CHAPTER III

CONCERNED WITH THE GREATEST PERFECTION OF THESE MICROSCOPES WHEREBY THEY MAY BE FREED FROM ALL CONFUSION

PROBLEM 1

308. *If the objective lens may be constructed from four convex lenses situated close to each other, such as are described above in problem* 4 *of the second section of the preceding chapter* [§ 177], *truly the remaining lenses shall be put in order thus, as we have described in the preceding chapter of this section, all confusion arising from the apertures of the lenses to be removed.*

SOLUTION

We have assumed we may treat these four lenses described in the place mentioned as equal to a single lens, to be substituted in place of the objective lens. Therefore since here the letter A shall pertain to the first lens, just as that enters into the determinations of the following lenses, the same may indicate, which was being indicated by the product $-ABCD$ in the place mentioned. But above we have designated this product by the letter θ, from which by regressing to the former place there θ will be the same as which is indicated here by us as by the A. Therefore we will designate the four focal lengths of these lenses by the letters p', p'', p''', p'''', and with whatever Germanic letters it may be agreed, which shall be $\mathfrak{A}'$, $\mathfrak{A}''$, $\mathfrak{A}'''$, $\mathfrak{A}''''$, from our A on account of $\theta = A$ we will have

$$\mathfrak{A}' = \frac{4A}{A+1}, \quad \mathfrak{A}'' = \frac{3A-1}{A+1}, \quad \mathfrak{A}''' = \frac{2A-2}{A+1}, \text{ and } \mathfrak{A}'''' = \frac{A-3}{A+1};$$

and from these Germanic letters , just as they are referring to the individual four lenses, together with the numbers λ, which for the individual lenses may be taken equal to unity, these individual lenses can be constructed by known formulas. But before we may be able to assign these focal lengths themselves, we will have to consider the minimum interval between these lenses; which since there it was put $= \zeta p$, lest this letter ζ may appear confusing in our formulas, we may put this interval $= \delta p'$; so that the letters used there P, Q, R, S may be defined thus :

$$\frac{1}{P} = 1 + \frac{(3A-1)\delta}{A+1}, \quad \frac{1}{PQ} = 1 + \frac{(5A-3)\delta}{A+1}, \quad \frac{1}{PQR} = 1 + \frac{(6A-6)\delta}{A+1};$$

from which the focal lengths themselves will be :

$$p' = \frac{4A}{A+1}\cdot a,\ \ p'' = \frac{4A}{A+1}\cdot a + \frac{4A(3A-1)\delta a}{(A+1)^2},$$
$$p''' = \frac{4A}{A+1}\cdot a + \frac{4A(5A-1)\delta a}{(A+1)^2},\ \ p'''' = \frac{4A}{A+1}\cdot a + \frac{4A(6A-6)\delta a}{(A+1)^2}.$$

With these observed, which above was *PQRS,* this is expressed by us with the letter *P* alone, thus so that now the interval from the objective lens as far as to our second lens shall be

$$Aa\left(1+\frac{6(A-1)\delta}{A+1}-\frac{1}{P}\right);$$

which therefore now may be considered as our first interval. Therefore on account of the magnification of the objective lens this first interval allows a certain alteration to arise from the fraction δ; indeed if the first lens were simple, this interval would be only $Aa(1-\frac{1}{P})$, but now that will be

$$Aa\left(1-\frac{1}{P}\right)+\frac{(6A-6)\delta A}{A+1}.$$

But since this increase generally is very small, that will be able to be ignored easily. Moreover the remaining intervals proceed in the established order; clearly the second will become

$$= -ABa\left(\frac{1}{P}-\frac{1}{PQ}\right) \text{ etc.}$$

and thence also the following focal lengths, clearly

$$q = -A\mathfrak{B}\cdot\frac{a}{P},\ \ r = AB\mathfrak{C}\cdot\frac{a}{PQ} \text{ etc.}$$

Now therefore the whole procedure returns to this, so that the confusion arising from the aperture of the lens may be reduced to zero, which happens, as we have found in the place mentioned, with the aid of this equation:

$$0 = \frac{(1+A)^3}{16A^3} - \frac{v(1+A)(5A^2-6A+5)}{16A^3} + \frac{\delta(1+A)^2(7A-5)}{32A^3}$$
$$-\frac{\delta v(1-A)(7A^2-18A+23)}{16A^3} - \frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{v}{B\mathfrak{B}}\right) + \frac{1}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{v}{C\mathfrak{C}}\right) \text{ etc.};$$

from which equation the fraction δ, from which the extremely small intervals between the objective lenses are determined, can be defined conveniently, but only if the letter A may denote a large enough number such as 60 and the lenses of the objective may be prepared from this kind of glass, for which there shall $v > \frac{1}{5}$. Indeed above we have shown [§ 182], if there shall become $A = 60$ and common glass may be used, for which there shall be

$n = 1,55$, then there becomes $\delta = \frac{1}{18}$, since in the case $A = 60$ the fractions arise from the following lenses as if they may vanish. Therefore the objective lens of this kind will be able to be composed from all the previous forms of microscopes, in which clearly the letter A will be present, and a value around 60 may be given to that. But when this happens in use, indeed there will be no need to be concerned further about this value of A ; for the following precept becomes known from the single lens construction, since that can be done without notification of the letter δ, the four intervals of the objective lenses may be left undefined and that at last thus will be determined by experiment, so that the confusion may become as small as possible, unless perhaps plainly zero. But then the radius of the aperture x of these lenses becomes known at once from the form which these lenses are capable of adopting, and thence the measure of the clarity $= \frac{20x}{\mathfrak{M}}$, with there being $\mathfrak{M} = \frac{ma}{h}$. But all the remaining moments present will remain the same, as has been set out in the previous chapter.

COROLLARY 1

309. Therefore if there were $A = 60$ and the four lenses of the objective may be made from common glass, for which there is $n = 1,55$, following the above [§ 183], we find the following construction of these lenses :

I. For the first lens

of which the focal length is $= 3,9333a$, the radius of the

$$\begin{cases} \text{anterior face} = -0,97756a \\ \text{posterior face} = +0,67332a, \end{cases}$$

the radius of the aperture $= 0,16833a$.

II. For the second lens

of which the focal length $= 4,5740a$, the radius of the

$$\begin{cases} \text{anterior face} = -1,7682a \\ \text{posterior face} = +1,0384a. \end{cases}$$

III. For the third lens

of which the focal length $r = 4,9965a$, the radius of the

$$\begin{cases}\text{anterior face} &= -4,3440a\\ \text{posterior face} &= +1,6833a.\end{cases}$$

IV. For the fourth lens

of which the focal length $s = 5,2006a$, the radius of the

$$\begin{cases}\text{anterior face} &= 18,1522a\\ \text{posterior face} &= \ \ 3,3955a.\end{cases}$$

Moreover the intervals between these four lenses will be able to be taken $= 0,2185a$, but that will be better to be defined by experiment. Then truly the radius of the aperture taken will be $x = 0,16833a$, from which the measure of the clarity

$$= \frac{20x}{\mathfrak{M}} = \frac{3,3666a}{\mathfrak{M}} = \frac{26,9328}{\mathfrak{M}},$$

clearly if the distance a may be expressed in inches.

COROLLARY 2

310. It is easily understood, even if the letter A may be taken either a little greater or less than 60, yet in the construction of these lenses scarcely any variation is going to arise, which certainly may be observed in practice ; only this will be required to be observed here, so that if there may be $A < 60$, then these lenses no longer can be properly put in place among themselves, as the destruction of the confusion demands. But if there shall be $A > 60$, then that matter will succeed there more beneficially.

SCHOLION

311. In general moreover it may be observed, where the number A were greater, there the construction would succeed better, since then the intervals of these lenses will be found to be not too much smaller; so that we may attend to this favorable condition, if perhaps several lenses may be prepared thus so far settling the confusion well enough, so that also these lenses may be more effective by being placed closer together. But in the final scholium of the preceding chapter [§ 807] the case occurred, where there became $A = \infty$ and $\mathfrak{B} = B = 0$, thus so that there must become $AB = -\frac{q}{a}$; therefore according to this case our quadruple objective lens will be able to be applied most conveniently; then indeed there will have to become

$$0 = \tfrac{1}{16} - \tfrac{5}{16}v + \tfrac{7}{32}\delta - \tfrac{7}{16}\delta v + \tfrac{a^3\lambda'}{q^3} + \tfrac{a^3}{3q^3}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{v}{C\mathfrak{C}}\right) \text{ etc.,}$$

from which the fraction δ is required to be defined; but then the four focal lengths of the first four lenses will be

$$p' = 4a, \ \ p'' = 4a + 12\delta a, \ \ p''' = 4a + 20\delta a, \ \ p'''' = 4a + 24\delta a;$$

which therefore certainly will depend also on the fraction δ, and where above we have said the lenses can be constructed without knowledge of δ, that is required to be understood from a more accurate value of that ; for in practice it suffices to know its value approximately. Then truly the values pertaining to these Germanic letters will become :

$$\mathfrak{A}' = 4, \ \ \mathfrak{A}'' = 3, \ \ \mathfrak{A}''' = 2, \ \ \mathfrak{A}'''' = 1;$$

truly all the letters λ are taken equal to unity. Therefore if we may take all the lenses made from common glass, for which $n = 1,55$, there will become $v = 0,2326$; and if in addition there shall become $\lambda' = 1$ and $\lambda'' = 1$, on account of $\mathfrak{C} = 1$ by resolving approximately our equation will adopt this form:

$$0 = \tfrac{-0,1630 + 1,8718\delta}{16} + \tfrac{4a^3}{3q^3},$$

where the following members, which above have been divided by $\mathfrak{M}$, with care will be allowed to return ; hence therefore we deduce

$$\delta = 0,08708 - 11,397 \cdot \tfrac{a^3}{q^3};$$

from which it is apparent there must become $\frac{q}{a} > 5,0771.$ Therefore if we may assume $q = 10a$, there will become

$$\delta = 0,07569 \ = \tfrac{3}{40} \text{ or } \delta = \tfrac{1}{13} \text{ approximately,}$$

which value is convenient enough in practice, which we will explain further in the following example.

EXAMPLE

312. If all the lenses may be prepared from common glass, in order that with all confusion removed, there must be taken $\delta = \frac{1}{13}$ and $q = 10a$, and the focal lengths of our four objective lenses will become

$$p' = 4a, \ \ p'' = 4,923a, \ \ p''' = 5,538a, \ \ p'''' = 5,846a;$$
$$\mathfrak{A}' = 4 \ , \ \mathfrak{A}'' = 3 \ , \ \mathfrak{A}''' = 2, \ \mathfrak{A}'''' = 1.$$

Thence the intervals between these four lenses will become $=\frac{4}{13}a=0,3077a.$ Then truly the interval fro this objective lens to the second principal lens will be approximately $=10\ \zeta a$, where ζ at this stage is left to our choice, provided there shall be $\zeta<\frac{3}{2}$. Then truly the remaining lenses and the remaining moments will remain evidently, as have been established in the final scholium [§ 307] of the preceding chapter. Hence therefore the individual lenses will be able to be formed and indeed also for the second and third lenses there must be put $\lambda'=1$ and $\lambda''=1$.

Therefore for the first of the objective lenses, the focal length of which is $p'=4a$ and $\mathfrak{A}'=4,$ the radius

$$\begin{cases}\text{of the anterior face } =\frac{p'}{\sigma-\mathfrak{A}'(\sigma-\rho)}=\frac{-p'}{4,1194}=-0,97101a\\ \text{of the posterior face}=\frac{p'}{\rho+\mathfrak{A}'(\sigma-\rho)}=\frac{p'}{5,9375}=+0,67370a.\end{cases}$$

For the second lens, the focal length of which is $p''=\ 4,923a$ and $\mathfrak{A}''=3,$ the radius

$$\begin{cases}\text{of the anterior face } =\frac{p''}{\sigma-\mathfrak{A}''(\sigma-\rho)}=\frac{-p''}{2,6827}=-1,8351a\\ \text{of the posterior face}=\frac{p''}{\rho+\mathfrak{A}''(\sigma-\rho)}=\frac{p''}{4,5008}=+1,0938a.\end{cases}$$

For the third lens, of which the focal length is $p'''=5,538a$ and $\mathfrak{A}'''=2,$ the radius

$$\begin{cases}\text{of the anterior face } =\frac{-p'''}{1,2460}=-4,4447a\\ \text{of the posterior face}=\frac{p'''}{3,0641}=+1,8074a.\end{cases}$$

For the fourth lens, the focal length of which is $p''''=\ 5,846a$ and $\mathfrak{A}''''=1,$ the radius

$$\begin{cases}\text{of the anterior face } =\frac{+p''''}{0,1907}=+30,6560a\\ \text{of the posterior face}=\frac{p''''}{1,6274}=+\ 3,5922a.\end{cases}$$

Truly for the following principal lens, [of which the focal length] is arbitrarily $=q,$ on account of $\mathfrak{B}=0$ and $\lambda'=1,$ the radius

$$\begin{cases}\text{of the anterior face } =\frac{q}{\sigma}=0,61448q\\ \text{of the posterior face}=\frac{q}{\rho}=5,2439q.\end{cases}$$

But for the third principal lens, of which the focal length is

$$r = \tfrac{1}{3}\mathfrak{C}q = \tfrac{1}{3}q,$$

on account of $\mathfrak{C} = 1$ approx. and $\lambda'' = 1$ the radius

$$\begin{cases} \text{of the anterior face} = \frac{q}{\rho} = 1,7479q \\ \text{of the posterior face} = \frac{q}{\sigma} = 0,20483q. \end{cases}$$

With which found, there follows

THE CONSTRUCTION OF A GENERAL MICROSCOPE

PREPARED FROM NINE LENSES MADE FROM COMMON GLASS

313. With the distance of the object taken as it pleases $= a$ the construction will be had thus :

I. For the first lens,

of which the focal length $= 4a$, the radius may be taken

$$\begin{cases} \text{of the anterior face} = -0,97101a \\ \text{of the posterior face} = +0,67370a; \end{cases}$$

which therefore allows an aperture, of which the radius $x = 0,1684a$.
The distance to the second lens $= 0,3077a$.

