CHAPTER II

CONCERNING COMPOSITE PERFECT OBJECTIVE LENSES

PROBLEM 1

59. *To describe the construction of simple objective lenses which may arise with the minimum confusion..*

SOLUTION

Since a simple lens cannot be prepared with the least confusion, unless $\lambda = 1$, we may put $\lambda = 1$ here at once, and since $a = \infty$ for these which have been treated above, it is easily understood this lens must be constructed thus, so that there shall be

$$\text{radius of}\begin{cases}\text{anterior face} = \frac{\alpha}{\sigma}\\ \text{posterior face} = \frac{\alpha}{\rho},\end{cases}$$

where the numbers α and ρ are required to be taken from the following table shown in §15. Therefore this construction thus will be performed for the various kinds of glass ; evidently, since if $\alpha = p$, there will be

n	radius of anterior face F	radius of posterior face G
1,50	0,58333p	3,4989p
1,51	0,58976p	3,7693p
1,52	0,59609p	4,0717p
1,53	0,60237p	4,4111p
1,54	0,60849p	4,8008p
1,55	0,61448p	5,2438p
1,56	0,62039p	5,7571p
1,57	0,62617p	6,3573p
1,58	0,63183p	7,0721p
1,59	0,63739p	7,9428p
1,60	0,64288p	9,0009p

COROLLARY 1

60. Since in the expression for the radius of confusion λ is multiplied by μ, from §15 it is understood the confusion to become smaller there with all else being the same, so that

the greater were the ratio of refraction *n,* thus so that this one shall be preferred to the rest, by considering the kind of glass which has the maximum refraction.

COROLLARY 2

61. Generally objective lenses are accustomed to be made each equally convex, for which case it will be worth the effort to investigate, how much greater the number λ shall be above unity ; but since there is

$$F = \frac{\alpha}{\sigma - \tau \cdot \sqrt{-1}} \quad \text{and} \quad G = \frac{\alpha}{\sigma + \tau \cdot \sqrt{-1}},$$

by putting $F = G$ there will be

$$\sqrt{\lambda - 1} = \frac{\sigma - \rho}{2\tau} = \frac{2(nn-1)}{n\sqrt{(4n-1)}};$$

then truly there will be found

$$\frac{1}{F} + \frac{1}{G} = \frac{\sigma + \rho}{\alpha} = \frac{1}{(n-1)\alpha} = \frac{2}{F}$$

or

$$F = G = 2(n-1)\alpha = 2(n-1)p.$$

But so that it may pertain to λ, for the case $n = 1,55$ there will be

$$\sqrt{\lambda - 1} = \frac{1,4025}{1,7673} = 0,79367$$

and hence $\lambda = 1,62991$; from which it is apparent, how much greater the confusion of such an objective lens may be apparent.

COROLLARY 3

62. If we wish to make a convex-plane face, so that its posterior face may become planar or $G = \infty$, there becomes

$$\sqrt{\lambda - 1} = \frac{-\rho}{\tau} \quad \text{and} \quad F = \frac{\alpha}{\sigma + \rho} = (n-1)\alpha$$

and in the case, where $n = 1,55$, $\lambda = 1,0444$, so that the confusion exceeds that by very little, which arises from the case $\lambda = 1$.

COROLLARY 4

63. But if the same plano-convex lens may be inverted, so that there shall become $F=\infty$ and thus $\sqrt{(\lambda-1)}=\frac{\sigma}{\tau}$ and $G=\frac{\alpha}{\sigma+\rho}=(n-1)\alpha$, there becomes for the case, in which $n=1{,}55,\ \lambda=4{,}2329$, thus so that such a lens may produce more than four times greater confusion than our recommended lens.

COROLLARY 5

64. Therefore it is apparent, if the plano-convex lens may be used, how much shall be the concern, whether its convex or concave face may be directed towards the object, since in the latter case around four times the confusion may arise.

PROBLEM 2

65. *To describe the construction of the duplicate objective lens, if indeed both lenses shall be made from the same kind of glass, so that the minimum confusion may be produced.*

SOLUTION

From § 114 of the previous book [Book I, Ch. 3, from which the following diagram is borrowed], since here there shall be $a=\infty$ and $\beta=p$, we may deduce the following construction:

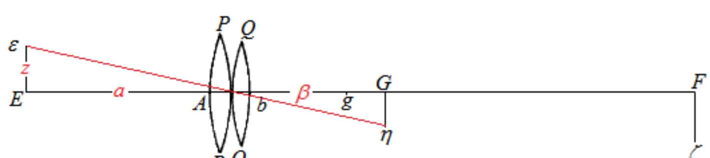

Fig. 7

for the first lens, the radius of the $\begin{cases}\text{anterior face} = \frac{2p}{\sigma}\\ \text{posterior face} = \frac{2p}{\rho}\end{cases}$

for the second lens, the radius of the $\begin{cases}\text{anterior face} = \frac{2p}{2\sigma-\rho}\\ \text{posterior face} = \frac{2p}{2\rho-\sigma},\end{cases}$

and if this double lens may be used in place of the objective, for this there will be $\lambda=\frac{1-v}{4}$, we will determine which values only for the particular kinds of glass.

