perfectly. Therefore we will be required to be content with a value almost satisfying, especially since the nature of the problem may not permit, that we can fully satisfy this condition; and we may assume as we have found before

$$\mathfrak{A} = -\frac{3}{7}, \quad \text{so that there shall become } A = -\frac{3}{10}$$

and hence

$$P = \frac{3}{3+10\eta}$$

and hence the focal lengths

$$p = -\frac{3}{7}a = \frac{-9}{21+70\eta}\cdot\frac{h}{m}, \quad q = \frac{3}{10}\cdot\frac{h}{m}$$

But for the confusion requiring to be remove there must be taken

$$\lambda = \frac{\mu}{\mu'}\cdot\frac{10^3}{7^3}\cdot\frac{3+10\eta}{3}\cdot\lambda' + \frac{30}{49}\nu;$$

in which form if there may be taken $\lambda' = 1$ and the letters $\mu,\ \mu'$ and ν may be assumed conveniently, there will be found :

$$\lambda = \frac{0,9875}{0,8724}\cdot\frac{10^3}{7^3}\cdot\frac{3+10\eta}{3} + \frac{30}{49}\cdot 0,2529 = 3,3001\left(1+\frac{10}{3}\eta\right) + \frac{30}{49}\cdot 0,25290$$
$$= 3,3001 + 11,0003\eta + 0,155$$

or

$$\lambda = 3,4551 + 11,0003\eta,$$

from which the construction of the first lens must be desired.

COROLLARY 1

77. Since there shall be

$$a = \frac{A}{A-\eta}\cdot\frac{h}{m} = \frac{3}{3+10\eta}\cdot\frac{h}{m},$$

it is apparent the distance of the object hereto be notably smaller than in the case of a simple lens, where there was $a = \frac{h}{m}$; for if we assume $\eta = \frac{1}{5}$, there is produced $a = \frac{3}{5}\cdot\frac{h}{m}$, nor truly can a smaller value be taken for η .

COROLLARY 2

78. Therefore in this manner the first of these inconveniences consisting in the nearness of the object, which we have mentioned above, need hardly be increased, truly the latter, indeed this arising from the confusion of the aperture, may be removed completely ; truly the focal lengths of the lenses will become so small, so that on putting $\eta = \frac{1}{5}$ there may be produced

$$p = -\frac{9}{35}\cdot\frac{h}{m},$$

while for the simple lens there had been $p = \frac{h}{m}$.

SCHOLIUM

79. Then also here an obstacle stands in the way, so that even if we may use two kinds of glass, yet the other confusion may not be able to be removed and thus may arrive at imaginary values ; from which this kind to be rejected and we may progress to establishing the other kind, for which the latter lens is assumed concave.

PROBLEM 2

80. *If a microscope may depend on two lenses of which the first is convex, the second truly concave, to produce such a microscope, so that the confusion arising from the aperture may vanish.*

SOLUTION

With the interval between the lenses put $=\eta a$ there becomes as before $P=\frac{A}{A-\eta}$, and since the focal lengths shall be $p=\mathfrak{A}a$ and $q=-\frac{A}{p}\cdot a$, both $\mathfrak{A}$ as well as A will be positive numbers and thus $\mathfrak{A}<1.$ Truly the magnification will give

$$m=P\cdot\frac{h}{a} \text{ or } a=P\cdot\frac{h}{m}=\frac{A}{A-\eta}\cdot\frac{h}{m}.$$

But the confusion arising from the apertures may vanish, if we may consider both lenses in turn made from diverse kinds of glass, if there were as before

$$\text{either} \quad \lambda=\frac{\mu'}{\mu}\cdot\frac{(1-\mathfrak{A})^3}{P}\cdot\lambda'-\nu\mathfrak{A}(1-\mathfrak{A})$$

$$\text{or} \quad \lambda'=\frac{\mu}{\mu'}\cdot\frac{P}{(1-\mathfrak{A})^3}\cdot\lambda+\frac{\mu\nu}{\mu'}\cdot\frac{P\mathfrak{A}}{(1-\mathfrak{A})^3},$$

and thus the other confusion vanishes, if there were

$$0=N\cdot\frac{1}{\mathfrak{A}}-\frac{N'}{P}\cdot\frac{1}{A}$$

and hence

$$\frac{\mathfrak{A}}{A}=\frac{N}{N'}\cdot P=1-\mathfrak{A};$$

from which, since $\mathfrak{A}<1$ and $1-\mathfrak{A}<1$, there must become $N'>N$, from which reason it will be convenient to make the first lens from crown glass and the second truly concave from crystal glass, thus so that there may become $P>1$, for that is required, so that there shall be $\frac{7}{10}P<1$, just as it is required to note, since $P>1$; or there must be $P<\frac{10}{7}$ and thus $\frac{A}{A-\eta} < \frac{10}{7}$, and consequently $\frac{\eta}{A} < \frac{3}{10}$. Therefore if we wish to satisfy this equation carefully, there must be $\frac{\eta}{A} < \frac{3}{10}$; from which, if we assume $\frac{\eta}{A} = \frac{1}{4}$, there will become $P = \frac{4}{3}$ and hence $1- \mathfrak{A} = \frac{14}{15}$ and $\mathfrak{A} = \frac{1}{15}$ and thus $A = \frac{1}{14}$ and $\eta = \frac{1}{56}$, from which the focal length of the lens produced $\eta a = \frac{1}{56a} = \frac{h}{42m}$; but since in the preceding chapter we have assumed the approximate interval $\frac{1}{5} \cdot \frac{h}{m.}$, it is apparent so small an interval cannot have a place in practice, thus so that in our case it may not be allowed to remove the other confusion. In short therefore it will be required to reject this condition, thus so that now likewise there shall be either lenses from the same kind of glass or perhaps from different kinds; therefore some may be made from the same kind of glass, thus so that there shall be $\mu' = \mu$, and for the first confusion to be removed, since it is agreed for $\mathfrak{A}$ not to be taken exceedingly small, we may put $\mathfrak{A} = \frac{1}{2}$ and hence $A = 1$; from which there becomes

$$P = \frac{1}{1-\eta}, \quad a = \frac{1}{1-\eta} \cdot \frac{h}{m}, \quad p = \frac{a}{2}, \text{ and } q = -(1-\eta)a.$$

But if now we may put $\eta a = \frac{1}{5} p$, it will be required to take $\eta = \frac{1}{10}$ and thus there will become

$$P = \frac{10}{9}, \quad a = \frac{10}{9} \cdot \frac{h}{m}, \quad p = \frac{5}{9} \cdot \frac{h}{m} \text{ and } q = -\frac{h}{m}.$$

and thus the first confusion vanishes, if there shall be

$$\lambda' = \frac{80}{9}\lambda + \frac{20}{9}\nu;$$

from which it will be easy to construct the lenses.

