CHAPTER VII

CONCERNING THE CONSTRUCTION OF DIOPTRIC INSTRUMENTS IN GENERAL

PROBLEM 1

329. *If a dioptric instrument may be composed from a single lens PP of any thickness, to define all the important properties pertaining to vision.*

SOLUTION

So that at first it is concerned with the structure of the lens itself, the distance of the object $E\varepsilon$ (Fig. 12 again) may be put before the lens at the distance $AE = a$ and the distance of the image $F\zeta$ projected after that $aF = \alpha$;

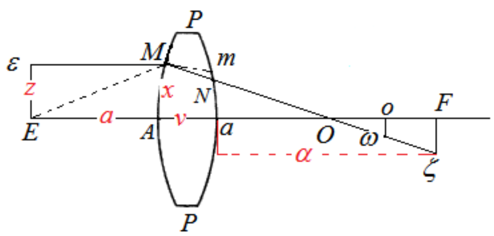

Fig. 12

then truly the thickness of the lens $Aa = v$ and the arbitrary quantity $= k$, from which there may be taken $\frac{k-v}{k+v} = i$. [Note: i is the ratio of the height of an arbitrary paraxial ray leaving the lens to that entering the lens.]
Hence the faces of the lens will be formed thus, so that there may be present $n = \frac{31}{20}$ (see § 68):

$$\text{radius of the anterior face } = \frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$$

$$\text{radius of the posterior face } = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$$

Again the radius of the object viewed may be put to be $E\varepsilon = z$ and the radius of the aperture in the anterior face $AM = x$, but in the posterior face the radius of the aperture shall not be smaller than *ix*. Finally the distance of the eye after the lens may be called $aO = O$, the true distance of which [*i.e.* distinct near vision for a nearby object, or a relaxed eye for a distant object] shall be $= l$. With these in place it will be required to consider the following matters of interest :

1. There must be $O=\alpha+l$, so that the distance of the image may be appropriate for viewing by the eye.

2. The multiplication [*i.e.* magnification] arising is required to be considered, which is defined from the ratio, which the diameter of the object viewed maintains to its diameter seen, if it may be viewed at a given distance $=h$ by the naked eye . So that therefore if here the exponent of the multiplication may be put $=m$, there will be

$$m=\frac{1}{i}\cdot\frac{\alpha h}{al}, \text{ for the inverted position.}$$

[That is, the magnification m is equal to the ratio of diameter seen through the lens at a distance l : diameter seen by the naked eye at a distance h . This ratio consists of 3 parts:
1. the ratio of the heights of the ray entering and leaving the lens, namely $\frac{1}{i}$;
2. the magnification effected by the lens, namely $\frac{\alpha}{a}$ from similar triangles for a thin lens;
3. the inverse ratio of the distances $\frac{l}{h}$, since halving the distance h doubles the size of the image viewed for paraxial.]

3. The degree of clarity may be determined from the radius of the light cone, which may be sent from some point in the object into the eye ; which if it may be put $=y$, will become

$$y=il\cdot\frac{x}{\alpha}=\frac{hx}{ma}.$$

$$[\text{Thus, } \frac{ix}{\alpha}\cdot l=\frac{\text{exit height}}{\alpha}\cdot l=\text{exit angle}\cdot l=\text{radius of cone } y.]$$

4. The confusion disturbing vision may be measured from the radius of the apparent circle, which is observed equally great by the naked eye and seen as individual points of the object through the lens. I have called this measure (§.194) the radius of confusion; for which being defined, if there may be put

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right),$$

the radius of confusion will become $=\frac{1}{4}i\cdot\frac{\alpha}{l}x^3P=\frac{1}{4}i^2\cdot\frac{ma}{h}x^3P.$

5. So that the eye may perceive the maximum apparent field of view, there must become

$$O=\frac{-i\alpha v}{n\alpha-iv}.$$

6. Which distance if it were positive, the radius of the field or of the object visible $E\varepsilon = -z$ thus depends on the aperture of the posterior face, so that on putting its radius $= \mathfrak{a}$, there shall be $z = \frac{n\alpha}{v}\mathfrak{a}$.

7. But if that assigned distance for O may have appeared negative, it will be agreed to place the eye next to the lens, so that there shall be $O = 0$; then truly for $\mathfrak{a}$ in place the radius of the pupil ω will be for the apparent field of view $z = \frac{n\mathfrak{a}}{v}\omega$.

8. In order that the object may appear without any colored fringe, with the distance O being found before positive, there must be $i = \frac{\alpha}{\alpha+v}$ or $k = 2\alpha + v$.

9. But if on account of that distance being produced negative there may be taken $O = 0$, in order that the colored fringe may disappear, it will be required that $\frac{v}{a} + 1 - i = 0$, from which there will become $k = -2\alpha - v$.

10. Finally all confusion from the different refrangibilities of all the rays may be removed at once, if in addition there may become $(\alpha + v)(a + \alpha + v) = 0$.

COROLLARY 1

330. If the exponent of the multiplication *m* [*i.e.* magnification] may be put in place with a level of greater clarity y, there will be $my = \frac{hx}{a}$, which property extends to any large number of lenses. Therefore since $\frac{h}{a}$ shall be a given quantity, x will be as *my;* then truly *y* will be as $\frac{x}{m}$ and *m* as $\frac{x}{y}$.

COROLLARY 2

331. Therefore just as a greater multiplication as well as a step of greater clarity demands a greater aperture. Truly with x increased the confusion is increased in the triplicate ratio, if indeed the quantity *P* may remain the same; whereby if *x* may be determined from the confusion and even now may be determined to be tolerable, likewise the quantity *my* is determined.

COROLLARY 3

332. Therefore so that both multiplication by *m* as well as the clarity *y* may be allowed to increase with the greatest safety without confusion, thus it will be agreed to define the arbitrary quantity k, so that the minimum value of the letter P may be acquired.

But since we may put $\frac{k-v}{k+v} = i$, the minimum value of P to be satisfying this equation will become:

$$2i^4\left(\tfrac{n}{\alpha}+\tfrac{1}{v}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)^2-\tfrac{2ni^3}{\alpha v}\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)-\tfrac{i^3}{v}\left(\tfrac{1}{\alpha}+\tfrac{2}{v}\right)^2$$
$$+\tfrac{2ni}{av}\left(\tfrac{1}{a}+\tfrac{1}{v}\right)+\tfrac{i}{v}\left(\tfrac{1}{a}+\tfrac{2}{v}\right)^2-2\left(\tfrac{n}{a}+\tfrac{1}{v}\right)\left(\tfrac{1}{a}+\tfrac{1}{v}\right)^2=0,$$

just as we found in § 45.

COROLLARY 4

333. If in place of i we may substitute its value $\frac{k-v}{k+v}=i$, the equation will be transformed into this form:

$$\tfrac{2n}{a^3}(k+v)^4+\tfrac{(2n+1)}{aav}(k+v)^3(k+3v)+\tfrac{4(n+2)}{av}(k+v)^3$$
$$-\tfrac{2n}{\alpha^3}(k-v)^4-\tfrac{(2n+1)}{\alpha\alpha v}(k-v)^3(k-3v)+\tfrac{4(n+2)}{\alpha v}(k-v)^3+32k=0,$$

which multiplied by v also can be applied to the case, where there is $v=0$:
but there is found

$$k=\frac{4(n+2)a\alpha}{(2n+1)(a-\alpha)}$$

and

$$P=\frac{n}{8(n-1)^2(n+2)}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left((4n-1)\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}\right).$$

SCHOLIUM

334. Here I do not consider generally all vision, which occurs with a single lens, but only that, by which the maximum field of view is observed; on account of which the position of the eye is to be defined thus, so that the maximum field of view must be brought to it. But if we may wish to be content with a smaller field of view, the eye put in place elsewhere after the lens likewise will be able to discern the object distinctly, just as it is accustomed to be done by large enough lenses, which are known by the name of tables of glasses. But this case I do not touch on here, since it may be set out easily by itself and the magnitude of the field of view to be considered chiefly in dioptric instruments.

PROBLEM 2

335. *If a dioptric instrument may be constructed from two lenses of any thickness, to define all the properties pertaining to vision.*

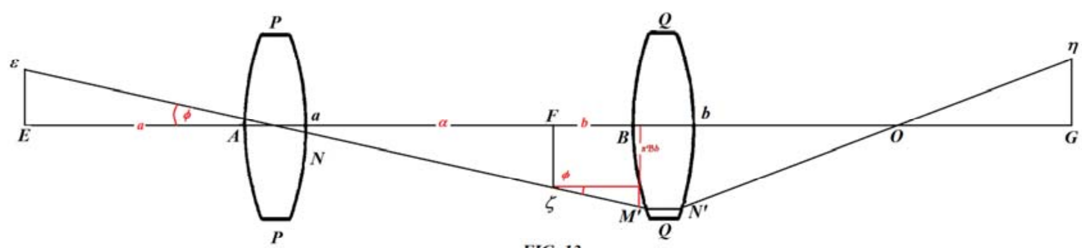

FIG. 13.

SOLUTION

With the object put in place at $E\varepsilon$ (Fig. 13 again) its images are projected through these lenses at $F\zeta$ and $G\eta$, and the quantities each lens determines are put in place

$AE = a$, $aF = \alpha$, thickness $Aa = v$ and the arbitrary distance $= k$,
$BF = b$, $bG = \beta$, thickness $Bb = v'$ and the arbitrary distance $= k'$,

and for the sake of brevity there is put $\frac{k-v}{k+v} = i$ and $\frac{k'-v'}{k'+v'} = i'$. Hence, with $n = \frac{31}{20}$, the construction of each lens itself will be had thus :

Radius of the face	anterior	posterior
for the lens PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k+v)}{k-v-2n\alpha}$
for the lens QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$.

Again the radius of the aperture of the first lens PP for the anterior face shall be $= x$, but in the posterior face it shall be greater than ix. Then truly the radius of the aperture of anterior face of the second lens QQ must be greater than $i.\frac{bx}{\alpha}$, truly in the posterior face greater than $ii'.\frac{bx}{\alpha}$. Then the radius of the object viewed shall be $E\varepsilon = z$, the distance of the eye $bO = O$ and its true distance $= l$. With these in place it will be required to attend to the following important details :

1. In order that the eye may look at the image $G\eta$ at the true distance [*i.e.* the point of distinct near-vision], there will be required to $O=\beta+l.$

2. With the distance put $=h$, to which the multiplication may be referred [*i.e.* magnification], the exponent of the multiplication $=m$, and above we have found $m=\frac{1}{ii'}\cdot\frac{\alpha\beta h}{abl}$ to be required for the erect situation.

3. With y indicating the order of the clarity, there will become

$$y=ii'l\cdot\frac{bx}{\alpha\beta} \text{ and thus } my=\frac{hx}{a}.$$

4. For the position with the least confusion therefore :

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right),$$
$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

there will be

$$\text{radius of confusion}=\frac{1}{4}ii'\cdot\frac{\beta}{l}\cdot\frac{b}{\alpha}x^3\left(\frac{1}{ii'}\cdot\frac{\alpha\alpha}{bb}P+ii\cdot\frac{bb}{\alpha\alpha}Q\right).$$

5. For the apparent field of view requiring to be defined, which depends on the apertures of the individual viewing lenses, if we may put

the radius of the aperture

for the lens PP $\begin{cases}\text{of the anterior face } =\mathfrak{A}=x\\ \text{of the posterior face}=\mathfrak{a}\end{cases}$

for the lens QQ $\begin{cases}\text{of the anterior face } =\mathfrak{B}=x\\ \text{of the posterior face}=\mathfrak{b},\end{cases}$

we will have the following equations :

$$\mathfrak{a}=\frac{v}{na}z,\quad \mathfrak{B}=\left(\frac{1}{i}\cdot\frac{\alpha+b}{a}-\frac{bv}{na\alpha}\right)z,\quad \mathfrak{b}=\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}\right)z,$$

from which three equations the minimum value of z produces the radius of the apparent field of view.

6. So that the eye actually may see which field of view, its position thus must be taken, so that there shall be

$$O=\frac{\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}}{\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}+\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}}\cdot\beta,$$

if indeed this distance were positive.

7. But if this distance were produced negative, it is agreed the eye be placed next to the lens *QQ*, so that there shall be $O=0$; then truly for the apparent field of view required to be found from the equations provided in n° 5, the radius of the pupil ω may be written in place of $\mathfrak{b}$, and from the same equations the minimum value of z elicited will give the radius of the apparent field of view.

8. Again the object will be perceived without a colored fringe, with the distance *O* found positive in n° 6 , if it may be satisfied by this equation:

$$0=1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}\left(1+\frac{b}{\alpha}\right)+\frac{(1-i')^2 b(\alpha+b)}{\alpha v'}+\frac{z-i-i'}{n}$$
$$-\frac{iv}{n\alpha}-\frac{i'v'}{n\beta}-\frac{ibv}{n\alpha\alpha}-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i(1-i')^2 bbv}{n\alpha\alpha v'}.$$

9. But if the distance may be taken $O=0$, the object will lack the colored fringe, if there were:

$$0=\frac{1-i}{in}+\frac{1-i'}{i'n}+\frac{v}{ina}+\frac{v'}{inb}+\alpha\left(1+\frac{\alpha}{iia}+\frac{(1-i)^2\alpha}{iiv}\right)\left(\frac{v'}{ni'bb}-\frac{1}{\alpha}-\frac{1}{b}\right).$$

10. Moreover all the confusion will be removed completely, which indeed arises from the different refrangibility of the rays, if it were possible to satisfy this equation

$d\beta=0$, or for this:

$$0=\frac{\alpha\alpha}{ii'bb}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)+\frac{i'}{\beta}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}.$$

COROLLARY 1

336. Because the separation of both lenses *aB* must be positive, it is required that there shall be $\alpha+b>0$. Then truly the apertures must be prepared thus, so that there shall be $\mathfrak{a}>ix$, $\mathfrak{B}>i\cdot\frac{bx}{\alpha}$ and $\mathfrak{b}>ii'\cdot\frac{bx}{\alpha}$.

COROLLARY 2

337. Therefore in the case, where on account of the value of *O* arising negative, the distance may be assumed to be $O = 0$, since then there may be put $\mathfrak{b} = \omega$, also there must be $\omega > ii' \frac{bx}{\alpha}$ or $x < \frac{\alpha\omega}{ii'b}$. Evidently in this case it may be supposed the first greater aperture to be useless, since the rays may not enter into the eye.

