CHAPTER IV

TELESCOPES OF THE FIRST KIND ARE WITHOUT REAL IMAGES AND REPRESENT OBJECTS UPRIGHT

PROBLEM 1

III. *If a telescope if the first kind may be composed from two lenses only, clearly the objective and the eyepiece, to establish its construction and to explain its properties.*

SOLUTION

Since here the quantity $\frac{\alpha}{b}$ shall be negative and $\alpha + b$ positive, if the ratio of the multiplication may be put $= m$, since $m > 1$ the distance α, as we have seen before, must be positive, truly the other distance b shall be negative, so that there shall become $b = \frac{-\alpha}{m}$, or with the focal distances introduced, $\alpha = p$ and $b = q = \frac{-p}{m}$, and the separation of the two lenses $\alpha + b = (\frac{m-1}{m})p$, from which it is apparent everything to be determined from the given multiplication m and the focal length p. Truly this distance p must be so great, that the objective lens may be allowed a given aperture, so that if the order of the clarity [i.e. *Ff*] may be put $= y$, then the radius must be $x = my$, from which now it may be apparent the focal length p must be greater than $4my$ or $5my$; from which, since y may be accustomed to be given in parts of a digit [roughly an old French inch] or $y = \frac{1}{50}$ dig. to be used, there becomes $x = \frac{m}{50}$, and $p > \frac{m}{10}$ dig.

Truly here the equation must be considered for the radius of the confusion [see Ch. I of this book, §. 42], which gives

$$\frac{mx^3}{4p^3}\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right) = \frac{1}{4k^3},$$

from which there is deduced:

$$p = kx\sqrt[3]{m\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right)} = kmy\sqrt[3]{m\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right)},$$

therefore which value, unless perhaps it may be less than $5my$, must be attribute to p itself, where, as we have observed above, the number k can be put equal to either 80, 40 or 50, as a greater or lesser order of clarity may be desired; and now from the given values λ and λ' since μ and μ' will depend on the kind of glass, both lenses will be able to be made and hence the whole telescope can be put together. For the properties requiring to be known we may seek initially the position of the eye or its distance from the ocular lens, and we come upon :

$$O=\frac{m-1}{m}\cdot q$$

(§.30), which since because $q<0$ shall be negative, it will be required to apply the eye next to the lens; from which the radius of the field of view is gathered, from §.37

$$\Phi=\frac{-\pi}{m-1} \text{ and } \pi=\frac{+\omega}{q},$$

with ω denoting the radius of the pupil [where the refraction due to the cornea has not been considered, making the pupil smaller than its true size] ; whereby on account of $q=\frac{-p}{m}$ there will become

$$\Phi=\frac{+m}{m-1}\cdot\frac{\omega}{p},$$

where it is to be noted initially the eye lens must be taken so large, that it may admit the aperture, the radius of which shall be $=\pi q=\omega$; from which it is necessary, that there may become $-q>4\omega$ or 5ω and hence also $p>4m\omega$ or $>5m\omega$, which condition now may be included in the first on account of $y<\omega$. So that finally it may be extended to the other source of confusion, since it shall require the destruction of the colored fringe, so that there shall be (§. 52)

$$0=\frac{dn}{n-1}B\pi\cdot p,$$

as since it may not be able to happen, unless the objective lens were perfect, it is evident the colored fringe cannot be removed. Finally for this confusion required to be removed completely there must become :

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot q \text{ or } \frac{dn}{n-1}=\frac{1}{m}\cdot\frac{dn'}{n'-1},$$

thus for this case, so that the objective lens is perfect, it will be unable to be satisfied on account of the first term vanishing; but since m is a large enough number, the other term by itself may become small enough, so that this confusion shall not require to be a source of concern.

COROLLARY 1

112. Since the focal length p must be greater than $5m\omega$, for the given magnification m the length of this telescope will be greater always than $5(m-1)\omega$, and, since there will be approximately $\omega=\frac{1}{20}$ dig., this length of the telescope cannot be made less than $\frac{m-1}{4}$ dig., evidently if we wish, so that there may become $m=50$, the length of the telescope cannot be less than $12\frac{1}{4}$ dig., even if the formula

$$p = mky\sqrt[3]{m\mu\lambda - \mu'\lambda'}$$

may be able to make it much smaller.

COROLLARY 2

113. For the apparent field of view $\Phi = \frac{m}{m-1}\cdot\frac{\omega}{p}$ we have found its radius, from which, since there shall be $p > 5m\omega$, the value of Φ certainly always will be less than $\frac{1}{5(m-1)}$, and in minutes of arc it will become $\Phi < \frac{687}{m-1}$ minutes of arc, which field of view we will be able to obtain easily, unless p were much greater than $5m\omega$.

COROLLARY 3

114. Since the colored margin cannot be removed, unless the objective lens were perfect, hence we understand at once, how great an improvement the use of perfect lenses shall be, as we have described above, thus so that from the benefit of these an order of perfection for telescopes may be able to be acquired.

