SECOND SECTION.

COMPOSITE MICROSCOPES, IN WHICH

NO REAL IMAGE OCCURS.

PROBLEM 1

99. *To construct a microscope from two lenses both with the magnification m as well as the distance of the object before the objective lens given, of which the objective lens shall be convex, and with the eyepiece truly concave.*

SOLUTION

Since the distance of the object a is given and equally the magnification m, the case of two lenses at once presents this equation $m = P \cdot \frac{h}{a}$; from which there is defined :

$$P = \frac{ma}{h};$$

hence the focal lengths of both lenses will be

$$p = \mathfrak{A}a, \quad q = -\frac{Ah}{m};$$

from which it is apparent both $\mathfrak{A}$ as well as A must be positive, from which it suffices, that A shall be positive. Truly the separation of the lenses will be

$$= Aa(1-\frac{h}{ma}) = \frac{A}{m}\left(ma-h\right),$$

from which it is evident there must be $ma > h$ or $m > \frac{h}{a}$; otherwise truly microscopes of this kind would not find a use. Thence for the area viewed within the object we will have its radius

$$z = a\Phi = \frac{\mathfrak{q}}{ma-h} \cdot ah\xi.$$

Therefore if we may assume $\xi = \frac{1}{4}$ and $\mathfrak{q} = 1$, which is the case, where the eyepiece lens allows the maximum aperture and thus each side is equally concave, therefore then there will be

$$z = \frac{1}{4} \cdot \frac{ah}{ma-h}.$$

Truly so that it may extend to the position of the eye, we gather from the above formulas:

$$O = \frac{\mathfrak{q}b}{Ma} \cdot \frac{h}{m};$$

truly there is

$$b = -\frac{\alpha}{P} = -\frac{Ah}{m}$$

and

$$M = \frac{\mathfrak{q}}{ma-h} \cdot h$$

and thus there becomes :

$$O = -\frac{Ah(ma-h)}{m^2 a};$$

which distance since it shall be negative, the eye will be required to be applied straight to the eyepiece lens; from which in order that the colored margin may vanish, it will be required to satisfy this equation:

$$0 = N(A+1)\mathfrak{q};$$

which since it may not be able to happen, it is evident the colored margin by no means may be able to be removed ; therefore this latter confusion much less will be able to be removed completely ; but the first confusion may be rendered insensitive with the aid of this equation:

$$\frac{mx^3}{a^2 h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu'\lambda'}{A^3 P}\right) = \frac{1}{k^3},$$

which therefore will be changed into this form :

$$\frac{mx^3}{a^2 h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu' h\lambda'}{A^3 ma}\right) = \frac{1}{k^3},$$

where since the eyepiece lens must be equally concave on each side, if for that kind of glass, from which the eyepiece is made, the corresponding numbers may be taken ρ', σ' and τ', there will become $\lambda' = 1 + \left(\frac{\sigma' - \rho'}{2\tau'}\right)^2$. Moreover from this equation the radius of the aperture of the objective lens x must be defined, evidently there will be

$$x\sqrt[3]{\left(\mu m\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu' h\lambda'}{A^3 a}\right)} = \frac{1}{k}\sqrt[3]{a^2 h},$$

unless perhaps it may produce a greater value for x , than the shape of the lens allows; hence therefore the most useful case may be, if there may be able to become

$$\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} = \frac{\mu'}{\mu} \cdot \frac{h\lambda'}{A^3 am};$$

according to which as suitable value will be required to be found for $\mathfrak{A}$ or A, which certainly thus will not be able to be found for large magnifications; but if the magnification m may be great, $A(1+A)^3+\nu A(A+1)$ must be equal to a very small fraction, which, since $A > 0$, cannot happen. But whatever it shall be, for the value found of x the order of the clarity will be $y=\frac{hx}{ma}$ and the measure of the clarity $=\frac{20hx}{ma}$; from which there more is required to be looked after, so that x may not arrive at a very small value.

COROLLARY 1

100. Hence it is apparent, so that x may obtain a greater value, for the most part to lead so that a small value may be given to the letter A ; but this exceedingly small value is not allowed to be assumed, since then the eyepiece lens will become exceedingly small, thus so that A may be agreed to be hardly less than unity.

COROLLARY 2

101. Since the formula $\lambda(1+A)^3+\nu A(A+1)$ certainly shall be greater than unity, because A cannot be less than unity, and therefore must rise beyond 8, this confusion will not be able to be removed completely, unless this formula $\frac{\mu'}{\mu}\cdot\frac{h\lambda'}{ma}$ also will exceed 8 , that is, unless on account of $\frac{\mu'}{\mu}=1$ approximately, there would be

$$\frac{h\lambda'}{ma}>8 \text{ or } m<\frac{h\lambda'}{8a}.$$

COROLLARY 3

102. These will become clearer, if on putting $h=8$ in. we may assume $a=\frac{1}{4}$ in.; and since there shall be approximately $\lambda'=\frac{3}{2}$, the limits found just now will give $m<6$; which magnification so very small cannot indeed produce microscopes of this kind, whereby now for certain it will be possible to affirm this confusion cannot be removed by any means.

EXAMPLE 1

102a. If the distance of the object must be $\frac{1}{4}$ in. and both lenses may be prepared from common glass $n=1,55$, then truly there may be put $A=1$ and hence $\mathfrak{A}=\frac{1}{2}$, in the first place we will have the focal lengths of the lenses $p=\frac{1}{2}a=\frac{1}{8}$ in. and $q=-\frac{8}{m}$ in.
the separation of the lenses will be

$$=\frac{1}{m}\left(\frac{1}{4}m-8\right)=\frac{1}{4}-\frac{8}{m} \text{ in.}$$

Truly the area viewed in the object will be

$$z = \frac{2}{m-32} \text{ in.}$$

Finally if as we have assumed up to now, $k = 20$, the latter equation will be

$$x\sqrt[3]{\mu\left(8\lambda m+2\nu m-32\lambda'\right)} = \frac{1}{20}\sqrt[3]{\frac{1}{2}}.$$

But here, as now we have seen often, there is $\lambda' = 1,6299$; truly besides since there shall be $\lambda = 1$ and $\mu = 1$ approx. , we will find

$$x = \frac{1}{20}\sqrt[3]{\frac{1}{16,9304m-104,3136}}.$$

Therefore if there were $m = 100$, in the first place there will become

$$p = \frac{1}{8} \text{ in. and } q = -\frac{2}{25} \text{ in.}$$

and the separation of the lenses

$$= \frac{17}{100} \text{ in.}$$

and

$$z = \frac{1}{34} \text{ in.},$$

since truly

$$x = 0,0043 \text{ in.},$$

from which there becomes

$$y = \frac{32}{100} = 0,0014 \text{ in.}$$

and a measure of the clarity is 0,028, which is around three times less than in almost the most simple microscope.

EXAMPLE 2

[102b]. Everything shall remain as in the previous example, except that a much greater value may be attributed to the letter A , so that we may observe, how the confusion then shall be going to be compared. Therefore there may be put $A = 5$ and there will become $\mathfrak{A} = \frac{5}{6}$ and the focal lengths will be

$$p = \frac{5}{6}a, \quad q = -5\cdot\frac{h}{m},$$

since as before there remains

$$P = \frac{ma}{h}$$

and the separation of the lenses

$$=\tfrac{5}{m}\left(ma-h\right);$$

then truly there will be for the area viewed

$$z=\tfrac{1}{4}\cdot\tfrac{ah}{ma-h}$$

as before. Finally so that confusion may not be perceived, there must become

$$x\sqrt[3]{\mu\left(\tfrac{216\lambda m}{125}+\tfrac{6\nu m}{25}-\tfrac{8\lambda'}{125a}\right)}=\tfrac{1}{10}\sqrt[3]{a^2}.$$

Therefore again on taking $a=\frac{1}{4}$ in., $\lambda=1$ and $\lambda'=1,6299$, if indeed both lenses may be made from common glass $n=1,55$, and on putting $\mu=1$ there will be had

$$x=\tfrac{1}{20}\sqrt[3]{\tfrac{1}{3,568m-0,8344}}.$$

Therefore if there were $m=100$, there will become

$$p=\tfrac{5}{24}\text{ in. and } q=-\tfrac{2}{5}\text{ in.}$$

and the separation of the lenses

$$=\tfrac{17}{20}\text{ in.}$$

and

$$z=\tfrac{1}{34}\text{ in.}$$

Then truly

$$x=\tfrac{1}{20}\sqrt[3]{\tfrac{1}{355,966}}=0,00705\text{ in.},$$

from which there becomes

$$y=0,00226\text{ in.}$$

and the measure of the clarity $=0,0452$.

SCHOLIUM

103. If we may compare these two examples with each other, the following will be required to be observed:

1. We see most interesting, that the greater the value to be attributed to the letter A, since then the expression for the confusion shall become much smaller, thus so that then

the letter x may come upon a greater value, from which likewise a greater clarity is obtained; indeed where the letter A is taken greater, there the letter $\mathfrak{A}$ approaches closer to unity, from which the first term $\frac{1}{\mathfrak{A}^2}$ will scarcely exceed unity, which, while there was $A=1$, was rising beyond 8.

2. Then also the objective lens will become greater on increasing the value of A, while its focal length p continually approaches closer to the object distance a .

3. But the maximum convenience is discerned in the eyepiece lens, which will be able to be increased as we please, however great the magnification will have been. Thus it can happen, that this lens may acquire a focal length such as of one inch; then clearly $A\cdot\frac{h}{m}$ $=1$ dig. and on account of $h=8$ in. there must be taken $A=\frac{m}{8}$; then indeed the length of the instrument will arise greater, evidently $=\frac{1}{8}\left(ma-h\right)$: but scarcely ever will it be so great, that it may not be easily tolerated.

4. Indeed in these examples we have assumed the distance of the object $a=\frac{1}{4}$ in., but nothing prevents that we may assume this distance greater, whereby the use of these instruments is rendered much more suitable, while especially convenient, as we expect from composite microscopes, in that it is allowed, so that there is no need for the object to be moved so close to the instrument; indeed since the letter a is allowed according to our choice, that will be allowed to be assumed to be as great as it would please.

5. Truly so that we may accept this greater distance a, we are forced to admit the degree of clarity to be going to be diminished; so that it may appear clearer, we shall depend on the value of the letter x with the remaining letters remaining the same to be proportional to the formula $\sqrt[3]{a^2}$ or the power $a^{\frac{2}{3}}$, thus so that, where a greater distance of the object may be put in place, also a greater aperture of the objective lens shall be going to be put in place; so that regarding this considerable convenience is required to be had; but for the degree of clarity, since y shall be proportional to the formula $\frac{x}{a}$, the clarity shall become proportional to the formula $\frac{1}{\sqrt[3]{a}}$, thus so that it may decrease in the inverse ratio of the cube root of the distance of the object *a;* truly this diminution is not itself required to be feared, provided that, if we may accept a distance of the object eight times larger, the clarity will become only twice as small, and from these it is clear, however much composite microscopes may excel over simple ones, and however many conveniences may be hoped for from these. Truly meanwhile this kind of microscopes treated here at this stage labour under a huge defect, since nothing can be freed from the colored margin. Just as we may see, either one or several more lenses shall be required to be added which may be able to remove this fault.

PROBLEM 2

104. *To insert a new lens between the objective and eyepiece lenses of the preceding kind of microscopes, so that the colored margin may be reduced to zero.*

SOLUTION

Since we have these three lenses, therefore the focal lengths of these will be expressed thus:

$$p = \mathfrak{A}a, \quad q = \left[\mathfrak{B}b = -\mathfrak{B}\tfrac{\alpha}{P} = -\mathfrak{B}\tfrac{A}{P}\cdot a\right] = -\tfrac{A\mathfrak{B}}{P}\cdot a, \quad r = \tfrac{AB}{PQ}\cdot a;$$

of which since the first must be convex, there will be $\mathfrak{A} > 0$, and since the third must be concave, there will be $AB < 0$ and thus of the letters *A* and *B* with one positive and the other negative ; indeed at this stage we may define nothing with regard to the middle lens; again, the separations of these lenses will be

$$\text{the former} = Aa(1-\tfrac{1}{P}) \text{ and the latter} = -\tfrac{ABa}{P}(1-\tfrac{1}{Q});$$

from which it is apparent there must become $Q > 1$. Truly the magnification *m* will give $PQ = \frac{ma}{h}$.

Moreover we will consider now that which is proposed by us, evidently so that the colored margin may vanish. Since the distance of the eye *O* is produced negative, it will be required to satisfy this equation:

$$0 = N(A+1)B\tau - \tfrac{N'}{P}((B+1)\mathfrak{r}+\mathfrak{q}),$$

which in the end the area visible within the object may be observed, for which there is

$$z = a\Phi = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot ah\xi;$$

in which, if each side of the eyepiece lens shall be equal, so that the maximum aperture may be allowed, there can be taken $\mathfrak{r} = 1$; then truly we have placed

$$\tfrac{\mathfrak{q}+1}{ma-h}\cdot h = M,$$

so that there shall be

$$z = Ma\xi.$$

Now therefore in the first place it is required to be seen, whether, if both lenses may be made from the same glass, we may be able to obtain the view sought. Therefore on putting $N = N'$ the equation for the margin gives us

$$B = \frac{\mathfrak{q}+\mathfrak{r}}{(A+1)\mathrm{P}\mathfrak{r}-\mathfrak{r}},$$

we may see whether which value may be consistent with the prescribed condition $AB < 0$. Hence in the end we may consider two cases, the one in which $A > 0$, the other truly, where not only $A < 0$, but also $1 + A < 0$, clearly so that $\mathfrak{A}$ may be produced positive. In the first case there will be $P > 1$ and thus in the value of B the denominator shall be positive and thus B will have a positive value, yet since on account of $AB < 0$ it must be negative; in the other case, where $A < 0$, there must be $P < 1$ and thus the denominator $(A+1)\mathrm{P}\mathfrak{r} - \mathfrak{r}$ shall become negative, even if $A+1$ may not be negative, thus so that the value of B in this case certainly may be made negative, since still on account of $AB < 0$ it must become positive.

But if the lenses may be made from different kinds of glass, it can be done, so that the colored margin may be removed completely and that in two ways, just as we will show in the adjoining cases. Moreover after the satisfaction of this condition, the following equation will be had for the aperture of the objective lens and thence for the depending clarity:

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''\lambda''}{A^3B^3PQ}\right)=\frac{1}{k^3},$$

where it is required to be noted to such an extent, that the value of λ'' thence must be given, so that the maximum aperture may be allowed for the eyepiece lens ; unity may be assumed conveniently for the two remaining λ and λ' and thus the solution of our problem will be found easily.

COROLLARY 1

105. Therefore since it shall be required to use here different kinds of glass, it is required to be understood that it is only for the first and second lens, to which the letters N and N' are referring; indeed for the third lens an account of the glass, from which it is made, plainly does not enter into the calculation, thus so that likewise it shall be the case that this lens may be constructed from whatever kind of glass.

COROLLARY 2

106. Therefore since for the colored margin being removed this equation may be used:

$$N(A+1)BP\mathfrak{r} = N'((B+1)\mathfrak{r}+\mathfrak{q}),$$

hence the value of the letter B must be deduced :

$$B=\frac{N'(\mathfrak{q}+\mathfrak{r})}{N(A+1)P\mathfrak{r}-N'\mathfrak{r}};$$

where there will be observed to be $\mathfrak{r}=1$ and $\mathfrak{q}+\mathfrak{r}$ by necessity greater than zero, in order that clearly the value of z may be produced positive ; then this value will be compared with that value, for which the product AB must be negative, which plainly cannot happen, as long as the letters N and N' are equal to each other, as we have show.

COROLLARY 3

107. Now the whole affair returns to this point, just as these two conditions may be able to be fulfilled, while the letters N and N' hold different values, so that as it were with the value of the letter A given the other letter B may obtain such a value, so that the product AB may become negative, where it is required to consider the formula $A(P-1)$ always must be positive, thus so that with A assumed positive there shall be $P>1$, but with A negative there must be taken $P < 1$.

SCHOLIUM

108. Therefore since we are agreed with two different kinds of glass to be use, without doubt it shall be required, so that these two kinds will differ between each other by the greatest ratio of refraction ; but since at this point no other kinds of glass of this description are known close to those which Dolland himself put in place, it will be required for us here to use the same. Indeed thus far with the same letters N and N', which may be agreed from these two species, we have attributed the ratio 7:10, which will be seen to conform the most from experiments, even if it may be possible for these to err notably from the truth. Whereby on account of the more convenient calculation here we may put in place this ratio to be rather as 2:3, certainly which differs from that minimally and involves a little greater discrimination; nor indeed hence is that to be feared, if perhaps the error may not be small enough, except that the colored margin may not be removed completely ; truly provided that this may arise much smaller, than is accustomed to happen from the single common kind of glass, we will be able to be content ; which in the end it will be agreed to examine two cases here more accurately, the one, where the letter A has a positive value, where it shall be negative, so that thence it may be apparent, how great a convenience may be able to be expected in practice.

[Need we remind the reader that there was no physical justification for Euler to introduce such formulas, which was due to his lack of understanding of the physical processes involved; the colored margin is greatest at the outside, but produces a certain confusion of the image, wherever it be located w.r.t. the optical axis; reducing the aperture helps sharpen the image ; Dollond's method to some extent actually cancels the effect of different colors having slightly different focal lengths due to the variation in the refractive index or dispersion, which in any case is only approximately linear in the visible region; however, it has seemed appropriate to include everything in this translation of Euler's Dioptrics; indeed, it is appropriate to indicate errors, but not to correct such translations, as clearly the historical context is lost.]

SETTING OUT THE FIRST CASE, WHERE THE LETTER *A* IS GIVEN A POSITIVE VALUE

109. Therefore in this case the letter $\mathfrak{A}$ will have a positive value also and indeed less than unity ; then truly the condition of the concave eyepiece lens demands, so that the letter *B* may obtain a negative value. Besides, on account of $A>0$ also there must be $P>1$, so that the first interval may be made positive. Now truly on account of the colored margin requiring to be removed the value of the letter *B* will be expressed thus, so that there shall become

$$B=\frac{N'(\mathfrak{q}+\mathfrak{r})}{N(A+1)P\mathfrak{r}-N'\mathfrak{r}};$$

where therefore on account of $\mathfrak{q}+\mathfrak{r}>0$ the denominator or the formula $N(A+1)P-N'$ must have a negative value ; which so that it may happen, since $(A+1)P$ certainly shall be greater than unity, it is necessary, that there may become $N'>N$ and thus so that the objective lens may be prepared from crown glass, the following truly from crystal glass. Whereby, since hence there may be produced $N:N'=2:3$ and hence there shall be

$$B=\frac{3(\mathfrak{q}+\mathfrak{r})}{2(A+1)P\mathfrak{r}-3\mathfrak{r}},$$

there will be required to become

$$2(A+1)P<3 \text{ or } P<\frac{3}{2(1+A)}.$$

But since there shall be $P>1$, it is evident the letter *A* must be taken so small, so that now also there shall be

$$\frac{3}{2(1+A)}>1 \text{ and thus } A+1<\frac{3}{2} \text{ and hence } A<\frac{1}{2};$$

if indeed there shall be $A=\frac{1}{2}$, there must be taken $P=1$ and the first interval plainly will vanish, that which is not allowed in practice ; from which likewise it is understood this letter *A* must be put in place so much smaller than $\frac{1}{2}$, so that now also the interval of the two first lenses shall be able to be returned in practice. Moreover the letter *A* will be assumed to be put in place with the letter *P* between the limits 1 and $\frac{3}{2(1+A)}$; but as we have seen just now it is not possible for the letter *P* to be taken equal to the lesser limit unity, or if it may be assumed equal to the upper limit, then *B* will become infinite and thus the length of the instrument will be extended to infinity. Therefore it may be agreed for P to be moved very close to the upper limit, so that the quantity AB at this stage may be able to be found. Then truly it remains now, that the latter equation may be satisfied, from which the aperture of the objective lens is defined; concerning which equation the following now will be able to be addressed:

1. Since $A<\frac{1}{2}$, there will be $\mathfrak{A}<\frac{1}{3}$, from which the value of its coefficient λ will be > 27; from which an enormous confusion will result, unless it may be diminished by the following term.
2. Truly since the coefficient of λ' for the second lens may itself become greater than 8 on account of $P=1$ approx., and since B always shall be a very large number, $\mathfrak{B}$ differs little from unity.
3. For the eyepiece lens the coefficient of λ'' will be very small, so that as if it may vanish before the remaining terms ; from which we may follow according to this convenience, that this whole confusion in short may be able to be reduced to zero, evidently duly by defining the letters λ and λ' ; whereby this case is worth mentioning, so that it may be illustrated by some examples.