SCHOLIUM

338. If we may wish to pursue the construction of dioptric instruments with several lenses in the same way, we may slip into much more intricate calculations, so that from these hardly anything may be able to be concluded. But this complication of the calculation arises from the thickness of the lenses, with which ignored everything becomes much neater. Whereby if we wish to use several lenses, we may assume the thickness so very small, so that it may be able to have the value zero without error, which in practice is accustomed to be observed with care. Besides also the determination of the apparent field of view and the position of the eye cannot be taken with geometrical rigor, and to be of some concern, if there may vary a little in that. Similar too is the account of the condition, by which the effect arising from the different refrangibilities of the rays is removed ; indeed it may suffice for these to be satisfied approximately, since it may not indeed have been able to hope for a complete destruction of this confusion. On account of which in the following, where we will establish instruments constructed from several lenses, we will ignore the thickness of the lenses completely: in which matter lest we may have the need for so much repetition, we may set out such problems, in which generally we may express only all the important matters and hence we may describe these successively for any number of lenses.

PROBLEM 3

339. *If a dioptric instrument may be composed from any number of lenses, the thickness of which may be ignored, the essential elements of its construction to be put in place, from which in turn the rules directing the construction may be able to be established.*

SOLUTION

With the object put in place at *E*, the determinable distances of the individual lenses together with the arbitrary numbers pertaining to the individual lenses may be put in place, so that there follows :

For the first	lens	$EA = a$,	$aF = \alpha$,	arb. num.	$= \lambda$
" " second	"	$FB = b$,	$bG = \beta$,	" "	$= \lambda'$
" " third	"	$GO = c$,	$cH = \gamma$,	" "	$= \lambda''$
" " fourth	"	$HD = d$,	$dI = \delta$,	" "	$= \lambda'''$
" " fifth	"	$IE = e$,	$eK = \varepsilon$,	" "	$= \lambda''''$
" " sixth	"	$KF = f$,	$fL = \zeta$,	" "	$= \lambda'''''$

etc.,

in whatever manner the individual lenses must be put in place, is set out above.
Now also for the sake of brevity, we may put

$$\alpha = Aa,\ \ \beta = Bb,\ \ \gamma = Cc,\ \ \delta = Dd,\ \ \varepsilon = Ee,\ \ \zeta = Ff \text{ etc.,}$$

then truly there may be put also

$$\frac{A}{A+1} = \mathfrak{A},\ \ \frac{B}{B+1} = \mathfrak{B},\ \ \frac{C}{C+1} = \mathfrak{C},\ \ \frac{D}{D+1} = \mathfrak{D},\ \ \frac{E}{E+1} = \mathfrak{E},\ \ \frac{F}{F+1} = \mathfrak{F},\ \text{etc.,}$$

thus so that the focal distances of the lenses shall be :

$$\mathfrak{A}a,\ \ \mathfrak{B}b,\ \ \mathfrak{C}c,\ \ \mathfrak{D}d,\ \ \mathfrak{E}e,\ \ \mathfrak{F}f\ \ \text{etc.}$$

[here we have $\mathfrak{A}a = \frac{aA}{A+1} = \frac{a\frac{\alpha}{a}}{\frac{\alpha}{a}+1} = \frac{a\alpha}{\alpha+a}$
or, $\frac{1}{\mathfrak{A}a} = \frac{1}{a} + \frac{1}{\alpha}$, in agreement with the customary thin lens formula
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$. Again, if $a \to \infty$, $\mathfrak{A} \to 0$, and $\mathfrak{A}a \to \alpha$.
Again, it is evident the aperture of a lens is defined by the angle $\mathfrak{A}$
it subtends at the front focus of the lens,
so that $a\mathfrak{A} = focal\ length,$ etc.]

Now the radius of the aperture of the first objective lens shall be $= x$, truly for the remaining lenses the ratio of the apertures may be expressed by the letters $\pi, \pi', \pi'', \pi''', \pi''''$ etc, thus so that the radius of each aperture must be taken greater than these following ratios; clearly it will be required to take

radius of the aperture
of the second lens $> \pi\ \mathfrak{B}b$
" " third " $> \pi'\ \mathfrak{C}c$
" " fourth " $> \pi''\mathfrak{D}d$
" " fifth " $> \pi'''\ \mathfrak{E}e$
" " sixth " $> \pi''''\ \mathfrak{F}f$
etc.

[We may assume here initially that a ray passing through the centre of the first lens is considered to be undeviated, as the thickness of the lens is negligible; the angle of this ray to the axis is Φ, taken as φ in the modified Fig. 13: in which case $\Phi = \frac{z}{a}$. This ray proceeds undeviated to the position of the image at the distance α further along the axis, where the height of the image become $\alpha\Phi$. This ray continues to the second lens from the point ζ of this image by the horizontal distance b and the vertical distance Φ where it encounters the aperture or stop is inserted to transmit central rays only; Hence, $\pi\mathfrak{B}b = F\xi + b\Phi$; $b = \frac{F\zeta}{\pi\mathfrak{B}-\Phi} = \frac{\alpha\Phi}{\pi\mathfrak{B}-\Phi} = \frac{Aa\Phi}{\pi\mathfrak{B}-\Phi}$. The images alternate between being inverted and erect and proceed as in the table below.]

So that if now the radius of the space in which the object may be viewed may be called $= z$ and there may be put $\frac{z}{a} = \Phi$, so that there shall become $z = a\Phi$, hence the determinable distances may be expressed thus, so that there shall be in the first place $\alpha = Aa$, then truly :

$$b = \frac{Aa\Phi}{\mathfrak{B}\pi-\Phi}; \qquad \beta = \frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}$$
$$c = \frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}; \qquad \gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$
$$d = \frac{ABCa\Phi}{\mathfrak{C}\pi''-\pi'+\pi-\Phi}; \qquad \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$
$$e = \frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}; \qquad \varepsilon = \frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$
$$f = \frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}; \qquad \zeta = \frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}$$
etc.

Concerning these expressions first it is required to be observed the sums $\alpha+b,\ \beta+c,\ \gamma+d,\ \varepsilon+e,\ \varepsilon+f$ etc. are required to be positive, clearly from which the intervals between the lenses are expressed. Evidently there will be

Separation of the lenses

I and II $= \frac{Aa\mathfrak{B}\pi}{\mathfrak{B}\pi-\Phi} > 0$

II and III $= \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$

III and IV $= \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$

IV and V $= \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$

V and VI $= \frac{ABCDEa\Phi(\mathfrak{F}\pi''''-(1-\mathfrak{E})\pi''')}{(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)} > 0$

etc.

In addition so that all the following lenses with the first excepted may transmit all the rays, there must be the radius of the aperture of the

second lens $> \frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

third lens $> \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

fourth lens $> \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

fifth lens $> \frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}x$

sixth lens $> \frac{\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Finally from § 214 on putting $\mu = 0,938191$ and $\nu = 0,232692$ the values of the capital letters *P, Q, R* etc. will be

$$P = \frac{\mu}{A^3a^3}(A+1)(\lambda(A+1)^2 + \nu A)$$
$$Q = \frac{\mu}{B^3b^3}(B+1)(\lambda'(B+1)^2 + \nu B)$$
$$R = \frac{\mu}{C^3c^3}(C+1)(\lambda''(C+1)^2 + \nu C)$$
$$S = \frac{\mu}{D^3d^3}(D+1)(\lambda'''(D+1)^2 + \nu D)$$
$$T = \frac{\mu}{E^3e^3}(E+1)(\lambda''''(E+1)^2 + \nu E)$$
$$V = \frac{\mu}{F^3f^3}(F+1)(\lambda'''''(F+1)^2 + \nu F)$$

etc.

Thus with these denominations in place *O* shall be the distance of the eye past the final lens, of which the true distance may be put $= l$. Then the exponent of the multiplication [magnification] may be put $= m$, relative to the distance *h*, thus so that the diameter of the object may be discerned seen through the instrument to be greater by *m*, than if the object may be seen at the distance $= h$ by the naked eye ; but in addition to the multiplication it will be agreed by indicating the situation, whether the object shall going to appear erect or inverted. Again the order of clarity *y* will denote the radius of the luminous cone, which is transmitted from some point on the object into the eye with the radius of the pupil put $= \omega$. Finally I call the radius of confusion the apparent radius of the circles, which are described of equal size seen by the naked eye, or the individual points of the object seen by the eye through the instrument.

COROLLARY 1

340. Since we have found double limits for the apertures of individual lenses, the radius of the aperture of each is assumed most conveniently equal to the sum of both limits or perhaps not less. But each limit, even if perhaps the other may be produced negative, must be accepted to be positive.

COROLLARY 2

341. Hence therefore we place together the following formulas for the apertures of the individual lenses :

Radius of the aperture

of the second lens $= \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

" third lens $= \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

" fourth lens $= \frac{ABC\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$

" fifth lens $= \frac{ABCD\mathfrak{E}a\pi''' \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$

" sixth lens $= \frac{ABCDE\mathfrak{F}a\pi''' \pm x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi} \cdot \Phi$

etc.

COROLLARY 3

342. From the two determinable distances with the arbitrary number any lens is constructed easily; as we may recall from above for the first lens. Evidently if there shall be $\lambda > 1$, a simple lens may be satisfied, the construction of which put in place for the sake of brevity

$$\rho = 0{,}190781, \quad \sigma = 1{,}627401 \text{ and } \quad \tau = 0{,}905133$$

there is had:

anterior posterior

$$\text{Radius of the face } \frac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}; \quad \frac{a\alpha}{\rho\alpha + \sigma a \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

or with the numbers substituted, if there shall be $\lambda = 1 + v$,

$$\text{radius of the face} \begin{cases} \text{anterior} = \dfrac{a\alpha}{+0{,}190781\alpha + 1{,}627401a \pm 0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterior} = \dfrac{a\alpha}{+1{,}627401\alpha + 0{,}190781a \mp 0{,}905133(a+\alpha)\sqrt{v}}. \end{cases}$$

COROLLARY 4

343. But if there shall be $\lambda < 1$, but still $> 0{,}191827$, the lens is required to be double or compounded from two simple lenses, the construction of which, if there may be put

$$\lambda = 0{,}191827 + v,$$

thus itself is had:

For the lens		Radius of the face
first	anterior	$=\dfrac{a\alpha}{-0{,}622919\alpha+0{,}813700a\pm 0{,}905133(a+\alpha)\sqrt{v}}$
	posterior	$=\dfrac{a\alpha}{+1{,}532010\alpha+0{,}095390a\mp 0{,}905133(a+\alpha)\sqrt{v}}$
second	anterior	$=\dfrac{a\alpha}{+0{,}095390\alpha+1{,}532010a\pm 0{,}905133(a+\alpha)\sqrt{v}}$
	posterior	$=\dfrac{a\alpha}{+0{,}813700\alpha-0{,}622919a\mp 0{,}905133(a+\alpha)\sqrt{v}}$.

COROLLARY 5

344. But if there shall be $\lambda = 0,042165 + v$, with the lens required to be used is the triple thus to be constructed:

For the lens		Radius of the face
first	anterior	$=\dfrac{a\alpha}{-0{,}894153\alpha+0{,}542467a\pm 0{,}905133(a+\alpha)\sqrt{v}}$
	posterior	$=\dfrac{a\alpha}{+1{,}500214\alpha+0{,}06354a\mp 0{,}905133(a+\alpha)\sqrt{v}}$
middle	anterior	$=\dfrac{a\alpha}{-0{,}415280\alpha+1{,}021340a\pm 0{,}905133(a+\alpha)\sqrt{v}}$
	posterior	$=\dfrac{a\alpha}{+1{,}021340\alpha-0{,}415280a\mp 0{,}905133(a+\alpha)\sqrt{v}}$
final	anterior	$=\dfrac{a\alpha}{+0{,}063594\alpha+1{,}500214a\pm 0{,}905133(a+\alpha)\sqrt{v}}$
	posterior	$=\dfrac{a\alpha}{+0{,}542467\alpha-0{,}894135a\mp 0{,}905133(a+\alpha)\sqrt{v}}$.

COROLLARY 6

345. Finally if there shall be $\lambda = -0,010216 + v$, the lens requiring to be made is quadruple requiring to be constructed thus:

For the lens

Radius of the face

first $\begin{cases} \text{anterior} = \dfrac{a\alpha}{-1{,}029770\alpha+0{,}406850a\pm0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterior} = \dfrac{a\alpha}{+1{,}484315\alpha+0{,}047695a\mp0{,}905133(a+\alpha)\sqrt{v}} \end{cases}$

second $\begin{cases} \text{anterior} = \dfrac{a\alpha}{-0{,}670615\alpha+0{,}766005a\pm0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterior} = \dfrac{a\alpha}{+1{,}125160\alpha-0{,}311460a\mp0{,}905133(a+\alpha)\sqrt{v}} \end{cases}$

third $\begin{cases} \text{anterior} = \dfrac{a\alpha}{-0{,}311460\alpha+1{,}125160a\pm0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterior} = \dfrac{a\alpha}{+0{,}766005\alpha-0{,}670615a\mp0{,}905133(a+\alpha)\sqrt{v}}. \end{cases}$

fourth $\begin{cases} \text{anterior} = \dfrac{a\alpha}{+0{,}047695\alpha+1{,}484315a\pm0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterior} = \dfrac{a\alpha}{+0{,}406850\alpha-1{,}029770a\mp0{,}905133(a+\alpha)\sqrt{v}}. \end{cases}$

SCHOLIUM

346. The matters which have been brought forwards first in these corollaries concerned with the construction of lenses, are seen to be easily adapted by the change of the letters a and α for the remaining lenses, in place of which, if the use may require it, so that the numbers λ', λ'' etc. shall be smaller than unity, also the lenses will have to be either double, triple, or quadruple. Furthermore these formulas extend to any number of lenses, thus so that for any proposed number of lenses so many letters shall be omitted, which will pertain to the following lenses. But in the denominations we have introduced at once the apparent field of view contained by the number Φ, since the radius shall be of the object viewed $z = a\Phi$. Yet meanwhile the apparent field of view may not be allowed to increase freely, if indeed that may be determined by reason of the multiplication and may be limited by the numbers π, π', π'' etc. But these numbers always stop less than $\frac{1}{2}$, or indeed $\frac{1}{3}$ and generally $\frac{1}{4}$, or thus cannot exceed $\frac{1}{6}$; whenever also they must be taken smaller, which in any case the formulas of corollary 2 will declare, from which the radius of each lens is expressed. Whereby with these denominations retained we may set out the main points for the determination of the construction by any number of lenses

PROBLEM 4

347. *If a dioptric instrument may be composed from two lenses, of which the thickness may be allowed to be ignored, to define the principles, by which the construction may be secured.*

SOLUTION

Since here the second lens is the final one, the principles of the construction thus will be had :

1. In order that the eye may view the image at the true distance of the eye, there must be $O=\beta+l$ and thus

$$O=l+\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi},$$

and from the condition of the apparent field of view :

$$O=\frac{\mathfrak{B}b\pi}{\pi-\Phi}.$$

2. The exponent of the multiplication *m* relative to the distance *h* produces this equation

$$m=AB\frac{h}{l} \text{ for the erect situation;}$$

but if the inverse situation may be desired, the number *m* is agreed to be taken negative.

