CHAPTER III

THE DIVISION OF TELESCOPES INTO

THREE PARTICULAR KINDS

DEFINITION 1

81. *The image is true [i.e. real], for which being formed the rays actually concur and then spread out again; while on the other hand these images may be called imaginary [i.e. virtual], for which the rays are only directed to be converging and not truly concurring to these actually being formed, or even may be said to be diverging further from these, and neither yet arising from these.*

COROLLARY 1

82. Therefore a real image makes use of this property, so that if a white sheet may be spread out at its place, upon that the images will be expressed by the incident rays, which does not happen with imaginary images used.

COROLLARY 2

83. Moreover the imaginary images are of two kinds; indeed either the rays progress thence by diverging further, if they have not yet departed thence, or for these trying to converge, nor yet actually thus arriving, but before that taking another direction by means of another lens.

SCHOLIUM

84. For these, which have been proposed so far, we will represent these figures thus, as if real images may be formed by these individual lenses, thus so that between some two successive lenses a true image may lie, nor has any imaginary image been indicated from these figures (Fig. 15, Book I). Moreover we have designated these true images by the letters $F\zeta, G\eta, H\theta$ etc., which all have been prepared thus, so that, if a white paper sheet may be placed there, an actual image of the object may be expressed there.

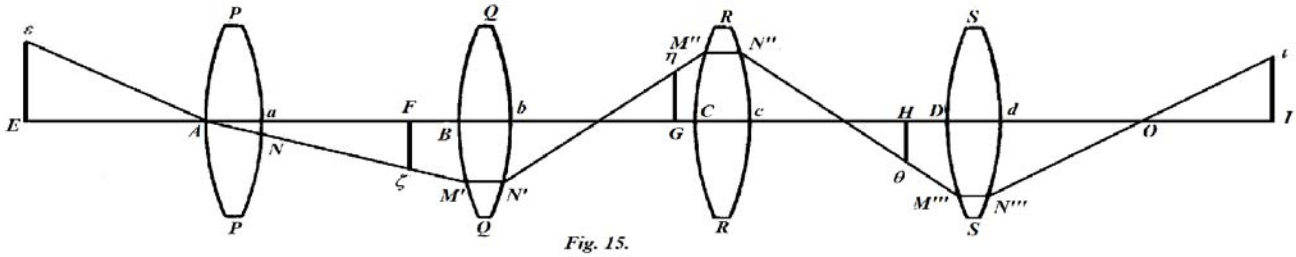

Fig. 15.

But it is evident by necessity real images must arise, if all the determinable distances, which we have put in place above, we may put in place above

$$aF = \alpha,\ FB = b,\ bG = \beta, Gc = c,\ cH = \gamma,\ HD = d \text{ etc.}$$

will have been positive; but then the images will be imaginary, when certain of these distances become negative, which we will explain further in the following theorems.

THEOREM 1

85. *Between two successive lenses, of any interval whatever, for instance cD, if the two parts of the interval* $cH = \gamma$ *and* $HD = d$ *were positive, thus so that there shall be* $cD = \gamma + d$ *, then a real inverted image will be shown at the point H.*

DEMONSTRATION

For the rays refracted by the lens *RR* act together towards forming the image $H\theta$ and, since the following lens *SS* has been placed beyond the place of the image H, from these rays a real image will be shown at *H*, thus so that, if a paper screen may be placed at $H\theta$, there these same rays may actually have been received and an effigy described on that ; therefore so that by necessity it must always happen, as often as the two parts r and d of this interval were positive. And if in turn the real image is represented by *H*, it is evident this cannot be done, unless the point *H* may lie after *C*, since otherwise the rays may not be extended there; then truly also it is clear an image cannot be formed, unless the following lens *D* may lie after *H*. Therefore since the distance $CH = \gamma$ must be positive and likewise the distance $cD = \gamma + d > \gamma$, and it is evident the distance *d* must be positive.

COROLLARY

86. Since until now we have considered the individual intervals between two successive lenses as composed from two parts placed together, truly the image $F\zeta$ falls between the first lens *A* and the second *B*, if both its parts α and *b* were positive; and truly the image may be found in a similar manner between the second *B* and third *C*, if both parts of this interval *BC* β and *c* were positive, and thus so on.

THEOREM 2

87. *If one of the two parts of this kind, constituting some interval such as cD, were negative, then the image $H\theta$ corresponding to this lens will be imaginary (for it cannot happen that both will become negative at the same time).*

DEMONSTRATION

Since the interval *cD* depends on the two parts $cH = \gamma$ and $HD = d$, we may take the first distance γ to be negative ; then the image $H\theta$ therefore lies before the lens *RR* and the rays transmitted by this lens thus will be refracted, as if diverging from that first image, yet since they will not have emanated thence ; on account of which this image will not be real, but imaginary. But if the other part *d* were negative, the image $H\theta$ finally may fall after the lens *RR* ; but since the rays transmitted through the lens *RR* , before they reach there, again are refracted by the lens *SS,* will not actually form that image, and thus this image again will be imaginary.

But both parts γ and *d* cannot be negative at the same time, because the sum of these $\gamma + d$, expressed the interval *cD* itself, which always be necessity must be positive.

[Thus we are made aware, perhaps rather painfully, that Euler does not consider the object position relative to the relevant focal point in his discussions on real and virtual images; the actual image position is determined algebraically by calculation from a given object position for some lens, which is necessary for precise measurements in any case.]