SCHOLIUM

115. The solution of this problem thus is general, thus so that it may be extended to all kind so glass, from which the lenses can be made ; why not also in the place of the objective lens not only simple lenses, but also double or triple lenses and for that perfect lenses can be substituted; from which many kinds of telescopes can be shown, as we have seen composed from two lenses only, particular kinds of which we may consider in the following examples.

EXAMPLE 1

116. To define the construction of a telescope of this kind, if both lenses were simple and made from the same kind of glass.

For this case the equation arises to be considered mainly :

$$p = mky\sqrt[3]{\mu(m\lambda - \lambda')},$$

which determines the focal length of the first lens, if indeed the value hence being produced were greater than $5m\omega$. But we will see, and at once if the magnification m were noteworthy, its value is going to be much greater than that limit $5m\omega$ or $\frac{1}{4}m$ dig., thus so that this formula shall have a greater effect than if it were small, which can happen; whereby at once we may make $\lambda = 1$, so that the objective lens must be made according to §. 59; which truly extends to the eyepiece, it will not be agreed to put $\lambda' = 1$, but rather from that again a greater value may be given to this letter itself ; but in the first place, so that this lens may be made of the largest aperture, that optimally is rendered concave on both sides, from which the number $\lambda' = 1,62991$ (§.61) for that kind of glass, for which $n = 1,55$ and which artisans generally are accustomed to use. Moreover for the other kinds not so much will differ, that there shall be a need for a different ratio, especially since the letters k and y cannot be defined with so great care. Therefore we may take $y = \frac{1}{40}$ dig., so that we may obtain a great enough clarity, which is seen to be necessary for this kind, and $k = 40$, so that the confusion may be reduced small enough, and there will be, on account of $\lambda = 1$, $\lambda' = 1\frac{5}{8}$:

$$p = m\sqrt[3]{\mu(m - 1\tfrac{5}{8})}\ ,$$

from which it is apparent here these kinds of glass must be preferred, for which a greater refraction n will correspond, since then the letter μ obtains smaller values. But since always μ may not differ much from unity and its cubic root therefore will differ even much less , whatever kind of glass we wish to use, with care it will be possible to take

$$p = m\sqrt[3]{(m - 1\tfrac{5}{8})}\ ;$$

but in this case the surrounding colored margin cannot be removed. Whereby, if hence we will have duly defined the focal length of the objective lens and n will denote the refraction of the glass, from which both lenses shall be prepared, the construction of the telescope will be had in the following manner:

I. The objective lens is to be prepared from the principles set out in §.59.
II. Each side of the eyepiece lens may be prepared equally concave by taking the radius of each face $= \frac{-2(n-1)p}{m}$ on account of $q = \frac{-p}{m}$.
III. These two lenses may be joined for the separation $AB = \frac{m-1}{m} \cdot p$ and placed in a tube, so that the eye may be able to be applied next to the concave lens.

IV. Here the tube offers a field of view, the ray of which will be at

$$\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p} = \frac{3437m}{m-1} \cdot \frac{\omega}{p} \text{ min.}$$

V. This telescope cannot be freed from the fault of the colored margin.

COROLLARY 1

117. But if the magnification may be so great, so that there may become $m=1\frac{5}{8}$, the formula defining p vanishes; truly with nothing less to be taken $p=5m\omega$ or as if $\frac{1}{4}m$ dig., and it will be possible for this value to be used, even if m may be somewhat greater, provided that formula does not exceed $\frac{1}{4}m$ dig., which happens, provided m may not exceed the limit $1\frac{41}{64}$, but which scarcely exceeds the value $1\frac{5}{8}$; and from which it is apparent at once both the magnification m shall be greater than $1\frac{5}{8}$, and the focal length p must be taken greater than $\frac{1}{4}m$ dig.

COROLLARY 2

118. Whereby, for example if there must be $m=5$, it will be required to take $p=7\frac{1}{2}$ dig. and $q=-\frac{3}{2}$, so that the radius of the apparent field of view produced :

$$\Phi=\frac{5}{4}\cdot\frac{2\omega}{15}=\frac{\omega}{6}=\frac{1}{120} \text{ on account of } \omega=\frac{1}{20} \text{ or } \Phi=\frac{3437}{120}\text{ min.}=29\text{ min.}$$

Therefore the length of the telescope will be 6 dig.

COROLLARY 3

119. If the magnification may be desired $m=10$, there is found $p=5\sqrt[3]{67}=20\frac{5}{16}$ dig. and hence $q=-2\frac{1}{32}$ dig., thus so that the length of the telescope shall be $18\frac{9}{32}$ dig.; then truly the radius of the apparent field of view, which is $\frac{\omega}{p+q}$, shall become $\Phi=\frac{32\omega}{585}=\frac{8}{2925}$ and in minutes $\Phi=9'24''$, which field now is so small, so that in no way may it be tolerated, whereby these kinds of telescopes cannot be applied for a magnification $m=10$.