3. For the order of clarity there is always $y=\frac{hx}{ma}$.

4. For the apparent field of view this equation will be had:

$$\pi-\Phi=\frac{-ma\Phi}{h}, \text{ from which there shall be } \Phi=\frac{-\pi h}{ma-h}.$$

5. If the distance O hence may be produced negative, there must be taken $O=0$ or $\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}=-l$; then truly π induces the same value, so that the aperture of the lens of the eye may not be greater than the aperture of the pupil; namely there will become $\pi\mathfrak{B}b=\omega$ and hence

$$\Phi=\frac{\mathfrak{B}\pi\omega}{A\mathfrak{B}a\pi+\omega} \quad \text{or} \quad (B+1)\omega=-\pi l.$$

6. The radius of confusion is expressed by the following formula :

$$\frac{Bbb}{4Aal}x^3\left(\frac{AAaa}{bb}P+\frac{bb}{AAaa}Q\right),$$

which with the substitutions introduce will turn into this :

$$\frac{\mu mx^3}{4aah}\left(\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\right).$$

7. In order that the object may appear without a colored fringe, if the distance O were positive, by necessity there shall be $\frac{\pi}{\mathfrak{B}\pi-\Phi}=0$.

8. But if there may be taken $O=0$, there must become $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ or $\frac{\pi}{\mathfrak{A}\Phi}=0$.

9. But all the confusion may be removed completely arising from the diverse refrangibility of the rays, if in the above there were $\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0$.

10. But in addition there will be the separation of the lenses $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, which must be positive.

11. Finally the radius of the aperture of the lens of the eye is $\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$, where of the sign $\pm$ that one must be taken, so that it will produce the maximum value, either positive or negative.

COROLLARY 1

348. Since there shall be (n$^{\text{o}}$ 4) $\Phi=\frac{-\pi h}{ma-h}$ and on account of the multiplication $B=\frac{ml}{Ah}$ and hence $\mathfrak{B}=\frac{ml}{ml+Ah}$, there will be had for the position of the eye $O=\frac{-Ahl(ma-h)}{mmal+Ahh}$.

Then truly there is $\mathfrak{B}\pi-\Phi=\frac{ml\pi}{ml+Ah}+\frac{\pi h}{ma-h}=\frac{\pi(mmal+Ahh)}{(ma-h)(ml+Ah)}$, from which the separation of the lenses will become

$$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}=\frac{mAa(ma-h)}{mmal+Ahh}l.$$

COROLLARY 2

349. Since there shall be $\frac{A}{A+1}=\mathfrak{A}$ and $A=\frac{\mathfrak{A}}{1-\mathfrak{A}}$, the radius of confusion thus can be expressed, so that there shall become

$$\frac{\mu mx^3}{4aah}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi\left(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B})\right)}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}\right).$$

COROLLARY 3

350. Since the separation of the lenses shall be $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, the object will be unable to be discerned without color, unless the separation of the lenses may vanish; if indeed the eye may be held in that place, which the apparent field of view demands. And because it is unable for this condition to be satisfied, much less total confusion will be able to be removed from n° 9 .

SCHOLIUM

351. I have began here at once with instruments constructed from two lenses, since it may be readily arranged as a single lens. For in the first place it is necessary that there shall be

$$O = l + \alpha = l + Aa.$$

Then truly the magnification produces $m = \frac{h}{a}$ for the erect position, and the apparent field of view is not restricted by assuming $Aa + l = 0$ or $O = 0$, thus so that the eye must be applied next to the lens. Truly the radius of confusion will be

$$= \frac{\alpha}{4l} x^3 \cdot P = \frac{Aax^3}{4l} \cdot P = -\frac{1}{4} x^3 \cdot P = \frac{-max^3}{4h} \cdot P \text{ since } l = -Aa \text{ and } ma = h.$$

Whereby the radius of confusion will be

$$\frac{\mu mx^3}{4aah}\left(\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}\right) = \frac{\mu mx^3}{4aah} \cdot \frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}.$$

Moreover so that for the degree of clarity there is always had $y = \frac{hx}{ma}$.

Again the object will be discerned without a colored fringe ; but the whole confusion cannot be removed, unless there shall be $\frac{1}{\mathfrak{A}} = 0$ or $A = -1$, which is contrary to the purpose of lenses. Whereby we may progress to the consideration of more lenses.

PROBLEM 5

352. *If a dioptric instrument may be constructed from three lenses, of which the thickness shall be so small that it may be ignored, to define all the principle concerns necessary to be taken into account for the construction.*

SOLUTION

Therefore here the third lens will be the final one or the eyepiece, and thus the main points will be had in the following manner:

1. In order that the image required to be seem may be offered to the eye at the true distance, there must be $O=\gamma+l$ and with the value substituted in place of γ :

$$O=l+\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

and from the condition of the apparent field of view :

$$O=\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}.$$

2. The exponent of the multiplication *m* relative to *h* presents this equation:

$$m=-\frac{ABCh}{l} \text{ for the upright position,}$$

from which if the inverse may be desired, the negative number *m* is agreed to be taken.

3. We have for the degree of clarity as always, $y=\frac{hx}{ma}$.

4. But for defining the apparent field of view we will have this equation :

$$\Phi=\frac{-\pi+\pi'}{ma-h}h.$$

5. But if the distance *O* hence may be produced negative, so that it may be required to take $O=0$ and thus $\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}=-l$, the apparent field of view must be defined from this equation $\pi'\mathfrak{C}c=\omega$ or

$$\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi'-\pi+\Phi}=\omega \quad \text{or} \quad (C+1)\omega=-\pi'l.$$

6. The radius of confusion truly may be expressed thus:

$$\frac{Ccc}{4ABal}x^3\left(\frac{AABBaa}{cc}P+\frac{BBb^4}{AAaacc}Q+\frac{cc}{AABBaa}R\right),$$

where on account of $\frac{1}{l}=\frac{-m}{ABCh}$ will become with the sign changed :

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}\\+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\+\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\end{array}\right).$$

7. So that the object may appear without the colored fringe, if indeed the distance O will have been produced positive, there is required to become :

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}=0.$$

8. But if on account of the same distance being produced negative there may be taken $O=0$, the colored fringe may vanish on making :

$$\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)}.$$

9. So that if the colored fringe may be allowed to be removed, in addition the vision will be freed from all confusion, if this equation may be satisfied:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=0.$$

10. Finally it must be put into effect, that all the separations of the lenses may become positive, from which there will be had :

Separations of the Lenses

I and II $=\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0$ [see § 348.]; II and III $=\frac{ABa\Phi\left(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi\right)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0.$

11. Finally it is required to take the numbers π and π' thus, so that the apertures of the lenses may not become exceedingly larger; truly there is :

The radius of the aperture

of the second lens $=\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$, of the third lens $=\frac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi.$

COROLLARY 1

353. Since there shall be $C=\frac{-ml}{ABh}$ and thus $\mathfrak{C}=\frac{-ml}{ABh-ml}$ and $\Phi=\frac{-\pi+\pi'}{ma-h}\cdot h$, if these values may be substituted, for the position of the eye there will be

$$O=\frac{ABhl(ma-h)\pi'}{(mmal-ABhh)\pi'+ma(ABh-ml)\pi}.$$

COROLLARY 2

354. The radius of confusion can be expressed thus also, so that there shall be

$$\frac{\mu mx^3}{4aah}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}+\frac{\Phi(\lambda''+\nu\mathfrak{C}(1-\mathfrak{C}))}{A^3B^3\mathfrak{C}^3(\mathfrak{C}\pi'-\pi+\Phi)}\right),$$

thus it is now evident the form of which to be in accord with the preceding, so that it may be extended easily to more lenses.

PROBLEM 6

355. *If a dioptric instrument shall be constructed from four lenses, the thickness of which does not merit to be introduced into the calculation, to define all the principles necessary to be directed for the construction.*

SOLUTION

Since here the fourth is the final lens,

1. So that the final image may be offered to the eye at the true distance, there must be $O=\gamma+l$ and thus,

$$O=l+\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

and from which condition the apparent field of view

$$O=\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}.$$

2. The relative exponent of the magnification [multiplication] m for the distance h, so that if at this point the object may be assumed to be shown situated upright, will be

$$m=+ABCD\cdot\frac{h}{l} \text{ for the erect position;}$$

but if the inverse position may be desired, the number *m* is required to be taken as negative.

3. For the degree of clarity there is obtained always $y=\frac{hx}{ma}$.

4. But the apparent field of view must be obtained from this equation

$$\Phi=\frac{-\pi+\pi'-\pi''}{ma-h}\cdot h \;;$$

hence indeed $a\Phi$ will produce the radius of the area visible in the object, if indeed the distance O were positive.

5. But if the distance O were produced negative, in which case it would be required to take $O=0$ of $\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=-l,$ the quantity Φ must be determined from this equation $\pi''\mathfrak{D}d=\omega$, or this

$$\frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=\omega, \quad \text{or} \quad (D+1)\omega=-\pi''l.$$

6. The radius of confusion thus expressed will be found :

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\+\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\end{array}\right).$$

7. In order that the object may appear without a colored fringe, if indeed the distance O shall be positive, there will be required to become :

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0.$$

[Thus, Euler is telling us that the colored fringes may be made to cancel by adjusting the fractions of the apertures of the lenses to zero: this of course cannot happen, the colored fringes will persist: the physical phenomena of dispersion cannot be made to vanish by means of algebraic manipulations, as Dollond pointed out to Euler at the time. Originally published in *Philosophical Transactions (London)* 48, 1754, pp. 287-296. Euler later (1761) to his credit produced the mathematics for the achromatic lens doublet (see E266).]

8. Truly if we may accept $O=0$, the colored fringe will vanish by satisfying this equation:

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}.$$

9. Moreover all the confusion arising from the diverse nature of all the rays may be removed completely, if it may be permitted for this equation to be satisfied :

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0.$$

10. Finally it is required to put into effect, that the separations of the lenses shall be positive, from which the following formulas arise:

Separation of the lenses

$$\text{I and II } =\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0\ ;\ \text{II and III } =\frac{ABa\Phi\left(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi\right)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0.$$

$$\text{III and IV } =\frac{ABCa\Phi\left(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi'\right)}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0.$$

$$\text{IV and V}=\frac{ABCDa\Phi\left(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi''\right)}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}>0.$$

11. Moreover the apertures of the individual lenses are required to be taken thus, so that there shall be :

Radius of the aperture

$$\text{of the second lens}=\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$$

$$\text{"}\quad\text{third}\quad\text{"}\quad=\frac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi$$

$$\text{"}\quad\text{fourth}\quad\text{"}\quad=\frac{ABC\mathfrak{D}a\pi''\pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\cdot\Phi.$$

COROLLARY 1

356. Since on account of the magnification, there shall be $D=\frac{ml}{ABCh}$ and thus $\mathfrak{D}=\frac{ml}{ABCh+ml}$ and $\Phi=\frac{-\pi+\pi'-\pi''}{ma-h}\cdot h$, this expression will be had for the position of the eye:

$$O=\frac{-ABChl(ma-h)\pi''}{(mmal+ABChh)\pi''-ma(ABCh+ml)(\pi'-\pi)}.$$

[Recall that *h* is the distance of the object viewed by the naked eye.]

COROLLARY 2

357. Since here all the letters occurring are required to be determined, it may be seen to be no doubt, why the formulas contained in n° 7 , 8 and 9 may not be able to be satisfied, and at any rate that can certainly happen, if more kinds of refracting mediums may be used.

PROBLEM 7

358. *If a dioptric instrument shall be constructed from five lenses, the thickness of which may be able to be ignored on account of smallness, to define all the principle concerns regarding the construction.*

SOLUTION

Since here the fifth lens may hold the final position, we will have:

1. For the position of the eye, so that the image viewed may be offered at the actual true position, there must be $O=\varepsilon+l$ and thus

$$O=l+\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi},$$

and from the property of the apparent field of view

$$O=\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}.$$

2. The exponent of the multiplication m relates to the distance $=h$, if indeed the object may be required to be seen upright,

$$m=-ABCDE\cdot\frac{h}{l};$$

but for the inverse situation the number m must be taken negative.

3. For the degree of clarity thus there is $y=\frac{hx}{ma}$.

4. The apparent field of view must be defined from this equation

$$\Phi=\frac{-\pi+\pi'-\pi''+\pi'''}{ma-h}\cdot h,$$

if indeed that distance O will have been produced positive.

5. But if this distance may be produced negative, so that there may be taken $O=0$ or $\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=-l$, the apparent field of view must not be defined from the formula n^o 4, but from this one $\pi'''\mathfrak{E}e=\omega$, from which there becomes

$$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=\omega \quad \text{or} \quad (E+1)\omega=-\pi'''l.$$

6. The radius of confusion arising from the apertures of the lenses is expressed in the following manner:

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{c}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\ +\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\\ +\frac{\Phi(E+1)\left(\lambda''''(E+1)^2+\nu E\right)}{A^3B^3C^3D^3E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\end{array}\right).$$

7. So that the object may appear without a colored fringe, if indeed the distance O were positive, will require to be satisfying this equation:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0.$$

8. But if the distance O shall be negative and there may be taken $O=0$, for this aim to be obtained there shall be required to be:

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=\frac{1}{ABCD\mathfrak{D}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right).$$

9. But with all the confusion removed completely from the nature of the different rays, if it were possible to become

$$\begin{array}{l}\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\\ +\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0.\end{array}$$

10. Then truly this being required to be done, so that all the intervals between the lenses shall be positive, from which the following formulas arise:

The interval of the lenses

$$\text{I ad II } =\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0 \ ; \ \text{II and III } =\frac{ABa\Phi\left(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi\right)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0.$$

$$\text{III and IV } = \frac{ABCa\Phi\left(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi'\right)}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0.$$

$$\text{IV and V} = \frac{ABCDa\Phi\left(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi''\right)}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0.$$

11. Finally the apertures of the individual lenses thus may be taken, so that there shall be :

The radius of the aperture

$$\text{of the second lens} = \tfrac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$$

$$\text{" " third " } = \tfrac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi$$

$$\text{" " fourth " } = \tfrac{ABC\mathfrak{D}a\pi''\pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\cdot\Phi$$

$$\text{" " fifth " } = \tfrac{ABCD\mathfrak{E}a\pi'''\pm x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\cdot\Phi.$$

COROLLARY

359. On account of the multiplication therefore there will be :

$$E = \tfrac{-ml}{ABCDh} \text{ and } \mathfrak{E} = \tfrac{-ml}{ABCDh-ml},$$

from which there becomes :

$$O = \frac{ABCDhl(ma-h)\pi'''}{(mmal-ABCDhh)\pi'''+ma(ABCDh-ml)(\pi''-\pi'+\pi)}.$$

SCHOLIUM

360. Thus far I have assumed the eye either to be kept in that place, which demands the apparent field of view, or to be placed next to the final lens; of which in each case it is required to effect, that the final image may be offered to the eye at the true distance [for the most distinct vision]. Since this position of the eye, which provides the field of view, I have taken together at once with the true distance of the image required to be seen; but it can be defined also without being had either with regard to this distance or according to a magnification. Indeed from above (§ 271) we have :

For the case of

one lens $O = 0$

two lenses $O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$

three lenses $O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$

four lenses $O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

five lenses $O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$

etc.

