BOOK TWO

THE CONSTRUCTION OF TELESCOPES

FIRST SECTION.

GENERAL KINDS OF TELESCOPES,

WHICH

EQUIPPED WITH A CONCAVE EYEPIECE REPRESENT OBJECTS UPRIGHT.

CHAPTER I

TELESCOPES IN GENERAL

DEFINITION 1

1. *The Telescope is an optical instrument serving to observe very distant objects.*

COROLLARY 1

2. Therefore since the distance of the object shall be very great, the quantity a in the calculation, by which the distance of the object from the lens may be designated, may be allowed to be observed as infinite, and thus α will denote the focal length of the objective lens, evidently with its thickness ignored.

COROLLARY 2

3. Since we will have put $\alpha = Aa$, on account of $a = \infty$ the number A is vanishing and thus both $A = 0$ and $\mathfrak{A} = \frac{A}{A+1} = 0$. Hence in the formulas treated above thus both the letters A and $\mathfrak{A}$ will be eliminated from the calculation, so that in place of Aa and $\mathfrak{A}a$

there is written α. [*i.e.* not to be confused with the very small arc *Aa* extended by the object at the surface of the lens; here Fig. 1 comes from Ch. I, Book I.]

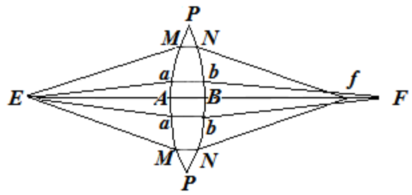

Fig. 1

DEFINITION 2

4. *In telescopes the apparent field of view cannot be estimated from the magnitude of that object viewed, but from the angle, within which this visible part may be viewed by the naked eye.*

COROLLARY

5. Therefore the letter Φ, as we have introduced above in our formulas, will denote the radius of the apparent field of view or rather its tangent ; but since this angle generally is very small, this itself may be used in place of the tangent without error, since it shall be notable especially in multiplication.

DEFINITION 3

6. *The multiplication* in telescopes is accustomed to be estimated from the ratio of the magnitude viewed through the instrument to the magnitude, by which the same object may be viewed at the same distance by the naked eye.*

[* This of course is called now the magnification of the telescope, a term not used by Euler at this time, as there does not appear to be any such single equivalent word in Latin, apart from that which he used. The word from which *to magnify* arises in Latin is concerned rather with extolling virtues and the like, and thus it could not be used. The fact that we do use this word now would seem to have been an error of judgement by someone.]

COROLLARY 1

7. Therefore since above we have referred to the manner of multiplication for the distance *h*, truly the distance of the object has been put $= a$, and also there will be $h = a$.

COROLLARY 2

8. Therefore in this case the exponent of the multiplication $=m$ may indicate, how many times the angle, under which we may discern the diameter of the object through the telescope, shall be greater than the angle, under which the same object may be discerned by the naked eye.

SCHOLIUM 1

9. Evidently this is required to be understood, as long as the angles discussed being small enough ; but when the angles are greater, the exponent of the multiplication m will indicate, not how many times this same angle may be increased, under which some object viewed through the telescope, but how many times its tangent shall be greater than the tangent of the angle, under which the same object may appear to the naked eye, thus so that, even if the multiplier m were to become infinite, yet the angle of view may not increase beyond 90°, evidently while the magnitude of the object may be estimated from the axis of the telescope.

SCHOLIUM 2

10. Therefore from these observations the formulas elucidated above are applied easily to telescopes, and from that they cannot emerge simpler. Truly in addition, even if for a variety of true distances of the eye shall be established maximally different, to be designated by the letter l, yet here this diversity is usually set aside, because the telescope adapted to one accommodation of eye in practice can be adapted easily to any another accommodation, and because generally the true distance l is large enough in comparison with the distance of the eye from the final lens and that thus for many eyes increases to infinity, we may put conveniently $l=\infty$. Hence, if the determinable distances of the lens shall be f and ζ, and after that the position of the eye $=O$, on account of $O=\zeta+l$ the distance ζ must be infinite, evidently $\zeta=O-l$, thus so that there shall become $\frac{\zeta}{l}=-1$ or $\frac{l}{\zeta}=-1$, and on account of $\zeta=\infty$ it is clear the focal length of the final lens to become $=f$. [Thus the image viewed by the eye through the final lens must consist either of diverging or parallel rays; the former corresponds to a virtual image behind the eyepiece, the latter situation to a real image and the relaxed eye.]

PROBLEM 1

11. *From however many lenses the telescope were composed, to set out the elements, from which we have made use above for its construction requiring to be determined, and likewise to represent the relation of their different elements among themselves.*

SOLUTION

For any lens we have considered 1° its ratio of refraction for rays of the mean kind ; 2° its two determinable distances with an arbitrary number λ ; 3° now also the focal length of each lens we will introduce into the calculation ; 4° also we have introduced the ratios of the apertures for the individual lenses indicated by the letters π. Which elements for the individual lenses we may put to be seen in the following manner:

Lenses	Refraction ratio	Determinable distances	Arbitrary number	Focal lengths	Aperture ratio
I[st]	$n:1$	$a,\ \alpha$	λ	p	π
II[nd]	$n':1$	$b,\ \beta$	λ'	q	π'
III[rd]	$n'':1$	$c,\ \gamma$	λ''	r	π''
IV[th]	$n''':1$	$d,\ \delta$	λ'''	s	π'''
V[th]	$n'''':1$	$e,\ \varepsilon$	λ''''	t	π''''
VI[th]	$n^{v}:1$	$f,\ \zeta$	λ^{v}	u	π^{v}

etc.

Thence also we have put :

$$A=\tfrac{\alpha}{a},\ B=\tfrac{\beta}{b},\ \ C=\tfrac{\gamma}{c},\ D=\tfrac{\delta}{d},\ \ F=\tfrac{\zeta}{f}\ \text{etc.},$$

then also truly

$$\mathfrak{A}=\tfrac{A}{A+1},\ \mathfrak{B}=\tfrac{B}{B+1},\ \mathfrak{C}=\tfrac{C}{C+1},\ \mathfrak{D}=\tfrac{D}{D+1},\ \mathfrak{E}=\tfrac{E}{E+1},\ \mathfrak{F}=\tfrac{F}{F+1}\ \text{etc.}$$

With these set out in this way we have seen before on account of $a=\infty$ to become $A=0$ and $\mathfrak{A}=0,$ thus with using these values so that there shall be $Aa=\alpha$ and $\mathfrak{A}a=p$ and $p=\alpha$ *;* moreover for the following lenses we will have :

$$q=\mathfrak{B}b,\ r=\mathfrak{C}c,\ s=\mathfrak{D}d,\ t=\mathfrak{E}e,\ u=\mathfrak{F}f\ \text{etc.},$$

from which in turn for the focal lengths there will be :

$$b = \frac{q}{\mathfrak{B}} = \frac{B+1}{B}q \text{ and } \beta = (B+1)q$$
$$c = \frac{r}{\mathfrak{C}} = \frac{C+1}{C}r \text{ and } \gamma = (C+1)r$$
$$d = \frac{s}{\mathfrak{D}} = \frac{D+1}{D}s \text{ and } \delta = (D+1)s$$
etc.

[All these latter formulas can be derived easily from the thin lens formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$; for example, $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$ and the magnfication $B=\frac{\beta}{b}$, from which $\left(1+\frac{b}{\beta}\right)q = \frac{\beta+b}{\beta}q = \frac{u+v}{v}f = \frac{u+v}{v}\cdot\frac{uv}{u+v} = u = b$, and their use follows from being able to be multiplied successively for rays passing through a series of lenses.]
Thence for the ratios of the apertures we had previously the following relations, clearly with the radius of the apparent field of view in place $=\Phi$:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi} = \frac{Aa}{b} = \frac{\alpha}{b} \text{ or } \frac{\mathfrak{B}\pi}{\Phi} = \frac{\alpha+b}{q}$$
$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi} = \frac{ABa}{c} = \frac{B\alpha}{c}$$
$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi} = \frac{ABCa}{d} = \frac{BC\alpha}{d}$$
$$\frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi} = \frac{ABCDa}{e} = \frac{BCD\alpha}{e}$$
etc.

and hence in turn we have elicited the following values :

$$a = \infty \qquad \alpha = p$$
$$b = \frac{p\Phi}{\mathfrak{B}\pi-\Phi} \qquad \beta = \frac{Bp\Phi}{\mathfrak{B}\pi-\Phi}$$
$$c = \frac{Bp\Phi}{\mathfrak{C}\pi'-\pi+\Phi} \qquad \gamma = \frac{BCp\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$
$$d = \frac{BCp\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \qquad \delta = \frac{BCDp\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$
$$e = \frac{BCDp\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi} \qquad \varepsilon = \frac{BCDEp\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$
etc.

COROLLARY 1

12. Now from the above it has been shown well enough, how it may be required to construct individual lenses from two determinable distances ; that finally it is necessary to recall the values of the letters ρ, σ, τ, to which also we have added μ and ν, which are

$$\rho+\sigma = \frac{1}{n-1},\ \sigma-\rho = \frac{2n+2}{n+2},\ \tau = \frac{n\cdot\sqrt{(4n-1)}}{2(n-1)(n+2)},$$
$$\mu = \frac{n(4n-1)}{8(n-1)^2(n+2)} \text{ and } \nu = \frac{4(n-1)^2}{4n-1},\ \mu\nu = \frac{n}{2(n+2)}.$$

[Recall from Book I, Ch. I, § 55, that for $n=\frac{31}{20}$,

$$\frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191=\mu,\quad \frac{4(n-1)^2}{(4n-1)}=0,232692=\nu;$$

$$\frac{4+n-2nn}{2(n-1)(n+2)}=\frac{149}{781}=0,190781=\rho,\quad \frac{n(2n+1)}{2(n-1)(n+2)}=\frac{1271}{781}=1,627401=\sigma,$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)}=0,905133=\tau.]$$

COROLLARY 2

13. From these values for any known ratio of refraction for the determinable distances α, a with the arbitrary number λ, the faces of the lens may be defined in the following manner:

$$\text{Radius of the anterior face }=\frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}},$$

$$\text{radius of the posterior face }=\frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}},$$

according to which example all the remaining lenses are required to be constructed.

COROLLARY 3

14. Since the confusion arising from such a lens may become a minimum by taking $\lambda=1$, it will be worthwhile to investigate, whatever the size of the number may be taken for λ, so that both faces of the lens may be made equal to each other ; in the end this is found:

$$\sqrt{(\lambda-1)}=\frac{a-\alpha}{a+\alpha}\cdot\frac{2(n+1)(n-1)}{n\cdot\sqrt{(4n-1)}}$$

and hence

$$\lambda=1+\frac{(a-\alpha)}{(a+\alpha)}\cdot\frac{4(nn-1)^2}{n^2(4n-1)};$$

now since there shall be

$$\frac{\overline{a-\alpha}^2}{\overline{a+\alpha}^2}=1-\frac{4a\alpha}{2(a+\alpha)^2},$$

there will be

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}-\frac{16a\alpha(nn-1)^2}{(a+\alpha)^2(4n-1)};$$

whereby, if there were either $a=\infty$ or $\alpha=\infty$, there will become:

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}.$$

SCHOLIUM

15. So that our investigation may be extended wider, we may attribute individual ratios of refraction to the singular lenses, since now it has been ascertained different kinds of glass differ between each other, thus so that yet the value of the number n may be contained between the limits 1,50 and 1,60 , on account of which for practical use the values of the letters ρ, σ, τ, μ, v and μv will be contained between the limits shown here ; which finally we will attach here in the following table :

n	ρ	σ	τ	μ	v	μv
1,50	0,2858	1,7143	0,9583	1,0714	0,2000	0,2143
1,51	0,2653	1,6956	0,9468	1,0420	0,2065	0,2151
1,52	0,2456	1,6776	0,9358	1,0140	0,2129	0,2159
1,53	0,2267	1,6601	0,9252	0,9875	0,2196	0,2168
1,54	0,2083	1,6434	0,9149	0,9622	0,2260	0,2176
1,55	0,1907	1,6274	0,9051	0,9381	0,2326	0,2182
1,56	0,1737	1,6119	0,8956	0,9151	0,2393	0,2192
1,57	0,1573	1,5970	0,8864	0,8932	0,2461	0,2199
1,58	0,1414	1,5827	0,8775	0,8724	0,2529	0,2206
1,59	0,1259	1,5689	0,8689	0,8525	0,2597	0,2214
1,60	0,1111	1,5555	0,8607	0,8333	0,2666	0,2221

So that truly for the differentials of the numbers n may attain, from these I define nothing, if indeed Dollond's experiments are true, except that, if $n = 1,53$ for crown glass, $n' = 1,58$ for crystal glass, there shall be from experiment:

$$dn : dn' = 2 : 3,\ \tfrac{dn}{n-1} : \tfrac{dn'}{n'-1} = 7 : 10 .$$

PROBLEM 2

16. *From however many telescope lenses it were composed, to define the conditions, so that the distances between the individual lenses may be made positive.*

SOLUTION

In whatever manner the determinable distances may be effected by the ratio of the signs $+$ and $-$, it is necessary always, that the magnitudes $\alpha + b$, $\beta + c$, $\gamma + d$, $\delta + e$ etc., by which the separations of the lenses are expressed, shall become positive; so that therefore the values found before may be substituted in place of these letters, it will be necessary for the conditions to be satisfied :

$$\alpha+b=\frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi-\Phi}>0$$
$$\beta+c=\frac{B\Phi p(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$$
$$\gamma+d=\frac{BC\Phi p(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0$$
$$\delta+e=\frac{BCD\Phi p(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi'''-\pi''+\pi'-\pi+\Phi)}>0$$
etc.;

concerning which distances it will be agreed certain of these also can become $=0$, evidently when two or more lenses are themselves in contact with each other in turn, just as we have seen above to be able to happen; but at no time must any of these distances become negative.
[See Vol. I, Ch. 7, § 339.]

