SECTION FOUR.

CONCERNING COMPOSITE MICROSCOPES

IN WHICH TWO REAL IMAGES OCCUR.

CHAPTER I

REGARDING SIMPLER MICROSCOPES OF THIS KIND

FOREWORD

Since the microscopes related to this section will represent the object situated erect again, the letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ etc. together with the magnification *m* will retain the same signs, as have been used in the general precepts.

PROBLEM 1

224. *To compose a microscope of this kind from three lenses and to investigate its qualities and deficiencies.*

SOLUTION

Since here only three lenses are present and thus two intervals, in each of which a real image is present, both the letters *P* and *Q* required to be put in place are negative; on account of which we may put $P=-k$ and $Q=-k'$, so that there shall become $kk'=\frac{ma}{h}$; truly the focal lengths of the lenses will be

$$p=\mathfrak{A}a,\quad q=\frac{A\mathfrak{B}a}{k},\quad r=\frac{ABa}{kk'}=AB\cdot\frac{h}{m},$$

truly the intervals of the lenses:

$$\text{first}=Aa\left(1+\frac{1}{k}\right),\qquad \text{second}=\frac{ABa}{k}\left(1+\frac{1}{k'}\right),$$

thus so that the first real image shall be removed from the first lens by the interval $=Aa$ and from the second lens by the interval $=\frac{Aa}{k}$; truly the latter real lies after the second lens by the interval $=\frac{Aa}{k}$ and before the third by the interval $=\frac{ABa}{kk'}$, and if the radius of the area in the object viewed shall be $=z$, the radius of the first image will be $=Az$, which is inverted, of the latter truly $=ABz$, which is erect again. Hence therefore it is apparent there must become $A>0$ and $B>0$, from which also there will become $\mathfrak{A}>0$ and $\mathfrak{B}>0$, thus however, so that there shall become $\mathfrak{A}<1$ and $\mathfrak{B}<1$. Then truly there will be

$$z = \frac{\mathfrak{q}+\mathfrak{r}}{ma-h} \cdot ah\xi \quad \text{and} \quad M = \frac{\mathfrak{q}+\mathfrak{r}}{ma-h} \cdot h,$$

so that there shall become $z = Ma\xi$, from which we find

$$\mathfrak{B}\mathfrak{q} = -(1+k)M;$$

from which it is evident, since $\mathfrak{B}$ shall be positive, $\mathfrak{q}$ to become negative and from that therefore the apparent field to be diminished; whereby, lest this finally may be reduced to zero, the maximum value will be attributed to the letter $\mathfrak{r}$, which is unity, and on putting $\mathfrak{q} = -\omega$ there must be $\omega < 1$, since there shall be

$$M = \frac{1-\omega}{ma-h} \cdot h;$$

thence on account of

$$\mathfrak{B} = \frac{1+k}{\omega} \cdot M = \frac{1+k}{\omega} \cdot \frac{1-\omega}{ma-h} \cdot h,$$

since $\mathfrak{B} < 1$, there will become

$$(1+k)(1-\omega)h < \omega(ma-h),$$

which condition will be implemented easily, if there were

$$\omega > \frac{(1+k)h}{ma+hk};$$

and since in addition there is $\omega < 1$, according to this it is required that there shall be $ma > h$, which condition will be found at once for the greater magnifications. But if we may wish to assume $\omega = \frac{(1+k)h}{ma+hk}$, $\mathfrak{B} = 1$ will be produced and hence $B = \infty$ and the instrument will become infinitely long; from which it is evident by necessity to be required to take $\omega > \frac{(1+k)h}{ma+hk}$.

Now also we may consider, whether the colored margin may be able to be destroyed ; which finally must be examined before the place of the eye must be determined by this equation [§ 18]:

$$O = \frac{r}{Ma} \cdot \frac{h}{m} \text{ on account of } \mathfrak{r} = 1.$$

Therefore since *r* is positive, certainly there will be $O > 0$, from which for the destruction of the margin this equation will be obtained :

$$0 = \frac{\omega}{k} + \frac{1}{kk'};$$

which since it cannot happen, it is evident microscopes of this kind must continue to be troubled by this significant fault, thus so that it would be superfluous to inquire about this in the remaining precepts of the construction.

COROLLARY 1

225. Since fewer than three lenses shall not be able to be used on account of the two real images, the construction contained in the problem certainly is the most simple, which may be able to be had ; whereby, since we may consider to reject that, it will be require at least four lenses to be used.

COROLLARY 2

226. Since the formula for the destruction of the colored margin depends on two positive parts, thus the confusion will be much greater than in telescopes and microscopes made from two lenses only, and thus will be much less able to be tolerated.

SCHOLIUM

227. Therefore with the three lenses proposed here it shall be required to add one in addition to the minimum, that will be done in a threefold manner; indeed in the first place this new lens between the objective lens and the first real image, secondly in addition between the first and the second real image, thus so that in this interval two lenses may be put in place, thirdly indeed it will be able to lie between the second real image and the eyepiece lens. Truly this third case will suffer from the same fault, because this it to be avoided; for the letters P and Q will retain the same values $-k$ and $-k'$, certainly from which only the third letter B is added, and thus the letter $\mathfrak{q}$ also will retain a negative value, which shall be $\mathfrak{q} = -\omega$, from which this equation will be had for the colored margin removal:

$$0 = \frac{\omega}{k} + \frac{r}{kk'} + \frac{\mathfrak{s}}{kk'R},$$

which by no means can be sustained, unless either $\mathfrak{r}$ or $\mathfrak{s}$ may be taken negative, but which, since now $\mathfrak{q}$ may have a negative value, by no means can be expedited, since otherwise the field of view will be returned exceedingly narrow, on account of which only the two first cases will remain to be established by us.

PROBLEM 2

228. *Thus, to compose microscopes of this kind from four lenses, so that the second lens shall lie before the first real image, the third lens truly between both images and thus alone past the second image of the eyepiece, by which that may be mainly effected, so that the colored margin may vanish.*

SOLUTION

Therefore here three intervals will be had and just as many letters *P*, *Q* and *R*, of which the latter two must be negative. And thus we may put $Q=-k$ and $R=-k'$, so that there shall become $Pkk'=\frac{ma}{h}$; again the focal lengths of these lenses will be

$$p=\mathfrak{A}a,\quad q=\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{Pk}\cdot a \quad\text{and}\quad s=-\frac{ABC}{Pkk'}\cdot a=-ABC\cdot\frac{h}{m},$$

then truly the intervals of the lenses will be :

$$\text{first}=Aa\left(1-\frac{1}{P}\right),\ \text{second}=-\frac{ABa}{P}\left(1+\frac{1}{k}\right),\quad \text{third}=-\frac{ABCa}{Pk}\left(1+\frac{1}{k'}\right),$$

from which it must be apparent $-AB>0$ and $C>0$. Then it may be observed the first image to lie past the second lens by the interval $=-\frac{ABa}{P}$ and before the third by the interval $=-\frac{ABa}{Pk}$, truly the latter image to lie after the third lens by the interval $=-\frac{ABCa}{Pk}$ and before the eyepiece by the interval $=-\frac{ABCa}{Pkk'}$, truly besides these the radius of the first inverted image to be $=ABz$, truly the radius of the latter erect image $=ABCz$, with there being

$$z=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma-h}\cdot ah\xi$$

and hence,

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma-h}\cdot h$$

thus so that there shall be $z=Ma\xi$, which quantity by hypothesis must be positive; from this value moreover, the following formulas are deduced:

$$\mathfrak{B}\mathfrak{q}=(P-1)M,\quad \mathfrak{C}\mathfrak{r}=-(Pk+1)M-\mathfrak{q}.$$

On account of the condition $C>0$ in the manner advanced moreover, there must become $\mathfrak{C}>0$ and $\mathfrak{C}<1$, from which it is evident either $\mathfrak{q}$ or $\mathfrak{r}$ must be negative. Therefore so that whichever may have a place, $\mathfrak{s}$ will be agreed to be taken positive and thus put $\mathfrak{s}=1$, so that there shall be

$$M = \frac{1+\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot h.$$

Hence the distance of the eye past the eyepiece lens will produce

$$O = \frac{\mathfrak{s}s}{M}\cdot\frac{h}{ma};$$

therefore since $s>0$, this distance will become positive and thus the colored margin will be destroyed with the aid of this equation:

$$0 = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} + \frac{1}{Pkk'};$$

which by no means shall be able to stand, if there were $\mathfrak{r}<0$, from which it is necessary, that there shall be $\mathfrak{q}<0$. There may be put in place $\mathfrak{q}=-\omega$ and there will become

$$\frac{1}{k'} = k\omega+\mathfrak{r}$$

and now we know there must be

$$\mathfrak{B} = \frac{1-P}{\omega}\cdot M \quad \text{and} \quad \mathfrak{C} = \frac{-(Pk+1)M+\omega}{\mathfrak{r}};$$

since which value must be positive, there will become

$$(Pk+1)M<\omega$$

and hence

$$\omega > \frac{(Pk+1)(1+\mathfrak{r})h}{ma+Pkh}.$$

But since there shall be

$$\frac{ma}{h} = Pkk' = \frac{Pk}{k\omega+\mathfrak{r}},$$

this equation will arise :

$$Pk^2\omega^2 + \omega(1+\mathfrak{r})(P-Pk-1)k - (Pk+1)(1+\mathfrak{r})\mathfrak{r} > 0,$$

which equation contains the condition, following which the letter ω must be defined. But with the letters ω and $\mathfrak{r}$ suitably defined and thence with the values $\mathfrak{B}$ and $\mathfrak{r}$ deduced, at least as an approximation, the remaining elements will become known; then truly nothing other remains, except that the aperture of the objective lens will be defined from the radius of the confusion.

COROLLARY 1

229. For the sake of brevity we may put

$$\frac{ma}{h} = \mathfrak{M},$$

so that there shall become

$$M = \frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1},$$

and we will have

$$Pk = \mathfrak{M}(k\omega+\mathfrak{r}),$$

and

$$\mathfrak{B}\omega = (1-P)\left(\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right),$$

and

$$(Pk+1)\left(\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right) = \frac{(\mathfrak{M}(k\omega+\mathfrak{r})+1)(1-\omega+\mathfrak{r})}{\mathfrak{M}-1} = \omega - \mathfrak{C}\mathfrak{r}\ ;$$

and from which there is apparent to become $w > \mathfrak{C}\mathfrak{r}$, when it is agreed to become

$$\mathfrak{C} > 0 \text{ and } \mathfrak{C} < 1.$$

COROLLARY 2

230. Therefore since notably ω ought to be greater than $\mathfrak{C}\mathfrak{r}$, we may see, whether it may be possible for $\omega = \mathfrak{r}$; to this end we may put $\omega = \mathfrak{r}$ and the final equation will become

$$\mathfrak{r}(1-\mathfrak{C}) = \frac{(\mathfrak{M}(1+k)+1)}{\mathfrak{M}-1};$$

from which it is concluded

$$\mathfrak{r} = \frac{-1}{\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}},$$

which cannot happen, since there must become $\mathfrak{r} > 0$, and thus also it is certain there must be $\omega > \mathfrak{r}$, thus so that the field may not return to its simple value, indeed $z = \frac{ah\xi}{ma-h}$ may be able to be increased on account of $1-\omega+\mathfrak{r} < 1$.

COROLLARY 3

231. Therefore since there shall be $\omega - \mathfrak{r} > 0$, it will be of the greatest interest to know, how the minimum value of this formula may be agreed on ; which in the end this may happen with the letters ω and $\mathfrak{r}$ viewed as variables, if there shall be $d\omega = d\mathfrak{r}$, according to which rule our equation may be differentiated

$$\left(\omega-\mathfrak{C}\mathfrak{r}\right)\left(\mathfrak{M}-1\right)=\left(\mathfrak{M}\left(k\omega+\mathfrak{r}\right)+1\right)\left(1-\omega+\mathfrak{r}\right)$$

and there will be produced

$$\left(1-\mathfrak{C}\right)\left(\mathfrak{M}-1\right)=\mathfrak{M}\left(1-\omega+\mathfrak{r}\right)\left(k+1\right),$$

from which we deduce

$$1-\omega+\mathfrak{r}=\frac{(1-\mathfrak{C})(\mathfrak{M}-1)}{\mathfrak{M}(k+1)},$$

thus so that there shall become

$$M=\left[\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right]=\frac{(1-\mathfrak{C})}{\mathfrak{M}(k+1)},$$

which formula provides the maxim field, which indeed it will be allowed to obtain. But here the maximum field will be obtained by taking

$$\omega=\mathfrak{r}+\frac{\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}}{\mathfrak{M}(k+1)},$$

which value substituted into our equation will give

$$\mathfrak{r}\left(1-\mathfrak{C}\right)\left(\mathfrak{M}-1\right)+\frac{(\mathfrak{M}-1)^2\mathfrak{C}+(\mathfrak{M}-1)(\mathfrak{M}k+1)}{\mathfrak{M}(k+1)}$$
$$=\frac{(1-\mathfrak{C})(\mathfrak{M}-1)}{(k+1)}\left(\mathfrak{r}\left(k+1\right)+\frac{(\mathfrak{M}-1)k\mathfrak{C}+\mathfrak{M}k^2+k}{\mathfrak{M}(k+1)}\right)$$

where the terms containing the letter $\mathfrak{r}$ cancel each other out; this equation remains :

$$\left(1-\mathfrak{C}\right)(1+\mathfrak{C}k)+\mathfrak{M}(\mathfrak{C}+k)\left(\mathfrak{C}k+1\right)=0,$$

which is reduced to this:

$$\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}=0;$$

which since it shall be impossible, it follows this maximum field certainly cannot be obtained. [There appears to be a mistake in this calculation, but presumably it does not affect the outcome.]

SCHOLIUM 1

232. Truly little remains, whether or not the field we seek obtains that maximum value, also since here the remedies may not be present for enlarging the remaining field as it pleases; whereby some cases remain from this investigation we may establish, which will be seen to be adapted especially in practice, and in the first place it is indeed evident the

letter *P* cannot be assumed exceedingly close to unity, since then the second lens may not be so close to the first, so that both will be able to be considered as a single lens ; from which the case treated by the second problem will result, which we have seen cannot be used in practice. On account of which the number to be taken for *P* will be agreed to be taken large enough; then also, since there shall be $\omega > \mathfrak{r}$ always, from that $\mathfrak{r}$ to be taken as small as possible; also finally, so that for the maximum field we may approach as far as it will be allowed to become, it will be agreed the letters *k* and $\mathfrak{C}$ to be as small as possible.

CASE 1 WHERE $P = \infty$

[232a] . Therefore in this case the first interval shall become $= Aa$ and thus $A > 0$ and $\mathfrak{A} < 1$ and the second lens lies on the first image itself; the focal length of which cannot become $= 0$, unless there shall be $\mathfrak{B} = \infty$, thus so that there shall be

$$\frac{P}{\mathfrak{B}} = -\zeta$$

and hence

$$q = \frac{Aa}{\zeta}.$$

Then since there shall be $r = -\frac{AB\mathfrak{C}a}{Pk}$, on account of $\mathfrak{B} = \infty$ there becomes $B = -1$ and on account of $\mathfrak{C} < 1$ it is evident there must be $k = 0$, in order that *Pk* shall become a finite quantity ; but since $k = 0$, there will be $\frac{1}{k'} = \mathfrak{r}$ and hence $Pk = \mathfrak{M}\mathfrak{r}$ with there being $\mathfrak{M} = \frac{ma}{h}$; from which there shall become

$$r = \frac{A\mathfrak{C}a}{\mathfrak{M}\mathfrak{r}} \quad \text{and} \quad s = AC \cdot \frac{h}{m};$$

from which for great magnifications *C* must be an extremely large number and hence $\mathfrak{C}$ to differ little from unity. Truly the remaining intervals will be

$$\text{second} = \frac{Aa}{\mathfrak{M}\mathfrak{r}} = \frac{r}{\mathfrak{C}}$$

and

$$\text{third} = \frac{ACa}{\mathfrak{M}\mathfrak{r}}(1+\mathfrak{r}) = (1+C)r + s.$$

