CHAPTER IV

CONCERNED WITH THE CONFUSION OF VISION FROM THE APPARENT MAGNITUDE AS WELL AS THE CLARITY

DEFINITION 1

159. *The vision is distinct, if all the rays arriving at the eye from some point of the object, may be gathered together again at the back of the eye into a single point on the retina.*

SCHOLIUM

160. For distinct vision it is required, that objects may be located at a certain fixed distance from the eye, which distance is usually especially diverse due to the nature of the eye, while being exceedingly small in myopic eyes, but large for these with strong eyes, and not only infinite for far-sighted people but for whom also negative images may arise for nearby objects [*i.e*., behind the retina], and as distances of this kind do not exist for real objects, such people must be satisfied to use spectacles. Therefore any eye is constructed for a certain fixed distance of objects, that I will call its true distance [*i.e.* the least distance of distinct near vision, or near point] ; where indeed a significant latitude may be found, on account of which the structure of the eye is skilled, in so far as by a certain contraction and elongation it may be able to accommodate itself to somewhat greater or lesser distances. Therefore vision is distinct when the objects may be found at true distances from the eye, while the individual points of the object may be expressed by individual points on the retina.

[The true nature of the accommodation of the eye was to be established later by Thomas Young, involving mainly changes in the shape of the lens and pupil, rather than changes in the shape of the eye as a whole.]

DEFINITION *2*

161. *The vision is confused, if the rays from any point of the object may not be sent to a single point in the retina, but may affect some area of the retina.*

COROLLARY 1

162. Thus the confusion will be greater, if this area on the retina were greater, by which the rays sent from the same point of the object may be scattered. From which the magnitude of this space will provide a true measure of the confusion.

COROLLARY 2

163. Therefore the vision will be confused, when the distance of the object seen were much different from the true distance from the eye. For a small difference by itself either will appear not to be confused, or the eye itself may prevail to exert a small influence to accommodate the distance of the object.

SCHOLIUM

164. If objects may be represented distinctly by lenses, the view of the image is to be maintained by the same law as of these objects themselves ; evidently the true distances of these from the eye will produce a distinct image, but otherwise different distances may well produce a confused image. Truly if an image were diffused by a certain amount, even if it may be moved back to the true distance from the eye, then it is still necessary that confusion of the image may arise. Since if indeed we may look not at the object itself, but at the images of this shown by one or more lenses, because of a twofold reason the image will be able to be confused; on the one hand, if the distance of the object from the eye were many times different from the true object distance, and on the other hand, if that image may diffuse out through a certain patch. Indeed the first cause is in our power to be removed, if indeed it may be allowed to position the lenses, so that they may show the image at its true distance from the eye; as here we may assume always the disposition of the lenses. On account of which I have decided in this chapter to investigate, how much confusion must arise from the diffusion space [or patch], and the amount of this thence determined will be required to be elaborated on, as well as the lens forms and dispositions, so that the confusion arising in the image may not exceed a given limit, which at this point is tolerable. In general indeed it is certain, where the greater were the diffusion patch, there thence the greater must be the confusion induced in the image seen; yet truly we will see soon the confusion of the image not to be in proportion to the diffusion patch, but generally to follow another law, as it will be of the greatest necessity to be determined accurately, since from this source the construction of all optical instruments shall be required to be deduced according to this image viewed being accommodated.

PROBLEM 1

165. *If the eye may look at the image of some object diffused by one or more lenses through the space Ll* (Fig. 10) *and the principal image L may be found at the true distance from the eye, to define the confusion, by which the vision of this image may be affected.*

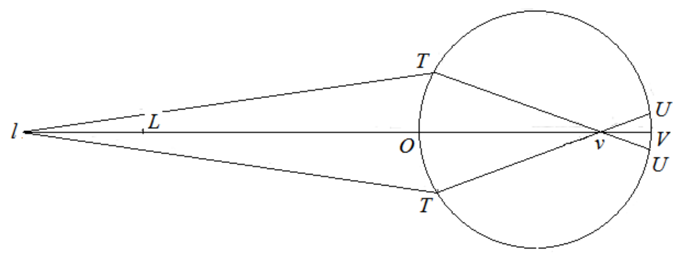

Fig. 10

SOLUTION

In the diffusion length *Ll,* in which the point *L* may be shown from the rays passing through the middle of the lens, truly the point *l* from the rays passing through the extremities of the lens, in the first place it will be required to consider the magnitude of this *Ll,* then the inclination of the rays to the axis concurring at *l*. But above we have seen this matter to depend on the aperture of the first lens, of which the radius shall be $=x$, so that the distance itself shall be $Ll = Vxx$ and the angle $OlT = \mathfrak{B}x$, clearly we have determined the values of these coefficients *V* and $\mathfrak{B}$ for any number of lenses . Now the circle *OV* may represent the eye, as here it is allowed to be considered as a small camera obscura, but thus perfect, so that the rays sent from one point may be gathered again into one point ; for even if there were several refractions in the eye, yet with that condition served by a single lens can be considered by as single lens in place of these, which shall be at *TOT,* from which the retina shall be moved back by the distance $OV = u$. Now since the principal image *L* is due to be present at some distance from the eye, which may be called $OL = l$, with the image of the point *L* incident in the eye on the retina itself and described distinctly at the point *V* : truly with the image of the point *l* not at *V*, but may be referred to at *v* before the retina, so that the interval *Vv,* if the thickness of the lens at *TOT* may vanish, thus may be expressed following § 62, so that there shall become

$$Vv = \frac{OV^2}{OL^2} \cdot Ll = \frac{uu}{ll} \cdot Vxx.$$

But since the rays forming the point *l* shall be inclined to the axis by the angle $OlT = \mathfrak{B}x$, these thus will enter through the lens of the eye by the points *TT*, so that, with the interval *Ll* ignored besides the distance $OL = l$, there shall be $OT = l\mathfrak{B}x$, from which these concurring at the point v with the axis will make the angle $OvT = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{1}{u}\mathfrak{B}x$. Hence therefore continuing above the retina tracing out that circle UU, the radius of which shall be

$$VU = \frac{1}{u}\mathfrak{B}x \cdot Vv = \frac{1}{u} \cdot \mathfrak{B}x \cdot \frac{uu}{ll} \cdot Vxx,$$

and this circular space above the retina will be filled by the middle points between L and l. Whereby the diffuse image through the distance Ll will be represented by the circle above the retina, of which radius $VU = \frac{u}{l} \cdot \mathfrak{B} \cdot Vx^3$, which circle takes the place of the true measure of the confusion. Evidently any point of an object, which is shown by lenses by the diffusion length Ll, is not expressed by a point above the retina, but by a circle of which the radius will be $= \frac{u}{l} \cdot \mathfrak{B} \cdot Vx^3$.

COROLLARY 1

166. Therefore we have seen the confusion by which vision is affected, to depend not only on the magnitude of the special diffusion $Ll = Vxx$, but also on the angle, by which the rays concurring at l may be inclined to the axis, which is $= \mathfrak{B}x$, and the radius of the circle of confusion measured by that to be in proportional to the product of this space by that angle.

COROLLARY 2

167. Therefore since with the radius of the aperture of the first lens put $= x$, the diffusion distance shall be as the square of xx, the radius of the circle measuring the confusion is as its cube x^3. And if the confusion may be considered proportional to the area of this circle, that will be as x^6 or as the cube of the aperture of the first lens.

COROLLARY 3

168. Therefore, it is hence understood by how great an amount the diffusion distance Vxx may be diminished by a suitable disposition of the lenses present ; for not only in the same ratio, by which that diffusion length Vxx or the quantity V is diminished, but thus the confusion seen will be rendered smaller in the duplicate ratio.

SCHOLIUM

169. To have assumed here the pupil of the eye to appear wide open, so that it may receive rays diverging from l ; but if the aperture of the pupil were made smaller, the rays arriving from the point l indeed will not be able to be captured : therefore in this case it will return the same, as if with the aperture of the first lens contracted the diffusion length there may be diminished as far as Ll, so that the pupil may be able to receive all the rays emitted by the image, and in this case it is evident smaller diffusion is going to be

produced. Truly it will be shown in the following both in telescopes as well as microscopes in this case scarcely a place can be found, since generally the cone of rays emitted by the point l around the entrance point in the eye shall be much narrower than the aperture of the pupil ; on account of which there is no need in this case, even if by itself it may be expedited easily, here to spend time setting it out. Moreover even if the eye may not conveniently be compared to a simple lens, of which the thickness may vanish, it may be able to be brought together and hence the expression of the distance Vv may be able to appear somewhat different to that following § 84, here we do not attend to this circumstance, since the distance Vv may have appeared to have increased or decreased according to a certain known ratio ; but here thus it is not necessary to know the size of the absolute confusion, provided we will have defined accurately the ratio which it follows. Indeed since we have assembled our calculation from experience, we get to know the term, because if our confusion may exceed our customary expression, it may increase in an intolerable manner ; and hence it will suffice always for the confusion expression to be reduced in a similar manner below this limit. Yet meanwhile it will help to have observed through the conformation of the confusion by the eye, to what extent it can be made much smaller.

PROBLEM 2

170. *With the same in place, which were assumed in the preceding problem, to define that shape of the eye, by which the minimum confusion may be perceived.*

SOLUTION

The whole of the eye is endowed with the ability to change its shape by a small amount, so that even objects may be seen distinctly, the distances of which may not differ exceedingly from the true one :

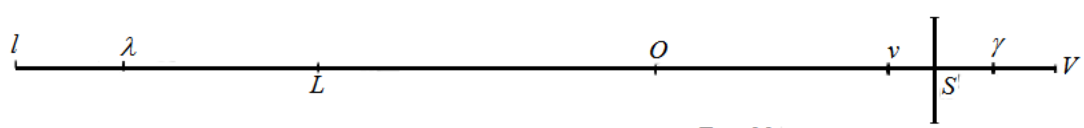

Fig. 11

which may be performed in some manner, thus so that it is possible to understand, as if the retina may be moved closer or further away from the pupil, or rather we may consider that lens taken in place of the eye to be distorted and to have moved either closer or further from that. Therefore since initially we may have considered the retina placed at V (Fig. 11), here we may suppose the retina itself to pass over to the point v, and it is evident the point l is going to be expressed distinctly on that. Truly since the point L does not emit other rays apart from those close to the axis, also the point L will be represented at v without any confusion, thus so that only the middle points between L and l shall be going to produce confusion. Therefore any intermediate point λ may be considered, which shall be the extreme in the confusion distance, if the radius of the aperture of the first lens may be less than x, which therefore may be put $= z$; and there will become $L\lambda = Vzz$ and the inclination of the rays to the axis concurring at $\lambda = \mathfrak{B}z$; hence the image of this point within the eye may be produced at γ, so that there shall be $V\gamma = \frac{uu}{ll}Vzz$, and thus on account of $Vv = \frac{uu}{ll}\mathrm{V}xx$ the interval will be $v\gamma = \frac{uu}{ll}\mathrm{V}(xx - zz)$ and the inclination of the rays to the axis converging at $\gamma = \frac{l}{u}\mathfrak{B}z$; from which the radius of the circle above the retina present at v expressed by the rays of the point λ will be

$$= \frac{u}{l}\mathfrak{B}Vz(xx - zz),$$

which vanishes, as we have now shown, either if $z = 0$ or $z = x$. Now that value of z is sought, where that circle may become a maximum ; which will happen, if $xx - 3zz = 0$ or $z = \frac{x}{\sqrt{3}}$; and thus the radius of the circle of confusion measured in this case will be $= \frac{2u}{3l\sqrt{3}} \cdot \mathfrak{B}Vx^3$, which is much less than in the preceding case, when the retina was at V.

But if the retina may be moved a little from v towards V, here the circle may be able to be made less at this stage. Indeed we may put the distance $VS = s$, so that there shall become

$$vS = \frac{uu}{ll}Vxx - s \quad \text{and} \quad S\gamma = s - \frac{uu}{ll}Vzz.$$

Therefore the point l, of which the image is shown at v, is referred to the circle S above the retina, of which the radius is $= \frac{l}{u}\mathfrak{B}x(\frac{uu}{ll}Vxx - s)$, truly the point λ, of which the image is at γ, with the circle S above the retina, of which the radius will be $= \frac{l}{u}\mathfrak{B}z(s - \frac{uu}{ll}Vzz)$; therefore now we will examine that point λ, from which this minimal circle may arise : which happens, if there shall be $s = \frac{3uu}{ll}Vzz$; and the radius of this circle will be $= \frac{2u}{l} \cdot \mathfrak{B}Vz^3$, which likewise may show confusion, but only if no greater circle may arise from the point l: but with the value found substituted for s the radius of this circle will be $\frac{u}{l}\mathfrak{B}Vx(xx - 3zz)$, which therefore may be put equal to that $\frac{2u}{l} \cdot \mathfrak{B}Vz^3$, from which this equation arises:

$$x^3 - 3xzz = 2z^3 \quad \text{and hence} \quad z = \tfrac{1}{2}x.$$

Whereby the radius of the circle, for which the minimum confusion may be measured, will be $= \frac{u}{4l}\mathfrak{B}Vx^3$, four times smaller, than if the retina were at V; and thus the confusion itself sixteen times smaller. But even if that cubic equation had three roots, besides $z = \frac{1}{2}x$ the remaining two are equal and $z = -x$ and thus $zz = xx$, from which no minima, but rather maximum confusion may arise.

COROLLARY 1

171. Therefore since there must be $z=\frac{1}{2}x$, so that the eye may experience minimum confusion, and thus there must become $s=\frac{3uu}{4ll}Vxx$, it is apparent the retina must be forced to be in that place *S*, so that there shall be $VS=\frac{3}{4}Vv$ and $vS=\frac{1}{4}Vv$.

COROLLARY 2

172. Therefore since the eye not only shall be endowed with the facility of changing itself for a short while, but also this facility may be accustomed to be used for avoiding confusion, there is no doubt, why the radius of the circle producing the confusion shall be $=\frac{u}{4l}\mathfrak{B}Vx^3$, and thus four times smaller, than we have found in the preceding problem, provided the image being viewed *Ll* may be found almost at the true distance.

COROLLARY 3

173. Therefore this expression $\frac{u}{4l}\mathfrak{B}Vx^3$ shows the true measure of our confusion, since it expresses the radius of the circle, by which the individual points of the object, in as much as that may be viewed through, may be represented on the retina. But only if the image may be found at the true distance from the eye.

SCHOLIUM

174. Clearly the vision shall be distinct, if these circles may vanish ; but this cannot happen, unless the interval of the diffusion itself *Vxx* may vanish : from which it is apparent, if the aperture of the first lens may be reduced to nothing, no confusion must be perceived. In truth no vision thus is revealed, so that in short no confusion shall be able to be allowed, but provided the confusion may not exceed a certain limit, it may be considered as if it may be zero. This same term or value, that the expression $\frac{u}{4l}\mathfrak{B}Vx^3$ must not exceed, must be sought from experiment rather than from theory, and that designated therefore will be contained according to this width : from which it will happen, that sometimes we may be content by looking into the diverse higher orders, but at other times we may examine the smaller order ; which circumstances, which we will approach as the need arises, we are going to examine more accurately. Moreover it is required to be observed for the interval *u,* from which as if the depth of the eye is shown, to be able to be assumed to be about one inch, indeed which measurement will be almost arbitrary, since henceforth we will put the limits of confusion by experience.

PROBLEM 3

175. *If the eye may view the object through a single lens, thus so that the image seen by the eye may be found at the true distance, to define the confusion, by which the vision may be affected.*

SOLUTION

Let the distance of the object Ee before the lens (Fig. 3 again) be $AE = a$, truly the principal image $F\zeta$ after the lens $BF = \alpha$, and the thickness of the lens $AB = v$, truly an arbitrary quantity shall be $= k$,

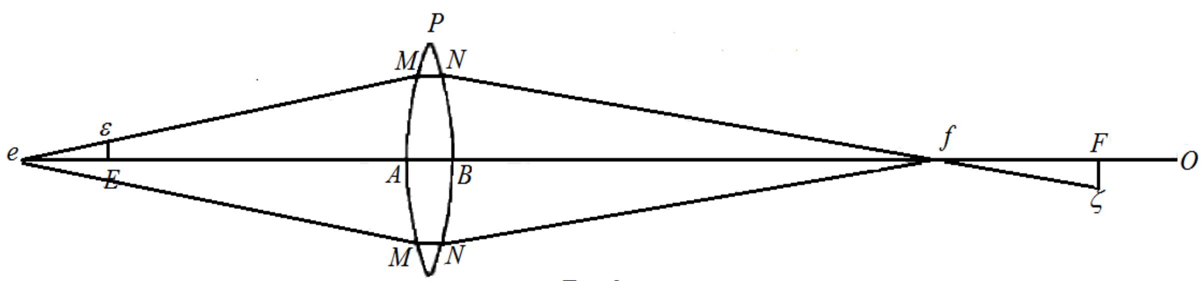

Fig. 3.

from which with both the determinable distances a and α, the face of the lens may be determined ; moreover for brevity there may be put $\frac{k-v}{k+v} = i$. But then with the ratio of the refraction put $= n$ there must become for the lens PP

$$\text{radius of the anterior face} = \frac{(n-1)a(k+v)}{k+v+2na},$$

$$\text{radius of the posterior face} = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha},$$

and if the radius of the aperture of the anterior face shall be $= x$, truly of the posterior $> x$, the diffusion space Ff will be $= P\alpha\alpha xx$, with there being

$$P = \frac{N}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right),$$

truly the inclination of the rays concurring at f to the axis $= i \cdot \frac{x}{\alpha}$. With these presented, which are established in § 86, the distance of the eye after the lens $BO = O$, and since the image $F\zeta$ is assumed to be present at the true distance before the eye, there will be $O = \alpha + l$ and thus $\alpha = O - l$, if the position of the eye may be considered as given.