COROLLARY 1

17. Hence it is evident, if there were $\pi=0$, then the distance between the first and second lens vanishes; and if besides there shall be $\pi'=0$, also the third lens will be joined next to the preceding lens, and the fourth lens in addition will be added to these, if there were also $\pi''=0$, which indeed happens with composite or multiple objective lenses, has now has been shown previously.

COROLLARY 2

18. But the separation between the first and second lens will become greater than nothing, either by attributing a positive value to π itself, clearly as often as $\frac{\mathfrak{B}p}{\mathfrak{B}\pi-\Phi}$ were a positive quantity, or by attributing a negative value to π, as often as $\frac{\mathfrak{B}p}{\mathfrak{B}\pi-\Phi}$ were a negative quantity.

COROLLARY 3

19. Since $\alpha=p$ is a positive quantity, the cases deserve to be observed:

$$\text{I.) } b=-p;\quad \text{II.) } b=0;\quad \text{III.) } b>0.$$

In the first case the interval vanishes, and thus there will be either $\pi=0$ or $\mathfrak{B}=0$, but since neither can happen, because there would become $B=0$ and thus $\frac{\beta}{b}=0$ and therefore $\beta=0$; but a lens of this kind cannot be given, unless also there shall become $b=0$; from which in this first case it will be necessary to have $\pi=0$. In the second case, where $b=0$, the second lens lies on that same image projected from the first lens, and there will become $\mathfrak{B}\pi - \Phi = \infty$, since neither p nor Φ can be $= 0$; from which for this case there will be produced $\mathfrak{B} = \infty$ and $B = -1$, that is $\beta = -b = 0$, from which in this case it is apparent both the determinable distances of the second lens vanish, truly with nothing less whatever to be retained except for the value of its focal length q, since there shall be $q = \mathfrak{B}b$, on account of $\mathfrak{B} = \infty$ and $b = 0$. Finally in the third case, where $b > 0$, there must become $\mathfrak{B}\pi - \Phi > 0$ or $\mathfrak{B} > \frac{\Phi}{\pi}$.

COROLLARY 4

20. What we have observed here in the second case, also prevails for any other lens, which may be put in the place of the image formed by the preceding lens ; then truly the anterior determination of its distance vanishes, and from which also the posterior distance by necessity must vanish ; for this may happen with the fourth lens, the determinable distances of which are d and δ and the focal length is s, and since there is $\frac{1}{s} = \frac{1}{d} + \frac{1}{\delta}$; therefore if there shall be $d = 0$, by necessity also there will become $\delta = 0$: for indeed there shall be $\delta = \frac{sd}{d-s}$, on putting $d = 0$ there becomes certainly $\delta = 0$; then truly hence also we know to become $\frac{\delta}{d} = -1 = D$, thus as in this case also there shall be $D = -1$ and $\mathfrak{D} = \infty$.

PROBLEM 3

21. *If a telescope were composed from some number of lenses, to define the apertures of the individual lenses, so that all the rays from the object entering through the objective lens likewise may be transmitted through all the following lenses.*

SOLUTION

Here not only some object is required to be considered, but so great, in order that it can be seen through the whole telescope, thus so that its apparent radius may agree with the radius of the field of view, as we have put in place $= \Phi$. So that if now the radius of the aperture of the objective lens may be put $= x$, we have shown above [Book I, § 341] the radii of the individual lenses following to be determined in the following way :

	Radius of the aperture :
II$^{\text{nd}}$ lens	$\frac{\mathfrak{B}\pi p \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$
III$^{\text{rd}}$ lens	$\frac{B\mathfrak{C}\pi' p \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$
IV$^{\text{th}}$ lens	$\frac{BC\mathfrak{D}\pi'' p \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$
V$^{\text{th}}$ lens	$\frac{BCD\mathfrak{E}\pi''' p \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$
	etc.

these individual expressions depend on two parts, and the sign of ambiguity $\pm$ indicate both parts to be taken positive, even if perhaps both or at least one or the other were negative. Moreover nothing stands in the way why these apertures may not be taken larger, even if this enlargement may be destitute of use. Since also it suffices for only some of the radii to be taken greater, which generally it is the first to be taken equal, because hence no other inconvenience is to be feared, unless the extremities of the field of view appearing may be observed a little obscured ; and the lenses shall be of so great sizes, so that very small fractions will be required to be taken for the letters $\pi,\ \pi',\ \pi'',\ \pi'''$ etc., either smaller by a $\frac{1}{4}$, or $\frac{1}{5}$ as hitherto has been established above .

COROLLARY 1

22. But the first parts of these formulas can be expressed much more neatly, if we may introduce focal lengths into the calculation; then indeed these will be expressed in the following way: $\pi q,\ \pi' r,\ \pi'' s,\ \pi''' t$ etc., which expressions follow at once from the nature of the letters $\pi,\ \pi',\ \pi''$ etc. established above [Book I, § 260].

COROLLARY 2

23. Hence also the other parts of these formulas will be able to be expressed more concisely, since there shall be

$$\frac{\Phi}{\mathfrak{B}\pi-\Phi}=\frac{b}{p}=\frac{q}{\mathfrak{B}p},\quad \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}=\frac{r}{B\mathfrak{C}p},\ \text{and}\ \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=\frac{s}{BC\mathfrak{D}p},$$

from which the above formulas can be represented thus:

Radius of the aperture

I$^{\text{st}}$	lens	$=x$
II$^{\text{nd}}$	lens	$=\pi q\pm\frac{qx}{\mathfrak{B}p}$
III$^{\text{rd}}$	lens	$=\pi' r\pm\frac{rx}{B\mathfrak{C}p}$
IV$^{\text{th}}$	lens	$=\pi'' s\pm\frac{sx}{BC\mathfrak{D}p}$
V$^{\text{th}}$	lens	$=\pi''' t\pm\frac{tx}{BCD\mathfrak{C}p}$,
		etc.

COROLLARY 3

24. Therefore so that it may eventuate, that some of the letters $\pi,\ \pi',\ \pi'',\ \pi'''$ etc. may vanish, then for the corresponding lens the radius of the aperture must be taken equal

only of the following part. Truly for other cases, for which the first part is greater than the second, it will suffice to define the aperture from the first part only.

SCHOLIUM

25. This same case, in which some of the letters $\pi,\ \pi',\ \pi''$ etc. shall be $=0$, then has a place, when the corresponding lens may be arranged in a place of this kind, which we have assigned above for a suitable place of the eye [Book I, § 233], in which clearly the rays from the extremity of the object transmitted through the centre of the lens again concur somewhere with the axis. For in this place the lens put in place will need no other aperture except that, which is required on account of the aperture of the objective lens. Whereby it is to be observed properly, as often as some lens may be set up in such a place, and in turn for this the corresponding value of π to become $=0$. Therefore since generally the part of the aperture depending on x becomes very small, lenses of this kind will be able to be used most conveniently in place of diaphragms, which commonly are accustomed to be connected to telescopes, so that stray light rays [*e.g.* from other sources] may be excluded on account of these exceedingly small apertures.

PROBLEM 4

26. *If a telescope were constructed from some number of lenses, to define the ratio of the multiplication m, by which objects seen through it may be observed to have increased.*

SOLUTION

From the formulas, which we have found above now [Book I, § 214] for multiplication, we will obtain the following formulas for the individual number of lenses :

Number of lenses	Ratio of multiplication
I	$m=+1$ from $\frac{\alpha}{l}=-1$
II	$m=-\frac{\alpha}{b}$ " $\frac{\beta}{l}=-1$
III	$m=+\frac{\alpha\beta}{bc}$ " $\frac{\gamma}{l}=-1$
IV	$m=-\frac{\alpha\beta\gamma}{bcd}$ " $\frac{\delta}{l}=-1$
V	$m=-\frac{\alpha\beta\gamma}{bcd}$ " $\frac{\varepsilon}{l}=-1$
	etc.;

here clearly it is noteworthy, if the value for m may be produced positive, the object will be going to be represented situated erect, but if negative then situated inverted. Therefore in turn, so that for example the telescope may multiply by one hundred times, two cases are required to be established, the one, where the representation is required to be erect, the other, where inverted; and in the first case we have put $m = +100$, truly for the latter $m = -100$, thus so that then it shall be clear enough, how for any number of lenses the values of the letters α, b; β, c; γ, d etc. ought to be prepared.

COROLLARY 1

27. If we may introduce slightly larger italic letters, for two lenses there will be $m = -\frac{\alpha}{b}$, for three lenses $m = +\frac{\alpha}{c}B$, for four $m = -\frac{\alpha}{d}BC$, for five $m = +\frac{\alpha}{e}BCD$ etc.

COROLLARY 2

28. Again there shall be $\alpha = p =$ focal length of the objective lens and in these formulas slightly smaller italic letters will denote the focal length of the final lens, these formulas for multiplication may be represented concisely in this manner :

I. $m = +1$

II. $m = -\frac{p}{q}$

III. $m = +\frac{p}{r}B$,

IV. $m = -\frac{p}{s}BC$

V. $m = +\frac{p}{t}BCD$

etc.

SCHOLIUM

29. In this problem we have found for the case of a single lens $m = +1$, where it is indicated the object not to be increased by a single lens, but for the nature of the amount to be observed ; that which evident by itself, since we assume the distance of the relaxed eye l to be infinite; for then there will be also $\alpha = p = \infty$, and thus this lens will have its faces parallel to each other, by which objects thence may be viewed by the naked eye ; then for the case of two lenses we have found $m = \frac{-p}{q}$; whereby, since p shall be positive, if q were negative, the telescope refers to upright objects and increased in the ratio $p : q$, or how many times the focal length of the objective lens were greater than the focal length of the concave eyepiece lens; but if the eyepiece lens also were convex, or q were positive, objects will be seen upside down and how many times greater, as many times as q will be contained in p. Then truly hence also it is clear, on account of $\alpha = p$ and $b = q$, as it is agreed well enough, the distance between these two lenses $\alpha + b$, or the length of the telescope to be equal to the magnitude $p + q$. But if several lenses may be used, the ratio of the multiplication may no longer be determined by the

focal lengths of the objective and eyepiece lenses, but in addition the ratio is required to be had of the numbers *B, C, D* etc. or of the intermediate lenses.

PROBLEM 5

30. *From however many lenses the telescope were composed, to define the position of the eye or its distance past the final eyepiece lens.*

SOLUTION

This distance we have indicated above [Book I, § 180] by the letter *O,* and we have seen at once for the case of a single lens to become $O=0$.

But for the case of two lenses we have found $O=\frac{\mathfrak{B}b\pi}{\pi-\Phi}$, which on account of $\beta=\infty$, [for the relaxed eye] and hence $B=\infty$ and $\mathfrak{B}=1$ will become this $O=\frac{b\pi}{\pi-\Phi}$. But again since there shall be $b=\frac{p\Phi}{\pi-\Phi}$ and thus $\pi-\Phi=\frac{p\Phi}{b}$, there will be had $O=\frac{b^2\pi}{p\Phi}=\frac{q^2\pi}{p\Phi}$ and because $m=-\frac{p}{q}$ or $p=-mq$ there will be $O=\frac{-q\pi}{m\Phi}$, and from $\frac{\pi}{\Phi}=\frac{p+1}{q}$, also there will be had

$$O=-\frac{(p+q)}{m}=+\frac{m-1}{m}q.$$

For the case of three lenses, on account of $\gamma=\infty$ and thus $C=\infty$ and $\mathfrak{C}=1$ we will have $O=\frac{c\pi'}{\pi'-\pi+\Phi}$; truly there is $c=r=\frac{Bp\Phi}{\pi'-\pi+\Phi}$ and $m=\frac{p}{r}B$ and hence $pB=mr$ and thus $c=\frac{mr\Phi}{\pi'-\pi+\Phi}$; from which there will become $O=\frac{\pi'}{m\Phi}r$.

For the case of four lenses on account of $\delta=\infty$ and thus $D=\infty$ and $\mathfrak{D}=1$ we find $O=\frac{d\pi''}{\pi''-\pi'+\pi-\Phi}$, that is $d=s=\frac{BCp\Phi}{\pi''-\pi'+\pi-\Phi}=\frac{-ms\Phi}{\pi''-\pi'+\pi-\Phi}$ and hence $\pi''-\pi'+\pi-\Phi=-m\Phi$ and thus $O=\frac{\pi''}{m\Phi}s$.

So that this may be adapted for more lenses, I have placed the following table below:

Number of lenses	Position of the eye
I	$O=0$
II	$O=\frac{b\pi}{\pi-\Phi}=-\frac{\pi}{m\Phi}q$
III	$O=\frac{c\pi'}{\pi'-\pi+\Phi}=\frac{\pi'}{m\Phi}r$
IV	$O=\frac{d\pi''}{\pi''-\pi'+\pi-\Phi}=-\frac{\pi''}{m\Phi}s$
V	$O=\frac{e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}=\frac{\pi'''}{m\Phi}t$

etc.