COROLLARY 1

81. Therefore for the construction of the lens, if common glass may be used, for which there is $n = 1,55$ and $\nu = 0,2326$, if there may be taken $\lambda = 1$, there will be

$$\lambda' = 9,406, \text{ from which there becomes } \tau\sqrt{(\lambda' - 1)} = 2,6242.$$

COROLLARY 2

82. Therefore for the first lens, of which the focal distance is $p = \frac{5}{9} \cdot \frac{h}{m} = \frac{40}{9m}$ in. on account of $h = 8$ in. and the numbers $\mathfrak{A} = \frac{1}{2}$ and $\lambda = 1$, the radius will be had

$$\begin{cases}\text{of the anterior face } = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)} = \frac{p}{0{,}9090} = 1{,}1001p = \frac{4{,}89}{m} \text{ in.}\\ \text{of the posterior face} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)} = \frac{p}{0{,}9090} = 1{,}1001p = \frac{4{,}89}{m} \text{ in.,}\end{cases}$$

thus so that this lens shall have both sides equally convex.

COROLLARY 3

83. For the other concave lens, of which the focal length $q = -\frac{8}{m}$ in. and the numbers $\mathfrak{B} = 1$ and $\lambda' = 9,406,$ the radius

$$\begin{cases}\text{of the anterior face } = \frac{q}{\rho+\tau\sqrt{(\lambda'-1)}} = \frac{q}{2{,}8149} = +0,35525q = -\frac{2{,}84}{m} \text{ in.}\\ \text{of the posterior face} = \frac{q}{\sigma-\tau\sqrt{(\lambda'-1)}} = \frac{-q}{0{,}9968} = -1,00321q = +\frac{8{,}03}{m} \text{ in.}\end{cases}$$

[Corrected from Euler's original values by E. Ch. in the *O O* edition.]

COROLLARY 4

84. Therefore the interval to be established between these two lenses will become

$$\eta a = \frac{1}{9}\cdot\frac{h}{m} = \frac{0{,}889}{m} \text{ in.,}$$

moreover the object is required to be located before the first lens at the distance

$$a = \frac{80}{9m} \text{ in.} = \frac{8{,}89}{m} \text{ in. ;}$$

but so that it may extend to the aperture, that will require to be defined from the minimum radius of both the lenses and thus its radius can be taken

$$x = \frac{0{,}71}{m} \text{ in.} = \frac{5}{7m} \text{ in.}$$

Hence there will become $y = \frac{hx}{ma} = \frac{3}{5m},$ for the clarity, from which the measure of the clarity, as established above, $= \frac{12}{m}$.

SCHOLIUM

85. Therefore a cure is brought forth for the inconvenience mentioned above for these microscopes, while it will permit the object to be removed to a greater distance ; truly on

the other hand, since the lenses will be required to be much smaller, which therefore allow only a much smaller aperture, hence a smaller clarity must be produced, which defect is seen to scarcely compensate that quality which has been removed from the first confusion. Therefore without doubt the maximum gain here is going to be had, if the other confusion also may be allowed to be removed, since by using only convex lenses that will not be able to considered. Therefore since this advantage cannot be obtained from two lenses, we will examine the case of three lenses, among which one shall be concave, which is understood from the preceding to be easy to use, which must be prepared from crystal glass, while the rest are constructed from crown glass ; then truly also no doubt can remain, why this concave lens may not be agreed to be used in the third position.

PROBLEM 3

86. *If a microscope depends on three lenses, between which the third lens shall be concave, truly the two anterior lenses convex, to produce such a telescope so that the confusion arising from the aperture may be removed completely.*

SOLUTION

Again the two intervals between these lenses may be put $=\eta a$ and we will have as in problem 2 of the preceding chapter

$$P=\frac{A}{A-\eta},\ \ Q=\frac{(A-\eta)B}{(A-\eta)B+\eta}.$$

Then truly, since the focal lengths shall be

$$p=\mathfrak{A}a,\ \ q=-\frac{A\mathfrak{B}}{P}\cdot a \text{ and } r=\frac{AB}{PQ}\cdot a,$$

from which in the first place $\mathfrak{A}>0$, and since there is

$$\frac{q}{r}=-\frac{Q\mathfrak{B}}{B},$$

on account of $q>0$ and $r<0$ there will have to become :

$$\frac{\mathfrak{B}}{B}>0 \text{ or if } 1-\mathfrak{B}>0 \text{ or } \mathfrak{B}<1.$$

Moreover it will be possible to satisfy these conditions in two ways:

I. If $\mathfrak{A}<1$ and thus $A>0$; and there will become $\mathfrak{B}<0$ and $B<0$.
II. If $\mathfrak{A}>1$ and hence $A<0$, there will become $B>0$ and $\mathfrak{B}>0$, provided $\mathfrak{B}<1$.

$P>1$ and $Q>1$ is produced in the first case, in the latter case truly there becomes $P<1$ and $Q>1$. But it is not defined whether PQ may become greater of smaller than unity. Moreover the magnification m will give $a=PQ\cdot\frac{h}{m}$, according to which it follows, that PQ will be much greater than unity, which happens in the first case, where $P>1$ and $Q>1$.