EXAMPLE 1

109a. Since there must be $A<\frac{1}{2}$, we may put $A=\frac{1}{3}$ and there will become $\mathfrak{A}=\frac{1}{4}$ and $\frac{3}{2(1+A)}=\frac{9}{8}$, thus so that P may be taken within the bounds 1 and $\frac{9}{8}$. Therefore there may be $P=\frac{10}{9}$, and there will become $B=-\frac{81(\mathfrak{q}+\mathfrak{r})}{\mathfrak{r}}$.
Now we will consider the fundamental equation, which is

$$\mathfrak{B}\mathfrak{q}=\frac{1}{9}\cdot\frac{(\mathfrak{q}+\mathfrak{r})}{ma-h}\cdot h.$$

But for the sake of brevity there is put $\frac{ma}{h}=1+\theta$, since there must be $ma>h$, and so that this kind of microscopes may be able to be established, there will be $\mathfrak{B}=\frac{\mathfrak{q}+\mathfrak{r}}{9\mathfrak{q}\theta}$.
Now since there shall be $\frac{1}{\mathfrak{B}}=1+\frac{1}{B}$, there will be had

$$\frac{9\mathfrak{q}\theta}{\mathfrak{q}+\mathfrak{r}}=1-\frac{\mathfrak{r}}{81(\mathfrak{q}+\mathfrak{r})},$$

from which there is deduced:

$$\mathfrak{q}=\frac{80\mathfrak{r}}{729\theta-81},$$

and thus there will be produced

$$B=\frac{-729\theta+1}{9\theta-1} \quad \text{and hence} \quad \mathfrak{B}=\frac{+729\theta-1}{729\theta}$$

with there being $\theta = \frac{ma}{h} - 1$ or if the magnification $m = \frac{h(1+\theta)}{a}$. Then truly on account of $m = PQ \cdot \frac{h}{a}$ there will become

$$Q = \frac{ma}{Ph} = \frac{9ma}{10h} = \frac{9}{10}(\theta + 1)$$

and hence the elements for the construction of the microscope will be :

1. $A = \frac{1}{3}$, $\mathfrak{A} = \frac{1}{4}$, $B = \frac{-729\theta+1}{9\theta-1}$, $\mathfrak{B} = \frac{729\theta-1}{720\theta}$

$$P = \frac{10}{9}, \quad Q = \frac{9}{10}(\theta+1).$$

2. Thence the focal lengths of the lenses :

$$p = \frac{1}{4}a, \quad q = \frac{-729\theta+1}{2400\theta} \cdot a \quad \text{and} \quad r = \frac{-729\theta+1}{(27\theta-3)(\theta+1)} \cdot a.$$

3. The separations of the lenses will become :

$$\text{the first} = \frac{1}{30}a, \quad \text{the second } \frac{(729\theta-1)a}{30(\theta+1)}.$$

4. In addition the radius of the area viewed in the object will be :

$$z = \frac{729\theta\mathfrak{r} - \mathfrak{r}}{(729\theta-81)\theta} \cdot a\xi;$$

just as if now we may put here $\mathfrak{r} = 1$ and $\xi = \frac{1}{4}$, which is allowed, if the eyepiece lens may become made with each face equally concave, there will become

$$z = \frac{729\theta-1}{324\theta(9\theta-1)} \cdot a.$$

5. Finally this equation will be considered:

$$\frac{mx^3}{a^2h}\left(\mu(64\lambda+12\nu) - \frac{243\mu'}{10}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu'}{B\mathfrak{B}}\right) + \frac{27\mu''\lambda''}{B^3(\theta+1)}\right) = \frac{1}{k^3},$$

where conveniently it arises in use, that this quantity may be able to be reduced to zero, for which finally we may put the third lens as the first to be made from crown glass, and there must be assumed

$$\lambda'' = 1,60006 \text{ and } \mu'' = \mu ;$$

then truly there may be assumed $\lambda = 1$, but λ' thus, so that there shall become

$$\frac{\mu'}{\mu}\cdot 24,3\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)=64\lambda+12\nu+\frac{27\cdot 1,60006}{B^3(1+\theta)}=66,6352$$

with there being

$$\frac{\mu'}{\mu}=\frac{0,8724}{0,9875},\quad \nu=0,2196 \text{ and } \nu'=0,2529.$$

Truly besides it may be noted for the greater magnifications, when evidently θ may become a small enough number, to become approx.

$$B=-81 \quad\text{and}\quad \mathfrak{B}=+\tfrac{81}{80};$$

from which there is deduced

$$0,96341\lambda'=0,00308+\frac{\mu}{\mu'}\cdot\frac{66,7352}{24,3}$$

and hence

$$\lambda'=3,22503 \text{ and } \tau\sqrt{(\lambda'-1)}=1,3089;$$

from which, since the focal length of this lens shall be

$$q=-\frac{729}{2400}a=-\frac{243}{800}a$$

and $\mathfrak{B}=\frac{81}{80}$, of this lens, the radius

$$\begin{cases}\text{of the anterior face } =\dfrac{q}{\sigma-\mathfrak{B}(\sigma-\rho)+\tau\sqrt{(\lambda'-1)}}=\dfrac{q}{1,4323}=0,69818q\\ \text{of the posterior face}=\dfrac{q}{\rho+\mathfrak{B}(\sigma-\rho)-\tau\sqrt{(\lambda'-1)}}=\dfrac{q}{0,2918}=3,42700q.\end{cases}$$

But for the first lens, of which the focal length is $p=\frac{1}{4}a$ and the numbers $\mathfrak{A}=\frac{1}{4}$ and $\lambda=1$, requiring to be made from crown glass, the radius

$$\begin{cases}\text{of the anterior face } =\dfrac{p}{\sigma-\mathfrak{A}(\sigma-\rho)}=\dfrac{p}{1,3017}=0,76823p\\ \text{of the posterior face}=\dfrac{p}{\rho+\mathfrak{A}(\sigma-\rho)}=\dfrac{p}{0,5851}=1,70911p.\end{cases}$$

CONSTRUCTION OF THIS KIND OF MICROSCOPE

110. With the object distance put $=a$ and with the magnification $m=(1+\theta)\frac{h}{a}$ there will be :

I. For the objective lens being made from crown glass, the radius of the

$$\begin{cases}\text{anterior face} = 0,1921a\\ \text{posterior face} = 0,4273a.\end{cases}$$

of which the focal length $p=\frac{1}{4}a$,
the radius of the aperture $x=0,0480a$
and the distance to the second lens will be $=\frac{1}{30}a$.

II. For the second lens being made from crystal glass, the radius of the

$$\begin{cases}\text{anterior face} = -0,2121a\\ \text{posterior face} = -1,0409a.\end{cases}$$

of which the focal length is $q=-\frac{243}{800}a=0,3037a$,
the radius of the aperture $x=0,0530a$ or is left indefinite, since it is greater than the radius of the aperture of the first lens, and the distance to the third lens $=24,3\cdot\frac{\theta}{\theta+1}\cdot a.$

III. For the third lens prepared from crown glass, of which the focal length

$$r=\frac{729}{27(\theta+1)}\cdot a=-\frac{27}{\theta+1}\cdot a,$$

the radius of each face will be

$$=-\frac{28,62}{\theta+1}\cdot a,$$

to which the eye will be required to be applied directly to the lens.

IV. The area within the object is perceived, the radius of which is given by

$$z=\frac{1}{4\theta}\cdot a$$

V. Finally since there shall be

$$x=0,0480a,$$

there will be

$$y = \frac{hx}{ma} = \frac{x}{\theta+1} = \frac{0,0480}{\theta+1} \cdot a$$

and hence the degree of clarity

$$20y = \frac{0,960}{\theta+1} \cdot a,$$

clearly if the distance *a* may be expressed in inches, which measure also is expressed thus:

$$0,960 \cdot \frac{h}{m} = \frac{7,68}{m}.$$

COROLLARY 1

111. Since the interval of these two first lenses does not depend on the proposed magnification and thus they can retain the same interval for all magnifications, thus so that for any magnification another eyepiece to be used, of which the focal length with the value $\frac{ma}{h}$ written in place of loco $\theta+1$ will be

$$r = -27\frac{h}{m} = -\frac{216}{m} \text{ in.},$$

thus so that this lens never may be made exceedingly small.

COROLLARY 2

112. But however the magnification may be varied, the separation of the second and third lens is changed very little, especially in the greater magnifications, since here the interval shall be

$$= 24,3 \cdot \frac{\theta}{\theta+1} \cdot a = 24,3\left(a - \frac{h}{m}\right),$$

thus so that the whole length of the instrument scarcely shall be required to be changed, and if the distance *a* of the object may be taken to be of 1 inch, the length of the instrument shall be around two feet.

SCHOLIUM

113. Since here the distance of the object may be allowed according to our choice, that without doubt is regarded as a significant convenience, since in this way the greatest fault of simple microscopes, which consists in the excessive closeness of the object, may be avoided with the most rewarding success, since with this distance increased however great certainly the measure of the clarity is not diminished, and equally the small space viewed in the object. Yet meanwhile on the other hand this kind of object will be able to be present in the first place, since the two first lenses may be able to be exceedingly close together; which scarcely deserves hardy any attention, since at this stage this interval shall be able to be observed easily in practice, unless the distance of the object *a* may be

established exceedingly small, but which no account suggests; truly the other objection is more serious, because, if the distance *a* may be taken greater by one inch, the length of this instrument thus now will exceed two feet, which deservedly can be seen as an inconvenience. Truly we will show soon, and in what manner this inconvenience may be able to occur. Moreover just as we have put in place this kind requiring to be defined by the letters *A* and *P*, that can be established in the first place, whether the difference of the numbers *N* and *N′* may be a little less than in the ratio 2:3, as we have assumed here, then further determinations generally cannot be done; if indeed in place of this ratio we may substitute that, as we have concluded from these experiments of Dollond, clearly to be used 7:10, so that there may become

$$B = \frac{10(\mathfrak{q}+\mathfrak{r})}{7(A+1)P\mathfrak{r}-10\mathfrak{r}},$$

then on taking $A = \frac{1}{3}$ and $P = \frac{10}{9}$, the denominator $7(A+1)P\mathfrak{r} - 10\mathfrak{r}$ will become $= \frac{280}{27} - 10$ and thus is no longer negative, as the nature of the matter demands; therefore this occurrence shall have a much smaller place, if the difference of the glass may have a much smaller dispersion at this point, so that indeed it will be seen to be hardly probable. On account of which, lest hence there shall be no reason to worry, it will be agreed to assume the letters *A* and *P* thus, so that the formula $(1+A)P$ may obtain a smaller value than in the case of the example advanced, clearly the fraction for the letter *A* will have to be assumed much smaller than $\frac{1}{3}$; then truly the value of P may increase as little as to unity, as the proximity of the lenses allows, which condition we will satisfy in the following example.

EXAMPLE 2

113a. Therefore here we may assume $\mathrm{A} = \frac{1}{5}$ and there will become $\mathfrak{A} = \frac{1}{6}$ and $p = \frac{1}{6}a$, moreover the interval between the first and second lens $= \frac{1}{5}(1-\frac{1}{P})a$; which so that it may be equal to the seventh part of p, there must be taken $P = \frac{42}{37} = \frac{8}{7}$ or approx. $\frac{9}{8}$; therefore we may suppose $P = \frac{9}{8}$, and since also here as in the preceding example the $\mathfrak{q}$ shall become much smaller than the letter $\mathfrak{r}$, with that ignored there will become

$$B = \frac{N'}{N(1+A)P-N'}$$

and on taking $N : N' = 7:10$ with these values substituted there will be $B = -\frac{200}{11}$ or $B = -18$, which value at this point would be produced larger, if the dispersion perceived now were smaller. Therefore since it shall be plausible enough at this point if this value may be perceived now to be smaller, we will hardly err from the aim, if we may put $B = -25$, and if any error hence may emerge from that, it will consist in the colored margin not being able to be removed completely; since indeed that may not be able to be aspired, it will require us to be content, if we have been able to render that small enough, which certainly we will obtain in this manner; moreover on taking $B=-25$ there will be $\mathfrak{B}=\frac{25}{24}$ and hence from the fundamental equation

$$\mathfrak{q}=\frac{3h\mathfrak{r}}{25ma-28h}$$

and hence

$$\mathfrak{q}+\mathfrak{r}=\frac{25ma\mathfrak{r}-25h\mathfrak{r}}{25ma-28h};$$

from which the radius of the viewed area is deduced to be :

$$z=\frac{25\mathfrak{r}}{25ma-28h}\cdot ha\xi\ ;$$

whereby, if there may be assumed $\xi=\frac{1}{4}$ and $\mathfrak{r}=1$, in which case it is required, that the eyepiece lens shall be equally concave on each side, and if we may put as before $\frac{ma}{h}=1+\theta$, there will become

$$z=\frac{25}{100\theta-12}\cdot a\ ;$$

moreover the remaining elements will be obtained in the following manner :

$$A=\frac{1}{5},\ \mathfrak{A}=\frac{1}{6},\ B=-25,\ \mathfrak{B}=\frac{25}{24},\ P=\frac{9}{8}\text{ and } Q=\frac{8}{9}(1+\theta)=\frac{8ma}{9h}$$

and hence the focal lengths :

$$p=\frac{1}{6}a,\quad q=-\frac{5}{27}a\ \text{ et }\ r=-\frac{5}{1+\theta}\cdot a=-\frac{5h}{m},$$

and the separation of the lenses

$$\text{I and II}=\frac{1}{45}a,\quad \text{II and III}=\frac{40ma-45h}{9m}.$$

Now we may accomplish, that also the confusion arising from the aperture may vanish, and since the first lens must be made from crown glass, the second from crystal glass, if the third also from crown glass, so that there shall become $\mu''=\mu$, there must be $\lambda''=1,60006$; then truly for the first lens there may be taken $\lambda=1$; this equation will be had :

$$\frac{\mu'}{\mu}\cdot\frac{5^3\cdot 8}{9}\left(\frac{24^3}{25^3}\lambda'-\frac{24\nu'}{25^2}\right)=6^3+30\nu-\frac{1,60006h}{5^3ma}.$$

But there is $\log.\frac{\mu'}{\mu}=0,0538214$ or

$$\frac{\mu'}{\mu}\left(98,304\lambda' - 4,2666\nu'\right) = 216 + 30\nu - 0,0128\cdot\frac{h}{ma}$$

or

$$98,304\lambda' = 253,034 - 0,0145\cdot\frac{h}{ma},$$

from which there is deduced

$$\lambda = 2,5740 \; - 0,00015\cdot\frac{h}{ma}$$

where the last term can be omitted with care on account of the small fraction $\frac{h}{ma}$.
Therefore on account of

$$\lambda' = 2,5740 \text{ and } \lambda' - 1 = 1,5740,$$

there will become

$$\tau\sqrt{(\lambda'-1)} = 1,1009 \;;$$

from which, since the focal length of this second lens shall be

$$q = -\tfrac{5}{27}a \text{ and the number } \mathfrak{B} = \tfrac{25}{24},$$

will the radius

$$\begin{cases} \text{of the anterior face } = \dfrac{q}{\sigma - \mathfrak{B}(\sigma-\rho) + \tau\sqrt{(\lambda'-1)}} = \dfrac{q}{1,1823} = 0,8458q \\ \text{of the posterior face} = \dfrac{q}{\rho + \mathfrak{B}(\sigma-\rho) - \tau\sqrt{(\lambda'-1)}} = \dfrac{q}{0,5418} = 1,8457q. \end{cases}$$

But for the first lens, of which the focal length

$$p = \tfrac{1}{6}a, \quad \mathfrak{A} = \tfrac{1}{6} \text{ and } \lambda = 1$$

and of crown glass, the radius

$$\begin{cases} \text{of the anterior face } = \dfrac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = \dfrac{p}{1,4212} = 0,7036p \\ \text{of the posterior face} = \dfrac{p}{\rho + \mathfrak{A}(\sigma-\rho)} = \dfrac{p}{0,4656} = 2,1478p. \end{cases}$$

Hence the following zero confusion construction of a microscope is produced :

114. With any desired distance of the object $= a$ put in place, we will have:

I. With the first lens being made from crown glass, the radius

$$\begin{cases}\text{of the anterior face } = 0{,}1173a\\ \text{of the posterior face } = 0{,}3579a,\end{cases}$$

of which the focal length is $\frac{1}{6}a = 0{,}1666a$;
the radius of the aperture of the aperture will be able to be taken $x = 0{,}0293a$,
the distance to the second lens $= \frac{1}{45}a = 0{,}022a$.

II. For the second lens being made from crystal glass, the radius

$$\begin{cases}\text{of the anterior face } = -0{,}1566a\\ \text{of the posterior face} = -0{,}3418a,\end{cases}$$

of which the focal length $= -\frac{5}{27}a = -0{,}1852a$,
the radius of the aperture $= -\ 0{,}0392a$;
the distance to the third lens will be $= \frac{40ma-45h}{9m} = 4\frac{4}{9}a - \frac{5h}{m}$.

III. For the third eyepiece lens made from crown glass, the focal length will be $-\frac{5h}{m}$ and hence the radius of each face $= -5{,}3 \cdot \frac{h}{m}$; but if it may be made from crystal glass, and the radius of each face may be taken $= -5{,}8 \cdot \frac{h}{m}$, the eye must be applied directly to this lens.

IV. Moreover the radius of the area viewed in the object will be

$$z = \frac{25}{100\theta - 12} \cdot a$$

with there being

$$\theta = \frac{ma}{h} - 1.$$

V. Since there may be allowed to take $x = 0{,}0293a$, there will be $y = 0{,}0293 \cdot \frac{h}{m}$ and the measure of the clarity $= 0{,}586 \cdot \frac{h}{m}$ and on putting $h = 8$ in., that will become $= \frac{4{,}688}{m}$.