COROLLARY 1

88. Therefore if for the first interval *aB* one of the parts α and *b* were negative, no real image falls between *a* and *B* ; if besides also for the second interval *bC* one of the parts β and *c* also were negative, no real image lies between *a* and *C* ; and if in addition of the parts of the interval *cD,* which are γ and *d,* either were negative, then certainly in the interval *aD* no real image thus can be found, so that clearly no real image may fall in the space between several lenses.

COROLLARY 2

89. Therefore by no means does the number of real images depend on the number of lenses, since it may happen that after any lens a real image may be manifest and equally that after several lenses plainly no real image may correspond.

COROLLARY 3

90. Therefore from however many lenses some telescope were composed, it can happen, so that through the whole of its length clearly no real image may be found, or only one or two or three etc., yet at no time can more be taken than there are lenses.

THEOREM 3

91. *After however many lenses finally a real image may be shown in the telescope, that always is inverted.*

DEMONSTRATION

Evidently when the image of the first lens is real at once, that is seen also to be inverted; but because that also may be going to be inverted again, if it may occur after so many lenses, can be shown in the following manner (Fig. 5, Book 1).

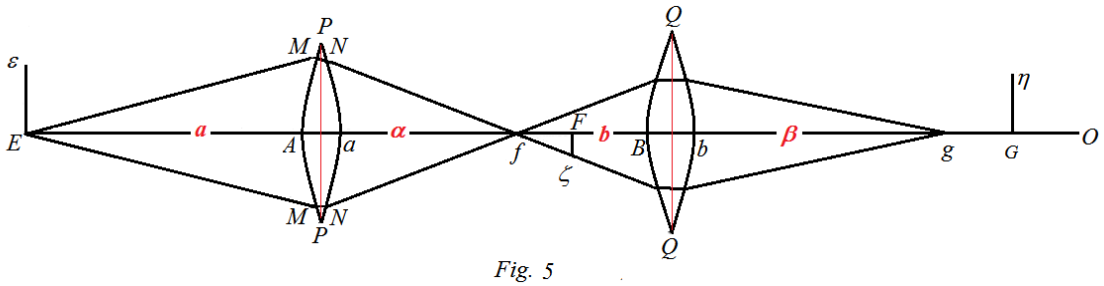

Fig. 5

Fig. 5 (again)

$[\, aF = \alpha,\ FB = b,\ bG = \beta. \,]$

The ray may be considered from the centre of the object E passing through an upper point M of the objective lens, and this same ray passing through the following lenses as long as it is situated above the axis, as long as it has extended to the first real image; indeed because the point E has left the axis, it will pass through the axis again at some point, there a true image of the object will be present (for here we do not consider either aberration or spreading of the rays), from which it is evident this ray is not going to arrive at the axis until it has been extended to the first real image, and since here it is going to be progressing from the upper to the lower region, the image expressed at this location will be inverted ; for since it shall have been advancing upwards from the object, but now it may be directed downwards from the image, the parts of the object inclined upwards now will be observed in the downwards position.

COROLLARY 1

92. In a similar manner it is possible to understand these rays by progressing so long below the axis to be going to turning up, while again they may extend to the axis, so that there shall be made into a second real image, from which again they may pass into the upper parts of the axis; from which a second erect image must be held in position, and thus again a third real image will be inverted, moreover an erect fourth, and so on thus.

COROLLARY 2

93. Therefore however many lenses there were, not only will it be required to be considered for the individual images corresponding to the individual lenses , but also the real images, since they may depend alternately to be placed erect and inverted only by being from real images, while imaginary images disturb nothing in this order.

SCHOLIUM

94. This property of the true image essentially affects the nature of telescopes so much, so that the main difference may seen to be desired from the number of true images, clearly with no account taken of the imaginary images, certainly which are of little concern in this matter. Indeed anyone who might wish to divide telescopes into kinds according to the number of lenses, will implicate himself in the greatest inconvenience; for in the first place these extremely small telescopes or rather opera glasses having a concave eyepiece lens and astronomical tubes are forced to be referred to the same class, while yet by their nature they may differ greatly between each other, since the former may be represented with the object situated upright, truly the latter with the object placed inverted, besides which the maximum difference in eyepiece is taken in each ; then if for some telescope, either to increase the field of view or for a greater order of resolution, that itself may be acquired by the addition of a single lens, the increased number of lenses at once may be required to be referred to another kind, which certainly equally must be seen to be an unsuitable criterion. From which careful considerations without doubt different kinds of telescopes to be constituted according to the number of real images, which occur in these, thus so that the first kind shall be going to include these telescopes, in which plainly no real image occurs, the second truly these, in which a single real image is found, the third these, which contain two real images; for which all three kinds of telescopes, which at this stage have been though out and made, will be required to be referred to ; and if we wish to proceed further, to recall the fourth kind it will be agreed these telescopes, in which three real images are taken; truly the preceding ones thus as widely apparent, so that clearly with these all the perfection, which may be desired at any time, may be able to be agreed on, thus so that plainly no reason may be present, why we may wish to put more real images in place. Therefore we will set out this division more distinctly in the following problems.