COROLLARY 1

31. From the above it will be convenient to repeat here, if the value of O may be produced positive, then a suitable place may be found for the eye, from which the whole perceptible field of view may be able to be seen, but a negative value may be produced for O, then the eye must be applied up to the final lens and in this case the apparent field of view to be determined by the aperture of the pupil.

COROLLARY 2

32. In the case of two lenses it may be allowed to show a neater distance O of the two lenses, since there shall be $O = \frac{m-1}{m} q = \left(1 - \frac{1}{m}\right) q$; from which it is at once apparent for an erect representation, where q is a negative quantity, the distance O equally to be negative and thus the eye must be applied next to the lens [*i.e.* the image is virtual] ; but if the eyepiece lens were convex and the image inverted, then the eye must be located at a certain distance after the ocular lens [*i.e.* the image is real].

PROBLEM 6

33. *From however many lenses a telescope were composed, if the distance of the eye past the eyepiece were produced positive, to define the apparent field of view or its radius* Φ [*i.e.* the angle in radians subtended by the object at the telescope], *which it will be permitted to view.*

SOLUTION

Since there shall be $h = a$, from the kind of formulas found above we will have the following determinations of the apparent field of view for any number of lenses :

Number of lenses	Apparent radius of the field of view
I	$\Phi =$ indeterminate
II	$\Phi = \frac{-\pi}{m-1}$
III	$\Phi = \frac{-\pi + \pi'}{m-1}$
IV	$\Phi = \frac{-\pi + \pi' - \pi''}{m-1}$
V	$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{m-1}$

etc.

COROLLARY 1

34. If *m* may denote a positive number, it shall indicate there always the image of the object to be erect ; but if in the telescope it may show an inverted image, than *m* is required to be expressed by a negative number always, as now has been cautioned above.

COROLLARY 2

35. Also it is apparent from these formulas, so that the greater were the multiplier *m,* thus the smaller will become the apparent field of view, with all other things being equal, and since the letters $\pi,\ \pi',\ \pi'',\ \pi''$ etc. shall not be able to denote fractions greater than $\frac{1}{4}$ or $\frac{1}{5}$, either positive or negative, it is clear by increasing the number of lenses the apparent field of view will be able to increase more continually.

SCHOLION

36. In this manner the radius of the apparent field of view is found to be expressed by a certain fraction, which is required to be observed as a part of the radius or of the total sine. Whereby, since the arc of the circle may contain a ratio approximately equal to 57° 17′ or 3437′, the fraction for Φ initially may be converted into minutes, if that may be multiplied by the number 3437, and in this way the distance in the heavens, which is observed by whatever telescope, will be readily defined in degrees and minutes, where in addition it is agreed to be observed this angle Φ here is to be regarded always as positive; indeed if it were produced, that always is an indication the ratio of multiplication *m* also to be required to be taken negative or for the image to be inverted. [Thus, the angle Φ in the above formulas is to be measured in radians.]

PROBLEM 7

37. *From whatever number of lenses the telescope were composed, if the distance of the eye past the eyepiece lens were produced negative, to define the apparent field of view or its radius Φ, which may be permitted to be observed.*

SOLUTION

If the distance *O* were negative and thus the eye shall be unable to be placed in this location, now above we have seen then the eye ought to be placed next to the lens, as if there were $O = 0$, and in this case the apparent field of view no further to be defined by the apertures of the lenses, but to depend mainly on the aperture of the pupil, the radius of which we have designated by the letter ω , which on account of the conspicuous variation can be increased from the twentieth part of a fingerwidth [digit] as far as to $\frac{1}{10}$ th, which is accustomed to happen, if the eye may be moved to a very dark place. Therefore for this case from the above treatment the determination of the apparent field of view will be found in the following manner:

For the case of two lenses on account of $\mathfrak{B}=1$ and $b=q$, in the first place there will be $\pi q=\omega$, then $\Phi=\frac{\pi\omega}{\pi p+\omega}$, which expression on account of $\pi=\frac{\omega}{q}$ will be changed into this:

$$\Phi=\frac{\omega}{p+q}=\frac{-\omega}{(m-1)q}=\frac{-\pi}{m-1}$$

on account of $p=-mq$, which expression, since in this case q holds a negative value, by itself becomes positive.

For the case of three lenses, initially from $\mathfrak{C}=1$ and $c=r$ there will be $\pi' r=\omega$; then truly there becomes fit $\frac{Bp\Phi\pi'}{\pi'-\pi+\Phi}=\omega$ or $\frac{mr\Phi\pi'}{\pi'-\pi+\Phi}=\omega$, from which there is found :

$$\Phi=\frac{(\pi'-\pi)\omega}{mr\pi'-\omega}=\frac{(\pi'-\pi)\omega}{(m-1)\pi' r}=\frac{\pi'-\pi}{m-1}.$$

For the case of four lenses on account of $\mathfrak{D}=1$ and $d=s$ there will be initially $\pi'' s=\omega$, then truly $\frac{BCp\pi''\Phi}{\pi''-\pi'+\pi-\Phi}=\omega=\frac{ms\pi''\Phi}{\pi''-\pi'+\pi-\Phi}$, from which there is found :

$$\Phi=\frac{(\pi''-\pi'+\pi)\omega}{\omega-ms\pi''}=\frac{-(\pi''-\pi'+\pi)\omega}{(m-1)\pi'' s}=-\frac{\pi''-\pi'+\pi}{m-1}.$$

Which determinations we will show in the following table :

For the number of lenses	there will be	and for the apparent field of view
II	$\pi=\frac{\omega}{q}$	$\Phi=\frac{-\omega}{(m-1)q}=\frac{-\pi}{m-1}$
III	$\pi'=\frac{\omega}{r}$	$\Phi=\frac{-(\pi-\pi')\omega}{(m-1)\pi' r}=\frac{-\pi+\pi'}{m-1}$
IV	$\pi''=\frac{\omega}{s}$	$\Phi=\frac{-(\pi-\pi'+\pi'')\omega}{(m-1)\pi'' s}=\frac{-\pi+\pi'-\pi''}{m-1}$
V	$\pi'''=\frac{\omega}{t}$	$\Phi=\frac{-(\pi-\pi'+\pi''-\pi''')\omega}{(m-1)\pi''' t}=\frac{-\pi+\pi'-\pi''+\pi'''}{m-1}$

etc.

COROLLARY 1

38. Hence it appears the formulas for the radius of the apparent field of view Φ does not differ from the preceding case ; but truly the distinction rests in this, because in the preceding case the last of the letters π, π', π'' etc. depended on our choice, provided it

was contained within the prescribed limit $\frac{1}{4}$ or $\frac{1}{5}$, but here that must be determined from the constitution of the pupil.

COROLLARY 2

39. Therefore so far in this case the apparent field of view shall appear smaller than in the preceding case, provided a smaller value must be attributed to the final of the letters $\pi,\ \pi',\ \pi''$ etc., which occurs if the fractions $\frac{\omega}{q},\ \frac{\omega}{r},\ \frac{\omega}{s}$ etc. were smaller than that limit $\frac{1}{4}$ or $\frac{1}{5}$. But if they have been produced equal to this limit, in each case the same apparent field of view will be apparent.

COROLLARY 3

40. But it cannot be concluded from this, if these fractions $\frac{\omega}{q},\ \frac{\omega}{r},\ \frac{\omega}{s}$ etc. may become greater than the prescribed limit, then in this latter case the field of view thus is seen to become greater, thus so that the nature of this final lens does not permit a greater value corresponding to the letter π. And for this reason it certainly cannot be agreed to allow so large ocular lenses, so that a final value of the letter π may be produced exceeding $\frac{1}{4}$ or $\frac{1}{5}$, since then the aperture of this lens must be taken smaller than that of the pupil.

SCHOLIUM

41. But nothing stands in the way, why the eyepiece lens may not be given a greater field of view than the pupil, since in that case no other inconvenience need be feared, except that not all the rays transmitted by that lens may not be going to enter the eye ; but as so much is absent, either that shall be considered an inconvenience or rather since then a significant gain might be able to be obtained; for then the pupil will be able to wander successively over the whole aperture of the lens, so that we may follow that convenience, as then we may look successively at other and still other parts of the object. That which cannot be done with telescopes according to the previous case. Therefore the determination of the final letters $\pi,\ \pi',\ \pi''$ etc. shown in the problem serve only to that end, so that from that the magnitude of the field of view may be defined only according to one point of view, since thus a significant gain may be expected, if the eyepiece lens may be given a much larger aperture; from which now a much clearer account can be seen, why it may be agreed to avoid very small eyepiece lenses.

PROBLEM 8

42. *If a telescope were composed from some number of lenses and thus the individual lenses shall be formed from different kinds of glass, to define the radius of confusion, by which the image of the object will be rendered imperfect.*

SOLUTION

Now at the beginning of this chapter we have attributed a special ratio of refraction to some lens and towards this end we have introduced the letters n, n', n'', n''' etc. into the calculation. Whereby there is a need only, so that we may transfer the formulas found by being added to the penultimate I[th] part found for the case of telescopes, so that there becomes $a=\infty,\ h=a$ and hence $A=0$ and $Aa=\alpha=p$; for which requiring to be effected from the denominators of the individual members, the factor A^3 may be taken with the common factor, so that there may become in its denominator $A^3\cdot aah=\alpha^3=p^3$. With which done for any number of lenses the radius of confusion will be expressed by the following formula :

$$\frac{mx^3}{4p^3}\left(\begin{array}{l}\mu\lambda+\frac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu'B\right)\Phi}{B^3(\mathfrak{B}\pi-\Phi)}+\frac{\mu''(B+1)\left(\lambda'(C+1)^2+\nu''B\right)\Phi}{B^3C^3(\mathfrak{C}\pi'-\pi-\Phi)}\\+\frac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu'''D\right)\Phi}{B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\text{etc.}\\\qquad+\frac{\Phi(E+1)\left(\lambda''''(E+1)^2+\nu E\right)}{A^3B^3C^3D^3E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\end{array}\right)$$

which, if the individual members may be separated into two parts, it will be more conveniently expressed on account of the values $\frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C}$ etc. Clearly there will become this expression :

$$\frac{mx^3}{4p^3}\left(\begin{array}{l}\mu\lambda+\frac{\mu'\Phi}{\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\Phi}{B^2\mathfrak{C}(\mathfrak{C}\pi'-\pi-\Phi)}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)\\+\frac{\mu'''\Phi}{B^3C^3\cdot\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right)\text{ etc.}\end{array}\right)$$

which again, with the aid of the formulas recalled from § 23, is reduced to this form :

$$\frac{mx^3}{4p^3}\left(\mu\lambda+\frac{\mu'q}{\mathfrak{B}^2p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''r}{B^4\mathfrak{C}^2p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)+\frac{\mu'''s}{B^4C^4\mathfrak{D}^2p}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right)\text{ etc.}\right)$$

and hence for any number of lenses the radius of confusion is expressed in the following manner:

For two lenses on account of $B=\infty,\ b=q,\ \mathfrak{B}=1$ the radius of confusion will become $=\frac{mx^3}{4p^3}\left(\mu\lambda+\frac{\mu'\lambda'q}{p}\right)$, which form from $p=-mq$ is reduced to this :

$$\frac{mx^3}{4p^3}\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right).$$

For three lenses on account of $C=\infty$ and $\mathfrak{C}=1$ and $Bp=mr$ the radius of confusion will become

$$=\frac{mx^3}{4p^3}\left\{\mu\lambda + \frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\lambda''}{B^3 m}\right\}.$$

For four lenses on account of $D=\infty$ and $\mathfrak{D}=1$ and $BCp=-ms$ the radius of confusion

$$=\frac{mx^3}{4p^3}\left\{\mu\lambda + \frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu'' r}{B^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''\lambda'''}{B^3C^3 m}\right\}.$$

For five lenses on account of $E=\infty$ and $\mathfrak{E}=1$ and $BCDp=mt$, the radius of confusion will become

$$=\frac{mx^3}{4p^3}\left\{\begin{array}{l}\mu\lambda + \frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu'' r}{B^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)\\ +\frac{\mu''' s}{B^4C^4\mathfrak{D}^2 p}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right)+\frac{\mu''''\lambda''''}{B^3C^3D^3 m}\end{array}\right\}.$$

COROLLARY 1

43. Therefore from the formulas the radius of confusion is found expressed by a number or by a certain numerical fraction, which fraction converted will show the number of degrees, minutes, and seconds, within how great an angle the individual points of the object may be viewed by the telescope, certainly within which the effect of confusion is evident.

COROLLARY 2

44. Therefore lest this confusion may become intolerable, it is necessary, that the radius of confusion may be contained within a certain limit; for which limit we have put in place above this formula $\frac{1}{4k^3}$, with k being around $k=40$ or $k=30$.

COROLLARY 3

45. Therefore so that if in conclusion we may put the number contained in the generation $=N$, it is required to insure, that $\frac{mx^3}{p^3}\cdot N$ may not exceed the limit $\frac{1}{4k^3}$, from which there will have to be established $\frac{mx^3}{p^3}\cdot N=\frac{1}{k^3}$, from which the magnitude p or the focal length of the objective lens is determined ; evidently there will become $p=k\cdot x\cdot\sqrt[3]{mN}$.