COROLLARY 1

115. Therefore lest the first two lenses may not be required to be made exceedingly small, it will be agreed to assume the distance of the object a to be so much greater; and if there may be put in place $a = 8$ in., these lenses will obtain a suitable enough magnitude and the magnification m will show, how much more the object will appear

through the microscope, than if we may view the same object at the same distance with the naked eye.

COROLLARY 2

116. Thence if we may assume $a = 8$ in., the length of the whole instrument will become around $35\frac{1}{2}$ in., which certainly is large enough; but that must be considered to be only $4\frac{1}{2}$ times greater in turn than the distance of the object, and that is reduced by half on assuming $a = 4$ in.; in which case the construction of the lens at this stage is accommodated well enough in practice, also there is no reason why the distance of the object cannot be assumed conveniently smaller at this stage so that indeed it may not exceed a whole foot.

SCHOLIUM 1

117. Quite a paradox will be observed, because the distance of the object clearly does not enter into the measure of the clarity; indeed certainly no one will believe, if the distance may be increased to several feet, that the object is going to appear with the same clarity, and that for the same magnification. Truly here our measure of clarity is required to be observed properly and referred to that order of clarity we may discern, where the same object is observed with the naked eye in the place where it actually is situated. Indeed if this measure may be produced equal to unity, it is required to understand we ourselves can see the object through the instrument with the same clarity by which it is going to appear at that same distance to the naked eye; moreover it is observed, where the object is moved away from us, its clarity to be diminished in the same natural ratio ; whereby, since our measure shall refer to the natural clarity, evidently which is seen in the object itself by the naked eye, it is evident, where we move the object away by increasing the distance a, so that its natural clarity to be diminished more, and then our measure only indicates, how much the clarity seen through the microscope shall be less than the natural clarity, and from this it is clearly seen the maximum clarity to be diminished, if we may assume the distance of the object a to be exceedingly great, thus so that for the use of the microscopes the distance of the object shall scarcely be agreed to be used extended beyond a few inches. In a similar manner the judgment concerned with the magnification is required to be understood, as we refer this to the distance $h = 8$ in. ; therefore so that if, e.g. the object were 16 in. distant, that now to the naked eye would appear twice as small as sat the distance of 8 inches; whereby, if the object were said to be increased by 100, thus this is to be understood, that the object shall appear two hundred times more than as viewed by the naked eye at the same distance.

SCHOLIUM 2

118. Hence therefore it is understood easily, if we may put the distance of the object great enough, then the microscope finally to be going to used as a telescope, which transition thus deserves more attention, where a greater distinction generally is put in place between the telescope and microscope, which certainly are accustomed to be considered clearly as different kinds. Therefore it is worth the effort to adjoin an example of this kind, concerning which there will be doubt, whether it shall be required to be referred to as a microscope, or as a telescope.

EXAMPLE 3

119. The distance of the object a shall be so great, so that with as small enough fraction taken for $\mathfrak{A}$ the product $\mathfrak{A}a = p$ may obtain as small value, or the fraction $\mathfrak{A} = \frac{p}{a}$ shall be very small and hence also $A = \frac{p}{a-p}$.
With these in place there shall be

$$B = \frac{N'}{N(A+1)P - N'} = \frac{10}{7(1+A)P-10},$$

taking $P = \frac{9}{8}$ as before, and lest we may ignore A completely, we may put

$$(1+A)P = \frac{8}{7} \text{ and there will become } B = -5;$$

and if perhaps the difference between the letters N and N' shall be minus, and lest we may neglect the letter $\mathfrak{q}$ completely, we may assume $B = -6$, so that there shall be $\mathfrak{B} = \frac{6}{5}$; therefore since in place of the letters a and A the focal length p is introduced into the calculation, so that there shall be either $\mathfrak{A}a = p$ or $Aa = p$, the remaining focal lengths will be

$$q = -\frac{15}{16}p \text{ and } r = \frac{6h}{ma} \cdot p.$$

Then truly the first interval $= \frac{1}{9}p$, the second $= \frac{16}{3}p\left(1 - \frac{9h}{8ma}\right)$.
Besides truly there is found :

$$\mathfrak{q} = \frac{5hr}{48ma - 53h}, \text{ hence } \mathfrak{q} + \mathfrak{r} = \frac{48ma\mathfrak{r} - 48h\mathfrak{r}}{48ma - 53h}$$

and the radius of the area viewed

$$z = \frac{12}{48ma - 53h} \cdot ah$$

and thus the angle

$$\frac{z}{a}=\Phi=\frac{12h}{48ma-53h},$$

which angle multiplied by 3437 expresses the angle Φ in minutes of arc.
Thence the radius of confusion, if the denominator $\mathfrak{A}^3$ may be freed fro the brackets [see § 104] and transferred into the common factor, thus there will be had :

$$\frac{a}{h}\cdot\frac{mx^3}{p^3}\left(\mu\lambda-\frac{8\mu'}{9}\left(\frac{5^3\lambda'}{6^3}-\frac{5\nu'}{36}\right)-\frac{\mu''\lambda''h}{6^3 ma}\right),$$

which may be reduced to nothing by taking

$$\frac{8\mu'}{9}\left(\frac{5^3\lambda'}{6^3}-\frac{5\nu'}{36}\right)=\mu\lambda-\frac{\mu''\lambda''h}{6^3 ma};$$

where the first lens must be made from crown glass, the second from crystal glass, and truly the third also equally from crown glass and thus $\lambda''=1,60006$ and $\mu''=\mu$; then truly there may be taken $\lambda=1$ and there will be found

$$\lambda'=0,24\nu'+\frac{\mu}{\mu'}(1,944-0,0144\cdot\frac{h}{ma})=2,2611$$

evidently with the final term ignored on account of its small size, from which there becomes $\tau\sqrt{(\lambda'-1)}=0,98542$; from which for the construction of this lens the radius

$$\begin{cases}\text{of the anterior face } =\frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)+\tau\sqrt{(\lambda-1)}}=\frac{q}{0,8385}=1,1926q\\ \text{of the posterior face}=\frac{q}{\rho+\mathfrak{B}(\sigma-\rho)-\tau\sqrt{(\lambda-1)}}=\frac{q}{0,8856}=1,1292q.\end{cases}$$

Truly for the first lens the radius

$$\begin{cases}\text{of the anterior face } =\frac{p}{\sigma}=0,6024p\\ \text{of the posterior face}=\frac{p}{\rho}=4,4111p\end{cases}$$

and hence the following

CONSTRUCTION EITHER OF MICROSCOPES OR TELESCOPES WITH ALL CONFUSION REMOVED

120. Here the distance of the object a is supposed so great, as in view of that the focal length of the first lens p shall be extremely small, and as if it may be able to be ignored.

I. Then therefore for the first lens requiring to be prepared from crown glass, the radius

$$\begin{cases}\text{of the anterior face} &= 0,6024p\\ \text{of the posterior face} &= 4,4111p,\end{cases}$$

of which the focal length $= p$,
the radius of the aperture $x = 0,1506p$,
the distance from the second lens $= \frac{1}{9}p$.

II. For the second lens requiring to be made from crystal glass, the radius

$$\begin{cases}\text{of the anterior face} &= -1,2721p\\ \text{of the posterior face} &= -1,2045p,\end{cases}$$

of which the focal length $= -\frac{15}{16}p$,
and an aperture can be given to that a little greater than of the first,
truly the distance to the eyepiece lens $= \frac{16}{3}p\left(1-\frac{9h}{8ma}\right)$.

III. For the third lens requiring to be made from crown glass, of which the focal length is $r = -\frac{6h}{ma}\cdot p$, the radius of each face will be $= -\frac{6,36h}{ma}\cdot p$, to which the eye will require to be applied directly.

IV. For the area viewed the radius we find now

$$z = \frac{12}{48ma-53h}\cdot ah$$

or the angle

$$\Phi = \frac{z}{a} = \frac{12h}{48ma-53h};$$

clearly is considered in the first way, if the instrument may be considered as a microscope, truly in the second way, if considered as a telescope.

V. Since it is allowed to take

$$x = 0,1506p,$$

there will be

$$y = \frac{0,1506h}{ma}\cdot p$$

and the measure of the clarity

$$=\frac{3{,}012}{ma}\cdot hp$$

if clearly the distances may be expressed in inches, from which it is apparent, were *p* may be taken greater, there greater clarity to be produced; but it is required to be remembered *p* must be taken small compared to *a*.

VI. Finally the length of the whole instrument will be

$$5\tfrac{4}{9}p-6\frac{h}{ma}\cdot p.$$

COROLLARY 1

121. So that if we may wish to view this instrument just as a microscope, certainly in the first place the distance *a* must nevertheless be large enough, so that a small part of that will suffice for the objective lens to be constructed ; then truly it is accustomed to take $k=8$ in., to which distance the magnification *m* is accustomed to be referred, and from the magnification estimated in this way is introduced into the calculation $\frac{h}{ma}$. But if we may wish to regard as a telescope and the distance *a* shall be just as great, so that also the value p may be taken large enough, then it is accustomed to take $h=a$ and nothing other present in the formulas found requiring to be changed, thus so that the whole distinction shall consist in estimating the magnification in varied accounts.

COROLLARY 2

122. So that this may be seen more clearly, we may put $\frac{ma}{h}=\zeta$ from which the construction may be determined fully ; and if the instrument may be viewed as a microscope, it is customary to estimate the magnification $m=\frac{h\zeta}{a}=\frac{8\zeta}{a}$, but if it is viewed as a telescope, then the magnification may be called $m=\zeta$ and thus the whole distinction may be reduced to the different way of discussing the instrument.

COROLLARY 3

123. For telescopes the measure of clarity thus can be increased as far as it pleases and thus to the full clarity unity; indeed there is a need only that there may be taken $p=\frac{m}{3{,}012}=\frac{m}{3}$. But generally we are accustomed to be content with the clarity $=\frac{2}{5}$, thus so that then there must be taken $p=\frac{2m}{15}$. But for microscopes it is not permitted to obtain so great a clarity ; for since on account of $h=8$ the measure of the clarity shall be $\frac{24}{m}\cdot\frac{p}{a}$

and the fraction $\frac{p}{a}$ by necessity is definitely small, and where a greater magnification may be desired, there it is necessary to produce a smaller clarity.

SCHOLIUM

124. Behold therefore beyond all expectation the elegant construction of a telescope, which may magnify objects in any ratio, and the construction of which will be outlined in the following manner.

Evidently for the magnification *m* the proposed focal length may be taken $p = \frac{2m}{15}$ in., so that clearly the measure of the clarity shall be produced $= \frac{2}{5}$.

Construction of the telescope free from all confusion:

I. For the first lens requiring to be made from crown glass the radius

$$\begin{cases} \text{of the anterior face} = 0,0803m \text{ in.} \\ \text{of the posterior face} = 0,5881m \text{ in.,} \end{cases}$$

the focal length $= \frac{2m}{15}$ in.,
the radius of the aperture $x = 0,0201m$ in. $= \frac{m}{50}$ in.,
the distance to the following lens will be $= \frac{2m}{135} = 0,01481m = \frac{m}{50}$ in.,

II. For the second lens requiring to be made from crystal glass, the radius

$$\begin{cases} \text{of the anterior face} = -\ 0,16961m \text{ in.} \\ \text{of the posterior face} = -0,16060m \text{ in.,} \end{cases}$$

of which the focal length $q = -0,1422m$,
and the aperture for that may be given a little greater than for the first lens,
the distance to the following lens $= (0,7111m - 0,8)$ in.

III. For the third lens, the focal length of which is $= -\frac{4}{5}$ in. $= -0,8$ in.,
if therefore this lens may be prepared from crown glass, the radius of each face $= -0,848$ in., but if constructed from common glass, where $n = 1,55$, the radius of each face will be $= -0,88$ in.; but if from crystal glass the radius of each face will be $= -0,928$ in.; to which the eye will be applied directly.

IV. The radius of the apparent field will be

$$\Phi = \frac{12}{48m-53};$$

but measured in angles it will become

$$\Phi = \frac{41244}{48m-53} \text{ min. or approx. } \frac{859}{m-1} \text{ min.}$$

V. Finally the length of the whole telescope will be

$$= (0,7259m - 0,8) \text{ in.}$$

Therefore this telescope is required to be recommended not so much because of its shortness, but thus, because the practical construction of this shall not be involved with so great difficulties as it will be much shorter, that what have been found above, especially since the letter λ' may not differ much from unity; which commendation therefore prevails also for microscopes of this kind.

EVOLUTION OF THE SECOND CASE (SEE § 108) WHERE THE LETTER A IS GIVEN A NEGATIVE VALUE

125. In this case will be unclear whether the letter $\mathfrak{A}$ is going to have either a positive or negative value ; but now the letter B must be positive, and since on account of the same ratio as in the preceding case the letter $\mathfrak{q}$ may be vanishing before $\mathfrak{r} = 1$, there will become

$$B = \frac{N'}{N(1+A)P - N'}$$

where there must become $P < 1$, and thus many more times will $(1+A)P < 1$; from which it is seen the letter N must be greater than N'. Whereby it will be required to make the first lens from crystal glass, truly the second from crown glass, so that there shall be $N : N' = 10 : 7$ and thus

$$B = \frac{7}{10(1+A)P - 1};$$

from which it is necessary, that there shall become $P > \frac{7}{10(1+A)}$, likewise truly $P < 1$; from which there must become the following :

$$7 < 10(1+A) \text{ or } 1+A > \frac{7}{10}.$$

Therefore we may put $A=-\alpha$ and to be assumed that $\alpha<\frac{3}{10}$ and certainly α must be taken to be notably less than $\frac{3}{10}$, since otherwise P must differ exceedingly little from unity and the distance between the two first lenses will be produced exceedingly small. But since α shall be a small enough fraction, there will become $\mathfrak{A}=\frac{\alpha}{1-\alpha}$ and hence the focal length of the first lens

$$p=-\frac{\alpha}{1-\alpha}\cdot a$$

Truly the interval between the first two lenses

$$=-\alpha a\left(1-\frac{1}{P}\right)=\alpha a(\frac{1}{P}-1),$$

which will be equal either to the ninth or to the tenth part of the distance $\frac{\alpha}{1-\alpha}\cdot a$, which will happen, if there may be taken $P=\frac{8}{9}$, thus so that there shall become $\alpha<\frac{17}{80}$, and lest we may be so anxious to hold on to this ratio 7:10 , if we may assume $\alpha=\frac{1}{6}$, there will become $B=\frac{189}{11}=17$. Moreover we may take rather $\alpha=\frac{1}{7}$ and there will become $B=\frac{441}{13}=11\frac{4}{14}$. Then without risk we may put then $\mathrm{A}=-\frac{1}{7}$ and $\mathfrak{A}=-\frac{1}{6}$ and hence the focal lengths

$$p=-\frac{1}{6}a,\quad q=\frac{27}{182}a\quad\text{and}\quad r=-\frac{12}{7}\cdot\frac{h}{m};$$

thence the intervals between the lenses

$$\text{I and II}=\frac{1}{56}a,\quad\text{II and III}=\frac{27}{14}a-\frac{12}{7}\cdot\frac{h}{m}\cdot$$

Now truly from the fundamental equation we will deduce

$$\mathfrak{q}=-\frac{13h}{108ma-95h},$$

hence

$$\mathfrak{q}+\mathfrak{r}=\frac{108ma-108h}{108ma-95h};$$

from which the radius of the area to be viewed is deduced

$$z=\frac{108}{108ma-95h}\cdot ah\xi=\frac{27ah}{108ma-95h}$$

evidently on taking $\mathfrak{r}=1$ and $\xi=\frac{1}{4}$.

Again the expression for the radius of the confusion is

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''\lambda''}{A^3B^3PQ}\right),$$

which may be reduced to zero. Finally these letters p and v will be noted for crystal glass, truly the letters μ' and v' refer to crown glass ; then truly there will be taken $\lambda' = 1$, and if the third lens also may be made from crown glass, so that there shall be $\mu'' = \mu'$, there must be taken $\lambda'' = 1,60006$, and hence the number λ can be defined in this way:

$$-\frac{\lambda}{\mathfrak{A}^3} - \frac{v}{A\mathfrak{A}} - \frac{\mu'}{\mu A^3 P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{v'}{B\mathfrak{B}}\right) + \frac{\mu''\lambda''}{\mu A^3 B^3 PQ}$$

or

$$\lambda = 0,0491 + \frac{\mu'}{\mu}\left(\frac{343\cdot 2197}{192\cdot 1728} + \frac{343\cdot 13\cdot 0,2196}{192\cdot 144} - \frac{343\cdot 1,60006}{216\cdot 1728}\cdot\frac{h}{ma}\right),$$

which on expansion gives

$$\lambda = 0,0491 + 2,5709 + 0,0401$$

with the final term ignored, or

$$\lambda = 2,6601,$$

from which there is deduced

$$\tau\sqrt{(\lambda - 1)} = 1,1306.$$

Hence the radius for the first lens will be

$$\begin{cases} \text{of the anterior face } = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho) - 1,1306} = \frac{p}{0,6923} = 1,4444p \\ \text{of the posterior face} = \frac{p}{\rho + \mathfrak{B}(\sigma-\rho) + 1,1306} = \frac{p}{1,0318} = 0,9692p. \end{cases}$$

Moreover for the second lens prepared from crown glass on account of $\mathfrak{B} = \frac{12}{13}$ and $\lambda' = 1$ the radius

$$\begin{cases} \text{of the anterior face } = \frac{q}{\sigma - \mathfrak{A}(\sigma-\rho)} = \frac{q}{0,3370} = 2,9673q \\ \text{of the posterior face} = \frac{q}{\rho + \mathfrak{B}(\sigma-\rho)} = \frac{q}{1,5498} = 0,6452q, \end{cases}$$

from which the following construction will be had:

CONSTRUCTION OF MICROSCOPES OF THIS KIND

FOR ANY MAGNIFICATION m

126. For whatever distance of the object put in place $= a$, there will be had

I. For the first lens requiring to be made from crystal glass, of which the focal length is $p = -\frac{1}{6}a$, the radius

$$\begin{cases} \text{of the anterior face} = -0,2407a \\ \text{of the posterior face} = -0,1615a, \end{cases}$$

of which the radius of the aperture will be able to be taken $x = 0,0404a$, except perhaps the second lens may demand a smaller one.

The interval to the second lens $= \frac{1}{56}a = 0,0178a.$

II. For the second lens requiring to be made from crown glass, of which the focal length is $q = \frac{27}{182}a$, the radius

$$\begin{cases} \text{of the anterior face} = 0,4402a \\ \text{of the posterior face} = 0,0957a, \end{cases}$$

of which the radius of the aperture cannot be greater than 0,0239*a;* to which value also the value of *x* must be equal to that of the first lens.

