CHAPTER IV

CONCERNED WITH THE FURTHER ENLARGEMENT OF THE FIELD OF VIEW REQUIRED TO BE ACQUIRED FOR THIS KIND OF MICROSCOPES

PROBLEM 1

206. *For an objective lens of whatever nature, after the real image two lenses besides thus to be put in place, so that the with the colored margin vanishing the maximum field of view may emerge.*

SOLUTION

Just as in the above chapter we have seen the nature of the objective lens, whether it shall be simple or multiple, to change nothing in the following lenses, thus so that in turn the multiplicity of the latter lenses by no means will affect the objective lens; on account of which we will consider here the objective lens as simple, since the limits which we will find, also will be applied equally to all multiples. Therefore since now three intervals will be had, the second of the letters P, Q, R will be negative and there may be put $Q = -k$, thus so that there shall be $PkR = \frac{ma}{h}$;

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{A}}{P}\cdot a, \quad r = -\frac{AB\mathfrak{C}}{Pk}\cdot a, \quad \text{and} \quad s = +ABC\cdot\frac{h}{m};$$

from which there is concluded to become $\mathfrak{C} > 1$, hence $C < 0$. Then truly the intervals will be

$$\text{first} = \mathfrak{A}a\left(1 - \frac{1}{P}\right),$$
$$\text{second} = -ABa\left(\frac{1}{P} + \frac{1}{Pk}\right),$$
$$\text{third} = ABCa\left(-\frac{1}{Pk} + \frac{h}{ma}\right);$$

from which $R < 1$ follows. Again since for the apparent field of view there shall be

$$z = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma+h}\cdot ah\xi,$$

in order that the maximum field may happen, there must be $\mathfrak{q} = 1$, $\mathfrak{r} = 1$ and $\mathfrak{s} = 1$, so that there may become

$$z = \frac{3ah}{ma+h}\cdot\xi,$$

from which there will be

$$M = \frac{3h}{ma+h}m$$

and hence the fundamental equations

$$1.\ -\mathfrak{B} = (P-1)M = \frac{3h(P-1)}{ma+h},$$
$$2.\ \mathfrak{C} = (Pk+1)M - 1.$$

Truly the distance of the eye

$$O = \frac{\mathfrak{s}s}{Ma}\cdot\frac{h}{m} = \frac{s}{Ma}\cdot\frac{h}{m} = \frac{1}{3}s\left(1+\frac{h}{ma}\right)$$

Moreover the colored margin will be removed with the aid of this equation [§ 23]:

$$0 = \frac{1}{P} - \frac{1}{Pk} - \frac{1}{PkR},$$

from which there is found

$$k = 1 + \frac{1}{R}.$$

Truly since there must be $R<1$, we may establish $R=\frac{1}{2}$ and there will become

$$k = 3 \text{ and } PkR = \frac{3}{2}P = \frac{ma}{h}$$

thus so that there shall be

$$P = \frac{2ma}{3h} \text{ and } Pk = \frac{2ma}{h},$$

from which there is concluded

$$\mathfrak{C} = \left(\frac{2ma+h}{h}\right)M - 1$$

or

$$\mathfrak{C} = \frac{6ma+3h}{ma+h} - 1 = \frac{5ma+2h}{ma+h};$$

therefore for great magnifications there will be $\mathfrak{C}=5$ and hence $C=-\frac{5}{4}$. Truly from the first equation there arises

$$\mathfrak{B} = \left(\frac{3h-2ma}{3h}\right)M = \frac{3h-2ma}{ma+h}$$

and for great magnifications

$$\mathfrak{B} = -2 \text{ et } B = -\frac{2}{3}.$$

Therefore we may establish

$$\mathfrak{B} = -2,\ B = -\frac{2}{3},\ \mathfrak{C} = 5 \text{ and } C = -\frac{5}{4},$$

while there is, as we have seen,

$$P = \frac{2ma}{3h},\ k = 3 \text{ and } R = \frac{1}{2},$$

and the focal lengths will become

$$p=\mathfrak{A}a,\quad q=\frac{3Ah}{m},\quad r=\frac{5Ah}{3m}\quad\text{and}\quad s=\frac{5Ah}{6m}=\frac{1}{2}r\,,$$

and the intervals of the lenses

$$\text{first}=Aa\left(1-\frac{3h}{2ma}\right),\quad \text{second}=\frac{4Ah}{3m},\ \text{third}=\frac{5Ah}{12m}.$$

Therefore lest the focal lengths of the final lenses may become exceedingly small, it is necessary, that A shall be a very large number and thus $\mathfrak{A}=1$ approximately, from which it is apparent these determinations do not affect the objective lens and likewise to prevail, whatever objective lens were to be prepared ; now on account of which in place of the letter A to introduce the focal length q into the calculation, so that there shall become $A=\frac{mq}{3h}$, and thus the focal lengths of the following lenses will become

$$r=\frac{5q}{9}\quad\text{and}\quad s=\frac{5q}{18}$$

and the intervals will be

$$\text{first}=\frac{mqa}{3h}\left(1-\frac{3h}{2ma}\right)=\frac{maq}{3h}-\frac{1}{2}q,\quad \text{second}=\frac{4q}{9},\quad \text{third}=\frac{5q}{36}$$

and the distance of the eye approximately

$$O=\frac{1}{3}s=\frac{5q}{54}.$$

Therefore in all these cases treated before in place of the two posterior lenses it will be allowed to use these three lenses, provided the intervals will be observed indicated here, and in this way that of the gain will be arising, so that the apparent field may be increased in the ratio 2 : 3, if indeed there is here

$$z=\frac{3ah}{ma+h}\cdot\xi.$$

COROLLARY 1

207. Since the letter R may be permitted by our choice, provided it shall be less than unity, we may put $R=\frac{2}{3}$ and there will become $k=\frac{5}{2}$ and on account of $PkR=\frac{ma}{h}$ there will be

$$Pk=\frac{3ma}{2h}\quad\text{and}\quad P=\frac{3ma}{5h}$$

from which it follows

$$\mathfrak{C}=\frac{7}{2}\quad\text{and}\quad \mathfrak{B}=-\frac{9}{5}$$

and hence

$$C = -\tfrac{7}{5} \text{ and } B = -\tfrac{9}{14}$$

COROLLARY 2

208 . Therefore in this case $R = \tfrac{2}{3}$ will become $q = \tfrac{3Ah}{m}$ and hence in turn $A = \tfrac{mq}{3h}$, from which the following focal distances will become :

$$r = \tfrac{3Ah}{2m} = \tfrac{1}{2}q \text{ and } s = \tfrac{3}{5}r = \tfrac{3}{10}q$$

and the intervals of the lenses:

$$\text{first} = \tfrac{maq}{3h} - \tfrac{5}{9}q, \text{ second} = \tfrac{1}{2}q \text{ and third} = \tfrac{1}{3}s = \tfrac{1}{10}q.$$

SCHOLIUM

209. Here evidently we may attribute these values from the fundamental equations to the letters $\mathfrak{B}$ and $\mathfrak{C}$, which will be obtained, if the magnification m actually may become infinitely great, nor hence truly can our solution be accused of being in error, not even for smaller magnifications; while indeed we may depart from the true values of the these letters in this way, nothing else thence is to be concerned, except that the apparent field may be going to be produced here not as large as we have supposed ; which error is required to be condoned, especially since for the greater magnifications it may not indeed become perceptible, just as we have observed now above ; but when in these determinations we may regard the letter *m* as if infinite, since P may involve that also on account of $M = \tfrac{3h}{ma}$, clearly according to the same hypothesis, we will have in general

$$\mathfrak{B} = -\tfrac{3h}{ma} \cdot P \text{ and } \mathfrak{C} = \tfrac{3h}{ma} \cdot Pk - 1;$$

where it is required to note this hypothesis $m = \infty$ only to be used with these values ; thence the letter *A*, for which a very large number is indicated, we have extracted from the calculation and in place of that we have introduced the focal length *q*, thus so that there shall become $A = \tfrac{mq}{3h}$, from which in general the rest will be

$$r = \tfrac{(B+1)\mathfrak{C}q}{k} \text{ and } s = -\tfrac{(C+1)Pkhr}{ma} = -\tfrac{(B+1)CPhq}{ma}.$$