Now truly we may begin by reducing the latter confusion to zero, which provides this equation, since we have assumed $N'=N$ for crown glass, while N'' shall refer to crystal glass:

$$0=N\cdot\frac{1}{\mathfrak{A}}-\frac{N}{P}\cdot\frac{1}{A\mathfrak{B}}+\frac{N''}{PQ}\cdot\frac{1}{AB}.$$

Therefore the two first convex lenses shall be made from crown glass, the third truly from crystal glass, so that there shall become $N:N''=7:10$, there will be had

$$0=\frac{1}{\mathfrak{A}}-\frac{1}{PA\mathfrak{B}}+\frac{10}{7}\cdot\frac{1}{PQAB};$$

in which if in place of P and Q the values found before may be substituted, there will be produced

$$0=\frac{1}{\mathfrak{A}}-\frac{A-\eta}{A^2\mathfrak{B}}+\frac{10}{7}\cdot\frac{(A-\eta)B+\eta}{A^2B^2},$$

which equation expanded out will be changed into this form:

$$0=A^2B^2+\frac{3}{7}AB+\eta\left(B^2-\frac{3}{7}B+\frac{10}{7}\right),$$

from which it is clear, if there were $\eta=0$, to become $AB=-\frac{3}{7}$ and thus $r=-\frac{3}{7PQ}\cdot a$, thus so that there shall become $-r<\frac{3}{7}a$ on account of $PQ>1$. But since the case $\eta=0$ cannot be used, while rather the value of this letter must be satisfied well enough, this form of the equation found may be given:

$$A^2B^2+\frac{3}{7}AB+\frac{9}{196}=\left(AB+\frac{3}{14}\right)^2=\frac{9}{196}-\eta\left(B^2-\frac{3}{7}B+\frac{10}{7}\right),$$

where it is clear the letter η cannot be greater than

$$\frac{9}{196\left(B^2-\frac{3}{7}B+\frac{10}{7}\right)},$$

if indeed this other confusion may be able to vanish directly; which limit since it shall be extremely small, we may establish

$$\eta = \frac{9}{28\left(7B^2-3B+10\right)}$$

and there will become $AB = -\frac{3}{14}$ and thus r twice as small as before, which is a nuisance in practice. But since it shall not be absolutely necessary to return that confusion directly to zero, thus this suggestion can be proposed, so that on putting $AB = -\frac{3}{14}$, so great a value may be taken for η, as the circumstances will permit, even it this is going to be greater than the limit proposed here.

Finally from these observations the first confusion may be reduced to zero, which happens with the aid of this equation:

$$0 = \frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} - \frac{\nu}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) + \frac{\mu''}{\mu}\cdot\frac{\lambda''}{A^3B^3PQ},$$

from which the number λ'' will be able to be defined by taking $\lambda = 1$ and $\lambda' = 1$.

COROLLARY 1

87. Therefore whatever interval ηa may be assumed, these microscopes will always labor under this defect, that the third lens will become exceedingly small, evidently to become almost five times smaller, than if we were to use a simple lens. Whereby since the smallness of the lens may impede greater magnification, this case will be much less suitable for producing greater magnifications.

COROLLARY 2

88. From that also the prescribed limits for η will produce an exceedingly small value, as may be able to be used in practice, even if the letter B may be chosen by us; indeed the value deduced from that limit

$$\eta = \frac{9}{28\left(7B^2-3B+10\right)}$$

approaches the maximum value, if there may be taken $B = \frac{3}{14}$, which therefore will become $\eta = \frac{9}{271}$ or approximately $\eta = \frac{1}{30}$, which clearly is as exceedingly small as may be allowed to be used in precise.

SCHOLIUM 1

89. Therefore this kind of microscopes promises little joy, even if each kind of confusion may be allowed to be reduced to zero, since each of the letters A may be defined B, both these lenses will be produced exceedingly small as well as an exceedingly small separation of the lenses. But if we do not wish to attend to the confusion arising from the different refrangibility, hence it will be allowed to deduce

these microscopes to be outstanding, among which the following kind may be seen mainly to deserve our attention.

Evidently there may be put $\mathfrak{A} = 1$, so that there shall become $A = \infty$; again there ay be taken $B = 0$, thus so that there shall become $AB = -\theta$, and hence our elements may be defined thus :

$$P = 1, \quad Q = \tfrac{\theta}{\theta - \eta}$$

and

$$p = a, \quad q = \theta a \quad \text{and} \quad r = -(\theta - \eta)a,$$

where θ thus may be taken readily, so that these focal lengths may not become exceedingly small and η also may be left to our choice. Then truly the distance α will become

$$\alpha = \tfrac{\theta}{\theta - \eta} \cdot \tfrac{h}{m}.$$

Thence moreover either all the lenses to be prepared likewise, whether all the lenses will be prepared from the same kind of glass or from different kinds ; yet meanwhile if θ may not differ much from the above given value $\tfrac{3}{14}$, we may follow on with a gain, if we may prepare the third lens from crystal glass, while the two anterior lenses are constructed from crown glass, certainly with which done the other confusion at least will be diminished. But then for the construction of the lens itself this equation is required to be resolved:

$$0 = \lambda + \tfrac{\lambda'}{\theta^3} - \tfrac{0{,}8724}{0{,}9875} \cdot \tfrac{\lambda''(\theta - \eta)}{\theta^4},$$

from which with $\lambda = 1$ and $\lambda' = 1$ there is deduced:

$$\lambda'' = \tfrac{0{,}9875}{0{,}8724} \cdot \tfrac{\theta^4}{\theta - \eta}\left(1 + \tfrac{1}{\theta^3}\right),$$

an example of this to be advanced will not go amiss.

EXAMPLE

90. There may be taken $\eta = \tfrac{1}{5}$ and there shall be $\theta = 1$; and hence we will have

$$P = 1, \quad Q = \tfrac{1}{1 - \eta} = \tfrac{5}{4},$$

$$p = a, \quad q = a, \quad r = -\tfrac{4}{5}a$$

with there being $a = \tfrac{5}{4} \cdot \tfrac{h}{m}$. Then truly the equation requiring to be resolved will be had in this case :

$$\lambda'' = \tfrac{0,9875}{0,8724} \cdot \tfrac{5}{2} = 2,8300;$$

from which for the crystal glass there is deduced :

$$\tau\sqrt{\lambda'' - 1} = 1,1870.$$

From which the following construction will be obtained of the three small-lens microscope .