COROLLARY 4

46. If again the order of clarity may be indicated by the number y, as we have made above, where we have seen there must be taken $x = m \cdot y$ and generally I have put $y = \frac{1}{50}$ dig., from which the observable order of clarity arises clear enough, and the manner found will be $p = m \cdot ky \cdot \sqrt[3]{mN}$; from which it is apparent with all else being equal the focal length of the objective lens p to follow the fractional account of multiplication m, where it is to be observed, since $y = \frac{1}{50}$ dig. and $k = 40$, to become around $ky = \frac{4}{5}$ dig. or as if 1 dig. greater or less according to the circumstances.
[The digit was a ancient unit for measuring lengths or angles.]

SCHOLIUM 1

47. Lest it may not offend, so that we have defined the value of p from this equation, since now this quantity still shall be present in the number N, it is required to be observed here not only that magnitude p itself as well as its ratio to the remaining focal lengths q, r, s etc. to be present in the number N ; which ratios since they may be able to become known from elsewhere, our equation is suitable everywhere, from which the absolute value of p may be determined, which happens for the magnitude x, which is obtained expressed in digits, since there shall be $x = my$ and y may be given in fractions of a digit or there may be taken $y = \frac{1}{50}$ dig. greater or less, just as a greater or less degree of clarity may be desired.

SCHOLIUM 2

48. Since it shall be wished especially, so that this confusion [*i.e.* blurring or the indistinct nature of the image] may be reduced completely to zero and there may become $N = 0$, if this were to be successful, for this it will be required to be shown, hence in what way the focal length of the objective lens p should be defined, if indeed for our case $N = 0$ our equation may give $p = 0$; which since it may not be able to happen, it is required to consider its aperture or the magnitude of x ; which since from the degree of clarity y on multiplying together by m has been given, for this lens by necessity so great a focal length p must be given, so that a large lens of so great an aperture may be made ; for the minimum clearly there must be $p > 5x$ and meanwhile accordingly for this to be greater, provided the faces of the lens may be produced with a greater curvature. But it is required to note nothing impedes the quantity p from becoming larger in the kind of lens, provided that it may not become smaller.

PROBLEM 9

49. *If a telescope may be made from some number of lenses and the distance of the eye past the final lens may be found to be positive, to define the nature of the lenses, so that objects may be viewed without colored fringes.*

SOLUTION

Since we have generally satisfied this condition above, where we may adapt the equation found for the present case of the telescope and we will see the goal to be obtained, if it were possible for this equation to be satisfied:

$$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{p\Phi}+\frac{cdn''}{n''-1}\cdot\frac{\pi'}{Bp\Phi}+\frac{ddn'''}{n'''-1}\cdot\frac{\pi''}{BCp\Phi}\ \text{etc.}$$

which we may apply to a particular number of lenses.

For two lenses on account of $b=q$ there will be

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi q}{\Phi p}=-\frac{dn'}{n'-1}\cdot\frac{\pi}{m\Phi}.$$

For three lenses on account of $c=r$ and $Bp=mr$ there will become

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' r}{B\Phi p}$$

or

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi'}{m\Phi}.$$

For four lenses on account of $d=s$ and $BCp=-ms$ there will be

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' c}{B\Phi p}-\frac{dn'''}{n'''-1}\cdot\frac{\pi''}{m\Phi}.$$

For five lenses on account of $e=t$ and $BCDp=mt$ there will be

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' c}{B\Phi p}+\frac{dn'''}{n'''-1}\cdot\frac{\pi'' d}{BC\Phi p}+\frac{dn''''}{n''''-1}\cdot\frac{\pi'''}{m\Phi}.$$

COROLLARY 1

50. For the case of two lenses there shall be $\frac{-\pi}{\Phi}=m-1$, this equation will be obtained : $0=\frac{dn'}{n'-1}\cdot\frac{m-1}{m}$; which since it is unable to be made, it is evident telescopes composed from two lenses cannot be freed from the error of colored fringes. [*i.e.* from two convex lenses]

COROLLARY 2

51. If all the lenses may be made from the same kind of glass, our equations will be allowed to be divided by the differential factors, and thus the same formulas will be found, which have been given for the above case.

PROBLEM 10

52. *If a telescope were made from some number of lenses and the distance of the eye past the final lens inserted were negative, to define the disposition of the lenses, so that the object may be viewed without colored fringes.*

SOLUTION

From previously for any number of lenses the following equations will require to be satisfied:

For two lenses, if the above equation may be multiplied by *A*, there will be found :

$$0=\tfrac{dn}{n-1}\cdot B\pi p,$$

which on account of $B=\infty$ cannot be done.

For three lenses on multiplying by *A* there will be had on account of $C=\infty$

$$0=\tfrac{dn}{n-1}\cdot B\pi' p+\tfrac{dn'}{n'-1}\cdot b\left((B+1)\pi'-\pi\right).$$

For four lenses on account of $D=\infty$ there will be had :

$$0=\tfrac{dn}{n-1}\cdot B\pi'' p+\tfrac{dn'}{n'-1}\cdot b\left((B+1)C\pi''-\pi\right)+\tfrac{dn''}{n''-1}\cdot c\left(\tfrac{(C+1)\pi''-\pi}{B}\right).$$

For five lenses on account of $E=\infty$ there will be had:

$$0=\tfrac{dn}{n-1}\cdot BCD\pi''' p+\tfrac{dn'}{n'-1}\cdot b\left((B+1)CD\pi'''-\pi\right)+\tfrac{dn''}{n''-1}\cdot c\left(\tfrac{(C+1)D\pi'''-\pi'}{B}\right)$$
$$+\tfrac{dn'''}{n'''-1}\cdot d\left(\tfrac{(D+1)\pi'''-\pi''}{BC}\right).$$

PROBLEM 11

53. *If a telescope may be constructed from whatever number of lenses, to define that disposition of the lenses, so that all the confusion may be removed arising from the rays of different refrangibility.*

SOLUTION

From the above treatment we are able to show the equation for any number of lenses, by which the proposed goal may be satisfied; for on multiplying by A^2 there will be had :

$$0=\frac{dn}{n-1}\cdot\alpha+\frac{dn'}{n'-1}\cdot\frac{b}{\mathfrak{B}}+\frac{dn''}{n''-1}\cdot\frac{c}{\mathfrak{C}B^2}+\frac{dn'''}{n'''-1}\cdot\frac{d}{\mathfrak{D}B^2C^2}+\frac{dn''''}{n''''-1}\cdot\frac{e}{\mathfrak{E}B^2C^2D^2}+\text{etc.}$$

which on account of $\alpha=p,\ b=\frac{q}{\mathfrak{B}},\ c=\frac{r}{\mathfrak{C}},\ d=\frac{s}{\mathfrak{D}}\text{etc.}$ will be changed into this :

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{d}{B^2C^2\mathfrak{D}^2}+\frac{dn''''}{n''''-1}\cdot\frac{e}{B^2C^2D^2\mathfrak{E}^2}+\text{etc.}$$

Hence for the individual numbers of lenses we obtain the following equations to be fulfilled:

For two lenses on account of $\mathfrak{B}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot q.$$

For three lenses on account of $\mathfrak{C}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2}.$$

For four lenses on account of $\mathfrak{D}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{s}{B^2C^2}.$$

For five lenses on account of $\mathfrak{E}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{s}{B^2C^2\mathfrak{D}^2}+\frac{dn''''}{n''''-1}\cdot\frac{t}{B^2C^2D^2}.$$

COROLLARY 1

54. Since there shall be $\mathfrak{B}=\frac{q}{b},\ \mathfrak{C}=\frac{r}{c},\ \mathfrak{D}=\frac{s}{d}$ etc., then truly $B=\frac{\beta}{b},\ C=\frac{\gamma}{c},\ D=\frac{\delta}{d}$ etc., the general equation divided by pp or $\alpha\alpha$ will be changed into the following form:

$$0=\frac{dn}{n-1}\cdot\frac{1}{p}+\frac{dn'}{n'-1}\cdot\frac{bb}{\alpha\alpha}\cdot\frac{1}{q}+\frac{dn''}{n''-1}\cdot\frac{bb\cdot cc}{\alpha\alpha\cdot\beta\beta}\cdot\frac{1}{r}+\frac{dn'''}{n'''-1}\cdot\frac{bb\cdot cc\cdot dd}{\alpha\alpha\cdot\beta\beta\cdot\gamma\gamma}\cdot\frac{1}{s}+\frac{dn''''}{n''''-1}\cdot\frac{bb\cdot cc\cdot dd\cdot ee}{\alpha\alpha\cdot\beta\beta\cdot\gamma\gamma\cdot\delta\delta}\cdot\frac{1}{t}\ \text{etc.}$$

which equation is seen to be more convenient than the preceding.

COROLLARY 2

55. So that for the number of these terms reached, it is clear that to be equal to the number of lenses, nor therefore is there a need, that we may adapt the formula separately for any number of lenses.

COROLLARY 3

56. If all the lenses were made from the same kind of glass, then this equation may be divided by the differential coefficients and will produce

$$0=\frac{1}{p}+\frac{b^2}{\alpha^2}\cdot\frac{1}{q}+\frac{b^2c^2}{\alpha^2\beta^2}\cdot\frac{1}{r}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2}\cdot\frac{1}{s}\text{ etc.,}$$

but which in no manner can be satisfied.

SCHOLION 1

57. Because this equation, when all the lenses have been prepared from the same glass, in no manner may be able to stop increasing, can be shown in the following manner. Since there shall be

$$\frac{1}{p}=\frac{1}{\alpha},\ \frac{1}{q}=\frac{1}{b}+\frac{1}{\beta},\ \frac{1}{r}=\frac{1}{c}+\frac{1}{\gamma},\ \frac{1}{s}=\frac{1}{d}+\frac{1}{\delta},$$

if these values may be substituted and the individual members after the first may be split into two parts, the equation will adopt this form:

$$0=\frac{1}{\alpha}+\frac{b}{\alpha^2}+\frac{b^2c}{\alpha^2\beta^2}+\frac{b^2c^2d}{\alpha^2\beta^2\gamma^2}+\frac{b^2}{\alpha^2\beta}+\frac{b^2c^2}{\alpha^2\beta^2\gamma}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta}\text{ etc.,}$$

now here again the each two terms may be joined together again and the equation thus arising will be prepared:

$$0=\frac{\alpha+b}{\alpha^2}+\frac{b^2(\beta+c)}{\alpha^2\beta^2}+\frac{b^2c^2(\gamma+d)}{\alpha^2\beta^2\gamma^2}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta};$$

since now $\alpha+b,\ \beta+c,\ \gamma+d$ as the separation of the lenses by necessity are positive, plainly all the terms as far as to the final one are positive; but the final term $\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta}$ on account of $\delta=\infty$ itself vanishes, evidently for the four lenses, which we have considered here.

SCHOLIUM 2

58. Therefore from these preparations we now will be able to progress to the different kinds of telescopes to be put in place and to provide instructions about the different kinds of constructions. But since the maximum use will be made in the perfection of telescopes from those, which have been treated above concerned with multiple lenses, while clearly multiple lenses will be used in place of simple lenses, which may suffer from much less confusion, here that deliberation is repeated and adapted for the telescope. But in the first place from the formula for the radius of confusion found it is apparent the objective lens in these to consist of particular parts, if indeed for that there were $\lambda = 1$; whereby, if in place of this lens a multiple lens were substituted, for which the value of the number λ shall be much smaller or thus may vanish, at once thus we gain the maximum suitability, while the total confusion is reduced to a very small level or perhaps to nothing. On which account in the following chapter we will enumerate particular composite lenses, which will be allowed to be substituted in place of the objective lens, and we will indicate for the individual value of λ, so that they may be deployed henceforth for the circumstances.

CAPUT I

DE TELESCOPIIS IN GENERE

DEFINITIO 1

1. *Telescopium est instrumentum dioptricum obiectis valde remotis spectandis*

inserviens.

COROLLARIUM 1

2. Cum ergo distantia obiecti sit valde magna, in calculo quantitatem *a,* qua distantia obiecti a lente obiectiva designatur, tamquam infinitam spectare licet, ideoque *a* denotabit distantiam focalem lentis obiectivae, neglecta scilicet eius crassitie.

COROLLARIUM 2

3. Cum posuerimus $\alpha = Aa$, ob $a = \infty$ erit numerus A evanescens ideoque et $A = 0$ et $\mathfrak{A} = \frac{A}{A+1} = 0$. Hinc ergo in formulis supra traditis litterae A et ita ex calculo eliminabuntur, ut loco Aa et $\mathfrak{A}a$ scribatur α.

DEFINITIO 2

4. *In telescopiis campus apparens non ex ipsa obiecti conspicui quantitate aestimatur, sed ex angulo, sub quo haec pars conspicua nudo oculo cerneretur.*

COROLLARIUM

5. Littera ergo Φ, quam supra in nostras formulas introduximus, denotabit semidiametrum campi adparentis vel potius eius tangentem; quia autem hic angulus plerumque est valde parvus, is ipse loco tangentis sine errore, praecipue si multiplicatio sit notabilis, usurpatur.

