Chapter V

Concerning the Apparent Field of View, with the Eye in the Most Suitable Place

PROBLEM 1

222. *If some ray εM, taken from a point of the object ε beyond the axis* (Fig. 12) *may be transmitted through the lens PP, to define its crossing O with the axes .*

SOLUTION

Let Fζ be the principal image projected through the lens, indeed here to be considered free from spreading, and we may put the determinable distances for the lens to be

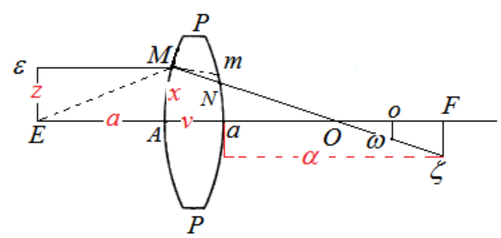

Fig. 12

$AE = a,\ aF = \alpha,$ the thickness of which $Aa = v$ and the arbitrary quantity $= k$, and for brevity there shall be put $\frac{k-v}{k+v} = i$. With these in place, if the image of the point ε may fall at ζ, and there may be called $E\varepsilon = z$, there will become $F\zeta = \frac{1}{i}\cdot\frac{\alpha z}{a}$; but for the point of the lens *M,* its distance from the axis $AM = x$; and we have shown above [See Ch. 1, Problem 5], if a ray may be transmitted from the point E through the point M, thus that to be progressing through *m* to the point *F*, so that there shall become $am = ix$ [*i.e.* for paraxial rays, the ratio $\frac{am}{AM} = i = \frac{k-v}{k+v}$]. Therefore the ray εM will be brought through the point *N* to ζ , thus so that, with the obliqueness of the rays incident on the faces of the lens always established to be very small, the angle $EM\varepsilon$ shall be to the angle *NMm* approximately as *n* to 1, on putting $n = \frac{31}{20}$. Hence $E\varepsilon$ [or *AM*] to *mN* will be in the ratio composed of these same angles and distances *AE* to *Aa,* or $E\varepsilon :\ mN = n\cdot a : v$, [thus, approx., $EMe \sim z : a;\ \ NMm \sim mN : v$] from which there becomes $mN = \frac{vz}{n\cdot a}$ and thus $aN = ix - \frac{vz}{n\cdot a}$. Now truly the ray from the point *N* proceeds straight to ζ and therefore will cut the axis at *O*, so that there shall be $aN + F\zeta : aF = aN : aO$, [*i.e.* $aO = aN\cdot aF : aN + F\zeta$]

$$aO = \left(i\alpha x - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{niia\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

and

$$FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}.$$

If x shall be the radius of the aperture for the anterior face of the lens, this intersection O will correspond to the case, in which the ray from ε may pass through the upper limit of the lens: but if this may be transmitted through the lowest, by taking x negative there will become

$$FO = \frac{n\alpha\alpha z}{-niiax - ivz + n\alpha z}.$$

But if the ray from ε may pass through the centre of the lens A, the point of intersection thus may fall at O, so that there shall be

$$FO = \frac{n\alpha\alpha}{n\alpha - iv}.$$

COROLLARY 1

223. Therefore if x may denote the radius of the aperture of the lens on the anterior face *PMAP,* so that all the rays incident on this face from the point ε may be transmitted through the lens, it is necessary, that the radius of the posterior face *PNaP* shall be greater than $\pm ix - \frac{vz}{na}$, with x taken both negative as well as positive. From which this radius cannot be smaller than $ix + \frac{vz}{n\alpha}$.

COROLLARY 2

224. If the aperture on the anterior face may vanish, so that there shall be $x = 0$, the rays from the point ε, whose distance from the axis $E\varepsilon = z$, will not be transmitted by the lens, unless the radius in the posterior face shall be $= \frac{vz}{na}$ or greater. From which it appears, where the thickness of the lens v were greater, there is a need for a greater aperture in the posterior face.

COROLLARY 3

225. Therefore in turn if the aperture of the posterior lens may be given, of which the radius shall be $= \frac{vz}{na}$, then likewise the extreme point ε in the object is determined, by which the ray falling on the centre of the lens A is transmitted.

COROLLARY 4

226. But this ray passing through A may occur after crossing the axis at O, so that on account of $x = 0$, the interval shall be $FO = \frac{n\alpha\alpha}{n\alpha - iv}$ or $aO = \frac{-i\alpha v}{n\alpha - iv}$. Therefore unless the

eye may be held in place on this axis, the transmitted ray will not enter into the eye ; certainly if the aperture of the pupil may be regarded as indefinitely small.

COROLLARY 5

227. But if the radius of the pupil may be put $=\omega$, also the eye placed at o will receive that ray, if there were $o\omega=\omega$. But since in the case $x=0$ there shall be

$$aN\left(\tfrac{vz}{na}\right):aO\left(\tfrac{iav}{n\alpha-iv}\right)=o\omega\left(\omega\right):Oo,\quad \text{there will be } Oo=\left(\tfrac{ni\alpha\alpha\omega}{z(n\alpha-iv)}\right).$$

Since which interval may be able to be taken equally positive or negative, for the position of the eye o we will have

$$ao=-\tfrac{i\alpha vz}{z(n\alpha-iv)}=\tfrac{-i\alpha(vz\pm na\omega)}{z(n\alpha-iv)}.$$

SCHOLlUM

228. In this treatment, where we investigate the apparent field and suitable place of the eye, we will take both the aperture of the objective lens in the anterior face as well as the size of the pupil as zero, so that we may obtain these bounded questions. Whereby we will put in place the following definitions of these elements, which are of great value in dioptric instruments for the greatest convenience in viewing.

DEFINITION 1

229. *The apparent field is the space within an object, from the individual points of which the rays incident on the centre of the objective lens are transmitted through all the remaining lenses. The radius of which interval, since it shall be circular, may be called the radius of the apparent field.* [: *here to be called the apparent field of view*]

COROLLARY 1

230. Therefore if $E\varepsilon=z$ were the radius of the apparent field, ε will be an extreme point of the object or maximally distant from the axis, from which at least the rays incident on the centre of the objective lens A are transmitted by all the lenses.

COROLLARY 2

231. Therefore the magnitude of the apparent field is determined by the refraction through the apertures of the following faces and perhaps by the aperture of one only, if evidently none of the rays arising from some point more distant than ε are transmitted, even if they will be transmitted by the remaining faces.

SCHOLIUM

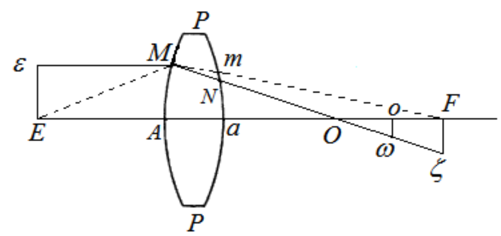

Fig. 12 again

232. If the rays may be considered from the single point E, which is the centre of the apparent field, certainly these will be transmitted which are incident at A, since they are progressing along the axis always, however small the aperture of these will have been ; and hence the apparent field at no time can vanish completely. But where the point ε may be taken more distant from the axis, so that the rays from that may be transmitted by all the faces, there a greater aperture is required for these; which since it may depend on the curvature of these and may not exceed a certain limit, hence even now the final point ε, from which the rays are transmitted, and therefore the radius of the apparent field may be determined. Indeed in the following propositions I may assume the apparent field or its radius $E\varepsilon = z$ as given, and I will investigate how great each face must become : for hence easily in turn, if each aperture were known, the apparent field itself will be allowed to be defined. Further in this definition I have assumed the aperture of the first face to be vanishing: from which it is readily understood with that increased it will be required to extend each part of the field ; truly after this increase nothing further is to be gained, which since it shall never amount to anything, I am not going to consider its account here; just as also in the following definition I will not have an account of the aperture of the pupil.

DEFINITION 2

233. *A suitable position of the eye is that point on the axis, at which the rays from the extremity of the apparent field transmitted by the lenses pass through the axis. Clearly the eye established in this place will observe the whole apparent field of view.*

COROLLARY 1

234. Therefore a suitable place for the eye hence will be assigned, if that intersection of the extreme rays with the axis may fall after the final lens ; but if this intersection may be found before the final lens, it cannot happen, as the eye held in that place therefore will be unable to consider the whole field of view.

COROLLARY 2

235. But if that intersection may fall behind the final lens, the eye will observe the whole apparent field of view, even if the pupil may be maximally contracted; nor yet on account of a greater pupil amplitude will a greater field of view prevail.

COROLLARY 3

236. Truly on account of the convenient amplitude of the pupil, we understand that the eye, even if it may be put placed beyond the most suitable place, provided the distance may not be too great, may yet be able to observe the whole apparent field of view : that which may arise in use especially in these cases, in which the most suitable place of the eye falls before the final refracting face. For then it will be able to happen, so that the eye applied next to this face still may perceive the whole field of view.

SCHOLIUM

237. When I speak about this vision, so that in general it is required to be understood, so that a ray may enter the eye from a point seen, should I care here if the vision shall be distinct or not ? Indeed in the following it will be shown, how lenses may be agreed to be set out, so that the eye put into the most suitable place also the true distance from the final distinct may be found [*i.e.* the near-point]. Therefore this I have without any regard for distinct vision when I assign the eye in place, where it may receive the rays from all the points contained in the apparent field of view ; and since it is agreed to set out the individual matters of interest separately, which pertain to vision, also here I do not consider the diffusion length, which indeed always vanish by themselves, if the aperture of the first face may be made vanishing.

PROBLEM 2

238. *If the object may be viewed through a single lens, to determine both the apparent field of view as well as the ideal place of the eye.*

SOLUTION

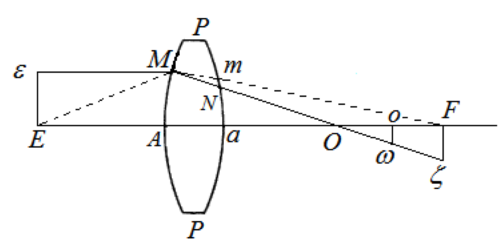

Fig. 12 again

As in the above problem the determinable distances of this lens shall be, evidently the distance of the object before the lens $AE = a$ (Fig.12) and of the image after the lens $aF = \alpha$, then truly the thickness of the lens $Aa = v$ and the arbitrary distance determining the structure of the lens completely $= k$. The we may put the radius of the apparent field of view $E\varepsilon = z$, thus so that with the aperture of the anterior face PAP put infinitely small even if now rays may be transmitted through the lens from the point ε. For the sake of brevity there shall be : $\frac{k-v}{k+v} = i$, and we have shown above the apparent field of view to be extended as far as to ε, if for the posterior face PaP the radius of the aperture were $aN = \frac{vz}{na}$ with the index of refraction being $n = \frac{31}{20}$; likewise indeed, this radius may be taken either positive or negative. Hence therefore in turn, if the radius of this aperture may be put $= \mathfrak{a}$, the radius of the apparent field of view will be $E\varepsilon = z = \frac{na}{v}\mathfrak{a}$.

So that the most suitable position of the eye may be attained, which shall be at O, since we have found $FO = \frac{n\alpha\alpha}{n\alpha - iv}$, the distance will be $aO = \frac{-i\alpha v}{n\alpha - iv}$ and thus with that negative, unless either i shall be a negative number, or $iv > n\alpha$. But if this distance aO were positive, the eye placed at O will observe the whole apparent field of view.

COROLLARY 1

239. If the thickness of the lens v may vanish, on account of $z = \frac{na}{v}\mathfrak{a}$ the apparent field of view becomes infinite or rather indeterminate; truly the distance aO will vanish. Therefore the eye applied immediately to the lens will observe a space of such a size as will prevail according to its individual nature.

COROLLARY 2

240. But if from the thickness of the lens $Aa = v$ the distance $aO = \frac{-i\alpha v}{n\alpha - iv}$ may be produced positive, the eye put at O will prevail to observe the whole apparent field of view or it will look at an object in the circular space, of which the radius

$$E\varepsilon = z = \frac{na}{v}\mathfrak{a} = \frac{31a\mathfrak{a}}{20v},$$

which therefore will be greater thus, where the lens were thinner.

COROLLARY 3

241. But if the distance aO may become negative, it will be unable to find a suitable position to place the eye, since this by necessity must be put after the last lens. But wherever that may be placed after the lens, it will not contain the whole apparent field of

view, but only a part of this, and indeed with that smaller, where it may be removed further beyond the lens, therefore so that in this way it departs more from a suitable location.

COROLLARY 4

242. Therefore in this case it will be agreed to apply the eye immediately to the posterior face of the lens, with which in place at this point it will accept only rays, as far as the pupil extends out; and thus the field of view will depend on the aperture of the pupil ; which if it may be zero, also the apparent field of view may vanish.

COROLLARY 5

243. Hence it is apparent, if the pupil may exceed the aperture of the face *PaP* or if there shall be $\omega > \mathfrak{a}$, with ω denoting the radius of the pupil, since then the eye applied to this face receives all the rays transmitted, that whole apparent field of view is going to be seen. But if there shall be $\omega < \mathfrak{a}$, only a part of the whole field may be seen, of which the radius will be $= \frac{na}{v}\omega = \frac{31a\omega}{20v}$, evidently the same, if the aperture of the face *PaP* shall be equal to that of the pupil.

SCHOLIUM

244. Yet meanwhile in this latter case if the aperture of the face of the lens *PaP,* to which the eye is applied, may be greater than the pupil, nothing stands in the way, whereby that may traverse successively the whole aperture, and thus step by step the whole apparent field of view will be able to be observed, even if it may not prevail to examine that at the same time. Moreover it is to be observed, if also the anterior face of the aperture may be assigned, thence the apparent field of view to be increased a little, but the more distant parts of the object, since they will not be transmitted by the medium of the lens, will remain more obscure, from which deservedly they may be excluded from the apparent field of view. But if the aperture of the anterior face may be given, of which the radius $= x$, so that it may transmit all the rays incident on that posterior face, its aperture must be so much greater according to the rules treated above; clearly its radius will be required to be $= ix + \mathfrak{a}$ or $= ix + \frac{vz}{na}$.

PROBLEM 3

245. *If a dioptric instrument may be composed of two lenses, to define the apparent field of view and a suitable place of the eye.*

SOLUTION

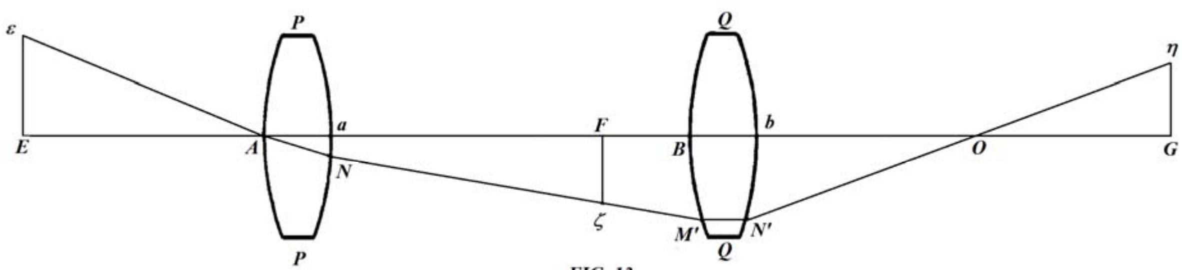

FIG. 13.

For these lenses there shall be as follows (Fig. 13)

$$\text{for } PP: \ AE = a,\ aF = \alpha,\ Aa = v; \text{ the arbitrary dist.} = k \text{ and } \tfrac{k-v}{k+v} = i$$

$$\text{for } QQ: BF = b,\ bG = \beta,\ Bb = v'; \text{ the arbitrary dist.} = k' \text{ and } \tfrac{k'-v'}{k'+v'} = i',$$

and, on putting the radius of the apparent field of view $E\varepsilon = z$, for the images there will be

$$F\zeta = \tfrac{1}{i}\cdot\tfrac{\alpha}{a}z \text{ and } G\eta = \tfrac{1}{ii'}\cdot\tfrac{\alpha\beta}{ab}z .$$

Now a ray may be considered from the point z incident at the centre of the first lens A and transmitted by the lenses, which certainly will pass through the extremities of the images ζ and η. Now from the preceding problem it is apparent to be $aN = \frac{vz}{2a}$, from which the point M' thus will be defined in the other lens, so that there shall be

$$F\zeta - aN : aF = BM' - F\zeta : BF$$

or

$$BM' = F\zeta + \frac{BF(F\zeta - aN)}{aF} = \frac{aB\cdot F\zeta - BF\cdot aN}{aF},$$

from which there becomes

$$BM' = \tfrac{1}{i}\cdot\tfrac{\alpha+b}{a}z - \tfrac{bvz}{na\alpha}.$$

Now the point N' hence may be defined likewise, and from the first problem , the point N will be determined from the point M *;* evidently there will be

$$bN' = i' \cdot BM' - \tfrac{v'}{n'b} \cdot F\zeta \text{ and thus } bN' = \tfrac{i'}{i} \cdot \tfrac{\alpha+b}{a} z - \tfrac{i'bv}{na\alpha} z - \tfrac{1}{i} \cdot \tfrac{\alpha v'}{n'ab} z.$$

Hence moreover the point O, where the most suitable point is located, will be assigned easily; for there will be

$$bN' + G\eta : bG = bN' : bO, \text{ and thus}$$

$$bO = \frac{bG \cdot bN'}{bN' + G\eta} = \frac{\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}}{\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab} + \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab}} \beta$$

So that if therefore we may put the radius of the opening

$$\text{for the lens } PP : \begin{cases} \text{anterior face} = \mathfrak{A} = x \\ \text{posterior face} = \mathfrak{a} \end{cases}$$

$$\text{for the lens } QQ : \begin{cases} \text{anterior face} = \mathfrak{B} \\ \text{posterior face} = \mathfrak{b}, \end{cases}$$

we will have

$$\mathfrak{A} = 0, \quad \mathfrak{B} = \tfrac{v}{na} z; \quad \mathfrak{B} = \left(\tfrac{1}{i} \cdot \tfrac{\alpha+b}{a} - \tfrac{bv}{na\alpha}\right) z, \quad \mathfrak{b} = \left(\tfrac{i'}{i} \cdot \tfrac{\alpha+b}{a} - \tfrac{i'bv}{na\alpha} - \tfrac{1}{i} \cdot \tfrac{\alpha v'}{n'ab}\right) z;$$

and if the distance of the eye after the lens QQ may be put $BO = O$, there will be:

$$O = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z} \cdot \beta .$$

COROLLARY 1

246. If the thicknesses of both the lenses may vanish, there will be $v = 0$, $v' = 0$ and $i = i' = 1$; therefore in which case the following will be changed in our formulas:

$$\mathfrak{A} = 0, \quad \mathfrak{a} = 0, \quad \mathfrak{B} = \tfrac{\alpha+b}{a} z \text{ and } \mathfrak{b} = \tfrac{\alpha+b}{a} z .$$

COROLLARY 2

247. Therefore in turn from the given treatment apertures of the lenses from that equation, for which the quantity z comes upon a minimum value, the apparent field of view may be defined.