Therefore initially we may consider the crown glass, for which $n=1,53$, and since there shall be $\rho=0,2267$, $\sigma=1,6601$, there will be $2\sigma-\rho=3,0935$ and $2\rho-\sigma=-1,2067$; then truly on account of $v=0,2196$ there arises $\lambda=0,1951$ and the following construction will be obtained :

For crown glass $n=1,58$

for the first lens, the radius of the $\begin{cases}\text{anterior face} = +1,2047p\\ \text{posterior face} = +8,8222p\end{cases}$

for the second lens, the radius of the $\begin{cases}\text{anterior face} = +0,6465\,p\\ \text{posterior face} = -1,6574p,\end{cases}$

and $\lambda=0,1951.$

Now we may put $n=1,55$ for the ordinary glass, and there well be

$\rho=0,1907$, $\sigma=1,6274$ and $2\sigma-\rho=3,0641$, $2\rho-\sigma=-1,2460, v=0,2326,$ hence $\lambda=0,1918$,

from which the following construction is elicited :

for the first lens, radius of the $\begin{cases}\text{anterior face} = +\ 1,2289p\\ \text{posterior face} = +10,4876p\end{cases}$

for the second lens, radius of the $\begin{cases}\text{anterior face} = +\ 0,6527p\\ \text{posterior face} = -1,\ 6051p,\end{cases}$

and $\lambda=0,1918.$

Again we may put $n=1,58$ for the crystal glass, and there will become $\rho=0,1414$, $\sigma=1,5827$, $2\sigma-\rho=\ 3,0240$, $2\rho-\sigma=-1,2999$, $v=0,2529$, and hence $\lambda=0,1868$, from which the following construction will be obtained :

For the crystal glass $n=1,58$

for the first lens, the radius of the $\begin{cases} \text{anterior face} = +\ 1,26366p \\ \text{posterior face} = +14,14427p \end{cases}$

for the second lens, the radius of the $\begin{cases} \text{anterior face} = +0,66137p \\ \text{posterior face} = -\ 1,58858p \end{cases}$

and $\lambda = 0,1868.$

PROBLEM 3

66. *To describe the construction of the triple lens, if indeed all three lenses may be made from the same kind of glass, which may produce the minimum confusion.*

SOLUTION

From § 136 of the above book [Ch. 3], since there shall be $a = \infty$ and $\gamma = p$, we may deduce this construction :

for the first lens, the radius of the $\begin{cases} \text{anterior face} = \frac{3p}{\sigma} \\ \text{posterior face} = \frac{3p}{\rho} \end{cases}$

for the second lens, the radius of the $\begin{cases} \text{anterior face} = \frac{3p}{2\sigma-\rho} \\ \text{posterior face} = \frac{3p}{2\rho-\sigma}, \end{cases}$

for the third lens, the radius of the $\begin{cases} \text{anterior face} = \frac{3p}{3\sigma-\rho} \\ \text{posterior face} = \frac{3p}{3\rho-\sigma}, \end{cases}$

for which the value of λ for the triple lens is $\lambda = \frac{3-8v}{3\cdot 9}$.

Whereby we may set out the values of these radii for these particular kinds of glass.

Therefore since there shall be $n = 1,53$ for the crown glass, there will be

$$\rho = 0,2267, \quad \sigma = 1,6601, \ 2\sigma - \rho = 3,0935,$$
$$2\rho - \sigma = -1,2067, \ 3\sigma - 2\rho = 4,5269, \ 3\rho - 2\sigma = -2,6401,$$

and on account of $v = 0,2196$ there is found $\lambda = 0,0461$, and the following construction is obtained:

For the crown glass $n = 1,53$

for the first lens, radius of the $\begin{cases}\text{anterior face} = +\ 1,8071p\\ \text{posterior face} = +13,2393p\end{cases}$

for the second lens, radius of the $\begin{cases}\text{anterior face} = +\ 0,9698p\\ \text{posterior face} = -\ 2,4861p\end{cases}$

for the third lens, radius of the $\begin{cases}\text{anterioris} = +0,6627p\\ \text{posterioris} = -1,1363p;\end{cases}$

then truly for this triple lens, there will be $\lambda = 0,0461$.

For the common glass, $n = 1,55$.

Since there shall be $\rho = 0,1907$, $\sigma = 1,6274$, $2\sigma - \rho = 3,0641$, $2\rho - \sigma = -1,2460$, $3\sigma - 2\rho = 4,5008$, $3\rho - 2\sigma = -2,6827$,
there will become:

for the first lens radius of the $\begin{cases}\text{anterior face} = +\ 1,8434p\\ \text{posterior face} = +15,7315p\end{cases}$

for the second lens, radius of the $\begin{cases}\text{anterior face} = +0,9791p\\ \text{posterior face} = -2,4077p\end{cases}$

for the third lens, radius of the $\begin{cases}\text{anterior face} = +0,6665p\\ \text{posterior face} = -1,1183p;\end{cases}$

and on account of $v = 0,2326$ there will be $\lambda = 0,0422$.