Hence we will have $V = P\alpha\alpha$ and $\mathfrak{B} = \frac{i}{\alpha}$ and thus $\mathfrak{B}V = i\alpha P.$ Consequently the radius of the circle measuring the confusion in the eye or, as we may say in the following, the measure of the confusion will be

$$= \frac{u}{4l} \cdot i\alpha x^3 \cdot P = \frac{iu}{4l}(O-l)x^3 P.$$

If the thickness of the lens v may vanish and for the determination of the lens the arbitrary number λ may be introduced in place of k, from § 91 there will become

$$P = \mu\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda(\tfrac{1}{a} + \tfrac{1}{\alpha})^2 + \tfrac{v}{a\alpha}\right).$$

and $i = 1$, from which with the cited in place also the construction of the lens itself is desired.

COROLLARY 1

176. Therefore if a lens were given, the position of the eye after the lens is defined thus, so that the distance must become $BO = O = \alpha + l$, with l being the true distance of the eye ; but without the position of the eye being given, for the construction of the lens the determinable distance α thus must be taken, so that there shall be $\alpha = O - l.$

COROLLARY 2

177. Whereby if the eye were prepared thus, so that the true distance may be required $l = \infty,$ there becomes $\alpha = -\infty,$ and the measure of the confusion will be $= -\frac{1}{4}iux^3 \cdot P$ or $= \frac{1}{4}iux^3 \cdot P,$ since the − ve sign changes nothing in the magnitude of the circle of confusion being produced.

COROLLARY 3

178. Therefore even if in this case, where $\alpha = -\infty,$ the diffusion length Ff is infinite, yet thence the confusion arising in the vision is finite, since this may remain finite with no opposing value of P : indeed there will become

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a} + \frac{2}{k+v}\right)(\tfrac{1}{a} + \tfrac{2}{k+v})^2 - \frac{8ii}{(k-v)^2}\right).$$

Moreover, the radius of the posterior face will become $= -\frac{(n-1)}{2n}(k - v).$

COROLLARY 4

179. Since we have $i=\frac{k-v}{k+v}$, the value of P also can be expressed in general thus, so that there shall be :

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right),$$

where there is $\frac{n}{2(n-1)^2}=\frac{310}{121}=2{,}561983$ on account of $n=\frac{31}{20}$. And thus also the values of the remaining letters *Q, R, S* etc. in § 86 will be able to be transformed.

PROBLEM 4

180. *If the eye may view the object* $E\varepsilon$ *through two lenses* (Fig. 5 again), *thus so that the image represented by these* $G\eta$ *shall be removed from the eye by the true distance* $OG=l$, *to define the confusion, by which the vision may be affected.*

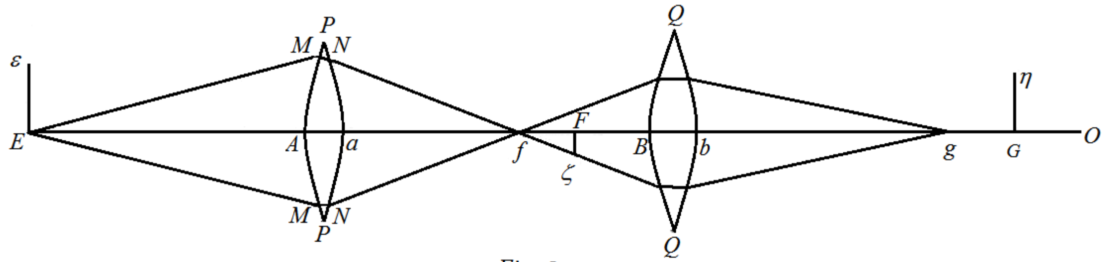

Fig. 5

SOLUTION

Initially, there shall be the determined distances for the lens *PP* $AE=a,\ aF=\alpha$, the thickness $Aa=v$ and the arbitrary distance $=k$, thus so that there shall be

$$\text{radius of the anterior face } =\frac{(n-1)a(k+v)}{k+v+2na},$$

$$\text{radius of the posterior face } =\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha};$$

then truly on placing $\frac{k-v}{k+v}=i$, there may be put

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right).$$

Then for the posterior lens QQ the determinable distances shall be $BF = b,\ bG = \beta$, the thickness of the lens $Bb = v'$ and the arbitrary distance $= k'$, so that there shall be

$$\text{the radius of the anterior face } = \frac{(n-1)b(k'+v')}{k'+v'+2nb},$$

$$\text{the radius of the posterior face} = \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta};$$

then truly on placing $\frac{k'-v'}{k'+v'} = i'$, there may be put

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right).$$

And now the diffusion length will be :

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii\cdot\frac{bb}{\alpha\alpha}Q\right) = Vxx,$$

and the inclination of the rays to the axis concurring at g will be

$$ii'\cdot\frac{bx}{\alpha\beta} = \mathfrak{B}x.$$

Now the eye shall be at O, with there being $OG = l$, and the distance may be put after the lens QQ, $bO = O$, there will be $O = \beta + l$ and thus $\beta = O - l$; with which in place, since the measure of the confusion shall be $= \frac{u}{4l}\mathfrak{B}x\cdot Vxx$, for our case there will become :

$$\frac{ii'u}{4l}\cdot\frac{b\beta}{\alpha}x^3\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii\cdot\frac{bb}{\alpha\alpha}Q\right),$$

or by the value $O-l$ put for β, and with the sign changed :

$$\frac{ii'u}{4l}\cdot\left(1-\frac{O}{l}\right)\frac{b}{\alpha}x^3\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii\cdot\frac{bb}{\alpha\alpha}Q\right).$$

COROLLARY 1

181. Therefore if for the eye there were $l = \infty$ and thus for $\beta = -\infty$, even if the diffusion distance Gg were infinite, yet on account of $\frac{O}{l} = 0$ the confusion affecting the vision nevertheless will be finite : and neither P nor Q becomes infinite on account of $\beta = -\infty$.

COROLLARY 2

182. If the thickness of the lens may vanish, so that there shall be $v=0$ and $v'=0$, there will become $i=1$ and $i'=1$ and thus in this case the measure of the confusion becomes

$$\frac{1}{4}\cdot u(1-\frac{O}{l})\frac{b}{\alpha}x^3\left(\frac{\alpha\alpha}{bb}P+\frac{bb}{\alpha\alpha}Q\right).$$

And if in place of k and k' the numbers λ and λ' may be introduced, there will become:

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2+\frac{v}{a\alpha}\right) \text{ etc. } Q=\mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'(\frac{1}{b}+\frac{1}{\beta})^2+\frac{v}{b\beta}\right),$$

with there being $\mu=0,938191$ and $v=0,232692$.

COROLLARY 3

183. But in this case the construction of the two lenses thus will be found likewise :

Radius of face

For lens PP $\begin{cases} \text{anterior} = \frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterior} = \frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

For lens QQ $\begin{cases} \text{anterior} = \frac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterior} = \frac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

with $\rho=0,190781$, $\sigma=1,627401$ and $\tau=0,905133$ being present.

PROBLEM

184. *If the eye may view an object $E\varepsilon$ through three lenses PP, QQ and RR* (Fig. 6), *thus so that the image represented by these $H\theta$ may be removed from the eye O by the true distance $OH = l$, to define the confusion, by which the vision may be affected.*

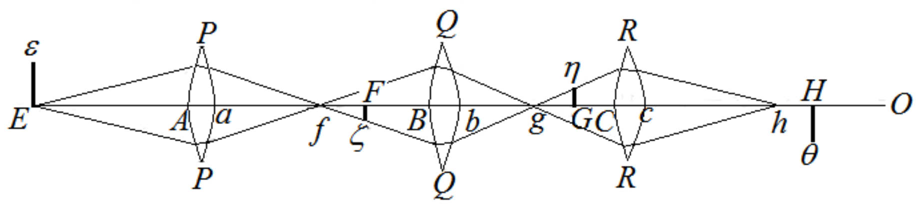

Fig. 6 (*iterata*)

SOLUTION

With the two first lenses *PP* and *QQ* put in place as in the preceding problem and thence with the values *P* and *Q* determined, the principal image represented by these two lenses may fall at $G\eta$, after which thus it may be put in place by the third lens *RR*, so that the determinable distances of this shall be
$CG = c,\ cH = \gamma,$ the thickness $Cc = v''$ and with the arbitrary quantity $= k''$, so that there shall be :

$$\text{radius of the anterior face} = \frac{(n-1)c(k''+v'')}{k''+v'+2nc},$$

$$\text{radius of the posterior face} = \frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}.$$

Then truly on putting $\frac{k''-v''}{k''+v''} = i''$, there may be put

$$R = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2 + \left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right),$$

and now the diffusion distance will be

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'\cdot i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii\cdot i''i''\cdot\frac{bbcc}{\alpha\alpha\beta\beta}R\right),$$

which is the value of Vxx. Truly the inclination of the rays to the axis concurring at *h* is :

$$ii'i''\cdot\frac{bcx}{\alpha\beta\gamma} = \mathfrak{B}x.$$

Now the eye shall be at *O,* and its distance after the lens may be put $RR = O$, there will be $O = \gamma + l$ and thus $\gamma = O - l$. Hence the measure of the confusion arising in the eye is deduced

$$\tfrac{1}{4} ii'i''u\left(1-\tfrac{O}{l}\right)\tfrac{bc}{\alpha\beta}x^3\left(\tfrac{1}{i'i'\cdot i''i''}\cdot\tfrac{\alpha\alpha\beta\beta}{bbcc}P+\tfrac{ii}{i''i''}\cdot\tfrac{bb\beta\beta}{\alpha\alpha cc}Q+ii\cdot i''i''\cdot\tfrac{bbcc}{\alpha\alpha\beta\beta}R\right).$$

COROLLARY 1

185. Here again as before it is apparent, if there were $l = \infty$ and therefore $\gamma = -\infty$ in which case the diffusion length is extended to infinity, the measure to be contained of the confusion of the finite terms, which also is the case from any number of lenses.

COROLLARY 2

186. If the thickness of the lens may be taken as vanishing, on account of $i = 1,\ i' = 1,\ i'' = 1$, the measure of the confusion may be expressed thus more simply, so that it shall become :

$$= \tfrac{1}{4}u\left(1-\tfrac{O}{l}\right)\tfrac{bc}{\alpha\beta}x^3\left(\tfrac{\alpha\alpha\beta\beta}{bbcc}P+\tfrac{bb\beta\beta}{\alpha\alpha cc}Q+\tfrac{bbcc}{\alpha\alpha\beta\beta}R\right).$$

But in this case in place of $k,\ k',\ k''$ with the numbers $\lambda,\ \lambda',\ \lambda''$ introduced, there will become

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda(\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{\nu}{a\alpha}\right)$$
$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'(\tfrac{1}{b}+\tfrac{1}{\beta})^2+\tfrac{\nu}{b\beta}\right)$$
$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''(\tfrac{1}{c}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{c\gamma}\right).$$

COROLLARY 3

187. Truly in the same case the construction of the lens thus will be required to be directed by these numbers $\lambda,\ \lambda',\ \lambda''$, so that there shall become

Radius of the face :

$$\text{For lens } PP\begin{cases}\text{anterior} = \dfrac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}\\[2ex] \text{posterior} = \dfrac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}\end{cases}$$

$$\text{For lens } QQ\begin{cases}\text{anterior} = \dfrac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}\\[2ex] \text{posterior} = \dfrac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}\end{cases}$$

$$\text{For lens } RR\begin{cases}\text{anterior} = \dfrac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}\\[2ex] \text{posterior} = \dfrac{b\beta}{\rho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}}\end{cases}.$$

SCHOLIUM

188. Hence it is clear enough, in whatever manner these formulas for several lenses may be presented ; truly before I may explain these, it will be appropriate that I may set out carefully other circumstances also, which hence are deduced readily ; evidently the magnitude of the object viewed and the number of rays transmitted into the eye from its individual points, so that these likewise may be able to be shown together with the confusion of vision henceforth for any number of lenses; with which agreed on we will be able to avoid several tedious repetitions. However, these two items are able to be defined easily from what has been advanced up to this stage, one of which is the amount, by which the image of the object represented by the lens is discerned by the eye, which amount is required to be estimated from the angle, according to which the image may be seen, so that henceforth this may be able to be compared with this angle, under which the object itself hence may be compared with that angle, according to which the object itself may be viewed at a given distance by the naked eye; from which, since it is understood the magnitude seen may be increased by reason of being viewed through the lens. The other item may depend on the amount of rays transmitted into the eye from the individual points of the object, from which the clarity perceived in the vision is held, evidently from

any point either a cone or cylinder of rays enter the eye ; which if the pupil may be fully open, the clarity will be carried out to the highest degree, unless perhaps it may be agreed for this object to be illuminated by a stronger light. But if the section of that cone or cylinder, by which the light may enter the eye, were with a smaller pupil, the clarity will decrease in the same order; as with all optical instruments, with which it is proposed to markedly increase the magnification of the vision, it may be usual to occur, that greater the amplitude of this cone or cylinder present which may enter the eye, the more accurately the image may be determined.

PROBLEM 6

189. *To define the amount, according to which some part of an object may be discerned by the eye by some number of lenses, with the final image removed to the true distance.*

SOLUTION

Let z be some line considered in the object, which shall be required to be defined, and by however many lens it shall become apparent to the eye. But it has been seen in the preceding, however many lenses there were, the principal image of this line again to become a line, the length to which to z may maintain a certain ratio depending on the determinable separations of the lenses and on the numbers i, i', i'', i''' etc. (§ 86). Therefore this length of the image shall be $= Mz$, which, since it may be seen to be removed from the eye by the distance l, it will appear subtended by the angle $= \frac{Mz}{l}$, or of which the tangent rather shall be $= \frac{Mz}{l}$; but since this angle is accustomed most rarely to arise beyond some level, the tangent is assumed by its arc without risk. But the image, which is expressed in the eye by this line Mz, will be $= \frac{Muz}{l}$; for here I have abstracted the idea from confusion, because from which it happens, so that the image may be expressed greater, therefore since the individual points of the circle may be shown. Now with the same determinations of the lenses put in place, as I have used above in § 86 for the various number of lenses the angle, under which the line *1* in the object will be assumed to be discerned, thus so that there will be had:

Angle of vision

For a single lens	$\frac{1}{i}\cdot\frac{\alpha}{a}\cdot\frac{z}{l}$	inverted
for two lenses	$\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}\cdot\frac{z}{l}$	erect
for three lenses	$\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}\cdot\frac{z}{l}$	inverted
for four lenses	$\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}\cdot\frac{z}{l}$	erect

etc.

Evidently if these formulas shall be positive, the eye will see the line z in place to be erect or inverted, just as has been noted ; but if it were negative, the situation indicated will be required to be turned into the opposite direction.

COROLLARY 1

190. If the same line z of the object may be discerned at the distance $= h$ from the naked eye, that will appear to be subtended by the angle $= \frac{z}{h}$, from which it is understood, by how much greater or less that may be seen through the lenses.

COROLLARY 2

191. If the distance of the eye from the final lens may be put $= O$, there will be in the case of a single lens, $\alpha = O - l$ and thus $\frac{\alpha}{l} = -(1 - \frac{O}{l})$, from which the optical angle of the line corresponding to the object z will be $= \frac{1}{i}(1 - \frac{O}{l})\frac{z}{a}$ for the erect situation, since we have changed the sign.

COROLLARY 3

192. In a similar manner in the case of two lenses on account of $\beta = O - l$ there will be this same angle $= \frac{1}{ii'}(1 - \frac{O}{l})\frac{\alpha z}{ab}$ for the inverse situation.

Truly in the case of three lenses on account of $\gamma = O - l$ there will be this same angle $= \frac{1}{ii'i''}(1 - \frac{O}{l})\frac{\alpha\beta z}{abc}$ for the erect situation.

In the case of four lenses on account of $\delta = O - l$ this same angle will be $= \frac{1}{ii'i''i'''}(1 - \frac{O}{l})\frac{\alpha\beta\gamma z}{abcd}$ for the inverted situation, and so on thus for more lenses.