COROLLARY 3

248. Therefore if the thickness of the lens may vanish, the apparent field from the aperture of the posterior lens may be determined easily. Indeed there will become $z=\frac{a\mathfrak{B}}{a+b}$; that which is required to be understood, if the distance $BO=O$ were positive and the eye may be located at O.

COROLLARY 4

249. But if the distance $bO=O$ may be produced negative and the eye may be applied at once to the final lens, then its aperture may not be greater than if it may be equal to the size of the pupil. Whereby if $\mathfrak{b}$ were greater than the radius of the pupil ω, in place of $\mathfrak{b}$ there may be written ω, and the value of z truly may be deduced from the final equation, unless perhaps yet from some of the remaining equations a smaller value may be going to be produced for z.

PROBLEM 4

250. *If a dioptric instrument may be constructed from three lenses, to determine both the apparent field of view as well as a suitable place for the eye.*

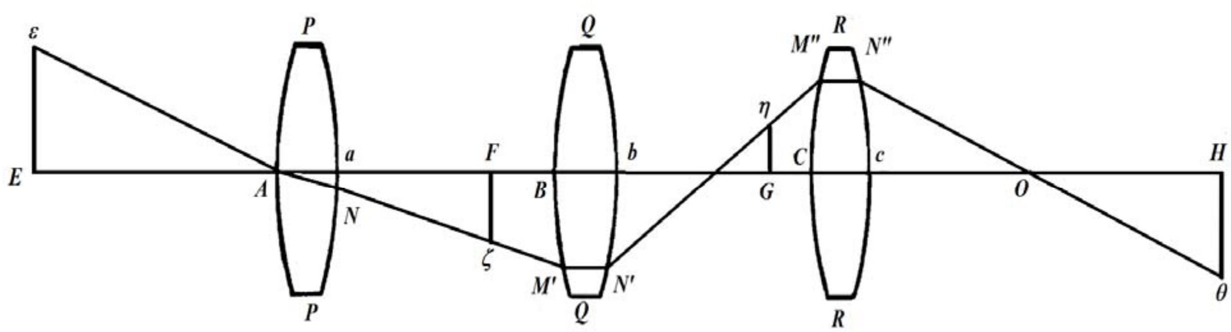

Fig. 14.

SOLUTION

With the images presented successively by these lenses (Fig. 14) represented by $F\zeta$, $G\eta$ and $H\theta$, truly with the object itself at $E\varepsilon$, we may put as follows :

	distances	thickness	arbitrary distance	and
for first lens PP	$AE = a,\ aF = \alpha$	$Aa = v$	k	$\frac{k-v}{k+v} = i$
for second lens QQ	$BF = b,\ bG = \beta$	$Bb = v'$	k'	$\frac{k'-v'}{k'+v'} = i'$
for third lens RR	$CG = c,\ cH = \gamma$	$Cc = v''$	k''	$\frac{k''-v''}{k''+v''} = i''$

truly the radii of the apertures

for the lens

$$PP \quad \begin{cases} \text{anterior face} = \mathfrak{A} = 0 \\ \text{posterior face} = \mathfrak{a} \end{cases}$$

$$QQ \quad \begin{cases} \text{anterior face} = \mathfrak{B} \\ \text{posterior face} = \mathfrak{b}, \end{cases}$$

$$RR \quad \begin{cases} \text{anterior face} = \mathfrak{C} \\ \text{posterior face} = \mathfrak{c}. \end{cases}$$

Then truly the radius of the apparent field shall become $E\varepsilon = z$, and we have shown above to become :

$$F\zeta = \frac{1}{i}\cdot\frac{\alpha}{a}z,\ \ G\eta = \frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z,\ \ H\theta = \frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z\,.$$

With these in place we will have $aN = \mathfrak{a} = \frac{vz}{na}$; from which, if it may be understood a right line may be drawn through F parallel to this to NM', there will be

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ or } BM' = \frac{aB\cdot F\zeta - BF\cdot aN}{aF}$$

and thus

$$BM' = \frac{\alpha+B}{\alpha}\cdot\frac{1}{i}\frac{\alpha}{a}z - \frac{bvz}{na\alpha} \quad \text{or} \quad \mathfrak{B} = \frac{1}{i}\cdot\frac{\alpha+b}{a}z - \frac{bv}{na\alpha}z\,.$$

Truly again from the first problem, there is $bN' = i'\cdot BM' - \frac{v}{nb}\cdot F\zeta$, and hence

$$\mathfrak{b} = \frac{i'}{i}\cdot\frac{\alpha+b}{a}z - \frac{i'bv}{na\alpha}z - \frac{1}{i}\cdot\frac{\alpha v'}{nab}z\,;$$

in the same way it may be understood a right line may be drawn through G parallel to this to $N'M''$, and there will become

$$bG : bN' + G\eta = CG : CM'' - G\eta$$

or

$$CM'' = \mathfrak{C} = G\eta + \tfrac{c}{\beta}(\mathfrak{b} + G\eta) = \tfrac{\beta+c}{\beta}\cdot G\eta + \tfrac{c}{\beta}\mathfrak{b},$$

so that there shall become

$$\mathfrak{C} = \tfrac{1}{ii'}\cdot\tfrac{\alpha(\beta+c)}{ab}z + \tfrac{i'}{i}\cdot\tfrac{c(\alpha+b)}{a\beta}z - \tfrac{i'bcv}{na\alpha\beta}z - \tfrac{1}{i}\cdot\tfrac{\alpha cv'}{nab\beta}z.$$

Finally form CM'' thus cN'' may be defined by the previous problem, so that there shall become:

$$cN'' = i''\cdot CM'' - \tfrac{v''}{nc}\cdot G\eta \;\text{ or }\; \mathfrak{c} = i''\cdot\mathfrak{C} - \tfrac{1}{ii''}\cdot\tfrac{\alpha\beta v''}{nabc}z,$$

and hence

$$\mathfrak{c} = \tfrac{i''}{ii'}\cdot\tfrac{\alpha(\beta+c)}{ab}z + \tfrac{i'i''}{i}\cdot\tfrac{c(\alpha+b)}{a\beta}z - i'i''\tfrac{bcv}{na\alpha\beta}z - \tfrac{i''}{i}\cdot\tfrac{\alpha cv'}{nab\beta}z - \tfrac{1}{ii'}\cdot\tfrac{\alpha\beta v''}{nabc}z.$$

Truly from these the following values are determined :

$$\mathfrak{A} = 0,\;\; \mathfrak{a} = \tfrac{v}{na}\cdot E\varepsilon;\;\; \mathfrak{B} = \tfrac{\alpha+b}{\alpha}\cdot F\zeta - \tfrac{b}{\alpha}\cdot\mathfrak{a},\;\; \mathfrak{b} = i'\cdot\mathfrak{B} - \tfrac{v'}{nB}\cdot F\zeta;$$

$$\mathfrak{C} = \tfrac{\beta+c}{\beta}\cdot G\eta - \tfrac{c}{\beta}\cdot\mathfrak{b},\;\; \mathfrak{c} = i''\cdot\mathfrak{C} - \tfrac{v''}{nc}\cdot G\eta.$$

Now since the point O may become the true point of the eye, if we may put $cO = O$, there will become $cN'' + H\theta : cH = cN'' : cO$, from which there may be provided

$$O = \tfrac{\gamma c}{c+H\theta} \;\text{ or }\; \tfrac{1}{O} = \tfrac{1}{\gamma} + \tfrac{1}{ii'i''}\cdot\tfrac{\alpha\beta}{ab}\cdot\tfrac{z}{c\mathfrak{c}}.$$

COROLLARY 1

251. From these equations there follows to become:

$$\tfrac{\mathfrak{B}}{b} + \tfrac{a}{\alpha} = \tfrac{\alpha+b}{\alpha b}\cdot F\zeta = \tfrac{1}{i}\cdot\left(\tfrac{\alpha}{b}+1\right)\tfrac{z}{a} \;\text{ and }\; \tfrac{\mathfrak{C}}{c} - \tfrac{\mathfrak{b}}{\beta} = \tfrac{\beta+c}{\beta c}\cdot G\eta = \tfrac{1}{ii'}\cdot\left(\tfrac{\alpha\beta}{bc} + \tfrac{\alpha}{b}\right)\tfrac{z}{a};$$

and hence again :

$$\tfrac{i\mathfrak{a}}{\alpha} + \tfrac{i\mathfrak{B}}{b} + \tfrac{ii'\mathfrak{b}}{\beta} - \tfrac{ii'\mathfrak{C}}{c} = \left(1 - \tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}.$$

COROLLARY 2

252. Therefore if the thickness of the lens may vanish, since there shall become $\mathfrak{a} = 0$, $\mathfrak{b} = \mathfrak{B}$ and $\mathfrak{c} = \mathfrak{C}$, the definition of the apparent field of view is reduced to these two equations :

$$\text{I.)}\ \mathfrak{B}\cdot\tfrac{1}{b}=\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}.\quad \text{II;)}\ \mathfrak{B}\cdot\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)-\mathfrak{C}\cdot\tfrac{1}{c}=(1-\tfrac{\alpha\beta}{bc})\tfrac{z}{a},$$

from which the smaller value of z provides the radius of the apparent field of view.

COROLLARY 3

253. Finally if the distance $cO=O$ may be produced negative, so that we may consider the eye placed immediately to the final lens, for $\mathfrak{c}$ it will be required to write the radius of the pupil x, and from the final equation the radius of the field of view may be defined z, unless perhaps a smaller value of z may be produced from another equation.

PROBLEM 5

254. *If a dioptric instrument may be constructed from four lenses set out on the same axis, to determine both the apparent field of view as well as a suitable place of the eye.*

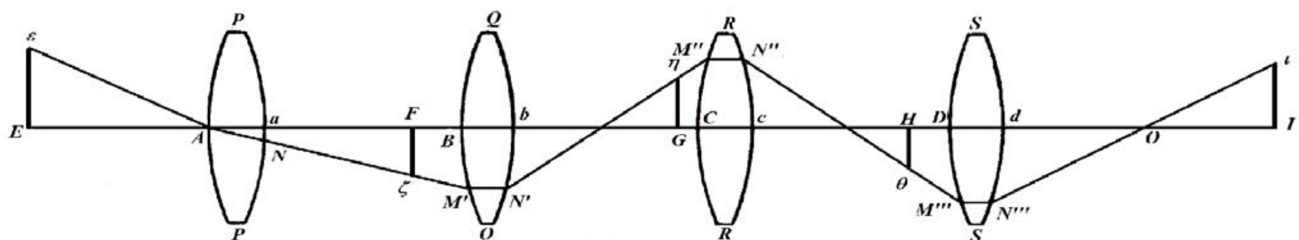

Fig. 15.

SOLUTION

With the object $E\varepsilon=z$ being present, the images presented by the successive lenses shall be $F\zeta$, $G\eta$, $H\theta$ and Ii, and we may put for the determination of the individual lenses this far as :

	distances	thickness	arbitrary distance	and
first lens *PP*	$AE=a,\ aF=\alpha$	$Aa=v$	k	$\frac{k-v}{k+v}=i$
second lens *QQ*	$BF=b,\ bG=\beta$	$Bb=v'$	k'	$\frac{k'-v'}{k'+v'}=i'$
third lens *RR*	$CG=c,\ cH=\gamma$	$Cc=v''$	k''	$\frac{k''-v''}{k''+v''}=i''$
fourth lens *SS*	$DH=d,\ dI=\delta$	$Dd=v'''$	v'''	$\frac{k'''-v'''}{k'''+v'''}=i'''$

Truly the radii of the apertures of the faces shall be :

first lens PP $\begin{cases}\text{anterior face} = \mathfrak{A} = 0\\ \text{posterior face} = \mathfrak{a}\end{cases}$

second lens QQ $\begin{cases}\text{anterior face} = \mathfrak{B}\\ \text{posterior face} = \mathfrak{b},\end{cases}$

third lens RR $\begin{cases}\text{anterior face} = \mathfrak{C}\\ \text{posterior face} = \mathfrak{c},\end{cases}$

and of the
fourth lens SS $\begin{cases}\text{anterior face} = \mathfrak{D}\\ \text{posterior face} = \mathfrak{d}.\end{cases}$

If now $E\varepsilon = z$ may show the radius of the apparent field of view, there will be, as now we have shown above:

$$F\zeta = \tfrac{1}{i}\cdot\tfrac{\alpha}{a}z,\ \ G\eta = \tfrac{1}{ii'}\cdot\tfrac{\alpha\beta}{ab}z,\ \ H\theta = \tfrac{1}{ii'i''}\cdot\tfrac{\alpha\beta\gamma}{abc}z \text{ and } Ii = \tfrac{1}{ii'i''i'''}\cdot\tfrac{\alpha\beta\gamma\delta}{abcd}z.$$

So that if now we may set up the reasoning as before, we will obtain the following equations :

$$\mathfrak{A} = 0, \qquad \mathfrak{a} = \tfrac{\nu}{na}\cdot E\varepsilon;$$
$$\mathfrak{B} = \left(1+\tfrac{b}{\alpha}\right)\cdot F\zeta - \tfrac{b}{\alpha}\cdot\mathfrak{a}, \quad \mathfrak{b} = i'\cdot\mathfrak{B} - \tfrac{\nu'}{nB}\cdot F\zeta;$$
$$\mathfrak{C} = \left(1+\tfrac{c}{\beta}\right)G\eta + \tfrac{c}{\beta}\cdot\mathfrak{b}, \quad \mathfrak{c} = i''\cdot\mathfrak{C} - \tfrac{\nu''}{nc}\cdot G\eta$$
$$\mathfrak{D} = \left(1+\tfrac{d}{\gamma}\right)H\theta + \tfrac{d}{\gamma}\cdot\mathfrak{c}, \quad \mathfrak{d} = i'''\cdot\mathfrak{D} - \tfrac{\nu'''}{nd}\cdot H\theta.$$

From the initial order of which there follows :

$$\tfrac{\mathfrak{B}}{b} + \tfrac{\mathfrak{a}}{\alpha} = \left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)F\zeta = \tfrac{1}{i}\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{C}}{c} - \tfrac{\mathfrak{b}}{\beta} = \left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)G\eta = \tfrac{1}{ii'}\left(\tfrac{\alpha}{b}+\tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{D}}{d} - \tfrac{\mathfrak{c}}{\gamma} = \left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)H\theta = \tfrac{1}{ii'i''}\left(\tfrac{\alpha\beta}{bc}+\tfrac{\alpha\beta\gamma}{bcd}\right)\tfrac{z}{a},$$

and truly from the latter order:

$$\mathfrak{a} = \frac{vz}{na}, \quad \mathfrak{b} = i'\mathfrak{B} - \frac{1}{i} \cdot \frac{\alpha}{b} \cdot \frac{v'z}{na}, \quad \mathfrak{c} = i''\mathfrak{C} - \frac{1}{ii'} \cdot \frac{\alpha\beta}{bc} \cdot \frac{v''z}{na},$$

$$\mathfrak{d} = i'''\mathfrak{D} - \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{bcd} \cdot \frac{v'''z}{na}.$$

And finally if we may put $dO = O$ for the position of the eye, there will be

$$O = \frac{\delta\mathfrak{d}}{\mathfrak{d}+I\iota} \quad \text{or} \quad \frac{1}{O} = \frac{1}{\delta} + \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{db}.$$

COROLLARY 1

255. From the first equations we may deduce the following :

$$\frac{\mathrm{i}\mathfrak{a}}{\alpha} + \frac{\mathrm{i}\mathfrak{B}}{b} = \left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{\mathrm{i}\mathfrak{a}}{\alpha} + \frac{\mathrm{i}\mathfrak{B}}{b} + \frac{\mathrm{ii}'\mathfrak{b}}{\beta} - \frac{\mathrm{ii}'\mathfrak{C}}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{\mathrm{i}\mathfrak{a}}{\alpha} + \frac{\mathrm{i}\mathfrak{B}}{b} + \frac{\mathrm{ii}'\mathfrak{b}}{\beta} - \frac{\mathrm{ii}'\mathfrak{C}}{c} - \frac{\mathrm{ii}'\mathrm{i}''\mathfrak{c}}{\gamma} + \frac{\mathrm{ii}'\mathrm{i}''\mathfrak{D}}{d} = \left(1 + \frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a};$$

from which is easy to see, how these may be continued to more lenses.