II. For the second lens,

of which the focal length is $= 4,923a$, the radius

$$\begin{cases} \text{of the anterior face} = -1,8351a \\ \text{of the posterior face} = +1,0938a, \end{cases}$$

of which the aperture is as the first lens and the distance to the third lens $= 0,3077a$.

III. For the third lens,

of which the focal length $= 5,536a$, the radius may be taken

$$\begin{cases} \text{of the anterior lens} = -4,4447a \\ \text{of the posterior} = +1,8074a, \end{cases}$$

the aperture as the first and the distance as the fourth $= 0,3077a$.

IV. For the fourth lens,

of which the focal length $=5,846a$, the radius may be taken

$$\begin{cases}\text{of the anterior face} = 30,6560a\\ \text{of the posterior face} = 3,5922a,\end{cases}$$

the aperture as the first lens, the distance to the fifth lens $=\zeta q$, where ζ is a number less than $\frac{3}{2}$, but q a quantity left to our choice.

V. For the fifth lens,

of which the focal length $=q$, the radius may be found

$$\begin{cases}\text{of the anterior face} = 0,61448q\\ \text{of the posterior face} = 5,2439q,\end{cases}$$

the radius of its aperture $=\frac{3}{4}\cdot\frac{\zeta}{\mathfrak{M}}\cdot q+x$

and the distance to the sixth lens $=\frac{4}{3}\cdot q$.

VI. For the sixth lens,

of which the focal length is $r=\frac{1}{3}q$, the radius

$$\begin{cases}\text{of the anterior face} = 1,7479q\\ \text{of the posterior face} = 0,2048q,\end{cases}$$

the radius of its aperture

$$=\frac{3}{4}\cdot\frac{(\zeta-4)}{\mathfrak{M}}\cdot r+\frac{1}{3}x=\frac{1}{4}\cdot\frac{(\zeta-4)}{\mathfrak{M}}\cdot q+\frac{1}{3}x$$

and the distance to the seventh lens

$$=\frac{1}{3}q\left(1-\frac{3(11-8\zeta)}{2\mathfrak{M}}\right).$$

VII. For the seventh lens,

of which the focal length is $s=\frac{3C}{\mathfrak{M}}\cdot q$

and of which each face must be equally convex, the radius of each face $=-1,1s$

and the radius of the aperture $r=\frac{1}{4}q$,

truly the distance to the eight lens $=\frac{12(3-2\zeta)}{17-8\zeta}\cdot\frac{C}{\mathfrak{M}}\cdot q.$

VIII. For the eighth lens,

of which the focal length $t = \frac{15}{17-8\zeta} \cdot \frac{C}{\mathfrak{M}} \cdot q$, the radius of each face $= 1,1t$ and the radius of the aperture $= \frac{1}{4}t$, truly the distance to the ninth lens $= \frac{15}{4(17-8\zeta)} \cdot \frac{C}{\mathfrak{M}} \cdot q$.

IX. For the ninth lens,

of which the focal length $u = \frac{15}{34-16\zeta} \cdot \frac{C}{\mathfrak{M}} \cdot q$, the radius of each face may be taken $= 1,1u$, the radius of the aperture $= \frac{1}{4}u$, and the distance to the eye will be $= \frac{1}{3}u$.

X. Then truly the radius of the area viewed in the object will be $z = \frac{3a}{4\mathfrak{M}}$, which is discerned with a degree of clarity, of which the measure is $= \frac{20x}{\mathfrak{M}} = \frac{3,368a}{\mathfrak{M}}$.

XI. Moreover here from the magnification put $= m$ there arises $\mathfrak{M} = \frac{ma}{h}$ on taking $h = 8$ in.; therefore the measure of the clarity expressed by m will become $= \frac{26,944}{m}$.

XII. Moreover besides the letters ζ and q will be depending on our choice, C can be taken as wished, provided it shall be a large enough number, and it serves to this end, that the final lenses may not become exceedingly small, clearly if the magnification were very large. But it may be observed, since on account of taking $A = \infty$ these formulas may differ a little from the calculation, errors can be avoided, if both the distance of the object as well as the four first intervals of the lenses must be determined by experimentation.

SCHOLIUM

314. Since this case, where there was taken $A = \infty$, may cause a delay, we may apply our quadruple objective lens to the case in the example following the third case [§ 300], where there was

$$P = \frac{3}{2},\ \mathfrak{B} = -1 \text{ and } B = -\frac{1}{2},$$

since conveniently the value of the letter A will be able to be given large enough. Therefore then the focal lengths of the lenses will remain, as they have been described there, likewise the intervals of these, except that now moreover, the first interval must be prepared for the removal of the confusion, and it will be required to satisfy this equation :

$$Aa\left(1+\frac{6(A-1)\delta}{A+1}-\frac{1}{P}\right) = Aa\left(\frac{1}{3}+\frac{6(A-1)\delta}{A+1}\right) = Aa\left(\frac{1}{3}+6\delta\right) \text{ [approx.];}$$

moreover for removing the confusion, it will be required now for this equation to be satisfied:

$$0=\frac{(1+A)^3}{16A^3}-\frac{v(1+A)(5A^2-6A+5)}{16A^3}+\frac{\delta(1+A)^2(7A-5)}{32A^3}$$
$$-\frac{\delta v(A-1)(7A^2-13A+23)}{16A^3}+\frac{2}{3A^3}(\lambda'-2v)+\frac{8\lambda''}{3A^3},$$

from which the fraction δ will be required to be defined ; where since it shall be more convenient, thus so that the fraction δ may not be found exceedingly small, we suppose all four lenses constituting the objective to be made from flint glass, thus so that there shall be $v=0,2529$, and thus now also there will be no need to put a large number for A ; if indeed we may put $A=50$, this equation will be found, clearly after it had been multiplied by $16A^3$,

$$0=-24768+242679\delta+48;$$

from which there is deduced

$$\delta=\frac{24720}{242679}=\frac{1}{10}\text{ approx.},$$

which value is suitable for the maximum in practice. Therefore by taking $A=50$ and $\delta=\frac{1}{10}$, the focal lengths of the four first lenses will become :

$$p'=3{,}922a,\quad p''=5{,}063a,\quad p'''=5{,}821a,\quad p''''=6{,}183a,$$

to which the numbers are associated:

$$\mathfrak{A}'=3{,}922,\ \ \mathfrak{A}''=2{,}922,\ \ \mathfrak{A}'''=1{,}922,\ \ \mathfrak{A}''''=0{,}922.$$

Truly the interval of these individual lenses is $\frac{1}{10}p'=0{,}392a$, then truly the interval from the objective lens to the second principal lens $=46{,}667a$.
From which therefore the four lenses constituting the objective will be constructed in this manner:

I. For the first lens

the focal length of which is $p'=3{,}922a$ and the number $\mathfrak{A}'=3{,}922=4-\frac{1}{13}$,
the radius

$$\begin{cases}\text{of the anterior face } =\frac{p'}{\sigma-\mathfrak{A}'(\sigma-\rho)}=-\frac{p'}{4{,}0717}=-0{,}9632a\\ \text{of the posterior face}=\frac{p'}{\rho+\mathfrak{A}'(\sigma-\rho)}=\frac{p'}{5{,}7958}=+0{,}6767a.\end{cases}$$

II. For the second lens
the focal length of which is $p'' = 5,063a$ and the number $\mathfrak{A}'' = 2,922 = 3 - \frac{1}{13}$,
the radius

$$\begin{cases} \text{of the anterior face} & = -\frac{p''}{2,6304} = -1,9248a \\ \text{of the posterior face} & = +\frac{p''}{4,3545} = +1,1628a. \end{cases}$$

III. For the third lens
the focal length of which is $p''' = 5,821a$ and $\mathfrak{A}''' = 1,922a$, the radius

$$\begin{cases} \text{of the anterior face} & = -\frac{p'''}{1,1819} = -4,922a \\ \text{of the posterior face} & = +\frac{p'''}{2,9132} = +1,998a. \end{cases}$$

IV. For the fourth lens
the focal length of which $p'''' = 6,183a$ and $\mathfrak{A}'''' = 0,922$, the radius

$$\begin{cases} \text{of the anterior face} & = -\frac{p''''}{2,2522} = 24,5160a \\ \text{of the posterior lens} & = +\frac{p''''}{2,4719} = +4,2007a. \end{cases}$$

Since nothing shall be different for the following lenses, which relates to the kind of glass from which they may be prepared, provided the three latter lenses shall be made equally convex on each side. But for the prior lenses some figure can be attributed to each side, provided the assigned focal lengths may be come upon. Therefore from these the following may be deduced :

THE CONSTRUCTION OF A MICROSCOPE COMPOSED FROM FOUR OBJECTIVE LENSES AND FIVE EYEPIECE LENSES

315. The four prior lenses constituting the objective are assumed to be prepared from crystal or flint glass, for which $n = 1,58$, but the remaining lenses will be permitted to be made from any kind of glass.
Moreover the construction itself will be had thus :

I. For the first lens
of which the focal length is 3,922*a*, the radius may be taken

$$\begin{cases} \text{of the anterior face} & = -0,9632a \\ \text{of the posterior face} & = +0,6767a; \end{cases}$$

the radius of which aperture will be able to be taken $x = 0{,}169a$ and the distance to the second lens $= 0{,}392a$.

II. For the second lens

of which the focal length $= 5{,}063a$, the radius

$$\begin{cases} \text{of the anterior face} = -1{,}9248a \\ \text{of the posterior face} = +1{,}1628a; \end{cases}$$

the aperture of which is as of the first, the distance to the following lens also $= 0{,}392a$.

III. For the third lens

of which the focal length $= 5{,}821a$, the radius

$$\begin{cases} \text{of the anterior face} = 4{,}8953a \\ \text{of the posterior face} = 1{,}9981a, \end{cases}$$

the aperture as before and the distance to the fourth lens again $= 0{,}392a$.

IV. For the fourth lens

of which the focal length $= 6{,}183a$, the radius

$$\begin{cases} \text{of the anterior face} = 24{,}5160a \\ \text{of the posterior face} = 4{,}2007a, \end{cases}$$

the aperture as of the first, but the distance to the following lens $= 46{,}667a$.

V. The focal length

of the fifth lens $q = 33{,}333a$

and the radius of the aperture $= \frac{3q}{8\mathfrak{M}} + \frac{2}{3}x$

and the distance to the sixth lens $= 25a$.

VI. The focal length

of the sixth lens $r = 8{,}333a$,

the radius of the aperture $= \frac{21r}{8\mathfrak{M}} + \frac{1}{3}x$,

the distance to the seventh lens $= 8{,}333Ca(1 - \frac{12}{\mathfrak{M}})$.

VII. The focal length

of the seventh lens is $s=\frac{75C}{\mathfrak{M}}\cdot a$, which lens must be equally convex on both sides and to be used in the two following lenses, and the maximum aperture and distance to the eighth lens $=\frac{48,21}{\mathfrak{M}}Ca.$

VIII. The focal length

of the eighth lens shall be $t=26,80\cdot\frac{C}{\mathfrak{M}}\cdot a$

and the distance to the ninth lens $=6,70\cdot\frac{C}{\mathfrak{M}}\cdot a.$

IX. The focal length

of the ninth and final lens is $u=13,40\cdot\frac{C}{\mathfrak{M}}\cdot a$

and the distance to the eye $=\frac{1}{3}u.$

X. Moreover the radius of the area viewed in the object will be $=\frac{3a}{4\mathfrak{M}}$, which will appear for the order of clarity, the measure of which is $=\frac{3,88a}{\mathfrak{M}}=\frac{27,04}{m}.$

SCHOLIUM

316. It will be desired in these microscopes, because the length of these may become exceedingly great, that the maximum part of this concern be situated in the ratio of that part, which was $P=\frac{3}{2}$. On account of which we will adapt our quadruple lens to example eight [§305] , where there is

$$P=24,\ \ \mathfrak{B}=-\frac{23}{4}\ \text{ and }\ B=-\frac{23}{27};$$

from which it is apparent, if again there may be taken $A=50$, the parts of the confusion arising from the lenses B and C to decrease more than in the preceding case; then truly also δ will retain the same value as before; hence therefore the following can be formed :

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM NINE LENSES, THE FIRST FOUR LENSES OF WHICH CONSTITUTE THE OBJECTIVE LENS SHALL BE MADE FROM CRYSTAL GLASS

317. In this construction the first four of the section related to the objective lenses remain the same as in the preceding kind, except that there must become

IV. The interval

of the fourth lens to the fifth $=76,74a$.

Again the remaining sections will become, as follows:

V. The focal length

of the fifth lens shall be $q = 11,98a$

and the radius of the aperture $= \frac{3q}{\mathfrak{M}} + \frac{1}{24}x$, with $x = 0,169a$,

the distance to the sixth lens $= 15,97a$.