I. The first lens is prepared from crown glass; the focal length of which since there shall be

$$p = a = \tfrac{5}{4} \cdot \tfrac{h}{m} = \tfrac{10}{m} \text{ in.},$$

and the numbers $\mathfrak{A} = 1$ and $\lambda = 1$, the radius

$$\begin{cases} \text{of the anterior face } = \tfrac{p}{\rho} = \tfrac{p}{0,2267} = 4,4111p = \tfrac{44,11}{m} \text{ in.} \\ \text{of the posterior face} = \tfrac{p}{\sigma} = \tfrac{p}{1,6601} = 0,6024p = \tfrac{6,02}{m} \text{ in.} \end{cases}$$

II. For the second lens also being prepared from crown glass, of which the focal length is :

$$q = a = \tfrac{5}{4} \cdot \tfrac{h}{m} = \tfrac{10}{m} \text{ in.}$$

and the numbers $\mathfrak{B} = 0$ and $\lambda' = 1$, the radius

$$\begin{cases} \text{of the anterior face } = \tfrac{q}{\sigma} = \tfrac{q}{1,6601} = 0,6024q = \tfrac{6,02}{m} \text{ in.,} \\ \text{of the posterior face} = \tfrac{q}{\rho} = \tfrac{q}{0,2267} = 4,4111q = \tfrac{44,11}{m} \text{ in.} \end{cases}$$

III. For the third lens being prepared from crystal glass, of which the focal length is

$$r = -\tfrac{4}{5}a = -\tfrac{h}{m} = -\tfrac{8}{m} \text{ in.}$$

and the numbers $\mathfrak{C} = 1$ and $\lambda'' = 2,8300$, the radius

$$\begin{cases}\text{of the anterior face } = \frac{r}{\rho+\tau\sqrt{(\lambda''-1)}} = \frac{r}{1,3284} = 0,75278r = -\frac{6,02}{m} \text{ in.,}\\ \text{of the posterior face} = \frac{r}{\sigma-\tau\sqrt{(\lambda''-1)}} = \frac{r}{0,3957} = 2,5272r = -\frac{20,22}{m} \text{ in.}\end{cases}$$

IV. From these lenses constructed the interval between the two lenses may be put in place

$$= \tfrac{1}{5}a = \tfrac{2}{m} \text{ in.}$$

and the object may be put in place at the distance $a = \frac{10}{m}$ in.

V. Since the first confusion shall be zero, from these lenses just as great an aperture can be attributed to these lenses, as the shape of these permits; whereby, since the minimum radius shall be $\frac{6,02}{m}$ in., the radius of the aperture may be put to be $x = \frac{1,50}{m}$ in., from which there will become for the clarity

$$y = \frac{hx}{ma} = \frac{4}{5}x = \frac{1,20}{m}$$

and hence the measure of the clarity will be $= 20y = \frac{24,0}{m}$ with 1 denoting full clarity.

SCHOLIUM 2

98. [There is some confusion in the section numbering here.] If we may compare this example with these, which we have found in the preceding chapter, this kind deserves the prerogative both on account of the distance of the object, which here certainly is a little greater, as well as on account of this reason, that here the other kind of confusion may be diminished more than a little, which indeed has not been introduced into the calculation previously. Truly if we may address the magnification of the lenses, these kinds which especially are made from four lenses, by far deserve to be mentioned, since there the focal lengths are much greater and thus these microscopes shall be able to be adapted for much greater magnifications, unless perhaps an excess of nearby objects may stand in the way. Therefore nor is there a need for this treatment to be extended at this stage to more lenses, since scarcely a greater perfection may be able to be expected for simple microscopes. Whereby if anyone may wish for greater perfections, it will be required to have recourse truly to composite microscopes, since with this composition the two inconveniences mentioned above will be occurring. Clearly in the first place, since there will be no need to have moved the object up so close, then as we may not need so small lenses, even if we may require the maximum magnification; for in this the compound microscopes will excel mainly over the simple ones, so that with the aid of these a magnification of any size may be able to be produced.

CAPUT III

DE MICROSCOPIIS SIMPLICIBUS AB OMNI PLANE CONFUSIONE IMMUNIBUS SIVE EX EODEM SIVE EX DIVERSO VITRI GENERE CONSTANTIBUS

PROBLEMA 1

76. *Si microscopium duabus lentibus priore concava, posteriore vero convexa proxime inter se iungendis constet, efficere, ut confusio ab apertura oriunda penitus destruatur.*

SOLUTIO

Quoniam hic duae tantum lentes occurrunt, earum intervallum $Aa(1-\frac{1}{P})$ statuatur minimum $=\eta a$ hincque fiet $P=\frac{A}{A-\eta}$; deinde cum distantiae focales sint

$$p=\mathfrak{A}a \text{ et } q=-\frac{A\mathfrak{B}}{P}\cdot a,$$

ob lentem priorem concavam debet esse negativum hincque etiam $A<0$; quare secunda lens sponte fit convexa ob $\mathfrak{B}=1$. Multiplicatio porro ita exprimetur, ut sit $m=P\cdot\frac{h}{a}$; unde colligitur distantia

$$a=P\cdot\frac{h}{m}=\frac{A}{A-\eta}\cdot\frac{h}{m},$$

ita ut distantia obiecti etiam minor sit capienda quam $\frac{h}{m}$. Nunc ut confusio ab apertura oriunda ad nihilum redigatur, huic aequationi satisfieri oportet:

$$\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{\lambda'}{A^3P}=0,$$

siquidem ambae lentes ex eodem vitro conficiantur. Sin autem ex diverso vitro parentur, pro secunda lente loco μ scribatur μ' et habebitur haec aequatio:

$$\frac{\mu\lambda}{\mathfrak{A}^3}+\frac{\mu\nu}{A\mathfrak{A}}-\frac{\mu'\lambda'}{A^3P}=0.$$

Quem casum hic evolvamus, quandoquidem casus vix fit complicatior, atque ex hac aequatione definire poterimus sive λ sive λ'; fiet scilicet

$$\text{vel}\quad \lambda=\frac{\mu'}{\mu}\cdot\frac{(1-\mathfrak{A})^3}{P}\cdot\lambda'-\nu\mathfrak{A}\left(1-\mathfrak{A}\right)$$

$$\text{vel} \quad \lambda' = \frac{\mu}{\mu'} \cdot \frac{P}{(1-\mathfrak{A})^3} \cdot \lambda + \frac{\mu\nu}{\mu'} \cdot \frac{P\mathfrak{A}}{(1-\mathfrak{A})^3},$$

ita ut littera $\mathfrak{A}$ adhuc arbitrio nostro relinquatur, dummodo negative capiatur; quare hanc litteram ita definire licebit, ut etiam altera confusio a diversa refrangibilitate oriunda tollatur; quem in finem si, ut supra fecimus, pro prima lente statuatur $\frac{dn}{n-1} = N$ et pro secunda $\frac{dn'}{n'-1} = N'$, huic aequationi erit satisfaciendum:

$$0 = N \cdot \frac{1}{\mathfrak{A}} - \frac{N'}{P} \cdot \frac{1}{A},$$

ex qua colligitur

$$\frac{\mathfrak{A}}{A} = \frac{N}{N'} \cdot P = 1 - \mathfrak{A},$$

ita ut ob $P = 1$ proxime fiat

$$\mathfrak{A} = 1 - \frac{N}{N'} = \frac{N'-N}{N'},$$

qui valor cum esse debeat negativus, necesse est, ut sit $N' < N$.