COROLLARY 2

256. If the thickness of the lens may vanish, there becomes $\mathfrak{a} = 0$, $\mathfrak{b} = \mathfrak{B}$, $\mathfrak{c} = \mathfrak{C}$, et $\mathfrak{d} = \mathfrak{D}$, again $i = i' = i'' = i''' = 1$, from which these equations will be changed into the following forms :

$$\frac{\mathfrak{B}}{b} = \left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b} + \frac{1}{\beta}\right) - \mathfrak{C}\frac{1}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b} + \frac{1}{\beta}\right) - \mathfrak{C}\left(\frac{1}{c} + \frac{1}{\gamma}\right) + \mathfrak{D}\frac{1}{d} = \left(1 + \frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a};$$

from which three equations, as in general, the value of *z,* which will have produced the minimum, truly will provide the radius of the apparent field of view.

COROLLARY 3

257. This whole apparent field of view actually will be seen with the eye put in place at the point *O*, provided that the distance $dO = O$ were positive. But if that shall be negative and the eye may be placed immediately after the final lens *SS*, there may be put $\mathfrak{d} = \omega$,

evidently the radius of the pupil, and from the final equation the radius may be deduced of the space actually to be viewed within the object .

SCHOLIUM 1

258. Hence therefore it may be observed, how the apparent field of view may depend on the apertures of the individual lenses ; and likewise it is apparent, how great the aperture of each individual lens must be, in order that the apparent field of view of the given magnitude may be obtained. For if the quantity z may be assumed as given, with the quantities pertaining to the determination of the lenses, will be defined by the successive formulas of the radii of the apertures for the individual lenses : where indeed henceforth it is required to be seen clearly, whether or not lenses of such apertures shall be large enough. Hence evidently the limits of the apparent field of view are defined, which may not be allowed to cross ; from which it follows a greater apparent field of view cannot be assumed, than the apertures allowed arising thence from the individual lenses. Moreover from these definitions it is evident, either to each lens itself, which was found there, an aperture may be attributed, to be either greater, while it may not be smaller, whenever here we may assume the aperture of the objective lens to be vanishing. Truly in addition an account of the clarity may be had, it is necessary, that the true aperture of each lens that, as we have assigned here, must become a little greater, and indeed by that amount, as we have shown above for the limits on account of the clarity required ; for unless this increase may be added, the extremities in the apparent field of view will be provided in a lesser light than normal. But then the apparent field of view will extend out more widely and the enclosing boundary will be more obscure; on account of which if we wish the boundaries around the extremities to be satisfied by a smaller light, indeed there is no need, that a greater aperture may be attributed to the lenses, than is defined by some of our formulas; and it would be superfluous for the apertures to be increased beyond these limits, thus so that hence for each lens a convenient aperture may be put in place.

SCHOLIUM 2

259. Even for the case, where the thickness of the lens is ignored, our formulas may not emerge more simply, as nevertheless usually arise in these most conveniently, as the coefficients of the letters $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, clearly $\frac{1}{b}+\frac{1}{\beta}$, $\frac{1}{c}+\frac{1}{\gamma}$, $\frac{1}{d}+\frac{1}{\delta}$ involving this same focal distance of each lens; but in practice the aperture is accustomed to be deduced well enough from the distance of the focus. For if the distance of the lens QQ from the focus may be put $=q$, there will become $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$ and thus $\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)=\frac{\mathfrak{B}}{q}$; and lest the arc in the aperture may be taken exceedingly large, it is necessary that there shall be $\mathfrak{B}<\frac{1}{2}q$; and for the

various forms of the lens the value of the fraction $\frac{\mathfrak{B}}{q}$ must be diminished as far as to $\frac{1}{4}$ or $\frac{1}{6}$. Whereby if we may put

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)=\pi,\ \ \mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)=\pi',\ \ \mathfrak{D}\left(\frac{1}{d}+\frac{1}{\delta}\right)=\pi'',$$

these letters $\pi,\ \pi',\ \pi''$ will denote fractions of this kind, of which the value at most will be either $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$; only this is required as a precaution, lest an exceedingly large value may be attributed to these letters. With which observed since the equations are left with our arbitrary numbers, it will be especially appropriate that these same letters be introduced into the calculation and the everything else to be determined from these ; indeed with the benefit of these the apparent field of view is defined easily. So that also the apparent field of view will be able to be introduced into the calculation, certainly its determination therefore may be set out by the most simple formula. This finally, so that the whole investigation may be reduced to pure numbers, I may put $\frac{z}{a}=\Phi$, thus so that Φ shall be the angle, under which the radius of the apparent field of view apparent from the eye may be viewed from the position of the objective lens. Therefore we may see, how the remaining quantities may be defined by these numbers $\pi,\ \pi',\ \pi''$ etc. and Φ.

DEFINITION 3

260. *The quotient which arises, if the radius of the aperture of any lens of which the thickness may be had as zero, may be divided by the focal length of the lens, will be called by me the ratio of the aperture.*

COROLLARY 1

261. Thus if b and β shall be the determinable distances of the lens [which of course we now call the object and image distances from a thin lens] and $\mathfrak{B}$ the radius of the same aperture, since the focal length is $=\frac{b\beta}{b+\beta}$, the ratio of the aperture will be $=\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)$.

COROLLARY 2

262. Therefore the ratio of the aperture of any lens is a fraction less than $\frac{1}{2}$, since we have established this rule, that neither face may contain an arc greater than 60^0.

COROLLARY 3

263. Evidently if both faces were of equal curvature, the ratio of the aperture will be able to increase as far as to $\frac{1}{2}$; but if either face were plane, the ratio of the aperture will hardly be able to exceed $\frac{1}{4}$; and if shall be a meniscus, this certainly will have to be made smaller.

COROLLARY 4

264. But when there shall be nothing, by which the aperture of the lens may be defined more accurately, that is left almost to our choice and in any case is determined most conveniently by experience, and it may suffice for that to be noted either by the fraction $\frac{1}{3}$, $\frac{1}{4}$, or even $\frac{1}{5}$ for an equal form of lens must be put in place.

SCHOLION

265. So that clearly in whatever case the ratio of the aperture may be defined correctly, it will be required to consider the radius of each face of the lens, which if they were *f* and *g*, the focal length will be $=\frac{fg}{(n-1)(f+g)}=\frac{20}{11}\cdot\frac{fg}{(f+g)}$ [*i.e.* the lens maker's formula for a thin lens, where Euler considers only a crown glass with this refractive index]. Now the radius of the aperture must be less than $\frac{1}{2}f$ or than $\frac{1}{2}g$, just as either *f* or *g* were smaller. There shall be $g<f$, and since the radius of the aperture must be smaller than $\frac{1}{2}f$, the ratio of the aperture is required to be taken smaller than $\frac{11}{40}(1+\frac{g}{f})$. From which it is apparent, if the lens shall be equally convex on both sides or $g=f$, the ratio of the aperture must be taken less than $\frac{11}{20}$; but if either face shall be plane or $f=\infty$, the other limited to become $\frac{11}{40}$, which actually may become smaller, if the lens shall be meniscus or $\frac{g}{f}$ a negative number. Finally if the ratio of the aperture shall be $=\pi$ and to that a suitable value may be given in this way, likewise it is the case, this must be taken either negative or positive : but always it will lead to a ratio of the aperture to be given a smaller value than following this rule, in part so that the obliquity of the incident rays may be determined, but partially mainly, so that on account of clarity the apertures of the lenses at this stage may be allowed to increase further.

PROBLEM 6

266. *If a dioptric instrument may be composed from so many lenses, of which the thickness may be considered as zero, and the ratio shall be given of the aperture for the individual lenses together with the apparent field of view, to define the determinable distances of the individual lenses.*

SOLUTION

The distance of the object before the first lens shall be $AE = a$ and the distance of the image represented through that $aF = \alpha$, and for the following lenses there may be put:

Lens	Determinable distances	Ratio of the aperture
second	$BF = b,\ bG = \beta$	π
third	$CG = c,\ cH = \gamma$	π'
fourth	$DH = d,\ dI = \delta$	π''
fifth	$EI = e,\ eK = \varepsilon$	π'''

etc.

Hence therefore, if the radii of the apertures of these lenses as before may be indicated by the letters $\mathfrak{B},\ \mathfrak{C},\ \mathfrak{D},\ \mathfrak{E}$ etc., there will be:

$$\pi = \mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right),\ \ \pi' = \mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right),\ \ \pi'' = \mathfrak{D}\left(\frac{1}{d}+\frac{1}{\delta}\right),\quad \pi''' = \mathfrak{E}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\ \text{etc.}$$

Then truly, if the radius of the apparent field of view shall be $= z$, also there may be put $\frac{z}{a} = \Phi$.
Therefore since hence there shall be :

$$\mathfrak{B} = \frac{\pi b\beta}{b+\beta},\ \ \mathfrak{C} = \frac{\pi' c\gamma}{c+\gamma},\ \ \mathfrak{D} = \frac{\pi'' d\delta}{d+\delta},\quad \mathfrak{E} = \frac{\pi''' e\varepsilon}{e+\varepsilon}\ \text{etc.,}$$

we will have the following equations from § 256 :

$$\frac{\pi\beta}{b+\beta} = \left(1+\frac{\alpha}{b}\right)\Phi,\ \ \pi - \frac{\pi'\gamma}{c+\gamma} = \left(1-\frac{\alpha\beta}{bc}\right)\Phi,$$

$$\pi - \pi' + \frac{\pi'' d\delta}{d+\delta} = \left(1+\frac{\alpha\beta\gamma}{bcd}\right)\Phi,\quad \pi - \pi' + \pi'' - \frac{\pi'''\varepsilon}{e+\varepsilon} = \left(1-\frac{\alpha\beta\gamma\delta}{bcde}\right)\Phi$$
etc.

So that now hence by $\pi,\ \pi',\ \pi''$ etc. and Φ the determinable distances of the lenses may be able to be found more easily, there may be put

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd\quad \varepsilon = Ee\ \text{etc.,}$$

thus so that the letters *A, B, C, D, E* etc. may denote absolute numbers, and our equations will adopt these forms :

$$\frac{B\pi}{B+1}=\left(1+\frac{Aa}{b}\right)\Phi,\quad \pi-\frac{C\pi'}{C+1}=\left(1-\frac{ABa}{c}\right)\Phi,$$

$$\pi-\pi'+\frac{D\pi''}{D+1}=\left(1+\frac{ABCa}{d}\right)\Phi,\qquad \pi-\pi'+\pi''-\frac{E\pi'''}{E+1}=\left(1-\frac{ABCDa}{e}\right)\Phi$$
etc.,

from which the following determinations are elicited :

$$b=\frac{A(B+1)a\Phi}{B\pi-(B+1)\Phi},\qquad c=\frac{AB(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)},$$

$$d=\frac{ABC(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)},\quad e=\frac{ABCD(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)},$$
etc.

Therefore besides the numbers $\Phi,\ \pi,\ \pi',\ \pi''$ etc. etc. with the numbers *A, B, C, D, E* etc. given with the distance of the object $AE=a$, by these formulas the distances *b, c, d, e* etc. are determined in addition to the others $\alpha,\ \beta,\ \gamma,\ \delta,\ \varepsilon,$ etc. in this manner :

$$\alpha=Aa,\ \ \beta=\frac{AB(B+1)a\Phi}{B\pi-(B+1)\Phi},\quad \gamma=\frac{ABC(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)},$$

$$\delta=\frac{ABCD(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)},\qquad \varepsilon=\frac{ABCDE(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)},$$
etc.

Hence we find the focal lengths of the lenses :

First $PP=\frac{Aa}{A+1}$

second $QQ=\frac{ABa\Phi}{B\pi-(B+1)\Phi}$

third $RR=\frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

fourth $SS=\frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

fifth $TT=\frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

COROLLARY 1

267. Thus the radius of the apparent field of view z is defined from the angle Φ with the distance of the object before the first lens $AE=a$, so that there shall be $z=a\Phi$: nor yet can the apparent field of view be assumed as it pleases, but this will be determined by multiplication, as we will see soon.

COROLLARY 2

268. Since all the numbers introduced here into the calculation may be able to be taken equally positive or negative, it is required to be observed these must be assumed always, that the spaces between the lenses, which are $\alpha+b,\ \beta+c,\ \gamma+d,\ \delta+e$ etc., must all be produced positive.

COROLLARY 3

269. So that for the aperture of each lens held in place, the radius of the field of view of which will be known, if its focal length may be multiplied by the ratio of the aperture designated by the letter π .

SCHOLIUM

270. So that we may make the formulas found here simpler, since we will no longer be in need of the letters $\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{D}$ etc., we may put for an abbreviation :

$$\frac{A}{A+1}=\mathfrak{A},\quad \frac{B}{B+1}=\mathfrak{B},\quad \frac{C}{C+1}=\mathfrak{C},\quad \frac{D}{D+1}=\mathfrak{D},\quad \frac{E}{E+1}=\mathfrak{E}\ \text{ etc.},$$

so that there shall be

$$A=\frac{\mathfrak{A}}{1-\mathfrak{A}},\quad B=\frac{\mathfrak{B}}{1-\mathfrak{B}},\quad C=\frac{\mathfrak{C}}{1-\mathfrak{C}},\quad D=\frac{\mathfrak{D}}{1-\mathfrak{D}},\quad E=\frac{\mathfrak{E}}{1-\mathfrak{E}}\ \text{ etc.},$$

and we will have :

$$\alpha=Aa \qquad b=\frac{Aa\Phi}{\mathfrak{B}\pi-\Phi},$$

$$\beta=\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi} \qquad c=\frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi},$$

$$\gamma=\frac{ABCa\Phi}{\mathfrak{c}\pi'--\pi+\Phi} \qquad d=\frac{ABCa\Phi}{D\pi''-\pi'-\pi+\Phi},$$

$$\delta=\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \qquad e=\frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi},$$

$$\varepsilon=\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi} \qquad \text{etc.}$$

Hence the focal lengths of the lenses may be defined again :

First $PP = Aa$

second $QQ = \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}$

third $RR = \frac{AB\mathfrak{C}a\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$

fourth $SS = \frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$

fifth $TT = \frac{ABCD\mathfrak{E}a\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

and the intervals between the lenses :

I and II $= \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$

II and III $= \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$

III and IV $= \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

IV and V $= \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}.$

Which intervals must be positive.

PROBLEM 7

271. *With these in place, which have been assumed in the previous problem, to define the suitable place of the eye, so that the whole apparent field of view may be able to be viewed.*

SOLUTION

All the denominations will remain as before, and since the aperture of the lens *PP* may be considered as zero, for the remaining lenses the radius of each aperture thus itself will be had from the given ratio of the aperture :

Lens	Radius of aperture
QQ	$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\Phi$
RR	$\frac{ABa\mathfrak{C}\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\Phi$
SS	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
TT	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi,$

where the letters $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ etc. hold the values assigned in the previous scholium.

Next it will be agreed to consider the magnitudes of the individual images, which on account of

$$E\varepsilon = z = a\Phi \text{ and } \alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd \text{ etc. will be}$$

$$F\zeta = a\Phi = Aa\Phi,\ G\eta = ABa\Phi,\ H\vartheta = ABCa\Phi,\ I\iota = ABCDa\Phi \text{ etc.}$$

Now for any number of lenses the most suitable position of the eye must be defined for each ; therefore with O denoting the distance of the eye after the final lens :

I. For a single lens

Since the thickness of the lens may be considered as zero, it is evident for a suitable place of the eye to be $O = 0$.

II. For two lenses

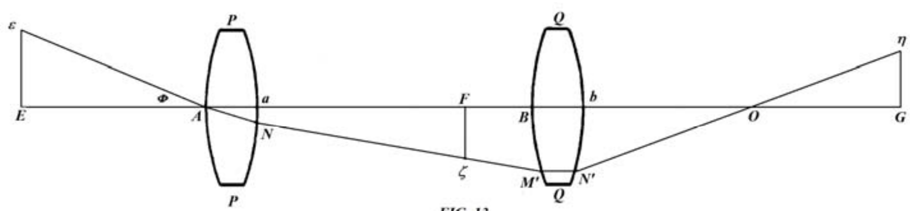

FIG. 13.

Since here (Fig. 13) there shall be $bN' + G\eta : bG = bN' : bO$, there will be

$$bO = O = \frac{bN'}{bN' + G\eta} \cdot \beta .$$

But there is $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi} a\Phi$ and $G\eta = ABa\Phi$, from which there becomes

$$bN' + G\eta = \frac{Aa\Phi \cdot (B+1)\mathfrak{B}\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi},$$

on account of $(\mathrm{B}+1)\mathfrak{B} = B$. Therefore there will be $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi}$, which fraction multiplied by $\beta = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$ gives the suitable place of the eye:

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}.$$

III. For three lenses

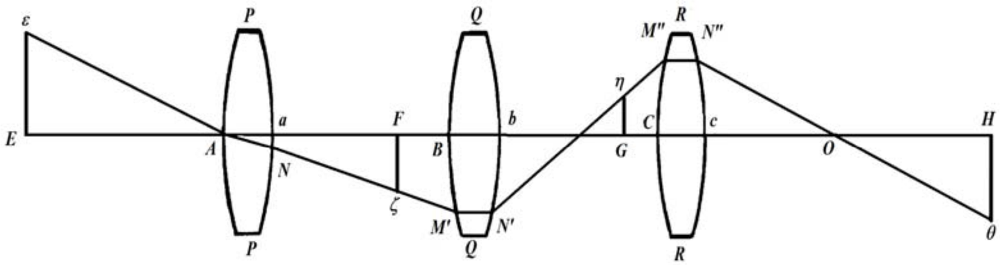

Fig. 14.