VI. The focal length

of the sixth lens shall be $r = 14,20a$

and the radius of the aperture $= \frac{1}{3}x$,

the distance to the seventh lens $= 14,20Ca(1 - \frac{3}{\mathfrak{M}})$.

VII. Each side

of the seventh lens must be equally convex as well as the two following lenses, and the focal length $s = 127,78 \cdot \frac{C}{\mathfrak{M}} \cdot a$; to which lens the maximum aperture may be attributed, and the distance to the eighth lens $= 47,92 \cdot \frac{C}{\mathfrak{M}} \cdot a$;

VIII. The focal length

of the eighth lens shall be $t = 79,86 \cdot \frac{C}{\mathfrak{M}} \cdot a$

and the distance to the ninth lens $= 19,97 \cdot \frac{C}{\mathfrak{M}} \cdot a$.

IX. The focal length

of the ninth and final lens is $u = 39,93 \cdot \frac{C}{\mathfrak{M}} \cdot a$ and the distance to the eye $= \frac{1}{3}u$.

X. Finally as before the radius of the area viewed in the object $= \frac{3a}{4\mathfrak{M}}$,
so that it will be apparent with a degree of clarity, the measure of which is $\frac{3,88a}{\mathfrak{M}} = \frac{27,04}{m}$.

SCHOLIUM 1

318. As both in these two kinds of microscopes as well as in others, which can be carried out in common in a similar way, chiefly in the first place it happens to be especially noteworthy, that the same lenses plainly can be used equally for all magnifications, proved large enough. For plainly since the number C may depend on our choice, provided it shall be moderately large, thus so that $\mathfrak{C} = \frac{C}{1+C}$ may be able to be had in practice for unity, just as the given fraction $\frac{C}{\mathfrak{M}}$ can be seen either as $= \frac{1}{10}$ or $= \frac{1}{20}$, so that the latter lenses may not become exceedingly small, thus so that this fraction shall not be considered to continue the magnification further. There with this observed only the interval between the sixth and seventh lenses will determine the magnification, thus so that with both the same lenses remaining as well as with the remaining intervals, only that interval for the varied magnification must be changed, but which change thus will not be

great. Indeed if the magnification $m = 400$ may be desired, with the distance of the object taken $a = \frac{1}{2}$ in. there will become $\mathfrak{M} = \frac{m}{16} = 25$ [Recall from § 229, $\frac{ma}{h} = \mathfrak{M}$, where $h = 8$ in.] and hence that same interval

$$= 12,48Ca = 6,24C \text{ dig.}$$

on account of $a = \frac{1}{2}$ in. But if we may wish a magnification $m = 800$, this interval will become

$$= 13,35Ca = 6,67C \text{ in.}$$

And thus if the interval may be taken

$$= 14,20Ca = 7,10C \text{ in.}$$

then an infinite magnification will be produced. But it will be advised always for these instruments except for the maximum magnifications to be used, since, unless $\mathfrak{M}$ may be a very great number, the letter C may not become so great, in order that unity may be taken for $\mathfrak{C}$.

SCHOLIUM 2

319. Now we may observe now also, whether these quadruple objective lenses may be able to be applied to the case, where the letter A shall become negative, for which requiring to be investigated we may tale the above example seven [§ 304], where there was $P = \frac{18}{25}$, and so that the interval

$$Aa\left(1 + \frac{6(A-1)\delta}{A+1} - \frac{1}{P}\right)$$

may be produced positive, on putting $A = -\alpha$, $\alpha a\left(\frac{7}{18} - \frac{6(\alpha+1)\delta}{\alpha-1}\right)$ will have to become positive and hence

$$\delta < \frac{7(\alpha-1)}{108(\alpha+1)}$$

and thus even more so, $\delta < \frac{7}{108}$ or $\delta < \frac{1}{15}$, which cannot be done in practice. But since this has not succeed on account of the value $P = \frac{18}{25}$, we may make an application to example 3 case 1 [§ 293], where there was $P = \frac{3}{7}$, $\mathfrak{B} = \frac{16}{21}$, $B = \frac{16}{5}$. Therefore in this case the interval

$$\alpha a\left(\frac{4}{3} - \frac{6(\alpha+1)}{\alpha-1}\delta\right)$$

must become positive from which not too small a value is required for δ. But it will be required to examine the equation, by which the confusion is removed, which multiplied by $16A^3$ will become:

$$0=-(\alpha-1)^2+v(\alpha-1)\left(5\alpha^3+6\alpha+5\right)$$
$$-\tfrac{\delta(\alpha-1)^2(7\alpha+5)}{2}+\delta v(\alpha+1)\left(7\alpha^2+18\alpha+23\right)-\tfrac{16}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3}+\tfrac{v}{B\mathfrak{B}}\right)-\tfrac{16}{\mathfrak{B}^3}\lambda'';$$

for which equation, indeed with suitable values both for α as well as δ, to be able to be satisfied well enough; but since it is liable to be an inconvenience for other kinds of microscopes, since for them a single lens has established a real posterior image, from which the representation we have observed above may be far from ideal, I think there is no need to undertake this investigation further, but rather the application of the formulas found thus more convenient to be shown for telescopes, since the use of these in microscopes may be seen not be desired so much, as in the same way, whether objects may be represented either inverted or erect; but which above have been advanced for telescopes of this kind, to which only two exceedingly special cases are required to be referred. Whereby from this, a further setting forth of this kind can be desired, and the following supplement cannot be dismissed.

APPLICATION OF THIS KIND OF PROBLEM TO TELESCOPES

320. Since the distance of the object $a=\infty$, the letter A will be required to be taken as vanishing, thus still, so that Aa will denote a finite distance, which shall be $=l$. Hence therefore the focal lengths of our four lenses thus themselves will be had:

$$p'=4l,\quad p''=4(1-\delta)l,\quad p'''=4(1-3\delta)\mathrm{l},\quad p''''=4(1-6\delta)l$$

with the common interval between these lenses $=4\delta l$. But the Germanic letters for these lenses requiring to be constructed are :

$$\mathfrak{A}'=0,\ \ \mathfrak{A}''=-1,\ \ \mathfrak{A}'''=-2,\ \ \mathfrak{A}''''=-3,$$

while all the other letters $\lambda,\ \lambda'$ etc. are $=1$. Then truly the interval from this quadruple lens to the following lens will be $=(1-6\delta-\tfrac{1}{P})l.$ Truly now for the confusion requiring to be destroyed, this equation will be had:

$$0=\tfrac{1}{16}-\tfrac{5v}{16}-\tfrac{5\delta}{32}+\tfrac{23\delta v}{16}-\tfrac{1}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3}+\tfrac{v}{B\mathfrak{B}}\right)+\tfrac{1}{B^3PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{v}{C\mathfrak{C}}\right)\ \text{ etc.,}$$

which, if the lenses may be made from crystal glass, where there is $v=0,2529$, will go into this:

$$0=\tfrac{-0,2645\,+\,3,3167\delta}{16}-\tfrac{1}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3}+\tfrac{v}{B\mathfrak{B}}\right)+\tfrac{1}{B^3PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{v}{C\mathfrak{C}}\right).$$

Hence therefore if the two last terms may vanish, there will become approximately $\delta=\frac{1}{13}$; but if with these smaller terms avoided from being added in, truly these terms to become so very small, that they will not surpass $\frac{0,2645}{16}$; moreover among the examples at the end of the above chapter nothing is met to be added on, which here may be used, since the components of the confusion arising from these lenses shall be much greater; nor also are the general formulas given there able to be adapted for this use, thus so that a quadruple objective lens of this kind plainly will be unable to have any use in this kind of telescope.

PROBLEM 2

321. *If the objective lens may depend on three lenses, the first of which shall be concave made from crystal glass, but the two remaining convex lenses made from crown glass, with all the remaining lenses remaining, as have been described in the preceding chapter, to construct a microscope, so that it shall be free from all confusion.*

SOLUTION

Here problem 2 of the preceding chapter III of the preceding section [§ 194] may be called into help, where the values are accepted for the three objective lenses themselves set out in the following :

$$\mathfrak{A}=-\tfrac{1}{2},\ \ A=-\tfrac{1}{3},\ \ \mathfrak{B}=2,\ \ B=-2,\ \ \mathfrak{C}=1\ \text{ and }\ C=\infty$$

or rather C to be undefined, while the number shall be large enough. Then

$$\tfrac{1}{P}=\tfrac{17}{14},\ \ \tfrac{1}{PQ}=\tfrac{37}{28};$$

from which the focal distances of these three lenses, which we will designate here by the letters p', p'', p''' , are defined thus :

$$p'=-\tfrac{1}{2}a,\ \ p''=\tfrac{17}{21}a,\ p'''=\tfrac{37}{42}a,$$

moreover the interval of these lenses $=\frac{1}{14}a$.

Therefore so that we may adapt these determinations to the present set up, which there was $ABC,$ here for us it is simply $A,$ thus so that there shall become $\frac{2}{3}C=A$, or there what was $C,$ here for us is $\frac{3}{2}A$, and since there C was indefinite, even now A will denote an indefinite number, provided it shall be great enough. Then what was there $PQR,$ here for us will be simply $P,$ which thus even now is indefinite; from which the interval from this triple objective lens as far as to the lens here will be for us

$$= Aa\left(\tfrac{37}{28}-\tfrac{1}{P}\right).$$

Therefore so that all the confusion may be removed, if the letter λ may refer to the first crystalline concave lens, to which the letters μ and ν shall correspond, truly for the following crown glass lenses the letters λ', λ'' etc. may be put equal to unity and for the crown glass the letters μ' and ν' may be appropriate; the letter λ must be defined from this equation:

$$8\lambda = 6\nu + \tfrac{\mu}{\mu'}\varLambda$$

with there being

$$\varLambda = \tfrac{27\cdot17}{8\cdot14}\left(1-2\nu'\right)+\tfrac{27\cdot37}{8\cdot28}\left(1+\tfrac{2\nu'}{3A}\right)-\tfrac{1}{A^3P}\left(\tfrac{1}{\mathfrak{B}^3}+\tfrac{\nu'}{B\mathfrak{B}}\right)+\tfrac{1}{A^3B^3PQ}\left(\tfrac{1}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right).$$

Therefore here from the above examples set out those will be able to be selected, in which the number $\varLambda$ can denote a large enough number and where $\frac{1}{P}<\frac{37}{28}$, and thus more kinds of this species of microscopes will be able to be found with all the confusion removed. Moreover there will be observed to be

$$\mu = 0,8724 \quad\text{and}\quad \nu = 0,2529,$$

but truly

$$\mu' = 0,9875 \quad\text{and}\quad \nu' = 0,2196,$$

thus so that there shall become

$$\log.\tfrac{\mu}{\mu'} = 0,0538214.$$

Hence therefore with the setting out done there will become

$$\varLambda = 2,2983+4,4598+\tfrac{0,6529}{A}-\tfrac{1}{A^3P}\left(\tfrac{1}{\mathfrak{B}^3}+\tfrac{\nu'}{B\mathfrak{B}}\right)+\tfrac{1}{A^3B^3PQ}\left(\tfrac{1}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right).$$

and from this value found there will become

$$\lambda = 0,1897+\tfrac{\mu}{8\mu'}\varLambda$$

with there being

$$\log.\tfrac{\mu}{8\mu'} = 9,1507314.$$

COROLLARY 1

322. Hence therefore it is apparent, if there may be taken $A = 10$ only and hence according to the above formulas $C = 15$, the latter parts of this formula will become so small, so that with care they may be able to be neglected, while the letters $\mathfrak{B}$ and B,

which are negative, may not become much smaller than unity, which will be obtained readily, only if P may be notably greater than unity. Then therefore there will be had precisely enough

$$\lambda = 0{,}1897 + 0{,}9654 - 1{,}1551.$$

COROLLARY 2

323. Because according to this hypothesis for the above formulas there was $C = 15$, since still we shall have assumed there $\mathfrak{C} = 1$, so that these may become more accurate, we shall have to assume there $\mathfrak{C} = \frac{15}{16}$, from which that third part 4,4598 will have become a little greater in the ratio $16^3 : 15^3$; with which factor substituted in place of this number there must become here 5,4125, from which it is concluded $\mathrm{d}\lambda = 1{,}2807$. Then truly for the third objective lens there will become $p''' = 0{,}826a$.

COROLLARY 3

824. But so that we may maintain some ratio of the following lenses, this same value of λ will be required to be increased a little and therefore we may assume that $\lambda = 1{,}29$. From which for the construction of the first crystalline lens there will become $\tau\sqrt{(\lambda - 1)} = 0{,}4725$.

Hence for the first objective lens, of which the focal length is $p' = -\frac{1}{2}a$ and the number $\mathfrak{A} = -\frac{1}{2}$, the radius

$$\begin{cases} \text{of the anterior face} = \dfrac{p'}{\sigma - \mathfrak{A}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = +\dfrac{p'}{1{,}83080} = -0{,}273a \\ \text{of the posterior face} = \dfrac{p'}{\rho + \mathfrak{A}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = -\dfrac{p'}{0{,}1067} = +\ 4{,}686a. \end{cases}$$

Truly for the second objective lens made from crown glass, of which the focal length is $p'' = \frac{17}{21}a = 0{,}8095a$, on account of $\mathfrak{B} = 2$ found above, the radius

$$\begin{cases} \text{of the anterior face} = -0{,}6708a \\ \text{of the posterior face} = +\ 0{,}2617a. \end{cases}$$

For the third objective lens, of which the focal length is $p''' = 0{,}826a$, on account of $\mathfrak{C} = \frac{15}{16}$, the radius

$$\begin{cases} \text{of the anterior face} = \dfrac{p'''}{\sigma - \mathfrak{C}(\sigma-\rho)} = \dfrac{p'''}{0{,}3163} = 2{,}611a \\ \text{of the posterior face} = \dfrac{p'''}{\rho + \mathfrak{C}(\sigma-\rho)} = \dfrac{p'''}{1{,}5705} = 0{,}526a. \end{cases}$$

To which three lenses, the intervals of which are $= \frac{1}{14}a = 0,071a$, the aperture will be given, of which the radius $x = 0,065a$; from which the clarity arises, of which the measure is $= \frac{10,4}{m}$.