For the crystal glass $n = 1,58$

$\rho = 0,1414$, $\sigma = 1,5827$, $2\sigma - \rho = 3,0240$,
$2\rho - \sigma = -1,2999$, $3\sigma - 2\rho = -4,4653$, $3\rho - 2\sigma = -2,7415$.

for the first lens, radius of the $\begin{cases}\text{anterior face} = +\ 1,8955p\\ \text{posterior face} = +21,2164p\end{cases}$

for the second lens, radius of the $\begin{cases}\text{anterior face} = +0,9921p\\ \text{posterior face} = -2,3085p\end{cases}$

for the third lens, radius of the $\begin{cases}\text{anterior face} = +0,6718p\\ \text{posterior face} = -1,0944p;\end{cases}$

and since there is $v = 0,2529$, there will be $\lambda = 0,0362$.

COROLLARY 1

67. Therefore if a lens of this kind whether duplicate or triplicate may be used in place of the objective lens, its maximum use consists in this, so that the radius of confusion on account of diminished value of λ may be rendered much smaller and hence the size of the focal length of the objective lens may be able to be taken much smaller.

COROLLARY 2

68. Thence also it is clear, so that the greater were the refraction or the number n for an objective lens of this kind, there the greater the reward in the construction of telescopes arises, since then not only the number λ becomes smaller, but also the number p, by which λ is required to be multiplied.

SCHOLIUM

69. But duplicate or triplicate lenses of this kind being substituted in place of the objective lens clearly do nothing for the other source of confusion, which arises from the diverse refrangibilities of the rays, requiring to be diminished, but the equations given in the first chapter for removing this kind of confusion therefore remain the same, just as if the objective lens may be simple ; truly the remaining duplicate or triplicate lenses, which we have mentioned in the above addition [VIII], also reduce to zero the first term in the equation for dispersion found before, in which a particular part of this confusion is contained. On account of which in this chapter it will be agreed to return to both kinds of these lenses, duplicate as well as triplicate.

DEFINITION 4

70. *An objective lens is perfect, which not only shows no confusion arising from the aperture, but plainly also gives rise to no dispersion of the rays.*

COROLLARY 1

71. Therefore if such a lens may be used, the number λ will vanish completely, so that the radius of confusion shall become much smaller than for the composite objective explained up to this point for lenses.

COROLLARY 2

72. Also from above it is understood well enough for perfect lenses of this kind required to be constructed to require at least two or more different kinds of glass, and since experiments are desired concerning other kinds of glass, other kinds are not permitted to be used at this stage besides crown and crystal glass, used by the illustrious Dollond.

PROBLEM 4

73. *A duplicate objective lens to be constructed composed partially from crown glass* $n = 1,53$*, and partially from crystal glass* $n = 1,58$.

SOLUTION

In adding to the appendix at the end of chapter VII to the preceding volume, we have deduced two perfect lenses of this kind, of which the first of the two has been made from crown glass, truly the other had been made from crystal glass; truly otherwise on the contrary the first lens from crystal glass, the latter from crown ; here we may refer to these as two kinds of perfect lenses.

I. Perfect double objective lens.

For the anterior lens prepared from crown glass $n = 1,53$

$$\text{radius of the}\begin{cases}\text{anterior face} = +0,1807p\\ \text{posterior face} = +1,3239p\end{cases}\text{Crown Glass.}$$

For the posterior lens prepared from crystal glass $n = 1,58$

$$\text{radius of the}\begin{cases}\text{anterior face} = -\ 0,4770p\\ \text{posterior face} = -\ 0,5191p\end{cases}\text{Flint Glass.}$$

Which has a maximum aperture, of which the radius is $x = 0,0452p$.

II. Perfect double objective lens

For the first lens prepared from crystal glass $n = 1,58$

$$\text{radius of the}\begin{cases}\text{anterior face} = -\ 2,0545p\\ \text{posterior face} = -\ 0,2828p\end{cases}\text{Flint Glass.}$$

For the posterior lens prepared from crown glass $n = 1,53$

$$\text{radius of the}\begin{cases}\text{anterior face} = +\ 0,4568p\\ \text{posterior face} = +\ 0,2438p\end{cases}\text{Crown Glass.}$$

Which lens is of the largest aperture, of which the radius $x = 0,0609p$, where it is required to be observed p designates the focal length of the duplicate lens.

COROLLARY 1

74. Therefore since the posterior of these lenses may allow a greater aperture than the anterior, this without doubt is to be preferred to that, since, as will be apparent below, by that the most perfect of all telescopes will be returned, so that the objective lens may allow the maximum aperture.

COROLLARY 2

75. Here it is required to be observed in each case the requiring to be prepared from crystal glass must be concave, then truly, the one which may be made from crown glass to be convex, as these actually prepared by Dollond.