Truly if we do not wish to consider the apparent field of view, it will be possible always to assign a place for the eye of such a kind, that the object may be discerned without a colored fringe. I cannot decide how this distance is to be determined without considering these matters, since in this case its distance from the location of the eye will be able to be judged more easily on account of the consideration of the field of view assumed, by how great a colored fringe the object may appear to be surrounded, if the eye may be maintained in that latter position. If indeed it were unable to satisfy the equation $n°$ 7 nearby, we understand nothing else, other than that the object is not going to appear without a colored fringe; but if in addition that place may be agreed, from which the object may be seen without a fringe of this kind, we will be able to collate the size of its margin.

PROBLEM 8

361. *If a dioptric instrument may be constructed from some number of lenses, the thickness of which may be neglected, to assign that place for the eye, from which the object may be viewed without a colored fringe.*

SOLUTION

This distance of the eye past the final lens may be put $=\Omega$, and from the given equations in §325 , if for O there may be written Ω, it will be possible to elicit this same distance Ω, which we seek. Just as for any number of lenses there will be, as follows :

I. For one lens

This equation will be had $\frac{\Omega d\alpha}{\alpha(\alpha-\Omega)} = 0$, from which there shall be

$\frac{\Omega}{\alpha(\alpha-\Omega)} = 0$ or $\frac{\Omega}{\alpha-\Omega} = 0.$

Truly also the apparent field of view emerges $O = 0$, from which in this case with both the position of the eye and O and Ω agreed on nor is there any need to be concerned about the colored fringe.

II. For two lenses

In this case this same equation is found $\frac{\Omega d\beta}{\beta(\beta-\Omega)} = d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, from which with the above assigned values substituted there arises

$$\frac{\Omega}{\beta(\beta-\Omega)} = \frac{\pi}{\mathfrak{A}\Phi} : (B+1)\, ABa\left(\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right),$$

or, if we may put

$$Y = \frac{\pi}{\mathfrak{A}\Phi} \quad \text{and} \quad Z = \frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)},$$

there will become

$$\frac{\Omega}{\beta-\Omega} = \frac{Y}{AB(B+1)aZ}\cdot\beta = \frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\mathfrak{B}\pi-\Phi}.$$

III. For three lenses

In the same manner, if we may put

$$Y = \frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} - \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)}$$

and

$$Z = \frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\Phi)},$$

for the case of three lenses there will become

$$\frac{\Omega}{\gamma-\Omega} = \frac{Y}{ABC(C+1)aZ}\cdot\gamma = \frac{Y}{(C+1)Z}\cdot\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi},$$

where it is required to be observed $Y = 0$ and $Z = 0$ to be these same equations, which we have reported on in the above problems under numbers 8 and 9.
[These being § 347 for two lenses, § 352 for three, § 355 for four, and § 358 for five lenses.]

IV. For four lenses

If now there may be put

$$Y = \frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} + \frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi-\Phi)} - \frac{1}{ABC\mathfrak{D}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi} + \frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\right)$$

$$Z = \frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)} + \frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)},$$

there will be

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{ABCD(D+1)aZ}\cdot\delta$$

and hence

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{(D+1)Z}\cdot\frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.$$

V. For five lenses

On putting the following equations shown in n° 8 and 9 (§ 358)

$$Y=\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$-\frac{1}{ABC\mathfrak{D}\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$$

we will have

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{ABCDE(E+1)aZ}\cdot\varepsilon$$

or

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{(E+1)Z}\cdot\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.$$

The law of these formulas for any greater number of lenses hence is clear enough.

COROLLARY 1

362. Therefore since above for any number of lenses we have shown the equations *n*° 8 and n° 9, the formula n° 8 will give the value Y and the formula n° 9 will give its value Z, with which known the position of the eye may become known easily, where the colored fringe vanishes.

COROLLARY 2

363. If in addition from equation n° 7 (§ 358) there may be set out

$$X=\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi},$$

certainly there will be for the case of five lenses

$$Y=\frac{\pi'''}{\Phi}Z-\frac{1}{ABCD\mathfrak{E}}X.$$

for four lenses $Y=\frac{\pi''}{\Phi}Z-\frac{1}{ABC\mathfrak{D}}X$

for three lenses $Y=\frac{\pi'}{\Phi}Z-\frac{1}{AB\mathfrak{C}}X$

for two lenses $Y=\frac{\pi}{\Phi}Z-\frac{1}{A\mathfrak{B}}X.$

SCHOLIUM

364. So that this location of the eye designated by the letter Ω may be allowed to be compared with the preceding letters designated by the letter O (§ 271), it is required to be noted :

For the case
of one lens: $\frac{O}{\alpha-O}=0,$

of two lenses: $\frac{O}{\beta-O}=\frac{\pi}{(B+1)(\mathfrak{B}\pi-\Phi)},$

of three lenses: $\frac{O}{\gamma-O}=\frac{\pi'}{(C+1)(\mathfrak{C}\pi'-\pi+\Phi)},$

of four lenses: $\frac{O}{\delta-O}=\frac{\pi''}{(D+1)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)},$

of five lenses: $\frac{O}{\varepsilon-O}=\frac{\pi'''}{(E+1)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)},$

etc.

Whereby so that they may agree with both places retained for any number of lenses with the letters Y and Z in the manner used, there shall be required:

For one lens $O=0$

for two lenses $\frac{Y}{Z}=\frac{\pi}{\Phi}$

for three lenses $\frac{Y}{Z}=\frac{\pi'}{\Phi}$

for four lenses $\frac{Y}{Z}=\frac{\pi''}{\Phi}$

for five lenses $\frac{Y}{Z}=\frac{\pi'''}{\Phi}$
etc.

from which generally there shall be $X = 0$, which equation n° 7 itself has brought forwards.

Therefore these are the general principles, from which the construction of dioptric instruments will be perfected, which instruments here indeed I have adapted chiefly for vision. Truly nothing less for the representation of objects in the camera obscura on a white screen can be used, where besides now the given precepts is to be extended to these effigies also to become apparent without a colored fringe, if equation $n°$ 7 may be satisfied, truly all the confusion arising from the diverse nature of the rays to be removed completely, if in addition it may satisfy the equations in n° 9.

SUPPLEMENT VII

If the ratio of refraction were different for individual lenses and the following order of the letters n, n', n'', n''' etc. of the lenses may be shown, then for the construction of lenses:

I. The corresponding letters ρ, σ, τ and for the confusion the letters μ and ν must be defined conveniently according to the formulas given above in § 55:

$$\rho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1, \quad \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1,$$

$$\tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)},$$

thus, so that there shall be

$$\rho + \sigma = \frac{1}{n-1} \quad \text{and} \quad \sigma - \rho = \frac{-2}{n+2} + 2 = \frac{2(n+1)}{n+2},$$

but again

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}, \quad \nu = \frac{4(n-1)^2}{4n-1},$$

from which it is understood, in what manner the corresponding letters $\rho', \sigma', \tau', \mu', \nu'$ will have to be defined from the ratio of refraction n'.

II. Because now with regard to the occasional element of the apparent field of view introduced π, π', π'' etc., the determinable distances of the lenses remain determined in the same manner as above, thus so that there shall be no need to transcribe these formulas here; meanwhile only the fundamental formulas will have to be remembered, from which these have arisen, which are :

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{Aa}{b}$$

$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{ABa}{c}$$

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{ABCa}{d}$$

$$\frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{ABCDa}{e}$$

etc.

III. Truly the values of the letters *P, Q, R* etc. are changed on account of the different refractions, as follows :

$$P=\frac{\mu}{A^3a^3}(A+1)\left(\lambda(A+1)^2+\nu A\right)$$
$$Q=\frac{\mu'}{B^3b^3}(B+1)\left(\lambda'(B+1)^2+\nu' B\right)$$
$$R=\frac{\mu''}{C^3c^3}(C+1)\left(\lambda''(C+1)^2+\nu'' C\right)$$
$$S=\frac{\mu'''}{D^3d^3}(D+1)\left(\lambda'''(D+1)^2+\nu''' D\right)$$

etc.

IV. Also the separations of the lenses remain as before together with the radii of the apertures of the individual lenses ; but the radii of the two faces of each lens are required to be adapted to the present case, so that for the lens, of which the refraction is n', the corresponding letters ρ', σ', and τ' for that will be required to be changed, and in a similar manner also for the following lenses, of which the refraction is indicated by the letters n'', n''' etc.

From these premises the individual principles, which are required to be observed in the construction of dioptric instruments, we will show in the following manner:

I. For the position of the eye or its distance past the final lens $= O$

we will have the following determinations for the individual number of lenses:

Number of lenses		
I	$O = 0 = \alpha + l$	and thus $\alpha = -l = Aa$
II	$O = \frac{\mathfrak{B}b\pi}{\pi-\Phi} = \beta + l$	and thus $\beta = \frac{\mathfrak{B}b\pi}{\pi-\Phi} - l = Bb$ hence $l = \frac{-Bb(\mathfrak{B}\pi-\Phi)}{\pi-\Phi}$;
III	$O = \frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi} = \gamma + l$	hence $\gamma = \frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi} - l = Cc$ hence $l = \frac{-Cc(\mathfrak{C}\pi'-\pi+\Phi)}{\pi'-\pi+\Phi}$;
IV	$O = \frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi} = \delta + l$	hence $\delta = \frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi} - l$ and thus $l = \frac{-Dd(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}{\pi''-\pi'+\pi-\Phi}$;
V	$O = \frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi} = \varepsilon + l$	hence $\varepsilon = \frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi} - l$ hence $l = \frac{-Ee(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}{\pi'''-\pi''+\pi'-\pi+\Phi}$;

etc

II. For the account of the magnification for the related distance h

we will have the following values of the letter m for any number of lenses :

Number of lenses		
I	$m = -A\frac{h}{l}$	for the erect position, if m hence will be produced positive; but for the inverted situation, if m will have obtained a negative value.
II	$m = +AB\frac{h}{l}$	
III	$m = -ABC\frac{h}{l}$	
IV	$m = +ABCD\frac{h}{l}$	
V	$m = -ABCDE\frac{h}{l}$	

etc

III. For the degree of clarity y

from the above it will be clear that is always to be expressed in the same manner, whatever were the number of lenses: indeed it will always be $y = \frac{hx}{ma}$.

IVa. For the apparent field of view

if indeed O may have a positive value, its radius Φ for any number of lenses may be defined in the following manner :

Number of lenses	
I	$\Phi = \infty$ or indefinite
II	$\Phi = \frac{-\pi}{ma-h} \cdot h$
III	$\Phi = \frac{-\pi+\pi'}{ma-h} \cdot h$
IV	$\Phi = \frac{-\pi+\pi'-\pi''}{ma-h} \cdot h$
V	$\Phi = \frac{-\pi+\pi'-\pi''+\pi'''}{ma-h} \cdot h$

etc

IVb. For the apparent field of view

if the distance O were negative, in which case the eye must be applied next to the final lens, Φ may be defined from the following equations :

Number of lenses	
I	$\Phi = \infty$ or indefinite
II	$\frac{A\mathfrak{B}a\Phi\pi}{\mathfrak{B}\pi-\Phi} = \omega$
III	$\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi'-\pi+\Phi} = \omega$

IV $\quad \dfrac{ABC\mathfrak{D}a\Phi\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=\omega$

V $\quad \dfrac{ABCD\mathfrak{E}a\Phi\pi'''}{\mathfrak{D}\pi'''-\pi''+\pi'-\pi-\Phi}=\omega$

etc

V. For the radius of confusion

the following expressions will be had:

Number of lenses

I
$$\frac{\mu m x^3}{4aah}\cdot\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}$$

II
$$\frac{\mu m x^3}{4aah}\left(\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\right)$$

III
$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\dfrac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\dfrac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\+\dfrac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi\end{array}\right)$$

IV
$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\dfrac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\dfrac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\+\dfrac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\dfrac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\end{array}\right)$$

V
$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\dfrac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\dfrac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\+\dfrac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\dfrac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\\+\dfrac{\mu''''(E+1)\left(\lambda''''(E+1)^2+\nu E\right)}{A^3B^3C^3D^3E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\end{array}\right).$$

etc

VI. For removing the colored fringe

if the distance O were produced positive, the object without the colored fringe will appear by satisfying the following equations:

Number of lenses	
I	$O = 0$
II	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$
III	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$
IV	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$
	etc

VII. For removing all the confusion

from the following equations above it is required to satisfy :

Number of lenses	
I	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A}$
II	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B}$
III	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2B^2C}$
IV	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2B^2C} + \frac{adn'''}{n'''-1} \cdot \frac{D+1}{A^2B^2C^2D}$ etc.
	etc

Moreover the separations of the lenses and the radii of the apertures may be defined as above; yet it may be noted the above formulas for the radius of confusion shown, unless they may be reduced completely to nothing, must be able to be put equal to the formula $\frac{1}{4\chi^2}$, with there being approximately $\chi = 40$ (§193) or less than that, just as the circumstances will have demanded.