Truly the interval to the third lens will be

$$\frac{27}{14}a - \frac{12}{7}\cdot\frac{h}{m} = 1,9285a - \frac{12}{7}\cdot\frac{h}{m}.$$

III. For the third lens, of which the focal length is

$$r = \frac{12}{7}\cdot\frac{h}{m} = -\frac{96}{7m}\text{ in.} = -\frac{13,71}{m}\text{ in.}$$

if it may be prepared from crown glass, the radius of each face will be

$$= \frac{14,53}{m}\text{ in.},$$

but if it may be prepared from common glass, $n = 1,55$, the radius of each face will be

$$= \frac{15,08}{m}\text{ in.},$$

but if it may be prepared from crystal glass, the radius of each face will be

$$= \frac{15,90}{m}\text{ in.}$$

IV. Again, the radius of the area viewed in the object will be

$$z = \frac{27ah}{108ma - 95h} = \frac{54a}{27ma - 190}\text{ in.}$$

V. But since here there shall be $x = 0,0239a$, there will be

$$y = \frac{hx}{ma} = \frac{0,1912}{m} \text{ dig.}$$

and hence the measure of the clarity will be

$$20y = \frac{3,824}{m}.$$

COROLLARY 1

127. Lest both the first lenses may not become very thin, the distance of the object a by necessity must undergo a small change of magnitude; just as if we may be unwilling that any radius of the face shall be less than the tenth part of an inch, by putting the minimum radius $0,0957a = \frac{1}{10}$ there will become $a = \frac{1}{0,957}$, or the distance a may not be agreed to be less than one inch.

COROLLARY 2

128. Therefore if there may be assumed $a = 1\frac{1}{2}$ in., in which case the first lenses now will be easy to prepare, the length of the whole instrument will become around 3 in., and since the focal length of the third lens shall be $-\frac{13,71}{m}$ in., it will be apparent the magnification scarcely able to be extended beyond 100, since otherwise this lens will become exceedingly small; which small size is a fault.

SCHOLIUM

129. But if we may desire very large magnifications, all these same microscopes are troubled by the same fault, because the eyepiece lens shall be required to be exceedingly small, and between these, which have been described in example 2 § 114, entertain the use of this rule, that the focal length of the third lens shall be $-\frac{40}{m}$ in., which therefore will be able to be applied to the magnification $m = 400$; but in the first example, which on account of the exceedingly great length of the instrument was seen to be rejected, the magnification can be increased much further; indeed since here the focal length of the third lens shall be $-\frac{216}{m}$ in., this provides the advantage to us, so that the magnification may be able to be increased beyond 1000, thus so that by this gain that inconvenience will be compensated maximally. From which it is able to deduce huge magnifications cannot be produced by microscopes of this kind, unless the length of these may become excessively great, for which it is necessary, that the letter B may obtain a very great

value, which indeed may be put in place most easily in the first case especially, where with q ignored there became

$$B = \frac{10}{7(A+1)P-10}$$

hence indeed on taking $A = \frac{1}{5}$ and $P = \frac{7}{6}$ there will be produced $B = -50$, and if with $A = \frac{1}{5}$ remaining, there may be taken $P = \frac{33}{28}$, from which there will become $B = -100$, thus so that then the focal length of the third lens shall become

$$r = -\frac{20h}{m} = -\frac{160}{m}$$

and thus the magnification may be able to be increased beyond 1000. But then the length of the instrument may become

$$-\frac{AB}{P}(1-\frac{1}{Q}) = 17 \text{ in.},$$

which indeed may be easily allowed. Truly here it is required to consider, if these values may be given to the letters A and P, to be able to happen easily, that the value of the letter B actually not only may be increased as far as to infinity, but also it may arise positive, evidently if the true ratio of the numbers N and N' were a little greater than 7:10. On account of which, thus we may advance the need for establishing microscopes of the two remaining kinds. Yet meanwhile also the greatest magnifications will be allowed to be produced suitably in the following manner.

PROBLEM 3

130. *To construct microscopes of this kind, which are required to be adapted for producing the maximum magnifications.*

SOLUTION

Since the whole investigation may be reduced to this, so that the letter B may obtain an extra large value, that can be obtained in two ways, just as either the first lens may be made from crown glass, the second truly from crystal glass, or in turn the first lens from crystal glass and the second from crown glass. Both these cases deserve to be treated separately.

THE PRIOR CASE IN WHICH THE FIRST LENS MAY BE PREPARED FROM CROWN GLASS AND THE SECOND FROM CRYSTAL GLASS

Since in this case there may be had

$$B = \frac{10}{7(A+1)P-10},$$

here certainly the denominator shall be reduced to zero, so that the value of B may become infinite; for then its enormous value is easily understood also with our aim going to be satisfied, especially since also for the case, in which the true ratio of the numbers N et N' may differ a little from the ratio 7:10 assumed, the value of the letter B will be going to be produced so much greater. Therefore we may put $P=\frac{8}{7}$, since on account of necessity, here a value of the interval between the first two lenses shall not be able to be put in place conveniently less ; and then there will require to be $A=\frac{1}{4}$; or if perhaps, as may probably be seen, the difference of refraction may not be so great as we have assumed, it may be agreed to assume A somewhat smaller; therefore we may put in place $A=\frac{1}{5}$, so that perhaps the value of B certainly shall be going to be produces much greater, thus so that we may have :

$$A=\tfrac{1}{5},\ \mathfrak{A}=\tfrac{1}{6},\ P=\tfrac{8}{7}\ \text{ and }\ Q=\tfrac{7ma}{8h}\ \text{ on account of }\ PQ=\tfrac{ma}{h}$$

and hence there will become

$$p=\tfrac{1}{6}a,\ q=-\tfrac{7\mathfrak{B}}{40}a\ \text{ and }\ r=\tfrac{B}{5}\cdot\tfrac{h}{m}.$$

Therefore here it is required to take care, so that the third lens may not become exceedingly small, even if the magnification m may be placed maximally; whereby we may assume the magnification ought to be $m=1000$, and since the distance of the object a may scarcely be able to be less than one inch, lest the first lenses may become exceedingly small, we may assume $a=1$ in., and since it may suffice to be putting in place $r=-\frac{1}{2}$ in., on account of $k=8$ in. hence there may become $B=-\frac{625}{2}$, which value certainly is great enough. Therefore again we may put $B=-300$, so that there shall become $\mathfrak{B}=\frac{300}{299}$, and there will be

$$q=\tfrac{420}{8\cdot 299}a\ \text{ or }\ q=-\tfrac{420}{2392}a\ \text{ and }\ r=-\tfrac{480}{m}\text{in.}$$

Then truly the intervals between the lenses will become :

$$\text{I and II}=\tfrac{1}{40}a,$$
$$\text{II and III}=60\left(\tfrac{7}{8}-\tfrac{h}{ma}\right)a=\left(\tfrac{105}{2}a-\tfrac{480}{m}\right)\text{in.}$$

Almost nothing will be required to be changed concerning the area viewed in the object ; indeed there will be found

$$z=\tfrac{1}{4}\cdot\tfrac{7ah}{7ma-8h}=\tfrac{14a}{7ma-64}\ \text{ in.}$$

Moreover for the aperture of the first lens requiring to be defined we may reduce the radius of confusion to zero with the aid of this equation:

$$0=\mu(216\lambda+30\nu)-\tfrac{7\cdot 125}{8}\mu'(0,99\lambda'-0,0008).$$

Hence if we may assume $\lambda=1$, there will become

$$0,99\lambda'=0,0008+\tfrac{\mu}{\mu'}\cdot 2,0351=2,3044$$

and thus

$\lambda'=2,3276$, from which there becomes $\tau\sqrt{(\lambda'-1)}=1,0111$.

From which the construction of this second lens will be :

It is evident the radius

$$\begin{cases}\text{of the anterior face } =\frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)+1,0111}=\frac{q}{1,1477}=0,8713q \quad (9,9401715)\\ \text{of the posterior face}=\frac{q}{\rho+\mathfrak{B}(\sigma-\rho)-1,0111}=\frac{q}{0,5764}=1,7349q, \quad (0,2392759).\end{cases}$$

Moreover for the first lens made from crown glass, on account of $\mathfrak{A}=\frac{1}{6}$ and $\lambda=1$, the radius

$$\begin{cases}\text{of the anterior face } =\frac{q}{\sigma-\mathfrak{A}(\sigma-\rho)}=\frac{p}{1,4212}=0,7036p\\ \text{of the posterior face}=\frac{q}{\rho+\mathfrak{A}(\sigma-\rho)}=\frac{p}{0,4656}=2,1478p.\end{cases}$$

THE LATTER CASE FOR WHICH THE FIRST LENS MAY BE PREPARED FROM CRYSTAL GLASS AND THE SECOND FROM CROWN GLASS

Since in this case there will be

$$B=\frac{7}{10(1+A)P-7},$$

the denominator again may be reduced to zero, and since P must be less than unity, there may be taken $P=\frac{7}{8}$ and there will be $A=-\frac{1}{5}$; but on account of the above ratio advanced there may be taken $A=-\frac{1}{6}$, so that there shall be $\mathfrak{A}=-\frac{1}{5}$, and hence the focal lengths :

$$p=-\tfrac{1}{5}a, \quad q=\tfrac{4\mathfrak{B}}{21}\cdot a, \quad \text{and} \quad r=-\tfrac{B}{6}\cdot\tfrac{h}{m}.$$

Here again we may compose, so that for the magnification $m=1000$ there may be produced $r=-\frac{1}{2}$ in. approx., and hence there will be produced $B=375$. Therefore we may assume $B=300$ and so that as before there will become $\mathfrak{B}=\frac{300}{301}$ and on account of $P=\frac{7}{8}$ there will be $Q=\frac{8ma}{7h}$; and hence the focal lengths

$$p=-\frac{1}{5}a,\quad q=\frac{4}{21}\cdot\frac{300}{301}a,\quad r=-50\frac{h}{m}.$$

Truly the intervals between the lenses will be

$$\text{I and II }=\frac{1}{42}a$$

and

$$\text{II and III }=50\left(\frac{8}{7}-\frac{h}{ma}\right)a=\frac{400}{7}a-\frac{50h}{m}=\left(57\frac{1}{7}a-\frac{400}{m}\right)\text{ in.}$$

Moreover for the area viewed in the object there will be

$$z=\frac{1}{4}\cdot\frac{8ah}{8ma-7h}=\frac{2a}{ma-7}\text{ in.},$$

so that the area is a little less than in the preceding case.

Finally for the aperture of the first lens requiring to be defined the radius of the confusion again is reduced to zero, so that this equation becomes :

$$\mu(125\lambda-30\nu)=\mu'\frac{8\cdot 216}{7}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right),$$

from which with $\lambda'=1$ assumed there is deduced

$$125\lambda=7,587+\frac{\mu'}{\mu}\cdot 246,857\cdot 1,0107=290,007$$

and hence

$$\lambda=2,3200\quad\text{and thus}\quad\tau\sqrt{(\lambda-1)}=1,0082.$$

Therefore the radius

$$\begin{cases}\text{of the anterior face }=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)-\tau\sqrt{(\lambda-1)}}=\frac{p}{0,2862}=3,4942p\\ \text{of the posterior face}=\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)+\tau\sqrt{(\lambda-1)}}=\frac{p}{1,4379}=0,6955p.\end{cases}$$

But for the second lens, of which the focal length is q and the numbers $\mathfrak{B}=\frac{300}{301}$ and $\lambda'=1$, the radius

$$\begin{cases}\text{of the anterior face } = \frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)} = \frac{q}{0,2315} = 4,3197q \quad (0,6354489) \\ \text{of the posterior face} = \frac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = \frac{q}{1,6553} = 0,6041q \quad (9,7811232).\end{cases}$$

So that it may pertain to the remaining principal parts, we will define these more carefully in the following constructions.

CONSTRUCTION OF THE PRIOR MICROSCOPE OF THIS KIND

131. With the distance of the object put $= a$, the construction will be had in the following manner :

I. For the first lens being made from crown glass, of which focal length $p = \frac{1}{6}a,$ the radius

$$\begin{cases}\text{of the anterior face } = 0,1173a \\ \text{of the posterior face} = 0,3579a.\end{cases}$$

The radius of the aperture of the aperture can be taken $x = 0,0293a$ and the distance to the second lens $= \frac{1}{40}a = 0,025a.$

II. For the second lens being prepared from crystal glass, its focal length $q = -\frac{420}{2392}a$, the radius

$$\begin{cases}\text{of the anterior face } = -\ 0,1530a \\ \text{of the posterior face} = -\ 0,3046a.\end{cases}$$

The radius of which aperture $x = 0,0382a$; which since it shall be greater than in the first lens, that value of x itself prevails and the distance to the eyepiece lens

$$= 52\tfrac{1}{2}a - \frac{480}{m} \text{ in.}$$

III. For the eyepiece lens, of which the focal length is

$$r = -\frac{480}{m} \text{ in .,}$$

the radius of each face will be, if this lens may be prepared from crown glass,

$$= -\frac{508,80}{m} \text{ in.,}$$

but if it may be made from crystal glass, the radius of each face will be $= -\frac{556,80}{m}$ dig.

The radius of its aperture x will be able to be taken $=\frac{120}{m}$ in., to which lens the eye must be applied directly.

IV. The radius of the area viewed in the object will be

$$z=\frac{14a}{7ma-64}\text{ in.}$$

V. For the clarity [*i.e.* the amount of light transmitted], since there shall be $x=0,0293a$, there will be

$$y=\frac{hx}{ma}=\frac{0,2344}{m}\text{ in.}$$

and hence the measure of the clarity $=\frac{4.688}{m}$.

VI. Lest the first lenses may become exceedingly small, the distance of the object a is seen to be scarcely able to take a value beyond an inch, unless perhaps the skilled constructor perhaps may prevail to make the smaller lenses exactly ; in which case the distance of the object will be less than one inch and thus the length of the instrument to be contracted.

CONSTRUCTION OF THE LATTER MICROSCOPE OF THIS KIND

132. Again with the distance of the object $=a$, the construction itself will be had thus :

I. For the first lens requiring to be made from crystal glass, of which the focal length $p=-\frac{1}{5}a$, the radius

$$\begin{cases}\text{of the anterior face} = -\ 0,6988a\\ \text{of the posterior face} = -\ 0,1391a.\end{cases}$$

The radius of this aperture $x=0,0348a$, unless the second lens may demand a smaller aperture.

The interval to the second lens $=\frac{1}{42a}$.

II. For the second lens requiring to be made from crown glass, of which the focal length is

$$q=\frac{4}{21}\cdot\frac{300}{301}a=\frac{400}{2107}a,$$

the radius

$$\begin{cases}\text{of the anterior face} = 0,8201a\\ \text{of the posterior face} = 0,1147a.\end{cases}$$

The radius of that aperture $x = 0,0286a$, from which also the aperture of the first lens will not be able to be taken greater, thus so that there must be taken $x = 0,0286a$.

The distance to the eyepiece lens $= 57\frac{1}{7}a - \frac{400}{m}$ in.

III. For the eyepiece lens, of which the focal length is $r = -\frac{400}{m}$ in., if that may be made from crown glass, the radius of each face must become $-\frac{400}{m}$ in., but if that may be made from crystal glass, that will be $= -\frac{464}{m}$ in.

The radius of this aperture x will be able to be taken $= \frac{100}{m}$ in. and the eye is required to be applied directly to this lens.

IV. For the area viewed in the object, we will find its radius $z = \frac{2a}{ma-7}$ in.

V. For the clarity, since here there shall be $x = 0,0286a$, there will become $y = \frac{0,2288}{m}$, and the measure of the clarity $= \frac{4576}{m}$.

VI. Since in this case the first lenses shall be a little bigger then in the preceding case, evidently with respect to the distance *a,* in this case nothing prevents, however smaller the distance *a* may be taken smaller than one inch, and thus the length of the instrument will be easily recalled to the length of the preceding one.

SCHOLIUM

133. Behold therefore at this point two kinds of microscopes, which by far are to be preferred to the others above, which also may be able to be adapted for the maximum magnifications. But the great length of these instruments deservedly will be considered with no small inconvenience; truly if the constructor may succeed with the making of the two first lenses for the distance $a = \frac{1}{2}$ in., a length of two feet will be able to be tolerated easily. But since here we may have used two kinds of glass, it will be worth the effort to investigate also, how great the other confusion shall be going to become arising from the different refractive indices besides that from the colored margin; this equation [§27] which in the end will have to be examined :

$$0 = N \cdot \frac{1}{p} + \frac{N'}{P^2} \cdot \frac{1}{q} + \frac{N''}{P^2 Q^2} \cdot \frac{1}{r},$$

the final term of which evidently will vanish before the preceding two, thus so that this condition will demand :

$$0 = N \cdot \frac{1}{p} + \frac{N'}{P^2} \cdot \frac{1}{q}.$$

Now since for the first case there shall be :

$$N=7,\quad N'=10,\quad p=\tfrac{1}{6}a,\quad \text{and}\ P=\tfrac{8}{7},\quad q=-\tfrac{420}{3292}a,$$

this formula will become $6-\frac{16744}{2688}$; of which the latter term since it almost removes the first, hence it is evident plainly no confusion is required to be a concern.

Truly for the other case, where there is

$$N=10,\quad N'=7,\quad P=\tfrac{7}{8},\ p=-\tfrac{1}{5}a,\quad q=\tfrac{400}{2107}a,$$

that formula shall become $-50+\frac{1204}{25}$, of which the two members maintain the ratio 25:24 between themselves, this is not so great a ratio of equality, thus so that the terms shall be agreed to cancel each other and in this case the other confusion may be agreed as if to vanish completely, and thus in the first case the confusion arising from this source will be a little smaller that from the latter case ; yet which difference will be exceedingly small, so that in practice the former case may be seen to be preferred to the latter.

SECTIO SECUNDA.
DE MICROSCOPIIS COMPOSITIS,
IN QUIBUS NULLA IMAGO REALIS OCCURRIT.