SCHOLIUM

76. Moreover this cannot be held back, that both these kinds require the greatest skill of the artisan ; indeed if he errs a little in the construction of these from the measures prescribed here, it can happen, that these may prevail less, than if simple lenses thus were used. Truly the following triple lenses demand much less skill, since for the individual simple lenses the number *A* may be equal to unity and thus small errors put in place in the construction shall not be a source of concern.

PROBLEM 5

77. *To construct a composite triple perfect objective lens partially from crown glass with* $n = 1,53$*, partially from crystal glass, with* $n = 1,58$.

SOLUTION

For this kind of perfection of the lenses we have given previously four species, which we may refer to here:

I. The perfect objective lens, of which the first and third lens has been prepared from crystal glass and the middle lens from crown glass

For the first lens, the radius of the $\begin{cases} \text{anterior face} = +\ 0,5039p \\ \text{posterior face} = +\ 5,6450p \end{cases}$ Flint Glass.

For the second lens, the radius of the $\begin{cases} \text{anterior face} = +\ 0,1364p \\ \text{posterior face} = -\ 0,9597p \end{cases}$ Crown Glass.

for the third lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 1,0699p \\ \text{posterior face} & = -\ 0,1404p \end{cases}$ Flint Glass.

Which lens is of the largest aperture, of which the radius $x = 0,0341p$.

II. The perfect objective lens, of which the first and third lens has been prepared from crown glass and the middle lens from crystal glass

For the first lens, the radius of the $\begin{cases} \text{anterior face} & = -0,1762p \\ \text{posterior face} & = -1,9741p \end{cases}$ Flint Glass.

for the second lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 2,5349p \\ \text{posterior face} & = +0,1696p \end{cases}$ Crown Glass.

for the third lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 0,6179p \\ \text{posterior face} & = +1,8532p \end{cases}$ Flint Glass.

Which lens is of the largest aperture, of which the radius $x = 0,0424p$.

III. The perfect objective lens, of which the first and third lens has been prepared from crown glass and the middle lens from crystal glass

For the first lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 0,5004p \\ \text{posterior face} & = +\ 3,6665p \end{cases}$ Crown Glass.

for the second lens, the radius of the $\begin{cases} \text{anterior face} & = -0,5107p \\ \text{posterior face} & = -0,4843p \end{cases}$ Flint Glass.

for the third lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 0,5219p \\ \text{posterior face} & = +\ 0,4757p \end{cases}$ Crown Glass.

The radius of the aperture $x = 0,1189p$.

IV. The perfect objective lens, of which the first and third lens has been prepared from crown glass and the middle lens from crystal glass

For the first lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 0,2829p \\ \text{posterior face} & = +\ 2,0729p \end{cases}$ Crown Glass

for the second lens, the radius of the $\begin{cases} \text{anterior face} & = -\ 1,6017p \\ \text{posterior face} & = -\ 0,2943p \end{cases}$ Flint Glass.

for the third lens, the radius of the $\begin{cases} \text{anterior face} & = +\ 0,5860p \\ \text{posterior face} & = +\ 1,7670p \end{cases}$ Crown Glass.

The radius of the aperture $x = 0,0707p$.

In these formulas the letter p denotes focal length of each perfect lens.

COROLLARY 1

78. Between these four lenses the third is especially noteworthy, since it allows the maximum aperture.

COROLLARY 2

79. Therefore if a perfect lens of this kind may be used in some telescope in place of the objective lens, in the expression for the radius of confusion the first term $\mu\lambda$ clearly vanishes; then truly also in the final equation for the dispersion requiring to be removed the first term also is reduced to nothing.

COROLLARY 3

80. Therefore not only perfect lenses of this kind are used in telescopes, but also with mirrors used in reflecting telescopes, which have been preferred for a long time, since mirrors shall not affected by the dispersion of the rays, and truly nor by the first kind of confusion, which arises from the aperture.

CAPUT II

DE LENTIBUS OBIECTIVIS COMPOSITIS ATQUE PERFECTIS

PROBLEMA 1

59. *Constructionem lentis obiectivae simplicis, quae minimam confusionem ariat, describere.*

SOLUTIO

Cum lens simplex minorem confusionem parere nequeat, quam si fuerit $\lambda = 1$, statuamus statim $\lambda = 1$, et cum sit $a = \infty$ ex iis, quae supra sunt tradita, facile intelligitur hanc lentem ita construi debere, ut sit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{\alpha}{\sigma}\\ \text{posterioris} = \frac{\alpha}{\rho},\end{cases}$$

ubi numeri α et ρ ex ratione refractionis sunt sumendi secundum tabulam § 15 exhibitam. Pro variis igitur vitri speciebus haec constructio ita se habebit; scilicet, cum sit $\alpha = p$, erit

pro n	radius faciei anterioris F	radius faciei posterioris G
1,50	0,58333p	3,4989p
1,51	0,58976p	3,7693p
1,52	0,59609p	4,0717p
1,53	0,60237p	4,4111p
1,54	0,60849p	4,8008p
1,55	0,61448p	5,2438p
1,56	0,62039p	5,7571p
1,57	0,62617p	6,3573p
1,58	0,63183p	7,0721p
1,59	0,63739p	7,9428p
1,60	0,64288p	9,0009p

COROLLARIUM 1

60. Cum in expressione pro semidiametro confusionis λ multiplicetur per μ, ex §15 intelligitur confusionem ceteris paribus eo fieri minorem, quo maior fuerit ratio

refractionis *n,* ita ut hoc respectu ea vitri species, quae maximam refractionem habet, reliquis sit anteferenda.