CAPUT VII

DE CONSTRUCTIONE INSTRUMENTORUM DIOPTRICORUM IN GENERE

PROBLEMA 1

329. *Si instrumentum dioptricum unica constet lente crassitiei cuiuscunque PP, definire omnia momenta ad visionem pertinentia.*

SOLUTIO

Quod primo ad ipsius lentis structuram attinet, ponatur obiecti $E\varepsilon$ (Fig. 12) ante eam distantia $AE = a$ imaginisque $F\zeta$ post eam proiectae $aF = \alpha$;

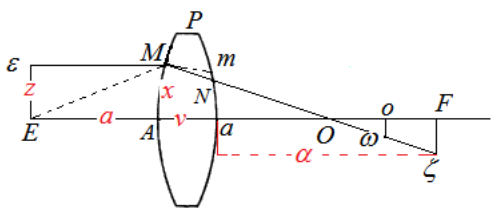

Fig. 12

tum vero lentis crassities $Aa = v$ et quantitas arbitraria $= k$, unde capiatur $\frac{k-v}{k+v} = i$. Hinc facies lentis ita erunt formatae, ut sit existante $n = \frac{31}{20}$ (conf. § 68):

$$\text{radius faciei anterioris } = \frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$$

$$\text{radius faciei posterioris } = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$$

Ponatur porro semidiameter obiecti conspicui $E\varepsilon = z$ et semidiameter aperturae in facie anteriori $AM = x$, in facie autem posteriori semdiameter aperturae non sit minor quam *ix.* Denique oculi post lentem distantia vocetur $aO = O$, cuius distantia iusta sit $= l$. His positis sequentia momenta perpendi oportet:

1. Debet esse $O = \alpha + l$, ut distantia imaginis visae naturae oculi conveniat.

2. Consideranda venit multiplicatio, quae ex ratione definitur, quam diameter obiecti visus tenet ad eiusdem diametrum visum, si nudo oculo in distantia data $= h$ spectaretur. Quodsi ergo hic exponens multiplicationis ponatur $= m$, erit

$$m = \frac{1}{i} \cdot \frac{\alpha h}{al} \quad \text{pro situ inverso.}$$

3. Gradus claritatis determinatur semidiametro coni luminosi, qui a quovis obiecti puncto in oculum immittitur; qui si ponatur $= y$, erit

$$y = il \cdot \frac{x}{\alpha} = \frac{hx}{ma}.$$

4. Confusio inquinans visionem mensuratur semidiametro apparente circuli, qui nudo oculo aeque magnus cernitur ac singula obiecti puncta per lentem spectata. Hanc mensuram vocavi (§.194) semidiametrum confusionis; ad quem definiendum si ponatur

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{ia} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{i}{\alpha} - \frac{2}{k+v}\right)^2\right),$$

erit semidiameter confusionis $= \frac{1}{4} i \cdot \frac{\alpha}{l} x^3 P = \frac{1}{4} i^2 \cdot \frac{ma}{h} x^3 P.$

5. Ut oculus maximum campum apparentem percipiat, debet esse

$$O = \frac{-i\alpha v}{n\alpha - iv}.$$

6. Quae distantia si fuerit positiva, semidiameter campi seu obiecti conspicui $E\varepsilon = -z$ ita pendet ab apertura faciei posterioris, ut posito huius semidiametro $= \mathfrak{a}$ sit $z = \frac{na}{v}\mathfrak{a}$.

7. Sin autem distantia illa pro O assignata prodierit negativa, oculum lenti immediate applicari conveniet, ut sit $O = 0$; tum vero pro a posito semidiametro pupillae ω erit pro campo apparente $z = \frac{na}{v}\omega.$.

8. Ut obiectum sine margine colorato appareat, existante distantia O ante inventa positiva, debet esse $i = \frac{\alpha}{\alpha + v}$ seu $k = 2\alpha + v$.

9. Sin autem ob illam distantiam negativam prodeuntem capiatur $O = 0$, ut margo coloratus evitetur, oportet esse $\frac{v}{a} + 1 - i = 0$, unde fit $k = -2\alpha - v$.

10. Omnis denique confusio a diversa radiorum refrangibilitate oriunda

prorsus tolletur, si insuper fieri posset $(\alpha+v)(a+\alpha+v)=0$.

COROLLARIUM 1

330. Si exponens multiplicationis *m* cum gradu claritatis *y* proponatur, erit $my=\frac{hx}{a}$, quae proprietas ad lentium numerum quantumvis magnum patet. Cum ergo $\frac{h}{a}$ sit quantitas data, erit *x* ut *my;* tum vero *y* ut $\frac{x}{2}$ ac *m* ut $\frac{x}{y}$.

COROLLARIUM 2

331. Tam maior ergo multiplicatio quam maior claritatis gradus postulat maiorem aperturam. Verum aucto *x* confusio augetur in ratione triplicata, siquidem quantitas *P* maneat eadem; quare si *x* ex confusione etiamnum tolerabili determinetur, simul quantitas *my* determinatur.

COROLLARIUM 3

332. Ut igitur tam multiplicationem *m* quam claritatem *y* salva confusione maxime augere liceat, quantitatem arbitrariam *k* ita definiri conveniet, ut litterae *P* minimus valor concilietur.

Cum autem posuerimus $\frac{k-v}{k+v}=i$, valor ipsius *P* fiet minimus huic aequationi satisfaciendo:

$$2i^4\left(\frac{n}{\alpha}+\frac{1}{v}\right)\left(\frac{1}{\alpha}+\frac{1}{v}\right)^2-\frac{2ni^3}{\alpha v}\left(\frac{1}{\alpha}+\frac{1}{v}\right)-\frac{i^3}{v}\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2$$
$$+\frac{2ni}{av}\left(\frac{1}{a}+\frac{1}{v}\right)+\frac{i}{v}\left(\frac{1}{a}+\frac{2}{v}\right)^2-2\left(\frac{n}{a}+\frac{1}{v}\right)\left(\frac{1}{a}+\frac{1}{v}\right)^2=0,$$

uti § 45 invenimus.

COROLLARIUM 4

333. Si loco *i* valor substituatur suus $\frac{k-v}{k+v}=i$, aequatio transibit in hanc formam:

$$\frac{2n}{a^3}(k+v)^4+\frac{(2n+1)}{aav}(k+v)^3(k+3v)+\frac{4(n+2)}{av}(k+v)^3$$
$$-\frac{2n}{\alpha^3}(k-v)^4-\frac{(2n+1)}{\alpha\alpha v}(k-v)^3(k-3v)+\frac{4(n+2)}{\alpha v}(k-v)^3+32k=0,$$

quae per *v* multiplicata etiam ad casum accommodari potest, quo est $v=0$: tum autem reperitur

$$k=\frac{4(n+2)a\alpha}{(2n+1)(a-\alpha)}$$

et

$$P=\frac{n}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left((4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}\right).$$

SCHOLION

334. Hic non generatim omnem visionem, quae fit per unicam lentem, considero, sed tantum eam, qua maximus campus apparens conspicitur; quamobrem locum oculi ita definivi, ut ipsi maximus campus adferatur. Sin autem minore campo velimus esse contenti, oculus quoque alibi post lentem constitutus obiecta distincte cernere poterit, quemadmodum per lentes satis amplas, quae tabularum vitrearum nomine sunt notae, fieri solet. Hunc autem casum, quoniam per se facile expeditur atque in instrumentis dioptricis magnitudo campi potissimum spectatur, hic non attingo.

PROBLEMA 2

335. *Si instrumentum dioptricum duabus constet lentibus cuiuscunque crassitiei, definire omnia momenta ad visionem pertinentia.*

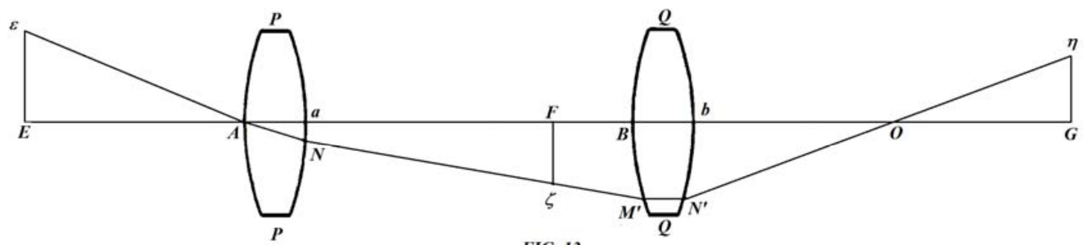

FIG. 13.

SOLUTIO

Obiecto constituto in $E\varepsilon$ (Fig. 13) eius imagines proiiciantur per istas lentes in $F\zeta$ et $G\eta$, ac ponantur quantitates utramque lentem determinantes

$AE=a,\ aF=\alpha$, crassities $Aa=v$ et distantia arbitraria $=k$,
$BF=b,\ bG=\beta$, crassities $Bb=v'$ et distantia arbitraria $=k'$,
ponaturque brevitatis gratia $\frac{k-v}{k+v}=i$ et $\frac{k'-v'}{k'+v'}=i'$. Hinc, existente $n=\frac{31}{20}$,
constructio utriusque lentis ita se habebit:

Radius faciei pro lente PP	anterioris $\frac{(n-1)a(k+v)}{k+v+2na}$	posterioris $\frac{(n-1)\alpha(k+v)}{k-v-2n\alpha}$

pro lente QQ $\Big|\ \frac{(n-1)b(k'+v')}{k'+v'+2nb}\ \Big|\ \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$.

Sit porro semidiameter aperturae lentis primae PP in facie anteriori $=x$, in facie autem posteriori maior sit quam ix. Tum vero lentis secundae QQ semidiameter aperturae in facie anteriori maior esse debet quam $i.\frac{bx}{\alpha}$, in posteriori vero maior quam $ii'.\frac{bx}{\alpha}$. Deinde sit semidiameter obiecti conspicui $E\varepsilon = z$, distantia oculi $bO = O$ eiusque distantia iusta $=l$. His positis ad sequentia momenta erit attendendum:

1. Ut oculus imaginem $G\eta$ in distantia iusta conspiciat, oportet esse $O=\beta+l$.

2. Posita distantia $=h$, ad quam multiplicatio referatur, sit exponens multiplicationis $=m$, ac supra invenimus esse oportere $m=\frac{1}{ii'}\cdot\frac{\alpha\beta h}{abl}$ pro situ erecto.

3. Denotante y gradum claritatis fiet

$$y = ii'l\cdot\frac{bx}{\alpha\beta} \text{ ideoque } my=\frac{hx}{a}.$$

4. Pro confusione posito brevitatis ergo

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right),$$
$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

erit

$$\text{semidiamer confusionis} = \frac{1}{4}ii'\cdot\frac{\beta}{l}\cdot\frac{b}{\alpha}x^3\left(\frac{1}{ii'}\cdot\frac{\alpha\alpha}{bb}P+ii\cdot\frac{bb}{\alpha\alpha}Q\right).$$

5. Pro campo apparente definiendo, qui pendet ab apertura singularum acierum lentium, si ponamus

semidiametrum aperturae

pro lente PP $\begin{cases}\text{faciei anterioris} = \mathfrak{A} = x\\ \text{faciei posterioris} = \mathfrak{a}\end{cases}$

pro lente QQ $\begin{cases}\text{faciei anterioris} = \mathfrak{B} = x\\ \text{faciei posterioris} = \mathfrak{b},\end{cases}$

habebimus sequentes aequationes

$$\mathfrak{a}=\frac{v}{na}z,\quad \mathfrak{B}=\left(\frac{1}{i}\cdot\frac{\alpha+b}{a}-\frac{bv}{na\alpha}\right)z,\quad \mathfrak{b}=\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}\right)z,$$

ex quibus tribus aequationibus valor ipsius z minimus praebet semidiametrum campi apparentis.

6. Quem campum ut oculus revera perspiciat, eius locus ita debet sumi, ut sit

$$O=\frac{\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}}{\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{nab}+\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}}\cdot\beta,$$

si quidem haec distantia fuerit positiva.

7. Sin autem haec distantia prodierit negativa, oculum lenti QQ immediate applicari convenit, ut sit $O=0$; tum vero pro campo apparente inveniendo in aequationibus n° 5 allatis loco $\mathfrak{b}$ scribatur semidiameter pupillae ω, atque ex iisdem valor ipsius z minimus erutus dabit semidiametrum campi apparentis.

8. Obiectum porro sine margine colorato cernetur, existante distantia O n° 6 inventa positiva, si huic aequationi satisfiat:

$$0=1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}\left(1+\frac{b}{\alpha}\right)+\frac{(1-i')^2 b(\alpha+b)}{\alpha v'}+\frac{z-i-i'}{n}$$
$$-\frac{iv}{n\alpha}-\frac{i'v'}{n\beta}-\frac{ibv}{n\alpha\alpha}-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i(1-i')^2 bbv}{n\alpha\alpha v'}.$$

9. Sin autem capiatur distantia $O=0$, obiectum margine colorato carebit, si fuerit:

$$0=\frac{1-i}{in}+\frac{1-i'}{i'n}+\frac{v}{ina}+\frac{v'}{inb}+\alpha\left(1+\frac{\alpha}{iia}+\frac{(1-i)^2\alpha}{iiv}\right)\left(\frac{v'}{ni'bb}-\frac{1}{\alpha}-\frac{1}{b}\right).$$

10. Omnis autem confusio penitus tolletur, quae quidem a diversa radiorum refrangibilitate proficiscitur, si huic aequationi satisfacere licuerit

$d\beta=0$ seu huic

$$0=\frac{\alpha\alpha}{ii'bb}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)+\frac{i'}{\beta}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}.$$

COROLLARIUM 1

336. Quia ambarum lentium distantia aB necessario est positiva, oportet sit $\alpha+b>0$. Tum vero aperturae ita debent esse comparatae, ut sit $\mathfrak{a}>ix$, $\mathfrak{B}>i\cdot\frac{bx}{\alpha}$ et $\mathfrak{b}>ii'\cdot\frac{bx}{\alpha}$.,

COROLLARIUM 2

337. Casu igitur, quo ob valorem ipsius O prodeuntem negativum distantia $O = 0$ assumitur, quia tum $\mathfrak{b} = \omega$ statuitur, etiam esse debet $\omega > ii'\frac{bx}{\alpha}$ seu $x < \frac{\alpha\omega}{ii'b}$. Scilicet hoc casu inutile esset primam aperturam maiorem sumere, quia radii non in oculum ingrederentur.