COROLLARY 4

325. As it may be extended to the remaining lenses, so that they may not enter into the calculation of the confusion, likewise it is, for whatever glass they may be prepared and whatever shape may be given to these, provided these may become equally convex on both sides, which must have the maximum aperture ; it need be observed only that the interval from the third objective lens to the following lens to be $= 10a\left(\frac{37}{28} - \frac{1}{P}\right)$.

EXAMPLE 1

326. We may adapt this to the final case of example 2 of the preceding chapter [§ 299], where the second lens clearly was removed and the first and second intervals coalesced into one interval, which now will be $= 16,54a$. Hence therefore we have the following :

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM SEVEN LENSES

Here besides the distance of the object $= a$ with the magnification m is assumed to be given, from which there becomes $\mathfrak{M} = \frac{ma}{h}$. Then truly also the number C is left to our choice, provided it shall be very great, which will be done, in order that the latter lenses may not become exceedingly small. Thus the construction therefore will be had :

I. For the first lens prepared from crystal glass, of which the focal length is $p' = -\frac{1}{2}a$, the radius

$$\begin{cases} \text{of the anterior face} = -0,273a \\ \text{of the posterior face} = +4,686a, \end{cases}$$

the radius of this aperture $x = 0,065a$ and the interval to the second lens $= 0,071a$.

II. For the second lens requiring to be prepared from crown glass, of which the focal length is $p'' = 0,809a$, the radius

$$\begin{cases} \text{of the anterior face} = -0,671a \\ \text{of the posterior face} = +0,262a, \end{cases}$$

the aperture to be as the first and the interval to the third lens also $= 0,071a$.

III. Likewise for the third lens required to be made from crown glass, of which the focal length is $p''' = 0,826a$, the radius

$$\begin{cases}\text{of the anterior face} = 2,611a\\ \text{of the posterior face} = 0,526a,\end{cases}$$

its aperture also being the same as of the first, but the interval to the fourth lens $= 16,54a$.

IV. Likewise the same for the fourth lens, may be prepared from whatever glass, provided its focal length shall be $r = \frac{10}{3}\mathfrak{C}a$ or $r = 3a$ approximately, nor also may there be much concern, whatever figure may be attributed to this lens.
The radius of its aperture $= \frac{3r}{\mathfrak{M}} + \frac{1}{3}x$ and the interval to the fifth lens

$$= \tfrac{10}{3} Ca\left(1 - \tfrac{1}{i\,\mathfrak{M}}\right) = \tfrac{10}{3} Ca\left(1 - \tfrac{33}{2\mathfrak{M}}\right)$$

on account of $i = \frac{2}{33}$.

V. For the fifth lens, since each side will be required to be equally convex, the focal length $s = 30\frac{C}{\mathfrak{M}} \cdot a$ and its aperture a maximum. The interval to the sixth lens

$$= \tfrac{15(3i+2)}{9i+1} \cdot \tfrac{C}{\mathfrak{M}} \cdot a = \tfrac{360}{17} \cdot \tfrac{C}{\mathfrak{M}} \cdot a .$$

VI. For the sixth lens equally each side to be equally convex and the focal length is :

$$t = \tfrac{225i}{9i+1} \cdot \tfrac{C}{\mathfrak{M}} \cdot a = \tfrac{150}{17} \cdot \tfrac{C}{\mathfrak{M}} \cdot a ,$$

and the interval to the seventh lens

$$= \tfrac{225i}{4(9i+1)} \cdot \tfrac{C}{\mathfrak{M}} \cdot a = \tfrac{75}{34} \cdot \tfrac{C}{\mathfrak{M}} \cdot a .$$

VII. For the seventh lens also each side to be equally convex and the focal length becomes

$$u = \tfrac{225i}{18i+2} \cdot \tfrac{C}{\mathfrak{M}} \cdot a = \tfrac{1}{2}t = \tfrac{75}{17} \cdot \tfrac{C}{\mathfrak{M}} \cdot a ,$$

and the distance to the eye $= \frac{1}{3}u$.

VIII. The radius of the area viewed in the object will be $= \frac{3a}{4\mathfrak{M}}$ and the measure of the clarity $= \frac{10,4}{m}$.

Here therefore the quantity C is left to our choice, provided it shall be a great enough number, thus so that the fraction $\frac{C}{\mathfrak{M}}$ may be considered as given; then it is apparent also to be $u = \frac{5}{34}s$ and thus with the same lenses remaining the same instrument will be able to be rendered suitable for all the magnifications, provided the interval between the lenses four and five may be varied, since also the remaining intervals will remain the same on account of the fraction $\frac{C}{\mathfrak{M}}$.

EXAMPLE 2

327. Our objective triple also will be allowed to be joined together with the above third example [§ 300], where there was $P = \frac{3}{2}$ and $\mathfrak{B} = -1$. Therefore with the three first articles as in the preceding example, except that in the end of the third section there must be written:

The interval from the third lens to the fourth

$$= \left(\tfrac{37}{28} - \tfrac{1}{P}\right) Aa = 6{,}55a.$$

IV. It is the same for the fourth lens, from whatever glass it may be prepared, provided its focal distance shall be

$$q = \tfrac{20}{3} a = 6{,}7a.$$

The radius of its aperture $= \frac{3}{8} \cdot \frac{q}{\mathfrak{M}} + \frac{2}{3}x$ and the interval to the fifth lens $= 5a$.

V. For the fifth lens, of which the focal length is

$$r = \tfrac{5}{3}\mathfrak{C}a = 1{,}667\mathfrak{C}a = 1{,}50a$$

clearly on taking $\mathfrak{C} = \frac{9}{10}$, if indeed it may not actually be equal to unity,

the radius of its aperture $= \frac{21r}{8\mathfrak{M}} + \frac{1}{3}x,$

the interval to the sixth lens $= \frac{5}{3}Ca\left(1 - \frac{12}{\mathfrak{M}}\right)$ or, if there may be put $\frac{C}{\mathfrak{M}} = \gamma$, so that there shall become $C = \gamma\mathfrak{M}$, this interval will be $= \frac{5}{3}\gamma a\left(\mathfrak{M} - 12\right)$, where γ is taken thus, so that the following lenses may not become exceedingly small.

VI. For the sixth lens, which since in the following each side must be equally convex, the focal length shall become $s = 15\gamma a$,

its maximum aperture or the radius of the aperture $= \frac{1}{4}s$

and the interval to the seventh lens $= \frac{135}{14}\gamma a = 9{,}643\gamma a.$

VII. For the seventh lens the focal length shall be $t = \frac{150}{28}\gamma a = 5{,}357\gamma a,$ the radius of the aperture $= \frac{1}{4}t$, and the interval to the eighth lens $= \frac{75}{56}\gamma a =$ $1{,}339\gamma a$.

VIII. For the eighth lens the focal length $u=\frac{75}{28}\gamma a=2,679\gamma a$,
the radius of the aperture $=\frac{1}{4}u$, and the distance to the eye $=\frac{1}{3}u$.

IX. The radius of the area observed in the object will be $=\frac{3a}{4\mathfrak{M}}$ and the measure of the clarity $=\frac{10,4}{m}$.

EXAMPLE 3

328. The above fourth example cannot be transferred here; but from the fifth [§ 302], where $P=\frac{18}{7}$ and $\mathfrak{B}=-\frac{11}{7}$, this construction arises:

The first three articles remain as in the first example; but to these there may be added on:

The interval from the third lens to the fourth $=9,325a$.

IV. For the fourth lens

The focal length $q=\frac{55}{9}a=6,11a.$

The radius of this aperture $=\frac{3}{4}\cdot\frac{q}{\mathfrak{M}}+\frac{7}{18}x.$

The distance to the fifth lens $=\frac{715}{162}a=4,414a.$

V. For the fifth lens

The focal length $r=1,83a$.

The radius of its aperture $=\frac{9}{4}\cdot\frac{r}{\mathfrak{M}}+\frac{1}{3}x.$

Distance to the sixth lens $=2,037\gamma a(\mathfrak{M}-10).$

VI. For the sixth lens

Focal length $s=\frac{55}{3}\gamma a=18,33\gamma a$.

Radius of the aperture $=\frac{1}{4}s.$

Distance to the seventh lens $=11,096\gamma a$.

VII. For the seventh lens

The focal length $t=\frac{275}{38}\gamma a=7,237\gamma a$.

Radius of the aperture $=\frac{1}{4}t.$

Interval to the eighth lens $=\frac{275}{152}\gamma a=1,809\gamma a$.

VIII. For the eighth lens

The focal length $u=\frac{1}{2}t=3,618\gamma a$.

The radius of the aperture $=\frac{1}{4}u$,

and the distance to the eye $=\frac{1}{3}u$.

IX. The field and clarity themselves are had as in the preceding examples.

EXAMPLE 4

329. Fro the above example eight [§ 305], where $P=24$ and $\mathfrak{B}=-\frac{23}{4}$,
this construction arises :
With the three first articles remaining to be adjoined as before:
Distance of the third lens to the fourth $=12,80a$.

IV. For the fourth lens

The focal length $q=\frac{115}{48}a=2,396a.$

The radius of the aperture $=\frac{3q}{\mathfrak{M}}+\frac{1}{24}x.$

Interval to the fifth lens $=\frac{115}{36}a=3,194a.$

V. For the fifth lens

The focal length $r=\frac{230}{81}\cdot\mathfrak{C}a=2,556a.$

The radius of the aperture $=\frac{1}{3}x$.

Interval to the sixth lens $=2,839\gamma a(\mathfrak{M}-3)$.

VI. For the sixth lens

The focal length $s=25,556\gamma a$.

The radius of the aperture $=\frac{1}{4}s$.

Interval to the seventh lens $=9,583\gamma a$.

VII. For the seventh lens

The focal length $t=15,972\gamma a$.

The radius of the aperture $=\frac{1}{4}t$.

Interval to the eighth lens $=\frac{1}{4}t=3,993\gamma a$.

VIII. For the eighth lens

The focal length $u=\frac{1}{2}t=7,986\gamma a$.

The radius of the aperture $=\frac{1}{4}u$.

Interval to the eye $=\frac{1}{3}u$.

IX. Field and clarity as in the preceding examples.

EXAMPLE 5

330. Finally with application made to the above example 9 [§ 306] this posterior construction arises:

With the three first lenses remaining as at present so that the third article may be adjoined:

The interval of the third and fourth lenses $=12,24a$.

IV. For the fourth lens

The focal length $q=\frac{325}{144}a=2,257a$.

The radius of the aperture $=\frac{3q}{\mathfrak{M}}+\frac{7}{72}x$.

Distance to the fifth lens $=3,009a$.

V. For the fifth lens

The focal length $r=2,097a$.

The radius of the aperture $=\frac{1}{3}x$.

interval to the seventh lens $=2,33\gamma a\left(\mathfrak{M}-1\right)$.

VI. For the sixth lens

The focal length $s=20,97\gamma a$.

The radius of the aperture $=\frac{1}{4}s$.

Interval to the seventh lens $=-\ 5,242\gamma a$.

VII. For the seventh lens

The focal length $t=15,726\gamma a$.

The radius of the aperture $=\frac{1}{4}t$.

Interval to the eighth lens $=\frac{1}{4}t=3,931\gamma a$.

VIII. For the eighth lens

The focal length $u=7,863\gamma a$.

The radius of the aperture $=\frac{1}{4}u$.

Distance to the eye $=\frac{1}{3}u$.

IX. Field and clarity as in the preceding examples.

END OF THE WORK.