COROLLARIUM 2

61. Vulgo lentes obiectivae utrinque aequaliter convexae confici solent, pro quo casu operae pretium erit investigare, quanto numerus λ unitatem sit superaturus; quia autem est

$$F=\frac{\alpha}{\sigma-\tau\cdot\sqrt{-1}} \quad \text{et} \quad G=\frac{\alpha}{\sigma+\tau\cdot\sqrt{-1}},$$

posito $F=G$ erit

$$\sqrt{\lambda-1}=\frac{\sigma-\rho}{2\tau}=\frac{2(nn-1)}{n\sqrt{(4n-1)}};$$

tum vero habebitur

$$\frac{1}{F}+\frac{1}{G}=\frac{\sigma+\rho}{\alpha}=\frac{1}{(n-1)\alpha}=\frac{2}{F}$$

seu

$$F=G=2(n-1)\alpha=2\left(n-1\right)p.$$

Quod autem ad λ attinet, pro casu $n=1,55$ erit

$$\sqrt{\lambda-1}=\frac{1,4025}{1,7673}=0,79367$$

hincque $\lambda=1,62991$; unde patet, quanto maiorem confusionem talis lens obiectiva pariat.

COROLLARIUM 3

62. Si lentem obiectivam convexo-planam facere velimus, ut eius facies posterior fiat plana seu $G=\infty$, erit

$$\sqrt{\lambda-1}=\frac{-\rho}{\tau} \quad \text{et} \quad F=\frac{\alpha}{\sigma+\rho}=(n-1)\alpha$$

et pro casu, quo $n=1,55,\ \ \lambda=1,0444$, unde confusio non nisi perparum superat illam, quae oritur ex casu $\lambda=1$.

COROLLARIUM 4

63. Sin autem eadem lens plano-convexa invertatur, ut sit $F=\infty$ ideoque $\sqrt{(\lambda-1)}=\frac{\sigma}{\tau}$ et $G=\frac{\alpha}{\sigma+\rho}=(n-1)\alpha$, erit, pro casu, quo $n=1,55,\ \lambda=4,2329$, ita ut talis lens plus quam quadruplo maiorem pariat confusionem quam nostra lens commendata.

COROLLARIUM 5

64. Patet ergo, si lens adhibeatur plano-convexa, quantum intersit, utrum facies eius convexa an plana versus obiectum dirigatur, cum posteriori casu confusio circiter quater maior fiat quam priori.

PROBLEMA 2

65. *Constructionem lentis obiectivae duplicatae, siquidem ambae lentes ex eadem vitri specie sint confectae, describere, quae minimam confusionem pariat.*

SOLUTIO

Ex § 114 libri superioris, cum hic sit $a=\infty$ et $\beta=p$, colligimus sequentem constructionem:

$$\text{pro lente priori} \qquad \text{radius faciei}\begin{cases}\text{anterioris} &= \frac{2p}{\sigma}\\ \text{posterioris} &= \frac{2p}{\rho}\end{cases}$$

$$\text{pro lente posteriori} \quad \text{radius faciei}\begin{cases}\text{anterioris} &= \frac{2p}{2\sigma-\rho}\\ \text{posterioris} &= \frac{2p}{2\rho-\sigma},\end{cases}$$

ac si haec lens duplicata loco lentis obiectivae adhibeatur, pro ea erit $\lambda=\frac{1-v}{4}$, quos valores pro praecipuis tantum vitri speciebus determinemus.

Contemplemur igitur primo vitrum coronarium, pro quo $n=1,53$, et cum sit $\rho=0,2267$, $\sigma=1,6601$, erit $2\sigma-\rho=3,0935$ et $2\rho-\sigma=-1,2067$; tum vero ob $v=0,2196$ prodit $\lambda=0,1951$ atque habetur sequens constructio:

Pro vitro coronario $n=1,58$

$$\text{pro lente priori} \qquad \text{radius faciei}\begin{cases}\text{anterioris} &= +1,2047p\\ \text{posterioris} &= +8,8222p\end{cases}$$

$$\text{pro lente posteriori} \quad \text{radius faciei}\begin{cases}\text{anterioris} &= +0,6465\,p\\ \text{posterioris} &= -1,6574p,\end{cases}$$

et $\lambda=0,1951.$

Ponamus nunc $n = 1,55$ pro vitro ordinario, eritque

$\rho = 0,1907$, $\sigma = 1,6274$ et $2\sigma - \rho = 3,0641$, $2\rho - \sigma = -1,2460$, $v = 0,2326$, hinc $\lambda = 0,1918$, unde elicitur sequens constructio:

$$\text{pro lente priori} \quad \text{radius faciei} \begin{cases} \text{anterioris} = +\ 1,2289p \\ \text{posterioris} = +10,4876p \end{cases}$$

$$\text{pro lente posteriori} \quad \text{radius faciei} \begin{cases} \text{anterioris} = +\ 0,6527p \\ \text{posterioris} = -1,\ 6051p, \end{cases}$$

et $\lambda = 0,1918$.

Ponamus porro $n = 1,58$ pro vitro crystallino, eritque
$\rho = 0,1414$, $\sigma = 1,5827$, $2\sigma - \rho = 3,0240$, $2\rho - \sigma = -1,2999$, $v = 0,2529$, hincque $\lambda = 0,1868$, unde habetur sequens constructio:

Pro vitro crystallino $n = 1,58$

$$\text{pro lente priori} \quad \text{radius faciei} \begin{cases} \text{anterioris} = +\ 1,26366p \\ \text{posterioris} = +14,14427p \end{cases}$$

$$\text{pro lente posteriori} \quad \text{radius faciei} \begin{cases} \text{anterioris} = +0,66137p \\ \text{posterioris} = -\ 1,58858p \end{cases}$$

et $\lambda = 0,1868$.

PROBLEMA 3

66. *Constructionem lentis triplicatae, siquidem omnes tres lentes ex eadem vitri specie sint confectae, describere, quae minimam confusionem pariat.*

SOLUTIO

Ex § 136 libri superioris, cum hic sit $a = \infty$ et $\gamma = p$, colligimus hanc constructionem:

$$\text{pro lente priori radius faciei}\begin{cases}\text{anterioris} = \frac{3p}{\sigma}\\ \text{posterioris} = \frac{3p}{\rho}\end{cases}$$

$$\text{pro lente secunda radius faciei}\begin{cases}\text{anterioris} = \frac{3p}{2\sigma-\rho}\\ \text{posterioris} = \frac{3p}{2\rho-\sigma},\end{cases}$$

$$\text{pro lente tertia radius faciei}\begin{cases}\text{anterioris} = \frac{3p}{3\sigma-\rho}\\ \text{posterioris} = \frac{3p}{3\rho-\sigma},\end{cases}$$

pro qua lente triplicata valor ipsius λ est $\lambda = \frac{3-8v}{3\cdot 9}$.

Quare pro praecipuis vitri speciebus valores horum radiorum evolvamus.

Cum igitur sit pro vitro coronario $n = 1,53$, erit

$$\rho = 0,2267, \quad \sigma = 1,6601, \; 2\sigma - \rho = 3,0935,$$
$$2\rho - \sigma = -1,2067, \; 3\sigma - 2\rho = 4,5269, \; 3\rho - 2\sigma = -2,6401,$$

atque ob $v = 0,2196$ reperitur $\lambda = 0,0461$, atque sequens habetur constructio:

Pro vitro coronario $n = 1,53$

$$\text{pro lente priori radius faciei}\begin{cases}\text{anterioris} = +\;1,8071p\\ \text{posterioris} = +13,2393p\end{cases}$$

$$\text{pro lente secunda radius faciei}\begin{cases}\text{anterioris} = +0,9698p\\ \text{posterioris} = -2,4861p\end{cases}$$

$$\text{pro lente tertia radius faciei}\begin{cases}\text{anterioris} = +0,6627p\\ \text{posterioris} = -1,1363p;\end{cases}$$

tum vero pro hac lente triplicata erit $\lambda = 0,0461$.

Pro vitro communi $n = 1,55$

Cum sit $\rho = 0,1907$, $\sigma = 1,6274$, $2\sigma - \rho = 3,0641$,
$2\rho - \sigma = -1,2460$, $3\sigma - 2\rho = 4,5008$, $3\rho - 2\sigma = -2,6827$,
erit:

$$\text{pro lente priori radius faciei}\begin{cases}\text{anterioris} = +\;1,8434p\\ \text{posterioris} = +15,7315p\end{cases}$$

$$\text{pro lente secunda radius faciei}\begin{cases}\text{anterioris} = +0,9791p\\ \text{posterioris} = -2,4077p\end{cases}$$

$$\text{pro lente tertia radius faciei}\begin{cases}\text{anterioris} = +0,6665p\\ \text{posterioris} = -1,1183p;\end{cases}$$

atque ob $v = 0,2326$ erit $\lambda = 0,0422$.