SCHOLION

338. Si simili modo instrumenta dioptrica pluribus lentibus instructa prosequi vellemus, in calculos tantopere intricatos delaberemur, ut ex iis nihil fere concludere liceret. Oritur autem haec calculi complicatio a crassitie lentium, qua neglecta omnia fiunt multo concinniora. Quare si pluribus lentibus uti velimus, crassitiem tam parvam assumamus, ut sine errore pro nihilo haberi possit, id quod in praxi etiam sedulo observari solet. Praeterea etiam campi apparentis locique oculi determinatio non capit rigorem geometricum, parumque refert, si in ea aliquantum aberretur. Similis quoque ratio est conditionum, quibus effectus a diversa radiorum refrangibilitate oriundus tollitur; sufficiet enim iis proxima satisfecisse, cum perfecta huius confusionis destructio ne sperari quidam possit. Quocirca in sequentibus, ubi instrumenta pluribus lentibus instructa evolvemus, crassitiem lentium in calculo penitus praetermittamus: in quo negotio ne eadem toties repetera opus habeamus, problema generale praemittamus, in quo omnia momenta generatim tantum exponamus eaque deinceps pro quovis lentium numero accurate describamus.

PROBLEMA 3

339. *Si instrumentum dioptricum compositum sit ex lentibus quotcunque, quarum crassitiem negligere liceat, elementa eius constructionem continentia exponere, ex quibus deinceps regulae dirigentes constructionem ipsam stabiliri possint.*

SOLUTIO

Obiecto in E constituto ponantur distantiae determinatrices singularum lentium una cum numeris arbitrariis ad singulas pertinentibus, ut sequitur:

Pro lente prima $EA = a$, $aF = \alpha$, num. arb. $= \lambda$
pro lente secunda $FB = b$, $bG = \beta$, num. arb. $= \lambda'$
pro lente tertia $GO = c$, $cH = \gamma$, num. arb. $= \lambda''$
pro lente quarta $HD = d$, $dI = \delta$, num. arb. $= \lambda'''$
pro lente quinta $IE = e$, $eK = \varepsilon$, num. arb. $= \lambda''''$
pro lente sexta $KF = f$, $fL = \zeta$, num. arb. $= \lambda'''''$
etc.,

ex quibus elementis quomodo singulae lentes debeant formari, supra est expositum. Nunc autem porro ponatur brevitatis gratia

$$\alpha = Aa,\ \ \beta = Bb,\ \ \gamma = Cc,\ \ \delta = Dd,\ \ \varepsilon = Ee,\ \ \zeta = Ff \text{ etc.},$$

tum vero statuatur etiam

$$\tfrac{A}{A+1} = \mathfrak{A},\ \ \tfrac{B}{B+1} = \mathfrak{B},\ \ \tfrac{C}{C+1} = \mathfrak{C},\ \ \tfrac{D}{D+1} = \mathfrak{D},\ \ \tfrac{E}{E+1} = \mathfrak{E},\ \ \tfrac{F}{F+1} = \mathfrak{F}, \text{ etc.},$$

ita ut sint distantiae focales lentium:

$$\mathfrak{A}a,\ \ \mathfrak{B}b,\ \ \mathfrak{C}c,\ \ \mathfrak{D}d,\ \ \mathfrak{E}e,\ \ \mathfrak{F}f \text{ etc.}$$

Nunc sit semidiameter aperturae primae lentis obiectivae $= x$, pro reliquis vero lentibus ratio aperturarum litteris $\pi, \pi', \pi'', \pi''', \pi''''$ etc. exponatur, ita ut semidiameter aperturae cuiusque maior accipi debeat quam secundum has rationes; scilicet capi oportebit

semidiametrum aperturae

lentis secundae $> \pi\ \mathfrak{B}b$
lentis tertiae $> \pi'\ \mathfrak{C}c$
lentis quartae $> \pi''\mathfrak{D}d$
lentis quintae $> \pi'''\ \mathfrak{E}e$
lentis sextae $> \pi''''\ \mathfrak{F}f$
etc.

Quod si iam semidiameter spatii in obiecto conspicui vocetur $= z$ fiatque $\frac{z}{a} = \Phi$, ut sit $z = a\Phi$, hinc distantiae determinatrices ita exprimentur, ut sit primo $\alpha = Aa$, tum vero:

$$b = \frac{Aa\Phi}{\mathfrak{B}\pi - \Phi}; \qquad \beta = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$$

$$c = \frac{ABa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}; \qquad \gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$d = \frac{ABCa\Phi}{\mathfrak{C}\pi'' - \pi' + \pi - \Phi}; \qquad \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$e = \frac{ABCDa\Phi}{\mathfrak{C}\pi''' - \pi'' + \pi' - \pi + \Phi}; \qquad \varepsilon = \frac{ABCDEa\Phi}{\mathfrak{C}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

$$f=\frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi};\quad \zeta=\frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}$$

etc.

Circa has expressiones primum notandum est aggregata
$\alpha+b,\ \beta+c,\ \gamma+d,\ \varepsilon+e,\ \varepsilon+f$ etc. esse oportere positiva, quippe quibus lentium intervalla exprimuntur. Erit nempe

Intervallum lentium

$$\text{I et II}=\frac{Aa\mathfrak{B}\pi}{\mathfrak{B}\pi-\Phi}>0$$

$$\text{II et III}=\frac{ABa\Phi\left(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi\right)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$$

$$\text{III et IV}=\frac{ABCa\Phi\left(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi'\right)}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0$$

$$\text{IV et V}=\frac{ABCDa\Phi\left(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi''\right)}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}>0$$

$$\text{V et VI}=\frac{ABCDEa\Phi\left(\mathfrak{F}\pi''''-(1-\mathfrak{E})\pi'''\right)}{(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)}>0$$

etc.

Praeterea ut lentes sequentes omnes radios a prima exceptos transmittant, debet esse

semidiameter aperturae

lentis secundae $>\frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

lentis tertiae $>\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

lentis quartae $>\frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

lentis quintae $>\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}x$

lentis sextae $>\frac{\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Denique ex § 214 posito $\mu=0,938191$ et $\nu=0,232692$ erunt litterarum maiuscularum. *P, Q, R* etc. valores

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2 + \nu A)$$
$$Q = \frac{\mu}{B^3 b^3}(B+1)(\lambda'(B+1)^2 + \nu B)$$
$$R = \frac{\mu}{C^3 c^3}(C+1)(\lambda''(C+1)^2 + \nu C)$$
$$S = \frac{\mu}{D^3 d^3}(D+1)(\lambda'''(D+1)^2 + \nu D)$$
$$T = \frac{\mu}{E^3 e^3}(E+1)(\lambda''''(E+1)^2 + \nu E)$$
$$V = \frac{\mu}{F^3 f^3}(F+1)(\lambda'''''(F+1)^2 + \nu F)$$
etc.

His ita constitutis denominationibus sit O distantia oculi post lentem ultimam, cuius distantia iusta ponatur $= l$. Deinde statuatur exponens multiplicationis $= m$, relatus ad distantiam h, ita ut diameter obiecti per instrumentum visi m vicibus maior cernatur, quam si idem obiectum a nudo oculo in distantia $= h$ aspiceretur; multiplicationi autem adiungi convenit situm indicando, utrum obiectum situ erecto an inverso sit appariturum. Porro gradus claritatis y denotet semidiametrum coni luminosi, qui a quovis obiecti puncto in oculum transmittitur posito semidiametro pupillae $= \omega$. Semidiametrum denique confusionis voco semidiametrum apparentem circulorum, qui in nudo oculo aeque magni depinguntur atque singula obiecti puncta per instrumentum in oculo.

COROLLARIUM 1

340. Quia pro apertura singularum lentium geminos limites invenimus, semidiameter aperturae cuiusque convenientissime aggregato amborum limitum aequalis assumitur vel saltem non minor. Uterque autem limes, etsi forte alter prodeat negativus, affirmative accipi debet.

COROLLARIUM 2

341. Hinc ergo sequentes consequimur formulas pro singularum lentium aperturis:

Semidiameter aperturae

lentis secundae $= \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

lentis tertiae $= \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

lentis quartae $= \frac{ABC\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$

lentis quintae $= \frac{ABCD\mathfrak{E}a\pi''' \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$

lentis sextae $= \frac{ABCDE\mathfrak{F}a\pi''' \pm x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi} \cdot \Phi$

etc.

COROLLARIUM 3

342. Ex distantiis binis determinatricibus cum numero arbitrario quaelibet lens facile construitur; id quod pro lente prima ex superioribus repetamus. Nempe si sit $\lambda > 1$, lens simplex satisfaciet, cuius constructio posito brevitatis gratia

$$\rho = 0{,}190781, \quad \sigma = 1{,}627401 \text{ and } \quad \tau = 0{,}905133$$

ita se habet:

$$\text{Radius faciei} \quad \overset{\text{anterioris}}{\frac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}}; \quad \overset{\text{posterioris}}{\frac{a\alpha}{\rho\alpha + \sigma a \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}}$$

seu numeris substitutis, si sit $\lambda = 1 + v$,

$$\text{radius faciei} \begin{cases} \text{anterioris} = \dfrac{a\alpha}{+0{,}190781\alpha + 1{,}627401a \pm 0{,}905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1{,}627401\alpha + 0{,}190781a \mp 0{,}905133(a+\alpha)\sqrt{v}}. \end{cases}$$

COROLLARIUM 4

343. Sin autem sit $\lambda < 1$, sed tamen $> 0{,}191827$, lens est duplicanda seu ex duabus simplicibus componenda, quarum constructio, si ponatur

$$\lambda = 0{,}191827 + v,$$

ita se habet:

Pro lente	Radius faciei
priori	anterioris $= \dfrac{a\alpha}{-0{,}622919\alpha + 0{,}813700a \pm 0{,}905133(a+\alpha)\sqrt{v}}$ posterioris $= \dfrac{a\alpha}{+1{,}532010\alpha + 0{,}095390a \mp 0{,}905133(a+\alpha)\sqrt{v}}$
posteriori	anterioris $= \dfrac{a\alpha}{+0{,}095390\alpha + 1{,}532010a \pm 0{,}905133(a+\alpha)\sqrt{v}}$ posterioris $= \dfrac{a\alpha}{+0{,}813700\alpha - 0{,}622919a \mp 0{,}905133(a+\alpha)\sqrt{v}}$.

COROLLARIUM 5

344. At si sit $\lambda = 0{,}042165 + v$, lente utendum est triplicata ita construenda:

Pro lente Radius faciei

$$\text{priori}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{-0{,}894153\alpha+0{,}542467a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+1{,}500214\alpha+0{,}06354a\mp 0{,}905133(a+\alpha)\sqrt{v}}\end{cases}$$

$$\text{media}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{-0{,}415280\alpha+1{,}021340a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+1{,}021340\alpha-0{,}415280a\mp 0{,}905133(a+\alpha)\sqrt{v}}\end{cases}$$

$$\text{posteriori}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{+0{,}063594\alpha+1{,}500214a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+0{,}542467\alpha-0{,}894135a\mp 0{,}905133(a+\alpha)\sqrt{v}}.\end{cases}$$

COROLLARIUM 6

345. Si denique sit $\lambda = -0{,}010216 + v$, lens facienda est quadruplicata ita construenda:

Pro lente Radius faciei

$$\text{prima}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{-1{,}029770\alpha+0{,}406850a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+1{,}484315\alpha+0{,}047695a\mp 0{,}905133(a+\alpha)\sqrt{v}}\end{cases}$$

$$\text{secunda}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{-0{,}670615\alpha+0{,}766005a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+1{,}125160\alpha-0{,}311460a\mp 0{,}905133(a+\alpha)\sqrt{v}}\end{cases}$$

$$\text{tertia}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{-0{,}311460\alpha+1{,}125160a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+0{,}766005\alpha-0{,}670615a\mp 0{,}905133(a+\alpha)\sqrt{v}}.\end{cases}$$

$$\text{quarta}\ \begin{cases}\text{anterioris} = \dfrac{a\alpha}{+0{,}047695\alpha+1{,}484315a\pm 0{,}905133(a+\alpha)\sqrt{v}}\\ \text{posterioris} = \dfrac{a\alpha}{+0{,}406850\alpha-1{,}029770a\mp 0{,}905133(a+\alpha)\sqrt{v}}.\end{cases}$$

SCHOLION

346. Quae in his corollariis de constructione lentis primae sunt allata, mutatis litteris *a* et *a* ad reliquas lentes facile accommodari manifestum est, quarum loco, si usus requirat, ut numeri λ', λ'' etc. sint unitate minores, etiam lentes sive duplicatae sive triplicatae sive adeo quadruplicatae adhiberi debebunt. Ceterum hae formulae ad lentes quotcunque patent, ita ut proposito lentium numero quocunque tantum litterae, quae ad lentes sequentes pertinerent, sint omittendae. In his autem denominationibus statim introduximus campum apparentem numero Φ contentum, cum sit semidiameter spatii in obiecto conspicui $z = a\Phi$. Interim tamen campum apparentem non ad lubitum augere licet, siquidem is multiplicationis ratione et numeris π, π', π'' etc. determinatur. Hi autem numeri semper infra $\frac{1}{2}$, imo $\frac{1}{3}$ subsistunt ac plerumque $\frac{1}{4}$ vel adeo $\frac{1}{6}$ superare nequeunt; quandoque etiam minores accipi debent, id quod formulae corollarii 2 quovis casu declarabunt, quibus semidiameter aperturae cuiusque lentis exprimitur. Quare retentis his denominationibus momenta constructionis pro quovis lentium numero determinato expendamus.

PROBLEMA 4

347. *Si instrumentum dioptricum duabus constet lentibus, quarum crassitiem negligere liceat, definire momenta, quibus constructio continetur.*

SOLUTIO

Cum hic lens secunda sit ultima, momenta constructionis ita se habebunt:

1. Ut oculus imaginem in distantia iusta conspiciat, debet esse $O = \beta + l$ ideoque

$$O = l + \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$$

et ex conditione campi apparentis

$$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi}.$$

2. Exponens multiplicationis *m* ad distantiam *h* relatus praebet hanc aequationem

$$m = AB\frac{h}{l} \text{ pro situ erecto;}$$

sin autem situs inversus desideretur, numerus *m* negative est accipi convenit.