CAPUT III

DE SUMMA HORUM MICROSCOPIORUM PERFECTIONE DUM EA AB OMNI CONFUSIONE LIBERANTUR

PROBLEMA 1

308. *Si lens obiectiva constet quatuor lentibus convexis proxime inter se iunctis, quales descriptae sunt supra in problemate* 4 *capitis secundi sectionis praecedentis* [§ 177], *reliquae vero lentes ita sint dispositae, uti in capitibus praecedentibus huius sectionis descripsimus, omnem confusionem a lentium apertura oriundam destruere.*

SOLUTIO

Quatuor illas lentes in loco citato descriptas hic loco lentis obiectivae substitui assumimus easque coniunctim in calculo instar unicae lentis tractamus. Cum igitur littera A hic ad primam lentem pertineat, quatenus ea in determinationes sequentium lentium ingreditur, idem significat, quod in loco citato per productum $-ABCD$ significabatur. At supra hoc productum designavimus littera θ, ex quo ad locum antecedentem regrediendo ibi θ idem erit, quod hic nobis est A. Ibi ergo quatuor illarum lentium distantias focales designemus litteris p', p'', p''', p'''', et cum cuilibet littera germanica conveniat, quae sit $\mathfrak{A}', \mathfrak{A}'', \mathfrak{A}''', \mathfrak{A}''''$, ex nostro A ob $\theta = A$ habebimus

$$\mathfrak{A}' = \frac{4A}{A+1}, \quad \mathfrak{A}'' = \frac{3A-1}{A+1}, \quad \mathfrak{A}''' = \frac{2A-2}{A+1}, \text{ et } \mathfrak{A}'''' = \frac{A-3}{A+1};$$

atque ex his litteris germanicis, quatenus ad singulas quatuor lentes priores referuntur, una cum numeris λ, qui pro singulis unitati aequales sumuntur, singulae hae lentes per formulas notas construi possunt. Antequam autem has ipsas distantias focales assignare queamus, intervallum minimum inter has lentes spectare debemus; quod cum ibi positum esset $= \zeta p$, ne haec littera ζ in nostris formulis confusionem pariat, statuamus hoc intervallum $= \delta p'$; unde litterae ibi usurpatae P, Q, R, S ita definientur:

$$\frac{1}{P} = 1 + \frac{(3A-1)\delta}{A+1}, \quad \frac{1}{PQ} = 1 + \frac{(5A-3)\delta}{A+1}, \quad \frac{1}{PQR} = 1 + \frac{(6A-6)\delta}{A+1};$$

unde ipsae distantiae focales erunt

$$p' = \frac{4A}{A+1} \cdot a, \quad p'' = \frac{4A}{A+1} \cdot a + \frac{4A(3A-1)\delta a}{(A+1)^2},$$

$$p''' = \frac{4A}{A+1} \cdot a + \frac{4A(5A-1)\delta a}{(A+1)^2}, \quad p'''' = \frac{4A}{A+1} \cdot a + \frac{4A(6A-6)\delta a}{(A+1)^2}.$$

His notatis, quod supra erat *PQRS,* hic nobis sola littera *P* exprimitur, ita ut iam intervallum a lente obiectiva usque ad nostram lentem secundam sit

$$Aa\left(1+\frac{6(A-1)\delta}{A+1}-\frac{1}{P}\right);$$

quod ergo iam spectatur ut nostrum intervallum primum. Ob multiplicationem ergo lentis obiectivae hoc primum intervallum quandam alterationem patitur a fractione δ natam; si enim prima lens esset simplex, hoc intervallum tantum esset $Aa(1-\frac{1}{P})$, nunc autem id erit

$$Aa\left(1-\frac{1}{P}\right)+\frac{(6A-6)\delta A}{A+1}.$$

Quia autem hoc augmentum plerumque est valde parvum, id facile negligi poterit. Reliqua autem intervalla ordine stabilito procedent; erit scilicet

$$\text{secundum} = -ABa\left(\frac{1}{P}-\frac{1}{PQ}\right) \text{ etc.}$$

et perinde ac sequentes distantiae focales, scilicet

$$q=-A\mathfrak{B}\cdot\frac{a}{P},\ \ r=AB\mathfrak{C}\cdot\frac{a}{PQ}\ \text{ etc.}$$

Nunc igitur totum negotium huc redit, ut confusio a lentium apertura orta penitus ad nihilum redigatur, quod fit, uti loco citato invenimus, ope huius aequationis:

$$0=\frac{(1+A)^3}{16A^3}-\frac{\nu(1+A)(5A^2-6A+5)}{16A^3}+\frac{\delta(1+A)^2(7A-5)}{32A^3}$$
$$-\frac{\delta\nu(1-A)(7A^2-18A+23)}{16A^3}-\frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)+\frac{1}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)\ \text{ etc.;}$$

ex qua aequatione fractio δ, qua exigua intervalla inter lentes obiectivas determinantur, commode definiri potest, si modo littera *A* numerum satis magnum veluti 60 denotet lentesque obiectivae ex eiusmodi vitri genere parentur, pro quo sit $\nu>\frac{1}{5}$. Supra [§ 182] enim ostendimus, si sit $A=60$ vitrumque commune adhibeatur, pro quo sit $n=1,55$, tum fieri $\delta=\frac{1}{18}$, quoniam casu $A=60$ partes a sequentibus lentibus ortae quasi evanescunt. Huius modi igitur lens obiectiva composita cum omnibus praecedentibus microscopiorum formis combinari poterit, in quibus scilicet inest littera *A,* eique valor circiter 60 tribuatur. Quando autem hoc usu venit, ne opus quidem erit in hunc valorem ipsius *A* anxie inquirere; constructis enim singulis lentibus secundum praecepta cognita, id quod sine notitia litterae δ fieri potest, intervalla quatuor lentium obiectivarum indefinita relinquantur eaque demum per experimenta ita determinentur, ut confusio fiat quam minima, nisi forte plane nulla , fieri nequeat. Tum autem ex forma harum lentium sponte innotescit semidiameter aperturarum *x,* cuius hae lentes sunt capaces, indeque mensura claritatis $=\frac{20x}{\mathfrak{M}}$ existente $\mathfrak{M}=\frac{ma}{h}$. Reliqua omnia autem momenta prorsus eadem manebunt, uti in praecedentibus capitibus est expositum.

COROLLARIUM 1

309. Quodsi ergo fuerit $A=60$ et quatuor lentes obiectivae ex vitro communi, pro quo est $n=1,55$, conficiantur, supra [§ 183] sequentem harum lentium constructionem invenimus:

I. Pro prima lente

cuius distantia focalis est $=3,9333a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -0,97756a\\ \text{posterioris} &= +0,67332a,\end{cases}$$

aperturae semidiameter $=0,16833a$.

II. Pro secunda lente

cuius distantia focalis $=4,5740a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -1,7682a\\ \text{posterioris} &= +1,0384a.\end{cases}$$

III. Pro tertia lente

cuius distantia focalis $r=4,9965a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -4,3440a\\ \text{posterioris} &= +1,6833a.\end{cases}$$

IV. Pro quarta lente
cuius distantia focalis $s=5,2006a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 18,1522a\\ \text{posterioris} &= 3,3955a.\end{cases}$$

Intervalla autem inter has quaternas lentes sumi poterunt $=0,2185a$, ea autem praestabit per experientiam definiri. Tum vero semidiameter aperturae capi poterit $x=0,16833a$, unde colligitur mensura claritatis

$$= \frac{20x}{\mathfrak{M}} = \frac{3,3666a}{\mathfrak{M}} = \frac{26,9328}{\mathfrak{M}},$$

si scilicet distantia a in digitis exprimatur.

COROLLARIUM 2

310. Facile intelligitur, etiamsi littera A sive aliquanto maior sive minor caperetur quam 60, tamen in constructione harum lentium vix ullam orituram esse variationem, quae quidem in praxi observari posset; id tantum hic notari oportet, quod si esset $A < 60$, tum lentes istas non amplius tam prope inter se constitui posse, quam confusionis destructio postulat. Sin autem $A > 60$, tum istud negotium eo felicius succedet.

SCHOLION

311. In genere autem notetur, quo maior fuerit numerus A, eo felicius constructionem succedere, quandoquidem tum intervalla harum lentium non amplius tantopere exigua reperientur; unde hoc commodi consequimur, si forte sequentes lentes adhuc satis notabilem confusionem pariant, ut etiam ea lentes has magis appropinquando destrui possit. In ultimo autem scholio praecedentis capitis [§ 807] casus occurrit, quo fiebat $A = \infty$ et $\mathfrak{B} = B = 0$, ita ut esset $AB = -\frac{q}{a}$; ad hunc igitur casum nostra lens obiectiva quadruplicata commodissime adplicari poterit; tum enim fieri debebit

$$0 = \frac{1}{16} - \frac{5}{16}v + \frac{7}{32}\delta - \frac{7}{16}\delta v + \frac{a^3\lambda'}{q^3} + \frac{a^3}{3q^3}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{v}{C\mathfrak{C}}\right) \text{ etc.},$$

ex qua fractionem δ definiri oportet; tum autem distantiae focales quatuor priorum lentium erunt

$$p' = 4a,\ p'' = 4a + 12\delta a,\ p''' = 4a + 20\delta a,\ p'''' = 4a + 24\delta a;$$

quae ergo utique etiam a fractione δ pendent, et ubi supra diximus lentes construi posse sine notitia ipsius δ, id tantum de eius valore adcurato est intelligendum; in praxi enim sufficit eius valorem proxima nosse. Deinde vero litterae germanicae ad has lentes pertinentes erunt

$$\mathfrak{A}' = 4,\ \mathfrak{A}'' = 3,\ \mathfrak{A}''' = 2,\ \mathfrak{A}'''' = 1;$$

omnes vero litterae λ unitati capiuntur aequales. Si igitur sumamus omnes lentes ex vitro communi confici, pro quo est $n = 1,55$, erit $v = 0,2326$; sique insuper sit $\lambda' = 1$ et $\lambda'' = 1$, ob $\mathfrak{C} = 1$ proxime aequatio nostra resolvenda induet hanc formam:

$$0 = \frac{-0,1630 + 1,8718\delta}{16} + \frac{4a^3}{3q^3},$$

ubi sequentia membra, quae insuper per $\mathfrak{M}$ sunt divisa, tuto reiicere licet; hinc ergo colligimus

$$\delta = 0,08708 - 11,397 \cdot \tfrac{a^3}{q^3};$$

unde patet esse debere $\frac{q}{a} > 5,0771$. Si ergo sumamus $q = 10a$, fiet

$$\delta = 0,07569 \;= \tfrac{3}{40} \text{ seu proxima } \delta = \tfrac{3}{13},$$

qui valor satis est ad praxin accommodatus, quem in sequente exemplo fusius evolvemus.

EXEMPLUM

312. Si omnes lentes ex vitro communi parentur, ut omnis confusio tollatur, sumi debet $\delta = \frac{1}{13}$ et $q = 10a\, a$ eruntque distantiae focales nostrarum quatuor lentium obiectivarum

$$p' = 4a,\;\; p'' = 4,923a,\;\; p''' = 5,538a,\;\; p'''' = 5,846a;$$
$$\mathfrak{A}' = 4\;,\; \mathfrak{A}'' = 3\;,\; \mathfrak{A}''' = 2,\; \mathfrak{A}'''' = 1.$$

Deinde intervalla inter has quatuor lentes erunt $= \frac{4}{13}a = 0,3077a.$ Deinde vero intervallum ab hac lente obiectiva ad secundam lentem principalem erit proxime $= 10\;\zeta a$, ubi ζ adhuc nostro arbitrio relinquitur, dummodo sit $\zeta < \frac{3}{2}$. Tum vero reliquae lentes et reliqua momenta manebunt prorsus, uti in scholio ultimo [§ 307] capitis praecedentis sunt exposita. Hinc igitur singulae lentes formari poterunt ac pro secunda ac tertia quidem etiam poni debet $\lambda' = 1$ et $\lambda'' = 1$.

Pro prima igitur harum lentium obiectivarum, cuius distantia focalis est $p' = 4a$ et $\mathfrak{A}' = 4,$ erit

$$\text{radius faciei} \begin{cases} \text{anterioris} \;= \frac{p'}{\sigma - \mathfrak{A}'(\sigma-\rho)} = \frac{-p'}{4,1194} = -0,97101a \\ \text{posterioris} = \frac{p'}{\rho + \mathfrak{A}'(\sigma-\rho)} = \frac{p'}{5,9375} = +0,67370a. \end{cases}$$

Pro secunda, cuius distantia focalis est $p'' = \;4,923a$ et $\mathfrak{A}'' = 3,$ erit

$$\text{radius faciei} \begin{cases} \text{anterioris} \;= \frac{p''}{\sigma - \mathfrak{A}''(\sigma-\rho)} = \frac{-p''}{2,6827} = -1,8351a \\ \text{posterioris} = \frac{p''}{\rho + \mathfrak{A}''(\sigma-\rho)} = \frac{p''}{4,5008} = +1,0938a. \end{cases}$$

Pro tertia lente, cuius distantia focalis est $p''' = 5,538a$ et $\mathfrak{A}''' = 2,$ erit

$$\text{radius faciei} \begin{cases} \text{anterioris} \;= \frac{-p'''}{1,2460} = -4,4447a \\ \text{posterioris} = \frac{p'''}{3,0641} = +1,8074a. \end{cases}$$

Pro quarta lente, cuius distantia focalis est $p'''' = 5,846a$ et $\mathfrak{A}'''' = 1$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{+p''''}{0,1907} = +30,6560a\\ \text{posterioris} = \frac{p''''}{1,6274} = +\ 3,5922a.\end{cases}$$

Pro secunda vero lente principali, [cuius distantia focalis] est arbitraria $= q$, ob $\mathfrak{B} = 0$ et $\lambda' = 1$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma} = 0,61448q\\ \text{posterioris} = \frac{q}{\rho} = 5,2439q.\end{cases}$$

Pro tertia autem lente principali, [cuius distantia focalis] est

$$r = \tfrac{1}{3}\mathfrak{C}q = \tfrac{1}{3}q,$$

ob $\mathfrak{C} = 1$ proxime et $\lambda'' = 1$ erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\rho} = 1,7479q\\ \text{posterioris} = \frac{q}{\sigma} = 0,20483q.\end{cases}$$

Quibus inventis sequitur

CONSTRUCTIO GENERALIS MICROSCOPII EX NOVEM LENTIBUS COMPOSITI EX VITRO COMMUNI PARANDIS

313. Sumta pro lubitu distantia obiecti $= a$ constructio ita se habebit:

I. Pro prima lente

cuius distantia focalis $= 4a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0,97101a\\ \text{posterioris} = +0,67370a;\end{cases}$$

quae ergo aperturam admittit, cuius semidiameter $x = 0,1684a$.
Distantia ad lentem secundam $= 0,3077a$.