EXAMPLE 2

120. If both lenses may be prepared from the same kind of glass, truly the double lens objective is required to be constructed following §. 65, so that there shall be $\lambda=\frac{1-\nu}{4}$, and if we may use for the common glass, that for which $n=1{,}55$, there will be $\lambda=0{,}1918$; and again by taking unity for $\sqrt[3]{\mu}$ and by putting as before $\lambda'=1\frac{5}{8}$, so that the lens of the eyepiece may become equally concave as before, there will become

$$p=m\sqrt[3]{(0{,}1918m-1\tfrac{5}{8})}$$

and as before $q = \frac{-p}{m}$ and hence the separation of the lenses $= \frac{m-1}{m} p$; whereby, if thence the value of the letter *p* may be defined for the given magnification, the construction thus will be had :

I. The objective lens requiring to be prepared from the formula §. 65 for $n = 1,55$.
II. The eyepiece lens made equally concave on both sides, with the radius being

$$= -2(n-1) \cdot \frac{p}{m} \quad = -\frac{11}{10} \cdot \frac{p}{m}.$$

III. The radius of the apparent field of view will be as before

$$\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p} = \frac{172m}{m-1} \cdot \frac{1}{p} \text{ min.}$$

IV. Moreover as before in this case of the equal pair the remedy of the colored margin can be brought to bear.

COROLLARY 1

121. But when that formula provides $p < \frac{1}{4} m$ dig., with nothing smaller than $p = \frac{1}{4} m$ dig. must be put in place, which happens in the first place, if there shall be $m = 8\frac{1}{2}$ roughly; from which there arises $p = 0$; whereby, unless the magnification may be desired greater, it will be able to take $p = \frac{1}{4} m$ dig., from which there becomes $q = -\frac{1}{4}$ dig., the length of the telescope $\frac{1}{4}(m-1)$ dig. and the radius of the apparent field of view $\Phi = \frac{688}{m-1}$ min.

COROLLARY 2

122. But if therefore the magnitude proposed shall be $m = 8\frac{1}{2}$, thus there will be required to construct the telescope :

I. On account of $p = \frac{17}{8}$ dig. $= 2\frac{1}{8}$ dig. the objective lens may be prepared according to the given precepts.
II. On account of $q = -\frac{1}{4}$ dig., the radius of each face will be

$$= -\frac{1}{2}(n-1) \text{ dig.},$$

from which the length of the telescope becomes $=1\frac{7}{8}$ dig., truly the radius of the apparent field of view $=1^{\circ}\ 31'$. Which telescope is seen to be most noteworthy not withstanding the color fringe.

COROLLARY 3

123. if the magnification $m=15$ may be desired, $p=16,15$ dig. is found at once, hence $q=-1,07$ dig., from which the length of the telescope $=15,08$ dig., and the radius of the apparent field of view will be $=\frac{172}{15,08}$ min. $=11'\ 24''$, from which it is apparent this telescope both on account of the exceedingly small field of view as well as on account of the exceedingly great length deserves to be rejected, while the preceding case on the other hand is seen to be especially recommended .

EXAMPLE 3

124. If both lenses are made from the same kind of glass, truly a triple objective may be required to be constructed following §. 66, so that there shall be $\lambda = \frac{3-8\nu}{3\cdot 9}$, to define the construction of this telescope.

We may use for the common glass, according to which $n=1,55$, and there will be $\lambda=0,0422$, and there may remain $\lambda'=1,629$; with unity taken again for $\sqrt[3]{\mu}$, there will be

$$p\ =m\sqrt[3]{(0,0422m-1,629)},$$

from which the remaining are determined, as in the preceding cases.

But initially this case deserves to be noted, so that there becomes $0,0422m=1,629$ or $m=38\frac{3}{5}$ for which the focal length of the objective lens must be taken $p=9\frac{13}{20}$ dig. with the remaining $q=-\frac{1}{4}$ dig. and hence the length of the tube $=9\frac{2}{5}$ dig., from which the radius of the field will be $=18'\ 17''$, indeed which field is small enough, but it can readily be tolerated on account of so notable a magnification, unless perhaps the colored margin may offend.

EXAMPLE 4

125. To describe the construction of the telescope, if a perfect lens may be taken for the objective, but the lens of the eyepiece may remain simple, and thus each side equally concave.

Since we have described above perfect lenses of this kind being put together partially from crown glass, and partially from crystal glass, here before everything it is required to consider, how great the size of any aperture shall be ; since for this magnification *m* there must be $x=\frac{m}{40}$ dig , it is required to be seen before all else, whether a perfect lens here being used will allow so great an aperture ; which may be required to be observed with great care, if the value of p in a certain case may appear $=0$, so that as before there must be taken $p=\frac{1}{4}m$, thus so that there may become $x=\frac{1}{10}p$, which may have a place in the third triple lens only. Truly there is no need, so that from this likewise to be concerned, since from the above root for this case $p=0$ can never be produced ; for since the lens is perfect, by the hypothesis there will be $\lambda=0$, thus so that there may become

$$p=m\sqrt[3]{-1,629},$$

from which it is apparent always to become $p>m$, evidently $p=1,17m$;whereby at once this conspicuous inconvenience follow, and so that soon a moderate value may be given for the magnification *m*, as a small value shall be produced for the apparent field of view, so that the telescope may be almost without any use ; the cause of which since the value shall be $\lambda=0$, may be required to be chosen here, so that a perfect lens even now will provide a certain small confusion, as that formula may provide $p=0$ for some magnification. But following the precepts given above such lenses are not difficult to find, which, while they may not produce any dispersion, yet they will produce some confusion ; truly an investigation of this kind may be put in place for these telescopes with one or two new lenses being added.