Since here (Fig. 14) there shall be $bN''+H\vartheta : cH = CN'' : CO$, there will be

$$cO = O = \frac{cN''}{cN''+H\vartheta}\cdot\gamma .$$

But there is $cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$ and $H\vartheta = ABCa\Phi$, and hence on account of $(C+1)\mathfrak{C} = C$ there will become

$$cN''+H\vartheta = \frac{ABCa\Phi(\pi'-\pi+\Phi)}{\mathfrak{C}\pi'-\pi+\Phi} \text{ and } \frac{cN''}{cN''+H\vartheta} = \frac{\mathfrak{C}\pi'}{C(\pi'-\pi+\Phi)} .$$

Now therefore on account of $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$ we will have the suitable distance of the eye :

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}.$$

IV. For four lenses

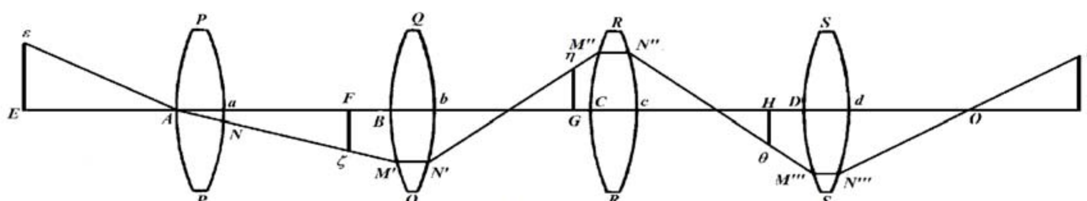

Fig. 15.

Since there shall be (Fig. 15) $dN'''+I\iota : dI = dN''' : dO$, there will be $dO = O = \frac{dN'''}{dN'''+I\iota}\cdot\delta$.
But there is $dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$ and $I\iota = ABCEa\Phi$, hence on account of $(D+1)\mathfrak{D} = D$ there will become $dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$ and $I\iota = ABCEa\Phi$, and hence on account of $(D+1)\mathfrak{D} = D$ there will become

$$dN'''+I\iota = \frac{ABCDa\Phi(\pi''-\pi'+\pi-\Phi)}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \text{ and } \frac{dN'''}{cN'''+Ii} = \frac{\mathfrak{D}\pi''}{D(\pi''-\pi'+\pi-\Phi)}.$$

Therefore on account of $\delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$ a suitable distance for the eye will be produced :

$$O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}.$$

V. For five lenses

If the account may be extended in a similar way to the case of five lenses, we will find a suitable distance to the eye :

$$O=\frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}.$$

VI. For six lenses

By progressing in the same manner a suitable distance for the eye is deduced for the case of six lenses :

$$O=\frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)}\ ;$$

and thus, it will be allowed to progress further, as far as desired.

COROLLARY 1

272. Therefore it is necessary in whatever case, so that a suitable distance of the eye may be produced positive : if indeed it were negative, the whole field of view nowhere will be able to be seen.

COROLLARY 2

273. But in these cases, in which the distance O becomes negative, it will be agreed to place the eye at once to the final lens; then truly the eye cannot discern more, than if the aperture of the final lens were equal to the size of the pupil.

COROLLARY 3

274. Therefore in this case the radius of the aperture of the final lens $=\omega$ may be put equal to the radius of the pupil, and from that equation the value of Φ itself may be elicited, with which found $a\Phi$ will be the radius of the apparent field of view, which is actually observed in the object.

PROBLEM 8

275. *With the same in place and to define that condition in the disposition of the lenses as in the preceding problems, so that the eye put in a suitable place the object likewise may be seen distinctly.*

SOLUTION

Since we assume the aperture of the objective lens to be vanishing, no other source of confusion can be found, unless in as much as the eye may not be considered to be present at the true distance from the final image, if the lenses thus may be set out, so that the final image may be put in place before the eye placed at O at the true distance, that we will designate by the letter l. Therefore since in figures the position of the eye may fall before the final image, this distance is required to be placed equal to the negative of l ; from which for any number of lenses we will have the following determinations :

I. For as single lens

Since here (Fig. 12) there shall be $O = 0$ and $OF = \alpha = Aa$, there is required to be $Aa = -l$ and thus $A = -\frac{l}{a}$ and $\alpha = -l$, from which the nature of this lens is determined, thus so that its focal length may become $= \frac{al}{l-a}$.

II. For two lenses

From the distance found $bO = O$ (Fig. 13) there will be $OG = \frac{O}{bN'} \cdot G\eta$. Therefore there is $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$ from which there shall be $\frac{ABa\Phi}{\pi-\Phi} = -l$, and hence for the second lens $B = \frac{-(\pi-\Phi)l}{Aa\Phi}$ and $\mathfrak{B} = \frac{B}{B+1}$. Or since there shall be $Aa\Phi = \frac{-(\pi-\Phi)l}{B}$, there will be for the position of the eye :

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi-\Phi)} l .$$

III. For three lenses

Here there is (Fig. 14) $OH = \frac{O}{cN''} \cdot H\vartheta = \frac{H\vartheta}{\pi'-\pi+\Phi}$, from which there is obtained $OH = \frac{ABCa\Phi}{\pi'-\pi+\Phi} = -l$, and thus for the final lens there will be had $C = \frac{-(\pi'-\pi+\Phi)}{ABa\Phi}$. But if for the determination there may be taken for the first lens $Aa\Phi = \frac{-(\pi'-\pi+\Phi)l}{BC}$, the distance of the eye will be :

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi'-\pi+\Phi)} l .$$

IV. For four lenses

Since there shall be (Fig. 15) $OI = \frac{O}{aN'''} \cdot I\iota = \frac{I\iota}{\pi'' - \pi' + \pi - \Phi}$, there will be had $OI = \frac{ABCDa\Phi}{\pi'' - \pi' + \pi - \Phi} = -l$. From which for the final lens $D = \frac{-(\pi'' - \pi' + \pi - \Phi)l}{ABCa\Phi}$. But since there shall be $ABCa\Phi = \frac{-(\pi'' - \pi' + \pi - \Phi)l}{D}$, with this value being required to be replaced for the final lens, for the place of the eye:

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} l.$$

V. For five lenses

In a similar manner for five lenses the final lens is required to be prepared thus, so that $E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCDa\Phi}$. But with the first defined thence there becomes

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} l.$$

VI. For six lenses

In the same manner it is apparent for six lenses to become $F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDEa\Phi}$, and if hence Aa may be defined :

$$O = \frac{-\mathfrak{F}\pi''''}{F(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)} l;$$

which formulas will be allowed to be continued as far as wished.

COROLLARY 1

276. If the true distance of the eye l were infinite, there will be, as follows :

I. For one lens	$A = \infty$ and $\mathfrak{A} = 1$
II. for two lenses	$B = \infty$ and $\mathfrak{B} = 1$
III. for three lenses	$C = \infty$ and $\mathfrak{C} = 1$
IV. for four lenses	$D = \infty$ and $\mathfrak{D} = 1$
etc.	

COROLLARY 2

277. Therefore in the case, where the true distance of the eye l is infinite, the distance of the eye past the final lens will be for any number of lenses :

I. For one lens $O = 0$

II. for two lenses $\quad O = \frac{Aa\pi\Phi}{(\pi-\Phi)^2}$

III. for three lenses $\quad O = \frac{ABa\pi'\Phi}{(\pi'-\pi+\Phi)^2}$

IV. for four lenses $\quad O = \frac{ABCa\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)^2}$

etc.

SCHOLIUM

278. Until now I have considered the angle Φ of the apparent field of view as given, and from that I have determined both the nature of the lenses as well as the disposition of these, so that the field of given amplitude may appear, and nothing prevents us from undertaking, why anything less may not satisfy this condition, since the numbers *A, B, C, D* etc. remain completely arbitrary, truly the ratios of the apertures π, π', π'', etc. below $\frac{1}{3}$ or $\frac{1}{4}$ may be accepted. In truth this account of account of the multiplication has not yet been introduced into the calculation, by which likewise the apparent field of view thus may be bound, so that it may not exceed a certain limit. Therefore since in all dioptric instruments it is usual to propose initially, we will establish in the following problem how the apparent field of view may be defined by that.

PROBLEM 9

279. *If a dioptric instrument were constructed from some number of lenses, of which indeed the thicknesses may be regarded as zero, truly likewise a ratio may be proposed from multiplication, to determine the apparent field of view.*

SOLUTION

With all the denominated items continuing, of which we have made use until now, thus so that $a\Phi$ may denote the radius of the apparent field of view, *h* shall be the distance, to which we may refer the multiplication. Therefore the magnitude $a\Phi$ at this distance $= h$ may be discerned by the naked eye under the angle, of which the tangent is $= \frac{a\Phi}{h}$. Whereby if the ratio of multiplication may be put $= m$, it is necessary, that the same magnitude $a\Phi$ may be seen through lenses under the angle, of which the angle shall be $= \frac{ma\Phi}{h}$. Now truly from these, which have been treated in the preceding problems, this angle may be assigned easily, and thus the angle Φ will be obtained and thence the radius of the apparent field of view $a\Phi$. But since this multiplication may not refer to these same angles, but to their tangents, it is clear only the smallest parts of the object situated around the centre *E* shall be multiplied in the proposed ratio, the more distant truly in a smaller ratio. With which observed we will consider this ratio of the multiplication *m* for any number of lenses.

I. For one lens

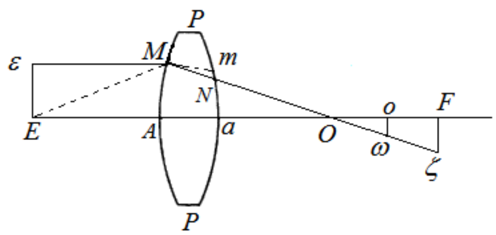

Fig. 12

The tangent of the angle (Fig. 12), by which the image $F\zeta$ may be observed by the eye at O, is $\frac{F\zeta}{OF}=\frac{F\zeta}{aF}$ on account of $aO=0$; therefore there will be $\frac{ma\Phi}{h}=\Phi$ or $ma=h$. Therefore in this case the apparent field of view is not determined, but the ratio of multiplication is $m=\frac{h}{a}$. Truly so that the vision shall be distinct, by the above problem there must be $A=-\frac{l}{a}$ and the distance of the eye past the lens $O=0$. But the object may be considered to be placed erect.

II. For two lenses

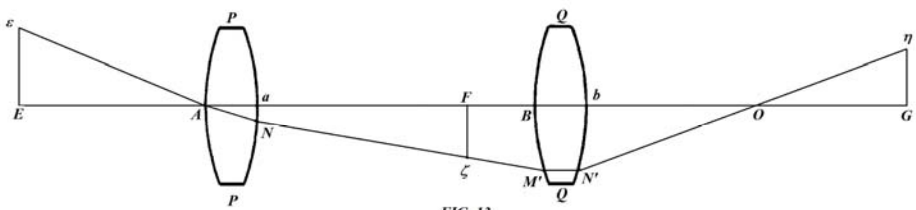

FIG. 13.

The tangent of the angle (Fig. 13), by which the image $G\eta$ may be discerned by the eye put in at O, is $=\frac{bN'}{bO}=\pi-\Phi=\frac{ma\Phi}{h}$; from which it follows the radius of the apparent field of view :

$$\Phi=\frac{\pi h}{ma+h}, \text{ for the invers position.}$$

From which found, so that the vision may be distinct, there is required to be $B=\frac{-ml}{Ah}$ and $\mathfrak{B}=\frac{-ml}{Ah-ml}$, and hence the distance of the eye past the ocular lens :

$$O=\frac{Ahl(ma+h)}{mmal+Ahh}.$$

III. For three lenses

The tangent of the angle (Fig. 14), by which the image $H\vartheta$ is discerned by the eye placed at O, is $\frac{cN''}{cO}=\pi'-\pi+\Phi=\frac{ma\Phi}{h}$, from which the radius of the apparent field of view becomes :

$$\Phi=\frac{(\pi'-\pi)h}{ma-h} \text{ for the erect position.}$$

Then, so that the vision shall be distinct, there is required to be $C=\frac{-ml}{ABh}$ and $\mathfrak{C}=\frac{-ml}{ABh-ml}$.

Therefore since there shall be $\pi'-\pi+\Phi=\frac{ma(\pi'-\pi)}{ma-h}$, there will be

$$\mathfrak{C}\pi'-\pi+\Phi=\pi'-\pi+\Phi-\frac{ABh\pi'}{ABh-ml}=\frac{ma(\pi'-\pi)}{ma-h}-\frac{ABh\pi'}{ABh-ml}$$

and hence for the position of the eye :

$$O=\frac{-ABhl\pi'}{(ABh-ml)(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{ABhl(ma-h)\pi'}{(mmal-ABhh)\pi'+ma(ABh-ml)\pi}.$$

IV. For four lenses

The tangent of the angle (Fig. 15), by which the image $I\iota$ may be discerned by the eye placed at O, is

$=\frac{dN'''}{dO}=\pi''-\pi'+\pi-\Phi=\frac{ma\Phi}{h}$; from which there is deduced :

$$\Phi=\frac{(\pi''-\pi'+\pi)h}{ma+h} \text{ for the inverted position.}$$

Hence again for distinct vision there must be $D=\frac{-ml}{ABCh}$ and $\mathfrak{D}=\frac{-ml}{ABCh-ml}$,

from which there becomes

$$\mathfrak{D}\pi''-\pi'+\pi-\Phi=\frac{ma(\pi''-\pi'+\pi)}{ma+h}-\frac{ABCh\pi''}{ABCh-ml}$$

and for the place of the eye :

$$O=\frac{ABChl(ma+h)\pi''}{(mmal-ABChh)\pi''+ma(ABCh-ml)(\pi'-\pi)}.$$

V. For five lenses

By progressing in the same manner there is found for the apparent field of view :

$$\Phi=\frac{(\pi'''-\pi''+\pi'-\pi)h}{ma-h} \text{ for the erect position,}$$

and so that the view may become distinct, $E=\frac{-ml}{ABCDh}$ and $\mathfrak{E}=\frac{-ml}{ABCDh-ml}$, from which for a suitable position of the eye there may be concluded:

$$O=\frac{ABCDhl(ma+h)\pi'''}{(mmal-ABCDhh)\pi'''+ma(ABCDh-ml)(\pi''-\pi'+\pi)}.$$

VI. For six lenses

Here the apparent field of view is defined thus, so that there shall be :

$$\Phi = \frac{(\pi'''' - \pi''' + \pi'' - \pi' + \pi)h}{ma+h} \text{ for the inverse situation ;}$$

truly for distinct vision there emerges $F = \frac{-ml}{ABCDEh}$, $\mathfrak{F} = \frac{-ml}{ABCDEh-ml}$ from which a suitable place for the eye shall be :

$$O = \frac{ABCDEhl(ma+h)\pi''''}{(mmal+ABCDEhh)\pi''''+ma(ABCDEh-ml)(\pi'''-\pi''+\pi'-\pi)};$$

and thus the progression for more lenses is evident.

COROLLARY 1.

280. Therefore from the given ratios of the apertures of the individual lenses $\pi,\ \pi',\ \pi''$ etc. together with the ratio of multiplication *m,* the distance *h,* to which the multiplication refers, and the distance of the object before the instrument *a,* the apparent field of view may be determined.

COROLLARY 2

281. Therefore so that the maximum apparent field of view may be obtained from the given multiplication, thus the maximum values will be agreed to be attributed to the letters $\pi,\ \pi',\ \pi''$ etc., so that they may be alternately positive and negative.

COROLLARY 3

282. Therefore if the values $\pi,\ \pi',\ \pi''$ etc. may be allowed to be increased to $\frac{1}{3}$, the maximum value of Φ for any number of lenses will be, as follows :

For the case of two lenses	$\Phi = \frac{h}{3(ma+h)}$
for the case of three lenses	$\Phi = \frac{2h}{3(ma-h)}$
for the case of four lenses	$\Phi = \frac{3h}{3(ma+h)}$
for the case of five lenses	$\Phi = \frac{4h}{3(ma-h)}$
etc.	

COROLLARY 4

288. Therefore so that more lenses may be used, there the apparent field of view must be increased more ; from which likewise it is apparent, where a greater multiplication may be desired, there the smaller must become the apparent field of view.

COROLLARY 5

284. The multiplying ratio m can be taken both positive and negative. If it is accepted positive, for an even number of lenses it will be declared to be inverse, but for an odd number it will be declared erect. Truly the contrary will eventuate, if m were a negative number.