II. Pro secunda lente

cuius distantia focalis est $= 4,923a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -1,8351a\\ \text{posterioris} &= +1,0938a,\end{cases}$$

cuius apertura est ut primae et distantia ad lentem tertiam $= 0,3077a$.

III. Pro tertia lente

cuius distantia focalis $= 5,536a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -4,4447a\\ \text{posterioris} &= +1,8074a,\end{cases}$$

apertura ut primae et distantia ad quartam $= 0,3077a$.

IV. Pro quarta lente

cuius distantia focalis $= 5,846a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 30,6560a\\ \text{posterioris} &= 3,5922a,\end{cases}$$

apertura ut primae, distantia ad quintam $= \zeta q$, ubi ζ est numeros minor quam $\frac{3}{2}$, at q quantitas arbitrio nostro relicta.

V. Pro quinta lente

cuius distantia focalis $= q$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,61448q\\ \text{posterioris} &= 5,2439q,\end{cases}$$

eius aperturae semidiameter $= \frac{3}{4}\cdot\frac{\zeta}{\mathfrak{M}}\cdot q + x$
et distantia ad lentem sextam $= \frac{4}{3}\cdot q$.

VI. Pro sexta lente

cuius distantia focalis est $r = \frac{1}{3}q$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 1,7479q\\ \text{posterioris} &= 0,2048q,\end{cases}$$

eius aperturae semidiameter

$$=\frac{3}{4}\cdot\frac{(\zeta-4)}{\mathfrak{M}}\cdot r+\frac{1}{3}x=\frac{1}{4}\cdot\frac{(\zeta-4)}{\mathfrak{M}}\cdot q+\frac{1}{3}x$$

et distantia ad lentem septimam

$$=\frac{1}{3}q\left(1-\frac{3(11-8\zeta)}{2\mathfrak{M}}\right).$$

VII. Pro septima lente

cuius distantia focalis est $s=\frac{3C}{\mathfrak{M}}\cdot q$

et quae debet esse utrinque aeque convexa, capiatur

radius utriusque faciei $=-1,1s$

et semidiameter aperturae $r=\frac{1}{4}q$,

distantia vero ad lentem octavam $=\frac{12(3-2\zeta)}{17-8\zeta}\cdot\frac{C}{\mathfrak{M}}\cdot q.$

VIII. Pro octava lente

cuius distantia focalis $t=\frac{15}{17-8\zeta}\cdot\frac{C}{\mathfrak{M}}\cdot q,$ capiatur

radius utriusque faciei $=1,1t$

et semidiameter aperturae $=\frac{1}{4}t$,

distantia vero ad lentem nonam $=\frac{15}{4(17-8\zeta)}\cdot\frac{C}{\mathfrak{M}}\cdot q.$

IX. Pro nona lente

cuius distantia focalis $u=\frac{15}{34-16\zeta}\cdot\frac{C}{\mathfrak{M}}\cdot q,$ capiatur

radius faciei utriusque $=1,1u$,

aperturae semi diameter $=\frac{1}{4}u$,

atque erit distantia ad oculum $=\frac{1}{3}u$.

X. Tum vero spatii in obiecto conspicui semidiameter erit $z=\frac{3a}{4\mathfrak{M}}$, quod cernetur claritatis gradu, cuius mensura est $=\frac{20x}{\mathfrak{M}}=\frac{3,368a}{\mathfrak{M}}.$

XI. Hic autem ex multiplicatione proposita $=m$ nascitur $\mathfrak{M}=\frac{ma}{h}$ sumto $h=8$ dig.; per m igitur expressa mensura claritatis erit $=\frac{26,944}{m}.$

XII. Praeterquam autem, quod litterae ζ et q ab arbitrio nostro pendent, etiam C pro lubitu assumi potest, dummodo sit numerus satis magnus, eique fini inservit, ut ultimae lentes non fiant nimis exiguae, si scilicet multiplicatio fuerit praemagna. Notetur autem, cum ob sumtum $A=\infty$ hae formulae aliquantum a calculo discrepant, errores evitari posse, si per experientiam tam distantia obiecti quam intervalla quatuor priorum lentium debite determinentur.

SCHOLION

314. Quoniam hic casus, quo erat $A=\infty$ moram facessere posset, applicemus nostram lentem obiectivam quadruplicatam ad casum in exemplo tertio casus secundi [§ 300] , ubi erat

$$P=\tfrac{3}{2},\ \mathfrak{B}=-1 \text{ et } B=-\tfrac{1}{2},$$

quoniam commode litterae A valor satis magnus tribui poterit. Tum ergo sequentium lentium distantiae focales manebunt, uti ibi sunt descriptae, perinde atque earum intervalla, nisi quod primum intervallum nunc debeat esse ad confusionem autem tollendam iam satisfieri oportet huic aequationi:

$$Aa\left(1+\frac{6(A-1)\delta}{A+1}-\frac{1}{P}\right)=Aa\left(\frac{1}{3}+\frac{6(A-1)\delta}{A+1}\right)=Aa\left(\frac{1}{3}+6\delta\right) \text{ [proxime];}$$

ad confusionem autem tollendam iam satisfieri oportet huic aequationi:

$$0=\frac{(1+A)^3}{16A^3}-\frac{v(1+A)\left(5A^2-6A+5\right)}{16A^3}+\frac{\delta(1+A)^2(7A-5)}{32A^3}$$
$$-\frac{\delta v(A-1)\left(7A^2-13A+23\right)}{16A^3}+\frac{2}{3A^3}\left(\lambda'-2v\right)+\frac{8\lambda''}{3A^3},$$

ex qua fractionem δ definiri oportet; quod quo commodius fieri possit, ita ut pro δ fractio non nimis exigua reperiatur, omnes quatuor lentes obiectivam constituentes ex vitro crystallino (Flint Glass) confici sumamus, ita ut nunc sit $v=0,2529$, atque nunc etiam pro A non opus erit numerum adeo magnum statuere; si enim statuamus $A=50$, reperietur ista aequatio, postquam scilicet per $16A^3$ fuerit multiplicata,

$$0=-24768+242679\delta+48;$$

unde colligitur

$$\delta=\frac{24720}{242679}=\frac{1}{10} \text{ proxima,}$$

qui valor ad praxin maxima est accommodatus. Sumto igitur $A=50$ et $\delta=\frac{1}{10}$ distantiae focales quatuor priorum lentium erunt

$$p'=3,922a,\quad p''=5,063a,\quad p'''=5,821a,\quad p''''=6,183a,$$

quibus iungantur numeri

$$\mathfrak{A}'=3,922,\ \ \mathfrak{A}''=2,922,\ \ \mathfrak{A}'''=1,922,\ \ \mathfrak{A}''''=0,922.$$

Intervallum vero singularum harum lentium est $\frac{1}{10}p' = 0,392a$, deinde vero intervallum a lente obiectiva ad lentem principalem secundam $= 46,667a$.
Ex his ergo quatuor lentes obiectivam constituentes sequenti modo construentur:

I. Pro prima lente

cuius distantia focalis est $p' = 3,922a$ et numerus $\mathfrak{A}' = 3,922 = 4 - \frac{1}{13}$,
erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p'}{\sigma - \mathfrak{A}'(\sigma-\rho)} = -\frac{p'}{4,0717} = -0,9632a\\ \text{posterioris} = \frac{p'}{\rho + \mathfrak{A}'(\sigma-\rho)} = \frac{p'}{5,7958} = +0,6767a.\end{cases}$$

II. Pro secunda lente

cuius distantia focalis est $p'' = 5,063a$ et numerus $\mathfrak{A}'' = 2,922 = 3 - \frac{1}{13}$,
erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = -\frac{p''}{2,6304} = -1,9248a\\ \text{posterioris} = +\frac{p''}{4,3545} = +1,1628a.\end{cases}$$

III. Pro tertia lente

cuius distantia focalis est $p''' = 5,821a$ et $\mathfrak{A}''' = 1,922a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = -\frac{p'''}{1,1891} = -4,8953a\\ \text{posterioris} = +\frac{p'''}{2,9132} = +1,9981a.\end{cases}$$

IV. Pro quarta lente

cuius distantia focalis $p'''' = 6,183a$ et $\mathfrak{A}'''' = 0,922$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = -\frac{p''''}{2,2522} = 24,5160a\\ \text{posterioris} = +\frac{p''''}{1,4719} = +4,2007a.\end{cases}$$

Quod ad sequentes lentes attinet, nihil interest, ex quonam vitri genere parentur, dummodo tres postremae utrinque fiant aeque convexae. Binis autem prioribus figura adeo quaecunque tribui potest, dummodo distantias focales assignatas adipiscantur. Ex his igitur colligitur sequens

CONSTRUCTIO MICROSCOPII EX QUATUOR LENTIBUS OBIECTIVIS ET QUINQUE OCULARIUS COMPOSITI

315. Quatuor lentes priores obiectivam constituentes ex vitro crystallino, pro quo est $n = 1,58$, parari sumuntur, reliquas autem lentes ex vitro quocunque conficere licebit. Constructio autem ipsa ita se habebit:

I. Pro lente prima

cuius distantia focalis est 3,922*a,* capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -0,9632a\\ \text{posterioris} &= +0,6767a;\end{cases}$$

cuius aperturae semidiameter sumi poterit $x = 0,169a$ et distantia ad lentem secundam $= 0,392a$.

II. Pro secunda lente

cuius distantia focalis $= 5,063a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -1,9248a\\ \text{posterioris} &= +1,1628a;\end{cases}$$

cuius apertura est ut primae, distantia ad lentem sequentem quoque $= 0,392a$.

III. Pro tertia lente

cuius distantia focalis $= 5,821a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 4,8953a\\ \text{posterioris} &= 1,9981a,\end{cases}$$

apertura ut ante et distantia ad quartam denuo $= 0,392a$.

IV. Pro quarta lente

cuius distantia focalis $= 6,183a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 24,5160a\\ \text{posterioris} &= 4,2007a,\end{cases}$$

apertura ut primae, at distantia ad lentem sequentem $= 46,667a$.

V. Quintae lentis

distantia focalis $q = 33,333a$

et semidiameter aperturae $= \frac{3q}{8\mathfrak{M}} + \frac{2}{3}x$

et distantia ad lentem sextam $= 25a$.

VI. Sextae lentis

distantia focalis sit $r = 8,333a$,

semidiameter aperturae $= \frac{21r}{8\mathfrak{M}} + \frac{1}{3}x$,

distantia ad lentem septimam $= 8,333Ca(1 - \frac{12}{\mathfrak{M}})$.

VII. Septimae lentis

quae utrinque debet esse aequaliter convexa uti et duae sequentes, distantia focalis est $s = \frac{75C}{\mathfrak{M}} \cdot a$ et apertura maxima et distantia ad lentem octavam $= \frac{48,21}{\mathfrak{M}} Ca$.

VIII. Octavae lentis

distantia focalis sit $t = 26,80 \cdot \frac{C}{\mathfrak{M}} \cdot a$

et distantia ad lentem nonam $= 6,70 \cdot \frac{C}{\mathfrak{M}} \cdot a$.

IX. Nonae denique lentis

distantia focalis est $u = 13,40 \cdot \frac{C}{\mathfrak{M}} \cdot a$

et distantia ad oculum $= \frac{1}{3}u$.

X. Spatii autem in obiecto conspicui semidiameter erit $= \frac{3a}{4\mathfrak{M}}$, quod apparebit gradu claritatis, cuius mensura est

$$= \frac{3,88a}{\mathfrak{M}} = \frac{27,04}{m}.$$

SCHOLION

816. In his microscopiis id desiderari poterit, quod eorum longitudo fiat nimis magna, cuius rei ratio maximam partem in eo est sita, quod erat $P = \frac{3}{2}$. Quamobrem nostram lentem quadruplicatam ad exemplum octavum [§ 305] accommodemus, ubi est

$$P = 24, \ \ \mathfrak{B} = -\frac{23}{4} \ \text{ et } \ B = -\frac{23}{27};$$

unde patet, si iterum capiatur $A = 50$, partes confusionis a lentibus B et C ortas magis evanescere quam casu praecedente; tum vero etiam littera δ eundem quem ante valorem retinebit; hinc ergo formari potest sequens

CONSTRUCTIO MICROSCOPII EX NOVEM LENTIBUS COMPOSITI QUARUM QUATUOR PRIORES LENTEM OBIECTIVAM CONSTITUENTES EX VITRO CRYSTALLINO SINT FACTAE

317. In hac constructione quatuor articuli priores ad lentes obiectivas relati manent iidem ut in genere praecedente, nisi quod statui debeat

IV. Intervallum quartae lentis ad quintam $=76,74a$.
Reliqui porro articuli erunt, ut sequuntur:

V. Quintae lentis

distantia focalis sit $q=11,98a$
et semidiameter aperturae $=\frac{3q}{\mathfrak{M}}+\frac{1}{24}x$ existente $x=0,169a$,
distantia ad lentem sextam $=15,97a$.

VI. Sextae lentis

distantia focalis sit $r=14,20a$
et semidiameter aperturae $=\frac{1}{3}x$,
distantia ad lentem septimam $=14,20Ca(1-\frac{3}{\mathfrak{M}})$.