Then truly also the intervals between the lenses

$$\text{first} = \frac{maq}{3h} - \frac{maq}{3hP},$$
$$\text{second} = (B+1)(1+\tfrac{1}{k})q = \frac{(k+l)}{\mathfrak{C}} \cdot r,$$
$$\text{third} = \left(1 - \frac{ma}{Pkh}\right)s = (1-R)s.$$

But since there shall be $P = \frac{ma}{hkR}$, the values here to be assigned may be expressed much more concisely in the following manner:

$$\mathfrak{B} = -\frac{3}{kR}, \quad \mathfrak{C} = \frac{3}{R} - 1, \; B = -\frac{3}{3+kR}, \quad C = -\frac{3-R}{3-2R}.$$

Thence the focal lengths :

$$r = \frac{3-R}{3+kR} \cdot q, \quad s = \frac{(3-R)q}{(3+kR)(3-2R)} \text{ or } s = \frac{r}{3-2R}$$

and finally the intervals

$$\text{first} = \frac{maq}{3h} - \frac{kRq}{3}, \quad \text{second} = \frac{R(k+l)\mathrm{r}}{3-R}, \; \text{third} = (1-R)s,$$

with the distance of the eye being approx. $O = \frac{1}{3}s.$

But up to this stage we have not yet given an account of the colored margin, the destruction of which demands

$$k = 1 + \frac{1}{R},$$

from which the formulas found will be changed into the following:

$$\mathfrak{B} = -\frac{3}{R+1}, \quad \mathfrak{C} = \frac{3}{R} - 1, \; B = -\frac{3}{R+4}, \quad C = -\frac{3-R}{3-2R},$$

$$r = \frac{3-R}{4+R} \cdot q, \quad s = \frac{r}{3-2R} = \frac{(3-R)q}{(3-2R)(4+R)},$$

and the intervals

$$\text{first} = \frac{maq}{3h} - \frac{(\mathrm{R}+1)}{3} \cdot q,$$
$$\text{second} = \frac{(2R+1)r}{3-R} = \frac{2R+1}{4+R} \cdot q,$$
$$\text{third} = (1-R)s.$$

Therefore these determinations with the individual kinds of microscopes, which we have described in the preceding chapters, will be allowed to be combined and thus the following will be obtained :

GENERAL CONSTRUCTION OF MICROSCOPES OF THIS KIND

BY WHICH THE FIELD OF THESE WILL BE INCREASED IN THE RATIO 3:2

210. Here again the distance of the object *a* can be assumed as it pleases and the magnification *m;* then truly also the focal length *q* is allowed according to our choice, as it is agreed to be assumed to be so large, in order that the final lens of the eyepiece may not become exceedingly small; truly besides also the fraction *R* will depend on our choice, provided that shall be less than unity; but here we may accept $R = 1$, which value is seen to be most suitable in practice.

I. Whether the lens actually shall be simple or composed from two or more lenses close to each other, here that will be viewed as a single lens, thus so that in place of this all the constructions given in the above chapters may be able to be substituted, and thence the radius of its aperture will be given $= x$; then truly its distance from the following lens will be $\frac{maq}{3h} - \frac{1}{2}q$; but so that the interval can be changed a little on account of the nature of the objective lens, yet of which the ratio in practice does not merit attention.

II. For the second lens it is required to be noted that and equally the following lenses to be able to be prepared from any kind of glass, provided they shall be equally convex on both sides , so that the greatest aperture may be able to be given to them. Therefore the focal length of the second lens shall be $= q$ and the distance to the third lens will be $= \frac{4}{9}q$.

III. For the third lens its focal length may be taken $r = \frac{5}{9}q$ and the distance to the fourth lens $= \frac{5}{36}q$.

IV. For the fourth lens its focal length may be taken $s = \frac{5}{18}q$ and the distance to the eye $O = \frac{1}{3}s$ approx.

V. But now the radius of the area viewed in the object will be

$$z = \frac{3ah}{ma+h} \cdot \xi = \frac{3}{4} \cdot \frac{ah}{ma+h}$$

and the measure of the clarity will remain as before, evidently $= 20 \cdot \frac{hx}{ma}$, while evidently the measures are expressed in inches.

COROLLARY

211. Therefore if we do not wish, that the eyepiece lens may become smaller than $\frac{1}{3}$ in., by putting $s=\frac{1}{3}$ in. there must be taken $q=\frac{6}{5}$ in. and hence the first interval

$$=\frac{2ma}{5h}-\frac{3}{5}\text{ in.;}$$

but if in the above the eyepiece lens also may be put $=\frac{1}{3}$ dig., the penultimate lens shall become 1 in. and roughly the same interval is produced ; from which it is apparent in the present case the length of the instrument to become notably shorter.

PROBLEM 2

212. *Whatever were nature the objective lens, at this stage after the real image thus to put in place three lenses, so that the maximum field may emerge without the colored margin* .

SOLUTION

Since here four intervals shall be had , again the second of the four letters *P, Q, R, S* shall be negative and therefore $Q=-k$, so that there may become $PkRS=\frac{ma}{h}$. Therefore the focal lengths will become

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{Pk},\quad s=\frac{ABC\mathfrak{D}a}{PkR},\quad t=-ABCD\cdot\frac{h}{m}=-\frac{ABCDa}{PkRS};$$

from which , if in place of A the letter q may be introduced into the calculation, on account of $A=-\frac{Pq}{\mathfrak{B}a}$ there will become

$$r=\frac{B\mathfrak{C}q}{\mathfrak{B}k},\quad s=-\frac{BC\mathfrak{D}q}{\mathfrak{B}kR},\quad t=\frac{BCDq}{\mathfrak{B}kRS}.$$

The intervals of the lenses thus may be expressed in a similar manner by q :

$$\text{first}=-\frac{Pq}{\mathfrak{B}}+\frac{1}{\mathfrak{B}}\cdot q,\quad \text{second}=\frac{B}{\mathfrak{B}}(1+\frac{1}{k})q,$$

$$\text{third}=\frac{BC}{\mathfrak{B}k}(1-\frac{1}{R})q,\quad \text{fourth}=-\frac{BCD}{\mathfrak{B}kR}(1-\frac{1}{S})q.$$

Now so that a maximum apparent field may be produced, the letters $\mathfrak{q}=\mathfrak{r}=\mathfrak{s}=\mathfrak{t}=1$ may be put in place, so that there may become $M=\frac{4h}{ma+h}$, with the radius of the field becoming

$$z = Ma\xi = \tfrac{4ah}{ma+h} \cdot \xi = \tfrac{ah}{ma+h}$$

on taking $\xi = \frac{1}{4}$; which field therefore becomes as if quadrupled, while in the preceding problem it was trebled, before truly only doubled. Hence therefore the fundamental equations will have

$$\mathfrak{B} = (1-P)M, \quad \mathfrak{C}\left(1+Pk\right)M - 1, \quad \mathfrak{D} = (1+PkR)M - 2.$$

But since it may be sufficient for these formulas to be satisfied approximately, since there is little concern, even if the field may become a little smaller, we will consider the magnification *m* by the number *P* as if infinite and then these letters may be expressed thus more neatly on account of $M = \frac{4h}{ma}$:

$$\mathfrak{B} = -\tfrac{4hP}{ma} \text{ and thus } \tfrac{\mathfrak{B}}{P} = -\tfrac{4h}{ma},$$
$$\mathfrak{C} = \tfrac{4hPk}{ma} - 1, \qquad \mathfrak{D} = \tfrac{4hPkR}{ma} - 2;$$

and since there shall become $P = \frac{ma}{hkRS}$, these expressions also will be expressed more conveniently thus:

$$\mathfrak{B} = -\tfrac{4}{kRS}, \quad \mathfrak{C} = \tfrac{4}{RS} - 1, \quad \mathfrak{D} = \tfrac{4}{S} - 2.$$

But on account of the condition, by which the colored margin must be destroyed, we will have this same equation:

$$0 = \tfrac{1}{P} - \tfrac{1}{Pk} - \tfrac{1}{PkR} - \tfrac{1}{PkRS},$$

from which there arises

$$k = 1 + \tfrac{1}{R} + \tfrac{1}{RS},$$

thus so that the letters *R* and *S* may be permitted by our choice. But since the two final intervals certainly must become small enough, the letters *R* and *S* can differ little from unity; from which the letters $\mathfrak{C}$ and $\mathfrak{D}$ evidently will become greater than unity and hence *C* and *D* negative, while on the other hand the letter $\mathfrak{B}$ itself and therefore also *B* are negative; whereby, so that our intervals of the lenses may become positive, it is evident there must become $S < 1$ and $R < 1$; with which condition observed all the moments now will be able to be determined easily.