Sumamus igitur priorem lentem concavam ex vitro crystallino, posteriorem vero ex vitro coronario confici et ob $N : N' = 10 : 7$ fiet $\mathfrak{A} = -\frac{3}{7}$, quo valore contenti esse possumus. Sin autem exactiorem desideremus, loco P eius valorem substituamus et nostra aequatio fiet

$$0 = \frac{10}{\mathfrak{A}} - 7\frac{(A-\eta)}{A^2} = 10A(A+1) - 7(A-\eta),$$

quae sumto ut ante $\eta = \frac{1}{5}$ dabit

$$A = -\frac{3}{20} \mp \sqrt{\frac{9}{400} - \frac{7}{50}};$$

qui valor manifesto est imaginarius, ita ut huic conditioni satisfieri nequeat, nisi distantia ηa multo minor capiatur, scilicet sumi deberet $\eta < \frac{1}{31}$; quia autem tantilla distantia in tam exiguis lenticulis locum habere nequit, etiam hanc conditionem perfecte implere non licebit. Contentos igitur nos esse oportebit valore saltim prope satisfaciente, praecipue cum ipsa rei natura non permittat, ut huic conditioni plene satisfaciamus; ac sumamus, ut ante repperimus

$$\mathfrak{A} = -\frac{3}{7}, \quad \text{ut sit} \quad A = -\frac{3}{10}$$

hincque

$$P = \frac{3}{3+10\eta}$$

hincque distantiae focales

$$p = -\frac{3}{7}a = \frac{-9}{21+70\eta} \cdot \frac{h}{m}, \quad q = \frac{3}{10} \cdot \frac{h}{m}$$

Pro confusione autem tollenda sumi debebit

$$\lambda = \frac{\mu}{\mu'} \cdot \frac{10^3}{7^3} \cdot \frac{3+10\eta}{3} \cdot \lambda' + \frac{30}{49} v;$$

in qua forma si sumatur $\lambda' = 1$ et litterae μ, μ' et v convenienter assumantur, reperietur

$$\lambda = \frac{0,9875}{0,8724} \cdot \frac{10^3}{7^3} \cdot \frac{3+10\eta}{3} + \frac{30}{49} \cdot 0,2529 = 3,3001\left(1 + \frac{10}{3}\eta\right) + \frac{30}{49} \cdot 0,25290$$
$$= 3,3001 + 11,0003\eta + 0,155$$

seu

$$\lambda = 3,4551 + 11,0003\eta,$$

ex quo constructio prioris lentis peti debet.

COROLLARIUM 1

77. Cum sit

$$a = \frac{A}{A-\eta} \cdot \frac{h}{m} = \frac{3}{3+10\eta} \cdot \frac{h}{m},$$

patet distantiam obiecti notabiliter hic minorem fore quam casu lentis simplicis, ubi erat $a = \frac{h}{m}$; nam si sumamus $\eta = \frac{1}{5}$, prodit $a = \frac{3}{5} \cdot \frac{h}{m}$ neque vero pro η minor valor accipi poterit.

COROLLARIUM 2

78. Hoc ergo modo prius eorum incommodorum in vicinitate obiecti consistens, quae supra commemoravimus, haud mediocriter augetur, posterius vero hic quidem penitus tolletur sublata confusione ab apertura oriunda; verum distantiae focales lentium tam fiunt exiguae, ut posito $\eta = \frac{1}{5}$ prodeat

$$p = -\frac{9}{35} \cdot \frac{h}{m},$$

cum pro lente simplici fuisset $p = \frac{h}{m}$.

SCHOLION

79. Deinde etiam hoc non parum obstat, quod, etiamsi duas vitri species adhibeamus, tamen altera confusio tolli nequeat atque adeo ad valores imaginarios perveniatur; unde hac specie repudiata ad -alteram evolvendam progrediamur, qua lens posterior concava assumitur.

PROBLEMA 2

80. *Si microscopium constet duabus lentibus, quarum prior convexa, posterior vero concava, efficere*, *ut confusio ab apertura oriunda evanescat.*

SOLUTIO

Posito lentium intervallo $=\eta a$ fiet ut ante $P=\frac{A}{A-\eta}$, et cum sint distantiae focales $p=\mathfrak{A}a$ et $q=-\frac{A}{p}\cdot a$, tam $\mathfrak{A}$ quam A erunt numeri positivi ideoque $\mathfrak{A}<1$.
Multiplicatio vero dabit

$$m=P\cdot\frac{h}{a} \text{ seu } a=P\cdot\frac{h}{m}=\frac{A}{A-\eta}\cdot\frac{h}{m}.$$

Confusio autem ab apertura oriunda, si ambas lentes iterum ut ex diverso vitro factas consideremus, evanescet, si fuerit ut ante

$$\text{vel} \quad \lambda=\frac{\mu'}{\mu}\cdot\frac{(1-\mathfrak{A})^3}{P}\cdot\lambda'-\nu\mathfrak{A}(1-\mathfrak{A})$$

$$\text{vel} \quad \lambda'=\frac{\mu}{\mu'}\cdot\frac{P}{(1-\mathfrak{A})^3}\cdot\lambda+\frac{\mu\nu}{\mu'}\cdot\frac{P\mathfrak{A}}{(1-\mathfrak{A})^3},$$