SCHOLIUM

193. Hence also the manner presents itself of estimating more distinctly the confusion arising in the eye on account of the aperture of the lens. Clearly since the individual points may be expressed in the eye by circles, of which the radius $= \frac{u}{4l}\mathfrak{B}Vx^3$, moreover truly a circle, of which the radius $= z$, with the eye present at the distance l may be referred to a circle in the eye, of which the radius $= \frac{u}{l}z$, the individual points of this object will appear equally large through the eye and to be circular arcs of radius $z = \frac{1}{4}\mathfrak{B}Vx^3$, if they may be observed at the distance from the eye $= l$. Or, since the radius of these arcs appearing shall be $= \frac{z}{l}$, on account of the confusion the image of the circles will be seen from the individual points of the object, of which the radius appearing

shall be $=\frac{1}{4l}\mathfrak{B}Vx^3$. Therefore the quantity u may be deleted in the expressions found before for the confusion, and the apparent radius of the expressed circle of confusion will be obtained. Hence it will be possible to judge, how small the confusion may become, so that it may not be sensed further ; evidently if the eye may not prevail to perceive further the interval for the circle, of which the radius shall be 1" or around the $\frac{1}{60^2}$ part of the radius, it is evident, if our expression were $\frac{1}{4l}\mathfrak{B}Vx^3=\frac{1}{60^2}$, the confusion to become imperceptible. And we understand from experimental deliberations much larger angles that much larger angles are no longer able to be perceived, thus so that no confusion shall be required to be measured, even if the expression $\frac{1}{4l}\mathfrak{B}Vx^3$ were notably larger than $\frac{1}{60^2}$; but here we may put without any doubt, the limit that the formula $\frac{1}{4l}\mathfrak{B}Vx^3$ may not exceed, to be $=\frac{1}{4\chi^3}$, thus so that there will be required to become $\frac{1}{l}\mathfrak{B}Vx^3<\frac{1}{\chi^3}$. Thereafter descending to practice we will be able to assume a number for χ either 40 or less, just as experience in any case will have demanded. So that therefore if in this manner we may have an account of the confusion, the depth of the eye will no longer enter into the calculation.

DEFINITION 3

194. *The radius of the confusion is the apparent radius of the circle, which is seen by the eye of equal magnitude, and the individual points themselves are apparent on account of the confusion.*

COROLLARY

195. Therefore we will find the radius of confusion easily, if we may divide the above formulas found for the confusion by the depth of the eye u, with which agreed on these formulas will be reduced to absolute numbers.

DEFINITION 4

196. *The multiplication* [*i.e. magnification*] *produced by the lenses from the ratio of the quantities, by which the objects are seen through the lenses, is to be estimated to the quantity, by which the same objects may be discerned by the naked eye at a given distance from the eye. But the same exposition of the multiplication will be found, if the magnitude, by which some line considered in the object may be seen through the lenses, may be divided by the magnitude, by which the same line at the given distance from the naked eye may be seen to be going to appear.*

COROLLARY 1

197. Therefore the judgement of the multiplication has involved a certain fixed distance, at which we have assumed the same object to be seen by the naked eye ; which provided it may be assumed different, the magnification is expressed in one way or another.

COROLLARY 2

198. If this fixed distance may be put $=h$, from which the multiplication may be judged, and the exponent of the multiplication $=m$, a certain line may be considered in the object $=z$, which therefore will appear to the naked eye at the distance h under the angle $=\frac{z}{h}$; but the same line will be viewed through the lens under the angle $=\frac{Mz}{l}$ (§ 88), from which the exponent of the multiplication $m=\frac{Mh}{l}$.

COROLLARY 3

199. Therefore the linear dimensions have been considered to be increased by the lenses in the ratio $m:1$; from which the surface will appear to have increased in the ratio $mm:1$ and thus the body in the ratio $m^3:1$. But with the exponent of the multiplication the position must be adjoined, this shall be either erect or inverted.

SCHOLIUM

200. But this fixed distance h, to which the multiplication is referred, cannot always be assumed to be in the same customary manner, clearly it may not be possible to be done for a diversity of objects. For if we may look at very distant or celestial objects, since at no time are we accustomed to see those at moderate distances, it is certainly convenient to compare that magnitude seen through lenses with that, by which the distances themselves may be discerned by the naked eye : and thus in these cases the fixed distance k is accustomed to be put equal to the distant a, by which the remote objects are distant from the lenses. Evidently if the distance a were extremely great, which is the case of telescopes, there is put $h=a$, and the magnitude seen by these instruments may be compared with the magnitude seen by the naked eye at the same most convenient distance. Thus it may be said of telescopes, however many times the diameters of celestial bodies may be multiplied, in each case the letter m will indicate the ratio of this exponent. But if we may consider closer objects, which is the use of microscopes, these generally thus will be moved close to the instrument, so that at that extremely small distance nothing will be able to be distinguished by the naked eye : therefore nor in these cases will it be agreed to have put $h=a$. Therefore another ratio introduced for h is accustomed to be taken for moderate distances of this kind, in which objects may be allowed to be seen distinctly and conveniently, which even if certainly for different kinds of eyes these must be taken differently, yet, so that something fixed may be put in place,

for h a distance of 8 inches is usually taken as the agreed maximum separation of the eyes, thus so that in these cases we may define, how much greater objects of this kind *may appear,* as if the same may be viewed with the naked eye at a distance of 8 inches. Yet meanwhile if the multiple were related to this distance, it will not be difficult to refer that to some other, thus so that this hypothesis shall not be considered to affect the nature of these instruments.

PROBLEM 7

201. *If an object may be viewed through some lenses, to define the amplitude of the cone or cylinder of light, which is transmitted from the individual points into the eye.*

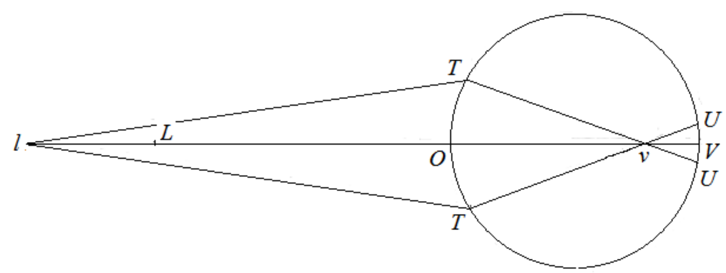

Fig. 10

(again).

SOLUTION

As above the eye is found at the true distance l (Fig. 10) after the last image represented by the lenses, which even if it is diffused a little by the distance Ll, yet this we have extracted thence mentally, since now we have determined the confusion separately. Therefore the light cone spreads from the point l towards the eye, the extreme rays of which are inclined to the axis at the angle *OlT,* which above we have put $=\mathfrak{B}x$. Therefore the section of this cone about the entrance into the eye may be considered, of which the radius will be $=\mathfrak{B}lx,$ from which for a various number of lenses here the radius will be defined in the following manner:

For for one lens $il\frac{x}{\alpha}$

for two lenses $ii'l\frac{bx}{\alpha\beta}$

for three lenses $ii'i''l\frac{bcx}{\alpha\beta\gamma}$

for four lenses $ii'i''i'''l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.;

and here it is the same, whether these formulae shall be positive of negative, because the circle, either with a positive or negative radius, will produce a circle of the same magnitude.

COROLLARY 1

202. If this same radius $\mathfrak{B}lx$ were greater than the radius of the pupil, the whole aperture of the pupil would be filled with rays, nor therefore can the vision be made clearer, unless perhaps the object may be rendered more brilliant by a stronger illumination.

COROLLARY 2

203. But if this quantity $\mathfrak{B}lx$ were less than the radius of the pupil, a measure of the clarity will arise, which there will be greater, when that quantity were made greater : while on the other hand with this quantity diminished the clarity can become extremely limited, so that it may no longer suffice for exciting the sense of vision.

DEFINITION 5

204. *The order of clarity perceived through lenses may be defined most conveniently from the radius of the luminous cone, which is transmitted from some point of the object into the eye.*

COROLLARY 1

205. Therefore the order of clarity is defined from the quantity found above $\mathfrak{B}lx$, thus so that if we may put the order of clarity $= y$, we shall have $y = \mathfrak{B}lx$, with which known we will be able to judge easily, for which order the vision may be had clear.

COROLLARY 2

206. Evidently if we may put the radius of the pupil $=\omega$, as long as $y>\omega$, we may experience the full clarity, which is capable of no increase, unless perhaps we may prevail to dilate the pupil itself more.

COROLLARY 3

207. But if $y<\omega$, certainly we will perceive a smaller clarity ; and if we may designate the full clarity by one, the resulting clarity from the case $y<\omega$ will be $=\frac{yy}{\omega\omega}$, on account of which the amount of rays sent into the eye is as the square of the radius y.

COROLLARY 4

208. Because if the degree of clarity y may decrease to that point, that the supply of rays may become exceedingly small, in order that it may excite the sense of vision, nothing will be able to be seen on account of the great gloom, from which it is evident to be required for vision, that the order of clarity will exceed some certain limit.

SCHOLlUM

209. Bothe in telescopes and microscopes it is of the greatest necessity, that objects may be shown with a certain degree of clarity, lest the image may become exceedingly obscure. But here the order depends most on the light from the objects themselves, which so that if they were illuminated, there a smaller degree of clarity would suffice for these to be seen clearly; and thus we are to be content observing the brighter stars of a smaller degree of clarity, truly terrestrial objects may demand a much higher degree of clarity. Therefore so that we may prevail to accommodate all these cases, I am going to multiply the degree of clarity into the calculation contained here by the indefinite letter y. On account of which I will show from these by the multiplication and clarity set out for these two elements and likewise with the confusion for any number of lenses : and indeed at first without ignoring the thickness of the lenses, then truly it will be appropriate to ignore the thickness of the same lenses set out.

PROBLEM 8

210. *If the eye may view an object $E\varepsilon$ through some number of lenses PP, QQ, RR, SS etc., thus so that the last image represented by these may be found before the eye at the true distance $=l$, to determine both the multiplication and clarity, as well as the confusion, by which the vision will be disturbed.*

SOLUTION

However many lenses there were, they shall be for the individual determinable distances as well as the thicknesses and with an arbitrary quantity, as follows :

For lens		Determinable distances	Thickness	Arbitrary quantity
first	PP	$EA = a, \quad aF = \alpha$	$Aa = v$	k
second	QQ	$FB = b, \quad bG = \beta$	$Bb = v'$	k'
third	RR	$GC = c, \quad cH = \gamma$	$Cc = v''$	k''
fourth	SS	$HD = d, \quad dI = \delta$	$Dd = v'''$	k'''

etc.

and hence on putting the ratio of the refraction $\frac{31}{20} = n$ $[=1.55]$ the construction of the lenses will be had thus :

		Radius of face anterior	Radius of face posterior
first lens	PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)a(k-v)}{k-v-2na}$
second lens	QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)b(k'-v')}{k'-v'-2nb}$
third lens	RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)c(k''-v'')}{k''-v''-2nc}$
fourth lens	SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)d(k'''-v''')}{k'''-v'''-2nd}$

etc.

Then on putting for brevity:

$$\frac{k-v}{k+v} = i, \quad \frac{k'-v'}{k'+v'} = i', \quad \frac{k''-v''}{k''+v''} = i'', \quad \frac{k'''-v'''}{k'''+v'''} = i''', \quad \text{etc.}$$

if the radius of the first lens on the anterior face were x, both for the posterior face as well as for each face of the following individual lenses, the apertures must be greater or at any rate not smaller, as the following table shows :

		Radius of aperture in face	
		anterior	posterior
First lens	PP	x	ix
second lens	QQ	$i\cdot\frac{bx}{\alpha}$	$ii'\cdot\frac{bx}{\alpha}$
third lens	RR	$ii'\cdot\frac{bcx}{\alpha\beta}$	$ii'i''\cdot\frac{bcdx}{\alpha\beta}$
fourth lens	SS	$ii'i''\cdot\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''\cdot\frac{bcdx}{\alpha\beta\gamma}$

etc.

Finally for abbreviation, there may be put:

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$
$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{ib}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$
$$R=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\alpha}-\frac{2}{k''+v''}\right)^2\right)$$
$$S=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\alpha}-\frac{2}{k'''+v'''}\right)^2\right)$$

With these in place we may put the eye to be located at the distance $=O$ from the final lens, thus so that it may be found at a distance $=l$ after the final image, moreover the magnitude observed to be compared with the magnitude, by which the same object may be discerned by the naked eye at the fixed distance $=h$, and the exponent of the multiplication may be put $=m$.

The for clarity the degree shall be $=y$, thus so that y will indicate the radius of the luminous cone interring into the eye.

But the confusion may be estimated by the radius of confusion defined above (§ 194).

Now for any number of lenses these three items thus will obtained :

I. For a single lens $O=\alpha+l$

1. Multiplication exponent $m=\frac{1}{i}\cdot\frac{\alpha h}{al}$, situated inversely;

2. Order of clarity $y=il\cdot\frac{x}{\alpha}$;

3. Radius of confusion $=\frac{1}{4}i\frac{\alpha}{l}x^3\cdot P$.

II. For two lenses $O=\beta+l$

1. Multiplication exponent $m=\frac{1}{ii'}\cdot\frac{\alpha\beta h}{abl}$ situated erect;

2. Order of clarity $y=ii'l\cdot\frac{bx}{\alpha\beta}$;

3. Radius of confusion $=\frac{1}{4}ii'\frac{\beta}{l}\cdot\frac{b}{\alpha}x^3\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P+ii\cdot\frac{bb}{\alpha\alpha}Q\right).$

III. For three lenses $O=\gamma+l$

1. Multiplication exponent $m=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma h}{abcl}$ situated inversely

2. Order of clarity $y=ii'i''l\cdot\frac{bcx}{\alpha\beta\gamma}$

3. Radius of confusion:

$$\frac{1}{4}ii'i''\frac{\gamma}{l}\cdot\frac{bc}{\alpha\beta}x^3\left(\frac{1}{i'i'\cdot i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}Q+ii\cdot i'i'\cdot\frac{bbcc}{\alpha\alpha\beta\beta}R\right).$$

IV. For four lenses $O=\delta+l$

1. Multiplication exponent $m=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta h}{abcdl}$ situated erect

2. Order of clarity $y=ii'i''i'''l\cdot\frac{bcdx}{\alpha\beta\gamma\delta}$

3. Radius of confusion:

$$\frac{1}{4}ii'i'''\frac{\delta}{l}\cdot\frac{bcd}{\alpha\beta\gamma}x^3\left\{\begin{matrix}\frac{1}{i'i'\cdot i''i''\cdot i'''i'''}\cdot\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}P+\frac{ii}{i''i''}i'''i'''\cdot\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}Q\\+\frac{ii\cdot i'i'}{i'''i'''}\cdot\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}R+i'i'\cdot i''i''\cdot i'''i'''\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}S\end{matrix}\right\}.$$

V. For five lenses $O=\varepsilon+l$

1. Multiplication exponent $m=\frac{1}{ii'i''i'''\cdot i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon h}{abcdel}$ situated inversely

2. Order of clarity $y=ii'i''i'''i''''l\cdot\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$

3. Radius of confusion:

$$\frac{1}{4} ii'i''i'''i'''' \frac{\varepsilon}{l} \cdot \frac{bcde}{\alpha\beta\gamma\delta} x^3 \left\{ \begin{array}{l} \frac{1}{i'i' \cdot i''i'' \cdot i'''i''' \cdot i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii}{i''i'' \cdot i'''i''' \cdot i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii \cdot i'i'}{i'''i''' \cdot i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{ii \cdot i'i' \cdot i''i''}{i''''i'''' \cdot i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S \\ + ii \cdot i'i' \cdot i''i'' \cdot i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{array} \right\}.$$

and hence also the progress to more lenses is evident.

COROLLARY 1

211. From all these cases it is evident to become generally $my = \frac{hx}{a}$. Clearly with the multiplication m given with the clarity y at once the aperture of the first lens surely $x = my \cdot \frac{a}{h}$. Evidently where both a greater multiple as well as clarity is wished, there the aperture of the first lens is required to be greater.

COROLLARY 2

212. But when x cannot be permitted to become larger, so that the confusion may be held within a certain limit, with the exponent x of the multiple m given, the clarity of the order may be defined by $y = \frac{hx}{ma}$, from which it is apparent with the rest equal, where a greater multiple may emerge, there we are required to be content with smaller clarity.

COROLLARY 3

213. But in the first place here it is required to observe these formulas equally can be made to work, whatever thickness the lenses will have. But they will emerge more manageable, if the thickness of the lenses may be ignored, which case deserves to be treated separately.

PROBLEM 9

214. *With the same in place, as in the preceding problem, if the thickness of the lenses may be considered as negligible, to determine both the multiplication as well as the clarity and confusion, by which the vision may be disturbed.*

SOLUTION

This treatment differs from the preceding in that, since the thicknesses of the lenses $v,\ v',\ v''$ etc. vanish and in place of the arbitrary quantities $k,\ k',\ k''$ etc. the arbitrary numbers $\lambda,\ \lambda',\ \lambda''$ etc. may be introduced into the calculation. Therefore there may be put:

		Determinable distances		Arbitrary number
first lens	PP	$EA = a,$	$aF = \alpha$	λ
second lens	QQ	$FB = b,$	$bG = \beta$	λ'
third lens	RR	$GC = c,$	$cH = \gamma$	λ''
fourth lens	SS	$HD = d$	$dI = \delta$	λ'''

etc.

Hence, if for brevity there shall be $\rho = 0,190781,\quad \sigma = 1,627401$ and $\tau = 0,905133,$ the construction of the lens is put in place :

		Radius of face anterior	posterior
first lens	PP	$\frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$	$\frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$
second lens	QQ	$\frac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}$	$\frac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$
third lens	RR	$\frac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}$	$\frac{c\gamma}{\rho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}}$
fourth lens	SS	$\frac{d\delta}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda'''-1)}}$	$\frac{d\delta}{\rho d+\sigma\delta\mp\tau(d+\delta)\sqrt{(\lambda'''-1)}}$

etc.