SCHOLIUM

126. The account of this conspicuous paradox, that here perfect lenses may not perform as well as the preceding double or triple lenses, evidently with this put in place, so that here we may not need an objective lens of this kind for which there shall be $\lambda=0$, but rather, so that $\lambda m-\lambda'$ may be able to be reduced to zero. Moreover it was easy above to find composite lenses of this kind, which, while they might remedy the confusion of the colors, for the first confusion the numbers λ had a given value for the first kind of confusion; truly here there is no need, that we may repeat that labour, but rather it will be convenient to accommodate this investigation for the present situation by another method; evidently two or more lenses, which will constitute one perfect lens, but we will consider here as if separate lenses; with which agreed on we will follow that suitably, so that not only each confusion lens but also the eyepiece to be taken completely into the calculation, but also perhaps the apparent field of view may be able to be extended further; which in the end will require the following problems to be presented.

PROBLEM 2

127. *To insert an additional lens between the objective and the eyepiece, so that it may remain a telescope required to be added to the telescopes of the first kind..*

SOLUTION

Therefore we may put the telescope to be constructed from the three lenses *PP, QQ, RR,* and indeed at first it is required, that these fractions $\frac{\alpha}{b}$, $\frac{\beta}{c}$ shall be negative, then truly, so that these intervals $\alpha+b$, $\beta+c$ shall be positive, with the magnification being $m=\frac{\alpha}{b}\cdot\frac{\beta}{c}$ or $m=\frac{\alpha}{c}\cdot B$ on account of $B=\frac{\beta}{b}$, which magnitude therefore will be positive. Now we may introduce other elements, which are retained by the above letters *B, C* and with the indices of the apertures π, π' with the radius of the field Φ, and for the first condition we will have

$$\frac{\alpha}{b}=\frac{\mathfrak{B}\pi-\Phi}{\Phi}<0,\ \frac{\beta}{c}=\frac{\mathfrak{C}\pi'-\pi+\Phi}{\mathfrak{B}\pi-\Phi}<0,$$

from which, since Φ by its nature shall always be positive, $\mathfrak{B}\pi-\Phi$ must be negative, but truly $\mathfrak{C}\pi'-\pi+\Phi$ positive; and because $\gamma=\infty$, thus $C=\infty$ and $\mathfrak{C}=1$, from which the latter condition gives $\pi'-\pi+\Phi>0$. But for the apparent field we find $\Phi=\frac{-\pi+\pi'}{m-1}$, from which, since Φ and $m-1$ shall be positive quantities, $-\pi+\pi'$ must be a positive quantity, by which preceding the condition also may be maintained.

So that as well as the separation of the lenses may become positive, from §. 16 we come upon these two conditions :

1. $\frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi-\Phi}>0$, from which since the denominator shall be negative, also the numerator must be negative or $\mathfrak{B}\pi p<0$; therefore just as the quantity p were positive or negative, thus $\mathfrak{B}\pi$ must be negative or positive.

2. $\frac{\mathfrak{B}\Phi p(\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\pi'-\pi+\Phi)}>0$, where, since Φ shall be positive, truly the whole denominator negative, also for the numerator $\mathfrak{B}p(\pi'-(1-\mathfrak{B})\pi)$ must be <0.

Therefore from these conditions, if we may substitute the value found in place of Φ, we may come upon the following conclusions :

1. $\pi'-\pi>0$
2. on account of $\mathfrak{B}\pi-\Phi<0$ there must become

$$(m-1)\mathfrak{B}\pi-\pi'+\pi<0 \text{ that is}$$
$$\pi'-\pi>(m-1)\mathfrak{B}\pi \text{ or } \pi'>((m-1)\mathfrak{B}+1)\pi$$

3. $\mathfrak{B}\pi p<0$
4. $Bp(\pi'-(1-\mathfrak{B})\pi)<0$;

since here therefore the formulas 3 and 4 are both negative, the latter divided by the former gives :

$$\frac{B(\pi'-(1-\mathfrak{B})\pi)}{\mathfrak{B}\pi} > 0;$$

from which, if the denominator were positive, the numerator also must be positive, and vice versa likewise.