COROLLARY 1

213. Therefore since both R as well as S shall be fractions smaller than unity, the value of the letter k certainly will be greater than three, since

$$\frac{1}{R} > 1 \text{ and } \frac{1}{RS} > \frac{1}{R}.$$

COROLLARY 2

214. Since there shall be

$$\frac{P}{\mathfrak{B}} = -\frac{ma}{4h},$$

the first interval will be

$$= \frac{maq}{4h} - \frac{kRS}{4} \cdot q,$$

of which the first part $\frac{maq}{4h}$ is less than in the case of the preceding problem, thus so that here the length of the instrument shall be going to be produced still less.

COROLLARY 3

215. Therefore it will be allowed to combine these four lenses together with all the objective lenses, which we have described above, whether they be simple or composite ; and to which we may assign this significant convenience, that the apparent field may be produced quadrupled, as only to be doubled in the preceding.

EXAMPLE 1

216. Since the letters R and S must be less than unity, we will consider the case as the most simple and put $R = \frac{1}{2}$ and $S = \frac{2}{3}$, thus so that there may become $RS = \frac{1}{3}$, and hence

$$k = 1 + 2 + 3 = 6 ;$$

therefore from these values, which are seen to be convenient enough in practice, the letters are deduced:

$$\mathfrak{B} = -2, \ \mathfrak{C} = 11, \ \mathfrak{D} = 4, \ B = -\frac{2}{3}, \ C = -\frac{11}{10}, \ D = -\frac{4}{3};$$

then the following will be defined from the focal length q :

$$r = \frac{11}{18}q, \ s = \frac{22}{45}q, \ t = \frac{11}{45}q = \frac{1}{2}s;$$

and truly finally the separation of the lenses will become:

$$\text{first} = \tfrac{maq}{4h} - \tfrac{1}{2}q, \quad \text{second} = \tfrac{17}{18}q, \quad \text{third} = \tfrac{11}{180}q = \tfrac{1}{4}t, \quad \text{fourth} = \tfrac{11}{135}q.$$

EXAMPLE 2

217. Now we may put both $R = 1$ as well as $S = \tfrac{1}{2}$ and there will be produced

$$k = 1+2+4 = 7$$

and there will become

$$\mathfrak{B} = -\tfrac{16}{7}, \quad \mathfrak{C} = 15, \quad \mathfrak{D} = 6, \quad B = -\tfrac{16}{23}, \quad C = -\tfrac{15}{14}, \quad D = -\tfrac{6}{5}$$

and thus the focal lengths will be expressed by q :

$$r = \tfrac{15}{23}q, \quad s = \tfrac{90}{161}q, \quad t = \tfrac{36}{161}q$$

and the intervals

$$\text{first} = \tfrac{maq}{4h} - \tfrac{7}{16}q, \quad \text{second} = \tfrac{8}{23}q, \quad \text{third} = \tfrac{15}{322}q, \quad \text{fourth} = \tfrac{18}{161}q.$$

So that in the end the place of the eye may be reached, here in general it will be

$$O = \tfrac{1}{4}t\left(1+\tfrac{h}{ma}\right) = \tfrac{1}{4}t \text{ approx.}$$

PROBLEM 3

218. *Whatever the nature of the objective lens, thus to put in place some number of lenses besides after the real image, the number of which shall be $= i$, so that the maximum field of view may emerge with the colored margin vanishing.*

SOLUTION

If the operation may be put in place as in the preceding problems, there will be always $Q = -k$ and the number of the following letters R, S, T etc. will be $i-1$ and thus the final $= Z$; then truly for the field there will be had here

$$M = \tfrac{(i+1)h}{ma+k}$$

and thus

$$z = \tfrac{(i+1)ah}{ma+h} \cdot \xi.$$

So that then also if we may consider as before, for the determination of the letters $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$, the magnification m with the number P as infinitely great, we will find

$$\mathfrak{B} = -\frac{(i+1)hP}{ma}, \quad \mathfrak{C} = \frac{(i+1)hP}{ma} - 1,$$
$$\mathfrak{D} = \frac{(i+1)hPkR}{ma} - 2, \quad \mathfrak{E} = \frac{(i+1)hPkRS}{ma} - 3 \text{ etc.}$$

Truly the removal of the colored margin will give

$$k = 1 + \tfrac{1}{R} + \tfrac{1}{RS} + \tfrac{1}{RST} + \ldots \tfrac{1}{RST\,..\,Z},$$

the number of terms of which is *i*.

Now truly we may define these letters thus, so that there may become

$$\tfrac{1}{R} = 2, \quad \tfrac{1}{RS} = 3, \quad \tfrac{1}{RST} = 4, \quad \tfrac{1}{RSTU} = 5,$$

and finally

$$\tfrac{1}{RSTU\,..\,Z} = i$$

and thus

$$R = \tfrac{1}{2}, \quad S = \tfrac{2}{3}, \quad T = \tfrac{3}{4}, \quad U = \tfrac{4}{5} z \ \cdots \text{ ac tandem } \mathrm{Z} = \tfrac{i-1}{i}.$$

Therefore since here there shall be produced

$$k = 1 + 2 + 3 \ldots + i,$$

that is

$$k = \tfrac{i(i+1)}{2},$$

and since there shall become $RST \ \cdots Z = \tfrac{1}{i}$, there will be

$$kRS \cdots Z = \tfrac{i+1}{2}$$

and hence

$$P = \tfrac{2ma}{(i+1)h} \text{ or } \tfrac{1}{P} = \tfrac{(i+1)h}{2ma},$$

and hence again

$$\tfrac{1}{Pk} = \tfrac{h}{ima}, \quad \tfrac{1}{PkR} = \tfrac{2h}{ima}, \quad \tfrac{1}{PkRS} = \tfrac{3h}{ima}, \quad \tfrac{1}{PkRST} = \tfrac{4h}{ima} \text{ etc.},$$

finally there may be arrived at :

$$\tfrac{1}{PkRST\,..Z} = \tfrac{h}{ma}.$$

Now from these formulas our Germanic letters are found $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc.

$$\mathfrak{B}=-2,\quad \mathfrak{C}=i^2+i-1,\quad \mathfrak{D}=\tfrac{i^2+i-4}{2},\quad \mathfrak{E}=\tfrac{i^2+i-9}{2},\quad \mathfrak{F}=\tfrac{i^2+i-16}{2}\ \text{etc.}$$

then finally $\mathfrak{Z}$ shall become $=1$, and hence

$$B=-\tfrac{2}{3},\quad C=\tfrac{i^2+i-1}{2-i-i^2},\quad D=\tfrac{i^2+i-4}{6-i-i^2},\quad E=\tfrac{i^2+i-9}{12-i-i^2},\quad F=\tfrac{i^2+i-16}{20-i-i^2}\ \text{etc.}$$