PROBLEM 1

95. *The particular properties of telescopes of the first kind are to be reviewed , in which no real image is present.*

SOLUTION

Since in these telescopes, however many lenses also they depend on, no true image shall be present, the individual intervals $aB = \alpha + b,\ bC = \beta + c,\ cD = \gamma + d$ etc. thus will be defined from the two parts, so that either of these shall be negative, and that as far as to the final eyepiece lens. And since these same intervals by necessity are positive, it is readily apparent all these fractions $\frac{\alpha}{b},\ \frac{\beta}{c},\ \frac{\gamma}{d}$ etc. must be negative, in which the nature of this essential kind of telescopes is required to be put in place. For in turn, if all these fractions were negative, in the whole telescope no real image will be found and thus will be required to be referred to our first kind. But another characteristic less essential of this kind may be present in this, so that these telescopes may represent upright objects, since on account of no real image we observe these objects themselves through the eyepiece, as if more closely.

COROLLARY 1

96. Therefore the simplest species of this kind will be established from two lenses, and since the quantity $\frac{\alpha}{b}$ shall be negative, the ratio of the multiplication shall become $m = \frac{-\alpha}{b}$, as the erect position demands ; hence it is necessary that there shall be $\alpha > b$ and thus the quantity α must be positive and b negative. But again since there must be $\beta = \infty$, we will have for the focal distance q of this eyepiece lens on account of $\frac{1}{q} = \frac{1}{b} + \frac{1}{f}$ the value $q = b$, and thus the eyepiece will be concave.

COROLLARY 2

97. Again since in this manner there shall be $m = \pm\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$ etc., of which the factors are our fractions, which all must be negative, hence it is evident, why the above signs $+$ and $-$ shall be found alternating, evidently so that a positive value may follow for the multiplication number m for any number of lenses.

COROLLARY 3

98. Also it can be shown none of these letters $\alpha,\ b,\ \beta,\ c,\ \gamma$ etc. to be taken as vanishing. Indeed if the distance b were forced to become a minimum, since either of the letters α and b must be negative, the sum of these truly $\alpha + b$ positive and finite, it is

necessary that there shall be $\alpha > 0,\ b < 0$; therefore there shall be $b = -\omega$, clearly for the vanishing quantity, and since $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$, there becomes $\beta = \frac{q\omega}{q+\omega} = \omega$ and thus positive ; therefore there will become $c < 0$, and hence $\beta + c$ may not express the interval cB ; from which it is apparent cases of this kind cannot occur. But it can happen, so that some of these quantities may become $= \infty$; if indeed there were, for example, $\beta = \infty$, on account of the interval $\beta + c =$ a finite number , for example $= k$, there will become $c = -\infty + k = -\infty$ and $\frac{\beta}{c} = -1$; but this does not hinder why the following fraction $\frac{\gamma}{d}$ may not obtain some value.

SCHOLIUM

99. This class of telescopes is most noteworthy, evidently because in the first place it is regarded as discovered by some artisan, while accidently he had combined a convex lens with a concave lens; nor yet is its essence required to be established here, only because it depends on two lenses. For if in place of the objective lens we may substitute a double or even a triple set of lenses, certainly nobody will think its kind to be changed, since multiple lenses of this kind are accustomed to be viewed as simple; in a like manner the eyepiece lens may be doubled or tripled without its kind being changed; but nevertheless since several simple lenses may be used, it is evident the nature of the kind of telescope cannot be considered to depend on the number of lenses. But in the following work we will show, how by adding new lenses this kind of telescope may be carried to greater perfection.

PROBLEM 2

100. *To review the special properties of telescopes related to the second kind, in which a single real image occurs.*

SOLUTION

It is evident if such a telescope were composed from some number of lenses, not all the fractions $\frac{\alpha}{b}$, $\frac{\beta}{c}$, $\frac{\gamma}{d}$ etc. arising from the intervals between the lenses must be negative, since otherwise no real image is going to be produced; but since one real image shall be present, it is necessary, that as also one of the fractions may become positive, since if $\frac{\gamma}{d}$ were in this situation both the letters γ and d must be real, while all the remaining fractions stay negative as before, and likewise it is the case, for whatever of these fractions may obtain a positive value, provided that not more than one shall be positive; and on this rests the essential nature of this kind of telescopes, among its properties this in addition is especially noteworthy, because the object will be shown inverted, since we have agreed to observe through telescopes of this kind not only the objects themselves but also a real image of these, which is inverted.

COROLLARY 1

101. If therefore a telescope of this kind may be agreed to consist of two lenses only, since without doubt is the simplest species of this kind, on account of the single interval *aB* the single fraction $\frac{\alpha}{b}$ will be had also, which therefore must be positive and thus also each distance α and *b* must be positive; which since on account of $a=\infty$ and $\beta=\infty$ bear the focal lengths of each lens, evidently each lens to be convex.

COROLLARY 2

102. Therefore since an inverse representation of this kind is appropriate, the exponent of the multiplication *m,* which has been found to be equal to the product of these fractions $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc., will produce a negative value, clearly contrary to that, which was produced in the preceding case.

COROLLARY 3

103. Moreover in this kind of telescope it can happen, that some of the magnitudes α, *b* etc. may vanish, which happens, if a lens may be set up in place of the real image. For if the real image may fall upon the third lens *C*, there will be $c=0$, or rather on putting $c=\omega$ on account of $\frac{1}{r}=\frac{1}{c}+\frac{1}{\gamma}$ there will be $\gamma=\frac{-r\omega}{r-\omega}=-\omega$ thus so that both the magnitudes *c* and γ may vanish, so that the distances β and *d* must be positive, and thus it is apparent that of the fractions $\frac{\beta}{c}$ and $\frac{\gamma}{d}$ one to become positive, the other negative, just as we would have wished ; indeed because we assume the image to fall on the lens *RR* itself, likewise, we wish to refer either to the interval *bC* or to the interval *cD* ; but in each case, if the fraction $\frac{\beta}{c}$ may become ∞, truly the fraction $\frac{\gamma}{d}=0$, and the product of both is $=-\frac{\beta}{d}$ always.