Now we will consider both the extreme cases, the one, where the middle lens of the objective lens is united, the other, where that of the eyepiece lens is united. In the first case, where clearly $\alpha + b = 0$, there becomes $\pi = 0$, just as now we have noted above for any number of lenses coalescing with the objective lens. In the latter case, where $\beta + c = 0$, there must be $\pi' - (1-\mathfrak{B})\pi = 0$ or $\pi' = (1-\mathfrak{B})\pi$, which value in the above second position will give $m\mathfrak{B}\pi < 0$ or $\mathfrak{B}\pi < 0$. But since the apparent field of view may depend chiefly on the eyepiece lens, to which the letter π' will correspond, this letter π by necessity is positive; whereby, so that the field may not be diminished, but rather increased, the number π will be required to be negative, from which the above conditions are better suited to be defined in this manner :

1st clearly $\pi' - \pi$ now at once shall become > 0 and thus can be omitted,
2nd there is $\pi' > ((m-1)\mathfrak{B}+1)\pi$, moreover from 3rd there follows $\mathfrak{B}p > 0$,
and 4th $\frac{B(\pi'-(1-\mathfrak{B})\pi)}{\mathfrak{B}\pi} > 0$.

At this point the position of the eye may be considered, the distance of which from the eyepiece becomes $O = \frac{\pi'}{m\Phi} r$, which will become positive on account of the positive $\frac{\pi'}{m\Phi}$, but only if r shall be positive; but since there shall be $r = c$ on account of $C = \infty$ and $\mathfrak{C} = 1$, there will be $r = \frac{Bp\Phi}{\pi'-\pi+\Phi}$; of which the denominator since it shall be positive, is required to be examined, whether Bp shall be positive or negative; but if Bp shall be positive, the distance O also will become positive, while if Bp were negative, the distance O also would be negative, and the eye must be applied next to the third lens, in which case the above given precepts are required to be observed.

COROLLARY 1

128. Since we have at once the multiplier m to be a moderate quantity, Φ is much less than π, and since $\mathfrak{B}\pi - \Phi$ shall be negative, the quantity $\mathfrak{B}\pi$ also must become negative, on account of $\pi < 0$, $\mathfrak{B}$ must be positive. Hence for the third condition $\mathfrak{B}\pi p < 0$, p will become positive (evidently with the case excepted, where π may have a minimum value and thus p also may become negative), and by the third and fourth conditions jointly on account of the negative denominator the numerator $B(\pi' - \pi + \mathfrak{B}\pi)$ also must be negative; therefore if there were $\pi' - \pi + \mathfrak{B}\pi > 0$, there will become $B < 0$, on the other hand truly $B > 0$.

COROLLARY 2

129. Therefore these conditions are able to be fulfilled in several ways, while several elements remain undetermined; for it is apparent at once the quantity α or p can take a positive as well as a negative value; and since $\mathfrak{B}p > 0$ on account of $\pi < 0$, if we may put p to be positive, $\mathfrak{B}$ also must be positive ; but if p may be taken negative, $\mathfrak{B}$ also must be taken negative; yet meanwhile, since $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$, even if $\mathfrak{B}$ shall be positive, the letter B now also can be both positive as well as negative; truly in the other case, where $\mathfrak{B}$ is negative, B shall always be negative also.

SCHOLIUM 1

130. In the same manner, so that we have resolved this problem, the conditions also can be found, when two or more lenses are inserted between the objective and the eyepiece or when a telescope of this kind has been made from four or more lenses; for we may put four lenses in place to construct it and the six following conditions are required to be fulfilled :

$$1.\ \tfrac{\alpha}{b} < 0, \qquad 2.\ \tfrac{\beta}{c} < 0, \qquad 3.\ \tfrac{\gamma}{d} < 0,$$

$$4.\ \alpha + b > 0,\ \ 5.\ \beta + c > 0,\ \ 6.\ \gamma + d > 0,$$

with $\delta = \infty$ and thus $D = \infty$ and $\mathfrak{D} = 1$; [Thus the eye itself is the final lens, considered in the relaxed condition, focused at infinity with the maximum aperture]; from which, if in place of these letters the above given values may be introduced, these six conditions give rise to the following formulas, in which Φ is put as positive always:

$$1.\ \mathfrak{B}\pi - \Phi < 0,\ \ 2.\ \tfrac{\mathfrak{C}\pi' - \pi + \Phi}{\mathfrak{B}\pi - \Phi} < 0, \quad 3.\ \tfrac{\pi'' - \pi' + \pi - \Phi}{\mathfrak{C}\pi' - \pi + \Phi} < 0,$$

which three conditions thus may be referred to more conveniently :

$$1.\ \mathfrak{B}\pi - \Phi < 0,\ \ 2.\ \mathfrak{C}\pi' - \pi + \Phi > 0, \quad 3.\ \pi'' - \pi' + \pi - \Phi < 0.$$

For the three remaining conditions, since they are negative in the individual denominators, it is required also for the numerators to be negative, from which the following conditions are fulfilled :

$$4.\ \mathfrak{B}\pi p < 0,$$
$$5.\ Bp(\mathfrak{C}\pi' - (1 - \mathfrak{B})\pi) < 0,$$
$$6.\ BCp(\pi'' - (1 - \mathfrak{C})\pi') < 0,$$

which, just as if p were either positive or negative, will have to be considered in two ways; but in this concern in the first place the expression will be required to be considered for the apparent field of view, which is $\Phi = \frac{-\pi+\pi'-\pi''}{m-1}$; which since it is desired to be as great as it can be able to become, it is required to be concerned, so that the fractions π and π'' may obtain negative values and the greatest of these, which still cannot exceed $\frac{1}{4}$ or $\frac{1}{6}$, and if perhaps this cannot happen and one or the other must be positive, then it will be required to effect that this shall be as small as possible.