VII. Septimae lentis

quae utrinque debet esse aequaliter convexa ut et duae sequentes, sit distantia focalis $s=127,78\cdot\frac{C}{\mathfrak{M}}\cdot a;$; cui lenti apertura tribuitur maxima et distantia ad lentem octavam $=47,92\cdot\frac{C}{\mathfrak{M}}\cdot a;$

VIII. Lentis octavae

distantia focalis sit $t=79,86\cdot\frac{C}{\mathfrak{M}}\cdot a$
et distantia ad lentem nonam $=19,97\cdot\frac{C}{\mathfrak{M}}\cdot a.$

IX. Nonae denique lentis

distantia focalis est $u=$ $39,93\cdot\frac{C}{\mathfrak{M}}\cdot a$ et distantia ad oculum $=\frac{1}{3}u$.

X. Denique ut ante est spatii in obiecto conspicui semidiameter $=\frac{3a}{4\mathfrak{M}}$,
quod apparebit gradu claritatis, cuius mensura est $\frac{3,88a}{\mathfrak{M}}=\frac{27,04}{m}$.

SCHOLION 1

318. Tam in his duabus microscopiorum speciebus quam in allis, quae simili modo in medium afferri possunt, id potissimum inprimis notatu dignum occurrit, quod eaedem lentes ad omnes plane multiplicationes, dummodo satis magnas, aeque adhiberi possunt. Cum enim numerus C plane ab arbitrio nostro pendeat, dummodo sit mediocriter magnus, ita ut $\mathfrak{C}=\frac{C}{1+C}$ in praxi pro unitate haberi queat, fractio $\frac{C}{\mathfrak{M}}$ tanquam data spectari potest veluti $=\frac{1}{10}$ vel $=\frac{1}{20}$, ut postremae lentes non fiant nimis exiguae, ita ut haec fractio multiplicationem non amplius continere sit censenda. Hoc igitur notato solum intervallum lentium sextae et septimae multiplicationem determinabit, ita ut manentibus tam iisdem lentibus quam reliquis intervallis solum istud intervallum pro varia multiplicatione mutari

debeat, quae autem mutatio non adeo erit magna. Si enim desideretur multiplicatio $m=400$, sumta distantia obiecti $a=\frac{1}{2}$ dig. erit $\mathfrak{M}=\frac{m}{16}=25$ hincque istud intervallum

$$=12,48Ca=6,24C \text{ dig.}$$

ob $a=\frac{1}{2}$ dig. Sin autem velimus multiplicationem $m=800$, hoc intervallum erit $=13,35Ca=6,67C$ dig.

Atque adeo si hoc intervallum sumeretur

$$=14,20Ca=7,10C \text{ dig.}$$

tunc multiplicatio infinita produceretur. Semper autem consultum erit his instrumentis non nisi ad multiplicationes maximas uti, quoniam, nisi $\mathfrak{M}$ esset numerus valde magnus, littera C tanta non foret, ut $\mathfrak{C}$ pro unitate haberi posset.

SCHOLION 2

319. Videamus nunc etiam, an hae lentes obiectivae quadruplicatae adeos casus adplicari possent, ubi littera A fit negativa, ad quod investigandum sumamus superius exemplum septimum [§ 304], ubi erat $P=\frac{18}{25}$, atque ut intervallum

$$Aa\left(1+\frac{6(A-1)\delta}{A+1}-\frac{1}{P}\right)$$

prodeat positivum, posito $A=-\alpha$ debebit esse $\alpha a\left(\frac{7}{18}-\frac{6(\alpha+1)\delta}{\alpha-1}\right)$ positivum hincque

$$\delta<\frac{7(\alpha-1)}{108(\alpha+1)}$$

ideoque multo magis $\delta<\frac{7}{108}$ sive $\delta<\frac{1}{15}$, id quod in praxi locum habere nequit. Quia autem hoc non successit ob valorem $P=\frac{18}{25}$, faciamus adplicationem ad exemplum 3 casus 1 [§ 293], ubi erat $P=\frac{3}{7}$, $\mathfrak{B}=\frac{16}{21}$, $B=\frac{16}{5}$. Hoc casu ergo positivum esse debet intervallum

$$\alpha a\left(\frac{4}{3}-\frac{6(\alpha+1)}{\alpha-1}\delta\right)$$

unde pro δ valor non nimis exiguus requiritur. Examinari autem oportet aequationem, qua confusio tollitur, quae per $16A^3$ multiplicata erit

$$0=-(\alpha-1)^2+\nu(\alpha-1)\left(5\alpha^3+6\alpha+5\right)$$
$$-\frac{\delta(\alpha-1)^2(7\alpha+5)}{2}+\delta\nu(\alpha+1)\left(7\alpha^2+18\alpha+23\right)-\frac{16}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)-\frac{16}{\mathfrak{B}^3}\lambda'';$$

cui quidem aequationi satis idoneis valoribus tam pro α quam δ satisfieri posset; sed quia haec microscopiorum species alii incommodo est obnoxia, quandoquidem una lens in ipsa imagine reali posteriori est constituta, unde repraesentationem non mediocriter inquinari iam supra observavimus, non opus esse duco hanc evolutionem suscipere, sed potius formularum inventarum adplicationem ad telescopia ostendi eo magis conveniet, quod earum usus in microscopiis non tantopere desiderari videtur, quoniam perinde est, sive obiecta inverse sive erecte repraesentantur; quae autem supra de telescopiis huius generis sunt allata, ad duos tantum casus nimis particulares sunt referenda. Quare ex hac ulteriori istius generis evolutione non contemnendum supplementum peti potest.

ADPLICATIO HUIUS PROBLEMATIS AD TELESCOPIA

320. Cum distantia obiecti $a=\infty$, litteram A evanescentem sumi oporteet, ita tamen, ut Aa distantiam finitam, quae sit $=l$, denotet. Hinc igitur nostrarum quatuor lentium obiectivantm distantiae focales ita se habebunt:

$$p'=4l,\quad p''=4(1-\delta)l,\quad p'''=4(1-3\delta)\mathrm{l},\quad p''''=4(1-6\delta)l$$

existente communi intervallo inter has lentes $=4\delta l$. Litterae autem germanicae ad has lentes construendas adhibendae sunt

$$\mathfrak{A}'=0,\quad \mathfrak{A}''=-1,\quad \mathfrak{A}'''=-2,\quad \mathfrak{A}''''=-3,$$

dum alterae litterae $\lambda,\ \lambda'$ etc. omnes sunt $=1$. Tum vero intervallum ab hac lente quadruplicata ad lentem sequentem erit $=(1-6\delta-\frac{1}{P})l.$ Verum ad confusionem destruendam nunc ista habebitur aequatio:

$$0=\frac{1}{16}-\frac{5v}{16}-\frac{5\delta}{32}+\frac{23\delta v}{16}-\frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{v}{B\mathfrak{B}}\right)+\frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{v}{C\mathfrak{C}}\right)\ \text{etc.},$$

quae, si lentes ex vitro crystallino conficiantur, ubi est $v=0,2529$, abit in hanc:

$$0=\frac{-0{,}2645\,+\,3{,}3167\delta}{16}-\frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{v}{B\mathfrak{B}}\right)+\frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{v}{C\mathfrak{C}}\right).$$

Hinc ergo si duo postrema membra evanescerent, foret circiter $\delta=\frac{1}{13}$; accedentibus autem istis membris minor evadet, verum tamen haec membra tam debent esse exigua, ut non superent $\frac{0{,}2645}{16}$; inter exempla autem in fine capitis superioris allata nullum occurrit, quod hic locum habere queat, cum confusionis partes ex his lentibus natae multo sint maiores; neque etiam formulae generales ibi datae ad hunc usum accommodari possunt, ita ut huiusmodi lens obiectiva quadruplicata in hoc telescopiorum genere nullum plane usum habere possit.

PROBLEMA 2

321. *Si lens obiectiva constet tribus lentibus, quarum prima sit concava ex vitro crystallino, duae autem reliquae convexae ex vitro coronario confectae, reliquis lentibus omnibus manentibus, uti in capite praecedente sunt descriptae, microscopium construere, quod ab omni confusione sit liberatum.*

SOLUTIO

Hic in subsidium vocetur problema 2 capitis III sectionis praecedentis [§ 194], ubi pro tribus istis lentibus obiectivis in evolutione sequentes sumti sunt valores:

$$\mathfrak{A}=-\tfrac{1}{2},\ \ A=-\tfrac{1}{3},\ \ \mathfrak{B}=2,\ \ B=-2,\ \mathfrak{C}=1\ \text{ et }\ C=\infty$$

seu potius C indefinitum, dum sit numerus satis magnus. Deinde

$$\tfrac{1}{P}=\tfrac{17}{14},\ \ \tfrac{1}{PQ}=\tfrac{37}{28};$$

unde trium harum lentium distantiae focales, quas hic litteris p', p'', p''' designemus, ita sunt definitae:

$$p'=-\tfrac{1}{2}a,\ \ p''=\tfrac{17}{21}a,\ p'''=\tfrac{37}{42}a,$$

intervalla autem harum lentium $=\tfrac{1}{14}a$.

Ut igitur has determinationes ad praesens institutum accommodemus, quod ibi erat ABC, hic nobis est simpliciter A, ita ut sit $\tfrac{2}{3}C=A$, seu quod ibi erat C, hic nobis est $\tfrac{3}{2}A$, et quia ibi C erat indefinitum, etiamnunc hic A denotabit numerum indefinitum, dummodo sit satis magnus. Deinde quod ibi erat PQR, hic nobis simpliciter erit P, quod ideo etiamnum est indefinitum; unde intervallum a lente obiectiva hac triplicata ad lentem usque sequentem nobis hic erit

$$=Aa\left(\tfrac{37}{28}-\tfrac{1}{P}\right).$$

Ut vero omnia confusio tollatur, si littera λ referatur ad lentem primam concavam crystallinam, cui respondeant litterae μ et ν, pro sequentibus vero lentibus coronariis litterae λ', λ'' etc. unitati aequales ponantur ac vitro coronario conveniant litterae μ' et ν', littera λ definiri debet ex hac aequatione:

$$8\lambda=6\nu+\tfrac{\mu}{\mu'}\Lambda$$

existente

$$\Lambda=\tfrac{27\cdot17}{8\cdot14}\left(1-2\nu'\right)+\tfrac{27\cdot37}{8\cdot28}\left(1+\tfrac{2\nu'}{3A}\right)-\tfrac{1}{A^3P}\left(\tfrac{1}{\mathfrak{B}^3}+\tfrac{\nu'}{B\mathfrak{B}}\right)+\tfrac{1}{A^3B^3PQ}\left(\tfrac{1}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right).$$

Hic ergo ex supra evolutis exemplis ea eligi poterunt, in quibus Λ numerum satis magnum denotare potest atque ubi $\frac{1}{P} < \frac{37}{28}$, sicque plures huiusmodi microscopiorum species omni confusione carentes inveniri poterunt. Notetur autem esse

$$\mu = 0,8724 \quad \text{et} \quad \nu = 0,2529,$$

at vero

$$\mu' = 0,9875 \quad \text{et} \quad \nu' = 0,2196,$$

ita ut sit

$$\log.\frac{\mu}{\mu'} = 0,0538214.$$

Hinc ergo evolutione facta erit

$$\Lambda = 2,2983 + 4,4598 + \frac{0,6529}{A} - \frac{1}{A^3P}\left(\frac{1}{\mathfrak{B}^3} + \frac{\nu'}{B\mathfrak{B}}\right) + \frac{1}{A^3B^3PQ}\left(\frac{1}{\mathfrak{C}^3} + \frac{\nu'}{C\mathfrak{C}}\right).$$

hocque valore invento erit

$$\lambda = 0,1897 + \frac{\mu}{8\mu'}\Lambda$$

existente

$$\log.\frac{\mu}{8\mu'} = 9,1507314.$$

COROLLARIUM 1

322. Hinc ergo patet, si modo capiatur $A = 10$ hincque in superioribus formulis $C = 15$, partes huius formulae posteriores tam fieri exiguas, ut tuto negligi queant, dummodo litterae $\mathfrak{B}$ et *B,* quae sunt negativae, non fiant unitate multo minores, quod facile obtinetur, si modo *P* unitatem notabiliter superet. Tum igitur habebitur satis exacte

$$\lambda = 0,1897 + 0,9654 - 1,1551.$$

COROLLARIUM 2

323. Quia in hac hypothesi pro superioribus formulis erat $C = 15$, cum tamen ibi sumsissemus $\mathfrak{C} = 1$, quo haec fiant accuratiora, debuissemus ibi sumere $\mathfrak{C} = \frac{15}{16}$, unde pars illa tertia 4,4598 aliquanto maior evasisset in ratione $16^3 : 15^3$; quo facto loco istius numeri substitui debet hic 5,4125, ex quo concluditur $\lambda = 1,2807$. Tum vero pro tertia lente obiectiva fiet $p''' = 0,826a$.

COROLLARIUM 3

824. Ut autem aliquam rationem teneamus sequentium lentium, istum valorem ipsius λ tantillum augeri oportebit eumque ergo sumamus $\lambda = 1,29$. Unde pro constructione primae lentis crystallinae fiet $\tau\sqrt{(\lambda - 1)} = 0,4725$.