Truly besides the distance of the eye will be

$$O = \frac{s}{M\mathfrak{M}}.$$

But now since there shall be

$$M = \frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1},$$

we may not substitute the value into the two formulas $\mathfrak{B}\omega$ and $\mathfrak{C}\mathfrak{r}$, but we may retain these in the letter *M* , so that we may be able to define that henceforth more easily; but then on account of $\frac{P}{\mathfrak{B}} = -\zeta$ from earlier we have found

$$\omega = \zeta M,$$

and truly from later

$$\mathfrak{C}\mathfrak{r} = -M - \mathfrak{M}\mathfrak{r}M + \zeta M$$

or

$$\mathfrak{r}\left(\mathfrak{C} + \mathfrak{M}M\right) = \left(\zeta - 1\right)M$$

and hence

$$\mathfrak{r} = \frac{(\zeta-1)M}{\mathfrak{C}+M\mathfrak{M}}$$

hence therefore we deduce

$$\omega - \mathfrak{r} = \frac{(\mathfrak{C}+M\mathfrak{M}-1)\xi M + M}{\mathfrak{C}+M\mathfrak{M}}$$

or

$$\omega - \mathfrak{r} = \frac{\left(M\mathfrak{M}\zeta - (1-\mathfrak{C})\zeta + 1\right)M}{\mathfrak{M}M + \mathfrak{C}};$$

and

$$1 - \omega + \mathfrak{r} = \frac{\mathfrak{M}M + \mathfrak{C} - \mathfrak{M}M^2\zeta + (1-\mathfrak{C})\xi M + M}{\mathfrak{M}M + \mathfrak{C}};$$

which expression must be equal to this $(\mathfrak{M}-1)M$; from which this equation arises:

$$M^2\mathfrak{M}\left(\mathfrak{M} - 1 + \zeta\right) - M\left(1 - \mathfrak{C}\right)\left(\mathfrak{M} - 1 + \zeta\right) - \mathfrak{C} = 0,$$

from which, since there shall be approximately $\mathfrak{C} = 1$, also we may deduce approximately

$$M = \frac{1}{\sqrt{\mathfrak{M}(\mathfrak{M}-1+\zeta)}};$$

truly with more care there will become

$$M = \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \sqrt{\frac{\mathfrak{C}}{\mathfrak{M}(\mathfrak{M}-1+\zeta)}},$$

but actually

$$M = \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}(\mathfrak{M}-1+\zeta)} + \frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)};$$

with which value found likewise the letters ω and $\mathfrak{r}$ will become known , from which all the rest will be determined. Finally, for the aperture of the objective lens required to be determined, since no account of different kinds of glass is proposed, it must be satisfied by this equation (§31):

$$\frac{1}{k^3} = \frac{\mathfrak{M}\mu x^3}{a^3}\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{v}{A\mathfrak{A}} - * + \frac{r\lambda''}{A^4 a} + *\right),$$

where the third term has vanished at once, truly the fifth can be rejected with care on account of the large number C ; from which, if here the latter factor may be put $= \Lambda$, there is found:

$$x=\frac{a}{k}\sqrt[3]{\frac{1}{\mathfrak{M}\mu\Lambda}}$$

COROLLARY 1

233. Since $\mathfrak{M}$ is a very large number, it will be allowed to write $\mathfrak{M}$ in place of the factor $\mathfrak{M}-1+\zeta$, if indeed ζ were not a very large number; moreover no account will be proposed for such a large number to be used for ζ ; indeed it suffices, that there may be taken $\zeta>1$, lest $\mathfrak{r}$ either may vanish or thus may become negative. Then truly there will become

$$M=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}^2}+\frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)}=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\frac{1+\mathfrak{C}}{2\mathfrak{M}}=\frac{1}{\mathfrak{M}},$$

from which there is deduced in turn

$$\omega=\frac{\zeta}{\mathfrak{M}} \text{ and } \mathfrak{r}=\frac{\zeta-1}{\mathfrak{M}(1+\mathfrak{C})}.$$

COROLLARY 2

234. Hence therefore we come upon the following determinations for the construction of this microscope itself:

1. The focal lengths of the lenses will be

$$p=\mathfrak{A}a,\quad q=\frac{Aa}{\zeta},\quad r=\frac{A\mathfrak{C}(1+\mathfrak{C})a}{\zeta-1}, \text{ and } s=AC\cdot\frac{h}{m}=\frac{ACa}{\mathfrak{M}}.$$

2. The intervals of the lenses:

$$\text{first }=Aa,\ \text{second } \frac{r}{\mathfrak{C}}=\frac{A(1+\mathfrak{C})a}{\zeta-1}$$

and

$$\text{third }=(1+C)r+s=\frac{AC(1+\mathfrak{C})a}{\zeta-1}+\frac{ACa}{\mathfrak{M}};$$

then truly the distance of the eye will be $O=s$.

3. For the aperture requiring to be found there will be

$$\Lambda=\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-*+\frac{(1+\mathfrak{C})}{A^3(\zeta-1)}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)+\frac{\lambda'''}{A^3C^3\mathfrak{M}},$$

where the final term clearly can be omitted.

SCHOLIUM

235. Now we are agreed no reason be proposed, why we may wish to accept an exceptionally large enough number for ζ ; yet meanwhile of the third interval, which is

$$\frac{AC(1+\mathfrak{C})a}{\zeta-1}+\frac{ACa}{\mathfrak{M}},$$

will be seen to become exceedingly large, unless ζ may be much greater than unity, since there must be agreed to assume a noteworthy large enough number for *C* and also for *Λ*. Yet meanwhile always it will be better to restrict the length of the instrument as far as the field of view will allow. Indeed even if we may accept a greater ζ , so that the measurements may be produced more convenient in practice, no other inconvenience thence may be a source of concern, unless the field may actually be going to become smaller than we have intended; which defect indeed will be applied as well to a few more lenses to be added on easily . But truly for the most part it is concerned that a noteworthy number *Λ* may be accepted , so that $\mathfrak{A}$ may be reduced close enough to unity, as it is necessary that *Λ* may be returned small enough and hence a greater degree of clarity may be obtained ; which in the end may be seen to suffice, provided there may be put in place $\Lambda=6$; hence indeed there becomes $\mathfrak{A}=\frac{6}{7}$ and thus $\frac{1}{\mathfrak{A}^2}=\frac{343}{216}\lambda$, which value with $\lambda=1$ assumed may not be much greater than $\frac{3}{2}$, which multiplied by $\mu<1$ certainly is reduced below $\frac{3}{2}$; so that now a convenient enough number for the value x may emerge.
Therefore if we may establish $\Lambda=6$, we may see, how much shall be required to be taken for *C*, lest *s* may become exceedingly small also for the assigned magnification $m=960$. And thus since then there becomes $s=\frac{48C}{960}$ in.$=\frac{C}{20}$ in., this distance may not be made smaller than 1 in., provided $C=5$; whereby, if we may put $C=6$, so that there shall become $\mathfrak{C}=\frac{6}{7}$, nothing will need to be concerned about from this part ; then truly the third interval will become $\frac{36\cdot13a}{7(\zeta-1)}$ with the other term, or $\frac{2\cdot36a}{\zeta-1}=\frac{72a}{\zeta-1}$, omitted; if the distance of the object shall be half an inch, this interval will become $\frac{36}{\zeta-1}$ in.; which therefore on taking $\zeta=3$ or $\zeta=4$ will be so small now , that no account arising from that need concern us.

CASE 2 WHERE $\mathfrak{r}=0$.

[235a]. Therefore in this case there will become $\frac{1}{k'}=k\omega$ and thus $P=\mathfrak{M}\omega$, then truly $M=\frac{1-\omega}{\mathfrak{M}-1}$. Hence the equations deduced from the field will be

$$\text{I. } \mathfrak{B}\omega=\frac{(1-\mathfrak{M}\omega)(1-\omega)}{\mathfrak{M}-1},$$

$$\text{II. } \mathfrak{Cr} = -\frac{(\mathfrak{M}k\omega+1)(1-\omega)}{\mathfrak{M}-1} + \omega = 0,$$

Therefore since $P = \mathfrak{M}\omega$; there will be $\omega = \frac{P}{\mathfrak{M}}$ and $1-\omega = \frac{\mathfrak{M}-P}{\mathfrak{M}}$; from which it is evident P must be smaller than $\mathfrak{M}$. Moreover this value substituted into the latter equation will give

$$k = \frac{\mathfrak{M}(P-1)}{P(\mathfrak{M}-P)},$$

from which, since $k > 0$, it is apparent there must be $P > 1$; hence moreover again there follows to become $A > 0$ and hence $\mathfrak{A} < 1$; then truly there is found :

$$\mathfrak{B} = -\frac{(P-1)(\mathfrak{M}-P)}{P(\mathfrak{M}-1)}, \quad B = -\frac{(P-1)(\mathfrak{M}-P)}{\mathfrak{M}(2P-1)-P^2};$$

from which B also will obtain a negative value, just as the nature of the matter demands. Finally there will be

$$k' = \frac{\mathfrak{M}}{Pk} = \frac{\mathfrak{M}-P}{P-1} \text{ and } M = \frac{\mathfrak{M}-P}{\mathfrak{M}(\mathfrak{M}-1)},$$

from which the field is understood; hence therefore it is apparent, where P may be taken smaller, there a greater field is going to be produced, and since P must be greater than unity, always there will be $M < \frac{1}{\mathfrak{M}}$. Therefore we will have from these values found :

The focal lengths

$$p = \mathfrak{A}a, \quad q = \frac{A(P-1)(\mathfrak{M}-P)a}{P^2(\mathfrak{M}-1)},$$

$$r = \frac{A\mathfrak{C}(\mathfrak{M}-P)^2 a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)} \quad \text{and} \quad s = \frac{AC(P-1)(\mathfrak{M}-P)a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}$$

and the intervals of the lenses

$$\text{first} = Aa\left(1-\tfrac{1}{P}\right), \quad \text{second} = \frac{A(\mathfrak{M}-P)a}{\mathfrak{M}P}, \quad \text{third} = \frac{AC(\mathfrak{M}-P)(\mathfrak{M}-1)a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}.$$

Then truly the distance of the eye will be

$$O = \frac{s}{M\mathfrak{M}} = \frac{\mathfrak{M}-1}{\mathfrak{M}-P} \cdot s$$

and finally the radius of the space viewed within the object will be

$$z = \frac{\mathfrak{M}-P}{\mathfrak{M}(\mathfrak{M}-1)} \cdot a\xi.$$

Truly the aperture of the objective lens will be required to be defined from the well known equation, truly the apertures of the following lenses it will be observed to be $\omega = \frac{P}{\mathfrak{M}}$ and $\mathfrak{r} = 0$. From which the radius is deduced of the aperture

$$\text{of the second lens} = \tfrac{1}{P} \cdot x + \tfrac{Pq}{4\mathfrak{M}},$$
$$\text{of the third lens} = \tfrac{x}{Pk} + 0 = \tfrac{\mathfrak{M}-P}{\mathfrak{M}(P-1)} \cdot x,$$
$$\text{of the fourth lens} = \tfrac{x}{\mathfrak{M}} + \tfrac{1}{4} s.$$

COROLLARY 1

236. Since the field of view postulates, that P may be taken small enough, for the greater magnifications it will be allowed to neglect P before $\mathfrak{M}$, from which, if P may not exceed unity by much, the focal lengths may be expressed thus:

$$p = \mathfrak{A}a, \quad q = \tfrac{A(P-1)}{P^2} \cdot a, \quad r = \tfrac{A\mathfrak{C}}{2P-1} \cdot a \quad \text{and} \quad s = \tfrac{AC(P-1)}{\mathfrak{M}(2P-1)} \cdot a;$$

then the intervals of the lenses will be for the

$$\text{first} = Aa\left(1 - \tfrac{1}{P}\right), \quad \text{second} = \tfrac{Aa}{P}, \quad \text{third} = \tfrac{ACa}{2P-1},$$

and the distance of the eye $O = s$.

COROLLARY 2

237. Therefore if we may put $P = 2$, the focal lengths will become

$$p = \mathfrak{A}a, \quad q = \tfrac{1}{4} Aa, \quad r = \tfrac{1}{3} A\mathfrak{C}a \quad \text{et} \quad s = \tfrac{1}{3} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a$$

and the intervals:

$$\text{first} = \tfrac{1}{2} Aa, \quad \text{second} = \tfrac{Aa}{2}, \quad \text{third} = \tfrac{ACa}{3},$$

and for the field of view:

$$z = \tfrac{a\xi}{\mathfrak{M}}.$$

COROLLARY 3

238. If, as above [Book II, § 314] we have made for telescopes, we may put $P = \sqrt{\mathfrak{M}}$ (because what was there m , here for us is $\mathfrak{M}$), the focal lengths will be expressed thus:

$$p = \mathfrak{A}a, \quad q = \frac{A\left(\sqrt{\mathfrak{M}}-1\right)}{\mathfrak{M}+\sqrt{\mathfrak{M}}} \cdot a,$$

$$r = \frac{A\mathfrak{C}\left(\sqrt{\mathfrak{M}}-1\right)}{2\mathfrak{M}} \cdot a, \quad s = \frac{AC\left(\sqrt{\mathfrak{M}}-1\right)}{2\mathfrak{M}} \cdot a$$

and the intervals of the lenses:

$$\text{first} = \frac{Aa\left(\sqrt{\mathfrak{M}}-1\right)}{\sqrt{\mathfrak{M}}}, \quad \text{second} = \frac{Aa\left(\sqrt{\mathfrak{M}}-1\right)}{\mathfrak{M}}, \quad \text{third} = \frac{ACa(\mathfrak{M}-1)}{2\mathfrak{M}\sqrt{\mathfrak{M}}}.$$

But for the apparent field of view there will become :

$$z = \frac{1}{\mathfrak{M}+\sqrt{\mathfrak{M}}} \cdot a\xi$$

and for the position of the eye:

$$O = \frac{\sqrt{\mathfrak{M}}+1}{\sqrt{\mathfrak{M}}} \cdot s = s\left(1+\frac{1}{\sqrt{\mathfrak{M}}}\right).$$

SCHOLIUM

239. The case established in the final corollary agrees extremely well with that, which we have treated above with telescopes, whereas the preceding cases, in which the letters *P* have used smaller values, we have excluded completely and for this reason, since the third interval would have been produced extremely large. Indeed for telescopes there shall be $h = a = \infty$ and it is necessary, that there shall become $\mathfrak{A} = 0 = A$, thus yet, so that there may become $\mathfrak{A}a = Aa = p$ and $\mathfrak{M} = m.$ But then in general the third interval will become

$$= \frac{C(\mathfrak{M}-P)(\mathfrak{M}-1)p}{\mathfrak{M}\left(\mathfrak{M}(2P-1)-P^2\right)},$$

so that, as if P may vanish besides $\mathfrak{M}$, the interval will become

$$= \frac{C}{2P-1} \cdot p;$$

whereby, since *C* must be a very large number, here only this interval of many large parts shall become the focal length *p* and thus the length of the telescope will be produced enormously large; therefore deservedly we have excluded the above case. But now, where it is concerned with microscopes, this account shall cease completely; nor indeed will the length of the instrument thus emerge enormously great on account of the third. If indeed, as we have observed before, also for the greatest magnification there may be taken $A = 6$ and $C = 6,$ then the third interval will become $= \frac{36a}{2P-1},$ and if *a* may be taken to be $\frac{1}{2}$ in., as it is accustomed to be become, here the interval will become $= \frac{18}{2P-1}$ in.; if there shall be only $P = 2$, this is reduced to 6 in., which in practice certainly can be allowed. On account of which in this case, concerning the development of microscopes, it will be agreed to exclude the course pursued in the third corollary and with that excepted, where there was $P=2$, if indeed in this way a greater field of view may be obtained; then why not also, if it were pleasing, $P=3$ can be taken, so that the focal lengths may be produced

$$p=\mathfrak{A}a,\quad q=\tfrac{2}{9}Aa,\quad r=\tfrac{1}{5}A\mathfrak{C}a \quad\text{and}\quad s=\tfrac{2}{5}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a$$

and the intervals of the lenses:

$$\text{first}=\tfrac{2}{3}Aa,\quad \text{second}=\tfrac{1}{3}Aa,\quad \text{third}=\tfrac{1}{5}ACa$$

with $O=s$ remaining approximately, and

$$z=\tfrac{\mathfrak{M}-3}{\mathfrak{M}(\mathfrak{M}-1)}\cdot a\xi.$$

But now lest *s* may become exceedingly small for great magnifications, the value of the letter *C* certainly will have to be given a greater value, thus so that now no account may be proposed, why we may wish to attribute the value 3 of the letter P rather than 2, since on putting $P=3$, the third interval is scarcely diminished.