PROBLEMA 1

99. *Datis tam multiplicatione m quam distantia obiecti ante lentem obiectivam microscopium ex duabus lentibus construere, quarum obiectiva sit convexa, ocularis vero concava.*

SOLUTIO

Cum distantia obiecti *a* detur aeque ac multiplicatio *m,* casus duarum lentium statim praebet hanc aequationem $m = P \cdot \frac{h}{a}$; unde definitur

$$P = \frac{ma}{h};$$

hinc distantiae focales ambarum lentium erunt

$$p = \mathfrak{A}a, \quad q = -\frac{Ah}{m};$$

unde patet tam $\mathfrak{A}$ quam *A* esse debere positiva, ad quod sufficit, ut *A* sit positivum. Intervallum vero lentium erit

$$= Aa(1-\frac{h}{ma}) = \frac{A}{m}\left(ma-h\right),$$

ex quo perspicuum est esse debere $ma > h$ seu $m > \frac{h}{a}$; alioquin enim huiusmodi microscopia locum habere non possent. Deinde pro spatio in obiectis conspicuo habebimus eius semidiametrum

$$z = a\Phi = \frac{\mathfrak{q}}{ma-h} \cdot ah\xi.$$

Si igitur sumamus $\xi = \frac{1}{4}$ et $\mathfrak{q} = 1$, qui est casus, quo lens ocularis maximam aperturam admittit ideoque utrinque aeque est concava, tum ergo erit

$$z = \frac{1}{4} \cdot \frac{ah}{ma-h}.$$

Quod vero ad locum oculi attinet, ex superioribus formulis generalibus colligimus

$$O = \frac{\mathfrak{q}b}{Ma} \cdot \frac{h}{m};$$

est vero

$$b = -\frac{\alpha}{P} = -\frac{Ah}{m}$$

et

$$M = \frac{\mathfrak{q}}{ma-h} \cdot h$$

sicque fit

$$O = -\frac{Ah(ma-h)}{m^2 a};$$

quae distantia cum sit negativa, oculum lenti oculari immediate adplicari oportet; unde ut margo coloratus evanescat, satisfieri debet huic aequationi:

$$0 = N(A+1)\mathfrak{q};$$

quod cum fieri nequeat, perspicuum est marginem coloratum neutiquam tolli posse; multo minus ergo haec confusio posterior penitus tolli poterit; prior autem confusio insensibilis reddetur ope huius aequationis:

$$\frac{mx^3}{a^2 h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu'\lambda'}{A^3 P}\right) = \frac{1}{k^3},$$

quae ergo abit in hanc formam:

$$\frac{mx^3}{a^2 h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu' h\lambda'}{A^3 ma}\right) = \frac{1}{k^3},$$

ubi cum lens ocularis debeat esse utrinque aequaliter concava, si pro ea vitri specie, ex qua lens ocularis conficitur, capiantur numeri respondentes ρ', σ' et τ', erit $\lambda' = 1 + \left(\frac{\sigma' - \rho'}{2\tau'}\right)^2$. Ex hac autem aequatione definiri debet semidiameter aperturae lentis obiectivae *x*, erit scilicet

$$x\sqrt[3]{\left(\mu m\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) - \frac{\mu' h\lambda'}{A^3 a}\right)} = \frac{1}{k}\sqrt[3]{a^2 h},$$

nisi forte hinc pro *x* prodeat valor maior, quam lentis figura permittit; hinc ergo casus utilissimus foret, si fieri posset

$$\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} = \frac{\mu'}{\mu} \cdot \frac{h\lambda'}{A^3 am};$$

ad quod idoneum valorem pro $\mathfrak{A}$ vel *A* quaeri oporteret, quod quidem pro non adeo magnis multiplicationibus fieri posset; at si multiplicatio *m* esset praegrandis, deberet $A(1+A)^3 + \nu A(A+1)$ aequari fractioni valde parvae, quod, cum $A > 0$, fieri non potest.

Quicquid autem sit, invento valore ipsius x gradus claritatis erit $=\frac{hx}{ma}$ et mensura claritatis $=\frac{20hx}{ma}$; unde eo magis curandum est, ut x non nimis parvum adipiscatur valorem.

COROLLARIUM 1

100. Hinc patet, ut x maiorem nanciscatur valorem, plurimum conducere, ut litterae A parvus tribuatur valor ; sed hunc valorem nimis parvum assumere non licet, quia tum lens ocularis nimis fieret parva, ita ut A vix unitate minus accipi conveniat.

COROLLARIUM 2

101. Cum formula $\lambda\left(1+A\right)^3+\nu A(A+1)$ certe sit unitate maior, quia A unitate minus esse nequit, atque adeo ultra 8 exsurgere debeat, haec confusio penitus tolli non poterit, nisi haec formula $\frac{\mu'}{\mu}\cdot\frac{h\lambda'}{ma}$ quoque 8 superet, hoc est, nisi ob $\frac{\mu'}{\mu}=1$ proxime fuerit

$$\frac{h\lambda'}{ma}>8 \text{ seu } m<\frac{h\lambda'}{8a}.$$

COROLLARIUM 3

102. Haec clariora fient, si posito $h=8$ dig. sumamus $a=\frac{1}{4}$ dig.; et cum sit circiter $\lambda'=\frac{3}{2}$, limes modo inventus daret $m<6$; quae multiplicatio tam exigua ne huiusmodi quidem microscopiis produci potest, quare nunc pro certo affirmare licet istam confusionem neutiquam tolli posse.

EXEMPLUM 1

102a. Si distantia obiecti debeat esse $\frac{1}{4}$ dig. et ambae lentes ex vitro communi $n=1,55$ parentur, tum vero statuatur $A=1$ hincque $\mathfrak{A}=\frac{1}{2}$, habebimus primo distantias focales lentium

$$p=\frac{1}{2}a=\frac{1}{8} \text{ dig. et } q=-\frac{8}{m} \text{ dig.}$$

lentiumque intervallum

$$=\frac{1}{m}\left(\frac{1}{4}m-8\right)=\frac{1}{4}-\frac{8}{m} \text{ dig.}$$

Spatium vero in obiecto conspicuum erit

$$z = \frac{2}{m-32} \text{ dig.}$$

Denique si ut hactenus sumamus $k = 20$, postrema aequatio erit

$$x\sqrt[3]{\mu\left(8\lambda m + 2\nu m - 32\lambda'\right)} = \frac{1}{20}\sqrt[3]{\frac{1}{2}}.$$

Hic autem, ut iam saepe vidimus, est $\lambda' = 1,6299$; praeterea vero cum sit $\lambda = 1$ et proxime $\mu = 1$, inveniemus

$$x = \frac{1}{20}\sqrt[3]{\frac{1}{16,9304m-104,3136}}.$$

Si ergo fuerit $m = 100$, fiet primo

$$p = \frac{1}{8} \text{ dig. et } q = -\frac{2}{25} \text{ dig.}$$

et lentium intervallum

$$= \frac{17}{100} \text{ dig.}$$

et

$$z = \frac{1}{34} \text{ dig.,}$$

tum vero

$$x = 0,0043 \text{ dig.,}$$

unde fit

$$y = \frac{32}{100} = 0,0014 \text{ dig.}$$

et mensura claritatis 0,028, quae circiter triplo minor est quam in microscopio fere simplici.

EXEMPLUM 2

[102b]. Maneant omnia uti in exemplo praecedente, praeterquam quod litterae A valor multo maior tribuatur, ut videamus, quomodo confusio tum futura sit comparata. Statuatur ergo $A = 5$ fietque $\mathfrak{A} = \frac{5}{6}$ et distantiae focales erunt

$$p = \frac{5}{6}a, \quad q = -5\cdot\frac{h}{m},$$

quia ut ante manet

$$P = \frac{ma}{h}$$

et lentium intervallum

$$= \frac{5}{m}\left(ma - h\right);$$

tum vero pro spatio conspicuo erit

$$z = \frac{1}{4} \cdot \frac{ah}{ma-h}$$

ut ante. Ut denique confusio non sentiatur, debet esse

$$x\sqrt[3]{\mu\left(\frac{216\lambda m}{125} + \frac{6\nu m}{25} - \frac{8\lambda'}{125a}\right)} = \frac{1}{10}\sqrt[3]{a^2}.$$

Sumto igitur iterum $a = \frac{1}{4}$ dig., $\lambda = 1$ et $\lambda' = 1,6299$, siquidem ambae lentes ex vitro communi $n = 1,55$ conficiantur, et posito $\mu = 1$ habebitur

$$x = \frac{1}{20}\sqrt[3]{\frac{1}{3,568m - 0,8344}}.$$

Si ergo fuerit $m = 100$, fiet

$$p = \frac{5}{24} \text{ dig. et } q = -\frac{2}{5} \text{ dig.}$$

et lentium intervallum

$$= \frac{17}{20} \text{ dig.}$$

et

$$z = \frac{1}{34} \text{ dig.}$$

Tum vero

$$x = \frac{1}{20}\sqrt[3]{\frac{1}{355,966}} = 0,00705 \text{ dig.},$$

unde fit

$$y = 0,00226 \text{ dig.}$$

et mensura claritatis $= 0,0452$.

SCHOLION

103. Si haec duo exempla inter se conferamus, sequentia observanda occurrent:

1. Videmus plurimum interesse, ut litterae A maior valor tribuatur, quia tum expressio pro confusione multo fit minor, ita ut littera x tum maiorem adipiscatur valorem, ex quo simul maior claritas obtinetur; quo maior enim littera A accipitur, eo propius littera $\mathfrak{A}$ ad unitatem accedit, ex quo primus terminus $\frac{1}{\mathfrak{A}^2}$ vix unitatem superabit, qui, dum erat $A = 1$, ultra 8 exsurgebat.

2. Deinde etiam valorem ipsius A augendo lens obiectiva fiet maior, dum eius distantia focalis p ad distantiam obiecti a continuo propius accedit.

3. Maximum autem commodum cernitur in lente oculari, quae hoc modo ad lubitum nostrum augeri poterit, quantumvis magna fuerit multiplicatio. Fieri adeo potest, ut haec

lens datam distantiam focalem adipiscatur veluti unius digiti; tum scilicet $A\cdot\frac{h}{m}$ ponatur $=1$ dig. et ob $h=8$ dig. capi debebit $A=\frac{m}{8}$; tum quidem longitudo instrumenti maior evadet, scilicet $=\frac{1}{8}(ma-h)$, sed vix: unquam ea tanta erit, ut non facile tolerari possit.

4. In his quidem exemplis assumsimus distantiam obiecti $a=\frac{1}{4}$ dig., sed nihil impedit, quominus hanc distantiam maiorem assumamus, quo ipso usus horum instrumentorum multo commodior redditur, dum praecipuum commodum, quod a microscopiis compositis exspectamus, in eo est situm, ut non opus sit obiecta tam prope ad instrumentum admovere; quia enim littera a arbitrio nostro permittitur, eam tantam assumere licebit, quantam lubuerit.

5. Verum quo maiorem hanc distantiam a accipiamus, fateri cogimur claritatis gradum inde diminutum iri; quod quo clarius appareat, perpendamus valorem litterae x reliquis litteris iisdem manentibus proportionalem esse formulae $\sqrt[3]{a^2}$ seu potestati $a^{\frac{2}{3}}$, ita ut, quo maior distantia obiecti statuatur, etiam apertura lentis obiectivae maior sit proditura; quod in se spectatum pro non exiguo commodo est habendum; at pro gradu claritatis, cum sit y formulae $\frac{x}{a}$ proportionalis, claritas proportionalis fiet formulae $\frac{1}{\sqrt[3]{a}}$, ita ut ea decrescat in ratione subtriplicata distantiae obiecti a; verum haec ipsa diminutio non adeo est pertimescenda, dum, si distantiam obiecti adeo octuplo maiorem accipiamus, claritas tantum duplo fit minor, atque ex his perspicuum est, quantopere microscopia composita simplicibus antecellant et quanta commoda ab iis exspectari possint. Interim vero haec species microscopiorum hic tractata adhuc ingenti defectu laborat, quod a margine colorato liberari neutiquam potest. Quocirca videamus, an unam pluresve lentes insuper adiiciendo istud vitium tolli queat.

PROBLEMA 2

104. *Inter lentes obiectivam et ocularem praecedentis microscopiorum speciei novam lentem ita inserere, ut margo coloratus ad nihilum redigatur.*

SOLUTIO

Quoniam igitur hic tres habemus lentes, earum distantiae focales ita erunt expressae:

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=\frac{AB}{PQ}\cdot a;$$

quarum cum prima debeat esse convexa, erit $\mathfrak{A}>0$, et cum tertia debeat esse concava, erit $AB<0$ ideoque altera litterarum A et B positiva, altera negativa; de lente enim media nihil adhuc definiamus; intervalla porro harum lentium erunt

$$\text{prius} = Aa(1-\tfrac{1}{P}) \ \text{ et posterius} = -\tfrac{ABa}{P}(1-\tfrac{1}{Q});$$

unde patet esse debere $Q>1$. Multiplicatio vero m dabit $PQ=\frac{ma}{h}$.

Nunc autem id consideremus, quod nobis est propositum, scilicet ut margo coloratus evanescat. Quoniam distantia oculi O prodit negativa, satisfieri oportet huic aequationi:

$$0 = N(A+1)B\tau - \tfrac{N'}{P}((B+1)\mathfrak{r}+\mathfrak{q}),$$

quem in finem spectetur spatium in obiecto conspicuum, pro quo est

$$z = a\Phi = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot ah\xi;$$

in qua, si lens ocularis utrinque fiat aequalis, ut maximam aperturam admittat, capi poterit $\mathfrak{r}=1$; tum vero posuimus

$$\tfrac{\mathfrak{q}+1}{ma-h}\cdot h = M,$$

ut sit

$$z = Ma\xi.$$

Nunc igitur primo videndum est, an, si ambae lentes ex eodem vitro parentur, scopum obtinere queamus. Posito igitur $N=N'$ aequatio pro margine nobis dabit

$$B = \tfrac{\mathfrak{q}+\mathfrak{r}}{(A+1)\mathrm{P}\mathfrak{r}-\mathfrak{r}}$$

qui valor an cum conditione praescripta $AB<0$ consistere possit, videamus. Hunc in finem duos casus perpendamus, alterum, quo $A>0$, alterum vero, quo non solum $A<0$, sed etiam $1+A<0$, ut scilicet prodeat $\mathfrak{A}$ positivum. Priore casu erit $P>1$ ideoque in valore ipsius B denominator fit positivus sicque B positivum habebit valorem, cum tamen ob $AB<0$ negativum esse debeat; altero casu, quo $A<0$, debet esse $P<1$ ideoque denominator $(A+1)\mathrm{P}\mathfrak{r}-\mathfrak{r}$ fit negativus, etiamsi $A+1$ non esset negativum, ita ut valor ipsius B hoc casu certo prodeat negativus, cum tamen ob $AB<0$ deberet esse positivus.

At si lentes ex diverso vitro conficiantur, fieri poterit, ut margo coloratus penitus tollatur idque duplici modo, quemadmodum in subiunctis casibus ostendemus. Postquam autem huic conditioni fuerit satisfactum, pro apertura lentis obiectivae indeque pendente claritate sequens habebitur aequatio:

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''\lambda''}{A^3B^3PQ}\right)=\frac{1}{k^3},$$

ubi tantum notandum est, ut lens ocularis maximam admittat aperturam, valorem λ'' inde esse datum; pro binis reliquis λ et λ' commode unitas assumitur sicque pro quovis casu problematis nostri solutio facile invenietur.

COROLLARIUM 1

105. Quod ergo hic diverso vitro uti oporteat, id intelligendum est tantum de lente prima et secunda, ad quas litterae N et N' referuntur; pro tertia enim lente vitri ratio, ex quo conficitur, hic plane in computum non ingreditur, ita ut perinde sit, ex quonam vitro haec lens conficiatur.

COROLLARIUM 2

106. Cum igitur pro margine colorato tollendo habeatur ista aequatio

$$N(A+1)BP\mathfrak{r} = N'((B+1)\mathfrak{r}+\mathfrak{q}),$$

hinc deduci debet valor litterae

$$B = \frac{N'(\mathfrak{q}+\mathfrak{r})}{N(A+1)P\mathfrak{r}-N'\mathfrak{r}};$$

ubi notetur esse $\mathfrak{r}=1$ et $\mathfrak{q}+\mathfrak{r}$ necessario maius nihilo, ut scilicet valor ipsius z prodeat positivus; tum iste valor comparetur cum ea conditione, qua productum AB debet esse negativum, id quod fieri plane non posse, quamdiu litterae N et N' sunt inter se aequales, iam ostendimus.

COROLLARIUM 3

107. Totum ergo negotium iam huc redit, quemadmodum hae duae conditiones impleri queant, dum litterae N et N' diversos obtinent valores, scilicet ut dato valore litterae A altera littera B talem sortiatur valorem, ut earum productum AB fiat negativum, ubi perpendendum est formulam $A(P-1)$ semper positivam esse debere, ita ut sumto A positivo sit $P>1$, sumto autem A negativo capi debeat $P<1$.

SCHOLION

108. Quoniam igitur duabus diversis vitri speciebus uti cogimur, optandum sine dubio esset, ut hae duae species ratione refractionis maxima inter se differrent; cum autem aliae adhuc eiusmodi species non sint cognitae praeter eas, circa quas DOLLONDUS experimenta sua instituit, easdem quoque nos hic adhibere oportebit. Hactenus quidem

litteris N et N', quae his duabus speciebus conveniunt, rationem 7:10 tribuimus, quae illis experimentis maxime videtur conformis, etiamsi ea satis notabiliter a veritate aberrare possit. Quamobrem ob calculi commoditatem hanc rationem hic potius ut 2:3 statuamus, quippe quae ab illa minime differt et aliquanto maius discrimen involvit; neque enim hinc aliud est metuendum, si forte error non satis esset exiguus, nisi quod margo coloratus non penitus tolleretur; verum dummodo is multo minor evadat, quam vulgo unicam vitri speciem adhibendo fieri solet, contenti esse poterimus; quem in finem duos casus hic accuratius examinare conveniet, alterum, quo littera A positivum habet valorem, alterum vero, quo negativum, ut inde pateat, quanta commoda hinc in praxi exspectari queant.

EVOLUTIO PRIMI CASUS

QUO LITTERAE A VALOR POSITIVUS TRIBUITUR

109. Hoc ergo casu littera $\mathfrak{A}$ valorem quoque positivum habebit et quidem unitate minorem; tum vero conditio lentis ocularis concavae postulat, ut littera B obtineat valorem negativum. Praeterea ob $A>0$ etiam esse debet $P>1$, ut intervallum prius fiat positivum. Nunc vero ob marginem coloratum tollendum valor litterae B ita exprimitur, ut sit

$$B=\frac{N'(\mathfrak{q}+\mathfrak{r})}{N(A+1)P\mathfrak{r}-N'\mathfrak{r}};$$

ubi igitur ob $\mathfrak{q}+\mathfrak{r}>0$ denominator seu formula $N(A+1)P-N'$ negativum habere debebit valorem; quod ut fieri possit, cum $(A+1)P$ certe sit unitate maius, necesse est, ut fiat $N'>N$ ideoque ut lens obiectiva ex vitro coronario, secunda vero ex crystallino conficiatur. Quare, cum hinc prodeat $N:N'=2:3$ hincque sit

$$B=\frac{3(\mathfrak{q}+\mathfrak{r})}{2(A+1)P\mathfrak{r}-3\mathfrak{r}},$$

oportebit esse

$$2(A+1)P<3 \text{ sive } P<\frac{3}{2(1+A)}.$$

Cum autem sit $P>1$, manifestum est litteram A tam parvam accipi debere, ut etiam nunc sit

$$\frac{3}{2(1+A)}>1 \text{ ideoque } A+1<\frac{3}{2} \text{ hinque } A<\frac{1}{2};$$

si enim esset $A=\frac{1}{2}$, capi deberet $P=1$ primumque intervallum plane evanesceret, id quod praxis non patitur; unde simul intelligitur hanc litteram A tanto minorem quam $\frac{1}{2}$ statui debere, ut etiam nunc intervallum duarum primarum lentium ad praxin revocari possit. Constituta autem littera A littera P sumi debebit inter limites 1 et $\frac{3}{2(1+A)}$; modo autem vidimus minori limiti, unitati, aequalem capi non posse, at si maiori limiti sumeretur aequalis, tum B fieret infinitum sicque longitudo instrumenti in infinitum extenderetur. Tam prope igitur P maiori limiti admoveri conveniet, ut quantitas AB adhuc in praxi locum habere possit. Tum vero adhuc superest, ut postremae aequationi satisfiat, qua apertura lentis obiectivae definitur; circa quam aequationem sequentia nunc annotasse iuvabit:

1. Cum $A < \frac{1}{2}$, erit $\mathfrak{A} < \frac{1}{3}$, unde ipsius λ coefficiens erit > 27; unde enormis confusio resultaret, nisi sequentibus terminis diminueretur.