Now since the radius of the aperture of the first lens PP shall be $= x$, et in whatever lens the ratio of each face shall be the same, so that all the rays may be transmitted by passing into the first likewise by passing into the rest, the apertures of the remaining must exceed the following limits:

Radius of the aperture

of the second lens $QQ > \frac{b}{\alpha}x$

of the third lens $RR > \frac{bc}{\alpha\beta}x$

of the fourth lens $SS > \frac{bcd}{\alpha\beta\gamma}x$

etc.

Then truly on putting for the sake of brevity $\mu = 0,938191$ and $v = 0,232692$ therefore there may be put in place:

$$P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2+\frac{v}{a\alpha}\right), \qquad Q = \mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'(\frac{1}{b}+\frac{1}{\beta})^2+\frac{v}{b\beta}\right)$$
$$R = \mu\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{v}{c\gamma}\right), \qquad S = \mu\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''(\frac{1}{d}+\frac{1}{\delta})^2+\frac{v}{d\delta}\right) \quad \text{etc.}$$

Now O shall be the distance of the eye past the final lens, thus so that it may be distant from the final image by the interval $= l$, the magnitude seen may be compared with that, by which the same object may be discerned by the naked eye at the fixed distance h, and the exponent of the multiplication shall be $= m$ and the order of clarity $= y$, with which in place there will be for some number of lenses, as follows :

I. For as single lens $O = \alpha + l$

l. Exponent of the multiplication $m = \frac{\alpha h}{al}$, in place inverted

2. Order of clarity $y = l \cdot \frac{x}{\alpha}$, hence $my = \frac{hx}{\alpha}$

3. Confusion radius $= \frac{\alpha}{4l} \cdot x^3 P$.

II. For two lenses $O = \beta + l$

1. Exponent of the multiplication $m = \frac{\alpha\beta h}{abl}$, in place erect

2. Order of clarity $y = l \cdot \frac{bx}{\alpha\beta}$, hence $my = \frac{hx}{a}$

3. Confusion radius $= \frac{\beta}{4l} \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q\right)$.

III. For three lenses $O = \gamma + l$

1. Exponent of the multiplication $m=\frac{\alpha\beta\gamma h}{abcl}$, in place inverted

2. Order of clarity $y=l\cdot\frac{bcx}{\alpha\beta\gamma}$, hence $my=\frac{hx}{a}$

3. Confusion radius :

$$\frac{\gamma}{4l}\cdot\frac{bc}{\alpha\beta}x^3\left(\frac{\alpha\alpha\beta\beta}{bbcc}P+\frac{bb\beta\beta}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

IV. For four lenses $O=\delta+l$

1. Exponent of the multiplication $m=\frac{\alpha\beta\gamma\delta h}{abcdl}$, in place erect

2. Order of clarity $y=l\cdot\frac{bcdx}{\alpha\beta\gamma\delta}$, hence $my=\frac{hx}{a}$

3. Confusion radius :

$$\frac{\delta}{4l}\cdot\frac{bcd}{\alpha\beta\gamma}x^3\left(\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}P+\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}Q+\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}R+\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}S\right).$$

V. For five lenses $O=\varepsilon+l$

1. Exponent of the multiplication $m=\frac{\alpha\beta\gamma\delta\varepsilon h}{abcdel}$ in place inverted.

2. Order of clarity $y=l\cdot\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$, hence $my=\frac{hx}{a}$

3. Confusion radius :

$$\frac{\varepsilon}{4l}\cdot\frac{bcde}{\alpha\beta\gamma\delta}x^3\left\{\begin{matrix}\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}P+\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}Q+\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}R\\+\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}S+\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}T\end{matrix}\right\},$$

from which without difficulty this formula will be continued to even more lenses.

COROLLARY 1

215. Simple lenses, by using the numbers $\lambda,\ \lambda',\ \lambda''$ etc., cannot accept values less than unity. Truly here, where double, triple or even quadruple lenses may be called into use in place of less simple lenses, with which agreed on the numbers $\lambda,\ \lambda',\ \lambda''$ etc. in these formulas not only to be diminished below unity but will be able to produced as far as zero. But then the construction of these multiple lenses sought in the above chapter and here must be adapted for positive determinable distances.

COROLLARY 2

216. Just as if for the first lens *PP* there must be $\lambda=0{,}191827$, hence it will be required for a lens from two components, so that there shall be :

For the lens	Radius of face anterior	Radius of face posterior
first	$\frac{2a\alpha}{(2\rho-\sigma)\alpha+\sigma a}$	$\frac{2a\alpha}{(2\sigma-\rho)\alpha+\rho a}$
second	$\frac{2a\alpha}{\rho\alpha+(2\sigma-\rho)a}$	$\frac{2a\alpha}{\sigma\alpha+(2\rho-\sigma)a}$

COROLLARY 3

217. But if we may wish, so that for the first lens there shall become $\lambda = 0,042165$, there will be on tripling, in this manner :

For lens	Radius of face anterior	Radius of face posterior
first	$\frac{3a\alpha}{(3\rho-2\sigma)\alpha+\sigma a}$	$\frac{3a\alpha}{(3\sigma-2\rho)\alpha+\rho a}$
middle	$\frac{3a\alpha}{(2\rho-\sigma)\alpha+(2\sigma-\rho)a}$	$\frac{3a\alpha}{(2\sigma-\rho)\alpha+(2\rho-\sigma)a}$
posterior	$\frac{3a\alpha}{\rho\alpha+(3\sigma-2\rho)a}$	$\frac{3a\alpha}{\sigma\alpha+(3\rho-2\sigma)a}$.

COROLLARY 4

218. And if for the first lens there may be required $\lambda = -0,010216$, thus in the construction the quadruple thus there will be required to be used:

For lens	Radius of face anterior	Radius of face posterior
first	$\frac{4a\alpha}{(4\rho-3\sigma)\alpha+\sigma a}$	$\frac{4a\alpha}{(4\sigma-3\rho)\alpha+\rho a}$
second	$\frac{4a\alpha}{(3\rho-2\sigma)\alpha+(2\sigma-\rho)a}$	$\frac{4a\alpha}{(3\sigma-2\rho)\alpha+(2\rho-\sigma)a}$
third	$\frac{4a\alpha}{(2\rho-\sigma)\alpha+(3\sigma-2\rho)a}$	$\frac{4a\alpha}{(2\sigma-\rho)\alpha+(3\rho-2\sigma)a}$
fourth	$\frac{4a\alpha}{\rho\alpha+(4\sigma-3\rho)a}$	$\frac{4a\alpha}{\sigma\alpha+(4\rho-3\sigma)a}$

COROLLARY 5

219. Moreover in a similar manner the second lens *QQ* will be required to be constructed by multiplication from its determinable distances *b* and β, and the number λ' corresponding to that must be either 0,191827, 0,042165, or − 0,010216; which likewise is to be understood concerning the remaining lenses.

SCHOLIUM 1

220. In practice here I would be unwilling to use other kinds of lenses, since these alone without doubt may be able to be constructed with enormous errors ; then truly, even if others from these not exceedingly discrepancies may be able to be introduced almost equally in practice, yet, even if the difference may not be exceedingly notable, we will be able to abstain from easily. Therefore since there shall be $\rho = 0{,}190781$ and $\sigma = 1{,}627401$ and thus

$$\begin{array}{ll} \rho = 0{,}190781, & \sigma = 1{,}627401 \\ 2\rho - \sigma = -1{,}245839, & 2\sigma - \rho = 3{,}064021 \\ 3\rho - 2\sigma = -2{,}682459, & 3\sigma - 2\rho = 4{,}500641 \\ 4\rho - 3\sigma = -4{,}119079, & 4\sigma - 3\rho = 5{,}937261, \end{array}$$

it will be agreed to show the same constructions numerically.

1. Therefore if for the lens *PP* there must be $\lambda = 1$, that will be a simple lens requiring to be constructed in this manner:

Radius of the face	
anterior	posterior
$\frac{a\alpha}{+0{,}190781\alpha + 1{,}627401a}$	$\frac{a\alpha}{+1{,}627401\alpha + 0{,}190781a}$

II. If for the lens *PP* there must be $\lambda = 0{,}191827$, that will be a double lens requiring to be constructed in this manner:

	Radius of the face	
For the	anterior	posterior
first lens	$\frac{a\alpha}{-0{,}622919\alpha+0{,}813700a}$	$\frac{a\alpha}{+1{,}532010\alpha+0{,}095890a}$
final lens	$\frac{a\alpha}{+0{,}096890\alpha+1{,}532010a}$	$\frac{a\alpha}{+0{,}813700\alpha-0{,}622919a}$.

III. If for the lens PP there must be $\lambda = 0{,}042165$, that will be triplicate to be constructed in this manner:

	Radius of the face	
	anterioris	posterioris
first lens	$\frac{a\alpha}{-0{,}894153\alpha+0{,}542467a}$	$\frac{a\alpha}{+1{,}500214\alpha+0{,}063594a}$
middle lens	$\frac{a\alpha}{-0{,}415280\alpha+1{,}021340a}$	$\frac{a\alpha}{1{,}021340\alpha-0{,}415280a}$
final lens	$\frac{a\alpha}{+0{,}063594\alpha+1{,}500214a}$	$\frac{a\alpha}{+0{,}542467\alpha-0{,}894153a}$.

IV. If for the lens PP there must be $\lambda = -0{,}010216$, that will be a quadruple requiring to be constructed in this manner:

	Radius of the face	
	anterior	posterior
first lens	$\frac{a\alpha}{-1{,}029770\alpha+0{,}406850a}$	$\frac{a\alpha}{+1{,}484315\alpha+0{,}047695a}$
second lens	$\frac{a\alpha}{-0{,}670615\alpha+0{,}766005a}$	$\frac{a\alpha}{+1{,}125160\alpha-0{,}311460a}$
third lens	$\frac{a\alpha}{-0{,}311460\alpha+1{,}1125160a}$	$\frac{a\alpha}{0{,}766005\alpha-0{,}670615a}$
fourth lens	$\frac{a\alpha}{0{,}047695\alpha+1{,}484315a}$	$\frac{a\alpha}{0{,}406850\alpha-1{,}029770a}$

SCHOLIUM 2

221. Yet meanwhile if the lenses we desire, in which the value of λ shall be greater, as has been assumed here, the construction of these will be able to be prepared easily from these formulas by addition. Evidently in the two fractions, in which the rays of the face of each lens are designated, the one to increase the denominator, the other truly to be diminished by the same amount, which quantity always $=\tau\left(a+\alpha\right)\sqrt{v}$,with v denoting the excess of the value of λ above that assumed before. Thus so that there must become :

$$\text{I.}\quad \lambda=1+v,\quad \text{II.}\quad \lambda=0,191827+v,\quad \text{III.}\ \lambda=0,042165+v$$

$$\text{or IV.}\quad \lambda=-0,010216+v,$$

the denominators if the fractions treated in the preceding scholium for any lens are simple to be increased or decreased by the quantity $0,905183(a+\alpha)\sqrt{v}$. From which if the quadruple lens were desired, fro which there shall be precisely $\lambda=0$, there will be $v=0,010216$ and $\tau\sqrt{v}=0,091487;$ and hence for any simple lens either there must be either an increase of the denominators, or truly to be diminished by this quantity $0,091487\alpha+0,091487a$, from which the construction of such a quadruple lens of this kind arises, for which there is $\lambda=0$:

	Radius of the face	
	anterior	posterior
first lens	$\frac{a\alpha}{-1,121267\alpha+0,316363a}$	$\frac{a\alpha}{+1,576802\alpha+0,139182a}$
second lens	$\frac{a\alpha}{-0,762102\alpha+0,674618a}$	$\frac{a\alpha}{+1,216647\alpha-0,219973a}$
third lens	$\frac{a\alpha}{-0,402947\alpha+1,033673a}$	$\frac{a\alpha}{0,857492\alpha-0,579128a}$
fourth lens	$\frac{a\alpha}{-0,043782\alpha+1,392828a}$	$\frac{a\alpha}{0,488337\alpha-0,938283a}$

But in this case any lens at this point can be constructed by another method: thus in the fourth order by a backwards step the exposition with the interchange of a and α will give another form of the first lens.

SUPPLEMENT III

TO PROBLEM 8

If the ratio of refraction may differ, so that the ratio of refraction shall become for the first lens $=n$, for the second $=n'$, for the third $=n''$ etc., thence neither in the multiplication *m* nor in the order of clarity is there any change; but truly in the radius of confusion the values of the letters *P, Q, R* etc. must be changed in the following manner:

$$P=\tfrac{n}{2(n-1)^2}\left(\left(\tfrac{n}{a}+\tfrac{2}{k+v}\right)\left(\tfrac{1}{ia}+\tfrac{2}{k-v}\right)^2+\left(\tfrac{n}{\alpha}-\tfrac{2}{k-v}\right)\left(\tfrac{i}{\alpha}-\tfrac{2}{k+v}\right)^2\right)$$
$$Q=\tfrac{n'}{2(n'-1)^2}\left(\left(\tfrac{n'}{b}+\tfrac{2}{k'+v'}\right)\left(\tfrac{1}{ib}+\tfrac{2}{k'-v'}\right)^2+\left(\tfrac{n'}{\beta}-\tfrac{2}{k'-v'}\right)\left(\tfrac{i'}{\beta}-\tfrac{2}{k'+v'}\right)^2\right)$$
$$R=\tfrac{n''}{2(n''-1)^2}\left(\left(\tfrac{n''}{c}+\tfrac{2}{k''+v''}\right)\left(\tfrac{1}{i''c}+\tfrac{2}{k''-v''}\right)^2+\left(\tfrac{n''}{\gamma}-\tfrac{2}{k''-v''}\right)\left(\tfrac{i''}{\alpha}-\tfrac{2}{k''+v''}\right)^2\right)$$
$$S=\tfrac{n'''}{2(n-1)^2}\left(\left(\tfrac{n'''}{d}+\tfrac{2}{k'''+v'''}\right)\left(\tfrac{1}{i'''d}+\tfrac{2}{k'''-v'''}\right)^2+\left(\tfrac{n'''}{\delta}-\tfrac{2}{k'''-v'''}\right)\left(\tfrac{i'''}{\alpha}-\tfrac{2}{k'''+v'''}\right)^2\right)$$
etc.

TO PROBLEM 9

If the lenses may differ in the ratio of refraction, in place of the letters $\rho,\ \sigma,\ \tau$ in the rays of the faces it will be required to write for the second lens $\rho',\ \sigma',\ \tau'$, for the third $\rho'',\ \sigma'',\ \tau''$ etc.; then truly the expressions for the multiplication and for the degree of clarity require no change; but for the confusion it will be required to be noted :

$$P=\mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda(\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{v}{a\alpha}\right),\ \ Q=\mu'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'(\tfrac{1}{b}+\tfrac{1}{\beta})^2+\tfrac{v'}{b\beta}\right),\ \ R=\mu''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''(\tfrac{1}{c}+\tfrac{1}{\gamma})^2+\tfrac{v''}{c\gamma}\right),\text{ etc.}$$

CAPUT IV

DE CONFUSIONE VISIONIS NEC NON DE MAGNITUDINE APPARENTE ET CLARITATE

DEFINITIO 1

159. *Visio est distincta, si omnes radii, qui ex quolibet obiecti puncto in oculum ingrediuntur, in fundo oculi super retina iterum in unum punctum congregantur.*

SCHOLION

160. Ad visionem distinctam requiritur, ut obiecta in certa quadam ab oculo distantia reperiantur, quae distantia pro varia oculorum indole maxime solet esse diversa, dum myopes exiguam, ii, qui oculis valent, ingentem, ac presbytes non solum infinitam, sed quandoque etiam negativam exigunt; cuiusmodi distantia cum in veris obiectis locum habere nequeat, ope perspicillorum sibi satisfacere soient. Quilibet ergo oculus ad certam quandam distantiam obiectorum est instructus, quam eius distantiam iustam appellabo; ubi quidem insignis latitudo locum habet, propterea quod structura oculi ita est artificiosa, ut contractione ac elongatione quadam se ad distantias aliquanto maiores et minores accommodare possit. Quando ergo obiecta in distantia ab oculo iusta reperiuntur, visio est distincta, dum singula obiectorum puncta super retina singulis punctis exprimuntur.

DEFINITIO *2*

161. *Visio est confusa, si radii ex quolibet obiecti puncto in oculum immissi non in uno retinae puncto congregantur, sed per aliquod spatium retinam afficiunt.*

COROLLARIUM 1

162. Eo maior ergo erit confusio, quo maius fuerit hoc spatium in retina, per quod radii ex eodem obiecti puncto emissi dissipantur. Ex quo huius spatii magnitudo veram confusionis mensuram suppeditabit.

COROLLARIUM 2

163. Visio ergo erit confusa, cum obiecti visi distantia multum fuerit diversa ab oculi distantia iusta. Parvum enim discrimen vel per se nullam confusionem parit vel oculus se ad obiecti distantiam sua, qua pollet, volubilitate accommodare valet.