DEFINITIO 3

6. *Multiplicatio in telescopiis ex ratione quantitatis per instrumentum visae ad quantitatem, qua idem obiectum in eadem distantia remotum nudo oculo cerneretur, aestimari solet.*

COROLLARIUM 1

7. Quia ergo supra in genere multiplicationem ad distantiam *h* retulimus, obiecti vero distantia posita est $= a$, erit quoque $h = a$.

COROLLARIUM 2

8. Exponens ergo multiplicationis $= m$ hoc casu indicat, quoties angulus, sub quo diametrum cuiuspiam obiecti per telescopium cernimus, maior sit angulo, sub quo idem obiectum nudis oculis cerneretur.

SCHOLION 1

9. Hoc scilicet intelligendum est, quamdiu de angulis satis parvis est sermo; quando autem anguli sunt maiores, exponens multiplicationis m declarabit, non quoties ipse angulus, sub quo obiectum quodpiam per telescopium cernitur, sed quoties eius tangens maior sit tangente eius anguli, sub quo idem obiectum nudis oculis esset appariturum, ita ut, etiamsi multiplicatio m foret infinita, tamen angulus visionis non ultra 90° excrescere posset, dum scilicet quantitas obiecti ab axe telescopii aestimatur.

SCHOLION 2

10. His igitur observatis formulae supra erutae facile ad telescopia accommodantur eoque non nihil simpliciores evadunt. Praeterea vero, etsi pro varia oculi constitutione distantia iusta littera l designata sit maxime diversa, tamen hic ista diversitas seponi solet, quia telescopium ad unam oculi speciem accomodatum in praxi facile ad quosvis alios oculos accommodatur, et quia plerumque distantia iusta l satis est magna prae oculi distantia ab ultima lente eaque adeo pro multis oculis in infinitum excrescit, commode statuemus $l = \infty$. Hinc, si ultimae lentis distantiae determinatrices sint f et ζ post eamque locus oculi $= O$, ob $O = \zeta + l$ distantia ζ debebit esse infinita, scilicet $\zeta = O - l$, ita ut sit $\frac{\zeta}{l} = -1$ sive $\frac{l}{\zeta} = -1$, atque ob $\zeta = \infty$. evidens est ultimae lentis distantiam focalem fore $= f$.

PROBLEMA 1

11. *Ex quotcunque lentibus telescopium fuerit compositum, elementa exponere, quibus supra usi sumus ad eius constructionem determinandum, simulque relationem eorum diversorem elementorum inter se repraesentare.*

SOLUTIO

Pro qualibet lente 1° consideravimus eius rationem refractionis pro radiis mediae naturae; 2° eius binas distantias determinatrices cum numero arbitrario λ; 3° nunc etiam cuiusque lentis distantiam focalem in calculum introducemus; 4° etiam introduximus rationes aperturarum pro singulis lentibus littera π indicatas. Quae elementa pro singulis lentibus sequenti modo ob oculos ponamus:

Lentes	Ratio refractiones	Distantiae determinatices	Numerus arbitrarius	Distantia focalis	Ratio aperturae
I^ma	$n:1$	$a,\ \alpha$	λ	p	π

II^da	$n':1$	$b,\ \beta$	λ'	q	π'
III^tia	$n'':1$	$c,\ \gamma$	λ''	r	π''
IV^ta	$n''':1$	$d,\ \delta$	λ'''	s	π'''
V^ta	$n'''':1$	$e,\ \varepsilon$	λ''''	t	π''''
VI^ta	$n^{v}:1$	$f,\ \zeta$	λ^{v}	u	π^{v}

etc,

Deinde etiam posuimus

$$A=\frac{\alpha}{a},\ B=\frac{\beta}{b},\ \ C=\frac{\gamma}{c},\ D=\frac{\delta}{d},\ \ F=\frac{\zeta}{f}\text{ etc.},$$

tum vero etiam

$$\mathfrak{A}=\frac{A}{A+1},\ \mathfrak{B}=\frac{B}{B+1},\ \mathfrak{C}=\frac{C}{C+1},\ \mathfrak{D}=\frac{D}{D+1},\ \mathfrak{E}=\frac{E}{E+1},\ \mathfrak{F}=\frac{F}{F+1}\text{ etc.}$$

His expositis modo ante vidimus ob $a=\infty$ fore $A=0$ et $\mathfrak{A}=0$, quibus valoribus ita est utendum, ut sit $Aa=\alpha$ et $\mathfrak{A}a=p$ atque $p=\alpha$; pro sequentibus autem lentibus habebimus

$$q=\mathfrak{B}b,\ r=\mathfrak{C}c,\ s=\mathfrak{D}d,\ t=\mathfrak{E}e,\ u=\mathfrak{F}f\text{ etc.},$$

unde vicissim per distantias focales erit

$$b=\frac{q}{\mathfrak{B}}=\frac{B+1}{B}q\text{ et }\beta=(B+1)q$$
$$c=\frac{r}{\mathfrak{C}}=\frac{C+1}{C}r\text{ et }\gamma=(C+1)r$$
$$d=\frac{s}{\mathfrak{D}}=\frac{D+1}{D}s\text{ et }\delta=(D+1)s$$
etc.

Deinde pro rationibus aperturarum habuimus supra sequentes relationes, posita scilicet semidiametro campi apparentis $=\Phi$:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{Aa}{b}=\frac{\alpha}{b}\text{ seu }\frac{\mathfrak{B}\pi}{\Phi}=\frac{\alpha+b}{q}$$
$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{ABa}{c}=\frac{B\alpha}{c}$$
$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{ABCa}{d}=\frac{BC\alpha}{d}$$
$$\frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{ABCDa}{e}=\frac{BCD\alpha}{e}$$
etc.

atque hinc vicissim valores sequentes elicuimus:

$$a = \infty \qquad \alpha = p$$
$$b = \frac{p\Phi}{\mathfrak{B}\pi - \Phi} \qquad \beta = \frac{Bp\Phi}{\mathfrak{B}\pi - \Phi}$$
$$c = \frac{Bp\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad \gamma = \frac{BCp\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$
$$d = \frac{BCp\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad \delta = \frac{BCDp\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$
$$e = \frac{BCDp\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \varepsilon = \frac{BCDEp\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc.

COROLLARIUM 1

12. In superioribus iam satis ostensum est, quomodo ex binis distantiis determinatricibus singulas lentes construi oporteat; quem in finem valores litterarum ρ, σ, τ, quibus etiam adiungimus μ et ν, recordari necesse est, qui sunt

$$\rho + \sigma = \frac{1}{n-1}, \ \sigma - \rho = \frac{2n+2}{n+2}, \ \tau = \frac{n\cdot\sqrt{(4n-1)}}{2(n-1)(n+2)},$$
$$\mu = \frac{n(4n-1)}{8(n-1)^2(n+2)} \ \text{ et } \ \nu = \frac{4(n-1)^2}{4n-1}, \ \mu\nu = \frac{n}{2(n+2)}.$$

COROLLARIUM 2

13. His valoribus pro quavis ratione refractionis cognitis pro distantiis determinatricibus α, a cum numero arbitrario λ facies lentis sequenti modo definientur:

$$\text{Radius faciei anterioris} = \frac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$\text{radius faciei posterioris} = \frac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

ad quod exemplum omnes reliquae lentes sunt construendae.

COROLLARIUM 3

14. Cum confusio ex tali lente oriunda fiat minima sumto $\lambda = 1$, operae pretium erit investigare, quantum numerum pro λ accipi oporteat, ut ambae lentis facies fiant inter se aequales; reperitur hunc in finem

$$\sqrt{(\lambda - 1)} = \frac{a-\alpha}{a+\alpha} \cdot \frac{2(n+1)(n-1)}{n\cdot\sqrt{(4n-1)}}$$

hincque

$$\lambda = 1 + \frac{(a-\alpha)}{(a+\alpha)} \cdot \frac{4(nn-1)^2}{n^2(4n-1)};$$

cum iam sit

$$\frac{\overline{a-\alpha}^2}{\overline{a+\alpha}^2} = 1 - \frac{4a\alpha}{2(a+\alpha)^2},$$

erit

$$\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} - \frac{16a\alpha(nn-1)^2}{(a+\alpha)^2(4n-1)};$$

quare, si fuerit vel $a = \infty$ vel $\alpha = \infty$ erit

$$\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)}.$$

SCHOLION

15. Quo nostra investigatio latius pateat, singulis lentibus peculiares refractionis rationes tribuamus, quoniam nunc quidem compertum est diversas vitri species ratione refractionis inter se discrepare, ita tamen, ut valor numeri *n* intra limites 1,50 et 1,60 contineatur, quamobrem pro praxi consultum erit pro singulis valoribus intra hos limites contentis valores litterarum $\rho, \sigma, \tau,\ \mu,\ \nu$ et $\mu\nu$ hic exhibera; quem in finem sequentem tabulam hic subiungemus.

n	ρ	σ	τ	μ	ν	$\mu\nu$
1,50	0,2858	1,7143	0,9583	1,0714	0,2000	0,2143
1,51	0,2653	1,6956	0,9468	1,0420	0,2065	0,2151
1,52	0,2456	1,6776	0,9358	1,0140	0,2129	0,2159
1,53	0,2267	1,6601	0,9252	0,9875	0,2196	0,2168
1,54	0,2083	1,6434	0,9149	0,9622	0,2260	0,2176
1,55	0,1907	1,6274	0,9051	0,9381	0,2326	0,2182
1,56	0,1737	1,6119	0,8956	0,9151	0,2393	0,2192
1,57	0,1573	1,5970	0,8864	0,8932	0,2461	0,2199
1,58	0,1414	1,5827	0,8775	0,8724	0,2529	0,2206
1,59	0,1259	1,5689	0,8689	0,8525	0,2597	0,2214
1,60	0,1111	1,5555	0,8607	0,8333	0,2666	0,2221

Quod vero ad differentialia numerorum *n* attinet, de iis nihil definio, si quidem experimenta Dollondi veritati sunt consentanea, praeterquam quod, si $n = 1,53$ pro vitro coronario, $n' = 1,58$ pro crystallino, sit per experimenta

$$dn : dn' = 2:3,\ \frac{dn}{n-1} : \frac{dn'}{n'-1} = 7:10.$$

PROBLEMA *2*

16. *Ex quotcunque lentibus telescopium fuerit compositum, definire conditiones, ut singularum lentium intervalia fiant positiva.*

SOLUTIO

Quomodocunque distantiae determinatrices lentium ratione signorum + et − sint adfectae, semper necesse est, ut quantitates $\alpha+b,\ \beta+c,\ \gamma+d,\ \delta+e$ etc., quibus distantiae lentium exprimuntur, fiant positivae; quodsi ergo loco harum litterarum valores ante exhibiti substituantur, sequentibus conditionibus satisfieri oportet:

$$\alpha+b=\frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi-\Phi}>0$$
$$\beta+c=\frac{B\Phi p(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$$
$$\gamma+d=\frac{BC\Phi p(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0$$
$$\delta+e=\frac{BCD\Phi p(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi'''-\pi''+\pi'-\pi+\Phi)}>0$$
etc.;

circa quas distantias observari convenit quasdam earum etiam fieri posse $=0$, quando scilicet duae pluresve lentes sibi invicem immediate iunguntur, quemadmodum in lentibus obiectivis evenire posse supra vidimus; nunquam autem ulla harum distantiarum fieri debet negativa.

COROLLARIUM 1

17. Hinc manifestum est, si fuerit $\pi=0$, tum distantiam inter lentem primam et secundam evanescere; ac si praeterea sit $\pi'=0$, etiam tertia lens praecedentibus immediate iungetur, et quarta lens insuper iis adiungetur, si quoque fuerit $\pi''=0$, quod quidem evenit in lentibus obiectivis compositis seu multiplicatis, uti supra iam est ostensum.

COROLLARIUM 2

18. Distantia autem inter lentem primam et secundam fiet maior nihilo, vel tribuendo ipsi π valorem positivum, quoties scilicet fuerit $\frac{\mathfrak{B}p}{\mathfrak{B}\pi-\Phi}$ quantitas positiva, vel tribuendo ipsi π valorem negativum, quoties $\frac{\mathfrak{B}p}{\mathfrak{B}\pi-\Phi}$ fuerit quantitas negativa.

COROLLARIUM 3

19. Quoniam $\alpha = p$ est quantitas positiva, casus notari merentur:

$$\text{I.) } b = -p; \quad \text{II.) } b = 0; \quad \text{III.) } b > 0.$$

Primo casu intervallum primum evanescit, ideoque erit vel $\pi = 0$ vel $\mathfrak{B} = 0$, quod autem fieri nequit, quia foret $B = 0$ ideoque $\frac{\beta}{b} = 0$ ac propterea $\beta = 0$; cuiusmodi autem lens non datur, nisi etiam sit $b = 0$; unde in hoc primo casu necessario habebitur $\pi = 0$. Secundo casu, quo $b = 0$, lens secunda cadet in ipsam imaginem a prima lente proiectam, fietque $\mathfrak{B}\pi - \Phi = \infty$, quia neque p neque Φ esse potest $= 0$; unde prodibit pro hoc casu $\mathfrak{B} = \infty$ et $B = -1$, hoc est $\beta = -b = 0$, unde patet hoc casu ambas distantias determinatrices secundae lentis evanescere, nihilo vero minus eius distantiam focalem q valorem quemcunque retinere posse, cum sit $q = \mathfrak{B}b$, ob $\mathfrak{B} = \infty$ et $b = 0$. Casu denique tertio, quo $b > 0$, fieri debet $\mathfrak{B}\pi - \Phi > 0$ seu $\mathfrak{B} > \frac{\Phi}{\pi}$.