adeoque adeo altera confusio evanescit, si fuerit

$$0=N\cdot\frac{1}{\mathfrak{A}}-\frac{N'}{P}\cdot\frac{1}{A}$$

hincque

$$\frac{\mathfrak{A}}{A}=\frac{N}{N'}\cdot P=1-\mathfrak{A};$$

unde, quia $\mathfrak{A}<1$ et $1-\mathfrak{A}<1$, debet esse $N'>N$, quamobrem hic lentem primam ex vitro coronario, secundam vero concavam ex crystallino parari conveniet, ita ut fiat $P>1$, ad quod requiritur, ut sit $\frac{7}{10}P<1$, quod ideo notari oportet, quia $P>1$; seu esse debet $P<\frac{10}{7}$ ideoque $\frac{A}{A-\eta}<\frac{10}{7}$, consequenter $\frac{\eta}{A}<\frac{3}{10}$. Huic igitur aequationi si adcurate satisfacere velimus, debet esse $\frac{\eta}{A}<\frac{3}{10}$; unde, si sumamus $\frac{\eta}{A}=\frac{1}{4}$, fiet $P=\frac{4}{3}$ hincque $1-\mathfrak{A}=\frac{14}{15}$ et $\mathfrak{A}=\frac{1}{15}$ ideoque $A=\frac{1}{14}$ et $\eta=\frac{1}{56}$, ex quo distantia lentium prodit $\eta a=\frac{1}{56a}=\frac{h}{42m}$; quia autem in praecedente capite assumsimus circiter intervallum $\frac{1}{5}\cdot\frac{h}{m.}$, patet tam exiguum intervallum in praxi locum habere non posse, ita ut nostro casu alteram confusionem tollere non liceat. Prorsus igitur isti conditioni renunciare oportet, ita ut iam perinde sit, sive lentes ex eodem vitro sive diverso conficiantur; fiant igitur ex eodem vitro quocunque, ita ut sit $\mu'=\mu$, et pro prima confusione tollenda, quoniam $\mathfrak{A}$ non nimis parvum sumi convenit, statuamus $\mathfrak{A}=\frac{1}{2}$ hincque $A=1$; unde fit

$$P=\frac{1}{1-\eta} \text{ et } a=\frac{1}{1-\eta}\cdot\frac{h}{m} \text{ et } p=\frac{a}{2} \text{ et } q=-(1-\eta)a.$$

Quodsi nunc statuamus $\eta a=\frac{1}{5}p$, sumi oportebit $\eta=\frac{1}{10}$ sicque fiet

$$P=\frac{10}{9},\quad a=\frac{10}{9}\cdot\frac{h}{m},\quad p=\frac{5}{9}\cdot\frac{h}{m} \text{ et } q=-\frac{h}{m}.$$

confusio prior itaque evanescit, si sit

$$\lambda' = \tfrac{80}{9}\lambda + \tfrac{20}{9}\nu;$$

unde facile erit lentes construere.

COROLLARIUM 1

81. Pro lentium igitur constructione, si vitrum adhibeatur commune, pro quo est $n = 1,55$ et $\nu = 0,2326$, si sumatur $\lambda = 1$, erit

$$\lambda' = 9,406, \text{ unde fit } \tau\sqrt{(\lambda'-1)} = 2,6242.$$

COROLLARIUM 2

82. Pro prima igitur lente, cuius distantia focalis est $p = \tfrac{5}{9}\cdot\tfrac{h}{m} = \tfrac{40}{9m}$ dig. ob $h = 8$ dig. numerique $\mathfrak{A} = \tfrac{1}{2}$ et $\lambda = 1$, habebitur

$$\text{radius faciei}\begin{cases}\text{anterioris } = \dfrac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = \dfrac{p}{0,9090} = 1,1001p = \dfrac{4,89}{m} \text{ dig.}\\ \text{posterioris} = \dfrac{p}{\rho + \mathfrak{A}(\sigma-\rho)} = \dfrac{p}{0,9090} = 1,1001p = \dfrac{4,89}{m} \text{ dig.,}\end{cases}$$

ita ut haec lens sit utrinque aequaliter convexa.

COROLLARIUM 3

83. Pro altera lente concava, cuius distantia focalis $q = -\tfrac{8}{m}$ dig. et numeri $\mathfrak{B} = 1$ et $\lambda' = 9,406$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = \dfrac{q}{\rho + \tau\sqrt{(\lambda'-1)}} = \dfrac{q}{2,8149} = +0,35525q = -\dfrac{2,84}{m} \text{ dig.}\\ \text{posterioris} = \dfrac{q}{\sigma - \tau\sqrt{(\lambda'-1)}} = \dfrac{-q}{0,9968} = -1,00321q = +\dfrac{8,03}{m} \text{ dig.}\end{cases}$$

COROLLARIUM 4

84. Intervallum inter has duas lentes statui ergo debebit

$$\eta a = \tfrac{1}{9}\cdot\tfrac{h}{m} = \tfrac{0,889}{m} \text{ dig.,}$$

obiectum autem ante lentem priorem est collocandum ad distantiam

$$a=\frac{80}{9m}\text{ dig.}=\frac{8,89}{m}\text{ dig. ;}$$

quod autem ad aperturam attinet, eam ex minimo radio ambarum lentium definiri oportet sicque eius semidiameter sumi poterit

$$x=\frac{0,71}{m}\text{ dig.}=\frac{5}{7m}\text{ dig.}$$

Hinc pro claritate fiet $y=\frac{hx}{ma}=\frac{3}{5m}$, unde mensura claritatis, ut supra est stabilita, $=\frac{12}{m}$.

SCHOLION

85. His ergo microscopiis priori incommodo supra memorato medela affertur, dum obiectum ad maiorem distantiam removere licet; contra vero, quia lentes multo minores requiruntur, quae propterea non nisi minorem aperturam admittunt, claritas minor prodire debet, qui defectus ea qualitate, quod confusio prior· penitus tollitur, vix compensari videtur. Maximum vero lucrum in hoc sine dubio situm esset futurum, si etiam alteram confusionem tollere licuisset, quandoquidem solis lentibus convexis adhibendis de hoc ne cogitari quidem poterat Quoniam igitur hoc lucrum duabus lentibus obtineri non potest, examinemus casum trium lentium, inter quas una sit concava, quae, uti facile ex praecedentibus intelligitur, ex vitro crystallino parari debet, dum reliquae ex coronario conficiuntur; tum vero etiam nullum dubium superesse potest, quin hanc lentem concavam loco tertio constitui conveniat.