SCHOLION

164. Si obiecta per lentes distincte repraesentarentur, visio imaginum eadem lege teneretur atque ipsorum obiectorum; iusta scilicet earum ab oculo distantia visionem distinctam, admodum autem diversa confusam produceret. Verum si imago per aliquod spatium fuerit diffusa, etiamsi ab oculo ad distantiam iustam sit remota, inde tamen in visione confusio oriatur necesse est. Quod si nempe non obiecta ipsa, sed earum imagines per unam pluresve lentes repraesentatas intueamur, ob duplicem causam visio poterit esse confusa; altera, si distantia imaginis ab oculo multum fuerit diversa a distantia iusta, altera vero, si ipsa imago per aliquod spatium diffundatur. Priorem quidem causam tollere in nostra est potestate, siquidem lentes ita disponere licet, ut imaginem in iusta ab oculo distantia exhibeant; quam lentium dispositionem propterea hic perpetuo assumamus. Quamobrem in hoc capite investigare constitui, quanta confusio in visione imaginum a spatio diffusionis oriri debeat, eiusque quantitate determinata deinceps in hoc erit elaborandum, quemadmodum lentes formatas ac dispositas esse oporteat, ut confusio inde in visione nata datum limitem, quo adhuc est tolerabilis, non excedat. In genere quidem certum est, quo maius fuerit spatium diffusionis, eo maiorem inde in visionem induci debere confusionam; verum tamen mox videbimus confusionem visionis non esse spatio diffusionis proportionalem, sed aliam omnino legem sequi, quam accurate determinasse maximi erit momenti, cum ex hoc fonte constructio omnium instrumentorum dioptricorum ad visionem accommodatorum sit repetenda.

PROBLEMA 1

165. *Si oculus aspiciat imaginem cuiuspiam obiecti ab una pluribusve lentibus per spatium Ll* (Fig. 10) *diffusam atque imago principalis L in iusta ab oculo distantia reperiatur, definire confusionem, qua visio huius imaginis afficietur.*

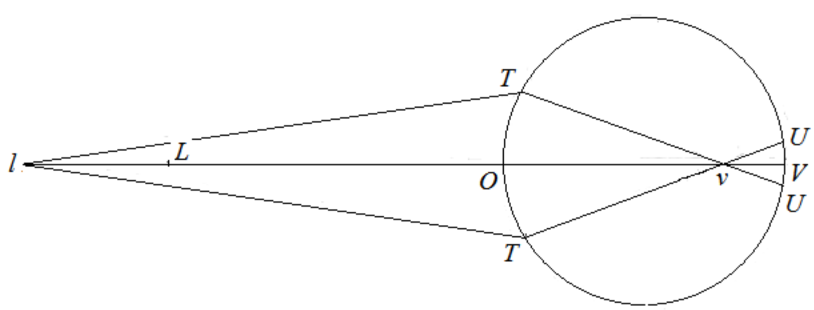

Fig. 10

SOLUTIO

In spatio diffusionis Ll, in quo punctum L a radiis per lentium medium, punctum l vero a radiis per lentium extremitates transmissis exhibeatur, considerari oportet primo eius magnitudinem Ll, deinde radiorum in l concurrentium inclinationem ad axem. Supra autem vidimus has res a primae lentis apertura, cuius semidiameter sit $= x$, ita pendere, ut sit ipsum spatium $Ll = Vxx$ et angulus $OlT = \mathfrak{B}x$, pro quovis scilicet lentium numero valores horum coefficientium V et $\mathfrak{B}$ determinavimus. Repraesentet iam circulus OV oculum, quem hic ut exiguam cameram obscuram spectare licet, sed ita perfectam, ut radios ex uno puncto emissos iterum in uno puncto colligat; etsi enim in oculo plures fiunt refractiones, tamen conditione illa servata unica lens earum loco considerari potest, quae sit in TOT, a qua retina remota sit intervalio $OV = u$. Cum nunc imago principalis L in debita ab oculo distantia, quae vocetur $OL = l$, existat, puncti L imago in oculo in ipsam retinam incidet et in V distincte depingetur: puncti vero l imago non in V, sed ante retinam in v referetur, quod intervallum Vv, si lentis in TOT conceptae crassities evanescat, secundum § 62 ita exprimitur, ut sit

$$Vv = \frac{OV^2}{OL^2} \cdot Ll = \frac{uu}{ll} \cdot Vxx.$$

Cum autem radii punctum l formantes ad axem inclinati sint angulo $OlT = \mathfrak{B}x$, ii per lentis oculi puncta TT ita intrabunt, ut, neglecto intervalio Ll prae distantia $OL = l$, sit $OT = l\mathfrak{B}x$, ex quo ii in puncto v concurrentes cum axe angulum facient $OvT = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{1}{u}\mathfrak{B}x.$ Hinc ergo ultra ad retinam pergentes super ea circulum UU effingent, cuius radius erit

$$VU = \frac{1}{u}\mathfrak{B}x \cdot Vv = \frac{1}{u} \cdot \mathfrak{B}x \cdot \frac{uu}{ll} \cdot Vxx,$$

et a punctis inter L et l mediis hoc spatium circulare super retina replebitur. Quare imago per spatium Ll diffusa super retina circello repraesentabitur, cuius radius $VU = \frac{u}{l} \cdot \mathfrak{B} \cdot Vx^3$. qui circellus veram confusionis mensuram suppeditat. Quodlibet scilicet obiecti punctum, quod lentibus per spatium Ll diffusum exhibetur, in oculo super retina non puncto, sed circello exprimetur, cuius radius erit $= \frac{u}{l} \cdot \mathfrak{B} \cdot Vx^3$.

COROLLARIUM 1

166. Videmus ergo confusionem, qua visio afficitur, non solum a quantitate spatii diffusionis $Ll = Vxx$, sed insuper ab angulo, quo radii in l concurrentes ad axem inclinantur, qui est $= \mathfrak{B}x$, pendere radiumque circelli confusionem metientis producto illius spatii per hunc angulum esse proportionalem.

COROLLARIUM 2

167. Cum igitur posito semidiametro aperturae primae lentis $= x$ spatium diffusionis sit ut eius quadratum xx, radius circelli confusionem metientis est ut eius cubus x^3. Ac si confusio ipsa areae huius circelli proportionalis aestimetur, erit ea ut x^6 seu ut cubus aperturae primae lentis.

COROLLARIUM 3

168. Hinc ergo intelligitur, quanti intersit per idoneam lentium dispositionem spatium diffusionis Vxx diminuisse; non solum enim in eadem ratione, qua spatium diffusionis Vxx seu quantitas V diminuitur, sed adeo in ratione duplicata ipsa confusio visa minor redditur.

SCHOLION

169. Assumsi hic pupillam tam late patere, ut radios ab l divergentes recipiat; at si apertura pupillae minor esset, radios a puncto l venientes ne quidem caperet: hoc ergo casu res eodem rediret, ac si contracta primae lentis apertura spatium diffusionis Ll eo usque diminueretur, quoad pupilla omnes radios ab imagine emissos recipere posset, hocque casu manifestum est confusionem minorem esse prodituram. Verum in sequentibus ostendetur tam in Telescopiis quam Microscopiis hunc casum vix unquam locum invenire, cum plerumque conus radiosus ex puncto l emissus circa ingressum in oculum multo tenuior sit quam pupillae apertura; quamobrem ne opus quidem est hunc casum, etsi per se facile expediretur, hic expendere. Ceterum tametsi oculus non commode cum lente simplici, cuius crassities evanescat, conferri possit hincque expressio spatii $V\,v$ secundum § 84 aliquantum diversa prodire potuisset, hic ad istam circumstantiam non attendimus, propterea quod spatium Vv secundum certam quandam rationem auctum vel diminutum prodiisset; hic autem non adeo necesse est ipsam quantitatem absolutam confusionis cognovisse, dummodo rationem, quam sequitur, accurate definiverimus. Cum enim nostrum calculum cum experientia contulerimus, terminum cognoscemus, quem si confusio nostro more expressa superavit, intolerabilis evadat; hincque perpetuo sufficiet confusionem simili modo expressam infra hunc terminum reduxisse. Interim tamen notasse iuvabit per oculi conformationem confusionem adhuc multo minorem effici posse.

PROBLEMA 2

170. *Positis iisdem, quae in praecedente problemate sunt assumta, eam definire oculi conformationem, qua minima confusio percipiatur.*

SOLUTIO

Oculus omnis facultate praeditus est sese aliquantillum aliter conformandi, ut etiam obiecta, quorum distantia a iusta non nimis differt, distincte

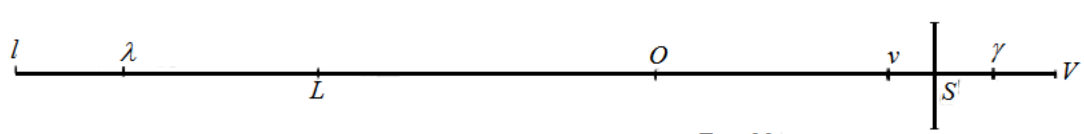

Fig. 11

videat: quod quomodocunque perficiatur, id ita fieri concipere licet, ac si retina ad pupillam seu potius eam lentem, quam oculi loco consideramus, propius admoveretur seu longius ab ea detorqueretur. Cum ergo ante retinam in *V* (Fig. 11) sitam simus contemplati, hic primo sumamus retinam per ipsum punctum *v* transire, ac manifestum est super ea punctum *l* distincte expressum iri. Verum quia punctum *L* alios radios nisi axi proximos non emittit, etiam punctum *L* sine ulla confusione in *v* repraesentabitur, ita ut tantum puncta media inter *L* et *l* confusionem sint paritura. Consideretur ergo quodvis punctum intermedium λ, quod esset extremum in spatio confusionis, si semidiameter aperturae primae lentis minor foret quam *x,* qui ergo ponatur $= z$; eritque $L\lambda = Vzz$ et inclinatio radiorum in λ concurrentium ad axem $= \mathfrak{B}z$; hinc istius puncti imago intra oculum referetur in γ, ut sit $V\gamma = \frac{uu}{ll}Vzz$, ideoque ob $Vv = \frac{uu}{ll}\mathrm{V}xx$ erit intervallum $v\gamma = \frac{uu}{ll}\mathrm{V}(xx-zz)$ et radiorum in γ convergentium inclinatio ad axem $= \frac{l}{u}\mathfrak{B}z$; ex quo circelli super retina in *v* existentis radiis puncti λ expressi semidiameter erit

$$= \frac{u}{l}\mathfrak{B}Vz(xx-zz),$$

qui evanescit, uti iam monuimus, sive sit $z = 0$ sive $z = x$. Quaeratur iam ille valor ipsius *z,* quo ille circellus fiat maximus; id quod eveniet, si $xx - 3zz = 0$ seu $z = \frac{x}{\sqrt{3}}$; sicque circelli confusionem hoc casu metientis semidiameter erit $= \frac{2u}{3l\sqrt{3}} \cdot \mathfrak{B}Vx^3$,, quod est multo minus quam casu praecedente, quo retina erat in *V.*

At si retina aliquantillum a *v* versus *V* removeatur, hic circellus adhuc minor effici poterit. Ponatur enim spatium $VS = s$, ut sit

$$vS = \frac{uu}{ll}Vxx - s \quad \text{et} \quad S\gamma = s - \frac{uu}{ll}Vzz.$$

Punctum ergo *l,* cuius effigies in *v* exhibetur, super retina *S* cirello referetur, cuius radius est $= \frac{l}{u}\mathfrak{B}x(\frac{uu}{ll}Vxx - s)$, punctum vero λ, cuius effigies est in γ, super retina *S* circello, cuius radius erit $= \frac{l}{u}\mathfrak{B}z(s - \frac{uu}{ll}Vzz)$; nunc igitur id punctum λ investigemus, unde iste circellus minimus evadat: quod fit, si $s = \frac{3uu}{ll}Vzz$; eritque huius circuli radius $= \frac{2u}{l} \cdot \mathfrak{B}Vz^3$, qui simul confusionem exhiberet, si modo a puncto *l* non maior circulus oriretur: sed substituto pro *s* valore invento huius circuli radius erit $\frac{u}{l}\mathfrak{B}Vx(xx-3zz)$, qui ergo illi $\frac{2u}{l} \cdot \mathfrak{B}Vz^3$ aequalis statuatur, unde nascitur haec aequatio:

$$x^3 - 3xzz = 2z^3 \quad \text{hincque} \quad z = \tfrac{1}{2}x.$$

Quare circelli, quo minima confusio mensuratur, radius erit $=\frac{u}{4l}\mathfrak{B}Vx^3$, quadruplo minor, quam si retina esset in *V;* indeque ipsa confusio sedecies minor. Etsi autem illa aequatio cubica tres habet radices, praeter $z=\frac{1}{2}x$ binae reliquae sunt aequales et $z=-x$ sicque $zz=xx$, unde non minima, sed quasi maxima confusio nasceretur.

COROLLARIUM 1

171. Cum igitur, ut oculus minimam confusionem sentiat, debeat esse $z=\frac{1}{2}x$ ideoque $s=\frac{3uu}{4ll}Vxx$, patet retinam in eum locum *S* cogi debere, ut sit $VS=\frac{3}{4}Vv$ et $vS=\frac{1}{4}Vv$.

COROLLARIUM 2

172. Cum ergo oculus non solum praeditus sit facultate sese parumper immutandi, sed etiam hac facultate uti soleat ad confusionem evitandam, nullum est dubium, quin circellorum confusionem producentium radius sit $=\frac{u}{4l}\mathfrak{B}Vx^3$, ideoque quadruplo minor, quam problemate praecedente inveneramus, dummodo imago spectanda *Ll* propemodum in distantia iusta reperiatur.

COROLLARIUM 3

173. Haec igitur expressio $\frac{u}{4l}\mathfrak{B}Vx^3$ iustam nobis exhibet mensuram confusionis, cum exprimat radium circellorum, quibus singula obiecti puncta, quatenus id per lentes spectatur, super retina repraesentantur. Si modo imago in distantia iusta ab oculo reperiatur.

SCHOLION

174. Si hi circelli evanescerent, visio plane esset distincta; hoc autem fieri nequit, nisi ipsum intervallum diffusionis *Vxx* evanescat: unde patet, si apertura primae lentis ad nihilum reduceretur, nullam confusionem sentiri debere. Verum visio non ita est delicata, ut prorsus nullam confusionem pati possit, sed dummodo confusio certum quendam terminum non superet, quasi esset nulla considerari potest. Iste terminus seu valor, quem expressio $\frac{u}{4l}\mathfrak{B}Vx^3$ excedere non debet, ex experientia potius peti debet quam ex theoria, isque idcirco insigni adhuc latitudine continetur: unde fit, ut pro diverso scopo modo maiore gradu confusionis contenti esse soleamus, modo autem minorem gradum exigamus; quas circumstantias, cum ad praxin propius accedemus, accuratius sumus examinaturi. Ceterum notaridum est pro intervalio *u,* quo quasi profunditas oculi exhibetur, unum circiter pollicem assumi posse, quae quidem mensura fere erit arbitraria, cum deinceps limites confusionis per experientiam constituerons.

PROBLEMA 3

175. *Si oculus per unicam lentem obiectum aspiciat, ita ut imago visa ab oculo in distantia iusta reperiatur, definire confusionem, qua visio afficietur.*

SOLUTIO

Sit distantia obiecti *Ee* (Fig. 3) ante lentem $AE = a$, imaginis vero principalis $F\zeta$ post lentem $BF = \alpha$, lentisque crassities $AB = v$, quantitas

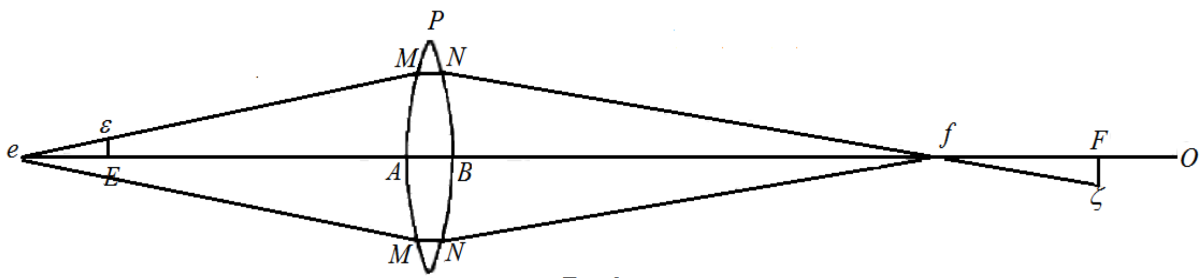

Fig. 3.

vero arbitraria, qua cum binis distantiis determinatricibus a et α lentis facies determinatur, sit $= k$; ponatur autem brevitatis gratia $\frac{k-v}{k+v} = i$. Tum autem posita ratione refractionis $= n$ debet esse lentis *PP*

$$\text{radius faciei anterioris} = \frac{(n-1)a(k+v)}{k+v+2na},$$

$$\text{radius faciei posterioris} = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha},$$

ac si semidiameter aperturae faciei anterioris sit $= x$, posterioris vero $> x$, spatium diffusionis Ff erit $= P\alpha\alpha xx$, existante

$$P = \frac{N}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{a} + \frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{1}{\alpha} - \frac{2}{k-v}\right)^2\right),$$

radiorum vero in *f* concurrentium inclinatio ad axem $= i \cdot \frac{x}{\alpha}$. His, quae § 86 sunt stabilita, praemissis, sit distantia oculi post lentem $BO = O$, et quia imago $F\zeta$ ante oculum in distantia iusta existere assumitur, erit $O = \alpha + l$ ideoque $\alpha = O - l$, si locus oculi ut datus consideretur. Hinc ergo habebimus $V = P\alpha\alpha$ et $\mathfrak{B} = \frac{i}{\alpha}$ ideoque $\mathfrak{B}V = i\alpha P$. Consequenter radius circellorum in oculo confusionem metientium seu, ut in posterum loquemur, mensura confusionis erit

$$=\tfrac{u}{4l}\cdot i\alpha x^3\cdot P=\tfrac{iu}{4l}(O-l)x^3P.$$

Si lentis crassities v evaneseat et pro determinatione lentis numerus arbitrarius λ loco k introducatur, erit ex § 91

$$P=\mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda(\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{\nu}{a\alpha}\right).$$

et $i=1$, with which cited in place the construction of the lens itself is to be performed.