3. Pro gradu claritatis semper est $y = \frac{hx}{ma}$.

4. Pro campo apparente habebitur haec aequatio:

$$\pi - \Phi = \frac{-ma\Phi}{h}, \text{ unde fit } \Phi = \frac{-\pi h}{ma - h}.$$

5. Si distantia O hinc prodeat negativa, capi debet $O=0$ seu $\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}=-l$; tum vero π eiusmodi valorem induit, ut apertura lentis ocularis non superet aperturam pupillae; fiet nempe $\pi\mathfrak{B}b=\omega$ hincque

$$\Phi=\frac{\mathfrak{B}\pi\omega}{A\mathfrak{B}a\pi+\omega} \quad \text{vel} \quad (B+1)\omega=-\pi l.$$

6. Semidiameter confusionis sequenti formula exprimetur:

$$\frac{Bbb}{4Aal}x^3\left(\frac{AAaa}{bb}P+\frac{bb}{AAaa}Q\right),$$

quae factis substitutionibus abit in hanc:

$$\frac{\mu mx^3}{4aah}\left(\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\right).$$

7. Ut obiectum sine margine colorato appareat, si distantia O fuerit positiva, necesse est sit $\frac{\pi}{\mathfrak{B}\pi-\Phi}=0.$

8. Sin autem capiatur $O=0$, fieri debet $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ seu $\frac{\pi}{\mathfrak{A}\Phi}=0.$

9. Omnis autem confusio a diversa radiorum refrangibilitate oriunda penitus tolletur, si insuper fuerit $\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0.$

10. Praeterea autem erit distantia lentium $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, quae debet esse positiva.

11. Denique semidiameter aperturae lentis ocularis est $\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$, ubi signorum $\pm$ id capi debet, quod valorem praebet maximum sive positivum sive negativum.

COROLLARIUM 1

348. Cum sit (n° 4) $\Phi=\frac{-\pi h}{ma-h}$ et ob multiplicationem $B=\frac{ml}{Ah}$ hincque $\mathfrak{B}=\frac{ml}{ml+Ah}$, habebitur pro loco oculi $O=\frac{-Ahl(ma-h)}{mmal+Ahh}.$

Tum vero est $\mathfrak{B}\pi-\Phi=\frac{ml\pi}{ml+Ah}+\frac{\pi h}{ma-h}=\frac{\pi(mmal+Ahh)}{(ma-h)(ml+Ah)}$, unde fit lentium distantia $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}=\frac{mAa(ma-h)}{mmal+Ahh}l.$

COROLLARIUM 2

349. Cum sit $\frac{A}{A+1}=\mathfrak{A}$ et $A=\frac{\mathfrak{A}}{1-\mathfrak{A}}$, semidiameter confusionis etiam ita exprimi potest, ut sit

$$\frac{\mu m x^3}{4aah}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi\left(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B})\right)}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}\right).$$

COROLLARIUM 3

350. Cum sit lentium distantia $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, obiectum sine coloribus cerni nequit, nisi lentium distantia evanescat; siquidem oculus in eo loco, quem campus apparens postulat, teneatur. Et quoniam huic conditioni satisfieri nequit, multo minus tota confusio ex n° 9 tolli poterit.

SCHOLION

351. Hic statim ab instrumentis duabus lentibus instructis incepi, quoniam lens unica facillime expeditur. Primum enim necesse est, ut sit

$$O=l+\alpha=l+Aa.$$

Tum vero multiplicatio praebet $m=\frac{h}{a}$ pro situ erecto, et campus apparens non terminatur sumendo $Aa+l=0$ seu $O=0$, ita ut oculus lenti immediate debeat applicari. Semidiameter vero confusionis erit

$$=\frac{\alpha}{4l}x^3\cdot P=\frac{Aax^3}{4l}\cdot P=-\frac{1}{4}x^3\cdot P=\frac{-max^3}{4h}\cdot P \text{ ob } l=-Aa \text{ et } ma=h.$$

Quare semidiameter confusionis erit

$$\frac{\mu m x^3}{4aah}\left(\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}\right)=\frac{\mu m x^3}{4aah}\cdot\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}.$$

Pro gradu claritatis autem habetur ut semper $y=\frac{hx}{ma}$.

Obiectum porro hoc casu sine margine colorato cernetur; tota autem confusio tolli nequit, nisi sit $\frac{1}{\mathfrak{A}}=0$ seu $A=-1$, id quod scopo lentium repugnat. Quare ad considerationem plurium lentium progrediamur.

PROBLEMA 5

352. *Si instrumentum, dioptricum tribus instructum sit lentibus, quarum crassities*

tam sit parva, ut negligi queat, definire cuncta momenta ad constructionem dirigendam necessaria.

SOLUTIO

Hic igitur lens tertia erit ultima seu ocularis, ideoque momenta sequenti modo se habebunt:

1. Ut oculo imago in distantia iusta spectanda offeratur, debet esse $O=\gamma+l$ ac loco γ valore substituto

$$O=l+\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

et ex conditione campi apparentis

$$O=\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}.$$

2. Exponens multiplicationis *m* ad distantiam *h* relatus praebet hanc aequationem:

$$m=-\frac{ABCh}{l}\text{ pro situ erecto,}$$

unde si situa inversus desideretur, numerum *m* negative accipi convenit.

3. Pro gradu claritatis habemus, ut semper, $y=\frac{hx}{ma}$.

4. Pro campo autem apparente definiendo habemus hanc aequationem:

$$\Phi=\frac{-\pi+\pi'}{ma-h}h.$$

5. At si distantia *O* hinc prodeat negativa, ut capi oporteat $O=0$ ideoque $\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}=-l$, campus apparens definiri debet ex hac aequatione $\pi'\mathfrak{C}c=\omega$ seu

$$\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi'-\pi+\Phi}=\omega \quad \text{vel} \quad (C+1)\omega=-\pi'l.$$

6. Semidiameter confusionis vero ita exprimetur:

$$\frac{Bbb}{4Aal}x^3\left(\frac{AAaa}{bb}P+\frac{bb}{AAaa}Q\right).$$

quae ob $\frac{1}{l}=\frac{-m}{ABCh}$ abit signo mutato in hanc :

$$\frac{\mu mx^3}{4aah}\left(\begin{array}{l}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}\\+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\+\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\end{array}\right).$$

7. Ut obiectum sine margine colorato appareat, si quidem distantia O prodierit positiva, esse oportet

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}=0.$$

8. Sin autem ob istam distantiam prodeuntem negativam capiatur $O=0$, margo coloratus evanescet faciendo:

$$\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)}.$$

9. Quod si marginem coloratum tollere liceat, praeterea visio ab omni confusione liberabitur, si huic aequationi satisfiat:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=0.$$

10. Denique effici debet, ut distantia lentium fiat positiva, unde habebitur:

Intervallum lentium

$$\text{I et II } =\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0\ ;\ \text{ II et III } =\frac{ABa\Phi\left(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi\right)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0.$$

11. Tandem numeros π et π' ita accipi oportet, ut aperturae lentium non fiant nimis magnae; est vero:

Semidiameter aperturae

$$\text{lentis secundae } =\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi,\quad \text{lentis tertiae } =\frac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi.$$

COROLLARIUM 1

353. Cum sit $C=\frac{-ml}{ABh}$ ideoque $\mathfrak{C}=\frac{-ml}{ABh-ml}$ et $\Phi=\frac{-\pi+\pi'}{ma-h}\cdot h$, si hi valores substituantur, erit pro loco oculi

$$O=\frac{ABhl(ma-h)\pi'}{(mmal-ABhh)\pi'+ma(ABh-ml)\pi}.$$

COROLLARIUM 2

354. Semidiameter confusionis etiam ita exprimi potest, ut sit

$$\frac{\mu m x^3}{4aah}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi\left(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B})\right)}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}+\frac{\Phi\left(\lambda''+\nu\mathfrak{C}(1-\mathfrak{C})\right)}{A^3B^3\mathfrak{C}^3(\mathfrak{C}\pi'-\pi+\Phi)}\right),$$

cuius formae cohaerentia cum praecedentibus iam ita est manifesta, ut ad plures lentes facile extendi queat.

PROBLEMA 6

355. *Si instrumentum dioptricum quatuor lentibus sit instructum, quarum crassities in computum duci non mereatur, definire omnia momenta ad constructionem dirigendam necessaria.*

SOLUTIO

Quia hic lens quarta est ultima,
1. Ut oculo imago postrema in distantia iusta offeratur, debet esse $O=\gamma+l$ ideoque

$$O=l+\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

et ex conditione campi apparentis

$$O=\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}.$$

2. Exponens multiplicationis *m* ad distantiam *h* relatus, si obiectum ut hactenus situ erecto exhiberi assumatur, erit

$$m=+ABCD\cdot\frac{h}{l}\text{ pro situ erecto;}$$

sin autem situs desideretur inversus, numerus *m* negative est accipiendus.

3. Pro gradu claritatis perpetuo habetur $y=\frac{hx}{ma}$.

4. Campus autem apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi+\pi'-\pi''}{ma-h}\cdot h \ ;$$

hinc enim $a\Phi$ praebebit semidiametrum spatii in obiecto conspicui, siquidem distantia O fuerit positiva.

5. Sin autem distantia O prodeat negativa, quo casu capi oportet $O=0$ seu $\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=-l,$ quantitas Φ determinari debet ex hac aequatione $\pi''\mathfrak{D}d=\omega$ seu hac

$$\frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=\omega \quad \text{sive} \quad (D+1)\omega=-\pi''l.$$

6. Semidiameter confusionis ita reperietur expressus:

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\+\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\end{array}\right).$$

7. Ut obiectum sine margine colorato appareat, siquidem distantia O sit positiva, esse oportet:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0.$$

8. Verum si acceperimus $O=0$, margo coloratus evanescet huic aequationi satisfaciendo:

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{ABC(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}.$$

9. Omnis autem confusio a diversa radiorum natura oriunda penitus tolletur, si satisfieri liceat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0.$$

10. Denique efficiendum est, ut distantiae lentium sint positivae, unde oriuntur formulae sequentes:

$$=\frac{ABCa\Phi\left(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi'\right)}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0.$$

11. Aperturae autem singularum lentium ita sunt capiendae, ut sit:

Semidiameter aperturae

lentis secundae $= \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

lentis tertiae $= \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

lentis quartiae $= \frac{ABC\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi.$

COROLLARIUM 1

356. Cum sit ob multiplicationem $D = \frac{ml}{ABCh}$ ideoque $\mathfrak{D} = \frac{ml}{ABCh + ml}$ et $\Phi = \frac{-\pi + \pi' - \pi''}{ma - h} \cdot h$, pro loco oculi habebitur haec expressio:

$$O = \frac{ABChl(ma-h)\pi''}{(mmal - ABChh)\pi'' - ma(ABCh - ml)(\pi' - \pi)}.$$

COROLLARIUM 2

357. Quia hic tot occurrunt litterae determinandae, dubium nullum esse videtur, quin formulis sub n° 7 vel 8 et 9 contentis satisfieri queat, et certum saltem est id fieri posse, si plura adhibeantur mediorum refringentium genera.

PROBLEMA 7

358. *Si instrumentum dioptricum quinque lentibus sit instructum, quarum crassities ob parvitatem negligi queat, definire omnia momenta constructionem dirigentia.*

SOLUTIO

Cum hic lens quinta ultimum locum teneat, habebimus:

1. Pro loco oculi, ut imago visa ipsi in iusta distantia offeratur, debet esse $O = \varepsilon + l$ ideoque

$$O = l + \frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

et ex conditione campi apparentis

$$O = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi}.$$

2. Exponens multiplicationis m ad distantiam $= h$ relatus, siquidem obiectum situ erecto sit spectandum,

$$m = -ABCDE \cdot \frac{h}{l};$$

pro situ autem inverso numerus m negative capi debet.

3. Pro gradu claritatis est ut hactenus $y=\frac{hx}{ma}$.

4. Campus apparens definiri debet ex hac aequatione

$$\Phi=\frac{-\pi+\pi'-\pi''+\pi'''}{ma-h}\cdot h,$$

siquidem distantia illa O prodierit positiva.

5. Sin autem haec distantia prodeat negativa, ut capiatur $O=0$ seu $\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=-l$, campus apparens non ex formula n^o 4, se ex hac $\pi'''\mathfrak{E}e=\omega$ definiri debet, unde fit

$$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=\omega \quad \text{seu} \quad (E+1)\omega=-\pi'''l.$$

6. Semidiameter confusionis ab apertura lentium oriundae sequenti modo exprimitur:

$$\frac{\mu mx^3}{4aah}\left(\begin{array}{c}\frac{(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\Phi(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\\+\frac{\Phi(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\\+\frac{\Phi(E+1)\left(\lambda''''(E+1)^2+\nu E\right)}{A^3B^3C^3D^3E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\end{array}\right).$$

7. Ut obiectum sine margine colorato appareat, siquidem distantia O fuerit positiva, huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0.$$

8. At si distantia O sit negativa capiaturque $O=0$, pro hoc scopo obtinendo oportet sit:

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=\frac{1}{ABCD\mathfrak{D}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right).$$

9. Omnis autem confusio a diversa radiorum natura oriunda penitus tolletur, si fieri possit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0.$$

10. Tum vero efficiendum est, ut intervalla lentium omnia sint positiva.,
unde oriuntur formulae sequentes:

Intervallum lentium

$$\text{I et II } = \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi} > 0 \text{ ; II et III } = \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0.$$

$$\text{III et IV } = \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0.$$

$$\text{IV et V} = \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0.$$

11. Aperturae denique singularum lentium ita sunt capiendae, ut sit:

Semidiameter aperturae

$$\text{lentis secundae} = \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi-\Phi}\cdot\Phi$$

$$\text{lentis tertiae } = \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi$$

$$\text{lentis quartiae } = \frac{ABC\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\cdot\Phi$$

$$\text{lentis quintiae } = \frac{ABCD\mathfrak{E}a\pi''' \pm x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\cdot\Phi.$$

COROLLARIUM

359. Ob multiplicationem ergo erit:

$$E = \frac{-ml}{ABCDh} \quad \text{et} \quad \mathfrak{E} = \frac{-ml}{ABCDh-ml},$$

unde oritur:

$$O = \frac{ABCDhl(ma-h)\pi'''}{(mmal-ABCDhh)\pi'''+ma(ABCDh-ml)(\pi''-\pi'+\pi)}.$$

SCHOLION

360. Hactenus supposui oculum vel in eo loco teneri, quem visio campi apparentis postulat, vel lenti postremae immediate applicari; quorum utroque casu efficiendum est, ut imago ultima oculo ad distantiam iustam offeratur. Hic quidam locum oculi, quem campus praebet, statim coniunxi cum iusta imaginis spectandae distantia; sed etiam sine

respectu sive ad hanc distantiam sive ad multiplicationem habito definiri potest. Ex superioribus enim (§ 271) habemus

Pro casu

unius lentis $O = 0$

duarum lentium $O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$

trium lentium $O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$

quatuor lentium $O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

quinque lentium $O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$

etc.