SCHOLIUM 1

285. But here initially it is required to note the value of Φ alone may provide the angle $EA\varepsilon$, when it were so very small, that no other order would be necessary ; for if the value of Φ may be produced much greater, then the tangent of this angle $EA\varepsilon$ is expressed. But generally, if a certain multiple shall be small, this value of Φ may be found to be small, so that it may be taken for the angle $EA\varepsilon$ without error. Therefore here the amplitude of the apparent field of view may be restricted for some other reason, so that it may not be able to exceed a certain limit ; for since the angle, to which the rays incident on the eye at O may be inclined to the axis, at no time may be able to be straight through nor perhaps to be scarcely greater than 60°, since indeed with the naked eye never may we wish to observe a space in the heavens greater than 120° ; if we may put in place that maximum angle which the eye may prevail to take, to be around 63°, so that its tangent shall be $=2$, for which we will have for any number of lenses $\frac{ma\Phi}{h}=2$, from which there becomes $\Phi=\frac{2h}{ma}$ and $a\Phi=\frac{2h}{m}$. Therefore with the multiple m and the distance h given, to which the radius of the region in the object observed refers at no time can be present greater than $\frac{2h}{m}$, however many lenses may be used and these thus may be put in place, so that the maximum field may be mad apparent. Therefore in telescopes, where there is taken $h=a$ and the radius of the field may be estimated from the angle Φ itself, its tangent never can be greater than $\frac{2}{m}$; from which I adjoin the following table, which for any multiple of the radius of the field of view shows the maximum apparent field of view, which at no time may be allowed to be exceeded.

Multiple m	Max. radius of apparent field of view	Multiple m	Max. radius of apparent field of view
5	21° 48'	60	1° 54' 33"
10	11° 18'	70	1° 38' 13"
15	7° 35'	80	1° 25' 57"
20	5° 42'	90	1° 16' 24"
25	4° 34'	100	1° 8' 45"
30	3° 49'	150	0° 45' 50"
35	3° $16\frac{1}{2}$'	200	0° 34' 22"
40	2° 52'	250	0° 27' 30"
45	2° 33'	300	0° 22' 55"
50	2° $17\frac{1}{2}$'	400	0° 17' 11"
		500	0° 13' 45"

So that therefore if the number of lenses requiring to be multiplied we have extended now almost to the whole apparent field of view, this in no way can be increased further.

SCHOLIUM 2

286. What pertains to a suitable place of the eye, which we have thus put in place at O, so that it may accept all the rays transmitted by the lenses, even if the pupil may be maximally constricted : from which it is apparent on account of the amplitude of the pupil the eye can be moved a little amount from that place without any detriment, thus so that it would be superfluous to consider this place with great care, unless perhaps the aperture of the final lens were exceedingly great. But if that may not surpass the pupil and thus shall be smaller than that, it is clear the eye may be applied immediately to that and all the rays to be received and the same field to be looked at, as if it were set up in the suitable place. Therefore from these cases, if perhaps the distance O from the position of the eye may be produced negative, nothing is lost from the apparent field of view, provided the eye may be applied directly to the final lens. Therefore from these, which pertain to the view in general through dioptric instruments, it remains to sort out, so that we may investigate, how much the vision may be disturbed on account of the different refraction of the rays and how we may seek to avoid this perturbation.

CAPUT V

DE CAMPO APPARENTE
OCULIQUE LOCO MAXIME IDONEO

PROBLEMA 1

222. *Si ex obiecti puncto extra axem sumto* ε (Fig. 12) *radius quicunque* εM *per lentem PP transmittatur, definire eius concursum cum axe O.*

SOLUTIO

Sit Fζ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices

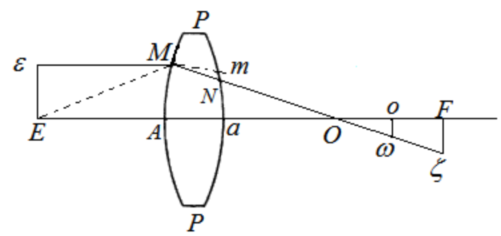

Fig. 12

$AE = a,\ aF = \alpha,$ eius crassitiem $Aa = v$ et quantitatem arbitrariam $= k$, sitque brevitatis ergo $\frac{k-v}{k+v} = i$. His positis si puncti ε imago cadat in ζ voceturque $E\varepsilon = z$, erit $F\zeta = \frac{1}{i}\cdot\frac{\alpha z}{a}$; at pro puncto lentis M statuatur eius distantia ab axe $AM = x$; ac supra ostendimus, si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, ut foret $am = ix$. Radius igitur εM per punctum N ad ζ feretur, ita
ut, cum obliquitas radiorum in facies lentis incidentium perpetuo valde parva statuatur, sit proxima angulus $EM\varepsilon$ ad NMm ut n ad 1 posito $n = \frac{31}{20}$. Hinc erit $E\varepsilon$ ad mN in ratione composita istorum angulorum et distantiarum AE ad Aa, seu $E\varepsilon : mN = na : v$, unde fit $mN = \frac{vz}{na}$ ideoque $aN = ix - \frac{vz}{na}$. Iam vero radius a puncto N recta ad ζ pergit et propterea axem ita in O secabit, ut sit $aN + F\zeta : aF = aN : aO$, sicque

$$aO = \left(i\alpha x - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{nii a\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

et

$$FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}.$$

Si x sit semidiameter aperturae lentis in facie anteriori, haec intersectio O respondet casui, quo radius ab ε per lentis terminum summum transeat: sin autem is per terminum imum transmittatur, sumto x negativo fiet

$$FO = \frac{n\alpha\alpha z}{-nii\alpha x - ivz + n\alpha z}.$$

At si radius ex ε per centrum lentis A transeat, punctum intersectionis ita cadet in O, ut sit

$$FO = \frac{n\alpha\alpha}{n\alpha - iv}.$$

COROLLARIUM 1

223. Si igitur x denotet semidiametrum aperturae lentis in facie anteriori $PMAP$, ut omnes radii a puncto ε in hanc faciem incidentes per lentem transmittantur, necesse est, ut faciei posterioris $PNaP$ semidiameter sit maior quam $\pm ix - \frac{vz}{na}$, sumto x tam negativo quam positivo. Unde hic semidiameter minor esse nequit quam $ix + \frac{vz}{n\alpha}$.

COROLLARIUM 2

224. Si apertura in facie anteriori evanescat, ut sit $x = 0$, radii a puncto ε, cuius ab axe distantia $E\varepsilon = z$, per lentem non transmittentur, nisi in facie posteriori semidiameter aperturae sit $= \frac{vz}{na}$ vel maior. Unde patet, quo maior fuerit lentis crassities v, eo maiori apertura in facie posteriori esse opus.

COROLLARIUM 3

225. Vicissim ergo si detur lentis apertura in facie posteriori, cuius semidiameter sit $= \frac{vz}{na}$, inde simul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam transmittatur.

COROLLARIUM 4

226. Hic autem radius per A immissus post transitum cum axe in O occurret, ut ob $x = 0$ sit intervallum $FO = \frac{n\alpha\alpha}{n\alpha - iv}$ seu $aO = \frac{-i\alpha v}{n\alpha - iv}$. Nisi ergo oculus in hoc axis loco teneatur, radius transmissus non in oculum ingredietur; si quidem apertura pupillae ut infinite parva spectetur.

COROLLARIUM 5

227. At si semidiameter pupillae ponatur $= \omega$, oculus etiam in o positus illum radium excipiet, si fuerit $o\omega = \omega$. Cum autem casu $x = 0$ sit

$$aN\left(\tfrac{vz}{an}\right):aO\left(\tfrac{iav}{n\alpha-iv}\right)=o\omega\left(\omega\right):Oo,\ \text{ erit } Oo=\left(\tfrac{ni\alpha\alpha\omega}{z(n\alpha-iv)}\right).$$

Quod intervallum cum aeque positivum ac negativum accipi possit, pro loco oculi o habebimus

$$ao=-\tfrac{i\alpha vz}{z(n\alpha-iv)}=\tfrac{-i\alpha(vz\pm na\omega)}{z(n\alpha-iv)}.$$

SCHOLlON

228. In hac tractatione, ubi campum apparentem et locum oculi idoneum investigamus, tam aperturam lentis obiectivae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, ut quaestiones obtineamus determinatas. Quare horum elementorum, quae in instrumentis dioptricis ad visionem aecommodatis maximi sunt momenti, sequentes definitiones constituemus.

DEFINITIO 1

229. *Campus apparens est spatium in obiecto, ex cuius singulis punctis radii in centrum lentis obiectivae incidentes per reliquas lentes omnes transmittuntur.*
Quod spatium cum sit circulare, eius radius vocatur semidiameter campi apparentis.

COROLLARIUM 1

230. Si ergo $E\varepsilon=z$ fuerit semidiameter campi apparentis, erit ε punctum obiecti extremum seu ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectivae incidentes per omnes lentes transmittuntur.

COROLLARIUM 2

231. Determinatur igitur magnitudo campi apparentis per aperturam sequentium facierum refringentium ac fortasse per aperturam unius, si scilicet radii a puncto quodam magis remoto quam ε venientes nullum transitum per eam invenirent, etiamsi per reliquas facies transmitterentur.

SCHOLION

232. Si radii ex solo puncto *E,* quod est centrum campi apparentis, considerentur, ii quidem, qui in A incidunt, quoniam perpetuo secundum axem progrediuntur, per omnes reliquas facies refringentes, quantumvis parva fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus evanescere potest. At quo magis punctum ε ab

axe distans accipitur, ut radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur; quae cum ab earum curvatura pendeat neque certum limitem superare debeat, hinc ultimum punctum ε, unde radii etiam nunc transmittuntur, ac propterea semidiameter campi apparentis determinatur. In sequentibus quidem propositionibus campum apparentem seu eius semidiametrum $E\varepsilon = z$ ut datum assumam, et quanta esse debeat cuiusque faciei apertura, investigabo: hinc enim facile vicissim, si quaeque apertura fuerit cognita, campum apparentem ipsum definire licebit. Ceterum in hac definitione assumsi aperturam primae faciei esse evanescentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum postea in lucrum cedet, quod cum nunquam soleat esse notabile, eius rationem hic non habendam censui; quemadmodum etiam in sequente definitione aperturae pupillae rationem non habebo.

DEFINITIO 2

233. *Locus oculi idoneus est id punctum in axe, in quo radii ab extremitate campi apparentis per lentes transmissi axem intersecant. Oculus scilicet in hoc loco constitutus totum campum apparentem conspiciet.*

COROLLARIUM 1

234. Hinc igitur idoneus locus oculo assignabitur, si illa intersectio radiorum extremorum cum axe post lentem ultimam cadat; sin autem haec intersectio ante lentem ultimam reperiatur, fieri nequit, ut oculus in eo loco teneatur, neque propterea totum campum contueri poterit.

COROLLARIUM 2

235. At si ista intersectio pone lentem ultimam cadat, oculus totum campum apparentem perspiciet, etiamsi pupilla maxime esset contracta; neque tamen ob maiorem pupillae amplitudinem maiorem campum perciperc valet.

COROLLARIUM 3

236. Verum ob amplitudinem pupillae hoc commodi assequimur, ut oculus, etiamsi extra locum idoneum constituatur, dummodo distantia non sit nimis magna, tamen totum campum apparentem conspicere possit: id quod egregie usu veniet iis casibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, ut oculus huic faciei immediate applicatus tamen totum campum percipiat.

SCHOLION

237. Quando hic de visione loquor, id ita in genere est interpretandum, ut a puncto viso radius in oculum ingrediatur, neque hic curo, utrum visio sit distincta nec ne? In sequentibus

enim docebitur, quomodo lentes disponi conveniat, ut oculus in loco idoneo positus etiam in iusta ab ultima imagine distantia reperiatur, quovisio distincta reddatur. Hic igitur sine ullo respectu ad visionem distinctam habito cum oculo locum assigno, ubi ab omnibus punctis in campo apparente contentis radios recipiat; et quoniam singula
momenta, quae ad visionem pertinent, seorsim expediri convenit, hic etiam non ad spatium diffusionis respicio, quod quidem semper per se evanescit, si apertara faciei primae evanescens statuatur.

PROBLEMA 2

238. *Si obiectum per unimicam lemtem aspiciatur, determinare tam campum apparentem quam locum idoneum oculi.*

SOLUTIO

Sint ut in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$ (Fig.12) et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$ et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita ut posita faciei anterioris *PAP* apertura infinite parva etiamnum a puncto ε radii per lentem transmittantur. Sit porro brevitatis gratia : $\frac{k-v}{k+v} = i$, atque supra demonstravimus campum apparentem ad ε usque extendi, si pro facie posteriori *PaP* fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existante $n = \frac{31}{20}$; perinde enim est, sive hic semidiameter affirmative accipiatur sive negative. Hinc ergo vicissim, si semidiameter huius aperturae ponatur $= \mathfrak{a}$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}\mathfrak{a}$.

Quod ad locum idoneum oculi attinet, qui sit in *O*, quoniam invenimus $FO = \frac{n\alpha\alpha}{n\alpha - iv}$, erit distantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ ideoque eoque negativa, nisi sit vel *i* numerus negativus vel $iv > n\alpha$. Sin autem haec distantia *aO* fuerit positiva, oculus in *O* positus totum campum apparentem perspiciet.

COROLLARIUM 1

239. Si crassities lentis *v* evanescat, ob $z = \frac{na}{v}\mathfrak{a}$ campus apparens evadet infinitus seu potius indeterminatus; distantia vero *aO* evanescet. Oculus igitur lenti immediate applicatus tantum spatium conspiciet, quantum per propriam indolem complecti valebit.

COROLLARIUM 2

240. Sin autem ob lentis crassitiem $Aa = v$ distantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat positiva, oculus in O positus totum campum apparentem aspicere valebit seu in obiecto spatium circulare spectabit, cuius semidiameter

$$E\varepsilon = z = \frac{na}{v}\mathfrak{a} = \frac{31a\mathfrak{a}}{20v},$$

quod ergo eo erit maius, quo tenuior fuerit lens.

COROLLARIUM 3

241. At si distantia aO resultat negativa, oculum in loco idoneo constitui non licet, quoniam is necessario post lentem teneri debet. Ubicunque autem is post lentem collocetur, non universum campum apparentem contuebitur, sed tantum eius partem, et quidem eo minorem, quo magis post lentem removeatur, propterea quod hoc modo magis a loco idoneo recedit.

COROLLARIUM 4

242. Hoc igitur casu conveniet oculom immediate ad faciem lentis posteriorem applicare, quo situ eatenus tantum radios accipiet, quatenus pupilla patet; sicque campus visus ab apertura pupillae pendebit; quae si esset nulla, etiam campus apparens evanesceret.

COROLLARIUM 5

243. Hinc patet, si pupilla excedat aperturam faciei PaP seu si sit $\omega > \mathfrak{a}$, denotante ω semidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmissos recipit, eum totum campum apparentem esse visurum. Sin autem sit $\omega < \mathfrak{a}$, partem tantum totius campi perspiciet, cuius semidiameter erit $= \frac{na}{v}\omega = \frac{31a\omega}{20v}$, scilicet non maiorem, quam si apertura faciei PaP aequalis esset pupillae.

SCHOLION

244. Interim tamen si hoc casu postremo apertura faciei lentis *PaP,* cui oculus est applicatus, maior sit quam pupilla, nihil obstat, quominus ea successive totam aperturam peragret, sicque pedetemtim totum campum apparentem conspicere poterit, etiamsi eum simul contueri non valeat. Ceterum notandum est, si etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, sed partes obiecti ulteriores, quia non per medium lentis radios transmittunt, obscuriores apparebunt, unde merito a campo apparente excluduntur. At si faciei anteriori tribuatur apertura, cuius semidiameter $= x$, ut facies posterior omnes radios in ilIam incidentes transmittat, eius apertura tanto maior esse debet secundum regulas supra traditas; scilicet eius semidiametrum esse oportet $= ix + \mathfrak{a}$ seu $= ix + \frac{vz}{na}$.

PROBLEMA 3

245. *Si instrumentum dioptricum duabus constet lentibus, definire campum apparentem et locun oculi idoneum.*

SOLUTIO

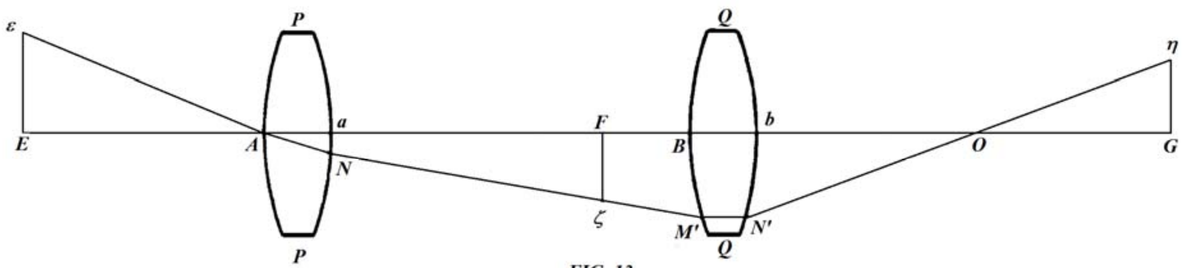

FIG. 13.