2. Verum cum pro secunda lente coefficiens ipsius λ' fiat maior quam 8 ob $P = 1$ proxima et quia B semper fit numerus valde magnus, $\mathfrak{B}$ parum ab unitate differt.

3. Pro lente oculari coefficiens ipsius λ'' tam erit parvus, ut prae reliquis terminis quasi evanescat; unde ad eo hoc commodi assequimur, ut tota haec confusio prorsus ad nihilum redigi queat, debite scilicet definiendo litteras λ et λ'; quare hic casus omnino meretur, ut aliquot exemplis illustretur.

EXEMPLUM 1

109a. Cum debeat esse $A < \frac{1}{2}$, ponamus $A = \frac{1}{3}$ fietque $\mathfrak{A} = \frac{1}{4}$ et $\frac{3}{2(1+A)} = \frac{9}{8}$, ita ut P capi debeat intra limites 1 et $\frac{9}{8}$. Sit ergo $P = \frac{10}{9}$ et fiet $B = -\frac{81(\mathfrak{q}+\mathfrak{r})}{\mathfrak{r}}$.
Consideremus nunc aequationem fundamentalem, quae est

$$\mathfrak{B}\mathfrak{q} = \frac{1}{9} \cdot \frac{(\mathfrak{q}+\mathfrak{r})}{ma-h} \cdot h.$$

Ponatur autem brevitatis gratia $\frac{ma}{h} = 1+\theta$, quandoquidem esse debet $ma > h$, ut haec microscopiorum species locum habere possit, eritque $\mathfrak{B} = \frac{\mathfrak{q}+\mathfrak{r}}{9\mathfrak{q}\theta}$.
Cum iam sit $\frac{1}{\mathfrak{B}} = 1 + \frac{1}{B}$, habebitur

$$\frac{9\mathfrak{q}\theta}{\mathfrak{q}+\mathfrak{r}} = 1 - \frac{\mathfrak{r}}{81(\mathfrak{q}+\mathfrak{r})},$$

unde elicitur

$$\mathfrak{q} = \frac{80\mathfrak{r}}{729\theta - 81},$$

sicque prodibit

$$B = \frac{-729\theta+1}{9\theta-1} \quad \text{hincque} \quad \mathfrak{B} = \frac{+729\theta-1}{729\theta}$$

existente $\theta = \frac{ma}{h} - 1$ sive multiplicatio $m = \frac{h(1+\theta)}{a}$. Tum vero ob $m = PQ \cdot \frac{h}{a}$ erit

$$Q = \frac{ma}{Ph} = \frac{9ma}{10h} = \frac{9}{10}(\theta+1)$$

atque hinc elementa pro microscopii constructione erunt

1. $A = \frac{1}{3}, \quad \mathfrak{A} = \frac{1}{4}, \quad B = \frac{-729\theta+1}{9\theta-1}, \quad \mathfrak{B} = \frac{729\theta-1}{720\theta}$

$$P = \frac{10}{9}, \quad Q = \frac{9}{10}(\theta+1).$$

2. Deinde distantiae focales lentium

$$p = \frac{1}{4}a, \quad q = \frac{-729\theta+1}{2400\theta}\cdot a \quad \text{et} \quad r = \frac{-729\theta+1}{(27\theta-3)(\theta+1)}\cdot a.$$

3. Lentium harum intervalla erunt

$$\text{prius} = \frac{1}{30}a, \quad \text{posterius} = \frac{(729\theta-1)a}{30(\theta+1)}.$$

4. Praeterea spatii in obiecto conspicui semidiameter erit

$$z = \frac{729\theta\mathfrak{r}-\mathfrak{r}}{(729\theta-81)\theta}\cdot a\xi;$$

quodsi iam hic sumamus $\mathfrak{r} = 1$ et $\xi = \frac{1}{4}$, id quod licet, si lens ocularis fiat utrinque aequaliter concava, erit

$$z = \frac{729\theta-1}{324\theta(9\theta-1)}\cdot a.$$

5. Denique consideretur haec aequatio:

$$\frac{mx^3}{a^2h}\left(\mu(64\lambda+12\nu) - \frac{243\mu'}{10}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{27\mu''\lambda''}{B^3(\theta+1)}\right) = \frac{1}{k^3},$$

ubi commode usu venit, ut haec quantitas ad nihilum revocari possit, quem in finem tertiam lentem uti primam ex vitro coronario fieri ponamus, sumique debebit

$$\lambda'' = 1,60006 \quad \text{et} \quad \mu'' = \mu \;;$$

tum vero sumatur $\lambda = 1$, at λ' ita, ut sit

$$\frac{\mu'}{\mu}\cdot 24,3\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right) = 64\lambda+12\nu+\frac{27\cdot 1,60006}{B^3(1+\theta)} = 66,6352$$

existente

$$\frac{\mu'}{\mu} = \frac{0,8724}{0,9875}, \quad \nu = 0,2196 \quad \text{et} \quad \nu' = 0,2529.$$

Praeterea vero notetur pro maioribus multiplicationibus, quando scilicet θ fit numerus satis modicus, fieri proxime

$$B = -81 \quad \text{et} \quad \mathfrak{B} = +\tfrac{81}{80};$$

unde colligitur

$$0,96341\lambda' = 0,00308 + \tfrac{\mu}{\mu'} \cdot \tfrac{66,7352}{24,3}$$

hincque

$$\lambda' = 3,22503 \quad \text{et} \quad \tau\sqrt{(\lambda'-1)} = 1,3089;$$

unde, cum huius lentis distantia focalis sit

$$q = -\tfrac{729}{2400}a = -\tfrac{243}{800}a$$

et $\mathfrak{B} = \frac{81}{80}$, erit huius lentis

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma - \mathfrak{B}(\sigma-\rho) + \tau\sqrt{(\lambda'-1)}} = \frac{q}{1,4323} = 0,69818q \\ \text{posterioris} = \frac{q}{\rho + \mathfrak{B}(\sigma-\rho) - \tau\sqrt{(\lambda'-1)}} = \frac{q}{0,2918} = 3,42700q.\end{cases}$$

Pro prima autem lente, cuius distantia focalis est $p = \frac{1}{4}a$ et numeri $\mathfrak{A} = \frac{1}{4}$ et $\lambda = 1$, ex vitro coronario facienda erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = \frac{p}{1,3017} = 0,76823p \\ \text{posterioris} = \frac{p}{\rho + \mathfrak{A}(\sigma-\rho)} = \frac{p}{0,5851} = 1,70911p.\end{cases}$$

CONSTRUCTIO HUIUSMODI MICROSCOPIORUM

110. Posita distantia obiecti $= a$ et multiplicatione $m = (1+\theta)\frac{h}{a}$ erit

I. Pro lente obiectiva ex vitro coronario facienda

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,1921a \\ \text{posterioris} = 0,4273a.\end{cases}$$

cuius distantia focalis $p = \frac{1}{4}a$,

semidiameter aperturae $x = 0,0480a$

et intervallum ad lentem secundam erit $=\frac{1}{30}a$.

II. Pro lente secunda ex vitro crystallino facienda

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -0,2121a\\ \text{posterioris} &= -1,0409a.\end{cases}$$

cuius distantia focalis est $q=-\frac{243}{800}a=0,3037a$,
semidiameter aperturae $x=0,0530a$ seu indefinita relinquitur, quia maior est semidiametro aperturae primae lentis, et intervallum ad lentem tertiam $=24,3\cdot\frac{\theta}{\theta+1}\cdot a$.

III. Pro lente tertia ex vitro coronario paranda, cuius distantia focalis

$$r=\frac{729}{27(\theta+1)}\cdot a=-\frac{27}{\theta+1}\cdot a,$$

erit

$$\text{radius faciei utriusque}=-\frac{28,62}{\theta+1}\cdot a,$$

cui lenti oculum immediate applicari oportet.

IV. Spatium in obiecto cernetur, cuius semidiameter

$$z=\frac{1}{4\theta}\cdot a$$

V. Denique cum sit

$$x=0,0480a,$$

erit

$$y=\frac{hx}{ma}=\frac{x}{\theta+1}=\frac{0,0480}{\theta+1}\cdot a$$

hincque mensura claritatis

$$20y=\frac{0,960}{\theta+1}\cdot a,$$

si scilicet distantia a in digitis exprimatur, quae mensura etiam ita exprimitur:

$$0,960\cdot\frac{h}{m}=\frac{7,68}{m}.$$

COROLLARIUM 1

111. Duae lentes priores cum earum intervallo plane non pendent a multiplicatione proposita ideoque pro omnibus multiplicationibus eaedem retineri possunt, ita ut tantum opus sit pro qualibet multiplicatione aliam lentem ocularem adhibere, cuius distantia focalis loco $\theta+1$ scripto valore $\frac{ma}{h}$ erit

$$r=-27\frac{h}{m}=-\frac{216}{m}\text{ dig.},$$

ita ut haec lens nunquam fiat nimis parva.

COROLLARIUM 2

112. Utcunque autem multiplicatio varietur, intervallum lentium secundae et tertiae parum admodum mutatur, praecipue in maioribus multiplicationibus, cum hoc intervallum sit

$$=24,3\cdot\frac{\theta}{\theta+1}\cdot a=24,3\left(a-\frac{h}{m}\right),$$

ita ut tota instrumenti longitudo vix sit mutanda, ac si distantia obiecti a capiatur 1 digiti, longitudo instrumenti erit circiter duorum pedum.

SCHOLION

113. Quod hic distantia obiecti arbitrio nostro permittatur, id sine dubio tamquam insigne commodum est spectandum, cum hoc modo maximum vitium microscopiorum simplicium, quod in nimia vicinitate obiecti consistit, felicissimo successu evitetur, quoniam quantumvis hac distantia aucta ne mensura quidem claritatis diminuitur, aeque parum ac spatium in obiecto conspicuum. Interim tamen contra hanc speciem obiici poterit, primo quod duae lentes priores nimis inter se propinquae esse debeant; quod tamen vix ullam attentionem meretur, cum adhuc hoc intervallum in praxi facile observari possit, nisi distantia obiecti a nimis parva statuatur, quod autem nulla ratio suadet; altera vero obiectio maioris est momenti, quod, si distantia a maior uno digito accipiatur, longitudo huius instrumenti duos adeo pedes iam superet, quae merito incommoda videri potest. Verum mox ostendemus, quomodo et huic incommodo facile occurri possit. Prouti autem hanc speciem litteris A et P definiendis constituimus, id inprimis obiici potest, quodsi diversitas numerorum N et N' tantillo minor fuerit quam in ratione 2:3, uti hic assumsimus, tum determinationes ulteriores locum omnino habere non posse; si enim loco huius rationis substituamus eam, quam supra ex ipsis DOLLONDI experimentis conclusimus, scilicet uti 7:10, ut foret

$$B=\frac{10(\mathfrak{q}+\mathfrak{r})}{7(A+1)P\mathfrak{r}-10\mathfrak{r}},$$

tum sumto $A=\frac{1}{3}$ et $P=\frac{10}{9}$, denominator $7(A+1)P\mathfrak{r}-10\mathfrak{r}$ fieret $=\frac{280}{27}-10$ ideoque non amplius negativus, ut natura rei postulat; multo igitur minus haec positio locum habere posset, si discrimen vitri ratione dispersionis adhuc esset minus, quod quidem non parum probabile videtur. Quamobrem, ne hinc quicquam sit pertimescendum, litteras A et P ita assumi conveniet, ut formula $(1+A)P$ multo minorem obtineat valorem quam casu exempli allati, pro littera scilicet A fractio sumi debebit multo minor quam $\frac{1}{3}$; tum vero valor ipsius P tam parum unitatem superet, quam lentium proximitas permittit, cui conditioni in sequenti exemplo satisfaciemus.

EXEMPLUM 2

113a. Sumamus igitur hic $A=\frac{1}{5}$ fietque $\mathfrak{A}=\frac{1}{6}$ et $p=\frac{1}{6}a$, intervallum autem primae et secundae lentis $=\frac{1}{5}(1-\frac{1}{P})a$; quod ut parti quasi septimae

ipsius p aequetur, sumi debet $P=\frac{42}{37}=\frac{8}{7}$ seu $\frac{9}{8}$ circiter; sumamus igitur $P=\frac{9}{8}$, et quia etiam hic uti in praecedente exemplo littera $\mathfrak{q}$ vehementer fit parva prae littera $\mathfrak{r}$, ea neglecta erit

$$B=\frac{N'}{N(1+A)P-N'}$$

et sumto $N:N'=7:10$ erit substitutis his valoribus $B=-\frac{200}{11}$ sive $B=-18$, qui valor adhuc maior prodiisset, si dispersionis discrimen adhuc minus fuisset. Cum igitur satis sit verisimile hoc discrimen adhuc esse minus, a scopo vix aberrabimus, si statuamus $B=-25$, et si ullus error hinc resultaret, is in eo consisteret, ut margo coloratus non perfecte tolleretur; quod cum ne sperari quidem possit, contentos nos esse oportet, si eum tantum satis parvum reddiderimus, id quod hoc modo certo obtinebimus; sumto autem $B=-25$ erit $\mathfrak{B}=\frac{25}{24}$ hincque ex aequatione fundamentali

$$\mathfrak{q}=\frac{3h\mathfrak{r}}{25ma-28h}$$

hincque

$$\mathfrak{q}+\mathfrak{r}=\frac{25ma\mathfrak{r}-25h\mathfrak{r}}{25ma-28h};$$

unde colligitur spatii conspicui semidiameter

$$z=\frac{25\mathfrak{r}}{25ma-28h}\cdot ha\xi\ ;$$

quare, si sumatur $\xi=\frac{1}{4}$ et $\mathfrak{r}=1$, quo casu requiritur, ut lens ocularis sit utrinque aeque concava, ac si ponamus ut ante $\frac{ma}{h}=1+\theta$, erit

$$z=\frac{25}{100\theta-12}\cdot a\ ;$$

reliqua autem elementa sequenti modo se habebunt:

$$A=\tfrac{1}{5},\ \mathfrak{A}=\tfrac{1}{6},\ B=-25,\ \mathfrak{B}=\tfrac{25}{24},\ P=\tfrac{9}{8}\ \text{et}\ Q=\tfrac{8}{9}(1+\theta)=\tfrac{8ma}{9h}$$

hincque distantiae focales

$$p=\tfrac{1}{6}a,\quad q=-\tfrac{5}{27}a\ \ \text{et}\ \ r=-\tfrac{5}{1+\theta}\cdot a=-\tfrac{5h}{m}$$

et lentium intervalla

$$\text{I et II}=\tfrac{1}{45}a,\quad \text{II et III}=\tfrac{40ma-45h}{9m}.$$

Faciamus nunc, ut etiam confusio ab apertura oriunda evanescat, et cum prima lens ex vitro coronario, secunda vero ex crystallino confici debeat, si tertia etiam ex coronario paretur, ut sit $\mu''=\mu$, debet esse $\lambda''=1,60006$; tum vero pro lente prima capiatur $\lambda=1$; habebitur ista aequatio:

$$\frac{\mu'}{\mu}\cdot\frac{5^3\cdot 8}{9}\left(\frac{24^3}{25^3}\lambda'-\frac{24\nu'}{25^2}\right)=6^3+30\nu-\frac{1,60006h}{5^3ma}.$$

Est autem $\log.\frac{\mu'}{\mu}=0,0538214$ seu

$$\frac{\mu'}{\mu}\left(98,304\lambda'-4,2666\nu'\right)=216+30\nu-0,0128\cdot\frac{h}{ma}$$

seu

$$98,304\lambda'=253,034-0,0145\cdot\frac{h}{ma},$$

unde colligitur

$$\lambda=2,5740\ -0,00015\cdot\frac{h}{ma}$$

ubi postremum membrum tuto omitti potest ob $\frac{h}{ma}$ fractionem exiguam.

Cum ergo sit

$$\lambda'=2,5740\ \text{et}\ \lambda'-1=1,5740,$$

erit

$$\tau\sqrt{(\lambda'-1)}=1,1009\ ;$$

unde, cum huius secundae lentis distantia focalis sit

$$q=-\tfrac{5}{27}a\ \text{et numerus}\ \mathfrak{B}=\tfrac{25}{24},$$

erit

$$\text{radius faciei}\begin{cases}\text{anterioris}=\dfrac{q}{\sigma-\mathfrak{B}(\sigma-\rho)+\tau\sqrt{(\lambda'-1)}}=\dfrac{q}{1,1823}=0,8458q\\[2ex]\text{posterioris}=\dfrac{q}{\rho+\mathfrak{B}(\sigma-\rho)-\tau\sqrt{(\lambda'-1)}}=\dfrac{q}{0,5418}=1,8457q.\end{cases}$$

Pro prima autem lente, cuius distantia focalis

$$p = \tfrac{1}{6}a \text{ et } \mathfrak{A} = \tfrac{1}{6} \text{ et } \lambda = 1$$

vitrumque coronarium, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)} = \frac{p}{1,4212} = 0,7036p \\ \text{posterioris} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)} = \frac{p}{0,4656} = 2,1478p.\end{cases}$$

.
Hinc ergo conficitur sequens

CONSTRUCTIO MICROSCOPII COMPOSITI NULLAM CONFUSIONEM PARIENTIS

114. Constituta pro lubitu distantia obiecti $= a$ habebimus

I. Pro prima lente ex vitro coronario facienda

$$\text{radius faciei}\begin{cases}\text{anterioris } = 0,1173a \\ \text{posterioris} = 0,3579a,\end{cases}$$

cuius distantia focalis est $\frac{1}{6}a = 0,1666a$;
aperturae semidiameter sumi poterit $x = 0,0293a$,
intervallum ad lentem secundam $= \frac{1}{45}a = 0,022a$.

II. Pro secunda lente ex vitro crystallino facienda erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = -0,1566a \\ \text{posterioris} = -0,3418a,\end{cases}$$

cuius distantia focalis $= -\frac{5}{27}a = -0,1852a$,
semidiameter aperturae $= -\ 0,0392a$;
intervallum ad lentem tertiam erit $= \frac{40ma-45h}{9m} = 4\frac{4}{9}a - \frac{5h}{m}$.

III. Pro lente tertia oculari ex vitro coronario paranda erit distantia focalis

$$-\frac{5h}{m}$$

hincque

$$\text{radius faciei utriusque } = -5,3\cdot\frac{h}{m};$$

sin autem ex vitro crystallino paretur, utriusque faciei radius sumatur $= -5,8 \cdot \frac{h}{m}$, huicque lenti oculus immediate adplicatur.

IV. Spatii autem in obiecto conspicui semidiameter erit

$$z = \frac{25}{100\theta - 12} \cdot a$$

existente

$$\theta = \frac{ma}{h} - 1.$$

V. Cum capere liceat $x = 0,0293a$, erit $y = 0,0293 \cdot \frac{h}{m}$ et mensura claritatis $= 0,586 \cdot \frac{h}{m}$ positoque $h = 8$ dig. fiet ea $= \frac{4,688}{m}$.

COROLLARIUM 1

115. Ne igitur primas lentes nimis exiguas confici oporteat, conveniet distantiam obiecti a tanto maiorem assumi; ac si statuatur $a = 8$ dig., hae lentes satis commodam magnitudinem obtinerent et multiplicatio m ostenderet, quanto maius obiectum appareat per microscopium, quam si idem obiectum in eadem distantia nudis oculis spectaremus.