COROLLARIUM 4

20. Quod hic de casu secundo notavimus, valet quoque de qualibet alia lente, quae in locum imaginis a lente praecedente formatae constituitur; tum enim eius distantiarum determinatricium anterior evanescit, unde et posterior necessario evanescere debet; eveniat enim hoc in lente quarta, cuius distantiae determinatrices sunt d et δ et distantia focalis s, et quia est $\frac{1}{s} = \frac{1}{d} + \frac{1}{\delta}$; si ergo sit $d = 0$, necessario quoque fiet $\delta = 0$: cum enim sit $\delta = \frac{sd}{d-s}$, posito $d = 0$ fiet utique $\delta = 0$; tum vero hinc etiam cognoscimus fore $\frac{\delta}{d} = -1 = D$, ita ut hoc quoque casu sit $D = -1$ et $\mathfrak{D} = \infty$.

PROBLEMA 3

21. *Si telescopium ex quotcunque lentibus fuerit compositum, definire aperturas singularum lentium, ut omnes radii ab obiecto per lentem obiectivam ingressi simul per omnes lentes sequentes transmittantur.*

SOLUTIO

Hic non obiectum quodcunque est intelligendum, sed tantum, quod per telescopium conspici potest totum, ita ut eius semidiameter apparens conveniat cum semidiametro campi apparentis, quam statuimus $= \Phi$. Quodsi iam lentis obiectivae ponatur semidiameter aperturae $= x$, supra [Liber I, § 341] ostendimus semidiametros aperturae singularum lentium sequentium sequenti modo determinari :

Semidiameter	aperturae
lentis II^dae^	$\frac{\mathfrak{B}\pi p\pm x}{\mathfrak{B}\pi-\Phi}.\Phi$
lentis III^tiae^	$\frac{B\mathfrak{C}\pi' p\pm x}{\mathfrak{C}\pi'-\pi+\Phi}\cdot\Phi$
lentis IV^tae^	$\frac{BC\mathfrak{D}\pi'' p\pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\cdot\Phi$
lentis V^tae^	$\frac{BCD\mathfrak{E}\pi''' p\pm x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\cdot\Phi$
	etc.

singulae hae expressiones constant duabus partibus, et signum ambiguum $\pm$ indicat ambas partes capi debere positivas, etiamsi forte ambae vel saltim alterutra fuerit negativa. Nihil autem impedit, quominus hae aperturae capiantur maiores, etiamsi haec amplificatio omni usu destituatur. Cum etiam sufficit has semidiametros maiori tantum parti, quae plerumque est prior, aequales sumsisse, quia hinc nullum aliud incommodum est metuendum, nisi quod extremitates campi adparentis aliquanto obscurius repraesententur; atque ut lentes tantae aperturae sint capaces, pro litteris $\pi,\ \pi',\ \pi'',\ \pi'''$ etc. tam exiguas fractiones sumi oportet, uti supra est expositum, veluti $\frac{1}{4},\ \frac{1}{5}$ vel adhuc minores.

COROLLARIUM 1

22. Priores partes harum formularum multo concinnius exprimi possunt, si distantias focales in calculum introducamus; tum enim eae sequenti modo exprimentur: $\pi q,\ \pi' r,\ \pi'' s,\ \pi''' t$ etc., quae expressiones immediate ex natura litterarum $\pi,\ \pi',\ \pi''$ etc. supra [Liber I, § 260] exposita sequuntur.

COROLLARIUM 2

23. Hinc etiam alterae partes illarum formularum concinnius exprimi poterunt, cum sit

$$\frac{\Phi}{\mathfrak{B}\pi-\Phi}=\frac{b}{p}=\frac{q}{\mathfrak{B}p} \text{ et } \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}=\frac{r}{B\mathfrak{C}p} \text{ et } \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=\frac{s}{BC\mathfrak{D}p},$$

unde superiores formulae ita repraesentari possunt:

Semidiameter aperturae

lentis $\text{I}^{\text{mae}} = x$

lentis $\text{II}^{\text{dae}} = \pi q \pm \frac{qx}{\mathfrak{B}p}$

lentis $\text{III}^{\text{tiae}} = \pi' r \pm \frac{rx}{B\mathfrak{C}p}$

lentis $\text{IV}^{\text{tae}} = \pi'' s \pm \frac{sx}{BC\mathfrak{D}p}$

lentis $\text{V}^{\text{tae}} = \pi''' t \pm \frac{tx}{BCD\mathfrak{E}p}$,

etc.

COROLLARIUM 3

24. Quodsi ergo eveniat, ut litterarum $\pi,\ \pi',\ \pi'',\ \pi'''$ etc. quaepiam evanescat, tum pro lente respondente semidiameter aperturae soli secundae parti aequalis sumi debet. Allis vero casibus, quibus pars prima maior est secunda, sufficit aperturam ex sola prima parte definiri.

SCHOLION

25. Casus iste, quo litterarum $\pi,\ \pi',\ \pi''$ etc. quaepiam fit $=0$, tum habet locum, quando lens respondens in eiusmodi loco collocatur, quem supra [Liber I, § 233] pro idoneo loco oculi assignavimus, in quo scilicet radii ab extremitate obiecti per centrum lentis primae transmissi iterum uspiam cum axe concurrunt. In hoc enim loco lens constituta nulla alia apertura indigebit nisi ea, quae ob aperturam lentis obiectivae requiritur. Quare probe notandum est, quoties quaepiam lens in tali loco collocatur, pro ea valorem ipsius π respondentis fore $=0$ et vicissim. Quoniam igitur plerumque pars aperturae ab x pendens fit valde parva, huiusmodi lentes commodissime loco diaphragmatum, quae vulgo in telescopiis adplicari soient, usurpari poterunt, ut earum tam exigua apertura radii peregrini excludantur.

PROBLEMA 4

26. *Ex quotcunque lentibus telescopium fuerit compositum, definire rationem multiplicationis m, qua obiecta per id visa aucta conspicientur.*

SOLUTIO

Ex formulis, quas iam supra [Liber I, § 214] pro multiplicatione invenimus, obtinebimus pro singulis lentium numeris sequentes formulas:

Pro numero lentium	Ratio multiplicationis
I	$m=+1$ ob $\frac{\alpha}{l}=-1$
II	$m=-\frac{\alpha}{b}$ ob $\frac{\beta}{l}=-1$
III	$m=+\frac{\alpha\beta}{bc}$ ob $\frac{\gamma}{l}=-1$
IV	$m=-\frac{\alpha\beta\gamma}{bcd}$ ob $\frac{\delta}{l}=-1$
V	$m=-\frac{\alpha\beta\gamma}{bcd}$ ob $\frac{\varepsilon}{l}=-1$

etc.;

hic scilicet notandum est, si pro *m* prodeat valor positivus, obiectum situ erecto, sin autem negativus, situ inverso repraesentatum iri. Vicissim igitur, si velimus, ut telescopium v. gr. centies multiplicet, duo casus sunt evolvendi, alter, quo repraesentatio requiritur erecta, alter, quo inversa; ac priori casu statuimus $m=+100$, posteriori vero $m=-100$, ita ut tunc satis sit perspicuum, quomodo pro quovis lentium numero valores litterarum α, b; β, c; γ, d etc. esse debeant comparati.

COROLLARIUM 1

27. Si litteras latinas maiusculas introducere velimus, erit pro duabus lentibus $m=-\frac{\alpha}{b}$, pro tribus $m=+\frac{\alpha}{c}B$, pro quatuor $m=-\frac{\alpha}{d}BC$, pro quinque $m=+\frac{\alpha}{e}BCD$ etc.

COROLLARIUM 2

28. Cum porro sit $\alpha=p=$ distantiae focali lentis obiectivae et littera latina minuscula in his formulis denotet distantiam focalem lentis ultimae, formulae istae pro multiplicatione concinnius hoc modo repraesentantur:

I. $m=+1$

II. $m=-\frac{p}{q}$

III. $m=+\frac{p}{r}B$,

IV. $m=-\frac{p}{s}BC$

V. $m=+\frac{p}{t}BCD$

etc.

SCHOLlON

29. In hoc problemate pro casu unius lentis invenimus $m=+1$, quo indicatur obiecta per unicam lentem non aucta, sed naturali quantitate spectari; id quod per se est manifestum, quoniam distantiam oculi iustam l infinitam assumimus; tum enim erit etiam $\alpha = p=\infty$, ideoque haec lens habebit suas facies inter se parallelas, per quam obiecta perinde cernuntur ac nudis oculis; deinde pro casu duarum lentium invenimus $m=\frac{-p}{q}$*;* quare, cum p sit positivum, si q fuerit negativum, telescopium referet obiecta situ erecto et aucta in ratione $p:q$*,* seu quoties distantia focalis lentis obiectivae maior fuerit quam distantia focalis lentis ocularis concavae; sin autem lens ocularis quoque fuerit convexa, seu q positivum, obiecta cernentur situ inverso ac toties aucta, quoties q continebitur in p*.* Tum vero hinc etiam liquet, ob $\alpha = p$ et $b=q$*,* distantiam inter has duas lentes $\alpha+b$ seu longitudinem telescopii fore aequalem quantitati $p+q$*,* uti satis constat. At si plures lentes adhibeantur, ratio multiplicationis non amplius per solas distantias focales lentium obiectivae et ocularis determinatur, sed insuper ratio est habenda numerorum *B, C, D* etc. seu lentium intermediarum.

PROBLEMA 5

30. *Ex quotcunque lentibus telescopium fuerit compositum, definire locum oculi seu eius distantiam post ultimam lentem ocularem.*

SOLUTIO

Hanc distantiam supra [Liber I, § 180, 347-358] littera O indicavimus statimque vidimus pro casu unicae lentis fore $O=0$.

Pro casu antem duarum lentium invenimus $O=\frac{\mathfrak{B}b\pi}{\pi-\Phi}$, quae ob $\beta=\infty$ hincque $B=\infty$ et $\mathfrak{B}=1$ abit in hanc $O=\frac{b\pi}{\pi-\Phi}$. Cum autem porro sit $b=\frac{p\Phi}{\pi-\Phi}$ ideoque $\pi-\Phi=\frac{p\Phi}{b}$, habebitur $O=\frac{b^2\pi}{p\Phi}=\frac{q^2\pi}{p\Phi}$ et ob $m=-\frac{p}{q}$ seu $p=-mq$ erit $O=\frac{-q\pi}{m\Phi}$, et ob $\frac{\pi}{\Phi}=\frac{p+1}{q}$ habebitur etiam

$$O=-\frac{(p+q)}{m}=+\frac{m-1}{m}q.$$

Pro casu trium lentium ob $\gamma=\infty$ ideoque $C=\infty$ et $\mathfrak{C}=1$ habebimus $O=\frac{c\pi'}{\pi'-\pi+\Phi}$; est vero $c=r=\frac{Bp\Phi}{\pi'-\pi+\Phi}$ et $m=\frac{p}{r}B$ atque hinc $pB=mr$ adeoque $c=\frac{mr\Phi}{\pi'-\pi+\Phi}$; unde erit $O=\frac{\pi'}{m\Phi}r.$

Pro casu quatuor lentium ob $\delta=\infty$ ideoque $D=\infty$ et $\mathfrak{D}=1$ invenimus $O=\frac{d\pi''}{\pi''-\pi'+\pi-\Phi}$, at est $d=s=\frac{BCp\Phi}{\pi''-\pi'+\pi-\Phi}=\frac{-ms\Phi}{\pi''-\pi'+\pi-\Phi}$ hincque $\pi''-\pi'+\pi-\Phi=-m\Phi$

adeoque $O=\frac{\pi''}{m\Phi}s$.

Quo haec ad plures lentes accommodari queant, tabulam sequentem subiungam:

Numerus lentium	Locus oculi
I	$O=0$
II	$O=\frac{b\pi}{\pi-\Phi}=-\frac{\pi}{m\Phi}q$
III	$O=\frac{c\pi'}{\pi'-\pi+\Phi}=\frac{\pi'}{m\Phi}r$
IV	$O=\frac{d\pi''}{\pi''-\pi'+\pi-\Phi}=-\frac{\pi''}{m\Phi}s$
V	$O=\frac{e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}=\frac{\pi'''}{m\Phi}t$

etc.

COROLLARIUM 1

31. Ex superioribus hic repeti conveniet, si valor ipsius O prodeat positivus, tum pro oculo locum idoneum inveniri, ex quo totus campus adparens conspici queat, sin autem pro O prodeat valor negativus, tum oculum lenti ultimae immediate adplicari debere hocque casu campum adparentem per aperturam pupillae determinari.

COROLLARIUM 2

32. Casu duarum lentium distantiam O concinnius exhibera licuit, cum esset $O=\frac{m-1}{m}q=\left(1-\frac{1}{m}\right)q$; unde statim patet pro repraesentatione erecta, ubi q est quantitas negativa, distantiam O pariter fore negativam ideoque oculum lenti oculari immediate adplicari debere; at si lens ocularis fuerit convexa et repraesentatio inversa, tum oculum in certa distantia post lentem ocularem collocari debere.