Therefore from these values we will find the focal lengths of all the lenses after the second to be defined by the focal length of this q itself, which can be easily put in place in the following manner :

$$\tfrac{r}{q}=\tfrac{B\mathfrak{C}}{\mathfrak{B}k}=\tfrac{2}{3}\cdot\tfrac{i^2+i-1}{i^2+i},\qquad \text{therefore}\quad r=\tfrac{2}{3}\cdot\tfrac{i^2+i-1}{i^2+i}\cdot q,$$
$$\tfrac{s}{r}=-\tfrac{C\mathfrak{D}}{\mathfrak{C}R}=\tfrac{i^2+i-4}{i^2+i-2},\qquad \text{therefore}\quad s=\tfrac{i^2+i-4}{i^2+i-2}\cdot r,$$
$$\tfrac{t}{s}=-\tfrac{D\mathfrak{E}}{\mathfrak{D}S}=\tfrac{i^2+i-9}{i^2+i-6},\qquad \text{therefore}\quad t=\tfrac{i^2+i-9}{i^2+i-6}\cdot s,$$
$$\tfrac{u}{t}=-\tfrac{E\mathfrak{F}}{\mathfrak{E}T}=\tfrac{i^2+i-16}{i^2+i-12},\qquad \text{therefore}\quad t=\tfrac{i^2+i-16}{i^2+i-12}\cdot t$$

and thus finally

$$v=\tfrac{i^2+i-25}{i^2+i-20}\cdot u,\quad w=\tfrac{i^2+i-36}{i^2+i-30}\cdot v\quad \text{etc.}$$

Finally the intervals between the lenses will be determined thus :

$$\text{first}=-\tfrac{1}{\mathfrak{B}}\left(P-1\right)q=\tfrac{maq}{(i+1)h}-\tfrac{1}{2}q,$$
$$\text{second}=\tfrac{1}{2}\cdot\tfrac{i^2+i+2}{i^2+i-1}\cdot r,$$
$$\text{third}=-\tfrac{s}{\mathfrak{D}}\left(R-1\right)=\tfrac{1}{i^2+i-4}\cdot s,$$
$$\text{fourth}=-\tfrac{t}{\mathfrak{E}}\left(S-1\right)=\tfrac{1}{i^2+i-9}\cdot t,$$
$$\text{fifth}=-\tfrac{u}{\mathfrak{F}}\left(T-1\right)=\tfrac{1}{i^2+i-16}\cdot u,$$
$$\text{sixth}=-\tfrac{v}{\mathfrak{G}}\left(U-1\right)=\tfrac{1}{i^2+i-25}\cdot v,\quad \text{etc.}$$

COROLLARY 1

219. If there shall be $i=1$, so that a single lens may be found after the real image, there will become $r=\tfrac{1}{3}q$ and the intervals

$$\text{first}=\tfrac{maq}{2h}-\tfrac{1}{2}q,\quad \text{second}=2r$$

and

$$O=\tfrac{1}{2}r(1+\tfrac{h}{ma}).$$

COROLLARY 2

220. If $i = 2$, so that there shall be two lenses after the real image, the focal lengths of these will be

$$r = \tfrac{5}{9}q \text{ and } s = \tfrac{1}{2}r = \tfrac{5}{18}q,$$

then truly the intervals will become

$$\text{first} = \tfrac{maq}{3h} - \tfrac{1}{2}q, \quad \text{second} = \tfrac{4}{5}r = \tfrac{4}{9}q, \ \text{third} = \tfrac{1}{2}s = \tfrac{5}{36}q$$

and

$$O = \tfrac{1}{3}s\left(1 + \tfrac{h}{ma}\right).$$

COROLLARY 3

221. If there shall be $i = 3$, so that three lenses may be found after the real image, there will be

$$r = \tfrac{11}{18}q, \quad s = \tfrac{4}{5}r = \tfrac{22}{45}q, \quad t = \tfrac{1}{2}s = \tfrac{11}{45}q;$$

then truly the intervals

$$\text{first} = \tfrac{maq}{4h} - \tfrac{1}{2}q, \qquad \text{second} = \tfrac{7}{11}r = \tfrac{7}{18}q,$$
$$\text{third} = \tfrac{1}{8}s = \tfrac{11}{180}q, \quad \text{fourth} \quad = \tfrac{1}{3}t = \tfrac{11}{135}q.$$

Finally, for the position of the eye:

$$O = \tfrac{1}{4}t\left(1 + \tfrac{h}{ma}\right).$$

COROLLARY 4

222. If there shall be $i = 4$, so that there may be found four lenses after the real image, the focal lengths of these will be

$$r = \tfrac{19}{30}q, \quad s = \tfrac{8}{9}r = \tfrac{76}{135}q, \quad t = \tfrac{11}{14}s = \tfrac{418}{945}q, \quad u = \tfrac{1}{2}t = \tfrac{209}{945}q;$$

then truly the intervals will be

$$\text{first} = \tfrac{maq}{5h} - \tfrac{1}{2}q, \quad \text{second} = \tfrac{11}{19}r = \tfrac{11}{30}q, \quad \text{third} = \tfrac{1}{16}s = \tfrac{19}{540}q,$$
$$\text{fourth} = \tfrac{1}{11}t = \tfrac{38}{945}q, \quad \text{fifth} = \tfrac{1}{4}u = \tfrac{209}{3780}q$$

and for the position of the eye

$$O=\tfrac{1}{5}t\left(1+\tfrac{h}{ma}\right).$$

COROLLARY 5

223. If there shall be $i=5$, so that five lenses may be placed after the real image, there will be

$$r=\tfrac{29}{45}q,\quad s=\tfrac{13}{14}r=\tfrac{13\cdot 29}{14\cdot 45}q,\quad t=\tfrac{7}{8}s=\tfrac{7\cdot 13\cdot 29}{8\cdot 14\cdot 45}q,$$
$$u=\tfrac{7}{9}t=\tfrac{7\cdot 13\cdot 29}{2\cdot 8\cdot 9\cdot 45}q,\quad v=\tfrac{1}{2}u=\tfrac{7\cdot 13\cdot 29}{4\cdot 8\cdot 9\cdot 45}q;$$

then truly the intervals will be

$$\text{first}=\tfrac{maq}{6h}-\tfrac{1}{2}q,\quad \text{second}=\tfrac{26}{29}r=\tfrac{16}{45}q,\quad \text{third}=\tfrac{1}{26}s=\tfrac{29}{2\cdot 14\cdot 45}q,$$
$$\text{fourth}=\tfrac{1}{21}t=\tfrac{13\cdot 19}{3\cdot 8\cdot 14\cdot 45}q,\quad \text{fifth}=\tfrac{1}{14}u=\tfrac{13\cdot 29}{4\cdot 8\cdot 9\cdot 45}q,\quad \text{sixth}=\tfrac{1}{5}v=\tfrac{7\cdot 13\cdot 29}{4\cdot 5\cdot 8\cdot 9\cdot 45}q$$

and for the place of the eye

$$O=\tfrac{1}{6}t\left(1+\tfrac{h}{ma}\right).$$

CAPUT IV

DE ULTERIORI AMPLIFICATIONE CAMPI

HUIC MICROSCOPIORUM GENERI CONCILIANDI

PROBLEMA 1

206. *Cuiuscunque indolis fuerit lens obiectiva, post imaginem realem duas adhuc lentes ita disponere, ut margine colorato evanescente campus maximus evadat.*