COROLLARY 1

176. Si igitur lens fuerit data, locus oculi post lentem ita definitur, ut debeat esse distantia $BO=O=\alpha+l$, existante l distantia oculi iusta ; sin autem locus oculi detur, pro lentis constructione distantia determinatrix α ita capi debet, ut sit $\alpha=O-l$.

COROLLARY 2

177. Quare si oculus ita fuerit comparatus, ut exigat distantiam iustam $l=\infty$, fiet $\alpha=-\infty$, et mensura confusionis erit $=-\tfrac{1}{4}iux^3\cdot P$ seu $=\tfrac{1}{4}iux^3\cdot P$, quia signum - nihil mutat in magnitudine circellorum confusionem producentium.

COROLLARY 3

178. Etsi ergo hoc casu, quo $\alpha=-\infty$, spatium diffusionis Ff est infinitum, tamen inde confusio in visione orta est finita, quia hoc non obstante valor ipsius P manet finitus: erit enim

$$P=\tfrac{n}{2(n-1)^2}\left(\tfrac{1}{ii}\left(\tfrac{n}{a}+\tfrac{2}{k+v}\right)(\tfrac{1}{a}+\tfrac{2}{k+v})^2-\tfrac{8ii}{(k-v)^2}\right).$$

Radius autem faciei posterioris fit $=-\tfrac{(n-1)}{2n}(k-v)$.

COROLLARY 4

179. Quia est $i=\tfrac{k-v}{k+v}$, valor ipsius P etiam in genere ita exprimi potest, ut sit:

$$P=\tfrac{n}{2(n-1)^2}\left(\left(\tfrac{n}{a}+\tfrac{2}{k+v}\right)(\tfrac{1}{ia}+\tfrac{2}{k-v})^2+\left(\tfrac{n}{\alpha}-\tfrac{2}{k-v}\right)(\tfrac{i}{\alpha}-\tfrac{2}{k+v})^2\right),$$

ubi est $\tfrac{n}{2(n-1)^2}=\tfrac{310}{121}=$ 2,561983 ob $n=\tfrac{31}{20}$. Sicque etiam valores reliquarum litterarum Q, R, S etc. in § 86 transformari poterunt.

PROBLEMA 4

180. *Si oculus per duas lentes obiectum* $E\varepsilon$ (Fig. 5) *aspiciat, ita ut imago per eas repraesentata* $G\eta$ *in distantia iusta* $OG = l$ *ab oculo sit remota, definire confusionem, qua visio afficietur.*

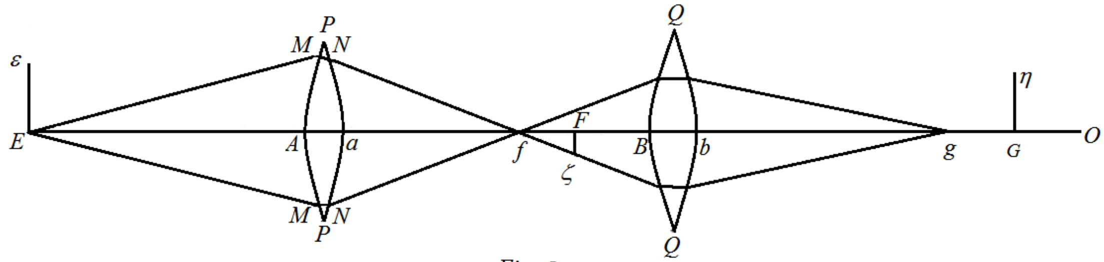

Fig. 5

SOLUTIO

Sint ut hactenus pro lente PP distantiae determinatrices
$AE = a,\ aF = \alpha,$ crassities $Aa = v$ et distantia arbitraria $= k$, ita ut sit

$$\text{radius faciei anterioris} = \frac{(n-1)a(k+v)}{k+v+2na},$$

$$\text{radius faciei posterioris} = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha};$$

tum vero posito $\frac{k-v}{k+v} = i$ ponatur

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right).$$

Deinde pro lente posteriori QQ sint distantiae determinatrices $BF = b,\ bG = \beta$, crassities lentis $Bb = v'$ et distantia arbitraria $= k'$, ut sit

$$\text{radius faciei anterioris} = \frac{(n-1)b(k'+v')}{k'+v'+2nb},$$

$$\text{radius faciei posterioris} = \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta};$$

tum vero posito $\frac{k'-v'}{k'+v'} = i'$, ponatur

$$Q=\tfrac{n}{2(n-1)^2}\Big(\big(\tfrac{n}{b}+\tfrac{2}{k'+v'}\big)(\tfrac{1}{i'b}+\tfrac{2}{k'-v'})^2+\big(\tfrac{n}{\beta}-\tfrac{2}{k'-v'}\big)(\tfrac{i'}{\beta}-\tfrac{2}{k'+v'})^2\Big).$$

Atque iam erit spatium diffusionis

$$Gg=\beta\beta xx\Big(\tfrac{1}{i'i'}\cdot\tfrac{\alpha\alpha}{bb}P+ii\cdot\tfrac{bb}{\alpha\alpha}Q\Big)=Vxx,$$

et radiorum in g concurrentium inclinatio ad axem

$$ii'\cdot\tfrac{bx}{\alpha\beta}=\mathfrak{B}x.$$

Sit iam oculus in O, existante $OG=l$, ac ponatur eius post lentem QQ distantia $bO=O$, erit $O=\beta+l$ ideoque $\beta=O-l$; quibus positis, cum sit mensura confusionis $=\frac{u}{4l}\mathfrak{B}x\cdot Vxx$, erit ea pro nostro casu:

$$\tfrac{ii'u}{4l}\cdot\tfrac{b\beta}{\alpha}x^3\Big(\tfrac{1}{i'i'}\cdot\tfrac{\alpha\alpha}{bb}P+ii\cdot\tfrac{bb}{\alpha\alpha}Q\Big)$$

vel pro β posito valore $O-l$ et signo mutato

$$\tfrac{ii'u}{4l}\cdot(1-\tfrac{O}{l})\tfrac{b}{\alpha}x^3\Big(\tfrac{1}{i'i'}\cdot\tfrac{\alpha\alpha}{bb}P+ii\cdot\tfrac{bb}{\alpha\alpha}Q\Big).$$

COROLLARY 1

181. Si ergo pro oculo fuerit $l=\infty$ ideoque et $\beta=-\infty$, etsi spatium diffusionis Gg fit infinitum, tamen ob $\frac{O}{i}=0$ confusio visionem afficiens nihilominus erit finita: neque enim P neque Q ob $\beta=-\infty$ fit infinita.

COROLLARY 2

182. Si crassities lentium evaneseat, ut sit $v=0$ et $v'=0$, erit $i=1$ et $i'=1$ ideoque hoc casu mensura confusionis

$$\tfrac{1}{4}\cdot u(1-\tfrac{O}{l})\tfrac{b}{\alpha}x^3\Big(\tfrac{\alpha\alpha}{bb}P+\tfrac{bb}{\alpha\alpha}Q\Big).$$

At si loco k et k' introducantur numeri λ et λ', erit

$$P=\mu\Big(\tfrac{1}{a}+\tfrac{1}{\alpha}\Big)\Big(\lambda(\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{v}{a\alpha}\Big)\ \text{ etc. }\ Q=\mu\Big(\tfrac{1}{b}+\tfrac{1}{\beta}\Big)\Big(\lambda'(\tfrac{1}{b}+\tfrac{1}{\beta})^2+\tfrac{v}{b\beta}\Big),$$

existante $\mu = 0,938191$ et $\nu = 0,232692$.

COROLLARY 3

183. Eodem autem hoc easu constructio binarum lentium ita se habebit:

Radius faciei

$$\text{Pro lente } PP \begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$$

$$\text{Pro lente } QQ \begin{cases} \text{anterioris} = \dfrac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$$

existante $\rho = 0,190781$, $\sigma = 1,627401$ et $\tau = 0,905133.$.

PROBLEMA 5

184. *Si oculus per tres lentes PP, QQ et RR* (Fig. 6) *obiectum $E\varepsilon$ aspiciat, ita ut imago per eas repraesentata $H\theta$ in distantia iusta $OH = l$ ab oculo O sit remota, definire confusionem, qua visio afficietur.*

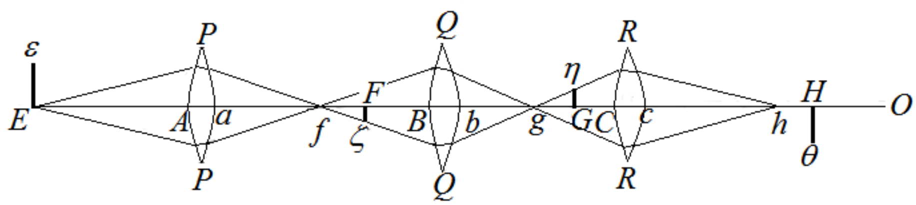

Fig. 6 (*iterata*)

SOLUTIO

Positis duabus prioribus lentibus *PP* et *QQ* ut in problemate praecedente indeque determinatis valoribus *P* et *Q*, cadat imago per has duas lentes repraesentata principalis in $G\eta$, post quam tertia lens *RR* ita collocata sit, ut sint eius distantiae determinatrices $CG = c$, $cH = \gamma$, crassities $Cc = v''$ et quantitas arbitraria $= k''$, ut sit:

$$\text{radius faciei anterioris} = \frac{(n-1)c(k''+v'')}{k''+v'+2nc},$$

$$\text{radius faciei posterioris} = \frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}.$$

Tum vero posito $\frac{k''-v''}{k''+v''} = i''$ ponatur

$$R = \frac{n}{2(n-1)^2}\left((\tfrac{n}{c} + \tfrac{2}{k''+v''})(\tfrac{1}{i''c} + \tfrac{2}{k''-v''})^2 + (\tfrac{n}{\gamma} - \tfrac{2}{k''-v''})(\tfrac{i''}{\gamma} - \tfrac{2}{k''+v''})^2 \right),$$

atque iam spatium diffusionis erit

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'\cdot i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii\cdot i''i''\cdot\frac{bbcc}{\alpha\alpha\beta\beta}R \right),$$

quod est valor ipsius *Vxx.* Radiorum vero in *h* concurrentium inclinatio ad axem est

$$ii'i''\cdot\frac{bcx}{\alpha\beta\gamma} = \mathfrak{B}x.$$

Sit iam oculus in *O,* ac ponatur eius distantia post lentem $RR = O$, erit $O = \gamma + l$ ideoque $\gamma = O - l$. Hinc mensura confusionis in oculo ortae colligitur

$$\tfrac{1}{4}ii'i''u\left(1-\tfrac{O}{l}\right)\tfrac{bc}{\alpha\beta}x^3\left(\frac{1}{i'i'\cdot i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii\cdot i''i''\cdot\frac{bbcc}{\alpha\alpha\beta\beta}R \right).$$

COROLLARY 1

185. Hic iterum ut ante patet, si fuerit $l = \infty$ ideoque et $\gamma = -\infty$ quo casu diffusionis spatium in infinitum extenditur, mensuram confusionis terminis finitis contineri, quod etiam locum habet pro quovis lentium numero.

COROLLARY 2

186. Si lentium crassities pro evanescente habeatur, ob $i = 1,\ i' = 1,\ i'' = 1$, mensura confusionis ita simplicius exprimetur, ut sit

$$\tfrac{1}{4}u\left(1-\tfrac{O}{l}\right)\tfrac{bc}{\alpha\beta}x^3\left(\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{bb\beta\beta}{\alpha\alpha cc}Q + \frac{bbcc}{\alpha\alpha\beta\beta}R \right).$$

At hoc casu loco $k,\ k',\ k''$ introductis numeris $\lambda,\ \lambda',\ \lambda''$ erit

$$P=\mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda(\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{\nu}{a\alpha}\right)$$
$$Q=\mu\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'(\tfrac{1}{b}+\tfrac{1}{\beta})^2+\tfrac{\nu}{b\beta}\right)$$
$$R=\mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''(\tfrac{1}{c}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{c\gamma}\right).$$

COROLLARY 3

187. Eodem vero casu constructio lentium per istos numeros λ, λ', λ'' ita erit dirigenda, ut sit

Radius faciei

$$\text{Pro lente } PP\begin{cases}\text{anterioris}=\dfrac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}\\[2ex]\text{posterioris}=\dfrac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}\end{cases}$$

$$\text{Pro lente } QQ\begin{cases}\text{anterioris}=\dfrac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}\\[2ex]\text{posterioris}=\dfrac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}\end{cases}$$

$$\text{Pro lente } RR\begin{cases}\text{anterioris}=\dfrac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}\\[2ex]\text{posterioris}=\dfrac{b\beta}{\rho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}}\end{cases}.$$

SCHOLION

188. Hinc satis manifestum est, quemadmodum hae formulae pro pluribus lentibus progrediantur; verum antequam eas exponam, conveniet alias quoque circumstantias, quae hinc facillime deducuntur, perpendi, scilicet magnitudinem obiecti visam et copiam radiorum a singulis eius punctis in oculum transmissorum, ut hae simul cum confusione visionis deinceps coniunctim pro quovis lentium numero exhiberi queant; quo pacto plures taediosas repetitiones evitabimus. Duae autem res ex hactenus allatis facile definiri possunt, quarum altera est quantitas, qua imago obiecti per lentes repraesentata ab oculo cernitur, quae quantitas aestimanda est ex angulo, sub quo imago videtur, ut is deinceps comparari possit cum eo angulo, sub quo ipsum obiectum in data distantia a nudo oculo spectaretur, unde, qua ratione magnitudo per lentes visa augeatur, intelligetur. Altera res in copia radiorum a singulis obiecti punctis in oculum transmissorum versatur, qua

claritas visione percepta continetur, a quolibet scilicet puncto conus seu cylindrus radiosus in oculum ingreditur; qui si pupillam penitus expleat, claritas ad summum gradum erit evecta, nisi forte maiori illustratione ipsi obiecto maiori lumen concilietur. At si sectio illius coni aut cylindri, qua in oculum intrat, minor fuerit pupilla, in eadem ratione claritas decrescet; quod cum in omnibus instrumentis dioptricis, quibus magnitudinem visam vehementer augere propositum est, usu venire soleat, plurimum intererit amplitudinem illius coni seu cylindri, qua in oculum penetrat, accurate determinasse.

PROBLEMA 6

189. *Definire quantitatem, sub qua quaevis obiecti portio per lentes quotcunque ab oculo in distantia iusta ab imagine ultima remoto cernetur.*

SOLUTIO

Sit z linea in obiecto concepta, quae, quanta per lentes oculo sit apparitura, definiri oporteat. Ostensum autem est in praecedentibus, quotcunque fuerint lentes, imaginem principalem huius lineae iterum esse lineam, cuius longitudo ad z certam teneat rationem a distantiis determinatricibus lentium et numeris i, i', i'', i''' etc. pendentem (§ 86). Sit ergo haee longitudo imaginis $= Mz$, quae, cum ab oculo in distantia l remoto aspiciatur, apparebit sub angulo $= \frac{Mz}{l}$, vel cuius tangens potius sit $= \frac{Mz}{l}$; sed quia hic angulus rarissime ultra aliquot gradus assurgere solet, tangens tuto pro ipso arcu assumitur. Effigies antem, quae ab hac linea Mz in oculo exprimitur, erit $= \frac{Muz}{l}$; hic enim cogitationes a confusione abstraho, qua utique fit, ut effigies maior imprimatur, propterea quod singula puncta circellis exhibentur. Iam positis iisdem lentium determinationibus, quibus supra § 86 sum usus, pro vario lentium numero angulus, sub quo linea *1* in obiecto sumta cernetur, ita se habebit:

Scilicet si hae formulae sint positivae, oculus lineam s situ sive erecto sive inverso videbit, prout est notatum; sin autem fuerint negativae, situm indicatum in contrarium verti oportet.

Angulus visionis

Pro unica lente	$\frac{1}{i}\cdot\frac{\alpha}{a}\cdot\frac{z}{l}$	situ inverso
pro duabus lentibus	$\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}\cdot\frac{z}{l}$	situ erecto
pro tribus lentibus	$\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}\cdot\frac{z}{l}$	situ inverso
pro quatuor lentibus	$\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}\cdot\frac{z}{l}$	situ erecto
	etc.	

COROLLARY 1

190. Si eadem obiecti linea z in distantia $= h$ ab oculo nudo cerneretur, ea apparitura esset sub angulo $= \frac{z}{h}$, unde perspicitur, quanto ea vel maior vel minor per lentes videatur.

COROLLARY 2

191. Si distantia oculi a postrema lente ponatur $= O$, erit casu unius lentis $\alpha = O - l$ ideoque $\frac{\alpha}{l} = -(1 - \frac{O}{l})$, unde angulus opticus lineae obiecti z respondens erit $= \frac{1}{i}(1 - \frac{O}{l})\frac{z}{a}$ pro situ erecto , quia signum mutavimus.