SCHOLION

104. Telescopes pertaining to this kind are accustomed to be called astronomical ; indeed since they show objects inverted, they are used mainly for astronomical observations, where it matters little, whether we may observe heavenly bodies either erect or inverted ; that which with terrestrial objects may itself be had otherwise, for the consideration of which, when telescopes of the first kind do not suffice, we are accustomed to have recourse to the third kind.

PROBLEM 3

105. *To review the particular properties of telescopes related to the third kind, in which two real images occur.*

SOLUTION

Since here two real images may occur, however many lenses may be used, thence between the fractions produced $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. by necessity two must be positive, truly all the rest are negative; from which, since two fractions of this kind at a minimum must be present and thus also two intervals of the lenses, it is evident for telescopes of this kind at a minimum three lenses of this kind to be required, in which case no such negative fractions will be had; from which negative fractions only occur so far, in as much as more than three lenses are called into use, and in this the essential character of this kind of telescopes is contained; but among the particular properties this especially required to be observed, so that through the telescope objects may be seen situated erect.

COROLLARY 1

106. If these telescopes may be formed from three lenses, all these four distances α, b, β, c must be positive, and since the distances a and γ shall be ∞, the three lenses must be convex ; if indeed the focal lengths of these shall be p, q and r, there will be had I. $p = \alpha$, II. $q = \frac{b\beta}{b+\beta}$ and III. $r = c$, which are all positive.

COROLLARY 2

107. Just as in the preceding case it has been allowed to put a lens in place of the real image, thus here also no reason stands in the way, why in each true image lenses may be put in place; but then these, which above are concerned with fractions treated sometimes by infinite increases, at other times by vanishing quantities, are required to be observed properly.

SCHOLION

108. Since finally for this kind of telescope it has been established how astronomical tubes may be adapted to consider terrestrial objects placed erect ; which indeed we have been able to determine able to happen with three lenses. But since with only three lenses used the apparent field of view will vanish almost completely and other inconveniences in addition may be involved, forthwith four lenses are accustomed to be used, which are joined together thus, so that they may refer to two connected astronomical tubes, and the three posterior lenses are called the eyepieces, to which also almost the same focal length

can be attributed. For the same kind also is required to be referred to are these new English recently invented by the most illust. Dollond, in which besides the double objective lenses, the arrangement of the eyepieces is seen to be far more diverse . Truly meanwhile this arrangement can be varied in an indefinite number of ways, so that these telescopes may be raised to a high degree of perfection.

PROBLEM 4

109. *To enumerate the particular properties of telescopes of the related fourth kind in which three real images occur.*

SOLUTION

Therefore in this kind, however many lenses may be used, between the fractions $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. corresponding to these, three must be positive, while the rest remain negative, from which it is evident for this class to be a minimum need for four lenses, and since the final real image, which as if it may be viewed with the eye, is inverted, the objects too will be observed inverted by all telescopes of this kind.

SCHOLION

110. Plainly since no account may persuade us, that we may wish to change the representation of the preceding kinds, and, as we will see, all the perfections of the preceding kinds of telescopes may be able to be brought together, we may gain nothing further, unless that telescopes may be made much longer, and we may multiply the number of lenses without any use, so that I may be silent about the conspicuous loss of the rays of light, which on account of so many lenses may deserve to be feared; and because of this, without doubt, this fourth kind to be completely rejected, from which also no doubt will remain, when thus we have dealt with the three preceding kinds, that we may be satisfied with everything considered on which the perfection of telescopes depends. But more to the point clearly what may constitute the kinds of telescopes, will deserve no more attention in the following.

CAPUT III

DE DISTRIBUTIONE TELESCOPIORUM IN TRIA GENERA PRAECIPUA

DEFINITIO 1

81. *Imago vera est, ad quam formandam radii revera concurrant indeque porro diffunduntur; dum contra eae imagines fictae vocantur, ad quas radii tantum convergendo diriguntur neque vero ad eas actu formandas concurrunt, vel etiam ab iis divergendo ulterius discedunt neque tamen ab iis prodierunt.*

COROLLARIUM 1

82. Imago igitur vera hac gaudet proprietate, ut, si in eius loco charta alba esset expansa, super ea effigies a radiis incidentibus exprimeretur, quod in imaginibus fictis usu non venit.

COROLLARIUM 2

83. Imagines autem fictae duplicis sunt generis; vel enim radii inde divergendo ulterius progrediuntur, cum tamen inde non discesserint, vel ad eas convergendo tendunt neque tamen eo revera perveniunt, sed ante ab alia lente aliam directionem accipiunt.

SCHOLION

84. Ad ea, quae hactenus sunt proposita, figuras ita repraesentavimus, quasi per singulas lentes imagines verae formarentur, ita ut inter binas quasque lentes successivas imago vera caderet, neque in his figuris ulla imago ficta est indicata (Fig. 15, Lib. I). Imagines autem illas veras litteris $F\zeta, G\eta, H\theta$ etc. designavimus, quae omnes ita sunt comparatae, ut, si ibi charta alba expanderetur, super ea effigies obiecti revera exprimeretur.