COROLLARIUM 2

116. Deinde si sumamus $a = 8$ dig., longitudo totius instrumenti fiet circiter $35\frac{1}{2}$ dig., quae utique satis est magna; sed perpendi debet eam tantum esse $4\frac{1}{2}$ vicibus maiorem quam distantiam obiecti, eaque ad dimidium reducetur sumendo $a = 4$ dig.; quo casu constructio lentium adhuc erit satis ad praxin accommodata, quin etiam distantia obiecti commode adhuc minor assumi poterit, ita ut longitudo instrumenti ne pedem quidem integrum superet.

SCHOLION 1

117. Non parum paradoxon videbitur, quod distantia obiecti plane non ingrediatur in mensuram claritatis; nemo enim certe arbitrabitur, si distantia ad plures pedes augeretur, obiectum semper eadem claritate esse appariturum idque pro eadem multiplicatione. Verum hic probe est observandum mensuram nostram claritatis ad eum claritatis gradum referri, quo idem obiectum in loco, ubi actu est, nudo oculo cemeremus. Si enim haec mensura prodeat aequalis unitati, intelligendum est nos per instrumentum conspicere obiectum eadem claritate, qua id in ea ipsa distantia nudo oculo esset appariturum; notum autem est, quo magis obiectum a nobis removetur, in eadem ratione eius claritatem naturalem diminui; quare, cum nostra mensura ad claritatem naturalem referatur, quae

scilicet in ipso obiecto nudis oculis conspicitur, manifestum est, quo magis idem obiectum removemus distantiam a augendo, eo magis claritatem naturalem diminui, ac tum nostra mensura tantum indicat, quoties claritas per microscopium visa minor sit naturali, atque ex hoc clare perspicitur claritatem visam maxime diminui, si distantiam obiecti a nimis magnam accipiamus, ita ut pro usu microscopiorum vix consultum sit distantiam obiecti ultra aliquot digitos extendere. Simili modo indicium de multiplicatione est intelligendum, quam hic ad distantiam $h = 8$ dig. referimus; quodsi ergo v. c. obiectum distaret 16 dig., id iam nudis oculis duplo minus appareret quam in distantia 8 digitorum; quare, si obiectum dicatur 100 augeri, id ita est intelligendum, ut obiectum ducenties maius appareat quam nudis oculis in eadem distantia.

SCHOLION 2

118. Hinc igitur facile intelligitur, si distantiam obiecti satis magnam statuamus, tum microscopium tandem in telescopium esse abiturum, qui transitus eo magis attendi meretur, quo maius discrimen vulgo inter telescopia et microscopia constituitur, quae quippe instrumenta ut plane heterogenea spectari solent. Operae igitur pretium erit eiusmodi exemplum subiungere, de quo dubium erit, utrum ad microscopia an ad telescopia sit referendum.

EXEMPLUM 3

119. Sit distantia obiecti a tanta, ut sumta pro $\mathfrak{A}$ satis exigua fractione productum $\mathfrak{A}a = p$ modicum obtineat valorem, seu sit $\mathfrak{A} = \frac{p}{a}$ fractio valde parva hincque etiam

$$A = \frac{p}{a-p}.$$

His positis cum sit

$$B = \frac{N'}{N(A+1)P-N'} = \frac{10}{7(1+A)P-10},$$

sumatur $P = \frac{9}{8}$ ut ante, et ne A penitus negligamus, ponamus

$$(1+A)P = \frac{8}{7} \text{ fietque } B = -5;$$

ac si forte discrimen inter litteras N et N' sit minus, ac ne litteram $\mathfrak{q}$ penitus negligamus, sumamus $B = -6$, ut sit $\mathfrak{B} = \frac{6}{5}$; quoniam igitur loco litterarum a et A distantia focalis p in calculum introducitur, ut sit sive $\mathfrak{A}a = p$ sive $Aa = p$, erunt reliquae distantiae focales

$$q = -\frac{15}{16} p \text{ et } r = \frac{6h}{ma} \cdot p.$$

Tum vero intervallum

$$\text{prius} = \tfrac{1}{9}p, \quad \text{posterius} = \tfrac{16}{3}p\left(1 - \tfrac{9h}{8ma}\right).$$

Praeterea vero reperitur

$$\mathfrak{q} = \frac{5hr}{48ma - 53h}, \quad \text{hinc} \quad \mathfrak{q} + \mathfrak{r} = \frac{48ma\mathfrak{r} - 48h\mathfrak{r}}{48ma - 53h}$$

et spatii conspicui semidiameter

$$z = \frac{12}{48ma - 53h} \cdot ah$$

ideoque angulus

$$\frac{z}{a} = \Phi = \frac{12h}{48ma - 53h},$$

quae fractio per 3437 multiplicata exprimet angulum Φ in minutis primis.
Deinde semidiameter confusionis, si ex vinculo denominator $\mathfrak{A}^3$ in factorem communem transferatur, ita se habebit:

$$\frac{a}{h} \cdot \frac{mx^3}{p^3}\left(\mu\lambda - \frac{8\mu'}{9}\left(\frac{5^3\lambda'}{6^3} - \frac{5\nu'}{36}\right) - \frac{\mu''\lambda''h}{6^3 ma}\right),$$

quae ad nihilum reducetur sumendo

$$\frac{8\mu'}{9}\left(\frac{5^3\lambda'}{6^3} - \frac{5\nu'}{36}\right) = \mu\lambda - \frac{\mu''\lambda''h}{6^3 ma};$$

ubi prima lens ex vitro coronario, secunda ex crystallino confici debet, tertia vero etiam ex coronario paretur eritque $\lambda'' = 1,60006$ et $\mu'' = \mu$; tum vero capiatur $\lambda = 1$ ac reperietur

$$\lambda' = 0,24\nu' + \frac{\mu}{\mu'}(1,944 - 0,0144 \cdot \frac{h}{ma}) = 2,2611$$

neglecto scilicet ob parvitatem membro ultimo, ex quo fit $\tau\sqrt{(\lambda' - 1)} = 0,98542$;
unde pro huius lentis constructione erit

$$\text{radius faciei}\begin{cases} \text{anterioris} = \dfrac{q}{\sigma - \mathfrak{B}(\sigma - \rho) + \tau\sqrt{(\lambda - 1)}} = \dfrac{q}{0,8385} = 1,1926q \\ \text{posterioris} = \dfrac{q}{\rho + \mathfrak{B}(\sigma - \rho) - \tau\sqrt{(\lambda - 1)}} = \dfrac{q}{0,8856} = 1,1292q. \end{cases}$$

Pro lente vero priore erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= \frac{p}{\sigma} = 0,6024p\\ \text{posterioris} &= \frac{p}{\rho} = 4,4111p\end{cases}$$

atque hinc deducitur sequens

CONSTRUCTIO SIVE MICROSCOPII SIVE TELESCOPII OMNIS CONFUSIONIS EXPERTIS

120. Hic distantia obiecti *a* tanta supponitur, ut prae ea distantia focalis primae lentis *p* vehementer sit parva et quasi negligi queat.

I. Tum ergo pro prima lente ex vitro coronario paranda erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,6024p\\ \text{posterioris} &= 4,4111p,\end{cases}$$

cuius distantia focalis $= p$,
aperturae semidiameter $x = 0,1506p$,
distantia a lente secunda $= \frac{1}{9}p$.

II. Pro lente secunda ex vitro crystallino facienda erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -1,2721p\\ \text{posterioris} &= -1,2045p,\end{cases}$$

cuius distantia focalis $= -\frac{15}{16}p$,
eique apertura tribui potest aliquanto maior quam primae,
distantia vero ad lentem ocularem $= \frac{16}{3}p\left(1 - \frac{9h}{8ma}\right)$.

III. Pro lente tertia ex vitro coronario facienda, cuius distantia focalis est $r = -\frac{6h}{ma}\cdot p$, erit
radius faciei utriusque $= -\frac{6,36h}{ma}\cdot p$,
cui oculum immediate adplicari oportet.

IV. Pro spatio conspicuo iam invenimus semidiametrum

$$z = \frac{12}{48ma - 53h}\cdot ah$$

seu angulum

$$\Phi = \frac{z}{a} = \frac{12h}{48ma-53h};$$

priore scilicet modo aestimatur, si instrumentum ut microscopium spectetur, posteriore vero, si ut telescopium.

V. Quia capere licet

$$x = 0{,}1506p,$$

erit

$$y = \frac{0{,}1506h}{ma} \cdot p$$

et mensura claritatis

$$= \frac{3{,}012}{ma} \cdot hp$$

si scilicet distantiae in digitis exprimantur, unde patet, quo maius capiatur *p,* eo maiorem prodire claritatem; sed meminisse oportet *p* valde parvum prae *a* esse debere.

VI. Longitudo denique totius instrumenti erit

$$5\tfrac{4}{9}p - 6\frac{h}{ma} \cdot p.$$

COROLLARIUM 1

121. Quodsi hoc instrumentum tanquam microscopium spectare velimus, primo quidem distantia *a* tam magna esse debet, ut eius exigua portio sufficiat pro lente obiectiva construenda; tum vero sumi solet $k = 8$ dig., ad quam distantiam multiplicatio *m* referri solet, atque ex multiplicatione hoc modo aestimata in calculum ingreditur $\frac{h}{ma}$. Sin autem ut telescopium spectare velimus et distantia *a* tam sit magna, ut etiam valor *p* satis magnus accipi possit, tum sumi solet $h = a$ nihilque aliud in formulis inventis mutandum occurrit, ita ut totum discrimen in varia ratione multiplicationem aestimandi consistat.

COROLLARIUM 2

122. Quo hoc clarius perspiciatur, statuamus $\frac{ma}{h} = \zeta$ unde constructio plene determinatur; ac si instrumentum ut microscopium spectetur, aestimari solet multiplicatio $m = \frac{h\zeta}{a} = \frac{8\zeta}{a}$, sin autem ut telescopium spectetur, tum dicetur multiplicatio esse $m = \zeta$ sicque totum discrimen ad diversitatem loquendi revocatur.

COROLLARIUM 3

123. Pro telescopiis mensura claritatis pro lubitu atque adeo usque ad unitatem seu claritatem plenam augeri potest; tantum enim opus est, ut capiatur $p = \frac{m}{3,012} = \frac{m}{3}$. Vulgo autem contenti esse solemus claritate $= \frac{2}{5}$, ita ut tum sumi debeat $p = \frac{2m}{15}$. Pro microscopiis autem tantam claritatem obtinere non licet; quia enim ob $h = 8$ mensura claritatis fit $\frac{24}{m} \cdot \frac{p}{a}$ et fractio $\frac{p}{a}$ necessario valde est parva, quo maior multiplicatio desideratur, eo minorem claritatem prodire necesse est.

SCHOLION

124. En ergo praeter omnem exspectationem elegantem constructionem telescopii, quod in ratione quacunque obiecta amplificat et cuius constructio sequenti modo se habebit.

Proposita scilicet multiplicatione m capiatur distantia focalis $p = \frac{2m}{15}$ dig., ut scilicet mensura claritatis prodeat $= \frac{2}{5}$.

Constructio telescopii ab omni confusione liberi

I. Pro prima lente ex vitro coronario facienda erit

$$\text{radius faciei} \begin{cases} \text{anterioris} = 0,0803m \text{ dig.} \\ \text{posterioris} = 0,5881m \text{ dig.,} \end{cases}$$

distantia focalis $= \frac{2m}{15}$ dig.,
aperturae semidiameter $x = 0,0201m$ dig. $= \frac{m}{50}$ dig.,
intervallum ad lentem sequentem erit $= \frac{2m}{135} = 0,01481m = \frac{m}{50}$ dig.,

II. Pro lente secunda ex vitro crystallino facienda erit

$$\text{radius faciei} \begin{cases} \text{anterioris} = -\ 0,16961m \text{ dig.} \\ \text{posterioris} = -0,16060m \text{ dig.,} \end{cases}$$

cuius distantia focalis $q = -0,1422m$,
eique apertura tribuitur aliquanto maior quam primae,
intervallum ad lentem sequentem $= (0,7111m - 0,8)$ dig.
III. Pro lente tertia, cuius distantia focalis est

$$= -\tfrac{4}{5} \text{dig.} = -0,8 \text{ dig.},$$

si ergo haec lens ex vitro coronario paretur, erit ,

$$\text{radius faciei utriusque} = -0,848 \text{ dig.,}$$

sin autem ex vitro communi, ubi $n = 1,55$, erit

$$\text{radius faciei utriusque} = -0,88 \text{ dig.},$$

sin autem ex vitro crystallino, erit

$$\text{radius faciei utriusque} = -0,928 \text{ dig.};$$

cui oculus immediate adplicetur.

IV. Semidiameter campi apparentis erit

$$\Phi = \frac{12}{48m-53};$$

in mensura angulorum autem erit

$$\Phi = \frac{41244}{48m-53} \text{ min. sive proxima } \frac{859}{m-1} \text{ min.}$$

V. Longitudo denique totius huius telescopii erit

$$= (0,7259m - 0,8) \text{ dig.}$$

Hoc ergo telescopium non tam ob brevitatem est commendandum, quam ideo, quod constructio eius practica non tantis difficultatibus sit involuta quam multo breviora, quae supra sunt inventa, propterea quod littera λ' non multum ab unitate discrepat; quae ergo commendatio etiam pro microscopiis huius generis valet.

EVOLUTIO SECUNDI CASUS (CONF. § 108) QUO LITTERAE *A* VALOR NEGATIVUS TRIBUITUR

125. Hoc casu an littera $\mathfrak{A}$ habitura sit valorem positivum an negativum, incertum est; at littera *B* nunc debet esse positiva, et cum ob eandam rationem ut casu praecedente littera $\mathfrak{q}$ prae $\mathfrak{r} = 1$ ut evanescens spectari possit, erit

$$B = \frac{N'}{N(1+A)P - N'}$$

ubi debet esse $P < 1$, sicque multo magis erit $(1+A)P < 1$; ex quo perspicuum est litteram *N* maiorem esse debere quam *N'*. Quare primam lentem ex vitro crystallino, secundam vero ex coronario confici oportebit, ut sit $N : N' = 10 : 7$ ideoque

$$B = \frac{7}{10(1+A)P-1};$$

unde necesse est, ut sit $P > \frac{7}{10(1+A)}$, simul vero $P < 1$; unde sequitur esse debere

$$7 < 10(1+A) \quad \text{seu} \quad 1+A > \frac{7}{10}.$$

Ponamus ergo $A=-\alpha$ sumique debet $\alpha<\frac{3}{10}$ et quidem α notabiliter minus capi debet quam $\frac{3}{10}$, quia alioquin P nimis parum ab unitate deficere deberet et intervallum duarum priorum lentium prodiret nimis parvum. Cum autem α fractio sit satis exigua, fiet $\mathfrak{A}=\frac{\alpha}{1-\alpha}$ hincque distantia focalis primae lentis

$$p=-\frac{\alpha}{1-\alpha}\cdot a$$

Intervallum vero binarum priorum lentium

$$=-\alpha a\left(1-\frac{1}{P}\right)=\alpha a(\tfrac{1}{P}-1),$$

quod parti sive nonae sive decimae distantiae $\frac{\alpha}{1-\alpha}\cdot a$ aequetur, quod fit, si sumetur $P=\frac{8}{9}$, ita ut esse debeat $\alpha<\frac{17}{80}$, et ne tam anxie huic rationi 7:10 inhaereamus, si sumamus $\alpha=\frac{1}{6}$, fiet $B=\frac{189}{11}=17$. Capiamus autem potius $\alpha=\frac{1}{7}$ fietque $B=\frac{441}{13}=11\frac{4}{14}$. Tuto igitur ponere poterimus tum vero $\mathrm{A}=-\frac{1}{7}$ et $\mathfrak{A}=-\frac{1}{6}$ hincque distantiae focales

$$p=-\frac{1}{6}a,\quad q=\frac{27}{182}a\quad \text{et}\quad r=-\frac{12}{7}\cdot\frac{h}{m};$$

deinde lentium intervalla

$$\text{I et II}=\frac{1}{56}a,\quad \text{II et III}=\frac{27}{14}a-\frac{12}{7}\cdot\frac{h}{m}\cdot$$

Nunc vero ex aequatione fundamentali colligemus

$$\mathfrak{q}=-\frac{13h}{108ma-95h},$$

hinc

$$\mathfrak{q}+\mathfrak{r}=\frac{108ma-108h}{108ma-95h};$$

unde deducitur spatii conspicui semidiameter

$$z=\frac{108}{108ma-95h}\cdot ah\xi=\frac{27ah}{108ma-95h}$$

sumto scilicet $\mathfrak{r}=1$ et $\xi=\frac{1}{4}$.

Expressio porro pro semidiametro confusionis est

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''\lambda''}{A^3B^3PQ}\right),$$

quae ad nihilum redigatur. Hunc in finem notetur litteras p et ν ad vitrum crystallinum, litteras vero μ' et ν' ad coronarium referri; tum vero capi poterit $\lambda'=1$, ac si tertia lens etiam ex vitro coronario fiat, ut sit $\mu''=\mu'$, sumi debet $\lambda''=1,60006$ hincque definiri poterit numerus λ hoc modo:

$$-\frac{\lambda}{\mathfrak{A}^3}-\frac{\nu}{A\mathfrak{A}}-\frac{\mu'}{\mu A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''\lambda''}{\mu A^3B^3PQ}$$

sive

$$\lambda = 0{,}0491 + \frac{\mu'}{\mu}\left(\frac{343\cdot 2197}{192\cdot 1728} + \frac{343\cdot 13\cdot 0{,}2196}{192\cdot 144} - \frac{343\cdot 1{,}60006}{216\cdot 1728}\cdot\frac{h}{ma}\right),$$

quae evoluta praebet

$$\lambda = 0{,}0491 + 2{,}5709 + 0{,}0401$$

neglecto termino ultimo seu

$$\lambda = 2{,}6601,$$

unde colligitur

$$\tau\sqrt{(\lambda-1)} = 1{,}1306.$$

Hincque pro prima lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)-1{,}1306} = \frac{p}{0{,}6923} = 1{,}4444p\\ \text{posterioris} = \frac{p}{\rho+\mathfrak{B}(\sigma-\rho)+1{,}1306} = \frac{p}{1{,}0318} = 0{,}9692p.\end{cases}$$

Pro lente secunda autem ex vitro coronario paranda ob $\mathfrak{B} = \frac{12}{13}$ et $\lambda' = 1$ erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma-\mathfrak{A}(\sigma-\rho)} = \frac{q}{0{,}3370} = 2{,}9673q\\ \text{posterioris} = \frac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = \frac{q}{1{,}5498} = 0{,}6452q,\end{cases}$$

unde habetur sequens

CONSTRUCTIO MICROSCOPIORUM HUIUS SPECIEI

PRO QUAVIS MULTIPLICATIONE *m*

126. Constituta pro lubitu distantia obiecti $= a$ habebitur

I. Pro prima lente ex vitro crystallino facienda, cuius distantia focalis est $p = -\frac{1}{6}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0{,}2407a\\ \text{posterioris} = -0{,}1615a,\end{cases}$$

cuius aperturae semidiameter sumi poterit $x = 0{,}0404a$, nisi forte secunda lens minorem postulet.