SCHOLIUM 2

131. We may observe from these premises, on account of the above inconvenience, how perfect lenses may be taken to supply a remedy for the useless telescopes of this kind. Therefore we will consider the telescope as composed from three lenses and at once we unite the first two, so that the interval $\alpha + b$ may vanish and thus the objective lens becomes a doublet; now indeed we may define the individual elements thus, so that not only may the confusion be removed for a single objective, but also for the whole telescope. Truly since there is a need for two kinds of glass for this, we are forced to use English glass of these two kinds, evidently crown and crystal. From which two main problems arise, just as the first lens were going to be prepared from crown and the second from crystal, or on the other hand the first from crystal and the second from crown ; but likewise almost in the same way concerning the third eyepiece lens, we wish to make either from crystal or crown glass, provided that each may be returned equally concave, whenever that in this way admits the maximum aperture, on which the apparent field depends.

PROBLEM 3

132. *If the objective lens of the telescope shall be a doublet and the former prepared from crown glass and the latter from crystal glass, moreover the eyepiece lens also from crystal glass, to describe the construction of this telescope for some magnification m.*

SOLUTION

Therefore since here there shall be $\alpha + b = 0$ or $\alpha = -b$ and $\frac{\alpha}{b} = -1$, the magnification will be $m = -\frac{\beta}{c}$ or $c = -\frac{\beta}{m}$, where the letter β expresses the focal length of the objective lens doublet and thus, so that it is apparent from problem 1, it must be positive ; from which the eyepiece lens will be concave. Therefore since there shall be $b = \frac{\beta}{B}$, $q = \mathfrak{B}b = \frac{\mathfrak{B}\beta}{B} = \frac{\beta}{B+1}$, there will be $\alpha = \frac{-\beta}{B}$, and the letters μ and ν together with μ'' from the refraction $n = 1,53$, truly the letters μ' and ν' from the refraction $n = 1,58$ are required to be taken; from which we will have this equation for the confusion from the aperture of the lens requiring to be destroyed:

$$\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\frac{\mu\lambda''}{mB^3}-\frac{\mu'\lambda'}{\mathfrak{B}B}=0.$$

But since there shall be $\frac{dn}{n-1}:\frac{dn'}{n'-1}=7:10$ and $n''=n$, on account of the value of the negative distance O for the colored margin requiring to be removed we arrive at this equation:

$$\pi'(3B+10)=10\pi;$$

then truly for this confusion being removed completely it will be required to satisfy this equation:

$$0=-7+\frac{10(B+1)}{B}-\frac{7}{mB} \text{ or } 0=-7B+10(B+1)-\frac{7}{m};$$

from which there is found $B=\frac{7-10m}{3m}$, $\mathfrak{B}=\frac{10m-7}{7m-7}$, from which the letter B is determined perfectly, thus so that from the first equation only the letters λ and λ' will remain to be defined, since on account of the eyepiece lens each side now is defined equal to λ''. Therefore thence the number λ' is defined most conveniently :

$$\lambda'=\frac{\mu\mathfrak{B}^3\lambda}{\mu'}+\frac{\mu\mathfrak{B}^3\lambda''}{m\mu'B^3}-\frac{\nu'\mathfrak{B}^2}{B},$$

indeed in which λ may be able to be taken as it pleases; but lest λ' may be exceedingly greater than one, it will be convenient to take $\lambda=1$, and thus now everything will have been determined, thus so that nothing further may remain, which may be able to be determined from the middle equation, since the ratio of the letters π and π' now are given from the premises. For since there shall be

$$b=\frac{\beta}{B}=\frac{p\Phi}{\mathfrak{B}\pi-\Phi}=\frac{-\beta\Phi}{B(\mathfrak{B}\pi-\Phi)}$$

and hence $\pi=0$, and since for the apparent field of view there shall be $\Phi=\frac{-\pi+\pi'}{m-1}$, there will be $\pi'=(m-1)\Phi$, from which the following equation is produced :

$$0=(m-1)\Phi(3B+10);$$

so that since there can be no other case apart from $3B+10=0$ or $\frac{7-10m}{m}+10=0$ and hence $m=\infty$, the colored fringe cannot be removed, unless the magnification shall be a maximum and thus it will not respond to the greater magnifications ; for which reason it may be agreed this telescope to be adapted, the colored margin will not be a concern and it will suffice, if we will have satisfied the first and third equations. Therefore with the quantities B, λ and λ' found for the given magnification m the order of clarity y may be assumed, as we would wish to be held, and thence the radius of the aperture of the first lens x will be found . If then the focal length of the whole objective lens, which is equal to β, as we may observe to be indefinite, thence we will have the focal length of the prior lens $\alpha = \frac{-\beta}{B}$; and for the posterior lens the determinable distances $b = \frac{\beta}{B}$ and β , from which we will be able to construct each lens with the numbers λ and λ' ; in which construction the minimum radius either of convexity or of concavity may be observed and its fifth or fourth part is equal to $x = my$; from which the quantity β will be determined. Hence again we deduce the focal length of the eyepiece lens $= c = \frac{-\beta}{m}$; from which if equal figures may be attributed to each side of this lens, so that evidently it may become large of the greatest aperture, its radius of curvature will be $= -\frac{2(n-1)\beta}{m}$, as we have shown now in § 61, where also we have found for this lens to become $\sqrt{(\lambda''-1)} = \frac{\sigma-\rho}{2\tau}$, from which the value of λ'' itself if defined.