PROBLEMA 3

86. *Si microscopium constet tribus lentibus, inter quas tertia sit concava, binae anteriores vero convexae, efficere, ut confusio ab apertura oriunda penitus destruatur.*

SOLUTIO

Ponantur iterum bina intervalla inter has lentes utrumque $=\eta a$ ac habebimus uti in problemate 2 capitis praecedentis

$$P=\frac{A}{A-\eta},\quad Q=\frac{(A-\eta)B}{(A-\eta)B+\eta}.$$

Tum vero, cum distantiae focales sint

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a\ \text{ et }\ r=\frac{AB}{PQ}\cdot a,$$

primo debebit esse $\mathfrak{A} > 0$, et quia est

$$\frac{q}{r} = -\frac{Q\mathfrak{B}}{B},$$

ob $q > 0$ et $r < 0$ debebit esse

$$\frac{\mathfrak{B}}{B} > 0 \text{ sive } 1 - \mathfrak{B} > 0 \text{ seu } \mathfrak{B} < 1.$$

His autem conditionibus duplici modo satisfieri potest:

I. Si $\mathfrak{A} < 1$ ideoque $A > 0$; atque fiet $\mathfrak{B} < 0$ et $B < 0$.
II. Si $\mathfrak{A} > 1$ hincque $A < 0$, fiet $B > 0$ et $\mathfrak{B} > 0$, attamen $\mathfrak{B} < 1$.

Priore casu prodit $P > 1$ et $Q > 1$, posteriore vero casu fit $P < 1$ et $Q > 1$. Utrum autem PQ fiat maius an minus unitate, non definitur. Multiplicatio m autem dabit $a = PQ \cdot \frac{h}{m}$, pro qua conducit, ut PQ notabiliter unitatem superet, quod evenit casu priore, ubi est $P > 1$ et $Q > 1$.

Nunc vero incipiamus a confusione posteriore ad nihilum redigenda, quae praebet hanc aequationem, quandoquidem assumimus $N' = N$ pro vitro coronario, dum N'' ad vitrum crystallinum referatur:

$$0 = N \cdot \frac{1}{\mathfrak{A}} - \frac{N}{P} \cdot \frac{1}{A\mathfrak{B}} + \frac{N''}{PQ} \cdot \frac{1}{AB}.$$

Si igitur duae priores lentes convexae ex vitro coronario, tertia vero ex crystallino sint factae, ut sit $N : N'' = 7 : 10$, habebitur

$$0 = \frac{1}{\mathfrak{A}} - \frac{1}{PA\mathfrak{B}} + \frac{10}{7} \cdot \frac{1}{PQAB};$$

in qua si loco P et Q valores ante inventi substituantur, prodibit

$$0 = \frac{1}{\mathfrak{A}} - \frac{A-\eta}{A^2\mathfrak{B}} + \frac{10}{7} \cdot \frac{(A-\eta)B+\eta}{A^2B^2}$$

quae aequatio evoluta abibit in hanc formam:

$$0 = A^2B^2 + \frac{3}{7}AB + \eta\left(B^2 - \frac{3}{7}B + \frac{10}{7}\right),$$

unde patet, si esset $\eta = 0$, fore $AB = -\frac{3}{7}$ ideoque $r = -\frac{3}{7PQ} \cdot a$, ita ut sit $-r < \frac{3}{7}a$ ob $PQ > 1$. Sed quia casus $\eta = 0$ locum habere nequit, dum potius huic litterae valor satis modicus tribui debet, tribuatur aequationi inventae haec forma:

$$A^2B^2 + \frac{3}{7}AB + \frac{9}{196} = \left(AB + \frac{3}{14}\right)^2 = \frac{9}{196} - \eta\left(B^2 - \frac{3}{7}B + \frac{10}{7}\right),$$

ubi evidens est litteram η maiorem esse non posse quam

$$\frac{9}{196\left(B^2-\frac{3}{7}B+\frac{10}{7}\right)},$$

siquidem haec altera confusio prorsus debeat evanescere; qui limes cum sit valde exiguus, statuamus

$$\eta=\frac{9}{28\left(7B^2-3B+10\right)}$$

fietque $AB=-\frac{3}{14}$ sicque r duplo minor quam ante, id quod praxi maxime obest. Cum autem non absolute necessarium sit istam confusionem prorsus ad nihilum redigere, res ita poterit proponi, ut posito $AB=-\frac{3}{14}$ pro η tantus capiatur valor, quam circumstantiae permittunt, etiamsi is maior sit futurus limite hic praescripto.

His observatis tandem prior confusio ad nihilum redigatur, id quod fit ope huius aequationis:

$$0=\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{\nu}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)+\frac{\mu''}{\mu}\cdot\frac{\lambda''}{A^3B^3PQ},$$

ex qua numerus λ'' definiri conveniet sumtis $\lambda=1$ et $\lambda'=1$.

COROLLARIUM 1

87. Utcunque igitur intervallum ηa assumatur, haec microscopia semper isto defectu laborabunt, ut tertia lens fiat nimis parva, scilicet fere quintuplo minor, quam si lente simplici uteremur. Quare cum parvitas lentis maiores multiplicationes impedivisset, hic casus multo minus erit aptus maioribus multiplicationibus producendis.

COROLLARIUM 2

88. Deinde etiam limes praescriptus pro η nimis parvum praebet valorem, quam ut in praxi locum habere possit, etiamsi littera B arbitrio nostro permittatur; valor enim ex illo limite deductus

$$\eta=\frac{9}{28\left(7B^2-3B+10\right)}$$

maximum adipiscitur valorem, si capiatur $B=\frac{3}{14}$, qui propterea erit $\eta=\frac{9}{271}$ seu proxime $\eta=\frac{1}{30}$, qui manifesto nimis est parvus, quam ut in praxi admitti possit.

SCHOLION 1

89. Parum igitur fructus haec microscopiorum species pollicetur, etiamsi utramque confusionem ad nihilum redigere liceat, cum, utcunque litterae A et B definiantur, tam ipsae lentes nimis prodeunt exiguae quam lentium intervalla nimis parva. Sin autem confusionem a diversa refrangibilitate oriundam non curare velimus, egregia hinc microscopia deducere licebit, inter quae sequens potissimum species nostram attentionem mereri videtur.