Sit pro his lentibus ut hactenus (Fig. 13)

$$\text{pro } PP:\ AE = a,\ aF = \alpha,\ Aa = v; \quad \text{dist. arb.} = k \ \text{ et } \ \tfrac{k-v}{k+v} = i$$

$$\text{pro } QQ:\ BF = b,\ bG = \beta,\ Bb = v'; \ \text{dist. arb.} = k' \ \text{ et } \ \tfrac{k'-v'}{k'+v'} = i',$$

ac, posito semidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit

$$F\zeta = \tfrac{1}{i} \cdot \tfrac{\alpha}{a} z \ \text{ et } \ G\eta = \tfrac{1}{ii'} \cdot \tfrac{\alpha\beta}{ab} z.$$

Consideretur iam radius a puncto z in centrum lentis primae A incidens et per lentes transmissus, qui utique per extremitates imaginum ζ et η transibit. Iam ex problemate praecedente patet esse $aN = \frac{vz}{2a}$, unde in altera lente punctum M' ita definietur, ut sit

$$F\zeta - aN : aF = BM' - F\zeta : BF$$

sive

$$BM' = F\zeta + \frac{BF(F\zeta - aN)}{aF} = \frac{aB \cdot F\zeta - BF \cdot aN}{aF},$$

unde fit

$$BM' = \tfrac{1}{i} \cdot \tfrac{\alpha + b}{a} z - \tfrac{bvz}{na\alpha}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$bN' = i' \cdot BM' - \tfrac{v'}{n'b} \cdot F\zeta \ \text{ ideoque } \ bN' = \tfrac{i'}{i} \cdot \tfrac{\alpha + b}{a} z - \tfrac{i'bv}{na\alpha} z - \tfrac{1}{i} \cdot \tfrac{\alpha v'}{n'ab} z.$$

Hinc autem punctum O, ubi est locus oculi idoneus, facile assignabitur; erit enim

$$bN' + G\eta : bG = bN' : bO, \text{ indeque}$$

$$bO = \frac{bG \cdot bN'}{bN' + G\eta} = \frac{\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}}{\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab} + \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab}} \beta$$

Quod si ergo ponamus semidiametrum aperturae

$$\text{pro lente } PP \text{ faciei } \begin{cases} \text{anterioris} = \mathfrak{A} = x \\ \text{posterioris} = \mathfrak{a} \end{cases}$$

$$\text{pro lente } QQ \text{ faciei } \begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b}, \end{cases}$$

habebimus

$$\mathfrak{A} = 0, \quad \mathfrak{B} = \frac{v}{na} z; \quad \mathfrak{B} = \left(\frac{1}{i} \cdot \frac{\alpha+b}{a} - \frac{bv}{na\alpha}\right) z, \quad \mathfrak{b} = \left(\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}\right) z;$$

ac si distantia oculi post lentem QQ ponatur $BO = O$, erit:

$$O = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z} \cdot \beta$$

COROLLARIUM 1

246. Si ambarum lentium crassities evanescat, erit $v = 0$, $v' = 0$ et $i = i' = 1$; quo ergo casu nostrae formulae in sequentes abibunt:

$$\mathfrak{A} = 0, \quad \mathfrak{a} = 0, \quad \mathfrak{B} = \frac{\alpha+b}{a} z \quad \text{et} \quad \mathfrak{b} = \frac{\alpha+b}{a} z .$$

COROLLARIUM 2

247. Datis ergo vicissim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiscitur, definietur campus adparens.

COROLLARIUM 3

248. Si igitur crassities lentium evanescat, campus apparens ex apertura lentis posterioris facillime determinatur. Erit enim $z = \frac{a\mathfrak{B}}{a+b}$; ; id quod intelligendum est, si distantia $BO = O$ fuerit positiva oculusque in O collocetur.

COROLLARIUM 4

249. Sin autem distantia $bO = O$ prodeat negativa oculusque ultimae lenti immediate applicetur, tum eius apertura plus non praestat, quam si amplitudini pupillae esset aequalis. Quare si $\mathfrak{b}$ maior fuerit semidiametro pupillae ω, loco $\mathfrak{b}$ scribatur ω, et ex ultima aequatione verus valor ipsius z elicietur, nisi forte ex aliqua reliquarum aequationum adhuc minor valor pro z esset proditurus.

PROBLEMA 4

250. *Si instrumentum dioptricum ex tribus constet lentibus, determinare cum campum apparentem tum locum oculi idoneum.*

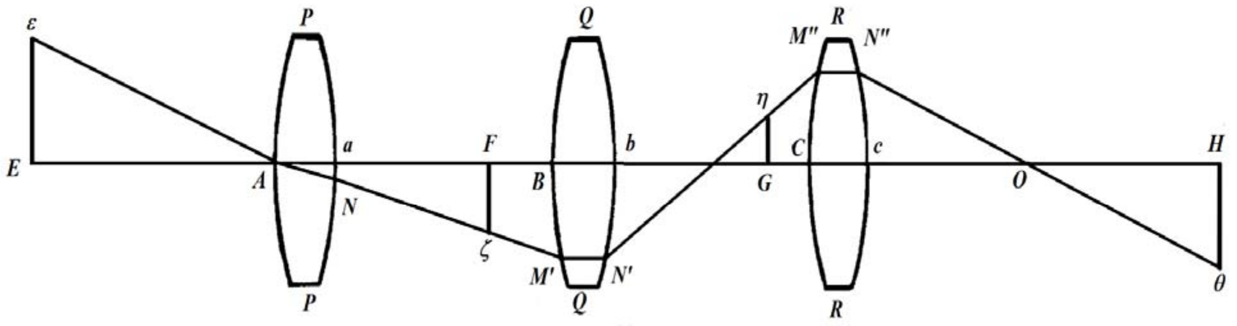

Fig. 14.

SOLUTIO

Existentibus imaginibus per has lentes (Fig. 14) successive repraesentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipso in $E\varepsilon$, ponamus ut hactenus

pro lente	distantias	crassitiem	dist. arb.	et
prima PP	$AE = a,\ aF = \alpha$	$Aa = v$	k	$\frac{k-v}{k+v} = i$
secunda QQ	$BF = b,\ bG = \beta$	$Bb = v'$	k'	$\frac{k'-v'}{k'+v'} = i'$
tertia RR	$CG = c,\ cH = \gamma$	$Cc = v''$	k''	$\frac{k''-v''}{k''+v''} = i''$

semidiametros vero aperturarum

pro lente

$$PP \text{ faciei } \begin{cases} \text{anterioris } = \mathfrak{A} = 0 \\ \text{posterioris} = \mathfrak{a} \end{cases}$$

$$QQ \text{ faciei } \begin{cases} \text{anterioris } = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b}, \end{cases}$$

$$RR \text{ faciei } \begin{cases} \text{anterioris } = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c}. \end{cases}$$

Tum vero sit semidiameter campi apparentis $E\varepsilon = z$, supraque ostendimus fore:

$$F\zeta = \tfrac{1}{i}\cdot\tfrac{\alpha}{a}z,\ \ G\eta = \tfrac{1}{ii'}\cdot\tfrac{\alpha\beta}{ab}z,\ \ H\theta = \tfrac{1}{ii'i''}\cdot\tfrac{\alpha\beta\gamma}{abc}z.$$

His positis habebimus $aN = \mathfrak{a} = \frac{vz}{na}$; unde, si per F recta ipsi NM' parallela, ducta intelligatur, erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ sive } BM' = \frac{aB\cdot F\zeta - BF\cdot aN}{aF}$$

ideoque

$$BM' = \tfrac{\alpha+B}{\alpha}\cdot\tfrac{1}{i}\tfrac{\alpha}{a}z - \tfrac{bvz}{na\alpha} \text{ seu } \mathfrak{B} = \tfrac{1}{i}\cdot\tfrac{\alpha+b}{a}z - \tfrac{bv}{na\alpha}z.$$

Porro vero est ex problemate primo $bN' = i'\cdot BM' - \frac{v}{nb}\cdot F\zeta$, hincque

$$\mathfrak{b} = \tfrac{i'}{i}\cdot\tfrac{\alpha+b}{a}z - \tfrac{i'bv}{na\alpha}z - \tfrac{1}{i}\cdot\tfrac{\alpha v'}{nab}z;$$

simili modo per G ducta intelligatur recta ipsi $N'M''$ parallela, eritque

$$bG : bN' + G\eta = CG : CM'' - G\eta$$

sive

$$CM'' = \mathfrak{C} = G\eta + \tfrac{c}{\beta}(\mathfrak{b} + G\eta) = \tfrac{\beta+c}{\beta}\cdot G\eta + \tfrac{c}{\beta}\mathfrak{b},$$

unde fit

$$\mathfrak{C} = \tfrac{1}{ii'}\cdot\tfrac{\alpha(\beta+c)}{ab}z + \tfrac{i'}{i}\cdot\tfrac{c(\alpha+b)}{a\beta}z - \tfrac{i'bcv}{na\alpha\beta}z - \tfrac{1}{i}\cdot\tfrac{\alpha cv'}{nab\beta}z.$$

Deinde ex CM'' ita definitur cN'' per problema primum, ut sit

$$cN'' = i''\cdot CM'' - \tfrac{v''}{nc}\cdot G\eta \text{ seu } \mathfrak{c} = i''\cdot\mathfrak{C} - \tfrac{1}{ii''}\cdot\tfrac{\alpha\beta v''}{nabc}z,$$

hincque

$$\mathfrak{c} = \tfrac{i''}{ii'}\cdot\tfrac{\alpha(\beta+c)}{ab}z + \tfrac{i'i''}{i}\cdot\tfrac{c(\alpha+b)}{a\beta}z - i'i''\tfrac{bcv}{na\alpha\beta}z - \tfrac{i''}{i}\cdot\tfrac{\alpha cv'}{nab\beta}z - \tfrac{1}{ii'}\cdot\tfrac{\alpha\beta v''}{nabc}z.$$

Isti ergo valores sequenti modo determinantur:

$$\mathfrak{A}=0,\ \ \mathfrak{a}=\tfrac{v}{na}\cdot E\varepsilon;\ \ \mathfrak{B}=\tfrac{\alpha+b}{\alpha}\cdot F\zeta-\tfrac{b}{\alpha}\cdot\mathfrak{a},\ \ \mathfrak{b}=i'\cdot\mathfrak{B}-\tfrac{v'}{nB}\cdot F\zeta;$$

$$\mathfrak{C}=\tfrac{\beta+c}{\beta}\cdot G\eta-\tfrac{c}{\beta}\cdot\mathfrak{b},\ \ \mathfrak{c}=i''\cdot\mathfrak{C}-\tfrac{v''}{nc}\cdot G\eta.$$

Cum iam punctum O praebeat locum oculi iustmn, si ponamus $cO=O$, erit $cN''+\ H\theta:cH=cN'':cO$, unde reperitur

$$O=\tfrac{\gamma c}{c+H\theta}\ \ \text{vel}\ \ \tfrac{1}{O}=\tfrac{1}{\gamma}+\tfrac{1}{ii'i''}\cdot\tfrac{\alpha\beta}{ab}\cdot\tfrac{z}{c\mathfrak{c}}.$$

COROLLARIUM 1

251. Ex his aequationibus sequitur fore

$$\tfrac{\mathfrak{B}}{b}+\tfrac{\mathfrak{a}}{\alpha}=\tfrac{\alpha+b}{\alpha b}\cdot F\zeta=\tfrac{1}{i}\cdot\left(\tfrac{\alpha}{b}+1\right)\tfrac{z}{a}\ \ \text{et}\ \ \tfrac{\mathfrak{C}}{c}-\tfrac{\mathfrak{b}}{\beta}=\tfrac{\beta+c}{\beta c}\cdot G\eta=\tfrac{1}{ii'}\cdot\left(\tfrac{\alpha\beta}{bc}+\tfrac{\alpha}{b}\right)\tfrac{z}{a};$$

hincque porro:

$$\tfrac{i\mathfrak{a}}{\alpha}+\tfrac{i\mathfrak{B}}{b}+\tfrac{ii'\mathfrak{b}}{\beta}-\tfrac{ii'\mathfrak{C}}{c}=\left(1-\tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}.$$

COROLLARIUM 2

252. Si ergo crassities lentium evanescat, cum sit $\mathfrak{a}=0$, $\mathfrak{b}=\mathfrak{B}$ et $\mathfrak{c}=\mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes:

$$\text{1.)}\ \mathfrak{B}\cdot\tfrac{1}{b}=\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}.\quad \text{II;)}\ \mathfrak{B}\cdot\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)-\mathfrak{C}\cdot\tfrac{1}{c}=(1-\tfrac{\alpha\beta}{bc})\tfrac{z}{a},$$

unde minor valor ipsius z praebet semidiametrum campi apparentis.

COROLLARIUM 3

253. Deinde si distantia $cO=O$ prodeat negativa, ut oculum cogamur lenti ultimae immediate applicare, pro $\mathfrak{c}$ scribi oportet semidiametrum pupillae x, et ex ultima aequatione definietur semidiameter campi z, nisi forte ex alia aequatione adhue minor valor pro z prodeat.

PROBLEMA 5

254. *Si instrumentum dioptricum ex quatuor lentibus super eodem axe dispositis constet, determinare cum campum opparentem tum locum oculi idoneum.*

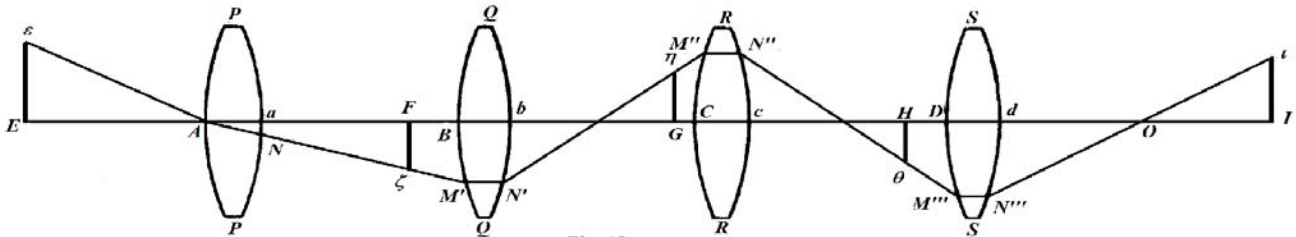

Fig. 15.

SOLUTIO

Existante obiecto $E\varepsilon = z$, sint imagines per lentes successive repraeseutae $F\zeta$, $G\eta$, $H\theta$ et Ii, ponamusque pro lentium singularum determinatione ut hactenus

pro lente	distantias	crassitiem	dist. arb.	et
prima PP	$AE = a,\ aF = \alpha$	$Aa = v$	k	$\frac{k-v}{k+v} = i$
secunda QQ	$BF = b,\ bG = \beta$	$Bb = v'$	k'	$\frac{k'-v'}{k'+v'} = i'$
tertia RR	$CG = c,\ cH = \gamma$	$Cc = v''$	k''	$\frac{k''-v''}{k''+v''} = i''$
quarta SS	$DH = d,\ dI = \delta$	$Dd = v'''$	v'''	$\frac{k'''-v'''}{k'''+v'''} = i'''$

Semidiametri vero aperturarum sint :

pro lente

PP faciei $\begin{cases}\text{anterioris} = \mathfrak{A} = 0\\ \text{posterioris} = \mathfrak{a}\end{cases}$

QQ faciei $\begin{cases}\text{anterioris} = \mathfrak{B}\\ \text{posterioris} = \mathfrak{b},\end{cases}$

RR faciei $\begin{cases}\text{anterioris} = \mathfrak{C}\\ \text{posterioris} = \mathfrak{c}\end{cases}$

SS faciei $\begin{cases}\text{anterioris} = \mathfrak{D}\\ \text{posterioris} = \mathfrak{d}.\end{cases}$

Si iam $E\varepsilon = z$ exhibeat semidiametrum campi apparentis, erit, ut iam supra ostendimus:

$$F\zeta = \frac{1}{i}\cdot\frac{\alpha}{a}z,\ \ G\eta = \frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z,\ \ H\theta = \frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z \ \text{ et } \ Ii = \frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z.$$

Quod si ratiocinium nunc ut ante instituamus, obtinebimus sequentes aequationes:

$$\mathfrak{A}=0, \qquad \mathfrak{a}=\tfrac{\nu}{na}\cdot E\varepsilon;$$
$$\mathfrak{B}=\left(1+\tfrac{b}{\alpha}\right)\cdot F\zeta-\tfrac{b}{\alpha}\cdot\mathfrak{a}, \quad \mathfrak{b}=i'\cdot\mathfrak{B}-\tfrac{\nu'}{nB}\cdot F\zeta;$$
$$\mathfrak{C}=\left(1+\tfrac{c}{\beta}\right)G\eta+\tfrac{c}{\beta}\cdot\mathfrak{b}, \qquad \mathfrak{c}=i''\cdot\mathfrak{C}-\tfrac{\nu''}{nc}\cdot G\eta$$
$$\mathfrak{D}=\left(1+\tfrac{d}{\gamma}\right)H\theta+\tfrac{d}{\gamma}\cdot\mathfrak{c}, \qquad \mathfrak{d}=i'''\cdot\mathfrak{D}-\tfrac{\nu'''}{nd}\cdot H\theta.$$

Ex quarum ordine priori consequimur:

$$\tfrac{\mathfrak{B}}{b}+\tfrac{\mathfrak{a}}{\alpha}=\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)F\zeta=\tfrac{1}{i}\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{C}}{c}-\tfrac{\mathfrak{b}}{\beta}=\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)G\eta=\tfrac{1}{ii'}\left(\tfrac{\alpha}{b}+\tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{D}}{d}-\tfrac{\mathfrak{c}}{\gamma}=\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)H\theta=\tfrac{1}{ii'i''}\left(\tfrac{\alpha\beta}{bc}+\tfrac{\alpha\beta\gamma}{bcd}\right)\tfrac{z}{a},$$

ex ordine vero posteriori:

$$\mathfrak{a}=\tfrac{\nu z}{na}, \quad \mathfrak{b}=i'\mathfrak{B}-\tfrac{1}{i}\cdot\tfrac{\alpha}{b}\cdot\tfrac{\nu' z}{na}, \quad \mathfrak{c}=i''\mathfrak{C}-\tfrac{1}{ii'}\cdot\tfrac{\alpha\beta}{bc}\cdot\tfrac{\nu'' z}{na},$$
$$\mathfrak{d}=i'''\mathfrak{D}-\tfrac{1}{ii'i''}\cdot\tfrac{\alpha\beta\gamma}{bcd}\cdot\tfrac{\nu''' z}{na}.$$

Si denique pro loco oculi idoneo ponamus $dO=O$, erit

$$O=\tfrac{\delta\mathfrak{d}}{\mathfrak{d}+I\iota} \quad \text{seu} \quad \tfrac{1}{O}=\tfrac{1}{\delta}+\tfrac{1}{ii'i''i'''}\cdot\tfrac{\alpha\beta\gamma}{abc}\cdot\tfrac{z}{db}.$$

COROLLARIUM 1

255. Ex aequationibus prioribus deducimus sequentes:

$$\tfrac{\mathfrak{i}\mathfrak{a}}{\alpha}+\tfrac{\mathfrak{i}\mathfrak{B}}{b}=\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{i}\mathfrak{a}}{\alpha}+\tfrac{\mathfrak{i}\mathfrak{B}}{b}+\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{b}}{\beta}-\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{C}}{c}=\left(1-\tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}$$
$$\tfrac{\mathfrak{i}\mathfrak{a}}{\alpha}+\tfrac{\mathfrak{i}\mathfrak{B}}{b}+\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{b}}{\beta}-\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{C}}{c}-\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{i}''\mathfrak{c}}{\gamma}+\tfrac{\mathfrak{i}\mathfrak{i}'\mathfrak{i}''\mathfrak{D}}{d}=\left(1+\tfrac{\alpha\beta\gamma}{bcd}\right)\tfrac{z}{a};$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

COROLLARIUM 2

256. Si lentium crassities evanescat, fiet $\mathfrak{a}=0$, $\mathfrak{b}=\mathfrak{B}$, $\mathfrak{c}=\mathfrak{C}$, et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, unde hae aequationes in sequentes formas abibunt:

$$\frac{\mathfrak{B}}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\frac{1}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)+\mathfrak{D}\frac{1}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a};$$

ex quibus tribus aequationibus, uti in genere, valor ipsius *z,* qui prodierit minimus, verum semidiametrum campi apparentis praebebit.