SCHOLIUM 2

240. With the more careful examination of these two cases we will be able to establish the general solution in a similar manner; indeed, for the sake of brevity, by putting

$$\tfrac{P-1}{\mathfrak{B}}=-\zeta \quad\text{or}\quad \mathfrak{B}=-\tfrac{(P-1)}{\zeta}$$

we will have at once $\omega=\zeta M$; then since there shall be $Pk=\mathfrak{M}(k\omega+\mathfrak{r})$, there will become $Pk=\zeta\mathfrak{M}Mk+\mathfrak{M}\mathfrak{r}$, which value which substituted into the other equation, which is

$$\mathfrak{C}\mathfrak{r}=\zeta M-PkM,$$

gives

$$\mathfrak{C}\mathfrak{r}=\zeta M-M-\zeta\mathfrak{M}M^2k-\mathfrak{M}M\mathfrak{r},$$

from which there is found

$$\mathfrak{r}=\tfrac{(\zeta-1)M-\zeta\mathfrak{M}M^2k}{\mathfrak{M}M+\mathfrak{C}}$$

and hence

$$1-\omega+\mathfrak{r}=\tfrac{-\zeta\mathfrak{M}(k+1)M^2+(\mathfrak{M}-1+\zeta(1-\mathfrak{C}))M+\mathfrak{C}}{\mathfrak{M}M+\mathfrak{C}}$$

Therefore since there shall be

$$M=\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}$$

there will be

$$1-\omega+\mathfrak{r}=M\left(\mathfrak{M}-1\right),$$

from which the following equation becomes available :

$$\mathfrak{M}\left(\mathfrak{M}-1+\zeta+\zeta k\right)M^{2}-\left(1-\mathfrak{C}\right)\left(\mathfrak{M}-1+\zeta\right)M-\mathfrak{C}=0;$$

from which, unless the numbers ζ and k were large enough, so that these with care may be ignored before $\mathfrak{M}$, there follows at least the approximation

$$\mathfrak{M}^{2}-\mathfrak{M}M\left(1-\mathfrak{C}\right)-\mathfrak{C}=0,$$

which is resolved into these factors:

$$\left(\mathfrak{M}M-1\right)\left(\mathfrak{M}M+\mathfrak{C}\right)=0,$$

from which clearly there is deduced

$$M=\frac{1}{\mathfrak{M}};$$

from which value, even if it may be only almost true, we will be able to use, since it returns too small a part, as to whether the field shall be somewhat greater or less, than the calculation indicates. But this condition will be observe properly, that ζ as well as k shall be small enough numbers, at least much smaller than $\mathfrak{M}$. For if ζk were so large a number, so that it would not be allowed to reject that before $\mathfrak{M}$, then a much smaller value for the letter M may be found than $\frac{1}{\mathfrak{M}}$ and thus the field may undergo a significant decrease, by which cause only it suffices, that greater values for the letters ζ and k may be excluded completely, and this rule is established, so that at no time may values be given to the letters ζ and k , which will be greater than two each, or so that ζk at least will not exceed four. Therefore since there shall become $Pkk'=\mathfrak{M}$ and k a number not differing much from unity, it is evident either P or k' or indeed each must be a large enough number. Therefore from these observed, thus so that there shall be $M=\frac{1}{\mathfrak{M}}$, we will have

$$\omega=\frac{\zeta}{\mathfrak{M}} \text{ and } \mathfrak{r}=\frac{\zeta-\zeta k-1}{\mathfrak{M}(1+\mathfrak{C})};$$

with which values substituted there becomes

$$Pk=k\zeta+\frac{\zeta-\zeta k-1}{1+\mathfrak{C}}$$

and hence

$$P=\zeta+\tfrac{\zeta-\zeta k-1}{(1+\mathfrak{C})k}=\tfrac{\zeta\mathfrak{C}k+\zeta-1}{(1+\mathfrak{C})k}$$

therefore with this value of P observed, the focal lengths will be

$$p=\mathfrak{A}a,\quad q=\tfrac{A(P-1)a}{P\zeta},\quad r=\tfrac{A(P-1)\mathfrak{C}a}{(\zeta+P-1)Pk},\quad s=\tfrac{A(P-1)Ca}{(\zeta+P-1)\mathfrak{M}}$$

and the separation of the lenses :

$$\text{first}=Aa\left(1-\tfrac{1}{p}\right),\quad \text{second}=\tfrac{A(P-1)a}{(\zeta+P-1)Pk}(k+1)$$

and

$$\text{third}\quad=\tfrac{A(P-1)Ca}{(\zeta+P-1)}\left(\tfrac{1}{Pk}+\tfrac{1}{\mathfrak{M}}\right)$$

and the distance of the eye $O=s$, and finally

$$z=\tfrac{1}{4}\cdot\tfrac{a}{\mathfrak{M}}.$$

Therefore with the distance of the object given $=a$ and with the magnification $=m$ or $\mathfrak{M}=\frac{ma}{h}$ the following quantities are left to our choice :

1. $\mathfrak{A}$, which it is agreed to assume to be not much less than unity; indeed the clarity condition requires this.

2. The number ζ which must be positive and of such a size, so that $\zeta+P-1$ shall become a positive number.

3. The letter *C*, but which it is agreed to be defined thus, so that the focal length may not become exceedingly small; but if this letter may be exceedingly large, it is evident the letter $\mathfrak{C}$ is going to be approaching close to unity.

4. Finally the letter *k*, which, as we have seen, is agreed to be taken exceedingly small.

But from the account of the value *P* it will be required to observe always there must be

$$\zeta+P-1>0,$$

then, for this condition requiring to be fulfilled at this point, there will become :

$$\zeta\mathfrak{C}\mathfrak{k}+P-1>0,$$

which is almost the same quantity, that we have ignored above before $\mathfrak{M}$; from which it is required to beware, lest that may exceed unity by some amount.

PROBLEM 3

241. *If a new lens may be put in place between the first and second images, to define all the moments , so that the colored margin may vanish and likewise the maximum field may be obtained.*

SOLUTION

Since here again four lenses are had and two of these may fall between the first and second images, here the first and third of the letters P, Q, R will be negative; therefore there may be put in place $P = -k$ and $R = -k'$; from which the focal lengths will be

$$p = \mathfrak{A}a, \quad q = \tfrac{A\mathfrak{B}}{k}\cdot a, \quad r = \tfrac{AB\mathfrak{C}}{kQ}\cdot a, \quad \text{and} \quad s = -\tfrac{ABC}{kQk'}\cdot a = -\tfrac{ABC}{\mathfrak{M}}\cdot a$$

on account of

$$kQk' = \mathfrak{M} = \tfrac{ma}{h}.$$

Truly, the intervals of the lenses will become

$$\text{first} = Aa\left(1+\tfrac{1}{k}\right), \quad \text{second} = \tfrac{ABa}{k}\left(1-\tfrac{1}{Q}\right), \quad \text{third} = -\tfrac{ABCa}{kQ}\left(1+\tfrac{1}{k'}\right)$$

and hence it follows that

$$A > 0, \quad B\left(1-\tfrac{1}{Q}\right) > 0, \quad BC < 0.$$

Again there will be

$$M = \tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{\mathfrak{M}-1},$$

so that there may become

$$z = Ma\xi.$$

And hence the distance of the eye

$$O = \tfrac{\mathfrak{s}s}{M\mathfrak{M}},$$

where, so that the maximum field may be returned, it will be agreed to propose $\mathfrak{s} = 1$, clearly if the eyepiece lens shall be prepared equal on both sides. Moreover then there will become

$$\mathfrak{B}q = -(1+k)M \quad \text{and} \quad \mathfrak{C}\mathfrak{r} = -(1+Qk)M - \mathfrak{q}.$$

If here as before for the sake of brevity there may be written

$$\tfrac{1+k}{\mathfrak{B}} = \zeta$$

so that there shall become

$$\mathfrak{q} = -\zeta M$$

then there will become

$$\mathfrak{C}\mathfrak{r} = -(1+Qk)M + \zeta M \quad \text{and} \quad \mathfrak{r} = \frac{(\zeta - Qk - 1)M}{\mathfrak{C}}$$

and hence

$$\mathfrak{q}+\mathfrak{r} = \frac{\left(\zeta(1-\mathfrak{C}) - Qk - 1\right)M}{\mathfrak{C}}$$

and

$$\mathfrak{q}+\mathfrak{r}+1 = \frac{\mathfrak{C} + \left(\zeta(1-\mathfrak{C}) - Qk - 1\right)M}{\mathfrak{C}}$$

Thence truly there becomes

$$\mathfrak{q}+\mathfrak{r}+1 = M\left(\mathfrak{M}-1\right),$$

from which there follows:

$$M = \frac{\mathfrak{C}}{(\mathfrak{M}-1)\mathfrak{C} - \zeta(1-\mathfrak{C}) + Qk + 1};$$

where therefore this quantity

$$\mathfrak{M} - 1 - \zeta \cdot \frac{(1-\mathfrak{C})}{\mathfrak{C}} + \frac{Qk+1}{\mathfrak{C}}$$

must be positive and just as small, as the circumstances permit.

Then truly so that the colored margin may vanish, this equation will be forthcoming:

$$0 = \frac{\mathfrak{q}}{P} + \frac{\mathfrak{r}}{PQ} + \frac{1}{PQR},$$

from which there is found :

$$\frac{1}{k'} = Q\mathfrak{q} + \mathfrak{r},$$

which quantity must be positive. Therefore since there shall be $kQk' = \mathfrak{M}$, there will become

$$\mathfrak{M} = \frac{kQ}{Q\mathfrak{q}+\mathfrak{r}} \quad \text{and hence} \quad kQ = \mathfrak{M}\left(Q\mathfrak{q} + \mathfrak{r}\right).$$

But before we may pursue this formula, several items remain to be investigated, whether perhaps $\mathfrak{t}$ may be put $=1$. Therefore we may put in place $\mathfrak{t} = 1$, so that there shall become

$$M = \frac{\mathfrak{q}+2}{\mathfrak{M}-1},$$

from which, on account of $\mathfrak{q} = -\zeta M$, there will become

$$\mathfrak{q} = \frac{-2\zeta - \zeta\mathfrak{q}}{\mathfrak{M}-1}$$

and thus

$$\mathfrak{q} = -\frac{2\zeta}{\mathfrak{M}+\zeta-1}$$

and hence

$$M = +\frac{2}{\mathfrak{M}+\zeta-1};$$

truly the other equation will give

$$\mathfrak{C} = \frac{-2(1+Qk)+2\zeta}{\mathfrak{M}+\zeta-1} = \frac{2(\zeta-Qk-1)}{\mathfrak{M}+\zeta-1}.$$

Then since there shall be $kQ = \mathfrak{M}Qq + \mathfrak{M}$, now there will become

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}\zeta Q}{\mathfrak{M}+\zeta-1},$$

from which there is found *k*, provided there shall be

$$1 > \frac{2\zeta Q}{\mathfrak{M}+\zeta-1}, \text{ that is } \quad Q < \frac{\mathfrak{M}+\zeta-1}{2\zeta}.$$

Again moreover there shall be

$$\mathfrak{B} = \frac{1+k}{\zeta} \text{ and [approximately] } \mathfrak{C} = \frac{4\mathfrak{M}\zeta q}{(\mathfrak{M}+\zeta-1)^2} - 2;$$

therefore, if there were $\mathfrak{B} > 1$, so that there shall be $B < 0$, then there must be taken $Q < 1$; then truly *C* must be positive and thus also $\mathfrak{C} > 0$ and $\mathfrak{C} < 1$; concerning which there must become

$$1.\ 2\mathfrak{M}\zeta Q > (\mathfrak{M}+\zeta-1)^2,$$

$$2.\ 2\mathfrak{M}\zeta Q < \tfrac{3}{2}(\mathfrak{M}+\zeta-1)^2,$$

which condition, from which there must be $Q < 1$, clearly disagrees with that. Therefore there must become $B > 0$ and hence $Q > 1$, but also there must become $C < 0$; which will happen, if $\mathfrak{C}$ also were negative, that is, if there were

$$Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta},$$

which condition can readily be consistent with the preceding $Q > 1$. But then there must be $\mathfrak{B} < 1$ and thus $\zeta > 1+k$. Therefore we will obtain so great a field, if we will take $\zeta > 1+k$, the letter *Q* truly within the bounds

$$1 \text{ and } \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta},$$

provided there were

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}\zeta Q}{\mathfrak{M}+\zeta-1}$$

and thus

$$Q < \frac{\mathfrak{M}+\zeta-1}{2\zeta},$$

since kQ is positive; which shall happen at once by the preceding condition, which is $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$. But besides there must become

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1};$$

but since there must be $\zeta > 1+k$, we will have now

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1},$$

from which this condition follows:

$$Q > \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{\zeta^2+\zeta(3\mathfrak{M}-2)},$$

which condition agrees very well with that, $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$.

But since besides there must be $\zeta > 1+k$, by substituting its value in place of k there will become

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1},$$

from which there must be concluded

$$Q > \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{\zeta^2+\zeta(3\mathfrak{M}-2)-\mathfrak{M}+1};$$

but just as we have seen to be $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$, here the bounds must be greater from that; from which it is deduced

$$\zeta^3+\zeta^2(4\mathfrak{M}-3)+\zeta\left(\mathfrak{M}^2-6\mathfrak{M}+3\right) > (\mathfrak{M}-1)^2,$$

from which it is clear, if $\mathfrak{M}$ shall be a very large number, there must become $\zeta > 1$. Therefore we may put in place

$$\zeta = 1 + \frac{\alpha}{\mathfrak{M}},$$

from which there becomes

$$\mathfrak{M}^2+\mathfrak{M}(\alpha-2)+2\alpha+1 > \mathfrak{M}^2 - 2\mathfrak{M}+1$$

and hence

$$\alpha(\mathfrak{M}+2) > 0;$$

but since it may always arise, it is clear, while $\zeta > 1$, the solution will be found always ; and if in these formulas in order that from these formulas $\mathfrak{M}$ may be considered as a very large number, the bounds for Q will be

$$Q > \frac{\mathfrak{M}}{3\zeta - 1} \text{ and } Q < \frac{\mathfrak{M}}{2\zeta}$$

and with these limits conveniently taken for Q again there will be

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M} + \zeta - 1}$$

and hence all the remaining elements will be defined readily. Moreover in the establishment of the following cases this will be rendered more clearly. Finally so that it may pertain to the apertures of the lenses, it will be allowed to define these easily for any case from the known formulas.

COROLLARY 1

242. Therefore we have seen, truly, how all these conditions may be able to evolve more clearly. And indeed in the first place, we may put at once $\mathfrak{r} = 1$, there shall become

$$M = \frac{2 + \mathfrak{q}}{\mathfrak{M} - 1} \text{ and thus } 2 + \mathfrak{q} = M(\mathfrak{M} - 1).$$

Again by putting

$$\frac{1+k}{\mathfrak{B}} = \zeta,$$

so that there shall become

$$\mathfrak{B} = \frac{1+k}{\zeta},$$

there will become $\mathfrak{q} = -\zeta M$; which value substituted there gives

$$2 - \zeta\mathfrak{M} = M(\mathfrak{M} - 1)$$

and hence

$$M = \frac{2}{\mathfrak{M} + \zeta - 1} \text{ and } \mathfrak{q} = -\frac{2\zeta}{\mathfrak{M} + \zeta - 1};$$

then indeed we find

$$kQ = \mathfrak{M}Q\mathfrak{q} + \mathfrak{M},$$

where the value of q substituted gives

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}Q\zeta}{\mathfrak{M} + \zeta - 1}$$

or

$$kQ(\mathfrak{M} + \zeta - 1) + 2\mathfrak{M}Q\zeta = \mathfrak{M}(\mathfrak{M} + \zeta - 1),$$

from which there is deduced conveniently

$$Q = \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}.$$

from which there becomes

$$1+kQ = \frac{k(\mathfrak{M}+1)(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}.$$

Therefore since there shall be

$$\mathfrak{C} = M(\zeta - Qk - 1),$$

there will become

$$\mathfrak{C} = \frac{2\mathfrak{M}\zeta(\zeta-1)-k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}{k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta}\cdot M,$$

or

$$\mathfrak{C} = \frac{4\mathfrak{M}\zeta(\zeta-1)-2k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}{\left(k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta\right)(\mathfrak{M}+\zeta-1)}.$$

COROLLARY 2

243. Now two cases are required to be considered in the account of the letter $\mathfrak{B}$, the one, where $\mathfrak{B} > 1$ and thus $B < 0$, the other truly, where $\mathfrak{B} < 1$ and thus $B > 0$. In the first case there will be $\zeta < 1+k$, and since B is less than zero, on that account there must be $Q < 1$; from which there becomes :

$$\mathfrak{M}(\mathfrak{M}+\zeta-1) < k(\mathfrak{M}+\zeta-1)+2\mathfrak{M}\zeta,$$

which cannot happen, since $\mathfrak{M}$ shall be much greater.

COROLLARY 3

243[a]. Therefore since it is unable for $\mathfrak{B} > 1$, we may put $\mathfrak{B} < 1$ or $\zeta > 1+k$, and since now $B > 0$, there will have to be $Q > 1$; that which clearly will happen at once for greater magnifications, to which we will attend here only. But then there must become $C < 0$, that which will happen, either if there were $\mathfrak{C} < 0$ or $\mathfrak{C} > 1$. In the first case, if $\mathfrak{C} < 0$, there will have to become :

$$k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1) > 2\mathfrak{M}\zeta(\zeta-1)$$

or

$$k > \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}.$$

Truly from the other condition $\zeta > 1+k$ there must become $k < \zeta - 1$; from which there must be deduced

$$\zeta-1>\frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}$$

or

$$(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)>2\mathfrak{M}\zeta,$$

which always happens, thus so that the letter ζ may be left to our choice, provided it may be taken greater than unity. But since there shall be $M=\frac{2}{\mathfrak{M}-\zeta+1}$ the magnitude of the field may require, that ζ may exceed unity by the smallest amount.

COROLLARY 4

244. The other case remains requiring to be examined, where there must become $\mathfrak{C}>1$; therefore then before all else there will have to remain a positive numerator, or

$$2\mathfrak{M}\zeta(\zeta-1)>k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)$$

and thus

$$k<\frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)};$$

then, so that also it will exceed unity, there must become

$$4\mathfrak{M}\zeta(\zeta-1)-2k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)>k(\mathfrak{M}+\zeta-1)^2+2\mathfrak{M}\zeta(\mathfrak{M}+\zeta-1)$$

and thus

$$k<\frac{2\mathfrak{M}\zeta(\mathfrak{M}-\zeta-1)}{(\mathfrak{M}+\zeta-1)(3\mathfrak{M}-\zeta+1)},$$

which since it shall be absurd on account of $k>0$, indicating this case cannot be used in practice, thus so that only the case in the preceding corollary is left for us.