Pro vitro crystallino $n = 1,58$

$$\rho = 0,1414,\ \sigma = 1,5827,\ 2\sigma - \rho = 3,0240,$$
$$2\rho - \sigma = -1,2999,\ 3\sigma - 2\rho = -4,4653,\ 3\rho - 2\sigma = -2,7415.$$

pro lente priori radius faciei $\begin{cases}\text{anterioris} = +\ 1,8955p\\ \text{posterioris} = +21,2164p\end{cases}$

pro lente secunda radius faciei $\begin{cases}\text{anterioris} = +0,9921p\\ \text{posterioris} = -2,3085p\end{cases}$

pro lente tertia radius faciei $\begin{cases}\text{anterioris} = +0,6718p\\ \text{posterioris} = -1,0944p;\end{cases}$

et quia est $v = 0,2529$, erit $\lambda = 0,0362$.

COROLLARIUM 1

67. Si ergo huiusmodi lens sive duplicata sive triplicata loco lentis obiectivae adhibeatur, summus eius usus in hoc consistit, ut semidiameter confusionis ob imminutum valorem ipsius λ multo minor reddatur hincque distantia focalis lentis obiectivae haud mediocriter minor sumi possit.

COROLLARIUM 2

68. Deinde etiam hinc patet, quo maior fuerit refractio seu numerus n pro huiusmodi lente obiectiva, eo maius lucrum in constructionem telescopiorum redundare, quia tum non solum numerus λ prodit minor, sed etiam numerus p, per quem λ multiplicari oportet.

SCHOLION

69. Huiusmodi autem lentes duplicatae et triplicatae in obiectivae lentis locum substituendae nihil plane conferunt ad alterum confusionis genus, quod ex diversa radiorum refrangibilitate nascitur, diminuendum, sed aequationes in capite primo datae pro hoc genere confusionis tollendo prorsus manent eaedem, ac si lens obiectiva esset simplex; verum reliquae lentes duplicatae et triplicatae, quas supra in additamento [VIII] commendavimus, primum etiam terminum in aequatione pro dispersione ante inventa ad

nihilum redigunt, in quo praecipua pars huius confusionis continetur. Quocirca in hoc capite illas lentium tam duplicatarum quam triplicatarum species repeti conveniet.

DEFINITIO 4

70. *Lens obiectiva perfecta est, quae non solum nullam parit confusionem ab apertura oriundam, sed etiam nullam plane radiorum dispersionem gignit.*

COROLLARIUM 1

71. Si igitur talis lens adhibeatur, numerus λ penitus evanescet, unde semidiameter confusionis multo fit minor quam pro lentibus obiectivis compositis hactenus explicatis.

COROLLARIUM 2

72. Ex superioribus etiam satis intelligitur ad huiusmodi lentes perfectas construendas duas ad minimum diversas vitri species requiri, et quia experimenta circa alias vitri species adhuc desiderantur, alias species adhibere non licet praeter vitrum coronarium et crystallinum, quibus Clarissimus DOLLONDUS est usus.

PROBLEMA 4

73. *Lentem Obiectivam duplicatam partim ex vitro coronario* $n = 1,53$*, partim ex crystalllino* $n = 1,58$ *compositam construere.*

SOLUTIO

In additamento ad calcem capitis VII partis praecedentis annexo [p. 249-253] duas huiusmodi lentes perfectas dedimus, quarum alterius lens prior ex vitro coronario, posterior vero ex vitro crystallino erat confecta; alterius vero contra lens prior ex vitro crystallino, posterior vero ex coronario; has duas lentium perfectarum species hic referamus.

I. Lens obiectiva perfecta duplicata

Pro lente priori ex vitro coronario $n = 1,53$ parata

$$\text{radius faciei} \begin{cases} \text{anterioris} = +0,1807p \\ \text{posterioris} = +1,3239p \end{cases} \text{Crown Glass.}$$

Pro lente posteriori ex vitro crystallino $n = 1,58$ parata

$$\text{radius faciei} \begin{cases} \text{anterioris} & = -\ 0,4770p \\ \text{posterioris} & = -\ 0,5191p \end{cases} \text{Flint Glass.}$$

Quae capax est aperturae, cuius semidiameter est $x = 0,0452p$.

II. Lens obiectiva perfecta duplicata

Pro lente priori ex vitro crystallino $n = 1,58$ parata

$$\text{radius faciei} \begin{cases} \text{anterioris} & = -\ 2,0545p \\ \text{posterioris} & = -\ 0,2828p \end{cases} \text{Flint Glass.}$$

Pro lente posteriori ex vitro coronario $n = 1,53$ parata

$$\text{radius faciei} \begin{cases} \text{anterioris} & = +\ 0,4568p \\ \text{posterioris} & = +\ 0,2438p \end{cases} \text{Crown Glass.}$$

Eritque semidiameter aperturae $x = 0,0609p$, ubi notandum est p designare distantiam focalem ipsius lentis duplicatae.

COROLLARIUM 1

74. Cum igitur harum lentium posterior maiorem admittat aperturam quam prior, haec illi sine dubio est anteferenda, quoniam, ut infra patebit, omnis telescopiorum perfectio eo redit, ut lens obiectiva quam maximam aperturam admittat.