COROLLARIUM 3

257. Totus iste campus apparens revera spectabitur ab oculo in puncto *O* constituto, dummodo distantia $dO=O$ fuerit positiva. Sed si ea sit negativa oculusque lenti ultimae *SS* immediate applicetur, ponatur $\mathfrak{b}=\omega$, scilicet semidiametro pupillae, et ex ultima aequatione elicietur semidiameter spatii in obiecto revera conspicui.

SCHOLION 1

258. Hinc igitur perspicitur, quomodo campus apparens a singularum lentium apertura pendeat; simulque patet, quanta esse debeat cuiusque lentis apertura, ut campus apparens datae magnitudinis obtineatur. Si enim quantitas *z* cum quantitatibus ad lentium determinationem pertinentibus pro data assumatur, per nostras formulas successive semidiametri aperturarum pro singulis lentibus definiuntur: ubi quidem deinceps est dispiciendum, num lentes tantae aperturae sint capaces. Hinc scilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; unde sequitur campum apparentem maiorem assumi non posse, quam ut aperturae inde pro singulis lentibus oriundae admitti queant. His autem definitis perinde est, sive cuique lenti ea ipsa, quae fuerit inventa, apertura tribuatur, sive maior, dum ne sit minor, quandoquidem hic aperturam lentis obiectivae evanescentem assumsimus. Verum si insuper claritatis ratio habeatur, necesse est, ut vera apertura cuiusque lentis eam, quam hic assignavimus, aliquantum superet, et quidem ea quantitate, quam supra pro limitibus ob claritatem requisitis exhibuimus; nisi enim hoc augmentum accesserit, extremitas in campo apparente minori lumine praedita erit quam medium. Tum autem campus apparens latius patebit oramque obscuriorem complectetur; quamobrem si circa extremitates minori lumine contenti esse velimus, ne opus quidem est, ut lentibus maior apertura, quam quidem per formulas nostras definitur; tribuatur; superfluumque foret aperturas ultra hos limites augere, ita ut hinc cuique lenti conveniens apertura constituatur.

SCHOLION 2

259. Etsi pro casu, quo lentium crassities negligitur, formulae nostrae non multo simpliciores evadunt, tamen in iis percommode usu venit, ut litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ coefficientes, scilicet $\frac{1}{b}+\frac{1}{\beta}$, $\frac{1}{c}+\frac{1}{\gamma}$, $\frac{1}{d}+\frac{1}{\delta}$ ipsam distantiam focalem involvant cuiusque lentis; in praxi autem apertura satis tuto ex distantia focali colligi solet. Nam si lentis *QQ* distantia focalis ponat $=q$, erit $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$ sicque $\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)=\frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, necesse est, ut sit $\mathfrak{B}<\frac{1}{2}q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ usque ad $\frac{1}{4}$ vel $\frac{1}{6}$ diminui debet. Quare si ponamus

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)=\pi,\quad \mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)=\pi',\quad \mathfrak{D}\left(\frac{1}{d}+\frac{1}{\delta}\right)=\pi'',$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor ut plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$; id scilicet tantum cavendum est, ne his litteris nimis magnus valor tribuatur. Quo observato cum arbitrio nostro relinquantur, imprimis conveniet has ipsas litteras in calculum introduci ex iisque reliquas determinari; earum enim beneficio campus apparens facillime definitur. Quin etiam ipse campus apparens statim quoque in calculum induci poterit, quippe cuius determinatio deinceps per formuIam simplicissimam expedietur. Hunc in finem, ut tota investigatio ad meros numeros redigatur, ponam $\frac{z}{a}=\Phi$, ita ut Φ, sit angulus, sub quo semidiameter campi apparentis ab oculo ad lentem obiectivam collocato spectaretur. Videamus ergo, quomodo per hos numeros π, π', π'' etc. et Φ, reliquae quantitates definiantur.

DEFINITIO 3

260. *Ratio aperturae cuiusque lentis mihi vocabitur quotus, qui oritur, si semidiameter aperturae dividatur per distantiam focalem lentis, eius crassitie pro nihilo habita.*

COROLLARIUM 1

261. Ita si b et β sint distantiae determinatrices lentis et $\mathfrak{B}$ semidiameter aperturae eiusdem, quia distantia focalis est $=\frac{b\beta}{b+\beta}$, ratio aperturae erit $=\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)$.

COROLLARIUM 2

262. Ratio igitur aperturae cuiusvis lentis est fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem sancivimus, ut neutrius faciei arcus 60 gradibus maior in apertura contineatur.

COROLLARIUM 3

263. Si scilicet ambae facies fuerint aeque curvae, ratio aperturae per hanc legem usque ad $\frac{1}{2}$ augeri poterit, sin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ superare vix poterit, ac si lens sit meniscus, ea adhuc minor statui debebit.

COROLLARIUM 4

264. Cum autem nihil sit, quo apertura lentium accuratius definiatur, ea fere arbitrio nostro relinquitur et quovis casu commodissime per experientiam determinatur, sufficietque notasse eam fractioni sive $\frac{1}{3}$ sive $\frac{1}{4}$ sive etiam $\frac{1}{5}$ pro forma lentis aequalem statui debere.

SCHOLION

265. Ut scilicet quovis casu ratio aperturae recte definiatur, radios utriusque faciei lentis contemplari oportet, qui si fuerint *f* et *g,* erit distantia focalis $=\frac{fg}{(n-1)(f+g)}=\frac{20}{11}\cdot\frac{fg}{(f+g)}$. Iam semidiameter aperturae minor esse debet quam $\frac{1}{2}f$ vel quam $\frac{1}{2}g$, prout vel *f* vel *g* fuerit minor. Sit $g<f$, et cum semidiameter aperturae minor esse debeat quam $\frac{1}{2}f$, ratio aperturae minor accipienda est quam $\frac{11}{40}(1+\frac{g}{f})\cdot$ Unde patet, si lens sit utrinque aeque convexa seu $g=f$, rationem aperturae capi debere infra $\frac{11}{20}$; sin autem sit altera facies plana seu $f=\infty$, illum limitem esse $\frac{11}{40}$, qui adhuc minor fiet, si lens sit meniscus seu $\frac{g}{f}$ numerus negativus. Ceterum si ratio aperturae sit $=\pi$ eique hoc modo idoneus valor tribuatur, perinde est, sive is negative sive positive accipiatur: semper autem conducet rationi aperturae minorem valorem tribui quam secundum hanc reguIam, partim ut obliquitas radiorum incidentinm diminuatur, partim vero potissimum, ut ob claritatem aperturas lentium adhuc ultra augere liceat.

PROBLEMA 6

266. *Si instrumentum dioptricum ex quotcunque lentibus, quarum crassitiem ut nullam spectare liceat, sit compositum dataque sit ratio aperturae pro singulis lentibus una cum campo apparente, definire distantias determinatrices singularum lentium.*

SOLUTIO

Sit distantia obiecti ante lentem primam $AE = a$ imaginisque per eam repraesentatae $aF = \alpha$, ac pro sequentibus lentibus ponatur:

pro lente	distantiae determinatrices	Ratio aperturae
secunda	$BF = b,\ bG = \beta$	π
tertia	$CG = c,\ cH = \gamma$	π'
quarta	$DH = d,\ dI = \delta$	π''
quinta	$EI = e,\ eK = \varepsilon$	π'''
	etc.	

Hinc ergo, si semidiametri aperturarum harum lentium ut ante indicentur litteris $\mathfrak{B},\ \mathfrak{C},\ \mathfrak{D},\ \mathfrak{E}$ etc., erit

$$\pi = \mathfrak{B}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right),\quad \pi' = \mathfrak{C}\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right),\quad \pi'' = \mathfrak{D}\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right),\quad \pi''' = \mathfrak{E}\left(\tfrac{1}{e}+\tfrac{1}{\varepsilon}\right) \text{ etc.}$$

Tum vero, si sit semidiameter campi apparentis $= z$, ponatur etiam $\frac{z}{a} = \Phi$.
Cum igitur hinc sit:

$$\mathfrak{B} = \tfrac{\pi b\beta}{b+\beta},\quad \mathfrak{C} = \tfrac{\pi' c\gamma}{c+\gamma},\quad \mathfrak{D} = \tfrac{\pi'' d\delta}{d+\delta},\quad \mathfrak{E} = \tfrac{\pi''' e\varepsilon}{e+\varepsilon} \text{ etc.},$$

habebimus ex § 256 sequentes aequationes:

$$\tfrac{\pi\beta}{b+\beta} = \left(1+\tfrac{\alpha}{b}\right)\Phi,\quad \pi - \tfrac{\pi'\gamma}{c+\gamma} = \left(1-\tfrac{\alpha\beta}{bc}\right)\Phi,$$

$$\pi - \pi' + \tfrac{\pi'' d\delta}{d+\delta} = \left(1+\tfrac{\alpha\beta\gamma}{bcd}\right)\Phi,\quad \pi - \pi' + \pi'' - \tfrac{\pi'''\varepsilon}{e+\varepsilon} = \left(1-\tfrac{\alpha\beta\gamma\delta}{bcde}\right)\Phi$$
etc.

Quo iam facilius hinc per $\pi,\ \pi',\ \pi''$ etc. et Φ, distantiae determinatrices lentium definiri queant, ponatur

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd\quad \varepsilon = Ee \text{ etc.},$$

ita ut litterae *A, B, C, D, E* etc. denotent numeros absolutos, ac nostrae aequationes induent has formas:

$$\tfrac{B\pi}{B+1} = \left(1+\tfrac{Aa}{b}\right)\Phi,\quad \pi - \tfrac{C\pi'}{C+1} = \left(1-\tfrac{ABa}{c}\right)\Phi,$$

$$\pi - \pi' + \tfrac{\pi'' D}{D+1} = \left(1+\tfrac{ABCa}{d}\right)\Phi,\quad \pi - \pi' + \pi'' - \tfrac{E\pi'''}{E+1} = \left(1-\tfrac{ABCDa}{e}\right)\Phi$$
etc.,

unde eliciuntur sequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi-(B+1)\Phi}, \qquad c = \frac{AB(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)},$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}, \quad e = \frac{ABCD(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)},$$
etc.

Datis ergo praeter numeros $\Phi,\ \pi,\ \pi',\ \pi''$ etc. etc. numeris *A, B, C, D, E* etc. cum distantia obiecti $AE = a$, per has formulas distantiae *b, c, d, e* etc. determinantur indeque insuper alterae $\alpha,\ \beta,\ \gamma,\ \delta,\ \varepsilon,$ etc. hoc modo:

$$\alpha = Aa,\ \ \beta = \frac{AB(B+1)a\Phi}{B\pi-(B+1)\Phi}, \quad \gamma = \frac{ABC(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)},$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}, \qquad \varepsilon = \frac{ABCDE(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)},$$
etc.

Hinc nanciscimur distantias focales lentium:

Primae $PP = \frac{Aa}{A+1}$
secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$
tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$
quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$
quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$
etc.

COROLLARIUM 1

267. Ex angulo Φ cum distantia obiecti ante lentem primam $AE = a$ ita definitur semidiameter campi apparentis *z,* ut sit $z = a\Phi$: neque tamen campus apparens pro lubitu assumi potest, sed is per multiplicationem determinabitur, ut mox videbimus.

COROLLARIUM 2

268. Cum omnes numeri hic in calculum introducti aeque negative ac positive accipi queant, observandum est eos perpetuo ita assumi debere, ut intervalla lentium, quae sunt $\alpha+b,\ \beta+c,\ \gamma+d,\ \delta+e$ etc., omnia prodeant positiva.

COROLLARIUM 3

269. Quod ad aperturam cuiusque lentis attinet, eius semidiameter habebitur, si eius distantia focalis multiplicetur per rationem aperturae littera π insignitam.

SCHOLION

270. Quo formulas hic inventas simpliciores reddamus, quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ etc. non amplius indigebimus, ponamus ad abbreviandum:

$$\frac{A}{A+1}=\mathfrak{A},\quad \frac{B}{B+1}=\mathfrak{B},\quad \frac{C}{C+1}=\mathfrak{C},\quad \frac{D}{D+1}=\mathfrak{D},\quad \frac{E}{E+1}=\mathfrak{E}\ \text{ etc.},$$

ut sit

$$A=\frac{\mathfrak{A}}{1-\mathfrak{A}},\quad B=\frac{\mathfrak{B}}{1-\mathfrak{B}},\quad C=\frac{\mathfrak{C}}{1-\mathfrak{C}},\quad D=\frac{\mathfrak{D}}{1-\mathfrak{D}},\quad E=\frac{\mathfrak{E}}{1-\mathfrak{E}}\ \text{ etc.},$$

atque habebimus:

$$\alpha = Aa \qquad b=\frac{Aa\Phi}{\mathfrak{B}\pi-\Phi},$$

$$\beta=\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi} \qquad c=\frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi},$$

$$\gamma=\frac{ABCa\Phi}{\mathfrak{c}\pi'--\pi+\Phi} \qquad d=\frac{ABCa\Phi}{D\pi''-\pi'-\pi+\Phi},$$

$$\delta=\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \qquad e=\frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi},$$

$$\varepsilon=\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

Primae $PP = Aa$

secundae $QQ = \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}$

tertiae $RR = \frac{AB\mathfrak{C}a\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$

quartae $SS = \frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$

quintae $TT = \frac{ABCD\mathfrak{E}a\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

et lentium intervalla:

$$\text{I et II} = \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$$

$$\text{II et III} = \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$$

$$\text{III et IV} = \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$\text{IV et V} = \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}.$$

Quae intervalla debent esse positiva.

PROBLEMA 7

271. *Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, unde totus campus apparens conspici queat.*

SOLUTIO

Maneant omnes denominationes ut ante, et quia apertura lentis *PP* ut nulla spectatur, pro reliquis lentibus ex data aperturae ratione semidiameter aperturae cuiusque ita se habebit:

$$\text{I et II} = \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$$

$$\text{II et III} = \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$$

$$\text{III et IV} = \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$\text{IV et V} = \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}.$$

Lentis	Semidiameter aperturae
secundae *QQ*	$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\Phi$
tertiae *RR*	$\frac{ABa\mathfrak{C}\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\Phi$
quartae *SS*	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
quintae *TT*	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi,$

ubi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ etc. ete. valores in praecedente scholio assignatos obtinent.

Deinde magnitudines singularum imaginum considerari convenit, quae ob

$$E\varepsilon = z = a\Phi \text{ et } \alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd \text{ etc. erunt}$$

$$F\zeta = a\Phi = Aa\Phi,\ G\eta = ABa\Phi,\ H\vartheta = ABCa\Phi,\ I\iota = ABCDa\Phi \text{ etc.}$$

Iam pro quolibet lentium numero locus oculi idoneus seorsim definiri debet; denotante ergo O distantiam oculi post ultimam lentem

I. Pro unica lente

Quia crassities lentis ut nulla spectatur, evidens est pro loco oculi idoneo fore $O = 0$.

II. Pro duabus lentibus

Cum hic (Fig. 13) sit $bN' + G\eta : bG = bN' : bO$, erit

$$bO = O = \frac{bN'}{bN' + G\eta} \cdot \beta .$$

Sed est $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi} a\Phi$ et $G\eta = ABa\Phi$, unde fit

$$bN' + G\eta = \frac{Aa\Phi \cdot (B+1)\mathfrak{B}\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi},$$

ob $(\mathrm{B}+1)\mathfrak{B} = B$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi}$ quae fractio per $\beta = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$ multiplicata dat locum oculi idoneum:

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}.$$

III. Pro tribus lentibus

Cum hic (Fig. 14) sit $bN'' + H\vartheta : cH = CN'' : CO$, erit

$$cO = O = \frac{cN''}{cN'' + H\vartheta} \cdot \gamma .$$

Sed est $cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ et $H\vartheta = ABCa\Phi$, hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\vartheta = \frac{ABCa\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } \frac{cN''}{cN'' + H\vartheta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)} .$$

Nunc igitur ob $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus distantiam oculi idoneum:

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}.$$

IV. Pro quatuor lentibus

Cum sit (Fig. 15) $dN''' + I\iota : dI = dN''' : dO$, erit $dO = O = \frac{dN'''}{dN''' + I\iota} \cdot \delta$.