EXAMPLE 1

245. Since the situation $\mathfrak{r}=1$ has been adapted for the maximum field and likewise the solution may be supplied so easily, that alone certainly deserves, that it may be applied in precise. But here in the first place to observe by no means can it be agreed to assume $\zeta=1$, since then there will become $k=0$, and the first interval at once becomes $=\infty$, so that from this inconvenience we may pass over the rest unmentioned, while clearly both the second and the third lens will have infinite focal lengths. From which it is necessary to assume a number greater than unity for ζ, thus so that the excess shall not be exceedingly small, since otherwise we may come upon the same inconvenience. On account of which, so that it may appear clearer, how it shall be required to proceed in this undertaking, we may suppose $\zeta=2$, so that there may become $M=\frac{2}{\mathfrak{M}+1}$. Then truly k must be contained between the limits

$$1 \text{ and } \tfrac{4\mathfrak{M}}{(\mathfrak{M}+1)(\mathfrak{M}-1)} \text{ or } 1 \text{ and } \tfrac{4}{\mathfrak{M}}.$$

But neither must k approach exceedingly close to unity, since otherwise B and hence the second interval will emerge exceedingly great. Therefore we may assume $k=\frac{1}{2}$; hence there will be

$$Q=\tfrac{2\mathfrak{M}(\mathfrak{M}+1)}{9\mathfrak{M}+1}=\tfrac{2}{9}\mathfrak{M} \text{ approximately.}$$

Truly then there will become

$$\mathfrak{B}=\tfrac{3}{4} \text{ and } B=3$$

and finally

$$\mathfrak{C}=\tfrac{16\mathfrak{M}-2(\mathfrak{M}+1)(\mathfrak{M}-1)}{(9\mathfrak{M}+1)(\mathfrak{M}+1)}=-\tfrac{2}{9}+\tfrac{16}{9\mathfrak{M}}=-\tfrac{2}{9} \text{ approximately.}$$

and hence $C=-\frac{1}{11}$; from which for the construction of the microscope we will have these focal lengths:

$$p=\mathfrak{A}a, \quad q=\tfrac{3}{2}Aa, \quad r=\tfrac{6}{\mathfrak{M}}\cdot Aa, \quad s=\tfrac{6}{11}\cdot\tfrac{Aa}{\mathfrak{M}}$$

and the intervals :

$$\text{first}=3Aa, \quad \text{second}=6Aa\left(1-\tfrac{9}{2\mathfrak{M}}\right), \quad \text{third}=\tfrac{60}{11}\cdot\tfrac{Aa}{\mathfrak{M}};$$

then truly the distance of the eye

$$O=\tfrac{s}{\mathfrak{M}M}=\tfrac{s(\mathfrak{M}+1)}{2\mathfrak{M}}=\tfrac{1}{2}s\left(1+\tfrac{1}{\mathfrak{M}}\right)=\tfrac{1}{2}s \text{ approximately.}$$

But for which the apparent field will become

$$z=Ma\xi=\tfrac{2a\xi}{\mathfrak{M}+1}=\tfrac{1}{2}\cdot\tfrac{a}{\mathfrak{M}+1} \text{ on account of } \xi=\tfrac{1}{4}.$$

Truly as it pertains to the letter A, which refers to the first lens, it is required to attend to A, so that it may not become such a large number that the eyepiece may become exceedingly small; from which it follow $\mathfrak{A}$ must be a fraction a little smaller than unity; with the value of which the aperture of the first lens must be defined from the known equation

$$\tfrac{1}{k^3}=\tfrac{\mu\mathfrak{M}x^3}{a^3}\left(\tfrac{\lambda}{\mathfrak{A}^3}+\tfrac{\nu}{A\mathfrak{A}}+\text{etc.}\right)$$

and thus we will obtain a noteworthy enough microscope, which can be seen after the fashion of the first example.

EXAMPLE 2

246. We will consider also the case $\zeta = 3$, so that there shall be $M = \frac{2}{\mathfrak{M}+2}$, and for k we will have these bounds 2 and $\frac{12}{\mathfrak{M}}$. Therefore we may put $k = 1$, so that there may become $\mathfrak{B} = \frac{2}{3}$ and $B = 2$. Then truly there will become $Q = \frac{\mathfrak{M}}{7}$ and $k' = 7$. Again indeed there will be $\mathfrak{C} = -\frac{2}{3}$ and $C = -\frac{2}{9}$, from which for these microscopes the focal lengths will become

$$p = \mathfrak{A}a, \quad q = \frac{2}{3}Aa, \quad r = 4\frac{Aa}{\mathfrak{M}} \quad \text{and} \quad s = \frac{4}{9}\cdot\frac{Aa}{\mathfrak{M}}$$

and the intervals

$$\text{first} = 2Aa, \quad \text{second} = 2Aa\left(1-\frac{7}{\mathfrak{M}}\right) \quad \text{and third} = \frac{32}{9}\cdot\frac{Aa}{\mathfrak{M}}$$

and the distance of the eye

$$O = s\cdot\frac{\mathfrak{M}+2}{2\mathfrak{M}} = \frac{1}{2}s\left(1+\frac{2}{\mathfrak{M}}\right) = \frac{1}{2}s \text{ approximately.}$$

Truly for the field there will become

$$z = \frac{1}{2}\cdot\frac{a}{\mathfrak{M}+2}.$$

Concerning the aperture they will prevail in the alleged manner. Moreover there will be for the ratio of the letter $\mathfrak{q}$ in the case of the preceding example $\mathfrak{q} = -\frac{4}{\mathfrak{M}+1}$ and in the case of this example $\mathfrak{q} = -\frac{6}{\mathfrak{M}+2}$; from which it is apparent the radius of the aperture of the second lens must be $= \frac{x}{P} = \frac{x}{k}$ and thus in the first case $= 2x$, here truly $= x$. But both the latter lenses will have to be equally convex on both sides.

SCHOLION 1

247. If we may refer these investigations to telescopes, because now thus there is a greater necessity, since above we have established only a particular case of this kind, which indeed will not present the maximum field, as we have done here, since here we have only to make $a = \infty$ and $\mathfrak{M} = m$, on account of $h = a$. But then there must be taken both $\mathfrak{A} = 0$ as well as $A = 0$, thus so that there may become $\mathfrak{A}a = Aa = p$; whereby, since all the rest may remain as before, from the first example with the focal length of the objective lens put $= p$, the focal lengths of the remaining lenses will be

$$q = \frac{3}{2}p, \quad r = \frac{6}{m}p, \quad \text{and} \quad s = \frac{6}{11}\cdot\frac{p}{m},$$

truly the intervals

$$\text{first} = 3p, \quad \text{second} = 6p\left(1-\tfrac{9}{2m}\right) \text{ and } \quad \text{third} = \tfrac{60}{11m}\cdot p$$

and

$$O = \tfrac{1}{2}s.$$

Then truly the radius of the field

$$\Phi = \tfrac{1}{2}\cdot\tfrac{1}{m+1} = \tfrac{1718}{m+1} \text{ min.}$$

Then truly on account of

$$\mathfrak{B} = \tfrac{3}{4}, \;\; B = 3, \;\; \mathfrak{C} = -\tfrac{2}{9} \;\text{ and }\; C = -\tfrac{2}{11}$$

the focal length p with the required aperture $x = \frac{1}{50}m$ in. must be defined with the aid of this equation:

$$p = m\sqrt[3]{m\mu}\left(\lambda + 2\left(\tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{\nu}{B\mathfrak{B}}\right) - \tfrac{9}{mB^3}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right) - \tfrac{\lambda'''}{mB^3C^3}\right).$$

But in the case of the other example the focal lengths will become

$$q = \tfrac{2}{3}p, \quad r = \tfrac{4}{m}p, \quad \text{and} \quad s = \tfrac{4}{9m}p$$

and the intervals:

$$\text{first} = 2p, \quad \text{second} = 2p\left(1-\tfrac{7}{2m}\right) \text{ and third} = \tfrac{32}{9m}p$$

then truly p will be defined thus, so that there shall become

$$p = m\sqrt[3]{m\mu}\left(\lambda + \tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{\nu}{B\mathfrak{B}} - \tfrac{7}{mB^3}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right) - \tfrac{\lambda'''}{mB^3C^3}\right),$$

indeed the rest will be as before.

Hence in place of the common terrestrial telescope we come upon the following construction from the latter case, if indeed we wish to make all four lenses from common glass, the refraction of which shall be $n = 1{,}55$.

CONSTRUCTION OF TELESCOPES TO BE SUBSTITUTED IN PLACE OF THE COMMON TERRESTRIAL TELESCOPES

248. For the given magnification m the focal distance $= p$ is sought for the first lens of the objective , certainly from this formula, to which the preceding approximation is reduced:

$$p = \tfrac{8}{5}m\sqrt[3]{m\mu} \text{ dig.};$$

then the construction itself will be had thus:

I. For the first lens, of which the focal length $= p$, the radius

$$\begin{cases}\text{of the anterior face} = 0,6145p\\ \text{of the posterior face} = 5,2438p,\end{cases}$$

the radius of which aperture $x = \frac{1}{50}m$ in.
and the distance to the next lens $= 2p$.

II. For the second lens, of which the focal length is $q = \frac{2}{3}p$ and the numbers $\mathfrak{B} = \frac{2}{3}$ and $\lambda' = 1$, the radius

$$\begin{cases}\text{of the anterior face} = \frac{q}{0,6696} = 0,99562p\\ \text{of the posterior face} = \frac{q}{1,1485} = 0,58047p,\end{cases}$$

the radius of its aperture $= x = \frac{1}{50}m$ in.
and the interval to the third lens $= 2p(1 - \frac{7}{m})$.

III. For the third lens, of which the focal length is $r = \frac{4}{m}p$, since that must be equally convex on both sides, the ratios of each face may be taken $= 1,1r = 4,4 \cdot \frac{p}{m}$, the radius of its aperture $= \frac{p}{m}$, and the distance to the fourth lens $= \frac{32}{9m}p$.

IV. For the fourth lens, of which the focal length is $s = \frac{4}{9} \cdot \frac{p}{m}$, likewise the radius of each face may be taken $= \frac{22}{45} \cdot \frac{p}{m}$, the radius of its aperture $= \frac{1}{9} \cdot \frac{p}{m}$, and the distance to the eye $= \frac{1}{2}s$.

V. Then truly the radius of the field $\Phi = \frac{1718}{m+2}$ min.

SCHOLIUM 2

249. These telescopes certainly are troubled by this significant fault, since the length of these clearly shall be very great, evidently more than 3*p*. But a remedy can be brought forth for this fault by attributing a greater value to the letter *k*; then also truly the letter ζ ought to be taken greater; from which certainly the field is diminished a little, yet which defect is scarcely noticeable in the greater magnifications. Therefore we may take $\zeta = 6$,

so that there may become $M = \frac{2}{\mathfrak{M}+5}$, and since the bounds for *k* shall be 5 and $\frac{60}{m}$, we may assume $k = 4$, so that there shall become $\mathfrak{B} = \frac{5}{6}$ and $B = 5$; then truly there will be $Q = \frac{m}{16}$ and $k' = 4$ and finally $\mathfrak{C} = -\frac{1}{2}$ and $C = -\frac{1}{3}$; hence moreover the following intervals will become :

$$\text{first} = \tfrac{5}{4}p, \quad \text{second} = \tfrac{5}{4}p\left(1-\tfrac{16}{m}\right), \quad \text{and the third } = \tfrac{25}{3}\cdot\tfrac{p}{m};$$

from which the length produced will be as if $2\frac{1}{2}p$, which at this stage can be seen as exceedingly great.

But we will be able to diminish this length greatly by taking $\zeta = 12$ and $k = 9$; hence indeed there becomes $\mathfrak{B} = \frac{5}{6}$ and $B = 5$ as before ; from which the following intervals arise:

$$\text{the first} = \tfrac{10}{9}p \text{ and the second} = \tfrac{5}{9}p,$$

thus so that the total length may become as if $1\frac{2}{3}p$, which does not exceed the common telescopes of this kind. If we may assume $\zeta = 12$ and $k = 8$, so that there shall become $\mathfrak{B} = \frac{3}{4}$ and $B = 3$, the interval will become :

$$\text{the first } = \tfrac{9}{8}p \text{ and the second} = \tfrac{3}{8}p,$$

thus so that the whole length may be as if $1\frac{1}{2}p$, which certainly can be allowed. Therefore this case deserves that it may set it out more fully.

But again there will become $Q = \frac{m}{32}$ and hence $k' = 4$. Again $\mathfrak{C} = -\frac{1}{2}$ and $C = -\frac{1}{3}$; from which

$$q = \tfrac{3}{32}p, \quad r = 6\tfrac{p}{m} \quad \text{and} \quad s = \tfrac{p}{m};$$

then truly the intervals become:

$$\text{the first} = \tfrac{9}{8}p, \quad \text{the second} = \tfrac{3}{8}p\left(1-\tfrac{32}{m}\right), \quad \text{the third} = 5\tfrac{p}{m};$$

and for the position of the eye : $O = \frac{s}{2}\left(1+\frac{11}{m}\right)$, and the radius of the field $= \frac{1718}{m+2}$ min. Therefore with $x = \frac{m}{50}$ in. assumed for the aperture of the objective lens and on taking $k = 50$ there will be approximately,

$$p = m\sqrt[3]{m\tfrac{4}{3}} = \tfrac{11}{10}m\sqrt[3]{m} \text{ in.};$$

from which the following

CONSTRUCTION OF THE COMMON TELESCOPE FROM COMMON GLASS

I. For the given magnification *m* there may be taken

$p=\frac{11}{10}m\sqrt[3]{m}$ approximately, or as if also $p=m\sqrt[3]{m}$ in.

II. For the first lens, of which the focal length $=p$, the radius may be taken

$$\begin{cases}\text{of the anterior face} = 0,6145p\\ \text{of the posterior face} = 5,2438p,\end{cases}$$

of which the radius of the aperture $=\frac{m}{50}$ in.
and the distance to the second lens $=1\frac{1}{8}p$.
III. For the second lens, of which the focal length is $q=\frac{3}{32}p$, the radius may be taken

$$\begin{cases}\text{of the anterior face} = \frac{q}{0,5499} = 0,17053p\\ \text{of the posterior face} = \frac{q}{1,2682} = 0,07388p,\end{cases}$$

of which the radius of the aperture $=\frac{1}{8}x=\frac{m}{400}$ in.
and the distance to the third lens $=\frac{3}{8}p\left(1-\frac{32}{m}\right)$.

IV. For the third lens, of which the focal length is $r=6\frac{p}{m}$, the radius of each face may be taken $=6,6\frac{p}{m}$, and the maximum aperture may be attributed to that, and truly the distance to the fourth lens will be $=5\frac{p}{m}$

V. For the fourth lens, of which the focal length $s=\frac{p}{m}$, the radius of each face may be taken $=1,1\frac{p}{m}$, and its distance from the eye $=\frac{s}{2}\left(1+\frac{11}{m}\right)$.

VI. The length will be $1\frac{1}{2}p-6\frac{1}{2}\cdot\frac{p}{m}$.
Truly the radius of the apparent field of view will be $=\frac{1718}{m+11}$ min.

Therefore this telescope is seen to deserve to be preferred before common terrestrial telescopes; indeed truly in practice it cannot be considered unless *m* shall be notably greater than 32.

But here we finish this chapter and to be going to progress to the following, where we will investigate more composite microscopes of this kind.

SECTIO QUARTA.

DE MICROSCOPIIS COMPOSITIS,

IN QUIBUS DUAE IMAGINES REALES OCCURRUNT.

CAPUT 1

DE MICROSCOPIIS SIMPLICIORIBUS HUIUS GENERIS

PRAEMONITUM

Cum microscopia ad hanc sectionem relata iterum situ erecto obiecta repraesentent, litterae $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ etc. una cum multiplicatione *m* eadem retinent signa, quae in praeceptis generalibus sunt usurpata.

PROBLEMA 1

224. *Microscopium huius generis ex tribus lentibus componere eiusque qualitates et defectus investigare.*

SOLUTIO

Cum hic tantum tres lentes occurrant ideoque duo intervalla, in quorum utroque imago realis existit, ambae litterae *P* et *Q* statuendae sunt negativae; quamobrem ponamus $P=-k$ et $Q=-k'$, ut sit $kk'=\frac{ma}{h}$; distantiae vero focales lentium erunt

$$p=\mathfrak{A}a,\quad q=\frac{A\mathfrak{B}a}{k},\quad r=\frac{ABa}{kk'}=AB\cdot\frac{h}{m},$$

intervalla vero lentium

$$\text{primum}=Aa\left(1+\tfrac{1}{k}\right),\quad \text{secundum}=\frac{ABa}{k}\left(1+\tfrac{1}{k'}\right),$$

ita ut prima imago realis distet a prima lente intervallo $=Aa$ et a secunda intervallo $=\frac{Aa}{k}$; posterior vero imago realis post lentem secundam cadit intervalo $=\frac{Aa}{k}$ et ante tertiam intervallo $=\frac{ABa}{kk'}$, ac si spatii in objecto conspicui semidiameter sit $=z$, semidiameter prioris imaginis erit $=Az$, quae est inversa, posterious vero $=ABz$, quae iterum est erecta. Hinc igitur patet esse debere $A>0$ et $B>0$, unde quoque fient $\mathfrak{A}>0$ et $\mathfrak{B}>0$, ita tamen, ut sit $\mathfrak{A}<1$ et $\mathfrak{B}<1$. Tum vero erit

$$z=\frac{\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot ah\xi \quad \text{et} \quad M=\frac{\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot h,$$

ut sit $z = Ma\xi$, unde nanciscimur

$$\mathfrak{B}\mathfrak{q} = -(1+k)M;$$

ex quo perspicuum est, cum $\mathfrak{B}$ sit positivum, fieri $\mathfrak{q}$ negativum eoque ergo campum apparentem diminui; quare, ne is penitus ad nihilum redigatur, tribui debebit litterae $\mathfrak{r}$ maximus valor, qui est unitas, et posito $\mathfrak{q} = -\omega$ debet esse $\omega < 1$, cum sit

$$M = \frac{1-\omega}{ma-h} \cdot h;$$

deinde ob

$$\mathfrak{B} = \frac{1+k}{\omega} \cdot M = \frac{1+k}{\omega} \cdot \frac{1-\omega}{ma-h} \cdot h,$$

quia $\mathfrak{B} < 1$, debebit esse

$$(1+k)(1-\omega)h < \omega(ma-h),$$

quae quidem conditio facile impletur, si fuerit

$$\omega > \frac{(1+k)h}{ma+hk};$$

et quia insuper est $\omega < 1$, ad hoc requiritur, ut sit $ma > h$, quae quidem conditio pro maioribus multiplicationibus sponte habet locum. Quodsi vellemus assumere $\omega = \frac{(1+k)h}{ma+hk}$, prodiret $\mathfrak{B} = 1$ hincque $B = \infty$ et instrumentum fieret infinite longum; ex quo perspicuum est necessario capi oportere $\omega > \frac{(1+k)h}{ma+hk}$.