COROLLARY 1

133. Since here the distance of the eye O after the final lens may become negative and thus the eye must be applied directly to this lens, in the formula by declaring the apparent field $\Phi = \frac{-\pi+\pi'}{m-1}$ the fraction π' must be assumed $= \frac{\omega}{c}$, so that clearly the field we may find, that we may observe at a glance ; but the aperture of this lens can be arranged only so great, according to how great a curvature of the face is allowed, and thus nothing stands in the way , by which a smaller value of π' may be given $= \frac{1}{4}$ or $= \frac{1}{5}$.

COROLLARY 2

134. Since here we have observed from the value of the final letters π, π', π'' etc., it is more widely apparent, so that evidently to that the value $\frac{1}{4}$ or $\frac{1}{5}$ shall be attributed, while in the computation of the apparent field its value may be reduced to $\frac{\omega}{c}$, if indeed this were smaller; certainly it may be seen at a glance in what way the field is defined. But when the aperture of the eyepiece lens were greater than the pupil, then that pupil may be considered as if by travelling successively through the whole field, which defines the true value of π' itself, and thus in the following we will be able to omit this limitation demanded of the pupil, provided it may be observed that in the case, where π' is greater than $\frac{\omega}{c}$, this field will not be evident at a single glance.

COROLLARY 3

135. Therefore with this agreed we obtain a telescope of the first kind, so that it will present the objective without any confusion either from the aperture of the lens or arising from different kinds of rays, thus so that in that nothing more is to be desired, except that

the apparent field of view shall be exceedingly small ; with which defect all the telescopes both of the Newtonian and Gregorian kinds are troubled equally.

SCHOLIUM 1

136. If we wish to apply this in practice, in the beginning a value will be required from the letter B , which the third equation may supply, evidently $B = \frac{7-10m}{3m}$, which will become, once m shall be a moderately large number, $B = -\frac{10}{3}$; but since this value $-\frac{7}{3}$ has been found from Dollond's experiments, from which we may deduce the ratio $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 7:10$, certainly nobody will be going to consider this ratio to correspond exactly to the truth, so that it shall be possible to differ from that to some extend ; as on this account plainly it would be a cause of ridicule, if we wished to be exceedingly careful about the value of this letter B ; nor also may it be seen that so much precision has been removed from this, since now it shall be considered to be most outstanding, so that this kind of confusion, which until now it is believed unable to be diminished in any way, will be able to be minimized for the most part, even if it will not have been reduced completely to zero ; therefore we are going boldly to put in place $B = -\frac{10}{3}$ for any magnification and thence there remains only, that the formula found for λ' may be set out ; in which generally nothing will be ignored, since, as we have shown above now, only the term $\frac{\mu\mathfrak{B}\lambda''}{m\mu' B^3}$ was so significant, so that one would not be able to hope for the completion of a perfect objective lens.

SCHOLIUM 2

137. Since in the following it will be of the greatest concern, that a shape of this kind may be given to the eyepiece lenses, which shall be capable of the largest aperture, and this evidently may arise, if both the faces of this lens may be returned equal; for a lens of this kind the value of the letter λ may be defined thus, so that there may become $\sqrt{\lambda - 1} = \frac{\sigma-\rho}{2\tau}$, which therefore we may show here for special kinds of glass :

n	$\sqrt{\lambda - 1}$	λ
1,53	0,77464	1,60006
1,55	0,79367	1,62991
1,58	0,82125	1,67445.

[Recall here also from § 55, Ch. 1, Book I, that the following quantities are defined with respect of the refractive index n of the lens:

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}, \quad \rho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1,$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}, \qquad \tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}.$$

The length λ is introduced initially in § 52 Ch. 1, Book I in the definition of the diffusion length. We should observe also that the refractive index of any kind of glass depends on the ratio of its ingredients; thus for example crystal glass depended on the proportion of lead oxide added in place of calcium oxide for soda glass, and so was quite variable from around 1.6 to 1.8 ; it is therefore naïve to set out tables for one particular refractive index of any kind of glass due to its variability. The refractive indices of the lenses used need to be measured independently.]

Therefore since now we may have the value $\lambda'' = 1,60006$, through that, which may be put in place in the problem we will have $\mu = 0,9875$, $\mu' = 0,8724$, $\nu' = 0,2529$; on taking $B = -\frac{10}{3}$ and $\mathfrak{B} = +\frac{10}{3}$ the first equation requiring to be resolved will adopt this form :

$$\lambda' = 3,3001\lambda - \frac{0,1426}{m} + 0,1548,$$

from which, lest besides by necessity the value of λ' may not produce an exceedingly large, we may put $\lambda = 1$ and there will become

$$\lambda' = 3,4549 - \frac{0,1426}{m},$$

the use of which equation we will show in some examples.