COROLLARIUM 2

75. Observandum hic est utroque casu lentem ex vitro crystallino parandam esse debere concavam, eam vero, quae ex vitro coronario conficitur, convexam, prouti eae revera a DOLLONDO parantur.

SCHOLlON

76. Ceterum hic non est reticendum ambas has species summam artificis sollertiam requirere; si enim tantillum in earum constructione a mensuris hic praescriptis aberretur, fieri potest, ut eae minus valeant, quam si lentes adeo simplices adhiberentur. Sequentes vero lentes triplicatae multo minorem sollertiam postulant, cum pro singulis lentibus simplicibus numerus A unitati aequetur ideoque leves errores in constructione commissi non adeo sint pertimescendi.

PROBLEMA 5

77. *Lentem obiectivam perfectam triplicatam partim ex vitro coronario* $n = 1{,}53$, *partim ex crystallino* $n = 1{,}58$ *compositam construere.*

SOLUTIO

Pro hoc lentium perfectarum genere supra quatuor dedimus species, quas hic referamus:

I. Lens obiectiva perfecta triplicata cuius lens prima et tertia ex vitro crystallino media ex coronario est parata

Pro lente prima radius faciei $\begin{cases} \text{anterioris} & = +\ 0{,}5039p \\ \text{posterioris} & = +\ 5{,}6450p \end{cases}$ Flint Glass.

pro lente secunda radius faciei $\begin{cases} \text{anterioris} & = +\ 0{,}1364p \\ \text{posterioris} & = -\ 0{,}9597p \end{cases}$ Crown Glass.

pro lente tertia radius faciei $\begin{cases} \text{anterioris} & = +\ 1{,}0699p \\ \text{posterioris} & = -\ 0{,}1404p \end{cases}$ Flint Glass.

Quae lens capax est aperturae, cuius semidiameter $x = 0{,}0341p$.

II. Lens obiectiva perfecta triplicata cuius lens prima et tertia ex vitro crystallino media ex coronario est parata

Pro lente prima radius faciei $\begin{cases} \text{anterioris} & = -0{,}1762p \\ \text{posterioris} & = -1{,}9741p \end{cases}$ Flint Glass.

pro lente secunda radius faciei $\begin{cases} \text{anterioris} & = +\ 2{,}5349p \\ \text{posterioris} & = +0{,}1696p \end{cases}$ Crown Glass.

pro lente tertia radius faciei $\begin{cases} \text{anterioris} & = +\ 0{,}6179p \\ \text{posterioris} & = +1{,}8532p \end{cases}$ Flint Glass.

Quae lens capax est aperturae, cuius semidiameter $x = 0{,}0424p$.

III. Lens obiectiva perfecta triplicata cuius lens prima et tertia ex vitro coronario media ex crystallino est parata

Pro lente prima radius faciei $\begin{cases}\text{anterioris} = +\ 0,5004p \\ \text{posterioris} = +\ 3,6665p\end{cases}$ Crown Glass.

pro lente secunda radius faciei $\begin{cases}\text{anterioris} = -0,5107p \\ \text{posterioris} = -0,4843p\end{cases}$ Flint Glass.

pro lente tertia radius faciei $\begin{cases}\text{anterioris} = +\ 0,5219p \\ \text{posterioris} = +\ 0,4757p\end{cases}$ Crown Glass.

Aperturae semidiametro $x = 0,1189p$.

IV. Lens obiectiva perfecta triplicata cuius lens prima et tertia ex vitro coronario media ex crystallino est parata

Pro lente prima radius faciei $\begin{cases}\text{anterioris} = +\ 0,2829p \\ \text{posterioris} = +\ 2,0729p\end{cases}$ Crown Glass

pro lente secunda radius faciei $\begin{cases}\text{anterioris} = -\ 1,6017p \\ \text{posterioris} = -\ 0,2943p\end{cases}$ Flint Glass.

pro lente tertia radius faciei $\begin{cases}\text{anterioris} = +\ 0,5860p \\ \text{posterioris} = +\ 1,7670p\end{cases}$ Crown Glass.

Semidiametro aperturae $x = 0,0707p$.

In his formulis littera p denotat distantiam focalem cuiusque lentis perfectae.

COROLLARIUM 1

78. Inter has quatuor lentes tertia imprimis est notatu digna, quod maximam aperturam admittat.

COROLLARIUM 2

79. Si ergo eiusmodi lens perfecta in quodam telescopio loco lentis obiectivae adhibeatur, in expressione pro semidiametro confusionis primus terminus $\mu\lambda$ prorsus evanescit; tum vero etiam in aequatione ultima pro dispersione destruenda terminus primus quoque ad nihilum redigitur.

COROLLARIUM 3

80. Huiusmodi igitur lentes perfectae etiam speculis, quibus in telescopiis catoptricis utuntur, longe sunt anteferendae, cum specula tantum a dispersione radiorum sint immunia, neutiquam vero a priori confusionis genere, quod ab apertura oritur.