COROLLARY 3

192. Simili modo casu duarum lentium ob $\beta = O - l$ erit iste angulus $= \frac{1}{ii'}(1 - \frac{O}{l})\frac{\alpha z}{ab}$ pro situ inverso.

Casu vero trium lentium ob $\gamma = O - l$ erit iste angulus $= \frac{1}{ii'i''}(1 - \frac{O}{l})\frac{\alpha\beta z}{abc}$ pro situ erecto. Casu quatuor lentium ob $\delta = O - l$ erit iste angulus $= \frac{1}{ii'i''i'''}(1 - \frac{O}{l})\frac{\alpha\beta\gamma z}{abcd}$ pro situ in verso, et ita porro pro pluribus lentibus.

SCHOLION

193. Hinc etiam modus se offert confusionem ob lentium aperturam in oculo natam distinctius aestimandi. Scilicet cum singula obiecti puncta in oculo exprimantur circulis, quorum radius est $= \frac{u}{4l}\mathfrak{B}Vx^3$, verus autem circulus, cuius radius $= z$, oculo in distantia l expositus in oculo referatur circulo, cuius radius $= \frac{u}{l}z$, singula puncta illius obiecti per lentes spectati aeque magna apparebunt ac orbes circulares radii $= \frac{1}{4}\mathfrak{B}Vx^3$ si in distantia ab oculo $= l$ spectarentur. Vel cum horum orrium semidiameter apparens sit $= \frac{z}{l}$, ob confusionem singula obiecti puncta iustar circulorum videbuntur, quorum semidiameter apparens esset $= \frac{1}{4l}\mathfrak{B}Vx^3$. In expressionibus igitur ante pro confusione inventis deleatur quantitas u, et habebitur semidiameter apparens circulorum confusionem exprimentium. Hinc iudicari poterit, quam parva esse debeat confusio, ut non amplius sentiatur; scilicet si oculus non amplius percipere valeat spatium circulare, cuius semidiameter esset 1" seu $\frac{1}{60^2}$ pars radii circiter, evidens est, si fuerit nostra expressio $\frac{1}{4l}\mathfrak{B}Vx^3 = \frac{1}{60^2}$, confusionem fore imperceptibilem. Ac experientiam consulentes deprehendimus multo maiores angulos non amplius percipi posse, ita ut confusio non sit metuenda, etiamsi

expressio $\frac{1}{4l}\mathfrak{B}Vx^3$ notabiliter maior fuerit quam $\frac{1}{60^2}$; ne autem hic temere quicquam statuamus, ponamus limitem, quem formula $\frac{1}{4l}\mathfrak{B}Vx^3$ excedera non debeat, esse $=\frac{1}{4u^3}$, ita ut esse oporteat $\frac{1}{l}\mathfrak{B}Vx^3<\frac{1}{u^3}$. Postea igitur ad praxin descendantes poterimus pro χ numerum vel 40 vel minorem assumere, prout experientia quovis casu postulaverit. Quod si ergo hoc modo confusionis rationem habeamus, profunditas oculi u non amplius in computum ingredietur.

DEFINITIO 3

194. *Semidiameter confusionis est semidiameter apparens circuli, qui ab oculo aeque magnus videtur, ac singula obiecti puncta ipsi ob confusionem apparent.*

COROLLARY

195. Inveniemus igitur facile semidiametrum confusionis, si formulas supra pro confusione repertas per profunditatem oculi u dividamus, quo pacto eae formulae ad numeros absolutos reducentur.

DEFINITIO 4

196. *Multiplicatio per lentes producta ex ratione quantitatis, qua obiecta per lentes spectantur, ad quantitatem, qua eadem obiecta in data distantia ab oculo nudo cernerentur, aestimatur. Exponens autem multiplicationis invenitur, si magnitudo, qua linea quaecunque in obiecto concepta per lentes videtur, dividatur per magnitudinem, qua eadem linea in data distantia ab oculo nudo spectata esset apparitura.*

COROLLARY 1

197. Involvit ergo diiudicatio multiplicationis distantiam quandam fixam, in qua eadem obiecta a nudo oculo aspici assumimus; quae prout diversa assumatur, multiplicatio alio atque alio modo exprimetur.

COROLLARY 2

198. Si haec distantia fixa, ex qua multiplicatio diiudicatur, ponatur $=h$ et exponens multiplicationis $=m$, sit linea quaepiam in obiecto concepta $=z$, quae ergo nudo oculo in distantia h appareret sub angulo $=\frac{z}{h}$; eadem autem linea per lentes spectetur sub angulo $=\frac{Mz}{l}$ (§ 88), ex quo erit exponens multiplicationis $m=\frac{Mh}{l}$.

COROLLARY 3

199. In ratione ergo m : 1 dimensiones lineares per lentes augeri sunt censendae; unde superficies auctae apparebunt in ratione $mm:1$ et ipsa corpora in ratione $m^3:1$. Cum exponente autem multiplicationis coniungi debet situa, quo obiecta apparent, sive is sit erectus sive inversus.

SCHOLION

200. Distantia haec fixa *h,* ad quam multiplicatio refertur, non eodem modo perpetuo assumi solet, quippe quod etiam pro diversitate obiectorum omnino fieri non posset. Nam si per lentes obiecta valde remota veluti coelestia contuemur, quoniam ea nunquam in distantia modica spectare solemus, convenit utique magnitudinem per lentes visam eum ea comparare, qua in ea ipsa a nobis distantia nudis oculis cernerentur: ideoque his casibus distantia fixa *k* ipsi distantiae *a,* qua a lentibus obiceta sunt remota, aequalis constitui solet. Scilicet si distantia *a* fuerit valde magna, qui est casus Telescopiorum, statuitur $h = a$, et magnitudo per haec instrumenta visa cum magnitudine per nudos oculos visa in eadem distantia commodissime comparatur. Sic de Telescopiis dicitur, quoties diametri corporum coelestium multiplicentur, eoque casu littera *m* exponentem huius rationis indicabit. Sin autem obiecta propiora contemplamur, qui est usus Microscopiorum, ea plerumque ita prope ad instrumentum admoventur, ut in tam exigua distantia nudis oculis nunquam distincte cerni possent: neque ergo his casibus statui $h = a$ conveniret. Aliam ergo rationem ineundo pro *h* sumi solet eiusmodi distantia modica, in qua obiecta commode ac distincte cernera liceat, quae etsi utique pro diversa oculorum indole diversa sumi deberet, tamen, ut aliquid fixi statuatur, pro *h* distantia 8 pollicum utpote maximae oculorum parti conveniens accipi solet, ita ut his casibus definiamus, quoties huiusmodi obiecta maiora *appareant,* quam si eadem nudis oculis in distantia 8 digitorum aspicerentur. Interim tamen si multiplicatio ad hanc distantiam fuerit relata, non difficile erit eam ad quamlibet aliam referre, ita ut haec hypothesis naturam horum instrumentorum afficere non sit censenda.

PROBLEMA 7

201. *Si obiectum per lentes quotcunque aspiciatur, definire amplitudinem coni seu cylindri luminosi, qui a singulis obiecti punctis in oculum transmittitur.*

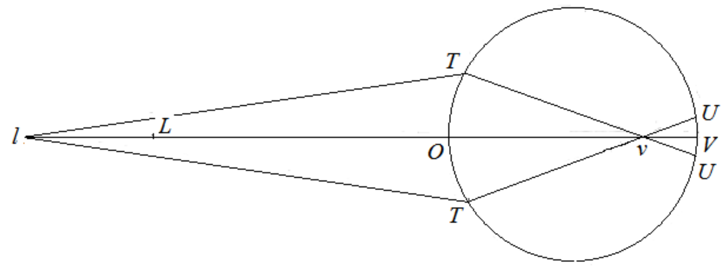

Fig. 10

(iterata).

SOLUTIO

Reperiatur ut supra oculus in distantia iusta l (Fig. 10) post imaginem postremam per lentes repraesentatam, quae etsi est per spatium aliquod Ll diffusa, hic tamen a confusione inde nata mentem abstrahimus, quoniam confusionem iam seorsim determinavimus. Ex puncto igitur l oculum versus diffunditur conus luminosus, cuius radii extremi ad axem inclinati sunt angulo OlT, quem posuimus supra $= \mathfrak{B}x$. Huius igitur coni sectio circa ingressum in oculum consideretur, cuius semidiameter erit $= \mathfrak{B}lx$, unde pro vario lentium numero hic semidiameter sequenti modo definietur:

Pro unica lente $\quad il\frac{x}{\alpha}$

pro duabus lentibus $\quad ii'l\frac{bx}{\alpha\beta}$

pro tribus lentibus $\quad ii'i''l\frac{bcx}{\alpha\beta\gamma}$

pro quatuor lentibus $\quad ii'i''i'''l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.;

hicque perinde est, sive hae formulae, sint positivae sive negativae, quia circulus, sive radio positivo sive negativo describatur, eiusdem prodit magnitudinis.

COROLLARY 1

202. Si iste semidiameter $\mathfrak{B}lx$ maior fuerit semidiametro pupillae, tota pupillae apertura radiis impletur, neque propterea visio clarior procurari poterit, nisi forte ipsum obiectum fortiori illustratione splendidius reddatur.

COROLLARY 2

203. Sin autem haec quantitas $\mathfrak{B}lx$ minor fuerit semidiametro pupillae, claritatis mensura existet, quae eo maior erit, quo maior fuerit ista quantitas: dum contra minuta hac quantitate claritas tam exigua evadere potest, ut non amplius sensui visus excitando sufficiat.

DEFINITIO 5

204. *Gradus claritatis per lentes perceptae commodissime definietur semidiametro coni luminosi, qui a quovis obiecti puncto in oculum tranmittitur.*

COROLLARY 1

205. Gradus ergo claritatis definitur quantitate supra inventa $\mathfrak{B}lx$, ita ut si gradum claritatis ponamus $= y$, habeamus $y = \mathfrak{B}lx$, quo cognito facillime iudicabimus, quonam gradu visio sit clara habenda.

COROLLARY 2

206. Scilicet si semidiametrum pupillae ponamus $= \omega$, quamdiu fuerit $y > \omega$, claritate plena fruemur, quae nullius augmenti est capax, nisi forte ipsam pupillam magis dilatare valeamus.

COROLLARY 3

207. At si fuerit $y < \omega$, claritatem utique minorem percipiemus; ac si claritatem plenam unitate designerons, claritas ex casu $y < \omega$ resultans erit $= \frac{yy}{\omega\omega}$, propterea quod copia radiorum in oculum immissorum est ut quadratum semidiametri y.

COROLLARY 4

208. Quod si gradus claritatis y eousque decrescat, ut copia radiorum nimis sit parva, quam ut sensum visus excitare possit, nihil ob summam caliginem percipi poterit, unde manifestum est ad visionem requiri, ut gradus claritatis certum quempiam limitem superet.

SCHOLlON

209. Tam in Telescopiis quam Microscopiis maxime necesse est, ut obiecta certo claritatis gradu exhibeantur, ne repraesentatio nimis fiat obscura. Hic autem gradus plurimum a lumine proprio obiectorum pendet, quae quo fuerint illustriora, eo minor

claritatis gradus iis satis clare videndis sufficit; ideoque stellas illustriores contemplantes minori gradu claritatis contenti esse possumus, terrestria vero obiecta multo maiorem claritatis gradum postulant. Quo igitur haec ad omnes casus accommodare valeamus, gradum claritatis hic littera indefinita y contentum in computum sum ducturus. Quamobrem his de multiplicatione et claritate praemissis haec duo elementa simul cum confusione pro quovis lentium numeto exhibebo: ac primo quidem non neglecta lentium crassitie, tum vero eadem seorsim lentium crassitie neglecta exponi conveniet.

PROBLEMA 8

210. *Si oculus per quotcunque lentes PP, QQ, RR, SS etc. Obiectum* $E\varepsilon$ *aspiciat, ita ut imago postrema per eas repraesentata ante oculum in iusta distantia* $= l$ *reperiatur, determinare tam multiplicationem et claritatem quam confusionem, qua visio perturrabitur.*

SOLUTIO

Quotcunque fuerent lentes, sint pro singulis distantiae determinatrices ut et crassities cum quantitate arbitraria, ut sequitur:

Pro lente	Distantiae determinatrices	Crassities	Quantitas arbitraria
prima PP	$EA = a, \quad aF = \alpha$	$Aa = v$	k
secunda QQ	$FB = b, \quad bG = \beta$	$Bb = v'$	k'
tertia RR	$GC = c, \quad cH = \gamma$	$Cc = v''$	k''
quarta SS	$HD = d, \quad dI = \delta$	$Dd = v'''$	k'''

etc.

atque hinc posita ratione refractionis $\frac{31}{20} = n$ constructio lentium ita se habebit:

Pro lente		Radius faciei anterioris	Radius faciei posterioris
prima	PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)a(k-v)}{k-v-2na}$
secunda	QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)b(k'-v')}{k'-v'-2nb}$
tertia	RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)c(k''-v'')}{k''-v''-2nc}$
quarta	SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)d(k'''-v''')}{k'''-v'''-2nd}$

etc.

Tum posito brevitatis gratia:

$$\frac{k-v}{k+v}=i,\quad \frac{k'-v'}{k'+v'}=i',\quad \frac{k''-v''}{k''+v''}=i'',\quad \frac{k'''-v'''}{k'''+v'''}=i''',\quad \text{etc.}$$

si aperturae primae lentis in facie anteriori semidiameter fuerit x, tam pro facie posteriori quam pro utraque facie singularum lentium sequentium aperturae maiores esse debent vel saltem non minores, quam sequens tabula ostendit:

Pro lente		Semidiameter aperturae in facie anteriori	Semidiameter aperturae in facie posteriori
prima	PP	x	ix
secunda	QQ	$i\cdot\frac{bx}{\alpha}$	$ii'\cdot\frac{bx}{\alpha}$
tertia	RR	$ii'\cdot\frac{bcx}{\alpha\beta}$	$ii'i''\cdot\frac{bcdx}{\alpha\beta}$
quarta	SS	$ii'i''\cdot\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''\cdot\frac{bcdx}{\alpha\beta\gamma}$

etc.

Denique ad abbreviandum ponatur:

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$
$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{ib}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$
$$R=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\alpha}-\frac{2}{k''+v''}\right)^2\right)$$
$$S=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\alpha}-\frac{2}{k'''+v'''}\right)^2\right)$$

His positis ponamus oculum in distantia $=O$ ab ultima lente locari, ita ut post imaginem ultimam reperiatur in distantia $=l$, magnitudinem autem visam comparari cum magnitudine, qua idem obicetum nudo oculo in distantia fixa $=h$ cerneretur, ac ponatur exponens multiplicationis $=m$.

Deinde pro claritate sit gradus claritatis $=y$, ita ut y indicet semidiametrum coni luminosi in oculum intrantis.

Confusio autem aestimetur per semidiametrum confusionis supra (§ 194) definitum.

Iam pro quovis lentium numero hae tres res ita se habebunt:

I. Pro unica lente $O=\alpha+l$

l. Exponens multiplicationis $m=\frac{1}{i}\cdot\frac{\alpha h}{al}$ situ inverso

2. Gradus claritatis $y=il\cdot\frac{x}{\alpha}$

3. Semidiameter confusionis $=\frac{1}{4}i\frac{\alpha}{l}x^3\cdot P.$

II. Pro duabus lentibus $O=\beta+l$

1. Exponens multiplieationis $m=\frac{1}{ii'}\cdot\frac{\alpha\beta h}{abl}$ situ erecto

2. Gradus claritatis $y=ii'l\cdot\frac{bx}{\alpha\beta}$;

3. Semidiameter confusionis $=\frac{1}{4}ii'\frac{\beta}{l}\cdot\frac{b}{\alpha}x^3\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P+ii\cdot\frac{bb}{\alpha\alpha}Q\right).$

III. Pro tribus lentibus $O=\gamma+l$

1. Exponens multiplicationis $m=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma h}{abcl}$ situ inverso

2. Gradus claritatis $y=ii'i''l\cdot\frac{bcx}{\alpha\beta\gamma}$

3. Semidiameter confusionis:

$$\frac{1}{4}ii'i''\frac{\gamma}{l}\cdot\frac{bc}{\alpha\beta}x^3\left(\frac{1}{i'i'\cdot i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}Q+ii\cdot i'i'\cdot\frac{bbcc}{\alpha\alpha\beta\beta}R\right).$$

IV. Pro quatuor lentibus $O=\delta+l$

1. Exponens multiplicationis $m=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta h}{abcdl}$ situ erecto

2. Gradus claritatis $y=ii'i''i'''l\cdot\frac{bcdx}{\alpha\beta\gamma\delta}$

3. Semidiameter confusionis:

$$\frac{1}{4}ii'i''i'''\frac{\delta}{l}\cdot\frac{bcd}{\alpha\beta\gamma}x^3\left\{\begin{array}{l}\frac{1}{i'i'\cdot i''i''\cdot i'''i'''}\cdot\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}P+\frac{ii}{i''i''}i'''i'''\cdot\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}Q\\+\frac{ii\cdot i'i'}{i'''i'''}\cdot\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}R+i'i'\cdot i''i''\cdot i'''i'''\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}S\end{array}\right\}.$$

V. Pro quinque lentibus $O=\varepsilon+l$

1. Exponens multiplicationis $m=\frac{1}{ii'i''i'''\cdot i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon h}{abcdel}$ situ inverso

2. Gradus claritatis $y=ii'i''i'''i''''l\cdot\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$

3. Semidiameter confusionis:

$$\frac{1}{4}ii'i''i'''i''''\frac{\varepsilon}{l}\cdot\frac{bcde}{\alpha\beta\gamma\delta}x^3\left\{\begin{array}{l}\frac{1}{i'i'\cdot i''i''\cdot i'''i'''\cdot i''''i''''}\cdot\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}P\\+\frac{ii}{i''i''\cdot i'''i'''\cdot i''''i''''}\cdot\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}Q\\+\frac{ii\cdot i'i'}{i'''i'''\cdot i''''i''''}\cdot\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}R\\+\frac{ii\cdot i'i'\cdot i''i''}{i''''i''''\cdot i''''i''''}\cdot\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}S\\+ii\cdot i'i'\cdot i''i''\cdot i'''i'''\cdot\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}T\end{array}\right\}.$$

atque hinc etiam progressus ad plures lentes est manifestus.