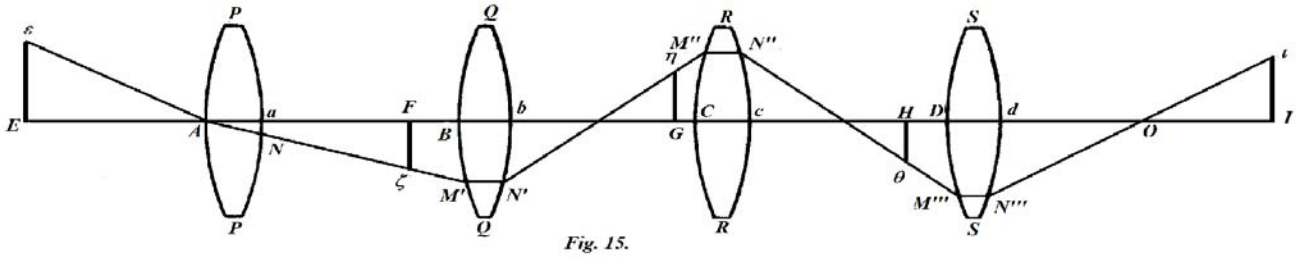

Fig. 15.

Perspicuum autem est imagines veras necessario oriri debere, si omnes distantiae, quas supra posuimus, determinatrices $aF = \alpha,\ FB = b,\ bG = \beta, Gc = c,\ cH = \gamma,\ HD = d$ etc. fuerint positivae; imagines autem tum erunt fictae, quando harum distantiarum quaedam fiunt negativae, id quod in sequentibus theorematibus fusius explicabimus.

THEOREMA 1

85. *Si intervalli inter binas lentes successivas cuiuscunque v. gr. cD binae partes* $cH = \gamma$ *et* $HD = d$ *, ita ut sit* $cD = \gamma + d$ *, fuerint positivae, imago vera in puncto H exhibebitur et contra.*

DEMONSTRATIO

Radii enim per lentem *RR* refracti ad imaginem $H\theta$ conformandam tendunt et, quia lens sequens *SS* ultra locum imaginis H est posita, ab his radiis imago vera in *H* repraesentabitur, ita ut, si per $H\theta$ charta alba esset expansa, ea istos radios revera exciperet super eaque effigies depingeretur; quod ergo necessario semper evenire debet, quoties binae partes huius intervalli *r* et *d* fuerint positivae. Ac si vicissim in *H* repraesentetur imago vera, manifestum est hoc fieri non posse, nisi punctum *H* post lentem *C* cadat, quia alioquin radii eo non porrigerentur; tum vero etiam liquet hanc imaginem efformari non posse, nisi sequens lens *D* post *H* cadat. Cum igitur esse debeat distantia $CH = \gamma$ positiva simulque distantia $cD = \gamma + d > \gamma$, evidens est et distantiam *d* esse debere positivam.

COROLLARIUM

86. Quoniam haetenus singula intervalla inter binas lentes successivas tanquam ex duabus partibus composita sumus contemplati, inter lentem primam *A* et secundam. *B* imago vera $F\zeta$ cadet, si ambae eius partes α et *b* fuerint positivae; similique modo inter lentem secundam *B* et tertiam *C* imago vera reperietur, si huius intervalli *BC* ambae partes β et *c* fuerint positivae, et ita porro.

THEOREMA 2

87. *Si binarum partium aliquod huiusmodi intervallum veluti cD constituentium alterutra fuerit negativa, tum imago* $H\theta$ *lenti a respondens erit ficta (fieri enim nequit, ut ambac simul sint negativae).*

DEMONSTRATIO

Cum intervallum *cD* binis partibus $cH = \gamma$ et $HD = d$ constat, sumamus primo distantiam γ esse negativam; tum igitur imago $H\theta$ ante lentem *RR* cadet et radii per hanc lentem transmissi ita refringentur, quasi ex ista imagine essent egressi, cum tamen inde non emanaverint; quamobrem ista imago non erit vera, sed ficta. Sin autem altera pars *d* fuerit negativa, imago $H\theta$ demum post lentem *RR* caderet; quia antem radii per lentem *RR* transmissi, antequam eo pertingunt, per lentem *SS* de novo refringuntur, istam effigiem non revera formabunt, ideoque haec imago erit ficta.

Ambae autem partes γ et *d* simul non possunt esse negativae, quia earum summa $\gamma + d$, ipsum intervallum *cD* exprimit, quod semper necessario est positivum.

COROLLARIUM 1

88. Si ergo pro primo intervallo *aB* partium α et *b* altera fuerit negativa, inter *a* et *B* nulla cadit imago vera; si praeterea etiam pro secundo intervallo *bC* partium β et *c* altera fuerit quoque negativa, inter *a* et *C* nulla cadet imago vera; ac si insuper partium intervalli *cD,* quae sunt γ et *d,* altera fuerit negativa, tum ne quidem in spatio *aD* reperietur imago vera sicque fieri potest, ut inter plurium lentium spatium nulla plane cadat imago vera.

COROLLARIUM 2

89. Neutiquam ergo numerus imaginum verarum a numero lentium pendet, cum aeque fieri possit, ut post quamlibet lentem imago vera repraesentetur atque ut pluribus lentibus nulla plane imago vera respondeat.

COROLLARIUM 3

90. Ex quotcunque igitur lentibus telescopium quodpiam fuerit compositum, fieri potest, ut per totum eius spatium vel nulla plane imago vera reperiatur vel unica tantum vel duae vel tres etc., nunquam tamen plures, quam sunt lentes, ultima demta.