PROBLEMA 6

33. *Ex quotcunque lentibus telescopium fuerit compositum, si distantia oculi post lentem ocularem prodierit positiva, definire campum apparentem seu eius semidiametrum* Φ, *quem conspicere licebit.*

SOLUTIO

Cum sit $h=a$, ex formulis generalibus supra inventis pro quovis lentium numero habebimus sequentes campi apparentis determinationes:

Numerus lentium	Semidiameter campi apparentis
I	$\Phi =$ indeterminatum
II	$\Phi = \frac{-\pi}{m-1}$
III	$\Phi = \frac{-\pi+\pi'}{m-1}$
IV	$\Phi = \frac{-\pi+\pi'-\pi''}{m-1}$
V	$\Phi = \frac{-\pi+\pi'-\pi''+\pi'''}{m-1}$

etc.

COROLLARIUM 1

34. Si *m* denotat numerum positivum, eo semper indicatur repraesentationem obiectorum esse erectam; sin autem telescopium in situ inverso repraesentet, tum semper *m* numero negativo est exprimendum, uti iam supra est monitum.

COROLLARIUM 2

35. Ex his formulis etiam patet, quo maior fuerit multiplicatio *m,* eo minorem fore ceteris paribus campum apparentem, et cum litterae π, π', π'', π'' etc. denotare possint fractiones non maiores quam $\frac{1}{4}$ vel $\frac{1}{5}$, sive positivas sive negativas, evidens est augendo numerum lentium campum apparentem continuo magis augeri posse.

SCHOLION

36. Hoc modo semidiameter campi apparentis per fractionem quandam reperietur expressa, quae tanquam pars radii seu sinus totius est spectanda. Quare, cum arcus circuli radio aequalis contineat circiter 57° 17′ seu 3437′, fractio pro Φ inventa in minuta prima convertetur, si ea multiplicetur per numerum 3437, hocque modo spatium in coelo, quod per telescopium quodcunque conspicitur, facillime in gradibus et minutis definietur, ubi insuper notari convenit angulum hunc Φ hic semper ut positivum spectari; si enim prodeat negativus, id semper est indicio rationem multiplicationis *m* quoque negative esse capiendam seu repraesentationem esse inversam.

PROBLEMA 7

37. *Ex quotcunque lentibus telescopium fuerit compositum, si distantia oculi post lentem ultimam prodierit negativa, definire campum apparentem seu eius semidiametrum Φ, quem conspicere licet.*

SOLUTIO

Si prodeat distantia O negativa ideoque oculus in hoc loco collocari nequeat, iam supra vidimus tum oculum lenti ultimae immediate adplicari debere, quasi esset $O = 0$, hocque casu campum apparentem non amplius per aperturas lentium definiri, sed potissimum ab apertura pupillae pendere, cuius semidiametrum littera w designavimus, quae ob insignem oculi variationem a parte vigesima digiti usque ad $\frac{1}{10}$ dig. augeri potest, quod evenire solet, si oculus in loco valde obscuro versetur. Pro hoc igitur casu ex supra traditis determinatio campi apparentis sequenti modo se habebit:

Pro casu duarum lentium ob $\mathfrak{B} = 1$ et $b = q$ erit primo $\pi q = \omega$, deinde $\Phi = \frac{\pi\omega}{\pi p + \omega}$, quae expressio ob $\pi = \frac{\omega}{q}$ abit in hanc

$$\Phi = \frac{\omega}{p+q} = \frac{-\omega}{(m-1)q} = \frac{-\pi}{m-1}$$

ob $p = -mq$, quae expressio, quia hoc casu q negativum valorem obtinet, per se fit positiva.

Pro casu trium lentium primo ob $\mathfrak{C} = 1$ et $c = r$ erit $\pi' r = \omega$; tum vero fit $\frac{Bp\Phi\pi'}{\pi' - \pi + \Phi} = \omega$ seu $\frac{mr\Phi\pi'}{\pi' - \pi + \Phi} = \omega$, unde invenitur

$$\Phi = \frac{(\pi' - \pi)\omega}{mr\pi' - \omega} = \frac{(\pi' - \pi)\omega}{(m-1)\pi' r} = \frac{\pi' - \pi}{m-1}.$$

Pro casu quatuor lentium ob $\mathfrak{D} = 1$ et $d = s$ erit primo $\pi'' s = \omega$, tum vero $\frac{BCp\pi''\Phi}{\pi'' - \pi' + \pi - \Phi} = \omega = \frac{ms\pi''\Phi}{\pi'' - \pi' + \pi - \Phi}$, unde invenitur

$$\Phi = \frac{(\pi'' - \pi' + \pi)\omega}{\omega - ms\pi''} = \frac{-(\pi'' - \pi' + \pi)\omega}{(m-1)\pi'' s} = -\frac{\pi'' - \pi' + \pi}{m-1}.$$

Quas determinationes in sequenti tabula repraesentemus:

Pro numerolentium	erit	et pro campo apparente
II	$\pi = \frac{\omega}{q}$	$\Phi = \frac{-\omega}{(m-1)q} = \frac{-\pi}{m-1}$
III	$\pi' = \frac{\omega}{r}$	$\Phi = \frac{-(\pi - \pi')\omega}{(m-1)\pi' r} = \frac{-\pi + \pi'}{m-1}$
IV	$\pi'' = \frac{\omega}{s}$	$\Phi = \frac{-(\pi - \pi' + \pi'')\omega}{(m-1)\pi'' s} = \frac{-\pi + \pi' - \pi''}{m-1}$
V	$\pi''' = \frac{\omega}{t}$	$\Phi = \frac{-(\pi - \pi' + \pi'' - \pi''')\omega}{(m-1)\pi''' t} = \frac{-\pi + \pi' - \pi'' + \pi'''}{m-1}$

etc.

COROLLARIUM 1

38. Hinc patet formulas pro semidiametro campi apparentis Φ non discrepare a casu praecedente; verum autem discrimen in hoc consistit, quod casu praecedente ultima

litterarum $\pi,\ \pi',\ \pi''$ etc. ab arbitrio nostro pendebat, dummodo intra limitem praescriptum $\frac{1}{4}$ vel $\frac{1}{5}$ contineretur, hic autem ea a constitutione pupillae determinari debeat.

COROLLARIUM 2

39. Eatenus ergo hoc casu campus apparens minor fit quam casu praecedente, quatenus litterarum $\pi,\ \pi',\ \pi''$ etc. ultimae minor valor tribui debet, id quod fit, si fractiones $\frac{\omega}{q},\ \frac{\omega}{r},\ \frac{\omega}{s}$ etc. minores fuerint quam limes ille $\frac{1}{4}$ vel $\frac{1}{5}$. Sin autem huic limiti prodierint aequales, utroque casu idem habebitur campus apparens.

COROLLARIUM 3

40. Hinc autem concludere non licet, si istae fractiones $\frac{\omega}{q},\ \frac{\omega}{r},\ \frac{\omega}{s}$ etc. maiores fiant limite praescripto, tum hoc posteriori casu campum adeo maiorem visum iri, propterea quod ipsa lentis postremae natura non permittit maiorem valorem litterae respondentis π. Atque ob hanc causam nequidem convenit tam exiguas lentes oculares admittere, ut valor ultimae litterae π limitem $\frac{1}{4}$ vel $\frac{1}{5}$ superans prodeat, quia tum ipsa huius lentis apertura minor capi deberet quam pupilla.

SCHOLION

41. Nihil autem obstat, quominus lenti oculari apertura maior tribuatur quam pupillae, quandoquidem inde nullum aliud incommodum esset metuendum, nisi quod non omnes radii per hanc lentem transmissi in oculum ingrederentur; quod autem tantum abest, ut sit incommodum, ut potius insigne lucrum inde obtineri possit; tum enim pupilla successive per totam lentis aperturam vagari poterit, quo id commodi consequemur, ut successive alias atque alias obiecti partes conspiciamus. Id quod in telescopiis ad praecedentem casum pertinentibus locum habere nequit. Determinatio igitur ultimae litterarum $\pi,\ \pi',\ \pi''$ etc. in problemate exhibita ei tantum fini inservit, ut inde magnitudo campi uno obtutu visi rite definiatur, cum adeo insigne lucrum exspectari queat, si lenti oculari multo maior apertura tribui queat; ex quo iam ratio multo clarius perspicitur, cur lentes oculares nimis parvas evitari conveniat.

PROBLEMA 8

42. *Si telescopium ex quotcunque lentibus fuerit compositum atque adeo singulae lentes ex diversis vitri speciebus sint formatae, definire semidiametrum confusionis, qua repraesentatio obiectorum erit inquinata.*

SOLUTIO

Iam in limine huius capitis cuilibet lenti peculiarem refractionis rationem tribuimus huncque in finem litteras n, n', n'', n''' etc. in calculum introduximus. Quare tantum opus est, ut formulas in additamento paenultimo I$^{\text{mae}}$ partis inventas [p. 238] ad casum telescopiorum, quo fit $a = \infty$, $h = a$ hincque $A = 0$ et $Aa = \alpha = p$, transferamus; ad quod efficiendum ex denominatoribus singulorum membrorum factor A^3 cum factore communi coniungatur, ut fiat in eius denominatore $A^3 \cdot aah = \alpha^3 = p^3$. Quo facto pro quolibet lentium numero semidiameter confusionis sequenti formula exprimetur:

$$\frac{mx^3}{4p^3}\left(\begin{array}{l} \mu\lambda + \dfrac{\mu'(B+1)\left(\lambda'(B+1)^2+\nu'B\right)\Phi}{B^3(\mathfrak{B}\pi-\Phi)} + \dfrac{\mu''(B+1)\left(\lambda'(C+1)^2+\nu''B\right)\Phi}{B^3C^3(\mathfrak{C}\pi'-\pi-\Phi)} \\ + \dfrac{\mu'''(D+1)\left(\lambda'''(D+1)^2+\nu'''D\right)\Phi}{B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} + \text{etc.} \end{array}\right)$$

quae, si singula membra in duas partes discerpantur, commodius exprimi poterit ob valores $\frac{B}{B+1} = \mathfrak{B}$, $\frac{C}{C+1} = \mathfrak{C}$ etc. Erit scilicet haec expressio:

$$\frac{mx^3}{4p^3}\left(\begin{array}{l} \mu\lambda + \dfrac{\mu'\Phi}{\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\left(\dfrac{\lambda'}{\mathfrak{B}^2} + \dfrac{\nu'}{B}\right) + \dfrac{\mu''\Phi}{B^2\mathfrak{C}(\mathfrak{C}\pi'-\pi-\Phi)}\left(\dfrac{\lambda''}{\mathfrak{C}^2} + \dfrac{\nu''}{C}\right) \\ + \dfrac{\mu'''\Phi}{B^3C^3\cdot\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\left(\dfrac{\lambda'''}{\mathfrak{D}^2} + \dfrac{\nu'''}{D}\right) \text{ etc.} \end{array}\right)$$

quae porro formulis § 23 in subsidium vocatis ad hanc formam redigitur:

$$\frac{mx^3}{4p^3}\left(\mu\lambda + \frac{\mu'q}{\mathfrak{B}^2p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''r}{B^4\mathfrak{C}^2p}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right) + \frac{\mu'''s}{B^4C^4\mathfrak{D}^2p}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu'''}{D}\right) \text{ etc.}\right)$$

atque hinc pro quovis lentium numero semidiameter confusionis sequenti modo exprimetur:

Pro duabus lentibus ob $B = \infty$, $b = q$, $\mathfrak{B} = 1$ erit semidiameter confusionis $= \frac{mx^3}{4p^3}\left(\mu\lambda + \frac{\mu'\lambda' q}{p}\right)$, quae forma ob $p = -mq$ reducitur ad hanc:

$$\frac{mx^3}{4p^3}\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right).$$

Pro tribus lentibus ob $C = \infty$ et $\mathfrak{C} = 1$ $Bp = mr$ erit semidiameter confusionis

$$\frac{mx^3}{4p^3}\left\{\mu\lambda + \frac{\mu'q}{\mathfrak{B}^2p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''\lambda''}{B^3m}\right\}.$$

Pro quatuor lentibus ob $D=\infty$ et $\mathfrak{D}=1$ $BCp=-ms$ erit semidiameter confusionis

$$=\frac{mx^3}{4p^3}\left\{\mu\lambda+\frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu'' r}{B^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''\lambda'''}{B^3C^3m}\right\}.$$

Pro quinque lentibus ob $E=\infty$ et $\mathfrak{E}=1$ $BCDp=mt$ erit semidiameter confusionis

$$=\frac{mx^3}{4p^3}\left\{\begin{aligned}&\mu\lambda+\frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu'' r}{B^4\mathfrak{C}^2 p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)\\&+\frac{\mu''' s}{B^4C^4\mathfrak{D}^2 p}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right)+\frac{\mu''''\lambda''''}{B^3C^3D^3m}\end{aligned}\right\}.$$

COROLLARIUM 1

43. His igitur formulis semidiameter confusionis per numerum seu fractionem quandam numericam expressa reperitur, quae fractio in gradus, minuta et secunda conversa indicabit, sub quanto angulo singula obiectorum puncta per telescopium conspiciantur, quippe in quo effectus confusionis existit.

COROLLARIUM 2

44. Ne igitur haec confusio fiat intolerabilis, necesse est, ut semidiameter confusionis infra certum limitem subsistat; pro quo limite supra hanc formulam constituimus $\frac{1}{4k^3}$, existante $k=40$ vel $k=30$ circiter.