Nunc etiam videamus, num margo coloratus destrui possit; quem in finem ante locus oculi examinari debet hac aequatione determinatus [§ 18]:

$$O = \frac{r}{Ma} \cdot \frac{h}{m} \text{ ob } \mathfrak{r} = 1.$$

Quoniam igitur *r* est positivum, utique erit $O > 0$, unde pro destructione marginis colorati habebitur ista aequatio:

$$0 = \frac{\omega}{k} + \frac{1}{kk'};$$

quod cum fieri nequeat, manifestum est huiusmodi microscopia insigni vitio marginis colorati laborare, ita ut superfluum foret in reliqua constructionis praecepta inquirere.

COROLLARIUM 1

225. Cum ob duas imagines reales pauciores quam tres lentes adhiberi nequeant, constructio in problemate contenta utique est simplicissima, quae locum habere queat; quare, cum eam repudiare cogamur, ad minimum quatuor lentibus uti oportebit.

COROLLARIUM 2

226. Quoniam formula pro destructione marginis colorati duabus constat partibus positivis, ista confusio multo erit maior quam in telescopiis et microscopiis ex duabus tantum lentibus formatis ideoque multo minus tolerari poterit.

SCHOLION

227. Cum igitur tribus lentibus hic propositis unam ad minimum insuper adiici oporteat, id triplici modo fieri poterit; primo enim haec nova lens inter lentem obiectivam et primam imaginem realem, secundo insuper inter imaginem realem primam et secundam, ita ut in hoc intervallo duae lentes constituantur, tertio vero inter imaginem realem secundam et lentem ocularem cadere poterit. Verum hic tertius casus eodem vitio laborabit, quod hic est reprehensum; litterae enim *P* et *Q* eosdem retinebunt valores $-k$ et $-k'$, quippe quibus tantum tertia littera *B* adiungitur, sicque littera $\mathfrak{q}$ retinebit quoque valorem negativum, qui sit $\mathfrak{q}=-\omega$, unde pro margine colorato destruendo habebitur ista aequatio:

$$0=\frac{\omega}{k}+\frac{\mathfrak{r}}{kk'}+\frac{\mathfrak{s}}{kk'R},$$

quae neutiquam subsistere potest, nisi vel $\mathfrak{r}$ vel $\mathfrak{s}$ capiatur negativum, quod autem, cum iam $\mathfrak{q}$ habeat valorem negativum, neutiquam expedit, quoniam alioquin campus nimis redderetur angustus, quocirca tantum bini casus priores nobis evolvendi relinquuntur.

PROBLEMA 2

228. *Microscopia huius generis ita ex quatuor lentibus componere, ut secunda adhuc ante priorem imaginem realem cadat, tertia vero inter ambas imagines ideoque sola ocularis post secundam imaginem, in quo id potissimum efficiatur, ut margo coloratus evanescat.*

SOLUTIO

Hic ergo habentur tria intervalla totidemque litterae *P, Q* et *R,* quarum duae posteriores debent esse negativae. Ponamus itaque $Q=-k$ et $R=-k'$, ut sit $Pkk'=\frac{ma}{h}$; distantiae porro focales harum lentium erunt

$$p=\mathfrak{A}a,\quad q=\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{Pk}\cdot a \quad\text{et}\quad s=-\frac{ABC}{Pkk'}\cdot a=-ABC\cdot\frac{h}{m},$$

tum vero intervalla lentium

$$\text{primum}=Aa\left(1-\frac{1}{P}\right),\ \text{secundum}=-\frac{ABa}{P}\left(1+\frac{1}{k}\right),\quad \text{tertium}=-\frac{ABCa}{Pk}\left(1+\frac{1}{k'}\right),$$

unde patet esse debere $-AB>0$ et $C>0$. Deinde notetur primam imaginem cadere post lentem secundam ad intervallum $=-\frac{ABa}{P}$ et ante tertiam intervalla $=-\frac{ABa}{Pk}$, posteriorem vero imaginem cadere post lentem tertiam intervallo $=-\frac{ABCa}{Pk}$ et ante ocularem interval $=-\frac{ABCa}{Pkk'}$, praeter ea vero imaginis prioris inversae radium esse $=ABz$, posterioris vero erectae $=ABCz$ existante

$$z=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma-h}\cdot ah\xi$$

hincque

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma-h}\cdot h$$

ita ut sit $z=Ma\xi$, quae quantitas per hypothesin debet esse positiva; ex hoc autem valore deductae sunt sequentes formulae:

$$\mathfrak{B}\mathfrak{q}=(P-1)M,\quad \mathfrak{C}\mathfrak{r}=-(Pk+1)M-\mathfrak{q}.$$

Ob conditionem $C>0$ autem modo allatam debet esse $\mathfrak{C}>0$ et $\mathfrak{C}<1$, ex quo perspicuum est vel $\mathfrak{q}$ vel $\mathfrak{r}$ esse debere negativum. Utrum igitur locum habeat, conveniet $\mathfrak{s}$ sumi positive atque adeo poni $\mathfrak{s}=1$, ut sit

$$M=\frac{1+\mathfrak{q}+\mathfrak{r}}{ma-h}\cdot h.$$

Hinc autem oculi distantia post lentem ocularem prodibit

$$O=\frac{\mathfrak{s}s}{M}\cdot\frac{h}{ma}.$$

quia igitur $s>0$, haec distantia fiet positiva ideoque margo coloratus destruetur ope huius aequationis:

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}+\frac{1}{Pkk'};$$

quae neutiquam subsistere posset, si esset $\mathfrak{r}<0$, unde necesse est, ut sit $\mathfrak{q}<0$. Statuatur $\mathfrak{q}=-\omega$ eritque

$$\frac{1}{k'}=k\omega+\mathfrak{r}$$

atque nunc novimus esse debere

$$\mathfrak{B}=\frac{1-P}{\omega}\cdot M\quad \text{et}\quad \mathfrak{C}=\frac{-(Pk+1)M+\omega}{\mathfrak{r}};$$

qui valor cum esse debeat positivus, erit

$$\left(Pk+1\right)M<\omega$$

hincque

$$\omega>\frac{\left(Pk+1\right)\left(1+\mathfrak{r}\right)h}{ma+Pkh}.$$

Cum autem sit

$$\frac{ma}{h}=Pkk'=\frac{Pk}{k\omega+\mathfrak{r}},$$

orietur haec aequatio:

$$Pk^2\omega^2+\omega\left(1+\mathfrak{r}\right)\left(P-Pk-1\right)k-\left(Pk+1\right)\left(1+\mathfrak{r}\right)\mathfrak{r}>0,$$

quae aequatio conditionem continet, secundum quam littera ω debet definiri. Definitis autem convenienter litteris ω et $\mathfrak{r}$ indeque deductis valoribus $\mathfrak{B}$ et $\mathfrak{r}$, saltim quam proxime, reliqua elementa innotescunt; tum vero nihil aliud superest, nisi ut apertura lentis obiectivae ex aequatione pro semidiametro confusionis determinetur.

COROLLARIUM 1

229. Ponamus brevitatis gratia

$$\frac{ma}{h}=\mathfrak{M},$$

ut sit

$$M=\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1},$$

et habebimus

$$Pk=\mathfrak{M}\left(k\omega+\mathfrak{r}\right)$$

et

$$\mathfrak{B}\omega=\left(1-P\right)\left(\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right)$$

et

$$\left(Pk+1\right)\left(\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}\right)=\frac{\left(\mathfrak{M}\left(k\omega+\mathfrak{r}\right)+1\right)\left(1-\omega+\mathfrak{r}\right)}{\mathfrak{M}-1}=\omega-\mathfrak{C}\mathfrak{r};$$

ex quo patet fore $w>\mathfrak{C}\mathfrak{r}$, ubi constat esse $\mathfrak{C}>0$ et $\mathfrak{C}<1$.

COROLLARIUM 2

230. Cum igitur ω notabiliter maius esse debeat quam $\mathfrak{C}\mathfrak{r}$, videamus, an fieri possit $\omega=\mathfrak{r}$; quem in finem ponamus $\omega=\mathfrak{r}$ et ultima aequatio fiet

$$\mathfrak{r}\left(1-\mathfrak{C}\right)=\frac{\left(\mathfrak{M}\left(1+k\right)+1\right)}{\mathfrak{M}-1};$$

unde concluditur

$$\mathfrak{r}=\frac{-1}{\mathfrak{M}\left(\mathfrak{C}+k\right)+1-\mathfrak{C}},$$

quod, cum esse debeat $\mathfrak{r}>0$, fieri nequit sicque etiam certum est esse debere $\omega>\mathfrak{r}$, ita ut campus ne ad valorem eius simplicem quidem $z=\frac{ah\xi}{ma-h}$ augeri possit ob $1-w+\mathfrak{r}<1$.

COROLLARIUM 3

231. Cum igitur sit $\omega-\mathfrak{r}>0$, plurimum interest nosse, quomodo isti formulae minimus valor concilietur; quem in finem litteris ω et $\mathfrak{r}$ ut variabilibus spectatis hoc eveniet, si sit $d\omega=d\mathfrak{r}$, cui regulae convenienter differentietur nostra aequatio

$$(\omega-\mathfrak{C}\mathfrak{r})(\mathfrak{M}-1)=(\mathfrak{M}(k\omega+\mathfrak{r})+1)(1-\omega+\mathfrak{r})$$

ac prodibit

$$(1-\mathfrak{C})(\mathfrak{M}-1)=\mathfrak{M}(1-\omega+\mathfrak{r})(k+1),$$

unde colligimus

$$1-\omega+\mathfrak{r}=\frac{(1-\mathfrak{C})(\mathfrak{M}-1)}{\mathfrak{M}(k+1)},$$

ita ut sit

$$M=\frac{(1-\mathfrak{C})}{\mathfrak{M}(k+1)},$$

quae formula praebet maximum campum, quem quidem obtinere licet. Hic autem campus maximus obtinebitur capiendo

$$\omega=\mathfrak{r}+\frac{\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}}{\mathfrak{M}(k+1)},$$

qui valor in nostra aequatione substitutus dabit

$$\mathfrak{r}(1-\mathfrak{C})(\mathfrak{M}-1)+\frac{(\mathfrak{M}-1)^2\mathfrak{C}+(\mathfrak{M}-1)(\mathfrak{M}k+1)}{\mathfrak{M}(k+1)}$$
$$=\frac{(1-\mathfrak{C})(\mathfrak{M}-1)}{(k+1)}\left(\mathfrak{r}(k+1)+\frac{(\mathfrak{M}-1)k\mathfrak{C}+\mathfrak{M}k^2+k}{\mathfrak{M}(k+1)}\right)$$

ubi membra litteram $\mathfrak{r}$ continentia se mutuo tollunt; relinquitur haec aequatio:

$$(1-\mathfrak{C})(1+\mathfrak{C}k)+\mathfrak{M}(\mathfrak{C}+k)(\mathfrak{C}k+1)=0,$$

quae reducitur ad hanc:

$$\mathfrak{M}(\mathfrak{C}+k)+1-\mathfrak{C}=0;$$

quae cum sit impossibilis, sequitur hunc campum maximum ne quidem obtineri posse.

SCHOLION 1

232. Parum vero refert, utrum campum illum maximum obtinere queamus necne, cum etiam hic non desint remedia campum pro lubitu amplificandi; quare relicta hac investigatione aliquot casus evolvamus, qui ad praxin inprimis accommodati videntur, ac primo quidem apparet litteram P unitati non nimis vicinam assumi posse, quia tum secunda lens primae tam esset propinqua, ut ambae tanquam una spectari possent; ex quo casus praecedente problemate tractatus resultaret, quem locum habere non posse vidimus. Quamobrem pro P numerum satis magnum accipi conveniet; deinde etiam, cum semper sit $\omega > \mathfrak{r}$, e re erit $\mathfrak{r}$ quam minimum accipere; denique etiam, ut ad campum maximum, quantum fieri licet, appropinquemus, conveniet litteras k et $\mathfrak{C}$ quam minimas assumi.

CASUS 1
QUO $P = \infty$

[232a] . Hoc ergo casu fit intervallum primum $= Aa$ ideoque $A > 0$ et $\mathfrak{A} < 1$ ac secunda lens cadet in ipsam imaginem primam; cuius distantia focalis ne fiat $= 0$, debet esse $\mathfrak{B} = \infty$ ita ut sit

$$\frac{P}{\mathfrak{B}} = -\zeta$$

hincque

$$q = \frac{Aa}{\zeta}.$$

Deinde cum sit $r = -\frac{AB\mathfrak{C}a}{Pk}$, ob $\mathfrak{B} = \infty$ fit $B = -1$ et ob $\mathfrak{C} < 1$ manifestum est esse debere $k = 0$, ut fieri possit Pk quantitas finita; at quia $k = 0$, erit $\frac{1}{k'} = \mathfrak{r}$ hincque $Pk = \mathfrak{M}\mathfrak{r}$ existante $\mathfrak{M} = \frac{ma}{h}$; ex quo erit

$$r = \frac{A\mathfrak{C}a}{\mathfrak{M}\mathfrak{r}} \quad \text{et} \quad s = AC \cdot \frac{h}{m};$$

unde pro magnis multiplicationibus esse debet C numerus praemagnus hincque $\mathfrak{C}$ ab unitate parum deficere. Reliqua vero intervalla erunt

$$\text{secundum} = \frac{Aa}{\mathfrak{M}\mathfrak{r}} = \frac{r}{\mathfrak{C}}$$

et

$$\text{tertium} = \frac{ACa}{\mathfrak{M}\mathfrak{r}}\left(1 + \mathfrak{r}\right) = \left(1 + C\right) r + s.$$

Praeterea vero distantia oculi erit

$$O = \frac{s}{M\mathfrak{M}}.$$

Nunc autem cum sit

$$M = \frac{1 - \omega + \mathfrak{r}}{\mathfrak{M} - 1},$$

hunc valorem in binis formulis $\mathfrak{B}\omega$ et $\mathfrak{C}\mathfrak{r}$ non substituamus, sed in iis litteram M retineamus, quo eam facilius deinceps definira queamus; tum autem ob $\frac{P}{\mathfrak{B}} = -\zeta$ ex priore invenimus

$$\omega = \zeta M,$$

ex posteriore vero

$$\mathfrak{C}\mathfrak{r} = -M - \mathfrak{M}\mathfrak{r}M + \zeta M$$

sive

$$\mathfrak{r}\left(\mathfrak{C} + \mathfrak{M}M\right) = \left(\zeta - 1\right)M$$

hincque

$$\mathfrak{r} = \frac{(\zeta-1)M}{\mathfrak{C}+M\mathfrak{M}}$$

hinc ergo colligimus

$$\omega - \mathfrak{r} = \frac{(\mathfrak{C}+M\mathfrak{M}-1)\xi M + M}{\mathfrak{C}+M\mathfrak{M}}$$

vel

$$\omega - \mathfrak{r} = \frac{\left(M\mathfrak{M}\zeta - (1-\mathfrak{C})\zeta + 1\right)M}{\mathfrak{M}M + \mathfrak{C}};$$

et

$$1 - \omega + \mathfrak{r} = \frac{\mathfrak{M}M + \mathfrak{C} - \mathfrak{M}M^2\zeta + (1-\mathfrak{C})\xi M + M}{\mathfrak{M}M + \mathfrak{C}};$$

quae expressio aequalis esse debet huic $\left(\mathfrak{M} - 1\right)M$; unde nascitur haec aequatio:

$$M^2\mathfrak{M}\left(\mathfrak{M} - 1 + \zeta\right) - M\left(1 - \mathfrak{C}\right)\left(\mathfrak{M} - 1 + \zeta\right) - \mathfrak{C} = 0,$$

ex qua, cum sit proxime $\mathfrak{C} = 1$, colligimus etiam proxime

$$M = \frac{1}{\sqrt{\mathfrak{M}(\mathfrak{M}-1+\zeta)}};$$

adcuratius vero erit

$$M = \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \sqrt{\frac{\mathfrak{C}}{\mathfrak{M}(\mathfrak{M}-1+\zeta)}},$$

revera autem

$$M = \frac{1-\mathfrak{C}}{2\mathfrak{M}} + \sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}(\mathfrak{M}-1+\zeta)} + \frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)};$$

quo valore invento simul innotescunt litterae ω et $\mathfrak{r}$, unde reliqua omnia determinabuntur. Denique pro apertura lentis obiectivae determinanda, quia nulla ratio vitri diversitatem suadet, satisfieri debet huic aequationi (§31):

$$\frac{1}{k^3}=\frac{\mathfrak{M}\mu x^3}{a^3}\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-*+\frac{r\lambda''}{A^4a}+*\right),$$

ubi terminus tertius sponte evanuit, quintus vero ob C numerum praemagnum tuto reiici potest; unde, si hic factor posterior ponatur $=\Lambda$, reperitur

$$x=\frac{a}{k}\sqrt[3]{\frac{1}{\mathfrak{M}\mu\Lambda}}$$

COROLLARIUM 1

233. Quia $\mathfrak{M}$ est numerus praemagnus, loco factoris $\mathfrak{M}-1+\zeta$ scribere licebit $\mathfrak{M}$, siquidem ζ non fuerit numerus valde magnus; nulla autem ratio suadet pro ζ tantum numerum adhibere; sufficit enim, ut capiatur $\zeta>1$, ne $\mathfrak{r}$ vel evanescat vel adeo negativum evadat. Tum igitur erit

$$M=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\sqrt{\left(\frac{\mathfrak{C}}{\mathfrak{M}^2}+\frac{(1-\mathfrak{C})^2}{4\mathfrak{M}^2}\right)}=\frac{1-\mathfrak{C}}{2\mathfrak{M}}+\frac{1+\mathfrak{C}}{2\mathfrak{M}}=\frac{1}{\mathfrak{M}},$$

unde vicissim colligitur

$$\omega=\frac{\zeta}{\mathfrak{M}}\quad\text{et}\quad\mathfrak{r}=\frac{\zeta-1}{\mathfrak{M}(1+\mathfrak{C})}.$$

COROLLARIUM 2

234. Hinc ergo sequentes adipiscimur determinationes pro ipsa microscopii constructione:

1. Distantiae focales lentium erunt

$$p=\mathfrak{A}a,\quad q=\frac{Aa}{\zeta},\quad r=\frac{A\mathfrak{C}(1+\mathfrak{C})a}{\zeta-1},\ \text{et}\ s=AC\cdot\frac{h}{m}=\frac{ACa}{\mathfrak{M}}.$$

2. Lentium intervalla

$$\text{primum}=Aa,\ \text{secundum}\ \frac{r}{\mathfrak{C}}=\frac{A(1+\mathfrak{C})a}{\zeta-1}$$

et

$$\text{tertium}=(1+C)r+s=\frac{AC(1+\mathfrak{C})a}{\zeta-1}+\frac{ACa}{\mathfrak{M}};$$

tum vero distantia oculi erit $O=s$.