SOLUTIO

Quemadmodum in superiori capite vidimus naturam lentis obiectivae, sive sit simplex sive multiplicata, nihil in lentibus posterioribus mutare, ita vicissim multiplicatio lentium posteriorum neutiquam lentem obiectivam adficiet; quamobrem considerabimus hic lentem obiectivam ut simplicem, quandoquidem determinationes, quas inveniemus, aeque ad omnes multiplicatas quoque erunt accommodatae. Cum igitur iam habeantur tria intervalla, litterarum *P, Q, R* secunda erit negativa hincque ponatur $Q=-k$, ut sit $PkR=\frac{ma}{h};$ distantiae igitur focales erunt

$$p=\mathfrak{A}a,\quad q=-\tfrac{A\mathfrak{A}}{P}\cdot a,\quad r=-\tfrac{AB\mathfrak{C}}{Pk}\cdot a,\quad \text{et}\quad s=+ABC\cdot\tfrac{h}{m};$$

unde concluditur fore $\mathfrak{C}>1,$ hinc $C<0$. Tum vero intervalla erunt

$$\begin{aligned}&\text{primum}=\mathfrak{A}a\left(1-\tfrac{1}{P}\right),\\&\text{secundum}=-ABa\left(\tfrac{1}{P}+\tfrac{1}{Pk}\right),\\&\text{tertium}=ABCa\left(-\tfrac{1}{Pk}+\tfrac{h}{ma}\right);\end{aligned}$$

unde sequitur $R<1$. Cum porro pro campo apparente sit

$$z=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma+h}\cdot ah\xi\,,$$

ut campus fiat maximus, debet esse $\mathfrak{q}=1,$ $\mathfrak{r}=1$ et $\mathfrak{s}=1$, ut fiat

$$z=\tfrac{3ah}{ma+h}\cdot\xi,$$

ex quo erit

$$M=\tfrac{3h}{ma+h}m$$

hincque aequationes fundamentales

$$1.\ -\mathfrak{B}=(P-1)M=\tfrac{3h(P-1)}{ma+h},$$
$$2.\ \mathfrak{C}=(Pk+1)M-1.$$

Pro loco oculi vero distantia

$$O=\tfrac{\mathfrak{s}s}{Ma}\cdot\tfrac{h}{m}=\tfrac{s}{Ma}\cdot\tfrac{h}{m}=\tfrac{1}{3}s\left(1+\tfrac{h}{ma}\right)$$

Margo autem coloratus destructur ope huius aequationis [§ 23]:

$$0=\tfrac{1}{P}-\tfrac{1}{Pk}-\tfrac{1}{PkR},$$

unde invenitur

$$k=1+\tfrac{1}{R}.$$

Quia vero debet esse $R<1$, statuamus $R=\tfrac{1}{2}$ fietque

$$k=3 \text{ et } PkR=\tfrac{3}{2}P=\tfrac{ma}{h}$$

ita ut sit

$$P=\tfrac{2ma}{3h} \text{ et } Pk=\tfrac{2ma}{h},$$

ex quo concluditur

$$\mathfrak{C}=\left(\tfrac{2ma+h}{h}\right)M-1$$

seu

$$\mathfrak{C}=\tfrac{6ma+3h}{ma+h}-1=\tfrac{5ma+2h}{ma+h};$$

pro magnis igitur multiplicationibus erit $\mathfrak{C}=5$ hincque $C=-\tfrac{5}{4}$. Ex priore vero aequatione prodit

$$\mathfrak{B}=\left(\tfrac{3h-2ma}{3h}\right)M=\tfrac{3h-2ma}{ma+h}$$

et pro magnis multiplicationibus

$$\mathfrak{B}=-2 \text{ et } B=-\tfrac{2}{3}.$$

Statuamus igitur

$$\mathfrak{B}=-2,\ B=-\tfrac{2}{3},\ \mathfrak{C}=5 \text{ et } C=-\tfrac{5}{4},$$

dum est, ut vidimus,

$$P=\tfrac{2ma}{3h},\ k=3 \text{ et } R=\tfrac{1}{2},$$

fientque distantiae focales

$$p=\mathfrak{A}a,\quad q=\tfrac{3Ah}{m},\quad r=\tfrac{5Ah}{3m}\quad \text{et}\quad s=\tfrac{5Ah}{6m}=\tfrac{1}{2}r,$$

et intervalla lentium

$$\text{primum}=Aa\left(1-\tfrac{3h}{2ma}\right),\quad \text{secundum}=\tfrac{4Ah}{3m},\ \text{tertium}=\tfrac{5Ah}{12m}.$$

Ne igitur distantiae focales posteriorum lentium fiant nimis parvae, necesse est, ut A sit numerus praemagnus ideoque $\mathfrak{A}=1$ proxima, unde patet has determinationes lentem obiectivam non adficere et perinde valere, utcunque lens obiectiva fuerit comparata; quamobrem iam conveniet loco litterae A distantiam focalem q in computum introducere, ut sit $A=\frac{mq}{3h}$, sicque fient distantiae focales sequentium lentium

$$r=\tfrac{5q}{9}\quad \text{et}\quad s=\tfrac{5q}{18}$$

et intervalla erunt

$$\text{primum}=\tfrac{mqa}{3h}\left(1-\tfrac{3h}{2ma}\right)=\tfrac{maq}{3h}-\tfrac{1}{2}q,\quad \text{secundum}=\tfrac{4q}{9},\quad \text{tertium}=\tfrac{5q}{36}$$

et distantia oculi proxima

$$O=\tfrac{1}{3}s=\tfrac{5q}{54}.$$

In omnibus igitur casibus antea tractatis loco binarum lentium posteriorum adhibere licebit has ternas lentes, dummodo intervalla hic indicata observentur, hocque modo id lucri nascetur, quod campus apparens augeatur in ratione 2 : 3, siquidem hic est

$$z=\tfrac{3ah}{ma+h}\cdot\xi.$$

COROLLARIUM 1

207. Cum littera R arbitrio nostro permittatur, dummodo sit unitate minor, ponamus $R=\frac{2}{3}$ eritque $k=\frac{5}{2}$ et ob $PkR=\frac{ma}{h}$ erit

$$Pk=\tfrac{3ma}{2h}\quad \text{et}\quad P=\tfrac{3ma}{5h}$$

unde sequitur

$$\mathfrak{C}=\tfrac{7}{2}\quad \text{et}\quad \mathfrak{B}=-\tfrac{9}{5}$$

hincque

$$C=-\tfrac{7}{5}\quad \text{et}\quad \mathrm{B}=-\tfrac{9}{14}$$

COROLLARIUM 2

208 . Hoc ergo casu $R=\frac{2}{3}$ fiet $q=\frac{3Ah}{m}$ neque viccissim $A=\frac{mq}{3h}$, unde sequentes distantiae focales fient:

$$r=\frac{3Ah}{2m}=\frac{1}{2}q \text{ et } s=\frac{3}{5}r=\frac{3}{10}q$$

et intervalla lentium

$$\text{primum}=\frac{maq}{3h}-\frac{5}{9}q, \text{ secundum}=\frac{1}{2}q \text{ et tertium}=\frac{1}{3}s=\frac{1}{10}q.$$

SCHOLION

209. Hic scilicet litteris $\mathfrak{B}$ et $\mathfrak{C}$ ex aequationibus fundamentalibus eos valores tribuimus, quos obtinerent, si multiplicatio *m* revera esset infinite magna, neque vero hinc nostra solutio erroris redargui potest, nequidem pro minoribus multiplicationibus; dum enim hoc modo a veris harum litterarum valoribus recedimus, nihil aliud inde est metuendum, nisi quod campus apparens non tantus sit proditurus, quam hic supposuimus; quod vitium facile est condonandum, praecipue quoniam pro maioribus multiplicationibus nequidem fiet sensibile, quemadmodum iam supra observavimus; quando autem in his determinationibus litteram *m* quasi infinitam spectamus, quoniam *P* eam quoque involvit ob $M=\frac{3h}{ma}$ in eadem scilicet hypothesi, habebimus in genere

$$\mathfrak{B}=-\frac{3h}{ma}\cdot P \text{ et } \mathfrak{C}=\frac{3h}{ma}\cdot Pk-1;$$

ubi probe notandum est hanc hypothesin $m=\infty$ tantum in his valoribus adhiberi; deinde litteram *A*, qua numerus praemagnus indicatur, ex calculo extrusimus eiusque loco distantiam focalem *q* introduximus, ita ut sit $A=\frac{mq}{3h}$, unde in genere reliquae erunt

$$r=\frac{(B+1)\mathfrak{C}q}{k} \text{ et } s=-\frac{(C+1)Pkhr}{ma}=-\frac{(B+1)CPhq}{ma}.$$