Statuatur scilicet $\mathfrak{A}=1$, ut sit $A=\infty$; sumatur porro $B=0$, ita ut sit $AB=-\theta$, hincque elementa nostra ita definiantur:

$$P=1,\quad Q=\tfrac{\theta}{\theta-\eta}$$

et

$$p=a,\quad q=\theta a\quad \text{et}\quad r=-\left(\theta-\eta\right)a,$$

ubi θ facile ita sumi potest, ut hae distantiae focales non fiant nimis parvae atque η etiam nostro arbitrio permittatur. Tum vero erit distantia

$$\alpha=\tfrac{\theta}{\theta-\eta}\cdot\tfrac{h}{m}.$$

Nunc autem perinde erit, sive omnes lentes ex eodem vitro sive ex diverso parentur; interim tamen si θ non multum a valore supra dato $\frac{3}{14}$ abludat, non parum lucri consequemur, si tertiam lentem ex vitro crystallino paremus, dum binae anteriores ex vitro coronario conficiuntur, quippe quo facto altera confusio saltim diminuetur. Tum autem pro ipsa lentium constructione haec aequatio est resolvenda:

$$0=\lambda+\tfrac{\lambda'}{\theta^3}-\tfrac{0,8724}{0,9875}\cdot\tfrac{\lambda''(\theta-\eta)}{\theta^4},$$

unde sumtis $\lambda=1$ et $\lambda'=1$ colligitur

$$\lambda''=\tfrac{0,9875}{0,8724}\cdot\tfrac{\theta^4}{\theta-\eta}\left(1+\tfrac{1}{\theta^3}\right),$$

cuius solutionis exemplum afferre non pigebit.

EXEMPLUM

90. Sumatur $\eta=\frac{1}{5}$ et sit $\theta=1$; atque hinc habebimus

$$P=1,\quad Q=\tfrac{1}{1-\eta}=\tfrac{5}{4},$$

$$p=a,\quad q=a,\quad r=-\tfrac{4}{5}a$$

existentia $a=\frac{5}{4}\cdot\frac{h}{m}$. Tum vero aequatio resolvenda pro hoc casu dabit

$$\lambda''=\tfrac{0,9875}{0,8724}\cdot\tfrac{5}{2}=2,8300;$$

ex quo pro vitro crystallino colligitur

$$\tau\sqrt{\lambda''-1}=1{,}1870.$$

Unde obtinetur sequens constructio microscopii trilenticularis.

I. Prima lens ex vitro coronario paratur; cuius distantia focalis cum sit

$$p=a=\tfrac{5}{4}\cdot\tfrac{h}{m}=\tfrac{10}{m}\text{ dig.}$$

et numeri $\mathfrak{A}=1$ et $\lambda=1$, erit

$$\text{radius faceie}\begin{cases}\text{anterioris }=\frac{p}{\rho}=\frac{p}{0{,}2267}=4{,}4111p=\frac{44{,}11}{m}\text{ dig.}\\ \text{posterioris}=\frac{p}{\sigma}=\frac{p}{1{,}6601}=0{,}6024p=\frac{6{,}02}{m}\text{ dig.}\end{cases}$$

II. Pro secunda lente etiam ex vitro coronario paranda, cuius distantia focalis est

$$q=a=\tfrac{5}{4}\cdot\tfrac{h}{m}=\tfrac{10}{m}\text{ dig.}$$

et numeri $\mathfrak{B}=0$ et $\lambda'=1$, erit

$$\text{radius faceie}\begin{cases}\text{anterioris }=\frac{q}{\sigma}=\frac{q}{1{,}6601}=0{,}6024q=\frac{6{,}02}{m}\text{ dig.}\\ \text{posterioris}=\frac{q}{\rho}=\frac{q}{0{,}2267}=4{,}4111q=\frac{44{,}11}{m}\text{dig.}\end{cases}$$

III. Pro tertia lente ex vitro crystallino paranda, cuius distantia focalis est

$$r=-\tfrac{4}{5}a=-\tfrac{h}{m}=-\tfrac{8}{m}\text{ dig.}$$

et numeri $\mathfrak{C}=1$ et $\lambda''=2{,}8300$, erit

$$\text{radius faceie}\begin{cases}\text{anterioris }=\frac{r}{\rho+\tau\sqrt{(\lambda''-1)}}=\frac{r}{1{,}3284}=0{,}75278r=-\frac{6{,}02}{m}\text{ dig.}\\ \text{posterioris}=\frac{r}{\sigma-\tau\sqrt{(\lambda''-1)}}=\frac{r}{0{,}3957}=2{,}5272r=-\frac{20{,}22}{m}\text{ dig.}\end{cases}$$

IV. His lentibus confectis intervallum inter binas statuatur

$$=\tfrac{1}{5}a=\tfrac{2}{m}\text{ dig .}$$

et obiectum exponatur ad distantiam $a=\frac{10}{m}$ dig.

V. Cum confusio prior sit nulla, his lentibus tanta apertura tribui potest, quantam, earum figura permittit; quare, cum minimus radius sit $\frac{6,02}{m}$ dig., statuatur semidiameter aperturae $x=\frac{1,50}{m}$ dig., unde pro claritate erit

$$y=\frac{hx}{ma}=\tfrac{4}{5}x=\frac{1,20}{m}$$

hincque mensura claritatis $=20y=\frac{24,0}{m}$ denotante 1 claritatem plenam.

SCHOLION 2

98. Si hanc speciem cum iis, quas in praecedente capite invenimus comparemus, haec species praerogativam meretur tam ratione distantiae obiecti, quippe quae hic est aliquanto maior, quam ob eam causam, quod hic etiam altera confusio non mediocriter diminuatur, quae ante ne in computum quidem est ducta. Verum si ad magnitudinem lentium attendamus, illae species, quae praecipue quatuor lentibus constant, longe anteferri merentur, cum ibi lentium distantiae focales multo sint maiores ideoque ea microscopia ad multo maiores multiplicationes accommodari possint, nisi forte nimia obiecti vicinitas obstaret. Neque igitur opus esse censeo hanc tractationem adhuc ad plures lentes extendere, cum vix maior perfectio in microscopiis simplicibus exspectari queat. Quare si quis maiores perfectiones desideret, necessario ad microscopia vere composita confugere debebit, quandoquidem hac compositione binis supra memoratis incommodis erit occurrendum. Primo scilicet, ut non opus sit obiecta tam prope admovere, deinde ut non tam exiguis lenticulis indigeamus, etiamsi multiplicationem maximam requiramus; in hoc enim microscopia composita potissimum simplicibus antecellunt, ut eorum ope multiplicatio quantumvis magna produci queat.