COROLLARY 1

211. His omnibus casibus evidens est fore generatim $my=\frac{hx}{a}$. Datis scilicet multiplicatione m cum claritate y statim definitur apertura primae lentis nempe $x=my\cdot\frac{a}{h}$. Quo maior scilicet tam multiplicatio quam claritas desideratur, eo maiorem esse oportet aperturam lentis primae.

COROLLARY 2

212. Quum autem x maius accipere non liceat, quam ut confusio infra certum limitem contineatur, dato x cum exponente multiplicationis m definitur claritatis gradus $y=\frac{hx}{ma}$, unde patet reliquis paribus, quo maior multiplicatio exigatur, eo minori claritate contentos nos esse oportere.

COROLLARY 3

213. Inprimis autem hic observandum est has formulas aeque negotium conficere, quamcunque crassitiem lentes habuerint. Evadent autem tractabiliores, si lentium crassities negligatur, qui casus seorsim tractari meretur.

PROBLEMA 9

214. *Iisdem positis, quae in problemate praecedente, si lentium crassities ut evanescens consideretur, determinare tam multiplicationem et claritatem quam confusionem, qua visio perturbabitur.*

SOLUTIO

Haec tractatio a praecedenti in isto differt, quod lentium crassities $v,\ v',\ v''$ etc. evanescant et loco quantitatum arbitrariarum $k,\ k',\ k''$ etc. numeri arbitrarii $\lambda,\ \lambda',\ \lambda''$ etc. in calculum introducantur. Ponatur ergo:

Pro lente		Distantiae determinatrices		Numerus arbitrarius
prima	PP	$EA = a,$	$aF = \alpha$	λ
secunda	QQ	$FB = b,$	$bG = \beta$	λ'
tertia	RR	$GC = c,$	$cH = \gamma$	λ''
quarta	SS	$HD = d$	$dI = \delta$	λ'''

etc.

Hinc, si sit brevitatis gratia $\rho = 0,190781, \sigma = 1,627401$ et $\tau = 0,905133,$ lentium constructio ita est instituenda:

Pro lente		Radius faciei anterioris	posterioris
prima	PP	$\frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$	$\frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$
secunda	QQ	$\frac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}$	$\frac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$
tertia	RR	$\frac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}$	$\frac{c\gamma}{\rho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}}$
quarta	SS	$\frac{d\delta}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda'''-1)}}$	$\frac{d\delta}{\rho d+\sigma\delta\mp\tau(d+\delta)\sqrt{(\lambda'''-1)}}$

etc.

Cum iam lentis primae *PP* semidiameter aperturae sit $=x$, et in qualibet lente utriusque faciei eadem sit ratio, ut omnes radii per primam ingressi simul per reliquas transmittantur, apertura reliquarum sequentes limites superare debet:

Semidiameter aperturae

lentis secundae $QQ>\frac{b}{\alpha}x$

lentis tertiae $RR>\frac{bc}{\alpha\beta}x$

lentis quartae $SS>\frac{bcd}{\alpha\beta\gamma}x$

etc.

Tum vero posito brevitatis ergo $\mu=0,938191$ et $\nu=0,232692$ statuatur:

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2+\frac{\nu}{a\alpha}\right),\qquad Q=\mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'(\frac{1}{b}+\frac{1}{\beta})^2+\frac{\nu}{b\beta}\right)$$
$$R=\mu\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{\nu}{c\gamma}\right),\qquad S=\mu\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''(\frac{1}{d}+\frac{1}{\delta})^2+\frac{\nu}{d\delta}\right)\quad \text{etc.}$$

Sit iam *O* distantia oculi post ultimam lentem, ita ut ab imagine postrema distet intervalio $=l$, comparetur magnitudo visa cum ea, qua idem obiectum in distantia fixa *h* nudo oculo cerneretur, sitque exponens multiplicationis $=m$ et gradus claritatis $=y$, quibus positis erit pro quovis lentium numero, ut sequitur:

I. Pro unica lente $O=\alpha+l$

l. Exponens multiplicationis $m=\frac{\alpha h}{al}$ situ inverso

2. Gradus claritatis $y=l\cdot\frac{x}{\alpha}$, hinc $my=\frac{hx}{\alpha}$

3. Semidiameter confusionis $=\frac{\alpha}{4l}\cdot x^3P.$

II. Pro duabus lentibus $O=\beta+l$

1. Exponens multiplieationis $m=\frac{\alpha\beta h}{abl}$ situ erecto

2. Gradus claritatis $y=l\cdot\frac{bx}{\alpha\beta}$, hinc $my=\frac{hx}{a}$

3. Semidiameter confusionis $=\frac{\beta}{4l}\cdot\frac{b}{\alpha}x^3\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P+\frac{bb}{\alpha\alpha}Q\right).$

III. Pro tribus lentibus $O=\gamma+l$

1. Exponens multiplicationis $m=\frac{\alpha\beta\gamma h}{abcl}$ situ inverso

2. Gradus claritatis $y=l\cdot\frac{bcx}{\alpha\beta\gamma}$, hinc $my=\frac{hx}{a}$

3. Semidiameter confusionis:

$$\frac{\gamma}{4l}\cdot\frac{bc}{\alpha\beta}x^3\left(\frac{\alpha\alpha\beta\beta}{bbcc}P+\frac{bb\beta\beta}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

IV. Pro quatuor lentibus $O=\delta+l$

1. Exponens multiplicationis $m=\frac{\alpha\beta\gamma\delta h}{abcdl}$ situ erecto

2. Gradus claritatis $y=l\cdot\frac{bcdx}{\alpha\beta\gamma\delta}$, hinc $my=\frac{hx}{a}$

3. Semidiameter confusionis:

$$\frac{\delta}{4l}\cdot\frac{bcd}{\alpha\beta\gamma}x^3\left(\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}P+\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}Q+\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}R+\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}S\right).$$

V. Pro quinque lentibus $O=\varepsilon+l$

1. Exponens multiplicationis $m=\frac{\alpha\beta\gamma\delta\varepsilon h}{abcdel}$ situ inverso

2. Gradus claritatis $y=l\cdot\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$, hinc $my=\frac{hx}{a}$

3. Semidiameter confusionis:

$$\frac{\varepsilon}{4l}\cdot\frac{bcde}{\alpha\beta\gamma\delta}x^3\left\{\begin{array}{l}\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}P+\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}Q+\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}R\\+\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}S+\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}T\end{array}\right\}.$$

unde non difficile erit has formulas ad lentes etiam plures continuare.

COROLLARY 1

215. Lentes simplices adhibendo numeros $\lambda,\ \lambda',\ \lambda''$ etc. unitate minores accipi non possunt. Verum hic nihil obstat, quo minus loco lentium simplicium lentes duplicatae, triplicatae vel etiam quadruplicatae in usum vocentur, quo pacto in his formulis numeri $\lambda,\ \lambda',\ \lambda''$ etc. non solum infra unitatem diminui, sed ad nihilum usque perduci poterunt. Tum autem constructio lentium harum multiplicatarum ex superiori capite peti et ad distantias determinatrices hic positas accommodari debet.

COROLLARY 2

216. Veluti si pro lente prima *PP* debeat esse $\lambda = 0{,}191827$, hanc lentem ita ex duabus componi oportet, ut sit:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
priori	$\frac{2a\alpha}{(2\rho-\sigma)\alpha+\sigma a}$	$\frac{2a\alpha}{(2\sigma-\rho)\alpha+\rho a}$
posteriori	$\frac{2a\alpha}{\rho\alpha+(2\sigma-\rho)a}$	$\frac{2a\alpha}{\sigma\alpha+(2\rho-\sigma)a}$

COROLLARY 3

217. Sin autem velimus, ut pro lente prima sit $\lambda = 0{,}042165$, ea erit triplicanda, hoc modo:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
priori	$\frac{3a\alpha}{(3\rho-2\sigma)\alpha+\sigma a}$	$\frac{3a\alpha}{(3\sigma-2\rho)\alpha+\rho a}$
media	$\frac{3a\alpha}{(2\rho-\sigma)\alpha+(2\sigma-\rho)a}$	$\frac{3a\alpha}{(2\sigma-\rho)\alpha+(2\rho-\sigma)a}$
posteriori	$\frac{3a\alpha}{\rho\alpha+(3\sigma-2\rho)a}$	$\frac{3a\alpha}{\sigma\alpha+(3\rho-2\sigma)a}$.

COROLLARY 4

218. At si pro lente prima requiratur $\lambda = -0,010216$, quadruplicata erit utendum ita construenda:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
prima	$\frac{4a\alpha}{(4\rho-3\sigma)\alpha+\sigma a}$	$\frac{4a\alpha}{(4\sigma-3\rho)\alpha+\rho a}$
secunda	$\frac{4a\alpha}{(3\rho-2\sigma)\alpha+(2\sigma-\rho)a}$	$\frac{4a\alpha}{(3\sigma-2\rho)\alpha+(2\rho-\sigma)a}$
tertia	$\frac{4a\alpha}{(2\rho-\sigma)\alpha+(3\sigma-2\rho)a}$	$\frac{4a\alpha}{(2\sigma-\rho)\alpha+(3\rho-2\sigma)a}$
quarta	$\frac{4a\alpha}{\rho\alpha+(4\sigma-3\rho)a}$	$\frac{4a\alpha}{\sigma\alpha+(4\rho-3\sigma)a}$

COROLLARIUM 5

219. Simili autem modo lens secunda *QQ* ex suis distantiis determinatricibus b et β per multiplicationem erit construenda, numerusque ei respondens λ' debet esse vel 0,191827 vel 0,042165 vel -0,010216; quod idem de reliquis lentibus est intelligendum.

SCHOLION 1

220. Alias lentium species hic nollem in praxi adhiberi, cum hae solae sine metu enormis erroris confici queant; tum vero, etsi aliae ab his non admodum discrepantes fere aequo successu in praxin introduci possent, tamen, quia discrimen non admodum est notabile, iis facile carere poterimus. Cum igitur sit $\rho = 0,190781$ et $\sigma = 1,627401$ ideoque

$$\begin{array}{ll} \rho = 0,190781, & \sigma = 1,627401 \\ 2\rho - \sigma = -1,245839, & 2\sigma - \rho = 3,064021 \\ 3\rho - 2\sigma = -2,682459, & 3\sigma - 2\rho = 4,500641 \\ 4\rho - 3\sigma = -4,119079, & 4\sigma - 3\rho = 5,937261, \end{array}$$

easdem constructiones in numeris evolutis exhiberi conveniet.

1. Si igitur pro lente *PP* debeat esse $\lambda = 1$, ea erit simplex hoc modo construenda:

Radius faciei	
anterioris	posterioris
$\frac{a\alpha}{+0,190781\alpha+1,627401a}$	$\frac{a\alpha}{+1,627401\alpha+0,190781a}$

II. Si pro lente PP debeat esse $\lambda = 0,191827$, ea erit duplicata hoc modo construenda:

Pro lente	Radius faciei anterioris	posterioris
priori	$\frac{a\alpha}{-0,622919\alpha+0,813700a}$	$\frac{a\alpha}{+1,532010\alpha+0,095890a}$
posteriori	$\frac{a\alpha}{+0,096890\alpha+1,532010a}$	$\frac{a\alpha}{+0,813700\alpha-0,622919a}$

III. Si pro lente PP debeat esse $\lambda = 0,042165$, ea erit triplicata hoc modo construenda:

Pro lente	Radius faciei anterioris	posterioris
priori	$\frac{a\alpha}{-0,894153\alpha+0,542467a}$	$\frac{a\alpha}{+1,500214\alpha+0,063594a}$
media	$\frac{a\alpha}{-0,415280\alpha+1,021340a}$	$\frac{a\alpha}{1,021340\alpha-0,415280a}$
posteriori	$\frac{a\alpha}{+0,063594\alpha+1,500214a}$	$\frac{a\alpha}{+0,542467\alpha-0,894153a}$.

IV. Si pro lente PP debeat esse $\lambda = -0,010216$, ea erit quadruplieata hoc modo construenda:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
prima	$\frac{a\alpha}{-1,029770\alpha+0,406850a}$	$\frac{a\alpha}{+1,484315\alpha+0,047695a}$
secunda	$\frac{a\alpha}{-0,670615\alpha+0,766005a}$	$\frac{a\alpha}{+1,125160\alpha-0,311460a}$
tertia	$\frac{a\alpha}{-0,311460\alpha+1,1125160a}$	$\frac{a\alpha}{0,766005\alpha-0,670615a}$
quarta	$\frac{a\alpha}{0,047695\alpha+1,484315a}$	$\frac{a\alpha}{0,406850\alpha-1,029770a}$

SCHOLION 2

221. Interim tamen si lentes desideremus, in quibus valor ipsius λ maior sit, quam hic est assumtus, constructio earum levi additione ex his ipsis formulis concinnari poterit. In binis scilicet fractionibus, quibus radii facierum cuiusque lentis designantur, alter denominator augeri, alter vero diminui debet eadem quantitate, quae quantitas semper $=\tau\left(a+\alpha\right)\sqrt{v}$, denotarite v excessum valoris ipsius λ supra ante assumtum. Ita si esse debeat

$$\text{I.}\quad \lambda=1+v,\quad \text{II.}\quad \lambda=0,191827+v,\quad \text{III.}\ \lambda=0,042165+v$$

$$\text{vel IV.}\ \lambda=-0,010216+v,$$

denominatores fractionum in scholio praecedente traditarum pro quavis lente simplici alternatim sunt augendi et minuendi quantitate $0,905183(a+\alpha)\sqrt{v}$. Unde si lens quadruplicata desideretur, pro qua sit praecise $\lambda=0$, erit
$v=0,010216$ et $\tau\sqrt{v}=0,091487$; hincque pro quavis lente simplici denominatorum alter augeri, alter vero diminui debet hac quantitate $0,091487\alpha+0,091487a$,
ex quo talis nascitur constructio huiusmodi lentis quadruplicatae, pro qua est $\lambda=0$:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
prima	$\frac{a\alpha}{-1,121267\alpha+0,316363a}$	$\frac{a\alpha}{+1,576802\alpha+0,139182a}$
secunda	$\frac{a\alpha}{-0,762102\alpha+0,674618a}$	$\frac{a\alpha}{+1,216647\alpha-0,219973a}$
tertia	$\frac{a\alpha}{-0,402947\alpha+1,033673a}$	$\frac{a\alpha}{0,857492\alpha-0,579128a}$
quarta	$\frac{a\alpha}{-0,043782\alpha+1,392828a}$	$\frac{a\alpha}{0,488337\alpha-0,938283a}$

Sed hoc casu quaelibet lens adhuc alio modo construi potest: sic quarta ordine retrogrado exposita permutatis a et α dabit alteram formam lentis primae.

SUPPLEMENTUM III

AD PROBLEMA 8

Si lentes ratione refractionis discrepent, ut sit ratio refractionis pro prima lente $=n$, pro secunda $=n'$, pro tertia $=n''$ etc., inde neque in multiplicatione m neque in gradu claritatis quicquam mutatur; at vero in semidiametro confusionis valores litterarum P, Q, R etc. sequenti modo immutari debent:

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$
$$Q=\frac{n'}{2(n'-1)^2}\left(\left(\frac{n'}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{ib}+\frac{2}{k'-v'}\right)^2+\left(\frac{n'}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$
$$R=\frac{n''}{2(n''-1)^2}\left(\left(\frac{n''}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n''}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\alpha}-\frac{2}{k''+v''}\right)^2\right)$$
$$S=\frac{n'''}{2(n-1)^2}\left(\left(\frac{n'''}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n'''}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\alpha}-\frac{2}{k'''+v'''}\right)^2\right)$$

etc.

AD PROBLEMA 9

Si lentes ratione refractionis discrepent, loco litterarum ρ, σ, τ in radiis facierum scribi oportet pro secunda lente ρ', σ', τ', pro tertia ρ'', σ'', τ'' etc.; tum vero expressiones pro multiplicatione et gradu claritatis nulla mutatione egent; pro confusione autem notari oportet fore:

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2+\frac{v}{a\alpha}\right),\quad Q=\mu'\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'(\frac{1}{b}+\frac{1}{\beta})^2+\frac{v'}{b\beta}\right),\quad R=\mu''\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{v''}{c\gamma}\right),\text{ etc.}$$