3. Pro apertura invenienda erit

$$\Lambda=\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-*+\frac{(1+\mathfrak{C})}{A^3(\zeta-1)}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)+\frac{\lambda'''}{A^3C^3\mathfrak{M}},$$

ubi membrum ultimum manifesto omitti potest.

SCHOLION

235. Iam innuimus nullam rationem suadere, cur pro ζ numerum satis notabilem accipere velimus; interim tamen tertium intervallum, quod est

$$\frac{AC(1+\mathfrak{C})a}{\zeta-1}+\frac{ACa}{\mathfrak{M}},$$

fieri videtur nimis magnum, nisi ζ unitatem multum superet, quoniam pro C numerum satis magnum assumi convenit atque etiam Λ numero satis notabili aequari debet. Interim tamen semper praestabit maiorem instrumenti longitudinem tolerare quam campum restringere. Verum etiamsi ζ maius acciperemus, ut mensurae prodeant ad praxin magis accommodatae, nullum aliud incommodum inde esset metuendum, nisi quod campus minor esset revera futurus, quam intendimus; quem vero defectum aliquot insuper lentibus adiungendis facile supplere licebit. At vero plurimum refert, ut numerus Λ satis notabilis accipiatur, ut $\mathfrak{A}$ satis prope ad unitatem reducatur, id quod necessarium est, ut Λ satis exiguum reddatur hincque maior claritatis gradus obtineatur; quem in finem sufficere videtur, dummodo statuatur $\Lambda=6$; hinc enim fit $\mathfrak{A}=\frac{6}{7}$ ideoque $\frac{1}{\mathfrak{A}^2}=\frac{343}{216}\lambda$, qui valor sumto $\lambda=1$ non multum superat $\frac{3}{2}$, qui per $\mu<1$ multiplicatus certe infra $\frac{3}{2}$ reducitur; unde iam satis notabilis valor pro x resultat. Si igitur statuatur $\Lambda=6$, videamus, quantum sumi oporteat C, ne s fiat nimis parvum etiam pro insigni multiplicatione $m=960$. Quia itaque tum fit $s=\frac{48C}{960}$ dig. $=\frac{C}{20}$ dig., haec distantia non infra 1 dig. deprimetur, dummodo $C=5$; quare, si statuamus $C=6$, ut sit $\mathfrak{C}=\frac{6}{7}$, ex hac parte nihil erit metuendum; tum vero tertium intervallum evadet $\frac{36\cdot 13a}{7(\zeta-1)}$ omisso altero membro sive $\frac{2\cdot 36a}{\zeta-1}=\frac{72a}{\zeta-1}$; unde, si distantia obiecti sit dimidii digiti, hoc intervallum erit $\frac{36}{\zeta-1}$ dig.; quod ergo sumto $\zeta=3$ vel $\zeta=4$ iam fit tam modicum, ut nulla possit esse ratio de eo conquerendi.

CASUS 2

QUO $\mathfrak{r}=0$

[235a]. Hoc ergo casu erit $\frac{1}{k'}=k\omega$ ideoque $P=\mathfrak{M}\omega$, tum vero $M=\frac{1-\omega}{\mathfrak{M}-1}$. Hinc aequationes ex campo deductae erunt

$$\text{I. } \mathfrak{B}\omega=\frac{(1-\mathfrak{M}\omega)(1-\omega)}{\mathfrak{M}-1},$$

$$\text{II. } \mathfrak{C}\mathfrak{r}=-\frac{(\mathfrak{M}k\omega+1)(1-\omega)}{\mathfrak{M}-1}+\omega=0,$$

Cum igitur $P=\mathfrak{M}\omega$; erit $\omega=\frac{P}{\mathfrak{M}}$ et $1-\omega=\frac{\mathfrak{M}-P}{\mathfrak{M}}$; unde patet P minus esse debere quam $\mathfrak{M}$. Hic autem valor in aequatione posteriore substitutus dabit

$$k=\frac{\mathfrak{M}(P-1)}{P(\mathfrak{M}-P)},$$

unde, cum $k>0$, patet esse debere $P>1$; hinc autem porro sequitur fore $A>0$ hincque $\mathfrak{A}<1$; deinde vero reperitur

$$\mathfrak{B}=-\frac{(P-1)(\mathfrak{M}-P)}{P(\mathfrak{M}-1)},\quad B=-\frac{(P-1)(\mathfrak{M}-P)}{\mathfrak{M}(2P-1)-P^2};$$

unde B etiam negativum valorem obtinet, uti rei natura postulat. Denique erit

$$k'=\frac{\mathfrak{M}}{Pk}=\frac{\mathfrak{M}-P}{P-1}\quad \text{et}\quad M=\frac{\mathfrak{M}-P}{\mathfrak{M}(\mathfrak{M}-1)},$$

unde campus cognoscitur; hinc igitur patet, quo minus capiatur P, eo maiorem proditurum esse campum, et cum P unitatem superare debeat, semper erit $M<\frac{1}{\mathfrak{M}}$. His igitur valoribus inventis habebimus:

Distantias focales

$$p=\mathfrak{A}a,\quad q=\frac{A(P-1)(\mathfrak{M}-P)a}{P^2(\mathfrak{M}-1)},$$

$$r=\frac{A\mathfrak{C}(\mathfrak{M}-P)^2a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}\quad \text{et}\quad s=\frac{AC(P-1)(\mathfrak{M}-P)a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}$$

et intervalla lentium

$$\text{primum}=Aa\left(1-\frac{1}{P}\right),\quad \text{secundum}=\frac{A(\mathfrak{M}-P)a}{\mathfrak{M}P},\quad \text{tertium}=\frac{AC(\mathfrak{M}-P)(\mathfrak{M}-1)a}{\mathfrak{M}(\mathfrak{M}(2P-1)-P^2)}.$$

Tum vero oculi distantia erit

$$O=\frac{s}{M\mathfrak{M}}=\frac{\mathfrak{M}-1}{\mathfrak{M}-P}\cdot s$$

ac denique spatii in obiecto conspicui erit semidiameter

$$z=\frac{\mathfrak{M}-P}{\mathfrak{M}(\mathfrak{M}-1)}\cdot a\xi.$$

Aperturam vero lentis obiectivae ex aequatione nota definira oportet, pro aperturis vero sequentium lentium notetur esse $\omega=\frac{P}{\mathfrak{M}}$ et $\mathfrak{r}=0$. Unde colligitur semidiameter aperturae lentis secundae $= \frac{1}{P} \cdot x + \frac{Pq}{4\mathfrak{M}}$,

lensis tertiae $= \frac{x}{Pk} + 0 = \frac{\mathfrak{M} - P}{\mathfrak{M}(P-1)} \cdot x$,

lentis quartae $= \frac{x}{\mathfrak{M}} + \frac{1}{4}s$.

COROLLARIUM 1

236. Quoniam campus postulat, ut P satis parvum accipiatur, pro maioribus multiplicationibus licebit P prae $\mathfrak{M}$ negligere, unde, si P unitatem non multum superet, distantiae focales ita exprimentur:

$$p = \mathfrak{A}a, \quad q = \frac{A(P-1)}{P^2} \cdot a, \quad r = \frac{A\mathfrak{C}}{2P-1} \cdot a \quad \text{et} \quad s = \frac{AC(P-1)}{\mathfrak{M}(2P-1)} \cdot a;$$

deinde intervalla lentium

$$\text{primum} = Aa\left(1 - \frac{1}{P}\right), \quad \text{secundum} = \frac{Aa}{P}, \quad \text{tertium} = \frac{ACa}{2P-1}$$

et distantia oculi $O = s$.

COROLLARIUM 2

237. Si ergo statuamus $P = 2$, fient distantiae focales

$$p = \mathfrak{A}a, \quad q = \frac{1}{4}Aa, \quad r = \frac{1}{3}A\mathfrak{C}a \quad \text{et} \quad s = \frac{1}{3} \cdot \frac{AC}{\mathfrak{M}} \cdot a$$

et intervalla

$$\text{primum} = \frac{1}{2}Aa, \quad \text{secundum} = \frac{Aa}{2}, \quad \text{tertium} = \frac{ACa}{3}$$

et pro campo

$$z = \frac{a\xi}{\mathfrak{M}}.$$

COROLLARIUM 3

238. Si, ut supra [Lib. ll § 314] pro telescopiis fecimus , statuamus $P = \sqrt{\mathfrak{M}}$ (quoniam, quod ibi erat m, hic nobis est $\mathfrak{M}$), distantiae focales ita exprimentur:

$$p = \mathfrak{A}a, \quad q = \frac{A\left(\sqrt{\mathfrak{M}} - 1\right)}{\mathfrak{M} + \sqrt{\mathfrak{M}}} \cdot a,$$

$$r = \frac{A\mathfrak{C}\left(\sqrt{\mathfrak{M}} - 1\right)}{2\mathfrak{M}} \cdot a, \quad s = \frac{AC\left(\sqrt{\mathfrak{M}} - 1\right)}{2\mathfrak{M}} \cdot a$$

et intervalla lentium

$$\text{primum} = \frac{Aa\left(\sqrt{\mathfrak{M}}-1\right)}{\sqrt{\mathfrak{M}}}, \quad \text{secundum} = \frac{Aa\left(\sqrt{\mathfrak{M}}-1\right)}{\mathfrak{M}}, \quad \text{tertium} = \frac{ACa(\mathfrak{M}-1)}{2\mathfrak{M}\sqrt{\mathfrak{M}}}.$$

Pro campo autem apparente erit

$$z = \frac{1}{\mathfrak{M}+\sqrt{\mathfrak{M}}} \cdot a\xi$$

et pro oculi loco

$$O = \frac{\sqrt{\mathfrak{M}}+1}{\sqrt{\mathfrak{M}}} \cdot s = s\left(1+\frac{1}{\sqrt{\mathfrak{M}}}\right).$$

SCHOLION

239. Casus in corollario ultimo evolutus apprime convenit cum eo, quem supra in telescopiis tractavimus, ubi praecedentes casus, in quibus litterae P minores valores sunt tributi, penitus exclusimus idque ob eam rationem, quia intervallum tertium enormiter magnum prodiisset. Cum enim pro telescopiis sit $h = a = \infty$ necesse est, ut sit $\mathfrak{A} = 0 = A$, ita tamen, ut fiat $\mathfrak{A}a = Aa = p$ et $\mathfrak{M} = m.$ Tum autem in genere erit tertium intervallum

$$= \frac{C(\mathfrak{M}-P)(\mathfrak{M}-1)p}{\mathfrak{M}\left(\mathfrak{M}(2P-1)-P^2\right)},$$

quod, si P prae $\mathfrak{M}$ quasi evanescat, fiet

$$= \frac{C}{2P-1} \cdot p;$$

quare, cum C debeat esse numerus praemagnus, hoc solum intervallum multis partibus excessurum esset distantiam focalem p ideoque longitudo telescopii prodiret enormiter magna; quos igitur casus merito supra exclusimus. Nunc autem, ubi de microscopiis agitur, haec ratio penitus cessat; neque enim longitudo instrumenti ob tertium intervallum adeo enormiter magna evadit. Si enim, ut ante notavimus, pro magnis etiam multiplicationibus sumatur $A = 6$ et $C = 6,$ tum tertium intervallum erit $= \frac{36a}{2P-1},$ ac si $a,$ ut fieri solet, capiatur $\frac{1}{2}$ dig., hoc intervallum fiet $= \frac{18}{2P-1}$ dig.; unde, si modo sit $P = 2$, id reducitur ad 6 dig., quod in praxi utique admitti potest. Quocirca in hac de microscopiis tractatione casum in tertio corollario evolutum excludi conveniet servato eo, ubi erat $P = 2$, siquidem hoc modo campus multo maior obtinetur; quin etiam, si lubuerit, sumi poterit $P = 3$, ut prodeant distantiae focales

$$p = \mathfrak{A}a, \quad q = \tfrac{2}{9}Aa, \quad r = \tfrac{1}{5}A\mathfrak{C}a \quad \text{et} \quad s = \tfrac{2}{5} \cdot \frac{AC}{\mathfrak{M}} \cdot a$$

et intervalla lentium

$$\text{primum} = \tfrac{2}{3}Aa, \quad \text{secundum} = \tfrac{1}{3}Aa, \quad \text{tertium} = \tfrac{1}{5}ACa$$

manente $O = s$ proxime et

$$z = \frac{\mathfrak{M}-3}{\mathfrak{M}(\mathfrak{M}-1)} \cdot a\xi.$$

Nunc autem ne *s* pro magnis multiplicationibus nimis fiat exiguum, litterae *C* utique maior valor tribui debebit, ita ut iam nulla ratio suadeat, cur litterae *P* potius valorem 3 quam 2 tribuere velimus, quandoquidem ponendo $P=3$ tertium intervallum vix diminuitur.