THEOREMA 3

91. *Post quotcunque demum lentes in telescopio prima imago vera exhibetur, ea semper est inversa.*

DEMONSTRATIO

Quando scilicet imago primae lentis statim est vera, perspicuum est eam quoque esse inversam; quod autem ea etiam futura sit inversa, si demum post plures lentes occurrat, sequenti modo ostendi potest (Fig. 5, Lib. 1).

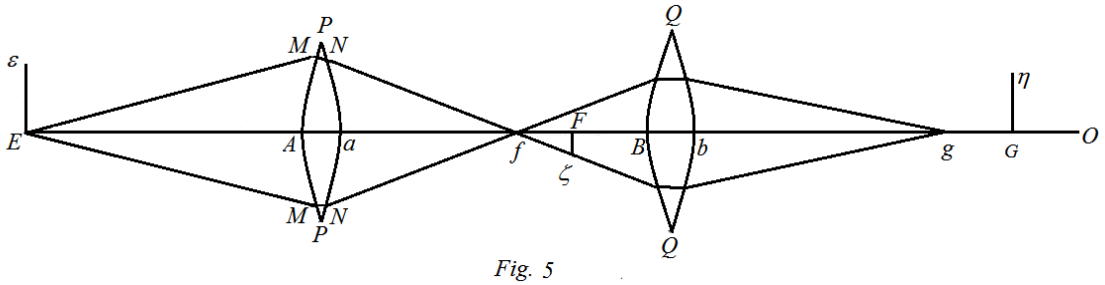

Fig. 5

Fig. 5 (iterate.).

Consideretur radius ex centro obiecti *E* per superius lentis obiectivae punctum *M* transiens, atque iste radius per sequentes lentes transiens tamdiu supra axem versabitur, donec ad primam imaginem veram pertigerit; quia enim ex axis puncto *E* est egressus, ubicunque iterum in axem incideret, ibi existeret imago obiecti vera (hic enim ad aberrationem vel diffusionem radiorum non respicimus), ex quo manifestum est hunc radium ante non ad axem esse perventurum, quam ad primam imaginem veram pertigerit, et quia ex regione superiori hic in axem incidit ad regionem inferiorem progressurus, imago in hoc loco expressa erit inversa; cum enim ex obiecto sursum sit progressus, nunc autem ex imagine deorsum dirigatur, partes obiecti sursum vergentes nunc deorsum sitae conspicientur.

COROLLARIUM 1

92. Simili modo intelligere licet radios illos ex imagine progredientes tamdiu infra axem esse versaturos, donec iterum ad axem pertingant, quod fit in imagine vera secunda, unde iterum in partes axis superiores transeunt; unde patet secundam imaginem situm erectum tenere debere, sicque porro tertia imago vera denuo erit inversa, quarta autem erecta, et ita porro.

COROLLARIUM 2

93. Quotcunque ergo fuerint lentes, non tam ad imagines singulis lentibus respondentes erit respiciendum quam ad imagines veras, cum alternato situs erecti et inversi pendeat tantum ab imaginibus veris, dum imagines fictae nihil in hoc ordine turbant.

SCHOLION

94. Haec proprietas imaginum verarum tam essentialiter naturam telescopiorum afficit, ut eorum discrimen potissimum a numero imaginum verarum petendum esse videatur, nulla plane ratione habita imaginum fictarum, quippe quae in hoc negotio parvi

sunt momenti. Qui enim voluerit telescopia secundum lentium numerum in genera distribuere, maximis incommodis se implicabit; primo enim exigua illa telescopia vel potius perspicilla lente oculari concava constantia et tubos astronomicos ad idem genus referre esset coactus, dum tamen sua natura maxima inter se discrepant, quandoquidem illis obiecta situ erecto, his vero situ inverso repraesentantur, praeterquam quod in loco oculi maxima utrinque deprehenditur diversitas; deinde si cuipiam telescopio sive ad campum apparentem augendum sive ad maiorem distinctionis gradum ipsi conciliandum unica lens insuper adiungeretur, statim ad longe aliud genus foret referendum, quod certe aeque incongruum videri debet. Quibus probe perpensis non dubito diversa telescopiorum genera secundum numerum imaginum verarum, quae in iis occurrunt, constituere, ita ut primum genus complexurum sit ea telescopia, in quibus nulla plane imago vera occurrit, secundum vero ea, in quibus unica imago vera reperitur, tertium vero ea, quae duas imagines veras continent; ad quae tria genera omnia telescopia, quae adhuc excogitata sunt et elaborata, erunt referenda; ac si ulterius progredi velimus, ad quartum genus revocari conveniet ea telescopia, in quibus tres imagines verae deprehenduntur; verum praecedentia iam tam late patent, ut iis omnes plane perfectiones, quae unquam desiderari queant, conciliari possint, ita ut nulla plane ratio adsit, cur plures imagines veras statuere velimus. Hanc igitur divisionem in sequentibus problematibus distinctius evolvamus.

PROBLEMA 1

95. *Telescopiorum ad primum genus relatorum, in quibus nulla inest imago vera, praecipuas proprietates recensere.*

SOLUTIO

Cum in his telescopiis, quotcunque etiam constant lentibus, nulla insit imago vera, singula intervalla $aB = \alpha + b$, $bC = \beta + c$, $cD = \gamma + d$ etc. ita ex binis partibus definientur, ut alterutra earum sit negativa, idque usque ad ultimam lentem ocularem. Et quoniam haec eadem intervalla necessario sunt positiva, facile patet omnes istas fractiones $\frac{\alpha}{b}$, $\frac{\beta}{c}$, $\frac{\gamma}{d}$ etc. debere esse negativas, in quo character essentialis huius generis telescopiorum est constituendus. Vicissim enim si omnes hae fractiones fuerint negativae, in toto telescopio nulla imago vera locum habebit ideoque ad nostrum primum genus erit referendum. Alius autem character minus essentialis huius generis in hoc consistit, quod haec telescopia situ erecto obiecta repraesentent, quia ob nullam imaginem veram ipsa obiecta quasi immediate adspicimus.