Sed est $dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ et $I\iota = ABCEa\Phi$, hinc ob $(D+1)\mathfrak{D} = D$ fiet

$dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ et $I\iota = ABCEa\Phi$, hincque ob $(D+1)\mathfrak{D} = D$ fiet

$$dN''' + I\iota = \frac{ABCDa\Phi(\pi''-\pi'+\pi-\Phi)}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \text{ et } \frac{dN'''}{cN'''+Ii} = \frac{\mathfrak{D}\pi''}{D(\pi''-\pi'+\pi-\Phi)}.$$

Ergo ob $\delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$ prodit distantia oculi idonea:

$$O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}.$$

V. Pro quinque lentibus

Si ratiocinium simili modo ad casum quinque lentium extendatur, reperiemus distantiam oculi idoneam:

$$O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}.$$

VI. Pro sex lentibus

Eodemque modo progrediendo colligitur fore pro casu sex lentium distantiam oculi idoneam:

$$O = \frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)};$$

sicque ulterius, quousque libuerit, progredi licet.

COROLLARIUM 1

272. Quovis ergo casu necesse est, ut distantia oculi idonea prodeat positiva: si enim fieret negativa, totus campus apparens nusquam conspici posset.

COROLLARIUM 2

273. Iis autem casibus, quibus distantia O fit negativa, oculum immediate ultimae lenti applicari conveniet; tum vero oculus plus non cernet, quam si ultimae lentis apertura aequalis esset amplitudini pupillae.

COROLLARIUM 3

274. Hoc ergo casu statuatur semidiameter aperturae ultimae lentis $= \omega$ semidiametro pupillae, ex eaque aequatione eliciatur valor ipsius Φ, quo invento erit $a\Phi$ semidiameter campi apparentis, qui in obiecto revera conspicietur.

PROBLEMA 8

275. *Positis iisdem atque in problematibus praecedentibus eam conditionem in lentium dispositione definire, ut oculus in loco idoneo positus obiectum simul distincte videat.*

SOLUTIO

Quia aperturam lentis obiectivae evanescentem assumimus, in visione alia confusio locum habere nequit, nisi quatenus oculus non in distantia iusta ab ultima imagine, quam intuetur, existit, quae ergo tolletur, si lentes ita disponantur, ut imago ultima ante oculum in O situm in distantia iusta, quam littera l designavimus, versetur. Cum igitur in figuris locus oculi ante imaginem ultimam cadat, haec distantia negative sumta ipsi l aequalis est ponenda; unde pro quovis lentium numero sequentes habebimus determinationes:

I. Pro unica lente

Cum hic (Fig. 12) sit $O = 0$ et $OF = \alpha = Aa$, oportet esse $Aa = -l$ ideoque $A = -\frac{l}{a}$ et $\alpha = -l$, unde indoles huius lentis determinatur, ita ut eius distantia focalis esse debeat $= \frac{al}{l-a}$.

II. Pro duabus lentibus

Ex inventa distantia $bO = O$ (Fig. 13) erit $OG = \frac{O}{bN'} \cdot G\eta$. Est vero $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$ unde fit $\frac{ABa\Phi}{\pi-\Phi} = -l$, hincque pro secunda lente $B = \frac{-(\pi-\Phi)l}{Aa\Phi}$ et $\mathfrak{B} = \frac{B}{B+1}$. Vel cum sit $Aa\Phi = \frac{-(\pi-\Phi)l}{B}$, erit pro loco oculi:

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi - \Phi)} l.$$

III. Pro tribus lentibus

Hic est (Fig. 14) $OH = \frac{O}{cN''} \cdot H\vartheta = \frac{H\vartheta}{\pi'-\pi+\Phi}$, unde obtinetur $OH = \frac{ABCa\Phi}{\pi'-\pi+\Phi} = -l$, sicque pro ultima lente habebitur $C = \frac{-(\pi'-\pi+\Phi)}{ABa\Phi}$. At si pro determinatione a primae lentis capiatur $Aa\Phi = \frac{-(\pi'-\pi+\Phi)l}{BC}$, erit distantia oculi:

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi'-\pi+\Phi)} l.$$

IV. Pro quatuor lentibus

Cum sit (Fig. 15, p. 151) $OI = \frac{O}{aN'''} \cdot I\iota = \frac{I\iota}{\pi''-\pi'+\pi-\Phi}$, habebitur $OI = \frac{ABCDa\Phi}{\pi''-\pi'+\pi-\Phi} = -l$.

Unde pro ultima lente $D=\frac{-(\pi''-\pi'+\pi-\Phi)l}{ABCa\Phi}$. Cum autem sit $ABCa\Phi=\frac{-(\pi''-\pi'+\pi-\Phi)l}{D}$, erit in loco oculi hoc valore surrogando:

$$O=\frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}l.$$

V. Pro quinque lentibus

Simili modo pro quinque lentibus ultima ita comparata esse debet, ut $E=\frac{-(\pi'''-\pi''+\pi'-\pi+\Phi)l}{ABCDa\Phi}$. Prima autem inde definita fit

$$O=\frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}l.$$

VI. Pro sex lentibus

Eodem modo patet pro sex lentibus fore $F=\frac{-(\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)l}{ABCDEa\Phi}$, atque si hinc Aa definiatur:

$$O=\frac{-\mathfrak{F}\pi''''}{F(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)}l;$$

quas formulas, quousque libuerit, continuare licet.

COROLLARIUM 1

276. Si distantia oculi iusta l fuerit infinita, erit, ut sequitur:

I. Pro una lente .	$A=\infty$ et $\mathfrak{A}=1$
II. pro duabus lentibus	$B=\infty$ et $\mathfrak{B}=1$
III. pro tribus lentibus	$C=\infty$ et $\mathfrak{C}=1$
IV. pro quatuor lentibus	$D=\infty$ et $\mathfrak{D}=1$
etc.	

COROLLARIUM 2

277. Casu ergo, quo distantia oculi iusta l est infinita, distantia oculi post lentem ultimam erit pro quovis lentium numero:

I. Pro unica lente	$O=0$
II. pro duabus lentibus	$O=\frac{Aa\pi\Phi}{(\pi-\Phi)^2}$
III. pro tribus lentibus	$O=\frac{ABa\pi'\Phi}{(\pi'-\pi+\Phi)^2}$
IV. pro quatuor lentibus	$O=\frac{ABCa\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)^2}$
etc.	

SCHOLION

278. Hactenus campum apparentem Φ ut datum consideravi, ex eoque tam lentium indolem quam earum dispositionem determinavi, ut campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat, cum numeri *A, B, C, D* etc. penitus arbitrio relinquantur, aperturarum vero rationes $\pi, \ \pi', \ \pi''$, etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta, qua simul campus apparens ita adstringitur, ut certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur, in sequente problemate exponamus.

PROBLEMA 9

279. *Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities ut nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.*

SOLUTIO

Manentibus omnibus denominationibus, quibus hactenus sumus usi, ita ut $a\Phi$ semidiametrum campi apparentis denotet, sit *h* distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac $= h$ nudo oculo cerneretur sub angulo, cuius tangens est $= \frac{a\Phi}{h}$ Quare si multiplicationis ratio statuatur $= m$, necesse est, ut eadem magnitudo $a\Phi$ per lentes spectetur sub angulo , cuius tangens sit $= \frac{ma\Phi}{h}$. Iam vero ex iis, quae in problematibus praecedentibus sunt tradita, iste angulus facile assignatur, sicque obtinebitur angulus Φ indeque semidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipsos angulos, sed eorum tangentes referatur, evidens est tantum partes obiecti minimas circa centrum *E* sitas in ratione proposita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem *m* pro quovis lentium numero contemplemur.

I. Pro unica lente

Tangens anguli (Fig. 12), quo imago $F\zeta$ ab oculo in *O* constituto conspicitur, est $\frac{F\zeta}{OF} = \frac{F\zeta}{aF}$ ob $aO = 0$; erit ergo $\frac{ma\Phi}{h} = \Phi$ seu $ma = h$. Hoc ergo casu campus apparens non determinatur, sed multiplicationis ratio est $m = \frac{h}{a}$. Verum ut visio sit distincta, per superius problema debet esse $A = -\frac{l}{a}$ et distantia oculi post lentem $O = 0$. At obiectum situ erecto cernetur.

II. Pro duabus lentibus

Tangens anguli (Fig. 13), quo imago $G\eta$ ab oculo in O constituto cernitur, est $=\frac{bN'}{bO}=\pi-\Phi=\frac{ma\Phi}{h}$; unde sequitur semdiameter campi apparentis:

$$\Phi=\frac{\pi h}{ma+h}, \text{ pro situ inverso.}$$

Quo invento, ut visio sit distincta, oportet esse $B=\frac{-ml}{Ah}$ et $\mathfrak{B}=\frac{-ml}{Ah-ml}$,
hincque prodit distantia oculi post lentem ocularem:

$$O=\frac{Ahl(ma+h)}{mmal+Ahh}.$$

III. Pro tribus lentibus

Tangens anguli (Fig. 14), quo imago $H\vartheta$ ab oculo in O constituto cernitur, est $\frac{cN''}{cO}=\pi'-\pi+\Phi=\frac{ma\Phi}{h}$, unde fit semidiameter campi apparentis :

$$\Phi=\frac{(\pi'-\pi)h}{ma-h} \text{ pro situ erecto.}$$

Deinde, ut visio sit distincta, oportet esse $C=\frac{-ml}{ABh}$ et $\mathfrak{C}=\frac{-ml}{ABh-ml}$. Cum igitur sit $\pi'-\pi+\Phi=\frac{ma(\pi'-\pi)}{ma-h}$, erit

$$\mathfrak{C}\pi'-\pi+\Phi=\pi'-\pi+\Phi-\frac{ABh\pi'}{ABh-ml}=\frac{ma(\pi'-\pi)}{ma-h}-\frac{ABh\pi'}{ABh-ml}$$

hincque pro loco oculi:

$$O=\frac{-ABhl\pi'}{(ABh-ml)(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{ABhl(ma-h)\pi'}{(mmal-ABhh)\pi'+ma(ABh-ml)\pi}.$$

IV. Pro quatuor lentibus

Tangens anguli (Fig. 15, p. 151), quo imago $I\iota$ ab oculo in O constituto cernituer, est $=\frac{dN'''}{dO}=\pi''-\pi'+\pi-\Phi=\frac{ma\Phi}{h}$; unde elicitur :

$$\Phi=\frac{(\pi''-\pi'+\pi)h}{ma+h} \text{ pro situ inverso.}$$

Hinc porro pro visione distincta esse debet $D=\frac{-ml}{ABCh}$ et $\mathfrak{D}=\frac{-ml}{ABCh-ml}$,
unde fit

$$\mathfrak{D}\pi''-\pi'+\pi-\Phi=\frac{ma(\pi''-\pi'+\pi)}{ma+h}-\frac{ABCh\pi''}{ABCh-ml}$$

et pro loco oculi:

$$O=\frac{ABChl(ma+h)\pi''}{(mmal-ABChh)\pi''+ma(ABCh-ml)(\pi'-\pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi=\frac{(\pi'''-\pi''+\pi'-\pi)h}{ma-h} \text{ pro situ erecto,}$$

et ut visio evadat distincta, $E=\frac{-ml}{ABCDh}$ et $\mathfrak{E}=\frac{-ml}{ABCDh-ml}$, unde pro loco oculi idoneo concluditur:

$$O=\frac{ABCDhl(ma+h)\pi'''}{(mmal-ABCDhh)\pi'''+ma(ABCDh-ml)(\pi''-\pi'+\pi)}.$$

VI. Pro sex lentibus

Hic campus apparens ita definitur, ut sit:

$$\Phi=\frac{(\pi''''-\pi'''+\pi''-\pi'+\pi)h}{ma+h} \text{ pro situ inverso;}$$

visio vero distincta exigit $F=\frac{-ml}{ABCDEh}$, $\mathfrak{F}=\frac{-ml}{ABCDEh-ml}$ unde pro loco oculi idoneo:

$$O=\frac{ABCDEhl(ma+h)\pi''''}{(mmal+ABCDEhh)\pi''''+ma(ABCDEh-ml)(\pi'''-\pi''+\pi'-\pi)};$$

sicque progressio ad plures lentes est manifesta.

COROLLARIUM 1.

280. Datis ergo rationibus aperturarum singularum lentium $\pi,\ \pi',\ \pi''$ etc. una cum ratione multiplicationis *m,* distantia *h,* ad quam multiplicatio refertur, et distantia obiecti ante instrumentum *a,* determinatur campus apparens.

COROLLARIUM 2

281. Ut ergo campus apparens pro data multiplicatione maximus obtineatur, litteris $\pi,\ \pi',\ \pi''$ etc. ita valores maximos tribui conveniet, ut alternatim sint positivi et negativi.

COROLLARIUM 3

282. Si igitur valores $\pi,\ \pi',\ \pi''$ etc. usque ad $\frac{1}{3}$ augeri liceat, maximus valor ipsius Φ pro quovis lentium numero erit, ut sequitur:

Pro casu duarum lentium $\Phi = \frac{h}{3(ma+h)}$

pro casu trium lentium $\Phi = \frac{2h}{3(ma-h)}$

pro casu quatuor lentium $\Phi = \frac{3h}{3(ma+h)}$

pro casu quinque lentium $\Phi = \frac{4h}{3(ma-h)}$

etc.

COROLLARIUM 4

288. Quo plures ergo lentes adhibentur, eo magis campus apparens augeri potest; simul vero patet, quo maior multiplicatio desideretur, eo minorem fieri campum apparentem.

COROLLARIUM 5

284. Ratio multiplicationis m tam positive quam negative capi potest. Si positive accipitur, pro lentium numero pari situm inversum, pro impari autem situm erectum declarat. Contrarium vero evenit, si m fuerit numerus negativus.

SCHOLION 1

285. Hic autem imprimis notandum est valorem ipsius Φ tum solum angulum $EA\varepsilon$ praebere, quando fuerit tam exiguus, ut aliquot gradus non superet; si enim valor ipsius Φ prodeat multo maior, tum tangentem huius anguli $EA\varepsilon$ exprimit. Plerumque autem, si quidam multiplicatio sit modica, iste valor ipsius Φ tam parvus reperitur, ut sine errore pro ipso angulo $EA\varepsilon$ accipi possit. Hic igitur ob aliam causam amplitudo campi apparentis restringitur, ut certum limitem superare nequeat; cum enim angulus, quo radii in oculum incidentes in *0* ad axem inclinantur, nunquam possit esse rectus neque fortasse vix 60° superare queat, quandoquidem ne nudo quidam oculo spatium in coelo maius quam 120° conspicere valeamus; si illum angulum maximum, quem oculus capere valeat, circiter 63° statuamus, ut eius tangens sit $=2$, pro quovis lentium numero habebimus $\frac{ma\Phi}{h} = 2$, unde fit $\Phi = \frac{2h}{ma}$ et $a\Phi = \frac{2h}{m}$. Data ergo multiplicatione m et distantia h, ad quam refertur semidiameter spatii in obiecto conspicui nunquam maior existera potest quam $\frac{2h}{m}$, quotcunque etiam adhibeantur lentes eaeque ita disponantur, ut maximum campum patefaciant. In Telescopiis ergo, ubi sumitur $h = a$ et semidiameter campi ex ipso angulo Φ aestimatur, eius tangens, nunquam maior esse potest quam $\frac{2}{m}$; unde sequentem tabellam adiungo, quae pro quavis multiplicatione semidiametrum campi apparentis maximi ostendit, quem nunquam superare
liceat.

Multiplicatio *m*	Semidiameter campi apparentis maximi	Multiplicatio *m*	Semidiameter campi apparentis maximi
5	21° 48'	60	1° 54' 33"
10	11 18	70	1 38 13
15	7 35	80	1 25 57
20	5 42	90	1 16 24
25	4 34	100	1 8 45
30	3 49	150	0 45 50
35	3 16$\frac{1}{2}$	200	0 34 22
40	2 52	250	0 27 30
45	2 33	300	1 2255
50	2 17$\frac{1}{2}$	400	0 17 11
		500	0 13 45

Quod si ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is ulterius nullo modo augeri poterit.

SCHOLION 2

286. Quod ad locum oculi idoneum attinet, eum ideo in O constituimus, ut omnes radios per lentes transmissos accipiat, etiamsi pupilla maxime esset constricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco sine ullo detrimento aliquantillum removeri posse, ita ut superfluum foret hunc locum nimis sollicite observare, nisi forte apertura ultimae lentis fuerit admodum magna. Sin ea autem pupilIam non superet eaque adeo sit minor, manifestum est oculum ei immediate applicatum aeque omnes radios excipere et eundem campum contueri, ac si in loco idoneo esset constitutus. His igitur casibus, si forte distantia O pro loco oculi prodeat negativa, nihil de campo apparente perit, dummodo oculus lenti ultimae immediate applicetur. His itaque, quae ad visionem per instrumenta dioptrica in genere pertinent, expeditis superest, ut investigemus, quantum visio ob diversam radiorum refrangibilitatem turbetur et quemadmodum hanc perturbationem evitare queamus.