SCHOLION 2

240. Evolutione horum duorum casuum attentius considerata poterimus simili modo solutionem generalem instituere; posito enim brevitatis gratia

$$\frac{P-1}{\mathfrak{B}}=-\zeta \quad \text{sive} \quad \mathfrak{B}=-\frac{(P-1)}{\zeta}$$

habebimus statim $\omega=\zeta M$; deinde cum sit $Pk=\mathfrak{M}(k\omega+\mathfrak{r})$, erit $Pk=\zeta\mathfrak{M}Mk+\mathfrak{M}\mathfrak{r}$, qui valor in altera aequatione, quae est

$$\mathfrak{C}\mathfrak{r}=\zeta M-PkM,$$

substitutus dat

$$\mathfrak{C}\mathfrak{r}=\zeta M-M-\zeta\mathfrak{M}M^2k-\mathfrak{M}M\mathfrak{r},$$

ex quo reperitur

$$\mathfrak{r}=\frac{(\zeta-1)M-\zeta\mathfrak{M}M^2k}{\mathfrak{M}M+\mathfrak{C}}$$

hincque

$$1-\omega+\mathfrak{r}=\frac{-\zeta\mathfrak{M}(k+1)M^2+(\mathfrak{M}-1+\zeta(1-\mathfrak{C}))M+\mathfrak{C}}{\mathfrak{M}M+\mathfrak{C}}$$

Cum igitur sit

$$M=\frac{1-\omega+\mathfrak{r}}{\mathfrak{M}-1}$$

erit

$$1-\omega+\mathfrak{r}=M(\mathfrak{M}-1),$$

unde sequens suppeditatur aequatio:

$$\mathfrak{M}(\mathfrak{M}-1+\zeta+\zeta k)M^2-(1-\mathfrak{C})(\mathfrak{M}-1+\zeta)M-\mathfrak{C}=0;$$

ex qua, nisi numeri ζ et k fuerint satis magni, ita ut eos prae $\mathfrak{M}$ tuto negligere liceat, sequitur fore saltim proxime

$$\mathfrak{M}^2-\mathfrak{M}M(1-\mathfrak{C})-\mathfrak{C}=0,$$

quae in hos factores resolvitur:

$$(\mathfrak{M}M-1)(\mathfrak{M}M+\mathfrak{C})=0,$$

unde manifesto colligitur

$$M = \frac{1}{\mathfrak{M}};$$

quo valore, etsi tantum prope vero, uti poterimus, quoniam parum refert, utrum campus aliquanto sit maior minorve, quam calculus indicat. Probe autem haec conditio· observetur, quod tam ζ quam k sint numeri satis exigui, saltim multo minores quam $\mathfrak{M}$. Si enim ζk tantus sit numerus, ut eum prae $\mathfrak{M}$ reiicere non liceat, tum littera M multo minorem nanciscetur valorem quam $\frac{1}{\mathfrak{M}}$ sicque campus insignem pateretur diminutionem, quae sola causa sufficit, ut maiores valores pro litteris ζ et k penitus excludantur haecque regula stabiliatur, ut nunquam litteris ζ et k valores tribuantur, qui binarium superent, vel ut saltim ζk quaternarium non superet. Cum igitur sit $Pkk' = \mathfrak{M}$ et k numerus ab unitate non multum discrepans, evidens est vel P vel k' esse debere numerum satis magnum vel adeo utrumque. His ergo observatis, ita ut sit $M = \frac{1}{\mathfrak{M}}$, habebimus

$$\omega = \frac{\zeta}{\mathfrak{M}} \quad \text{et} \quad \mathfrak{r} = \frac{\zeta - \zeta k - 1}{\mathfrak{M}(1+\mathfrak{C})};$$

quibus valoribus substitutis fit

$$Pk = k\zeta + \frac{\zeta - \zeta k - 1}{1+\mathfrak{C}}$$

hincque

$$P = \zeta + \frac{\zeta - \zeta k - 1}{(1+\mathfrak{C})k} = \frac{\zeta \mathfrak{C} k + \zeta - 1}{(1+\mathfrak{C})k}$$

hoc igitur valore ipsius P notato erunt distantiae focales

$$p = \mathfrak{A}a, \quad q = \frac{A(P-1)a}{P\zeta}, \quad r = \frac{A(P-1)\mathfrak{C}a}{(\zeta + P - 1)Pk}, \quad s = \frac{A(P-1)Ca}{(\zeta + P - 1)\mathfrak{M}}$$

et lentium intervalla

$$\text{primum} = Aa\left(1 - \frac{1}{p}\right), \quad \text{secundum} = \frac{A(P-1)a}{(\zeta + P - 1)Pk}(k+1)$$

et

$$\text{tertium} \quad = \frac{A(P-1)Ca}{(\zeta + P - 1)}\left(\frac{1}{Pk} + \frac{1}{\mathfrak{M}}\right)$$

et distantia oculi $O = s$ ac denique

$$z = \frac{1}{4} \cdot \frac{a}{\mathfrak{M}}.$$

Datis ergo distantia obiecti $= a$ et multiplicatione $= m$ sive $\mathfrak{M} = \frac{ma}{h}$ arbitrio nostro relinquuntur sequentes quantitates:

1. $\mathfrak{A}$, quam unitate non multo minorem assumi convenit; hanc enim conditionem claritas postulat.

2. Numerus ζ qui esse debet positivus ac tantus, ut $\zeta + P - 1$ fiat numerus positivus.

3. Littera *C*, quam autem ita definiri convenit, ut distantia focalis ne fiat nimis exigua; sin autem haec littera sit valde magna, evidens est litteram $\mathfrak{C}$ ad unitatem proxima esse accessuram.

4. Littera denique *k*, quam, ut vidimus, admodum parvam accipi convenit.

Ratione autem valoris *P* observari oportet semper esse debere

$$\zeta + P - 1 > 0,$$

deinde haec conditio adhuc implenda erit

$$\zeta \mathfrak{C} \mathfrak{k} + P - 1 > 0,$$

quae est fere eadem quantitas, quam supra prae $\mathfrak{M}$ negleximus; ex quo cavendum est, ne ea aliquot unitates superet.

PROBLEMA 3

241. *Si nova lens inter imaginem primam et secundam disponatur, omnia momenta ita definire, ut margo coloratus evanescat simulque maximus campus obtineatur.*

SOLUTIO

Quoniam hic iterum quatuor habentur lentes earumque duae intra imaginem primam et secundam cadant, litterarum *P, Q, R* prima et tertia hic erunt negativae; statuatur igitur $P = -k$ et $R = -k'$; unde distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = \tfrac{A\mathfrak{B}}{k} \cdot a, \quad r = \tfrac{AB\mathfrak{C}}{kQ} \cdot a, \quad \text{et} \quad s = -\tfrac{ABC}{kQk'} \cdot a = -\tfrac{ABC}{\mathfrak{M}} \cdot a$$

ob

$$kQk' = \mathfrak{M} = \tfrac{ma}{h}.$$

Intervalla vero lentium erunt

$$\text{primum} = Aa\left(1+\tfrac{1}{k}\right), \quad \text{secundum} = \tfrac{ABa}{k}\left(1-\tfrac{1}{Q}\right), \quad \text{tertium} = \tfrac{ABCa}{kQ}\left(1+\tfrac{1}{k'}\right)$$

hincque sequitur

$$A>0,\quad B\left(1-\tfrac{1}{Q}\right)>0,\quad BC<0.$$

Porro erit

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{\mathfrak{M}-1},$$

ut fiat

$$z=Ma\xi.$$

Hincque distantia oculi

$$O=\frac{\mathfrak{s}s}{M\mathfrak{M}},$$

ubi, ut campus reddatur maximus, sumi conveniet $\mathfrak{s}=1$, si scilicet lens ocularis utrinque aequalis paretur. Tum autem erit

$$\mathfrak{B}q=-(1+k)M \quad \text{et} \quad \mathfrak{C}\mathfrak{r}=-\ (1+Qk)M-\mathfrak{q}.$$

Si hic ut ante brevitatis gratia scribatur

$$\frac{1+k}{\mathfrak{B}}=\zeta$$

ut sit

$$\mathfrak{q}=-\zeta M$$

tum igitur erit

$$\mathfrak{C}\mathfrak{r}=-(1+Qk)M+\zeta M \quad \text{et} \quad \mathfrak{r}=\frac{(\zeta-Qk-1)M}{\mathfrak{C}}$$

hincque

$$\mathfrak{q}+\mathfrak{r}=\frac{\left(\zeta(1-\mathfrak{C})-Qk-1\right)M}{\mathfrak{C}}$$

et

$$\mathfrak{q}+\mathfrak{r}+1=\frac{\mathfrak{C}+\left(\zeta(1-\mathfrak{C})-Qk-1\right)M}{\mathfrak{C}}$$

Inde vero est

$$\mathfrak{q}+\mathfrak{r}+1=M\left(\mathfrak{M}-1\right),$$

unde sequitur

$$M=\frac{\mathfrak{C}}{(\mathfrak{M}-1)\mathfrak{C}-\zeta(1-\mathfrak{C})+Qk+1};$$

ubi ergo haec quantitas

$$\mathfrak{M}-1-\zeta\cdot\frac{(1-\mathfrak{C})}{\mathfrak{C}}+\frac{Qk+1}{\mathfrak{C}}$$

debet esse positiva et tam parva, quam circumstantiae permittunt
Deinde vero ut margo coloratus evanescat, habetur haec aequatio:

$$0=\frac{\mathfrak{q}}{P}+\frac{\mathfrak{r}}{PQ}+\frac{1}{PQR},$$

ex qua reperitur

$$\frac{1}{k'}=Q\mathfrak{q}+\mathfrak{r},$$

quae quantitas debet esse positiva. Cum igitur sit $kQk' = \mathfrak{M}$, erit

$$\mathfrak{M} = \frac{kQ}{Q\mathfrak{q}+\mathfrak{r}} \text{ hincque } kQ = \mathfrak{M}\left(Q\mathfrak{q}+\mathfrak{r}\right).$$

Antequam autem hanc formulam prosequamur, plurimum intererit investigare, num forte $\mathfrak{t}$ possit poni $=1$. Statuamus igitur $\mathfrak{t}=1$, ut sit

$$M = \frac{\mathfrak{q}+2}{\mathfrak{M}-1},$$

unde ob $\mathfrak{q} = -\zeta M$ fiet

$$\mathfrak{q} = \frac{-2\zeta-\zeta\mathfrak{q}}{\mathfrak{M}-1}$$

adeoque

$$\mathfrak{q} = -\frac{2\zeta}{\mathfrak{M}+\zeta-1}$$

hincque

$$M = +\frac{2}{\mathfrak{M}+\zeta-1};$$

altera vero aequatio iam dabit

$$\mathfrak{C} = \frac{-2(1+Qk)+2\zeta}{\mathfrak{M}+\zeta-1} = \frac{2(\zeta-Qk-1)}{\mathfrak{M}+\zeta-1}$$

Deinde cum sit $kQ = \mathfrak{M}Qq + \mathfrak{M}$, fiet nunc

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}\zeta Q}{\mathfrak{M}+\zeta-1},$$

unde invenitur *k*, dummodo sit

$$1 > \frac{2\zeta Q}{\mathfrak{M}+\zeta-1}, \text{ hoc est } \quad Q < \frac{\mathfrak{M}+\zeta-1}{2\zeta}$$

Porro autem fiet

$$\mathfrak{B} = \frac{1+k}{\zeta} \text{ et [proxima] } \mathfrak{C} = \frac{4\mathfrak{M}\zeta q}{(\mathfrak{M}+\zeta-1)^2} - 2;$$

si igitur fuerit $\mathfrak{B} > 1$, ut sit $B < 0$, sumi debet $Q < 1$; tum vero C debet esse positivum ideoque etiam $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$; quocirca debet esse

$$1.\ 2\mathfrak{M}\zeta Q > (\mathfrak{M}+\zeta-1)^2,$$

$$2.\ 2\mathfrak{M}\zeta Q < \tfrac{3}{2}(\mathfrak{M}+\zeta-1)^2,$$

quae conditio illi, qua debet esse $Q < 1$, manifesta repugnat. Debet ergo esse $B > 0$ hincque $Q > 1$, tum autem esse debet $C < 0$; quod eveniet, si fuerit $\mathfrak{C}$ etiam negativum, id est, si fuerit

$$Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta},$$

quae conditio cum praecedente $Q > 1$ facile consistere potest. Tum autem esse debet $\mathfrak{B} < 1$ ideoque $\zeta > 1+k$. Tantum igitur campum obtinebimus, si capiamus $\zeta > 1+k$, litteram Q vero intra limites

$$1 \text{ et } \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta},$$

dummodo fuerit

$$kQ = \mathfrak{M} - \frac{2\mathfrak{M}\zeta Q}{\mathfrak{M}+\zeta-1}$$

adeoque

$$Q < \frac{\mathfrak{M}+\zeta-1}{2\zeta},$$

quia est kQ positivum; quod sponte fit per conditionem praecedentem, qua est $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$. Praeterea autem debet esse

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1};$$

quia autem esse debet $\zeta > 1+k$, habebimus nunc

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1},$$

unde sequitur haec conditio:

$$Q > \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{\zeta^2+\zeta(3\mathfrak{M}-2)},$$

quae conditio cum illa $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$ egregie consistit.

Cum autem praeterea esse debeat $\zeta > 1+k$, loco k substituendo eius valorem fiet

$$\zeta > 1 + \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M}+\zeta-1},$$

unde concluditur esse debere

$$Q > \frac{\mathfrak{M}(\mathfrak{M}+\zeta-1)}{\zeta^2+\zeta(3\mathfrak{M}-2)-\mathfrak{M}+1};$$

quia autem modo vidimus esse $Q < \frac{(\mathfrak{M}+\zeta-1)^2}{2\mathfrak{M}\zeta}$, hic limes illo debet esse maior; unde colligitur

$$\zeta^3 + \zeta^2(4\mathfrak{M}-3) + \zeta\left(\mathfrak{M}^2 - 6\mathfrak{M}+3\right) > (\mathfrak{M}-1)^2.$$

unde patet, si $\mathfrak{M}$ sit numerus praemagnus, esse debere $\zeta > 1$. Statuamus ergo

$$\zeta = 1 + \frac{\alpha}{\mathfrak{M}},$$

unde fit

$$\mathfrak{M}^2 + \mathfrak{M}(\alpha - 2) + 2\alpha + 1 > \mathfrak{M}^2 - 2\mathfrak{M} + 1$$

hincque porro

$$\alpha(\mathfrak{M} + 2) > 0;$$

quod cum semper eveniat, patet, dummodo $\zeta > 1$, solutionem semper locum habere; ac si in illis formulis $\mathfrak{M}$ ut numerus praemagnus spectetur, limites pro Q erunt

$$Q > \frac{\mathfrak{M}}{3\zeta - 1} \text{ et } Q < \frac{\mathfrak{M}}{2\zeta}$$

sumtaque Q his limitibus convenienter erit porro

$$k = \frac{\mathfrak{M}}{Q} - \frac{2\mathfrak{M}\zeta}{\mathfrak{M} + \zeta - 1}$$

hincque reliqua elementa omnia facillime definientur. Ceterum in evolutione sequentium casuum haec clariora reddentur. Quod denique ad lentium aperturas attinet, eas pro quovis casu ex cognitis formulis facile definira licet.

COROLLARIUM 1

242. Videamus vero, quomodo omnes hae conditiones clarius evolvi queant. Ac primo quidem, statim ac statuimus $\mathfrak{r} = 1$, fit

$$M = \frac{2 + \mathfrak{q}}{\mathfrak{M} - 1} \text{ ideoque } 2 + \mathfrak{q} = M(\mathfrak{M} - 1).$$

Posito autem

$$\frac{1 + k}{\mathfrak{B}} = \zeta,$$

ut sit

$$\mathfrak{B} = \frac{1 + k}{\zeta},$$

fiet $\mathfrak{q} = -\zeta M$; qui valor ibi substitutus dat

$$2 - \zeta\mathfrak{M} = M(\mathfrak{M} - 1)$$

hincque

$$M = \frac{2}{\mathfrak{M} + \zeta - 1} \text{ et } \mathfrak{q} = -\frac{2\zeta}{\mathfrak{M} + \zeta - 1};$$

deinde vero invenimus

$$kQ = \mathfrak{M}Q\mathfrak{q} + \mathfrak{M},$$

ubi valor ipsius q substitutus dat

$$kQ=\mathfrak{M}-\frac{2\mathfrak{M}Q\zeta}{\mathfrak{M}+\zeta-1}$$

sive

$$kQ\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}Q\zeta=\mathfrak{M}\left(\mathfrak{M}+\zeta-1\right),$$

unde commode deducitur

$$Q=\frac{\mathfrak{M}\left(\mathfrak{M}+\zeta-1\right)}{k\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta}.$$

unde fit

$$1+kQ=\frac{k\left(\mathfrak{M}+1\right)\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta}{k\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta}.$$

Cum igitur sit

$$\mathfrak{C}=M\left(\zeta-Qk-1\right),$$

erit

$$\mathfrak{C}=\frac{2\mathfrak{M}\zeta\left(\zeta-1\right)-k\left(\mathfrak{M}+\zeta-1\right)\left(\mathfrak{M}-\zeta+1\right)}{k\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta}\cdot M,$$

seu

$$\mathfrak{C}=\frac{4\mathfrak{M}\zeta\left(\zeta-1\right)-2k\left(\mathfrak{M}+\zeta-1\right)\left(\mathfrak{M}-\zeta+1\right)}{\left(k\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta\right)\left(\mathfrak{M}+\zeta-1\right)}.$$

COROLLARIUM 2

243. Iam ratione litterae $\mathfrak{B}$ duo casus sunt considerandi, alter, quo $\mathfrak{B}>1$ ideoque $B<0$, alter vero, quo $\mathfrak{B}<1$ ideoque $B>0$. Priori casu erit $\zeta<1+k$, et quia B est minus nihilo, ob secundum intervallum debet esse $Q<1$; unde fit

$$\mathfrak{M}\left(\mathfrak{M}+\zeta-1\right)<k\left(\mathfrak{M}+\zeta-1\right)+2\mathfrak{M}\zeta,$$

id quod fieri nequit, cum sit $\mathfrak{M}$ numerus valde magnus.

COROLLARIUM 3

243[a]. Cum igitur esse nequeat $\mathfrak{B}>1$, statuamus $\mathfrak{B}<1$ sive $\zeta>1+k$, et quia iam $B>0$, debebit esse $Q>1$; id quod sponte evenit pro maioribus scilicet multiplicationibus, ad quas hic solas attendimus. Tum autem esse debet $C<0$, id quod evenit, vel si fuerit $\mathfrak{C}<0$ vel $\mathfrak{C}>1$. Priori casu, si $\mathfrak{C}<0$, debebit esse

$$k\left(\mathfrak{M}+\zeta-1\right)\left(\mathfrak{M}-\zeta+1\right)>2\mathfrak{M}\zeta\left(\zeta-1\right)$$

sive

$$k>\frac{2\mathfrak{M}\zeta\left(\zeta-1\right)}{\left(\mathfrak{M}+\zeta-1\right)\left(\mathfrak{M}-\zeta+1\right)}.$$

Ex illa vero conditione $\zeta > 1+k$ debet esse $k < \zeta - 1$; unde porro colligitur esse debere

$$\zeta - 1 > \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)}$$

seu

$$(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1) > 2\mathfrak{M}\zeta,$$

quod etiam semper evenit, ita ut littera ζ arbitrio nostro relinquatur, dummodo unitate maior accipiatur. Cum autem sit $M = \frac{2}{\mathfrak{M}-\zeta+1}$ campi magnitudo postulat, ut ζ quam minime unitatem superet.