COROLLARIUM 1

96. Simplicissima ergo species huius generis duabus constabit lentibus, et cum sit $\frac{\alpha}{b}$ quantitas negativa, fiet ratio multiplicationis $m = \frac{-\alpha}{b}$, uti situs erectus postulat; hinc necesse est, ut sit $\alpha > b$ ideoque α quantitas positiva et b negativa. Cum autem porro

esse debeat $\beta = \infty$, pro huius lentis ocularis distantia focali *q* habebimus ob $\frac{1}{q} = \frac{1}{b} + \frac{1}{f}$ valorem $q = b$ sicque lens ocularis erit concava.

COROLLARIUM 2

97. Cum porro in genere sit $m = \pm \frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$ etc., cuius factores sunt nostrae fractiones, quae omnes esse debent negativae, hinc manifestum est, cur supra signa + et − sint alternantia inventa, ut scilicet pro quovis lentium numero multiplicatio *m* valorem positivum consequatur.

COROLLARIUM 3

98. Ostendi etiam potest nullam harum litterarum α, b, β, c, γ etc. sumi posse evanescentem. Si enim v.c. distantia *b* esset minima, quia altera litterarum α et *b* debet esse negativa, earum summa vero $\alpha + b$ positiva et finita, necesse est, ut sit $\alpha > 0$, $b < 0$; sit igitur $b = -\omega$, quantitati scilicet evanescenti, et quia est $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$, fiet $\beta = \frac{q\omega}{q+\omega} = \omega$ ideoque positivum; foret ergo $c < 0$, hincque $\beta + c$ intervallum *cB* exprimere non posset; unde patet huiusmodi casus locum habere non posse. Fieri autem potest, ut quaepiam harum quantitatum fiat $= \infty$; si enim fuerit v. gr. $\beta = \infty$, ob intervallum $\beta + c =$ finito, puta $= k$, erit $c = -\infty + k = -\infty$ et $\frac{\beta}{c} = -1$; hoc autem non impedit, quominus sequens fractio $\frac{\gamma}{d}$ valorem obtineat quemcunque.

SCHOLION

99. Notissimum est hoc telescopiorum genus, quippe quod primum ab artifice quodam inventum perhibetur, dum casu lentem convexam cum concava combinaverat; neque tamen eius essentia in hoc est statuenda, quod tantum duabus constat lentibus. Si enim loco lentis obiectivac simplicis substituamus duplicatam vel adeo triplicatam, nemo certe putabit ipsum eius genus mutatum esse, quoniam huiusmodi lentes multiplicatae ut simplices spectari solent; simili modo lens ocularis posset duplicari vel triplicari ipso genera non mutato; cum autem nihilominus plures lentes simplices adhibeantur, manifestum est ipsam generis indolem non a numero lentium pendere censeri posse. In sequentibus autem inprimis operam dabimus, ut novis lentibus addendis hoc genus ad maiorem perfectionem evehamus.

PROBLEMA 2

100. *Telescopiorum ad secundum genus relatorum, in quibus unica imago vera occurrit, praecipuas proprietates recensere.*

SOLUTIO

Ex quotcunque lentibus tale telescopium fuerit compositum, evidens est non omnes fractiones ex singulis lentium intervallis natas $\frac{\alpha}{b}$, $\frac{\beta}{c}$, $\frac{\gamma}{d}$ etc. negativas esse debere, quia alioquin nulla imago vera esset proditura; cum autem unica adsit vera, necesse est, ut etiam unica illarum fractionum fiat positiva, quia si fuerit v. c. $\frac{\gamma}{d}$ ambae litterae γ et d positivae esse debebunt, dum reliquae fractiones omnes manent ut ante negativae, atque perinde est, quaenam illarum fractionum valorem positivum nanciscatur, dummodo plus una non sit positiva; atque in hoc consistit character essentialis huius generis telescopiorum, inter cuius proprietates haec insuper inprimis est notanda, quod obiecta situ inverso repraesentet, quandoquidem per huiusmodi telescopia non tam ipsa obiecta quam eorum imaginem veram, quae est inversa, conspicere sumus censendi.

COROLLARIUM 1

101. Si ergo huiusmodi telescopium duabus tantum constet lentibus, quia sine dubio simplicissima huius generis est species, ob unicum intervallum *aB* unica quoque habetur fractio $\frac{\alpha}{b}$, quae propterea positiva esse debet ideoque etiam utraque distantia α et *b;* quae cum ob $\alpha = \infty$ et $\beta = \infty$ praebeant distantiam focalem utriusque lentis, manifestum est utramque lentem fore convexam.

COROLLARIUM 2

102. Quia igitur huic generi repraesentatio inversa est propria, exponens multiplicationis *m,* quae producto harum fractionum $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. aequalis est inventa, valorem negativum obtinebit, contrarium scilicet ei, qui casu praecedenti prodierat.