Tum vero etiam intervalla lentium

$$\text{primum}=\frac{maq}{3h}-\frac{maq}{3hP},$$
$$\text{secundum}=(B+1)(1+\tfrac{1}{k})q=\frac{(k+1)}{\mathfrak{C}}\cdot r,$$
$$\text{tertium}=\left(1-\frac{ma}{Pkh}\right)s=(1-R)s.$$

Cum autem sit $P=\frac{ma}{hkR}$, valores hic assignati sequenti modo multo concinnius exprimentur:

$$\mathfrak{B}=-\frac{3}{kR}, \quad \mathfrak{C}=\frac{3}{R}-1, \quad B=-\frac{3}{3+kR}, \quad C=-\frac{3-R}{3-2R}.$$

Deinde distantiae focales

$$r=\tfrac{3-R}{3+kR}\cdot q,\quad s=\frac{(3-R)q}{(3+kR)(3-2R)}\ \text{seu}\ s=\tfrac{r}{3-2R}$$

ac denique intervalla

$$\text{primum}=\tfrac{maq}{3h}-\tfrac{kRq}{3},\quad \text{secundum}=\tfrac{R(k+1)r}{3-R},\ \text{tertium}=\left(1-R\right)s,$$

existente distantia oculi proxima $O=\tfrac{1}{3}s$.

Hactenus autem nondum rationem habuimus marginis colorati, cuius destructio postulat

$$k=1+\tfrac{1}{R},$$

unde formulae inventae in sequentes abibunt:

$$\mathfrak{B}=-\tfrac{3}{R+1},\quad \mathfrak{C}=\tfrac{3}{R}-1,\ B=-\tfrac{3}{R+4},\quad C=-\tfrac{3-R}{3-2R},$$

$$r=\tfrac{3-R}{4+R}\cdot q,\quad s=\tfrac{r}{3-2R}=\frac{(3-R)q}{(3-2R)(4+R)},$$

et intervalla

$$\text{primum}=\tfrac{maq}{3h}-\tfrac{(R+1)}{3}\cdot q,$$
$$\text{secundum}=\tfrac{(2R+1)r}{3-R}=\tfrac{2R+1}{4+R}\cdot q,$$
$$\text{tertium}=(1-R)\ s.$$

Has igitur determinationes cum singulis microscopiorum speciebus, quas in praecedentibus capitibus descripsimus, combinare licebit sicque obtinebitur sequens

CONSTRUCTIO GENERALIS MICROSCOPIORUM HUIUS GENERIS QUA EORUM CAMPUS IN RATIONE SESQUIALTERA AUGETUR

210. Hic iterum distantia obiecti a pro lubitu assumi potest perinde ac multiplicatio m; tum vero etiam distantia focalis q arbitrio nostro permittitur, quam tantam assumi convenit, ut postrema lens ocularis non fiat nimis parva; praeterea vero quoque fractio R ab arbitrio nostro pendet, dummodo ea unitate sit minor; hic autem accipiamus $R=1$, qui valor ad praxin maxime accommodatus videtur.

I. Sive lens obiectiva revera sit simplex sive ex duabus pluribusve lentibus proxime sibi iunctis composita, ea hic ut unica spectetur, ita ut eius loco omnes constructiones in superioribus capitibus datae substitui possint, atque inde dabitur eius aperturae semidiameter $=x$; tum vero eius a secunda lente distantia erit $\tfrac{maq}{3h}-\tfrac{1}{2}q$; quod autem intervallum ob indolem lentis obiectivae aliquantum immutari potest, cuius tamen ratio in praxi non attendi meretur.

II. Pro secunda lente notandum est eam aeque ac sequentes ex quovis vitri genere parari posse, dummodo sint utrinque aequaliter convexae, ut ipsis maxima apertura tribui possit. Sit igitur secundae lentis distantia focalis $=q$ eritque distantia ad lentem tertiam $\frac{4}{9}q$.

III. Pro tertia lente eius distantia focalis capiatur $r=\frac{5}{9}q$ et distantia ad quartam lentem $=\frac{5}{36}q$.

IV. Pro quarta lente eius distantia focalis capiatur $s=\frac{5}{18}q$ et distantia ad oculum $O=\frac{1}{3}s$ proxime.

V. Nunc autem spatii in obiecto conspicui erit semidiameter

$$z=\frac{3ah}{ma+h}\cdot\xi=\frac{3}{4}\cdot\frac{ah}{ma+h}$$

et mensura claritatis eadem manebit ut ante, scilicet $=20\cdot\frac{hx}{ma}$, dum nempe mensurae in digitis exprimuntur.

COROLLARIUM

211. Si ergo nolimus, ut lens ocularis minor fiat quam $\frac{1}{3}$ dig., posito $s=\frac{1}{3}$ dig. sumi debebit $q=\frac{6}{5}$ dig. hincque intervallum primum

$$=\frac{2ma}{5h}-\frac{3}{5}\text{ dig.;}$$

at si in superioribus lens ocularis etiam statuatur $=\frac{1}{3}$ dig., penultima fit 1 dig. et idem intervallum prodit circiter; unde patet praesenti casu longitudinem instrumenti notabiliter fore minorem.

PROBLEMA 2

212. *Cuiuscunque indolis fuerit lens obiectiva, post imaginem realem tres adhuc lentes ita disponere, ut margine colorato evanescente campus evadat maximus.*

SOLUTIO

Cum hic habeantur quatuor intervalla, litterarum *P, Q, R, S* secunda iterum erit negativa sitque ergo $Q=-k$, ut fiat $PkRS=\frac{ma}{h}$. Distantiae ergo focales erunt

$$p = \mathfrak{A}a,\quad q = -\tfrac{A\mathfrak{B}}{P}\cdot a,\quad r = -\tfrac{AB\mathfrak{C}}{Pk},\quad s = \tfrac{ABC\mathfrak{D}a}{PkR},\quad t = -ABCD\cdot\tfrac{h}{m} = -\tfrac{ABCDa}{PkRS};$$

unde, si loco A littera q in calculum introducatur, ob $A = -\frac{Pq}{\mathfrak{B}a}$ erit

$$r = \tfrac{B\mathfrak{C}q}{\mathfrak{B}k},\quad s = -\tfrac{BC\mathfrak{D}q}{\mathfrak{B}kR},\quad t = \tfrac{BCDq}{\mathfrak{B}kRS}.$$

Simili modo intervalla lentium per q ita reperientur expressa:

$$\text{primum} = -\tfrac{Pq}{\mathfrak{B}} + \tfrac{1}{\mathfrak{B}}\cdot q,\quad \text{secundum} = \tfrac{B}{\mathfrak{B}}(1+\tfrac{1}{k})q,$$

$$\text{tertium} = \tfrac{BC}{\mathfrak{B}k}(1-\tfrac{1}{R})q,\quad \text{quartum} = -\tfrac{BCD}{\mathfrak{B}kR}(1-\tfrac{1}{S})q.$$

Iam ut campus apparens prodeat maximus, statuantur litterae $\mathfrak{q} = \mathfrak{r} = \mathfrak{s} = \mathfrak{t} = 1$, ut fiat $M = \frac{4h}{ma+h}$, campi semidiametro existante

$$z = Ma\xi = \tfrac{4ah}{ma+h}\cdot\xi = \tfrac{ah}{ma+h}$$

sumto $\xi = \frac{1}{4}$; qui ergo campus quasi fit quadruplicatus, dum in problemate praecedente erat triplicatus, antea vero tantum duplicatus. Hinc ergo aequationes fundamentales dabunt

$$\mathfrak{B} = (1-P)M,\quad \mathfrak{C}\left(1+Pk\right)M - 1,\quad \mathfrak{D} = (1+PkR)M - 2.$$