Intervallum ad lentem secundam $= \frac{1}{56}a = 0{,}0178a.$

II. Pro secunda lente ex vitro coronario facienda, cuius distantia focalis est $q=\frac{27}{182}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,4402a\\ \text{posterioris} &= 0,0957a,\end{cases}$$

cuius aperturae semidiameter maior esse nequit quam 0,0239*a;* cui ergo etiam pro prima lente valor ipsius *x* aequari debet.

Intervallum vero ad lentem tertiam erit

$$\tfrac{27}{14}a-\tfrac{12}{7}\cdot\tfrac{h}{m}=1{,}9285a-\tfrac{12}{7}\cdot\tfrac{h}{m}.$$

III. Pro tertia lente, cuius distantia focalis est

$$r=\tfrac{12}{7}\cdot\tfrac{h}{m}=-\tfrac{96}{7m}\text{ dig.}=-\tfrac{13{,}71}{m}\text{dig.}$$

si ex vitro coronario paretur, erit

$$\text{radius faciei utriusque }=\tfrac{14{,}53}{m}\text{ dig .,}$$

sin autem ex vitro communi $n=1{,}55$ paretur, erit

$$\text{radius faciei utriusque }=\tfrac{15{,}08}{m}\text{ dig .,}$$

at si ex vitro crystallino paretur, erit

radius utriusque faciei $=\frac{15{,}90}{m}$ dig.

IV. Spatii porro in obiecto conspicui erit semidiameter

$$z=\tfrac{27ah}{108ma-95h}=\tfrac{54a}{27ma-190}\text{ dig.}$$

V. Cum autem hic sit $x=0{,}0239a$, erit

$$y=\tfrac{hx}{ma}=\tfrac{0{,}1912}{m}\text{ dig.}$$

hincque mensura claritatis erit

$$20y=\tfrac{3{,}824}{m}.$$

COROLLARIUM 1

127. Ne ambae lentes priores fiant nimis parvae, distantia obiecti a necessaria modicae magnitudinis statui debet; veluti si nolimus, ut ullus radius faciei minor sit parte decima digiti, posito minimo radio $0,0957a = \frac{1}{10}$ fiet $a = \frac{1}{0,957}$, seu distantiam a minorem uno digito capi non conveniet.

COROLLARIUM 2

128. Si ergo sumatur $a = 1\frac{1}{2}$ dig., quo casu primae lentes adhuc commode parari poterunt, longitudo totius instrumenti fiet circiter 3 dig., et cum distantia focalis lentis tertiae sit $-\frac{13,71}{m}$ dig., apparet multiplicationem vix ultra 100 extendi posse, quia alioquin haec lens fieret nimis parva; quod exiguum est vitium.

SCHOLION

129. Quodsi ingentes multiplicationes desideremus, omnia haec microscopia isto laborant vitio, quod lens ocularis nimis exigua requiratur, et inter ea, quae § 114 in exemplo 2 sunt descripta, hac praerogativa gaudent, quod distantia focalis tertiae lentis sit $-\frac{40}{m}$ dig., quae ergo ad multiplicationem $m = 400$ accommodari poterunt; at in primo exemplo, quod ob nimis magnam instrumenti longitudinem reiiciendum videbatur, multiplicatio multo longius augeri potest; cum enim ibi distantia focalis tertiae lentis esset $-\frac{216}{m}$ dig., ea hoc lucrum nobis praestat, ut multiplicatio ultra 1000 possit augeri, ita ut hoc lucro illud incommodum maxime compensetur. Ex quo colligere licet ingentes multiplicationes huiusmodi microscopiis produci non posse, nisi eorum longitudo valde fiat magna, ad quod necesse est, ut littera B valde magnum obtineat valorem, id quod quidem facillime praestatur in priore praecipue casu, ubi neglecto $\mathfrak{q}$ erat

$$B = \frac{10}{7(A+1)P-10}$$

hinc enim sumto $A = \frac{1}{5}$ et $P = \frac{7}{6}$ prodit $B = -50$, ac si manente $A = \frac{1}{5}$ capiatur $P = \frac{33}{28}$, orietur $B = -100$, ita ut tum foret distantia focalis tertiae lentis

$$r = -\frac{20h}{m} = -\frac{160}{m}$$

ideoque multiplicatio longe ultra 1000 augeri posset. Tum autem longitudo instrumenti foret

$$-\frac{AB}{P}(1-\frac{1}{Q}) = 17 \text{ dig.},$$

quae quidem facile admitti posset. Verum hic perpendendum est, si litteris A et P isti valores tribuantur, facile fieri posse, ut valor litterae B revera non solum in infinitum usque augeatur, sed etiam positivus evadat, si scilicet vera ratio numerorum N et N' tantillo maior fuerit quam 7:10. Quamobrem eo maiorem operam adhibeamus in microscopiis duorum reliquorum generum evolvendis. Interim tamen etiam maximas multiplicationes sequenti modo non incongrue producere licebit.

PROBLEMA 3

130. *Microscopia huius generis construere, quae ad maximas multiplicationes producendas sint accommodata.*

SOLUTIO

Cum totum negotium eo redeat, ut littera B praegrandem valorem nanciscatur, id duplici modo obtineri potest, prouti vel prima lens ex vitro coronario, secunda vero ex crystallino conficiatur vel vice versa prima lens ex crystallino, secunda vero ex coronario. Hos ambos casus seorsim pertractasse operae erit pretium.

CASUS PRIOR QUO PRIMA LENS EX VITRO CORONARIO SECUNDA VERO EX CRYSTALLINO PARATUR

Cum hoc casu habeatur

$$B = \frac{10}{7(A+1)P-10}$$

denominator hic prorsus ad nihilum redigatur, ut valor ipsius B infinitus evadat; tum enim facile intelligitur praegrandem eius valorem scopo nostro etiam esse satisfacturum, praecipue cum etiam casu, quo vera ratio numerorum N et N' a ratione assumta 7:10 parumper discrepat, valor litterae B tantum valde magnus erit proditurus. Ponamus igitur $P = \frac{8}{7}$, quoniam ob necessarium binarum priorum lentium intervallum hic valor non commode minor statui potest; ac tum esse oportebit $A = \frac{1}{4}$; at si forte, uti probabile videtur, discrimen refractionis non sit tantum, uti assumimus, conveniet A aliquanto minus assumi; statuamus ergo $A = \frac{1}{5}$, ut saltem valor ipsius B certe valde magnus sit proditurus, ita ut habeamus

$$A = \frac{1}{5},\ \mathfrak{A} = \frac{1}{6},\ P = \frac{8}{7} \ \text{ et } \ Q = \frac{7ma}{8h} \ \text{ ob } \ PQ = \frac{ma}{h}$$

hincque erit

$$p = \frac{1}{6}a,\ q = -\frac{7\mathfrak{B}}{40}a \ \text{ et } \ r = \frac{B}{5}\cdot\frac{h}{m}.$$

Hic igitur curandum est, ut lens tertia non fiat nimis parva, etiamsi multiplicatio m maxima statuatur; quare sumamus multiplicationem esse debere $m = 1000$, et cum distantia obiecti a vix minor uno digito esse possit, ne primae lentes fiant nimis exiguae, sumamus $a = 1$ dig., et cum sufficiat statuisse $r = -\frac{1}{2}$ dig., ob $k = 8$ dig. fiet hinc

$B=-\frac{625}{2}$, qui valor certe est satis magnus. Statuamus igitur porro $B=-300$, ut sit $\mathfrak{B}=\frac{300}{299}$, eritque

$$q=\frac{420}{8\cdot 299}a \quad \text{seu} \quad q=-\frac{420}{2392}a \quad \text{et} \quad r=-\frac{480}{m}\text{dig.}$$

Tum vero intervalla lentium erunt

$$\text{I et II}=\frac{1}{40}a,$$
$$\text{II et III}=60\left(\frac{7}{8}-\frac{h}{ma}\right)a=\left(\frac{105}{2}a-\frac{480}{m}\right)\text{dig.}$$

Circa spatium in obiecto conspicuum nihil fere in praecedentibus formulis erit mutandum; invenietur enim

$$z=\frac{1}{4}\cdot\frac{7ah}{7ma-8h}=\frac{14a}{7ma-64}\text{dig.}$$

Pro apertura autem primae lentis definienda semidiametrum confusionis ad nihilum redigamus ope huius aequationis:

$$0=\mu\left(216\lambda+30\nu\right)-\frac{7\cdot 125}{8}\mu'\left(0,99\lambda'-0,0008\right).$$

Hinc si sumamus $\lambda=1$, erit

$$0,99\lambda'=0,0008+\frac{\mu}{\mu'}\cdot 2,0351=2,3044$$

adeoque

$$\lambda'=2,3276 \text{, ex quo fit } \tau\sqrt{(\lambda'-1)}=1,0111.$$

Unde huius secundae lentis constructio erit:

Radius scilicet faciei

$$\begin{cases}\text{anterioris} =\frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)+1,0111}=\frac{q}{1,1477}=0,8713q \quad (9,9401715)\\ \text{posterioris}=\frac{q}{\rho+\mathfrak{B}(\sigma-\rho)-1,0111}=\frac{q}{0,5764}=1,7349q, \quad (0,2392759).\end{cases}$$

Pro prima autem lente ex vitro coronario ob $\mathfrak{A}=\frac{1}{6}$ et $\lambda=1$ erit

$$\begin{cases}\text{anterioris} = \frac{q}{\sigma-\mathfrak{A}(\sigma-\rho)} = \frac{p}{1{,}4212} = 0{,}7036p \\ \text{posterioris} = \frac{q}{\rho+\mathfrak{A}(\sigma-\rho)} = \frac{p}{0{,}4656} = 2{,}1478p.\end{cases}$$

CASUS POSTERIOR QUO PRIMA LENS EX VITRO CRYSTALLINO SECUNDA EX CORONARIO PARATUR

Cum hoc casu sit

$$B = \frac{7}{10(1+A)P-7},$$

denominator iterum ad nihilum redigatur, et cum P debeat esse unitate minus, sumatur $P=\frac{7}{8}$ eritque $A=-\frac{1}{5}$; at ob rationem supra allegatam sumatur $A=-\frac{1}{6}$, ut sit $\mathfrak{A}=-\frac{1}{5}$, hincque distantiae focales

$$p=-\frac{1}{5}a,\quad q=\frac{4\mathfrak{B}}{21}\cdot a \quad \text{et} \quad r=-\frac{B}{6}\cdot\frac{h}{m}.$$

Hic iterum faciamus, ut pro multiplicatione $m=1000$ prodeat circiter $r=-\frac{1}{2}$ dig., atque hinc prodibit $B=375$. Sumamus igitur $B=300$ ut ante fietque $\mathfrak{B}=\frac{300}{301}$ et ob $P=\frac{7}{8}$ erit $Q=\frac{8ma}{7h}$; atque hinc distantiae focales

$$p=-\frac{1}{5}a,\quad q=\frac{4}{21}\cdot\frac{300}{301}a,\quad r=-50\frac{h}{m}.$$

Intervalla vero lentium erunt

$$\text{I et II } = \frac{1}{42}a$$

atque

$$\text{II et III } = 50\left(\frac{8}{7}-\frac{h}{ma}\right)a = \frac{400}{7}a-\frac{50h}{m} = \left(57\tfrac{1}{7}a-\frac{400}{m}\right) \text{ dig.}$$

Pro spatio autem in obiecto conspicuo erit

$$z=\frac{1}{4}\cdot\frac{8ah}{8ma-7h}=\frac{2a}{ma-7} \text{ dig.},$$

quod spatium aliquantillo minus est quam casu praecedente.

Pro apertura denique primae lentis definienda semidiameter confusionis iterum ad nihilum redigatur, quod fit hac aequatione:

$$\mu(125\lambda-30\nu)=\mu'\,\frac{8\cdot216}{7}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right),$$

ex qua sumto $\lambda'=1$ colligitur

$$125\lambda = 7,587 + \tfrac{\mu'}{\mu} \cdot 246,857 \cdot 1,0107 = 290,007$$

hincque

$$\lambda = 2,3200 \quad \text{adeoque} \quad \tau\sqrt{(\lambda-1)} = 1,0082.$$

Pro prima igitur lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)-\tau\sqrt{(\lambda-1)}} = \frac{p}{0,2862} = 3,4942p\\ \text{posterioris} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)+\tau\sqrt{(\lambda-1)}} = \frac{p}{1,4379} = 0,6955p.\end{cases}$$

Pro secunda autem lente, cuius distantia focalis est q et numeri $\mathfrak{B} = \frac{300}{301}$ et $\lambda' = 1$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = \frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)} = \frac{q}{0,2315} = 4,3197q \quad (0,6354489)\\ \text{posterioris} = \frac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = \frac{q}{1,6553} = 0,6041q \quad (9,7811232).\end{cases}$$

Quod ad reliqua momenta attinet, ea in sequentibus constructionibus accuratius definiemus.

CONSTRUCTIO PRIORIS MICROSCOPII HUIUS GENERIS

131. Posita obiecti distantia $= a$ constructio sequenti modo se habebit:

I. Pro prima lente ex vitro coronario facienda, cuius distantia focalis $p = \frac{1}{6}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = 0,1173a\\ \text{posterioris} = 0,3579a.\end{cases}$$

Aperturae semidiameter sumi poterit $x = 0,0293a$ et distantia ad lentem secundam $= \frac{1}{40}a = 0,025a.$

II. Pro secunda lente ex vitro crystallino paranda, cuius distantia focalis $q = -\frac{420}{2392}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = -\ 0,1530a\\ \text{posterioris} = -\ 0,3046a.\end{cases}$$

Eius aperturae semidiameter $x = 0,0382a$; quae cum sit maior quam in prima lente, valor ille ipsius x valet et distantia ad lentem ocularem

$$= 52\tfrac{1}{2}a - \tfrac{480}{m}\text{dig.}$$

III. Pro lente oculari, cuius distantia focalis est

$$r = -\tfrac{480}{m} \text{ dig .,}$$

erit, si haec lens ex vitro coronario paretur,

$$\text{radius faciei utriusque } = -\tfrac{508,80}{m} \text{ dig .,}$$

sin autem ex vitro crystallino conficiatur, erit

$$\text{radius utriusque faciei } -\tfrac{556,80}{m} \text{ dig.}$$

Eius aperturae semidiameter sumi poterit $x = \tfrac{120}{m}$ dig., cui lenti oculus immediate est applicandus.

IV. Spatii in obiecto conspicui semidiameter erit

$$z = \tfrac{14a}{7ma-64}\text{dig.}$$

V. Pro claritate, cum sit $x = 0,0293a$, erit

$$y = \tfrac{hx}{ma} = \tfrac{0,2344}{m} \text{ dig.}$$

hincque mensura claritatis $= \tfrac{4.688}{m}$.

VI. Ne priores lentes nimis fiant parvae, distantia obiecti a vix infra digitum sumi posse videtur, nisi forte artifex lenticulas adhuc minores exacte elaborare valeat; quo casu distantia obiecti uno digito minor sicque longitudo instrumenti contrahi poterit.

CONSTRUCTIO MICROSCOPII POSTERIORIS HUIUS GENERIS

132. Posita iterum obiecti distantia $= a$ constructio ita se habebit:

I. Pro prima lente ex vitro crystallino facienda, cuius distantia focalis $p = -\tfrac{1}{5}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = -\ 0,6988a\\ \text{posterioris} & = -\ 0,1391a.\end{cases}$$

Eius aperturae semidiameter $x = 0,0348a$, nisi lens secunda minorem aperturam postulet.

Intervallum ad lentem secundam $= \frac{1}{42a}$.

II. Pro lente secunda ex vitro coronario facienda, cuius distantia focalis est

$$q = \tfrac{4}{21} \cdot \tfrac{300}{301} a = \tfrac{400}{2107} a,$$

$$\text{radius faciei} \begin{cases} \text{anterioris} & = 0,8201a \\ \text{posterioris} & = 0,1147a. \end{cases}$$

Eius aperturae semidiameter $x = 0,0286a$, unde etiam prioris lentis apertura maior accipi non poterit, ita ut sumi debeat $x = 0,0286a$.

Distantia ad lentem ocularem $= 57\tfrac{1}{7}a - \frac{400}{m}$ dig.

III. Pro lente oculari, cuius distantia focalis est $r = -\frac{400}{m}$ dig., si ea ex vitro coronario paretur, radius utriusque faciei esse debet

$$-\tfrac{400}{m} \text{ dig.,}$$

sin autem ea ex vitro crystallino fiat, erit is

$$= -\tfrac{464}{m} \text{ dig.}$$

Eius aperturae semidiameter capi poterit $x = \frac{100}{m}$ dig. huicque lenti oculus immediate est applicandus.

IV. Pro spatio in obiecto conspicuo reperimus eius semidiametrum $z = \frac{2a}{ma-7}$ dig.

V. Pro claritate, cum hic sit $x = 0,0286a$, erit $y = \frac{0,2288}{m}$ et mensura m claritatis $= \frac{4576}{m}$.

VI. Cum hoc casu lentes priores aliquanto sint maiores quam casu praecedente, respectu scilicet distantiae *a,* hoc casu nihil impedit, quominus distantia *a* uno digito minor capiatur, sicque longitudo instrumenti facile ad praecedentem revocabitur.

SCHOLION

133. En ergo duas adhuc huiusmodi microscopiorum species, quae supra allatis ideo longe sunt anteferendae, quod etiam ad maximas multiplicationes accommodari queant. Ingens autem horum instrumentorum longitudo merito non parum incommoda videbitur; verum si artifici succedat binarum lentium priorum elaboratio pro distantia $a = \frac{1}{2}$ dig., longitudo duorum pedum facile tolerari poterit. Cum autem hic duplici vitro simus usi, operae quoque pretium erit investigare, quanta sit futura altera confusio praeter marginem

coloratum ex diversa refractione oriunda; quem in finem spectari debebit haec aequatio [§ 27]:

$$0 = N \cdot \frac{1}{p} + \frac{N'}{P^2} \cdot \frac{1}{q} + \frac{N''}{P^2Q^2} \cdot \frac{1}{r},$$

cuius ultimus terminus manifesto evanescit prae prioribus, ita ut haec conditio postulet

$$0 = N \cdot \frac{1}{p} + \frac{N'}{P^2} \cdot \frac{1}{q}.$$

Cum nunc pro priore casu sit

$$N = 7, \quad N' = 10, \quad p = \frac{1}{6}a \quad \text{et} \quad P = \frac{8}{7} \quad \text{et} \quad q = -\frac{420}{3292}a,$$

haec formula fiet $6 - \frac{16744}{2688}$; cuius posterior terminus quia fere priorem tollit, manifestum est hinc nullam plane confusionem esse metuendam.

Pro altero vero casu, quo est

$$N = 10, \quad N' = 7, \quad P = \frac{7}{8}, \quad p = -\frac{1}{5}a, \quad q = \frac{400}{2107}a,$$

formula illa fiat $-50 + \frac{1204}{25}$, cuius bina membra inter se tenant rationem 25:24, hoc est tantum non rationem aequalitatis, ita ut se mutuo destruere sint censenda hocque casu altera confusio adeo penitus quasi evanescat, sicque priori casu confusio ex hoc fonte oriunda paululo minor erit quam casu posteriori; quod discrimen tamen nimis exiguum erit, quam ut in praxi alter casus alteri anteferendus videatur.