COROLLARIUM 3

45. Quodsi ergo in genere numeros in clausulis contentos ponamus $=N$, efficiendum est, ut $\frac{mx^3}{p^3}\cdot N$ non excedat limitem $\frac{1}{4k^3}$, ex quo statui debebit $\frac{mx^3}{p^3}\cdot N=\frac{1}{k^3}$, unde quantitas p seu distantia focalis lentis obiectivae determinatur; fiet scilicet $p=k\cdot x\cdot\sqrt[3]{mN}$.

COROLLARIUM 4

46. Si porro gradus claritatis littera y indicetur, ut supra fecimus, ubi vidimus capi debere $x=m\cdot y$ et vulgo statui $y=\frac{1}{50}$ dig., unde satis notabilis gradus claritatis oritur, aequatio modo inventa erit $p=m\cdot ky\cdot\sqrt[3]{mN}$; unde patet ceteris paribus distantiam focalem lentis obiectivae p sequi rationem sesquitriplicatam multiplicationis m, ubi notandum, quia $y=\frac{1}{30}$ dig. et $k=40$, fore propemodum $ky=\frac{4}{5}$ dig. seu quasi 1 dig. plus vel minus secundum circumstantias.

SCHOLION 1

47. Ne quem offendat, quod ex hac aequatione valorem ipsius p definivimus, cum tamen haec quantitas iam insit in numero N, notandum est hic non tam ipsam quantitatem p quam eius rationem ad reliquas distantias focales q, r, s etc. in numerum N ingredi; quae rationes cum aliunde ut iam cognitae spectari possint, nostra aequatio utique est idonea, ex qua valor absolutus ipsius p determinetur, id quod fit ex quantitate x, quae in digitis expressa habetur, cum sit $x = my$ et y in partibus digiti detur seu capiatur $y = \frac{1}{50}$ dig. sive maior sive minor, prout maior vel minor claritatis gradus desideratur.

SCHOLION 2

48. Cum maxime sit optandum, ut haec confusio penitus ad nihilum redigatur fiatque $N = 0$, si hoc suceesserit, ostendendum adhuc est, quomodo hinc distantia focalis lentis obiectivae p definiri debeat, siquidem pro casu $N = 0$ nostra aequatio daret $p = 0$; quod cum fieri nequeat, ad eius aperturam seu quantitatem x est respiciendum; quae quia ex gradu claritatis y cum multiplicatione m coniuncto est data, huic lenti necessario tantam distantiam focalem p tribui oportet, ut lens tantae aperturae fiat capax; ad minimum scilicet debet esse $p > 5x$ atque interdum adhuc maius, prout lentis facies magis prodeunt incurvatae. In genere autem observandum est nihil impedire, quominus maior statuatur quantitas p, dummodo non fiat minor.

PROBLEMA 9

49. *Si telescopium quotcunque lentibus constet oculique distantia post ultimam lentem inventa fuerit positiva, definire lentium dispositionem, ut obiecta sine margine colorato conspiciantur.*

SOLUTIO

Quoniam huic conditioni iam supra generatim satisfecimus, aequationem ibi inventam ad casum praesentem telescopiorum accommodemus ac videbimus scopum obtineri, si huic aequationi satisfieri possit:

$$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{p\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{Bp\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{BCp\Phi} \text{ etc.}$$

quam ad singulos lentium numeros applicemus.

Pro duabus lentibus ob $b = q$ erit

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi q}{\Phi p} = -\frac{dn'}{n'-1} \cdot \frac{\pi}{m\Phi}.$$

Pro tribus lentibus ob $c=r$ et $Bp=mr$ erit

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' r}{B\Phi p}$$

sive

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi'}{m\Phi}.$$

Pro quatuor lentibus ob $d=s$ et $BCp=-ms$ erit

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' c}{B\Phi p}-\frac{dn'''}{n'''-1}\cdot\frac{\pi''}{m\Phi}.$$

Pro quinque lentibus ob $e=t$ et $BCDp=mt$ erit

$$0=\frac{dn'}{n'-1}\cdot\frac{\pi b}{\Phi p}+\frac{dn''}{n''-1}\cdot\frac{\pi' c}{B\Phi p}+\frac{dn'''}{n'''-1}\cdot\frac{\pi'' d}{BC\Phi p}+\frac{dn''''}{n''''-1}\cdot\frac{\pi'''}{m\Phi}.$$

COROLLARIUM 1

50. Cum pro casu duarum lentium sit $\frac{-\pi}{\Phi}=m-1$, habebitur haec aequato $0=\frac{dn'}{n'-1}\cdot\frac{m-1}{m}$; quod cum fieri non possit, manifiestum est telescopia ex duabus lentibus composita a vitio marginis colorati liberari non posse.

COROLLARIUM 2

51. Si omnes lentes ex eadem vitri specie sint factae, aequationes nostras per factores differentiales dividere licebit, indeque eaedem formulae reperiuntur, quae pro hoc casu supra sunt datae.

PROBLEMA 10

52. *Si telescopium quotcunque constet lentibus oculique distantia post ultimam lentem inserta fuerit negativa, definire lentium dispositionem,* ut O*biecta sine margine colorato conspiciantur.*

SOLUTIO

Ex superioribus pro quovis lentium numero sequentibus aequationibus erit satisfaciendum:
Pro duabus lentibus, si superior aequatio per A multiplicetur, habebitur

$$0=\frac{dn}{n-1}\cdot B\pi p,$$

quod ob $B=\infty$ fieri nequit.

Pro tribus lentibus multiplicando per A habebitur ob $C=\infty$

$$0=\frac{dn}{n-1}\cdot B\pi' p+\frac{dn'}{n'-1}\cdot b\Big(\big(B+1\big)\pi'-\pi\Big).$$

Pro quatuor lentibus ob $D=\infty$ habebitur

$$0=\frac{dn}{n-1}\cdot B\pi'' p+\frac{dn'}{n'-1}\cdot b\Big(\big(B+1\big)C\pi''-\pi\Big)+\frac{dn''}{n''-1}\cdot c\left(\frac{(C+1)\pi''-\pi}{B}\right).$$

Pro quinque lentibus ob $E=\infty$ habebitur

$$0=\frac{dn}{n-1}\cdot BCD\pi''' p+\frac{dn'}{n'-1}\cdot b\Big(\big(B+1\big)CD\pi'''-\pi\Big)+\frac{dn''}{n''-1}\cdot c\left(\frac{(C+1)D\pi'''-\pi'}{B}\right)$$
$$+\frac{dn'''}{n'''-1}\cdot d\left(\frac{(D+1)\pi'''-\pi''}{BC}\right).$$

PROBLEMA 11

58. *Si telescepium ex quotcunque lentibus sit compositum, eam definire lentium dispositionem, ut omnis confusio a diversa radiorum refrangibilitate oriunda penitus tollatur.*

SOLUTIO

Ex supra traditis pro omni lentium numero aequationem exhibera possumus, qua scopo proposito satisfiet; multiplicando enim per A^2 habebitur

$$0=\frac{dn}{n-1}\cdot\alpha+\frac{dn'}{n'-1}\cdot\frac{b}{\mathfrak{B}}+\frac{dn''}{n''-1}\cdot\frac{c}{\mathfrak{C}B^2}+\frac{dn'''}{n'''-1}\cdot\frac{d}{\mathfrak{D}B^2C^2}+\frac{dn''''}{n''''-1}\cdot\frac{e}{\mathfrak{E}B^2C^2D^2}+\text{etc.}$$

quae ob $\alpha=p,\ b=\frac{q}{\mathfrak{B}},\ c=\frac{r}{\mathfrak{C}},\ d=\frac{s}{\mathfrak{D}}$ etc. abit in hanc:

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{d}{B^2C^2\mathfrak{D}^2}+\frac{dn''''}{n''''-1}\cdot\frac{e}{B^2C^2D^2\mathfrak{E}^2}+\text{etc.}$$

Hinc ergo pro singulis lentium numeris nanciscimur sequentes aequationes adimplendas:

Pro duabus lentibus ob $\mathfrak{B}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot q.$$

Pro tribus lentibus ob $\mathfrak{C}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2}.$$

Pro quatuor lentibus ob $\mathfrak{D}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{s}{B^2C^2}.$$

Pro quinque lentibus ob $\mathfrak{E}=1$

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot\frac{q}{\mathfrak{B}^2}+\frac{dn''}{n''-1}\cdot\frac{r}{B^2\mathfrak{C}^2}+\frac{dn'''}{n'''-1}\cdot\frac{s}{B^2C^2\mathfrak{D}^2}+\frac{dn''''}{n''''-1}\cdot\frac{t}{B^2C^2D^2}.$$

COROLLARIUM 1

54. Cum sit $\mathfrak{B}=\frac{q}{b},\ \mathfrak{C}=\frac{r}{c},\ \mathfrak{D}=\frac{s}{d}$ etc. etc., tum vero $B=\frac{\beta}{b},\ C=\frac{\gamma}{c},\ D=\frac{\delta}{d}$ etc., aequatio generalis per pp seu $\alpha\alpha$ divisa abibit in sequentem formam:

$$0=\frac{dn}{n-1}\cdot\frac{1}{p}+\frac{dn'}{n'-1}\cdot\frac{bb}{\alpha\alpha}\cdot\frac{1}{q}+\frac{dn''}{n''-1}\cdot\frac{bb\cdot cc}{\alpha\alpha\cdot\beta\beta}\cdot\frac{1}{r}+\frac{dn'''}{n'''-1}\cdot\frac{bb\cdot cc\cdot dd}{\alpha\alpha\cdot\beta\beta\cdot\gamma\gamma}\cdot\frac{1}{s}+\frac{dn''''}{n''''-1}\cdot\frac{bb\cdot cc\cdot dd\cdot ee}{\alpha\alpha\cdot\beta\beta\cdot\gamma\gamma\cdot\delta\delta}\cdot\frac{1}{t}\text{ etc.}$$

quae aequatio commodior videtur praecedente.

COROLLARIUM 2

55. Quod ad numerum horum terminorum attinet, perspicuum est eum esse numero lentium aequalem, neque igitur opus est, ut hanc formulam seorsim ad quemlibet lentium numerum accommodemus.

COROLLARIUM 3

56. Si omnes lentes ex eadem vitri specie essent confectae, tum haec aequatio per coefficientes differentiales dividi posset prodiretque

$$0=\frac{1}{p}+\frac{b^2}{\alpha^2}\cdot\frac{1}{q}+\frac{b^2c^2}{\alpha^2\beta^2}\cdot\frac{1}{r}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2}\cdot\frac{1}{s}\text{ etc.,}$$

cui autem nullo modo satisfieri potest.

SCHOLION 1

57. Quod haec aequatio, quando omnes lentes ex eadem vitri specie sunt paratae, nullo modo subsistere queat, sequenti modo ostendi potest. Cum sit

$$\frac{1}{p}=\frac{1}{\alpha},\ \frac{1}{q}=\frac{1}{b}+\frac{1}{\beta},\ \frac{1}{r}=\frac{1}{c}+\frac{1}{\gamma},\ \frac{1}{s}=\frac{1}{d}+\frac{1}{\delta},$$

si hi valores substituantur singulaque membra post primum in duas partes discerpantur, aequatio induet hanc formam:

$$0=\frac{1}{\alpha}+\frac{b}{\alpha^2}+\frac{b^2c}{\alpha^2\beta^2}+\frac{b^2c^2d}{\alpha^2\beta^2\gamma^2}+\frac{b^2}{\alpha^2\beta}+\frac{b^2c^2}{\alpha^2\beta^2\gamma}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta}\text{ etc.},$$

hic iam iungantur iterum bini termini et aequatio prodiens ita erit comparata:

$$0=\frac{\alpha+b}{\alpha^2}+\frac{b^2(\beta+c)}{\alpha^2\beta^2}+\frac{b^2c^2(\gamma+d)}{\alpha^2\beta^2\gamma^2}+\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta};$$

quia nunc $\alpha+b,\ \beta+c,\ \gamma+d$ ut lentium distantiae necessario sunt positivae, omnes plane termini usque ad ultimum necessario positivi sunt; ultimus autem terminus $\frac{b^2c^2d^2}{\alpha^2\beta^2\gamma^2\delta}$ ob $\delta=\infty$ per se evanescit, scilicet pro casu quatuor lentium, quem hic consideravimus.

SCHOLION 2

58. His igitur praeparatis iam possemus ad diversa genera telescopiorum constituenda progredi singularumque specierum constructionem docere. Sed quoniam ea, quae supra de lentibus multiplicatis sunt tradita, maximum usum in perficiendis telescopiis habere possunt, dum scilicet loco lentium simplicium multiplicatae adhibentur, quae multo minorem confusionem pariant, consultum videtur ea hic repetere et ad telescopia accommodare. Inprimis autem ex formula pro semidiametro confusionis inventa patet lentem obiectivam in ea praecipuas partes tenere, siquidem pro ea fuerit $\lambda=1$; quare, si eius loco lens multiplicata substituatur, pro qua valor numeri λ vehementer sit minor vel adeo evanescat, statim maximum inde commodum adipiscimur, dum tota confusio ad valde exiguum vel fortasse ad nihilum redigitur. Quocirca in capite sequente praecipuas lentes compositas, quas in locum lentis obiectivae substituere licebit, enumerabimus et pro singulis valorem ipsius λ indicabimus, ut deinceps pro circumstantiis hinc depromi possint.