COROLLARIUM 4

244. Examinandus restat alter casus, quo debet esse $\mathfrak{C} > 1$; tum ergo esse deberet ante omnia numerator positivus seu

$$2\mathfrak{M}\zeta(\zeta-1) > k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)$$

ideoque

$$k < \frac{2\mathfrak{M}\zeta(\zeta-1)}{(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1)};$$

deinde, ut etiam unitatem superet, debet esse

$$4\mathfrak{M}\zeta(\zeta-1) - 2k(\mathfrak{M}+\zeta-1)(\mathfrak{M}-\zeta+1) > k(\mathfrak{M}+\zeta-1)^2 + 2\mathfrak{M}\zeta(\mathfrak{M}+\zeta-1)$$

ideoque

$$k < \frac{2\mathfrak{M}\zeta(\mathfrak{M}-\zeta-1)}{(\mathfrak{M}+\zeta-1)(3\mathfrak{M}-\zeta+1)},$$

quod cum sit absurdum ob $k > 0$, indicio est hunc casum locum habere non posse, ita ut nobis solus casus in corollario praecedente evolutus relinquatur.

EXEMPLUM 1

245. Quoniam positio $\mathfrak{r} = 1$ ad campum maxime est accommodata simulque solutionem tam facilem suppeditat, ea utique sola meretur, ut ad praxin adplicetur. Hic autem primum observari convenit nequaquam sumi posse $\zeta = 1$, quia tum foret $k = 0$ statimque primum intervallum $= \infty$, ut reliqua incommoda taceamus, dum scilicet tam secunda quam tertia lens haberent distantias focales infinitas. Ex quo necesse est pro ζ sumi numerum unitate maiorem, ita ut excessus non sit nimis parvus, quia alioquin ad eadem incommoda appropinquaremus. Quamobrem, quo clarius appareat, quomodo in hoc negotio sit procedendum, sumamus $\zeta = 2$, ut fiat $M = \frac{2}{\mathfrak{M}+1}$. Tum vero k contineri debet intra hos limites

$$1 \text{ et } \tfrac{4\mathfrak{M}}{(\mathfrak{M}+1)(\mathfrak{M}-1)} \quad \text{vel } 1 \text{ et } \tfrac{4}{\mathfrak{M}}.$$

Neque autem k ad unitatem nimis prope accedera debet, quod alioquin B hincque secundum intervallum nimis evaderet magnum. Sumamus igitur $k=\tfrac{1}{2}$; hinc erit

$$Q=\tfrac{2\mathfrak{M}(\mathfrak{M}+1)}{9\mathfrak{M}+1}=\tfrac{2}{9}\mathfrak{M} \text{ proxime.}$$

Deinde vero erit

$$\mathfrak{B}=\tfrac{3}{4} \text{ et } B=3$$

ac denique

$$\mathfrak{C}=\tfrac{16\mathfrak{M}-2(\mathfrak{M}+1)(\mathfrak{M}-1)}{(9\mathfrak{M}+1)(\mathfrak{M}+1)}=-\tfrac{2}{9}+\tfrac{16}{9\mathfrak{M}}=-\tfrac{2}{9} \text{ proxime.}$$

hincque $C=-\tfrac{1}{11}$; unde pro microscopii constructione habebimus has distantias focales :

$$p=\mathfrak{A}a, \quad q=\tfrac{3}{2}Aa, \quad r=\tfrac{6}{\mathfrak{M}}\cdot Aa, \quad s=\tfrac{6}{11}\cdot\tfrac{Aa}{\mathfrak{M}}$$

et intervalla

$$\text{primum}=3Aa, \quad \text{secundum}=6Aa\left(1-\tfrac{9}{2\mathfrak{M}}\right), \quad \text{tertium}=\tfrac{60}{11}\cdot\tfrac{Aa}{\mathfrak{M}};$$

tum vero distantia oculi

$$O=\tfrac{s}{\mathfrak{M}M}=\tfrac{s(\mathfrak{M}+1)}{2\mathfrak{M}}=\tfrac{1}{2}s\left(1+\tfrac{1}{\mathfrak{M}}\right)=\tfrac{1}{2}s \text{ proxime.}$$

Pro campo autem apparente

$$z=Ma\xi=\tfrac{2a\xi}{\mathfrak{M}+1}=\tfrac{1}{2}\cdot\tfrac{a}{\mathfrak{M}+1} \quad \text{ob} \quad \xi=\tfrac{1}{4}.$$

Quod vero ad litteram A attinet, quae ad primam lentem refertur, curandum est, ut A tantus fiat numerus, ut lens ocularis non fiat nimis exigua; unde sequitur $\mathfrak{A}$ esse debere fractionem parum ab unitate deficientem; cuius valore stabilito apertura primae lentis definiri debet ex aequatione nota

$$\tfrac{1}{k^3}=\tfrac{\mu\mathfrak{M}x^3}{a^3}\left(\tfrac{\lambda}{\mathfrak{A}^3}+\tfrac{\nu}{A\mathfrak{A}}+\text{etc.}\right)$$

sicque obtinemus microscopium satis notatu dignum, quod instar primi exempli spectari potest.

EXEMPLUM 2

246. Consideremus etiam casum $\zeta = 3$, ut sit $M = \frac{2}{\mathfrak{M}+2}$, et pro k habebuntur hi limites 2 et $\frac{12}{\mathfrak{M}}$. Statuamus ergo $k = 1$, ut fiat $\mathfrak{B} = \frac{2}{3}$ et $B = 2$. Tum vero erit $Q = \frac{\mathfrak{M}}{7}$ et $k' = 7$. Porro vero erit $\mathfrak{C} = -\frac{2}{3}$ et $C = -\frac{2}{9}$, unde pro his microscopiis erunt distantiae focales

$$p = \mathfrak{A}a, \quad q = \tfrac{2}{3}Aa, \quad r = 4\tfrac{Aa}{\mathfrak{M}} \quad \text{et} \quad s = \tfrac{4}{9}\cdot\tfrac{Aa}{\mathfrak{M}}$$

et intervalla

$$\text{primum} = 2Aa, \quad \text{secundum} = 2Aa\left(1-\tfrac{7}{\mathfrak{M}}\right) \quad \text{et} \quad \text{tertium} = \tfrac{32}{9}\cdot\tfrac{Aa}{\mathfrak{M}}$$

et distantia oculi

$$O = s\cdot\tfrac{\mathfrak{M}+2}{2\mathfrak{M}} = \tfrac{1}{2}s\left(1+\tfrac{2}{\mathfrak{M}}\right) = \tfrac{1}{2}s \ \text{ proxime.}$$

Pro campo vero

$$z = \tfrac{1}{2}\cdot\tfrac{a}{\mathfrak{M}+2}.$$

Circa aperturam modo allegata valent. Ratione autem litterae q erit casu exempli praecedentis $\mathfrak{q} = -\frac{4}{\mathfrak{M}+1}$ et casu huius exempli $\mathfrak{q} = -\frac{6}{\mathfrak{M}+2}$; unde patet secundae lentis semidiametrum aperturae esse debere $= \frac{x}{P} = \frac{x}{k}$ ideoque priori casu $= 2x$, hoc vero $= x$. Binae postremae lentes autem fieri debent utrinque aeque convexae.

SCHOLION 1

247. Si haec ad telescopia referamus, quod nunc eo magis est necessarium, quoniam supra huius generis casum tantum maxime particularem evolvimus, qui ne campum quidem maximum, ut hic fecimus, praebebat, tantum faciamus $a = \infty$ et $\mathfrak{M} = m$ ob $h = a$. Tum autem capi debet tam $\mathfrak{A} = 0$ quam $A = 0$, ita ut fiat $\mathfrak{A}a = Aa = p$; quare, cum reliqua omnia maneant ut ante, ex exemplo priore posita lentis obiectivae distantia focali $= p$ erunt reliquarum lentium distantiae focales

$$q = \tfrac{3}{2}p, \quad r = \tfrac{6}{m}p, \quad \text{et} \quad s = \tfrac{6}{11}\cdot\tfrac{p}{m},$$

intervalla vero

$$\text{primum} = 3p, \quad \text{secundum} = 6p\left(1-\tfrac{9}{2m}\right) \quad \text{et} \quad \text{tertium} = \tfrac{60}{11m}\cdot p$$

et

$$O = \tfrac{1}{2}s.$$

Tum vero campi semidiameter

$$\Phi = \tfrac{1}{2}\cdot\tfrac{1}{m+1} = \tfrac{1718}{m+1} \ \text{min.}$$

Deinde vero ob

$$\mathfrak{B} = \tfrac{3}{4}, \quad B = 3, \quad \mathfrak{C} = -\tfrac{2}{9} \quad \text{et} \quad C = -\tfrac{2}{11}$$

distantia focalis p ex requisita apertura $x = \frac{1}{50}m$ dig. definiri debet ope huius aequationis:

$$p = m\sqrt[3]{m\mu}\left(\lambda + 2\left(\tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{\nu}{B\mathfrak{B}}\right) - \tfrac{9}{mB^3}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right) - \tfrac{\lambda'''}{mB^3C^3}\right).$$

In casu autem alterius exempli erunt distantiae focales

$$q = \tfrac{2}{3}p, \quad r = \tfrac{4}{m}p, \quad \text{et} \quad s = \tfrac{4}{9m}p$$

et intervalla

$$\text{primum} = 2p, \quad \text{secundum} = 2p\left(1 - \tfrac{7}{2m}\right) \;\text{ et }\; \text{tertium} = \tfrac{32}{9m}p$$

tum vero p ita definietur, ut sit

$$p = m\sqrt[3]{m\mu}\left(\lambda + \tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{\nu}{B\mathfrak{B}} - \tfrac{7}{mB^3}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right) - \tfrac{\lambda'''}{mB^3C^3}\right),$$

reliqua vero erunt ut ante.

Hinc igitur loco communium telescopiorum terrestrium nansciscimur ex casu posteriore sequentem constructionem, siquidem omnes quatuor lentes ex vitro communi, cuius refractio sit $n = 1,55$, conficere velimus.

CONSTRUCTIO TELESCOPIORUM LOCO VULGARIUM TERRESTRIUM SUBSTITUENDORUM

248. Pro data multiplicatione m quaeratur primo lentis obiectivae distantia focalis $= p$, ex hac nempe formula, ad quam praecedens proxima reducitur:

$$p = \tfrac{8}{5}m\sqrt[3]{m\mu}\ \text{dig.};$$

deinde constructrio ita se habebit:

I. Pro prima lente, cuius distantia focalis est $x = \frac{1}{50}m$ dig. , capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,6145p\\ \text{posterioris} = 5,2438p,\end{cases}$$

eius aperturae semidiameter $x = \frac{1}{50}m$ dig.
et distantia ad lentem sequentem $= 2p$.

II. Pro secunda lente, cuius distantia focalis est $q = \frac{2}{3}p$ et numeri $\mathfrak{B} = \frac{2}{3}$ et $\lambda' = 1$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= \frac{q}{0,6696} = 0,99562\,p\\ \text{posterioris} &= \frac{q}{1,1485} = 0,58047\,p,\end{cases}$$

eius aperturae semidiameter $= x = \frac{1}{50} m$ dig.
et intervallum ad tertiam lentem $= 2p(1 - \frac{7}{m})$.

III. Pro tertia lente, cuius distantia focalis est $r = \frac{4}{m} p$, quoniam ea debet esse utrinque aequaliter convexa, capiatur

eius uterque radius $= 1,1r = 4,4 \cdot \frac{p}{m}$,
eius aperturae semidiameter $= \frac{p}{m}$
et distantia ad quartam lentem $= \frac{32}{9m} p$.

IV. Pro quarta lente, cuius distantia focalis est $s = \frac{4}{9} \cdot \frac{p}{m}$, capiatur itidem
uterque radius $= \frac{22}{45} \cdot \frac{p}{m}$,
eius aperturae semidiameter $= \frac{1}{9} \cdot \frac{p}{m}$
et distantia ad oculum $= \frac{1}{2} s$.

V. Tum vero erit semidiameter campi $\Phi = \frac{1718}{m+2}$ min.

SCHOLION 2

249. Telescopia haec utique insigni vitio laborant, propterea quod eorum longitudo fit plane enormis, maior scilicet quam $3p$. Huic autem vitio medela afferri poterit litterae k maiorem valorem tribuendo; tum vero etiam littera ζ maior accipi debebit; unde quidem campus aliquantillum diminuitur, qui tamen defectus in maioribus multiplicationibus vix percipietur. Sumamus igitur $\zeta = 6$, ut fiat $M = \frac{2}{\mathfrak{M}+5}$, et cum limites pro k sint 5 et $\frac{60}{m}$, sumamus $k = 4$, ut fiat $\mathfrak{B} = \frac{5}{6}$ et $B = 5$; tum vero erit $Q = \frac{m}{16}$ et $k' = 4$ tandemque $\mathfrak{C} = -\frac{1}{2}$ et $C = -\frac{1}{3}$; hinc autem erit intervallum

$$\text{primum} = \frac{5}{4} p, \quad \text{secundum} = \frac{5}{4} p\left(1 - \frac{16}{m}\right) \quad \text{et} \quad \text{tertium} = \frac{25}{3} \cdot \frac{p}{m};$$

unde longitudo prodiret quasi $2\frac{1}{2} p$, quae adhuc nimis magna videri potest.
Hanc longitudinem autem non mediocriter diminuera poterimus sumendo $\zeta = 12$ et $k = 9$; hinc enim fit $\mathfrak{B} = \frac{5}{6}$ et $B = 5$ ut ante; unde sequitur intervallum

$$\text{primum} = \tfrac{10}{9}p \text{ et secundum} = \tfrac{5}{9}p,$$

ita ut tota longitudo quasi fiat $1\frac{2}{3}p$, quae non excedit telescopia huius generis vulgaria. Si sumsissemus $\zeta = 12$ et $k = 8$, ut fiat $\mathfrak{B} = \frac{3}{4}$ et $B = 3$, foret intervallum

$$\text{primum} = \tfrac{9}{8}p \text{ et secundum} = \tfrac{3}{8}p,$$

ita ut tota longitudo quasi sit $1\frac{1}{2}p$, quae utique admitti poterit. Hic ergo casus meretur, ut plenius evolvatur.

Fiet autem porro $Q = \frac{m}{32}$ hincque $k' = 4$. Porro $\mathfrak{C} = -\frac{1}{2}$ et $C = -\frac{1}{3}$;
unde

$$q = \tfrac{3}{32}p, \quad r = 6\tfrac{p}{m} \quad \text{et} \quad s = \tfrac{p}{m};$$

tum vero intervalla

$$\text{primum} = \tfrac{9}{8}p, \quad \text{secundum} = \tfrac{3}{8}p\left(1 - \tfrac{32}{m}\right), \quad \text{tertium} = 5\tfrac{p}{m}$$

et pro loco oculi $O = \frac{s}{2}\left(1 + \frac{11}{m}\right)$,
semidiameter campi $= \frac{1718}{m+2}$ min.
Sumto igitur pro apertura lentis obiectivae $x = \frac{m}{50}$ dig. et $k = 50$ capi debebit circiter

$$p = m\sqrt[3]{m\tfrac{4}{3}} = \tfrac{11}{10}m\sqrt[3]{m} \text{ dig.};$$

unde conficitur sequens

CONSTRUCTIO TELESCOPII COMMUNIS EX VITRO COMMUNI

I. Pro data multiplicatione m sumatur

$p = \frac{11}{10}m\sqrt[3]{m}$ circiter sive etiam $p = m\sqrt[3]{m}$ dig.

II. Pro prima lente, cuius distantia focalis $= p$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,6145p\\ \text{posterioris} = 5,2438p,\end{cases}$$

eius aperturae semidiameter $= \frac{m}{50}$ dig.
et distantia ad lentem secundam $= 1\frac{1}{8}p$.
III. Pro lente secunda, cuius distantia focalis est $q = \frac{3}{32}p$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{0,5499} = 0,17053p\\ \text{posterioris} = \frac{q}{1,2682} = 0,07388p,\end{cases}$$

eius aperturae semidiameter $= \frac{1}{8}x = \frac{m}{400}$ dig.
et distantia ad lentem tertiam $= \frac{3}{8}p\left(1 - \frac{32}{m}\right)$.

IV. Pro lente tertia, cuius distantia focalis est $r = 6\frac{p}{m}$, capiatur
uterque radius $= 6,6\frac{p}{m}$,
eique apertura maxima tribuatur,
distantia vero a lente quarta erit $= 5\frac{p}{m}$

V. Pro lente quarta, cuius distantia focalis $s = \frac{p}{m}$, capiatur

uterque radius $= 1,1\frac{p}{m}$

eiusque ab oculo distantia $= \frac{s}{2}\left(1 + \frac{11}{m}\right)$.

VI. Longitudo erit $1\frac{1}{2}p - 6\frac{1}{2}\cdot\frac{p}{m}$.
Campi vero apparentis semidiameter erit $= \frac{1718}{m+11}$ min.

Hoc ergo telescopium vulgaribus terrestribus merito anteferendum videtur; notetur vero id in praxi locum habere non posse, nisi sit *m* notabiliter maius quam 32.

Hic autem istud caput finimus ad sequens progressuri, ubi microscopia magis composita huius generis investigabimus.