Cum autem sufficiat his formulis proxima satisfecisse, quia parum interest, etiamsi campus aliquantum fiat minor, spectemus multiplicationem m cum numero P quasi infinitam ac tum istae litterae concinnius ita exprimentur ob $M = \frac{4h}{ma}$:

$$\mathfrak{B} = -\tfrac{4hP}{ma}\ \text{ adeoque }\ \tfrac{\mathfrak{B}}{P} = -\tfrac{4h}{ma},$$
$$\mathfrak{C} = \tfrac{4hPk}{ma} - 1,\qquad \mathfrak{D} = \tfrac{4hPkR}{ma} - 2;$$

et cum sit $P = \frac{ma}{hkRS}$, hae expressiones etiam commodius ita exprimentur:

$$\mathfrak{B} = -\tfrac{4}{kRS},\quad \mathfrak{C} = \tfrac{4}{RS} - 1,\quad \mathfrak{D} = \tfrac{4}{S} - 2.$$

At ob conditionem, qua margo coloratus destrui debet, habebimus istam aequationem:

$$0=\frac{1}{P}-\frac{1}{Pk}-\frac{1}{PkR}-\frac{1}{PkRS},$$

ex qua nascitur

$$k=1+\frac{1}{R}+\frac{1}{RS},$$

ita ut litterae R et S arbitrio nostro permittantur. Cum autem bina ultima intervalla fiant certe satis exigua, litterae R et S parum ab unitate discrepare possunt; unde litterae $\mathfrak{C}$ et $\mathfrak{D}$ manifesto fient unitate maiores hincque C et D negativae, dum e contrario littera $\mathfrak{B}$ ipsa ac propterea etiam B sunt negativae; quare, ut nostra intervalla lentium fiant positiva, evidens est esse debere $S<1$ et $R<1$; qua conditione observata nunc omnia momenta facile determinari poterunt.

COROLLARIUM 1

213. Cum igitur tam R quam S sint fractiones unitate minores, litterae k valor certe ternarium superabit, quoniam

$$\frac{1}{R}>1 \text{ et } \frac{1}{RS}>\frac{1}{R}.$$

COROLLARIUM 2

214. Cum sit

$$\frac{P}{\mathfrak{B}}=-\frac{ma}{4h},$$

erit primum intervallum

$$=\frac{maq}{4h}-\frac{kRS}{4}\cdot q,$$

cuius pars prior $\frac{maq}{4h}$ minor est quam casu praecedentis problematis, ita ut hic longitudo instrumenti adhue minor sit proditura.

COROLLARIUM 3

215. Has ergo quaternas lentes etiam cum omnibus lentibus obiectivis sive simplicibus sive compositis, quas supra descripsimus, combinare licebit; unde hoc insigne commodum assequemur, ut campus apparens prodeat quadruplicatus, cum in praecedentibus tantum esset duplicatus.

EXEMPLUM 1

216. Cum litterae R et S debeant esse unitate minores, consideremus casum quasi simplicissimum et ponamus $R=\frac{1}{2}$ et $S=\frac{2}{3}$, ut fiat $RS=\frac{1}{3}$, hincque

$$k=1+2+3=6;$$

ex his igitur valoribus, qui ad praxin satis accommodati videntur, colliguntur litterae

$$\mathfrak{B}=-2,\ \ \mathfrak{C}=11,\ \ \mathfrak{D}=4,\ B=-\tfrac{2}{3},\ \ C=-\tfrac{11}{10},\ \ D=-\tfrac{4}{3};$$

deinde ex distantia focali q sequentes ita definientur:

$$r=\tfrac{11}{18}q,\ \ s=\tfrac{22}{45}q,\ \ t=\tfrac{11}{45}q=\tfrac{1}{2}s;$$

denique vero intervalla lentium

$$\text{primum}=\tfrac{maq}{4h}-\tfrac{1}{2}q,\ \ \text{secundum}=\tfrac{17}{18}q,\ \ \text{tertium}=\tfrac{11}{180}q=\tfrac{1}{4}t,\ \ \text{quartum}=\tfrac{11}{135}q.$$

EXEMPLUM 2

217. Statuamus nunc tam $R=1$ quam $S=\frac{1}{2}$ ac prodibit

$$k=1+2+4=7$$

eritque

$$\mathfrak{B}=-\tfrac{16}{7},\ \ \mathfrak{C}=15,\ \ \mathfrak{D}=6,\ \ B=-\tfrac{16}{23},\ \ C=-\tfrac{15}{14},\ \ D=-\tfrac{6}{5}$$

et distantiae focales ita per q exprimentur:

$$r=\tfrac{15}{23}q,\ \ s=\tfrac{90}{161}q,\ \ t=\tfrac{36}{161}q$$

et intervalla

$$\text{primum}=\tfrac{maq}{4h}-\tfrac{7}{16}q,\ \ \text{secundum}=\tfrac{8}{23}q,\ \ \text{tertium}=\tfrac{15}{322}q,\ \ \text{quartum}=\tfrac{18}{161}q.$$

Quod tandem ad locum oculi attinet, hic in genere erit

$$O=\tfrac{1}{4}t\left(1+\tfrac{h}{ma}\right)=\tfrac{1}{4}t\ \text{proxime.}$$

PROBLEMA 3

218. *Cuiuscunque indolis fuerit lens obiectiva, post imaginem realem quotcunque adhuc lentes, quarum numerus sit* $=i$, *ita disponere, ut evanescente margine colorato campus maximus evadat.*

SOLUTIO

Si operatio instituatur ut in problematibus antecedentibus, erit semper $Q=-k$ litterarumque sequentium *R, S, T* etc. numerus erit $i-1$ sitque ultima = *Z;* tum vero pro campo hic habebitur

$$M=\frac{(i+1)h}{ma+k}$$

ideoque

$$z=\frac{(i+1)ah}{ma+h}\cdot\xi.$$

Quodsi deinde etiam ut ante pro determinatione litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ multiplicationem *m* cum numero *P* ut infinite magnam consideremus, reperiemus

$$\mathfrak{B}=-\frac{(i+1)hP}{ma},\quad \mathfrak{C}=\frac{(i+1)hP}{ma}-1,$$
$$\mathfrak{D}=\frac{(i+1)hPkR}{ma}-2,\quad \mathfrak{E}=\frac{(i+1)hPkRS}{ma}-3 \text{ etc.}$$

Destructio vero marginis colorati dabit

$$k=1+\frac{1}{R}+\frac{1}{RS}+\frac{1}{RST}+\ldots\frac{1}{RST\,..\,Z},$$

quorum terminorum numerus est *i*.