Verum si nolimus ad campum apparentem respicere, semper eiusmodi locum oculo assignare licet, unde obiectum sine margine colorato cernatur. Haud abs re autem fore arbitror hunc locum determinasse, quia ex eius distantia a loco oculi ob rationem campi assumto facilius iudicare licebit, quanto margine cinctum apparere debeat obiectum, si oculus in loco isto posteriori teneatur. Si enim aequationi $n°$ 7 appositae satisfieri nequeat, nil aliud intelligimus, nisi obiectum non sine margine colorato esse appariturum; at si insuper constat ille locus, unde obiectum sine huiusmodi margine spectari posset, facilius quantitatem istius marginis colligere poterimus.

PROBLEMA 8

361. *Si instrumentum dioptricum ex lentibus quotcunque constet, quarum crassitiem negligere liceat, eum pro oculo locum assignare, unde obiectum sine margine colorato conspiciatur.*

SOLUTIO

Ponatur haec oculi post lentem ultimam distantia $= \Omega$, atque ex aequationibus § 325 datis, si pro O scribatur Ω, haec ipsa distantia Ω, quam quaerimus, elici potest. Scilicet pro quovis lentium numero erit, ut sequitur:

I. Pro unica lente

Habetur haec aequatio $\frac{\Omega d\alpha}{\alpha(\alpha-\Omega)} = 0$, unde fit $\frac{\Omega}{\alpha(\alpha-\Omega)} = 0$ seu $\frac{\Omega}{\alpha-\Omega} = 0$.

Verum etiam campus apparens exigit $O=0$, unde hoc casu ambo loca oculi O et Ω conveniunt neque margo coloratus est pertimescendus.

II. Pro duabus lentibus

Hoc casu reperta est ista aequatio $\frac{\Omega d\beta}{\beta(\beta-\Omega)}=d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, unde substitutis valoribus supra assignatis oritur

$$\frac{\Omega}{\beta(\beta-\Omega)}=\frac{\pi}{\mathfrak{A}\Phi}:(B+1)ABa\left(\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right),$$

seu, si ponamus

$$Y=\frac{\pi}{\mathfrak{A}\Phi} \quad \text{et} \quad Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)},$$

erit

$$\frac{\Omega}{\beta-\Omega}=\frac{Y}{AB(B+1)aZ}\cdot\beta=\frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\mathfrak{B}\pi-\Phi}.$$

III. Pro tribus lentibus

Eodem modo, si ponamus

$$Y=\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}-\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)}$$

et

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\Phi)},$$

erit pro casu trium lentium

$$\frac{\Omega}{\gamma-\Omega}=\frac{Y}{ABC(C+1)aZ}\cdot\gamma=\frac{Y}{(C+1)Z}\cdot\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi},$$

ubi notandum est $Y=0$ et $Z=0$ esse eas ipsas aequationes, quas in superioribus problematibus sub numeris 8 et 9 retulimus.

IV. Pro quatuor lentibus

Si iam ponatur

$$Y=\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi-\Phi)}-\frac{1}{ABC\mathfrak{D}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)},$$

erit

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{ABCD(D+1)aZ}\cdot\delta$$

hincque

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{(D+1)Z}\cdot\frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.$$

V. Pro quinque lentibus

Posito secundum aequationes n° 8 et 9 (§ 358) exhibitas

$$Y=\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$-\frac{1}{ABC\mathfrak{D}\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$$

habebimus

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{ABCDE(E+1)aZ}\cdot\varepsilon$$

vel

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{(E+1)Z}\cdot\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.$$

Lex harum formularum pro quovis maiori lentium numero hinc satis est perspicua.

COROLLARIUM 1

362. Cum igitur supra pro quovis lentium numero exhibuerimus aequationes n° 8 et n° 9, formula n° 8 dabit valorem Y et formula n° 9 valorem ipsius Z, quibus cognitis locus oculi, ubi margo coloratus evanescit, facile innotescit.

COROLLARIUM 2

363. Si praeterea ex aequatione n° 7 (§ 358) exposita ponatur

$$X=\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi},$$

erit quidam pro casu quinque lentium

$$Y=\frac{\pi'''}{\Phi}Z-\frac{1}{ABCD\mathfrak{E}}X.$$

pro quatuor lentibus $Y = \frac{\pi''}{\Phi} Z - \frac{1}{ABC\mathfrak{D}} X$

pro tribus lentibus $Y = \frac{\pi'}{\Phi} Z - \frac{1}{AB\mathfrak{C}} X$

pro duabus lentibus $Y = \frac{\pi}{\Phi} Z - \frac{1}{A\mathfrak{B}} X.$

SCHOLION

364. Quo facilius haec oculi loca littera Ω designata cum praecedentibus littera O designatis (§ 271) comparare liceat, notandum est esse:

Pro casu
unius lentis $\frac{O}{\alpha - O} = 0$

duarum lentium $\frac{O}{\beta - O} = \frac{\pi}{(B+1)(\mathfrak{B}\pi - \Phi)}$

trium lentium $\frac{O}{\gamma - O} = \frac{\pi'}{(C+1)(\mathfrak{C}\pi' - \pi + \Phi)}$

quatuor lentium $\frac{O}{\delta - O} = \frac{\pi''}{(D+1)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

quinque lentium $\frac{O}{\varepsilon - O} = \frac{\pi'''}{(E+1)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$
etc.

Quare ut ambo loca congruant retentis pro quovis lentium numero litteris Y et Z modo adhibitis, oportet sit:

Pro una lente $O = 0$

pro duabus lentibus $\frac{Y}{Z} = \frac{\pi}{\Phi}$

pro tribus lentibus $\frac{Y}{Z} = \frac{\pi'}{\Phi}$

pro quatuor lentibus $\frac{Y}{Z} = \frac{\pi''}{\Phi}$

pro quinque lentibus $\frac{Y}{Z} = \frac{\pi'''}{\Phi}$
etc.

unde generatim fit $X = 0$, quae est ipsa aequatio n° 7 allata.

Haec igitur sunt principia generalia, ex quibus constructio instrumentorum dioptricorum erit perficienda, quae quidem instrumenta hic potissimum ad visionem accommodavi. Nihilo vero minus ad repraesentationem obiectorum in camera obscura super tabula alba adhiberi possunt, ubi praeter iam data praecepta tenendum est has effigies etiam sine margine colorato esse apparituras, si aequationibus $n°$ 7 satisfiat, omnem vero confusionem a diversa radiorum natura oriundam penitus tolli, si insuper aequationibus n° 9 satisfiat.

SUPPLEMENTUM VII

Si ratio refractionis in singulis lentibus fuerit diversa et secundum lentitium ordinem litteris n, n', n'', n''' etc. exhibeatur, inde pro lentium constructione

I. Litterae respondentes ρ, σ, τ et pro confusione litterae μ et ν convenienter definiri debent secundum formulas supra datas § 55:

$$\rho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1, \quad \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1,$$

$$\tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)},$$

ita ut sit

$$\rho + \sigma = \frac{1}{n-1} \quad \text{et} \quad \sigma - \rho = \frac{-2}{n+2} + 2 = \frac{2(n+1)}{n+2},$$

porro autem

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}, \quad \nu = \frac{4(n-1)^2}{4n-1},$$

unde intelligitur, quemadmodum ex ratione refractionis n' litterae respondentes $\rho', \sigma', \tau', \mu', \nu'$ definiri debeant.

II. Quod iam ad elementa occasione campi apparentis introducta π, π', π'' etc. attinet, distantiae determinatrices lentium inde eodem modo manent determinatae ut supra, ita ut non opus sit eas formulas hic transcribere; interim tamen meminisse iuvabit formulas primitivas, unde illae sunt natae,
quae sunt:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{Aa}{b}$$

$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{ABa}{c}$$

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{ABCa}{d}$$

$$\frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{ABCDa}{e}$$

etc.

III. Verum valores litterarum *P, Q, R* etc. ob diversas refractiones mutantur, ut sequitur:

$$P=\frac{\mu}{A^3a^3}(A+1)\left(\lambda(A+1)^2+\nu A\right)$$
$$Q=\frac{\mu'}{B^3b^3}(B+1)\left(\lambda'(B+1)^2+\nu' B\right)$$
$$R=\frac{\mu''}{C^3c^3}(C+1)\left(\lambda''(C+1)^2+\nu'' C\right)$$
$$S=\frac{\mu'''}{D^3d^3}(D+1)\left(\lambda'''(D+1)^2+\nu''' D\right)$$

etc.

IV. Distantiae lentium etiam manent ut ante una cum semidiametris aperturarum singularum lentium; at radii binarum facierum utriusque lentis ita sunt ad praesentem casum accommodandae, ut pro lente, cuius refractio est n', ei respondentes litterae ρ', σ', et τ' et usurpari debeant, similique modo etiam pro sequentibus lentibus, quarum refractio litteris n'', n''' etc. indicatur.

His praemissis singula momenta, quae in constructione instrumentorum dioptricorum sunt observanda, sequenti modo repraesentabimus:

I. Pro loco oculi seu eius distantia post ultimam

lentem $=O$ habebimus pro singulis lentium numeris sequentes determinationes:

Numerus lentium		
I	$O=0=\alpha+l$	ideoque $\alpha=-l=Aa$
II	$O=\frac{\mathfrak{B}b\pi}{\pi-\Phi}=\beta+l$	ideoque $\beta=\frac{\mathfrak{B}b\pi}{\pi-\Phi}-l=Bb$ hinc $l=\frac{-Bb(\mathfrak{B}\pi-\Phi)}{\pi-\Phi}$;
III	$O=\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}=\gamma+l$	hinc $\gamma=\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}-l=Cc$

		hinc $l = \frac{-Cc(\mathfrak{C}\pi' - \pi + \Phi)}{\pi' - \pi + \Phi}$;
IV	$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} = \delta + l$	hinc $\delta = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} - l$ ideoque $l = \frac{-Dd(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}{\pi'' - \pi' + \pi - \Phi}$;
V	$O = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \varepsilon + l$	hinc $\varepsilon = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} - l$ hinc $l = \frac{-Ee(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}{\pi''' - \pi'' + \pi' - \pi + \Phi}$;

etc

II. Pro ratione multiplicationis ad distantiam *h* relata

habebimus pro quolibet lentium numero sequentes valores litterae *m*:

Numerus lentium		
I	$m = -A\frac{h}{l}$	pro situ erecto, si *m* hinc prodierit positivum; pro situ autem inverso, si *m* obtinuerit valorem negativum.
II	$m = +AB\frac{h}{l}$	
III	$m = -ABC\frac{h}{l}$	
IV	$m = +ABCD\frac{h}{l}$	
V	$m = -ABCDE\frac{h}{l}$	

etc

III. Pro gradu claritatis *y*

ex superioribus liquet eum semper pari modo exprimi, quantuscunque fuerit lentium numerus: perpetuo emin erit $y = \frac{hx}{ma}$.

IV. Pro campo apparente

si quidem *O* habeat valorem positivum, eius semidiameter Φ pro quovis lentium numero sequenti modo definietur:

Numerus lentium	
I	$\Phi = \propto$ seu indefinitum

II	$\Phi = \frac{-\pi}{ma-h} \cdot h$
III	$\Phi = \frac{-\pi+\pi'}{ma-h} \cdot h$
IV	$\Phi = \frac{-\pi+\pi'-\pi''}{ma-h} \cdot h$
V	$\Phi = \frac{-\pi+\pi'-\pi''+\pi'''}{ma-h} \cdot h$
	etc

IV. Pro campo apparente

si distantia O fuerit negativa, quo casu oculus lenti ultimae immediate adplicari debet, Φ definietur ex aequationibus sequentibus:

Numerus lentium	
I	$\Phi = \infty$ seu indefinitum
II	$\frac{A\mathfrak{B}a\Phi\pi}{\mathfrak{B}\pi-\Phi} = \omega$
III	$\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi'-\pi+\Phi} = \omega$
IV	$\frac{ABC\mathfrak{D}a\Phi\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} = \omega$
V	$\frac{ABCD\mathfrak{E}a\Phi\pi'''}{\mathfrak{D}\pi'''-\pi''+\pi'-\pi-\Phi} = \omega$
	etc

V. Pro semidiametro confusionis

habebuntur sequentes expressiones:

Numerus lentium	
I	$\frac{\mu m x^3}{4aah} \cdot \frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}$
II	

$$\frac{\mu m x^3}{4aah}\left(\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\right)$$

III

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\ +\frac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi\end{array}\right)$$

IV

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\ +\frac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\end{array}\right)$$

V

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\frac{\mu(A+1)\left(\lambda(A+1)^2+\nu A\right)}{A^3}+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu B\right)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\Phi\\ +\frac{\mu''(C+1)\left(\lambda''(C+1)^2+\nu C\right)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu D\right)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\\ \quad +\frac{\mu''''(E+1)\left(\lambda''''(E+1)^2+\nu E\right)}{A^3B^3C^3D^3E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\end{array}\right).$$

etc

VI. Pro tollendo margine colorato

si distantia O prodierit positiva, obiectum sine margine colorato apparebit satisfaciendo sequentibus aequationibus:

Numerus lentium	
I	$O=0$
II	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{Aa\Phi}$
III	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{Aa\Phi}+\frac{cdn''}{n''-1}\cdot\frac{\pi'}{ABa\Phi}$
IV	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{Aa\Phi}+\frac{cdn''}{n''-1}\cdot\frac{\pi'}{ABa\Phi}+\frac{ddn'''}{n'''-1}\cdot\frac{\pi''}{ABCa\Phi}$

etc

VII. Pro tollenda confusione omni
insuper sequentibus aequationibus est satisfaciendum:

Numerus lentium	
I	$0=\frac{adn}{n-1}\cdot\frac{A+1}{A}$
II	$0=\frac{adn}{n-1}\cdot\frac{A+1}{A}+\frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B}$
III	$0=\frac{adn}{n-1}\cdot\frac{A+1}{A}+\frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B}+\frac{cdn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}$
IV	$0=\frac{adn}{n-1}\cdot\frac{A+1}{A}+\frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B}+\frac{cdn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}+\frac{adn'''}{n'''-1}\cdot\frac{D+1}{A^2B^2C^2D}$
	etc.
	etc

Distantiae autem lentium et semidiametri aperturarum perinde definiuntur ac supra; tantum notetur insuper formulas pro semidiametro confusionis exhibitas, nisi penitus ad nihilum redigi queant, aequales poni debere formulae $\frac{1}{4\chi^2}$, existante circite $\chi=40$ (§193) vel adhuc minore, prout circumstantiae postulaverint.