Hinc pro prima lente obiectiva, cuius distantia focalis est $p' = -\frac{1}{2}a$ et numerus $\mathfrak{A} = -\frac{1}{2}$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p'}{\sigma - \mathfrak{A}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = +\frac{p'}{1,83080} = -0,273a\\ \text{posterioris} = \frac{p'}{\rho + \mathfrak{A}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = -\frac{p'}{0,1067} = +\,4,686a.\end{cases}$$

Pro secunda vero lente obiectiva ex vitro coronario, cuius distantia focalis est $p'' = \frac{17}{21}a = 0,8095a$, ob $\mathfrak{B} = 2$ supra inventus est

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0,6708a\\ \text{posterioris} = +\,0,2617a.\end{cases}$$

Pro tertia autem lente obiectiva, cuius est distantia focalis $p''' = 0,826a$, ob $\mathfrak{C} = \frac{15}{16}$ erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p'''}{\sigma - \mathfrak{C}(\sigma-\rho)} = \frac{p'''}{0,3163} = 2,611a\\ \text{posterioris} = \frac{p'''}{\rho + \mathfrak{C}(\sigma-\rho)} = \frac{p'''}{1,5705} = 0,526a.\end{cases}$$

Quibus tribus lentibus, quarum intervalla sunt $= \frac{1}{14}a = 0,071a$, tribui poterit apertura, cuius semidiameter $x = 0,065a$; unde nascitur claritas, cuius mensura $= \frac{10,4}{m}$.
est

COROLLARIUM 4

325. Quod ad reliquas lentes attinet, quoniam in calculum confusionis non ingrediuntur, perinde est, ex quonam vitro parentur et quaenam figura ipsis tribuatur, dummodo eae utrinque fiant aeque convexae, quae maximam aperturam habere debent; id tantum notetur intervallum a tertia lente obiectiva ad sequentem lentem esse debere $= 10a\left(\frac{37}{28} - \frac{1}{P}\right)$.

EXEMPLUM 1

326. Adplicemus haec ad exemplum 2 postremi casus capitis praecedentis [§ 299], ubi secunda lens plane tollebatur primumque et secundum intervallum in unum coalescebat, quod nunc erit $= 16,54a$. Hinc ergo sequitur

CONSTRUCTIO MICROSCOPII EX SEPTEM LENTIBUS COMPOSITI

Hic praeter distantiam obiecti $= a$ multiplicatio m ut data assumitur, unde fit $\mathfrak{M} = \frac{ma}{h}$. Tum vero etiam numerus C arbitrio nostro relinquitur, dummodo sit praemagnus, quo effici poterit, ut postremae lentes non fiant nimis exiguae. Constructio igitur ita se habebit:

I. Pro prima lente ex vitro crystallino paranda, cuius distantia focalis est $p' = -\frac{1}{2}a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0,273a\\ \text{posterioris} = +4,686a,\end{cases}$$

eius aperturae semidiameter $x = 0,065a$ et intervallum ad lentem secundam $= 0,071a$.

II. Pro secunda lente ex vitro coronario paranda, cuius distantia focalis est $p'' = 0,809a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0,671a\\ \text{posterioris} = +0,262a,\end{cases}$$

eius apertura ut primae et intervallum ad lentem tertiam etiam $= 0,071a$.

III. Pro tertia lente itidem ex vitro coronario paranda, cuius distantia focalis est $p''' = 0,826a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = 2,611a\\ \text{posterioris} = 0,526a,\end{cases}$$

eius apertura etiam ut primae, at intervallum ad lentem quartam $= 16,54a$.

IV. Pro quarta lente perinde est, ex quonam vitro paretur, dummodo sit eius distantia focalis $r = \frac{10}{3}\mathfrak{C}a$ sive proxime $r = 3a$, neque etiam multum refert, quaenam huic lenti figura tribuatur.

Eius aperturae semidiameter $= \frac{3r}{\mathfrak{M}} + \frac{1}{3}x$ et intervallum ad lentem quintam

$$= \frac{10}{3}Ca\left(1 - \frac{1}{i\,\mathfrak{M}}\right) = \frac{10}{3}Ca\left(1 - \frac{33}{2\mathfrak{M}}\right)$$

ob $i = \frac{2}{33}$.

V. Pro quinta lente, quam utrinque aeque convexam esse oportet, distantia focalis $s = 30\frac{C}{\mathfrak{M}} \cdot a$ eiusque apertura maxima. Intervallum ad lentem sextam

$$=\frac{15(3i+2)}{9i+1}\cdot\frac{C}{\mathfrak{M}}\cdot a=\frac{360}{17}\cdot\frac{C}{\mathfrak{M}}\cdot a=\frac{15(3i+2)}{9i+1}\cdot\frac{C}{\mathfrak{M}}\cdot a.$$

VI. Pro sexta lente pariter utrinque aeque convexa distantia focalis est

$$t=\frac{225i}{9i+1}\cdot\frac{C}{\mathfrak{M}}\cdot a=\frac{150}{17}\cdot\frac{C}{\mathfrak{M}}\cdot a$$

et intervallutn ad lentem septimam

$$=\frac{225i}{4(9i+1)}\cdot\frac{C}{\mathfrak{M}}\cdot a=\frac{75}{34}\cdot\frac{C}{\mathfrak{M}}\cdot a$$

VII. Pro septima lente etiam utrinque aeque convexa distantia focalis est

$$u=\frac{225i}{18i+2}\cdot\frac{C}{\mathfrak{M}}\cdot a=\frac{1}{2}t=\frac{75}{17}\cdot\frac{C}{\mathfrak{M}}\cdot a$$

et distantia ad oculum $=\frac{1}{3}u.$

VIII. Spatii in obiecto conspicui erit semidiameter $=\frac{3a}{4\mathfrak{M}}$ et mensura claritatis $=\frac{10{,}4}{m}$.

Hic igitur quantitas C arbitrio nostro relinquitur, dummodo sit numerus satis magnus, ita ut fractio $\frac{C}{\mathfrak{M}}$ tanquam data spectari possit; deinde patet etiam esse $u=\frac{5}{34}s$ sicque ipsis lentibus iisdem manentibus idem instrumentum ad omnes multiplicationes aptum reddi poterit, dummodo intervallum inter lentem quartam et quintam varietur, cum etiam reliqua intervalla maneant eadem ob fractionem $\frac{C}{\mathfrak{M}}$.

EXEMPLUM 2

327. Lentem nostram obiectivam triplicatam etiam coniungere licebit cum superiori exemplo tertio [§ 300], ubi erat $P=\frac{3}{2}$ et $\mathfrak{B}=-1$. Manebunt igitur tres priores articuli uti in exemplo praecedente, nisi quod in fine tertii scribi debet:
Intervallum a tertia lente ad quartam

$$=\left(\frac{37}{28}-\frac{1}{P}\right)Aa=6{,}55a.$$

IV. Pro quarta lente perinde est, ex quonam vitro paretur, dummodo sit eius distantia focalis

$$q=\frac{20}{3}a=6{,}7a.$$

Eius aperturae semidiameter $=\frac{3}{8}\cdot\frac{q}{\mathfrak{M}}+\frac{2}{3}x$
et intervallum ad lentem quintam $=5a$.

V. Pro quinta lente, cuius distantia focalis est

$$r = \tfrac{5}{3}\mathfrak{C}a = 1,667\mathfrak{C}a = 1,50a$$

sumto scilicet $\mathfrak{C} = \frac{9}{10}$, siquidem unitati prorsus aequari non potest,

eius aperturae semidiameter $= \frac{21r}{8\mathfrak{M}} + \frac{1}{3}x$,

intervallum ad lentem sextam $= \frac{5}{3}Ca\left(1 - \frac{12}{\mathfrak{M}}\right)$ seu, si ponatur $\frac{C}{\mathfrak{M}} = \gamma$, ut sit $C = \gamma\mathfrak{M}$, hoc intervallum erit $= \frac{5}{3}\gamma a\left(\mathfrak{M} - 12\right)$, ubi γ ita sumitur, ut lentes sequentes non fiant nimis exiguae.

VI. Pro sexta lente, quae cum sequentibus debet esse utrinque aeque convexa, distantia focalis sit $s = 15\gamma a$,

eius apertura maxima seu semidiameter aperturae $= \frac{1}{4}s$

et intervallum ad lentem septimam $= \frac{135}{14}\gamma a = 9,643\gamma a$.

VII. Pro septima lente distantia focalis $t = \frac{150}{28}\gamma a = 5,357\gamma a$,

aperturae semidiameter $= \frac{1}{4}t$,

intervallum ad lentem octavam $= \frac{75}{56}\gamma a = 1,339\gamma a$.

VIII. Pro lente octava distantia focalis $u = \frac{75}{28}\gamma a = 2,679\gamma a$,

aperturae semidiameter $= \frac{1}{4}u$,

distantia ad oculum $= \frac{1}{3}u$.

IX. Spatii in obiecto conspicui erit semidiameter $= \frac{3a}{4\mathfrak{M}}$ et mensura claritatis $= \frac{10,4}{m}$.

EXEMPLUM 3

328. Superius quartum exemplum huc transferri nequit; ex quinto [§ 302] autem, ubi $P = \frac{18}{7}$ et $\mathfrak{B} = -\frac{11}{7}$, nascitur haec constructio:

Tres articuli priores manent ut in exemplo primo; iis autem subiungatur:

Intervallum a tertia lente ad quartam $= 9,325a$.

IV. Pro quarta lente

Distantia focalis $q = \frac{55}{9}a = 6,11a$.

Eius aperturae semidiameter $= \frac{3}{4}\cdot\frac{q}{\mathfrak{M}} + \frac{7}{18}x$.

Distantia ad lentem quintam $=\frac{715}{162}a=4,414a.$

V. Pro quinta lente

Distantia focalis $r=1,83a$.

Eius aperturae semidiameter $=\frac{9}{4}\cdot\frac{r}{\mathfrak{M}}+\frac{1}{3}x.$

Distantia ad lentem sextam $=2,037\gamma a(\mathfrak{M}-10).$

VI. Pro sexta lente

Distantia focalis $s=\frac{55}{3}\gamma a=18,33\gamma a$.

Aperturae semidiameter $=\frac{1}{4}s.$

Distantia ad lentem septimam $=11,096\gamma a$.

VII. Pro septima lente

Distantia focalis $t=\frac{275}{38}\gamma a=7,237\gamma a$.

Aperturae semidiameter $=\frac{1}{4}t.$

Intervallum ad lentem octavam $=\frac{275}{152}\gamma a=1,809\gamma a$.

VIII. Pro octava lente

Distantia focalis $u=\frac{1}{2}t=3,618\gamma a$.

Aperturae semidiameter $=\frac{1}{4}u$

et distantia ad oculum $=\frac{1}{3}u$.

IX. Campus et claritas se habent uti in praecedentibus exemplis.

EXEMPLUM 4

329. Ex superiori exemplo octavo [§ 305], ubi $P=24$ et $\mathfrak{B}=-\frac{23}{4}$, nascitur haec constructio:
Tribus prioribus articulis manentibus ut ante subiungatur:
Distantia tertiae lentis ad quartam $=12,80a$.

IV. Pro quarta lente

Distantia focalis $q=\frac{115}{48}a=2,396a.$

Aperturae semidiameter $=\frac{3q}{\mathfrak{M}}+\frac{1}{24}x.$

Intervallum ad quintam lentem $=\frac{115}{36}a=3,194a.$

V. Pro quinta lente

Distantia focalis $r = \frac{230}{81} \cdot \mathfrak{C}a = 2,556a.$

Aperturae semidiameter $= \frac{1}{3}x$.

Intervallum ad sextam lentem $= 2,839\gamma a(\mathfrak{M} - 3)$.

VI. Pro sexta lente

Distantia focalis $s = 25,556\gamma a$.

Aperturae semidiameter $= \frac{1}{4}s$.

Intervallum ad septimam lentem $= 9,583\gamma a$.

VII. Pro septima lente

Distantia focalis $t = 15,972\gamma a$.

Aperturae semidiameter $= \frac{1}{4}t$.

Intervallum ad lentem octavam $= \frac{1}{4}t = 3,993\gamma a$.

VIII. Pro octava lente

Distantia focalis $u = \frac{1}{2}t = 7,986\gamma a$.

Aperturae semidiameter $= \frac{1}{4}u$.

Intervallum ad oculum $= \frac{1}{3}u$.

IX. Campus et claritas ut in praecedentibus exemplis.

EXEMPLUM 5

330. Facta denique applicatione ad superius exemplum 9 [§ 306] postremo nascitur haec constructio:

Tribus prioribus lentibus manentibus ut hactenus subiungatur tertia articulo:

Intervallum tertiae et quartae lentis $= 12,24a$.

IV. Pro quarta lente

Distantia focalis $q = \frac{325}{144}a = 2,257a$.

Aperturae semidiameter $= \frac{3q}{\mathfrak{M}} + \frac{7}{72}x.$

Distantia ad quintam lentem $= 3,009a$.

V. Pro quinta lente

Distantia focalis $r = 2,097a$.

Aperturae semidiameter $= \frac{1}{3}x$.

lntervallum ad sextam lentem $= 2,33\gamma a(\mathfrak{M} - 1)$.

VI. Pro sexta lente

Distantia focalis $s = 20,97\gamma a$.

Aperturae semidiameter $= \frac{1}{4}s$.

Intervallum ad septimam lentem $= -\ 5,242\gamma a$.

VII. Pro septima lente

Distantia focalis $t = 15,726\gamma a$.

Aperturae semidiameter $= \frac{1}{4}t$.

Intervallum ad lentem octavam $= \frac{1}{4}t = 3,931\gamma a$.

VIII. Pro octava lente

Distantia focalis $u = 7,863\gamma a$.

Aperturae semidiameter $= \frac{1}{4}u$.

Distantia ad oculum $= \frac{1}{3}u$.

IX. Campus et claritas se habent ut in praecedentibus exemplis.

FINIS OPERIS