Nunc vero has litteras ita definiamus, ut fiat

$$\frac{1}{R}=2,\quad \frac{1}{RS}=3,\quad \frac{1}{RST}=4,\quad \frac{1}{RSTU}=5,$$

atque ultimus

$$\frac{1}{RSTU\,..\,Z}=i$$

ideoque

$$R=\frac{1}{2},\quad S=\frac{2}{3},\quad T=\frac{3}{4},\quad U=\frac{4}{5}z\ \cdots \text{ ac tandem } Z=\frac{i-1}{i}.$$

Cum igitur hinc prodeat

$$k=1+2+3\ldots.+i,$$

hoc est

$$k=\frac{i(i+1)}{2},$$

et cum sit $RST\cdots Z=\frac{1}{i}$, erit

$$kRS\cdots Z=\frac{i+1}{2}$$

hincque

$$P=\frac{2ma}{(i+1)h}\ \text{ seu }\ \frac{1}{P}=\frac{(i+1)h}{2ma},$$

et hinc porro

$$\frac{1}{Pk}=\frac{h}{ima},\quad \frac{1}{PkR}=\frac{2h}{ima},\quad \frac{1}{PkRS}=\frac{3h}{ima},\quad \frac{1}{PkRST}=\frac{4h}{ima} \text{ etc.,}$$

donec perveniatur ad

$$\frac{1}{PkRST\ ..Z}=\frac{h}{ma}.$$

Iam ex his formulis litterae nostrae germanicae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc. reperiuntur

$$\mathfrak{B}=-2,\quad \mathfrak{C}=i^2+i-1,\quad \mathfrak{D}=\tfrac{i^2+i-4}{2},\quad \mathfrak{E}=\tfrac{i^2+i-9}{2},\quad \mathfrak{F}=\tfrac{i^2+i-16}{2}\ \text{etc.}$$

donec ultimus $\mathfrak{Z}$ fiat $=1$ hincque

$$B=-\tfrac{2}{3},\quad C=\tfrac{i^2+i-1}{2-i-i^2},\quad D=\tfrac{i^2+i-4}{6-i-i^2},\quad E=\tfrac{i^2+i-9}{12-i-i^2},\quad F=\tfrac{i^2+i-16}{20-i-i^2}\ \text{etc.}$$

Ex his igitur valoribus poterimus distantias focales omnium lentium post secundam per huius ipsius distantiam focalem q definira, quod facile praestabitur sequenti modo:

$$\frac{r}{q}=\frac{B\mathfrak{C}}{\mathfrak{B}k}=\frac{2}{3}\cdot\frac{i^2+i-1}{i^2+i},\qquad \text{ergo}\quad r=\frac{2}{3}\cdot\frac{i^2+i-1}{i^2+i}\cdot q,$$

$$\frac{s}{r}=-\frac{C\mathfrak{D}}{\mathfrak{C}R}=\frac{i^2+i-4}{i^2+i-2},\qquad \text{ergo}\quad s=\frac{i^2+i-4}{i^2+i-2}\cdot r,$$

$$\frac{t}{s}=-\frac{D\mathfrak{E}}{\mathfrak{D}S}=\frac{i^2+i-9}{i^2+i-6},\qquad \text{ergo}\quad t=\frac{i^2+i-9}{i^2+i-6}\cdot s,$$

$$\frac{u}{t}=-\frac{E\mathfrak{F}}{\mathfrak{E}T}=\frac{i^2+i-16}{i^2+i-12},\qquad \text{ergo}\quad t=\frac{i^2+i-16}{i^2+i-12}\cdot t$$

sicque ulterius

$$v=\frac{i^2+i-25}{i^2+i-20}\cdot u,\qquad w=\frac{i^2+i-36}{i^2+i-30}\cdot v\qquad \text{etc.}$$

Lentium intervalla denique ita determinabuntur:

$$\text{primum}=-\tfrac{1}{\mathfrak{B}}(P-1)q=\tfrac{maq}{(i+1)h}-\tfrac{1}{2}q,$$

$$\text{secundum}=\tfrac{1}{2}\cdot\tfrac{i^2+i+2}{i^2+i-1}\cdot r,$$

$$\text{tertium}=-\tfrac{s}{\mathfrak{D}}(R-1)=\tfrac{1}{i^2+i-4}\cdot s,$$

$$\text{quartum}=-\tfrac{t}{\mathfrak{E}}(S-1)=\tfrac{1}{i^2+i-9}\cdot t,$$

$$\text{quintum}=-\tfrac{u}{\mathfrak{F}}(T-1)=\tfrac{1}{i^2+i-16}\cdot u,$$

$$\text{sextum}=-\tfrac{v}{\mathfrak{G}}(U-1)=\tfrac{1}{i^2+i-25}\cdot v,\quad \text{etc.}$$

COROLLARIUM 1

219. Si igitur sit $i=1$, ut lens unica post imaginem realem reperiatur, erit $r=\frac{1}{3}q$ et intervalla

$$\text{primum} = \frac{maq}{2h} - \tfrac{1}{2}q, \quad \text{secundum} = 2r$$

et

$$O = \tfrac{1}{2}r(1 + \tfrac{h}{ma}).$$

COROLLARIUM 2

220. Si $i = 2$, ut sint duae lentes post imaginem realem, earum distantiae focales erunt

$$r = \tfrac{5}{9}q \text{ et } s = \tfrac{1}{2}r = \tfrac{5}{18}q,$$

tum vero intervalla

$$\text{primum} = \frac{maq}{3h} - \tfrac{1}{2}q, \quad \text{secundum} = \tfrac{4}{5}r = \tfrac{4}{9}q, \ \text{tertium} = \tfrac{1}{2}s = \tfrac{5}{36}q$$

et

$$O = \tfrac{1}{3}s\left(1 + \tfrac{h}{ma}\right).$$

COROLLARIUM 3

221. Si sit $i = 3$, ut tres lentes post imaginem realem reperiantur, erit

$$r = \tfrac{11}{18}q, \quad s = \tfrac{4}{5}r = \tfrac{22}{45}q, \quad t = \tfrac{1}{2}s = \tfrac{11}{45}q;$$

tum vero intervalla

$$\text{primum} = \frac{maq}{4h} - \tfrac{1}{2}q, \quad \text{secundum} = \tfrac{7}{11}r = \tfrac{7}{18}q,$$
$$\text{tertium} = \tfrac{1}{8}s = \tfrac{11}{180}q, \quad \text{quartum} = \tfrac{1}{3}t = \tfrac{11}{135}q.$$

Pro loco denique oculi

$$O = \tfrac{1}{4}t\left(1 + \tfrac{h}{ma}\right).$$

COROLLARIUM 4

222. Si sit $i = 4$, ut quatuor lentes post imaginem realem reperiantur, earum distantiae focales erunt

$$r = \tfrac{19}{30}q, \quad s = \tfrac{8}{9}r = \tfrac{76}{135}q, \quad t = \tfrac{11}{14}s = \tfrac{418}{945}q, \quad u = \tfrac{1}{2}t = \tfrac{209}{945}q;$$

tum vero intervalla erunt

$$\text{primum} = \frac{maq}{5h} - \frac{1}{2}q, \quad \text{secundum} = \frac{11}{19}r = \frac{11}{30}q, \quad \text{tertium} = \frac{1}{16}s = \frac{19}{540}q,$$
$$\text{quartum} = \frac{1}{11}t = \frac{38}{945}q, \quad \text{quintum} = \frac{1}{4}u = \frac{209}{3780}q$$

et pro loco oculi

$$O = \frac{1}{5}t\left(1 + \frac{h}{ma}\right).$$

COROLLARIUM 5

223. Si sit $i = 5$, ut quinque lentes post imaginem realem disponantur, erit

$$r = \frac{29}{45}q, \quad s = \frac{13}{14}r = \frac{13\cdot29}{14\cdot45}q, \quad t = \frac{7}{8}s = \frac{7\cdot13\cdot29}{8\cdot14\cdot45}q,$$
$$u = \frac{7}{9}t = \frac{7\cdot13\cdot29}{2\cdot8\cdot9\cdot45}q, \quad v = \frac{1}{2}u = \frac{7\cdot13\cdot29}{4\cdot8\cdot9\cdot45}q;$$

tum vero intervalla erunt

$$\text{primum} = \frac{maq}{6h} - \frac{1}{2}q, \quad \text{secundum} = \frac{26}{29}r = \frac{16}{45}q, \quad \text{tertium} = \frac{1}{26}s = \frac{29}{2\cdot14\cdot45}q,$$
$$\text{quartum} = \frac{1}{21}t = \frac{13\cdot19}{3\cdot8\cdot14\cdot45}q, \quad \text{quintum} = \frac{1}{14}u = \frac{13\cdot29}{4\cdot8\cdot9\cdot45}q, \quad \text{sextum} = \frac{1}{5}v = \frac{7\cdot13\cdot29}{4\cdot5\cdot8\cdot9\cdot45}q$$

et pro loco oculi

$$O = \frac{1}{6}t\left(1 + \frac{h}{ma}\right).$$