COROLLARIUM 3

103. In hoc autem genere evenire potest, ut quaepiam quantitatum α, b etc. evanescat, quod fit, si in loco ipsius imaginis verae lens constituatur. Cadat enim imago vera in ipsam lentem tertiam *C,* erit $c = 0$, vel potius posito $c = \omega$ ob $\frac{1}{r} = \frac{1}{c} + \frac{1}{\gamma}$ erit $\gamma = \frac{-r\omega}{r-\omega} = -\omega$ ita ut ambae quantitates *c* et γ evanescant, unde distantiae β et *d* debent esse positivae, sicque patet fractionum $\frac{\beta}{c}$ et $\frac{\gamma}{d}$ alteram fore positivam, alteram negativam, prout voluerimus; quoniam enim imaginem in ipsam lentem *RR* cadere assumimus, perinde est, sive eam ad intervallum *bC* sive ad intervallum *cD* velimus referre; utroque autem casu, etsi fractio $\frac{\beta}{c}$ fiat ∞, fractio vero $\frac{\gamma}{d} = 0$, productum ambarum semper est $= -\frac{\beta}{d}$.

SCHOLION

104. Telescopia ad hoc genus pertinentia vocari solent astronomica ; quoniam enim obiecta situ inverso repraesentant, potissimum ad observationes astronomicas adhibentur, ubi parum refert, sive obiecta in coelo situ erecto sive inverso conspiciamus; id quod in obiectis terrestribus secus se habet, ad quorum contemplationem, quando telescopia primi generis non sufficiunt, ad tertium genus recurrere solemus.

PROBLEMA 3

105. *Telescopiorum ad tertium genus relatorum, in quibus duae imagines verae occurrunt, praecipuas proprietates recensere.*

SOLUTIO

Cum hic duae imagines verae occurrant, quotcunque lentes adhibeantur, inter fractiones inde natas $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. duae necessario debent esse positivae, reliquae vero omnes negativae; unde, cum duae ad minimum eiusmodi fractiones adesse debeant adeoque etiam duo lentium intervalla, evidens est ad huiusmodi telescopia tres ad minimum lentes requiri, quo casu nullae tales fractiones negativae habebuntur; unde fractiones negativae eatenus tantum occurrent, quatenus plures tribus lentes in usum vocantur, atque in hoc essentialis character huius generis telescopiorum continetur; inter praecipuas autem proprietates haec inprimis est notanda, quod per telescopia obiecta in situ erecto conspiciantur.

COROLLARIUM 1

106. Si haec telescopia ex tribus lentibus formentur, omnes hae quatuor distantiae α, b, β, c esse debent positivae, et cum distantiae a et γ sint ∞ omnes tres lentes debent esse convexae ; si enim earum distantiae focales sint p, q et r, habebitur
I. $p = \alpha$, II. $q = \frac{b\beta}{b+\beta}$ et III. $r = c$, quae omnes sunt positivae.

COROLLARIUM 2

107. Quemadmodum praecedenti casu licuit in ipsum locum imaginis verae lentem constituere, ita etiam hic nulla ratio obstat, quominus in utraque imagine vera lentes collocentur; tum autem ea, quae supra sunt de fractionibus modo in infinitum excrescentibus modo evanescentibus tradita, probe sunt observanda.

SCHOLION

108. Hoc genus cum in finem est excogitatum, ut tubi astronomici ad obiecta terrestria situ erecto contemplanda accommodarentur; quod quidem tribus lentibus fieri posse iam annotavimus. Sed quoniam tribus tantum lentibus adhibendis campus apparens fere totus evanescit aliaque incommoda se insuper admiscent, statim quatuor lentes usurpari sunt solitae, quae ita sunt iunctae, ut duos tubos astronomicos connexos referant, et tres lentes posteriores nomine ocularium appellatae sunt, quibus etiam fere eadem distantia focalis tribui potest. Ad idem quoque genus referenda sunt nova illa telescopia anglica a Clariss. DOLLONDO nuper inventa, in quibus praeter lentes obiectivas duplicatas longe diversa lentium ocularium dispositio cernitur.
Interim vero haec dispositio infinitis modis variari potest atque adeo debet, ut haec telescopia ad summum perfectionis gradum evehantur.

PROBLEMA 4

109. *Telescopiorum ad quartum genus relatorum, in quibus tres imagines verae occurrunt, praecipuas proprietates enumerare.*

SOLUTIO

In hoc ergo genere, quotcunque lentes adhibeantur, inter fractiones iis respondentes $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. tres debent esse positivae, dum reliquae manent negativae, ex quo perspicuum est ad hoc genus ad minimum opus esse quatuor lentibus, et quia ultima imago vera, quae quasi ab oculo spectatur, est inversa, obiecta quoque per omnia telescopia huius generis inversa conspicientur.

SCHOLION

110. Quoniam nulla plane ratio suadet, ut repraesentationem praecedentis generis denuo invertere velimus, atque, uti videbimus, omnes perfectiones praecedentibus generibus conferri queunt, nihil aliud lucraremur, nisi ut telescopia multo fierent longiora, et numerum lentium sine ullo usu multiplicaremus, ut taceam iacturam insignem radiorum lucidorum, quae ob tot lentes merito esset metuenda; atque hanc ob rationem non dubito genus hoc quartum penitus reiicere, de quo etiam nullum supererit dubium, quando tria praecedentia genera ita pertractaverimus, ut omnibus momentis, quibus perfectio telescopiorum innititur, satisfecerimus. Multo magis autem sequentia, quae constitui possent genera, nullam plane attentionem merebuntur.