EXAMPLE 1

138. To construct a telescope of this kind, so that an object may be increased twenty five times; or there shall be $m = 25$.
Since there shall be

$\lambda = 1$, there will be $\lambda' = 3,4492$, $\lambda' - 1 = 2,4492$, and $\sqrt{(\lambda' - 1)} = 1,5649$;

and hence the construction of the following individual lenses may be deduced:

I. For the first lens made from crown glass on account of its focal length $p = \alpha = +\frac{3\beta}{10}$ and $\sqrt{(\lambda - 1)} = 0$:

the radius of the $\begin{cases}\text{anterior face will become} = 0,1807\beta \\ \text{posterior face will become} = 1,3233\beta.\end{cases}$

II. For the second lens made from crystal glass, since the determinable distances shall become $b=\frac{\beta}{B}=-\frac{3}{10}\beta$ and the letters

$$\rho=0,1414,\ \sigma=1,5827,\ \tau=0,8775 \text{ and } \sqrt{(\lambda'-1)}=1,5649,$$

if we may put the letters F and G for the radiii of the anterior and posterior faces, we will have

$$F=\frac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}},\qquad G=\frac{b\beta}{\rho\beta+\sigma b\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$$

and hence

$$\frac{1}{F}=\frac{3\sigma-10\rho\mp7\tau\sqrt{(\lambda'-1)}}{3\beta},\qquad \frac{1}{G}=\frac{3\rho-10\sigma\pm7\tau\sqrt{(\lambda'-1)}}{3\beta}$$

with which expanded there is produced:

$$\frac{1}{F}=\frac{3,3341\mp9,6124}{3\beta},\qquad \frac{1}{G}=\frac{-15,4028\pm9,6124}{3\beta}$$

therefore so that the radii may not become exceedingly small, it will be required to use the upper signs, from which we will obtain

$$\frac{1}{F}=\frac{-6,2783}{3\beta},\quad F=-0,4779\beta,\quad \frac{1}{G}=\frac{-5,7904}{3\beta},\quad G=-0,5181\beta.$$

III. For the third eyepiece lens requiring to be made the construction is most simple, while the radius of each face must become

$$=2(n-1)r=+\ 1,06r=-0,0424\beta.$$

The two first lenses are themselves joined together in contact, so that as if they make a single lens, of which the radius of the aperture cannot be greater than around a quarter of the smallest radius, which is $=0,0452\beta$, and we will have $x=0,0452\beta$. But there must be $x=my$ with y denoting the degree of clarity of the image, and now we have indicated to put in place $y=\frac{1}{50}$ dig., thus so that in this case we may have $x=\frac{1}{2}$ dig., on account of which the value of β itself thus will be determined, so that there shall be $\beta=11,1$ dig.; at least β must not be taken less than this limit, from which the above measurements become known absolutely. But the radius of the apparent field of view on account of $\pi=0$ will be $\Phi=\frac{\pi'}{m-1}=\frac{\pi'}{24}$; on taking $\pi'=\frac{1}{4}$ it will become $35\frac{3}{4}$ minutes of arc, which field may discerned by the eye, if the radius of the pupil were $\pi'r=0,1110$. But when it is smaller, also so much less will be seen by the eye. Moreover, the length of this telescope will be $=10\frac{3}{4}$ digits [*i.e.* inches].

SCHOLION

139. Therefore this telescope is seen to be adapted well enough in practice, since its length shall be less than eleven inches and yet it may magnify an object twenty five times, with the apparent field present being no too small; and hence also apparent, and to what extent the perfect lens here must be owed to have changed, so that also the confusion arising from the eyepiece may be removed. Truly here it is required to be observed the construction of this instrument to require craftsmen of the highest skill and the smallest error introduced to render the whole work useless, whereby not except after several attempts can it be hoped to succeed. But the will be a need for much greater skill, if we may desire a greater magnification also, as will be evident from the following example.

EXAMPLE 2

140. To construct a telescope of this kind, so that it may multiply the object fifty times; or there shall be $m = 50$.

For this case there will be $\lambda' = 3,4521$ and $\sqrt{(\lambda'-1)} = 1,5659$, which value exceeds the preceding by $\frac{1}{1000}$, that is by its $\frac{1}{1565}$ part, thus so that the above formula $\sqrt{(\lambda'-1)}$ multiplied by $1+\frac{1}{1565}$ gives the true value, and since the remaining elements may remain as before, there will become :

$$\text{I. For the first lens, } \begin{cases} \text{the radius of the anterior face} = 0,1807\beta \\ \text{the radius of theposterior face} = 1,8233\beta. \end{cases}$$

II. For the second lens we will have

$$\frac{1}{F} = \frac{3,3341 \mp 9,6185}{3\beta}, \quad \frac{1}{G} = \frac{-15,4028 \pm 9,6185}{3\beta}$$

and with the upper signs taken we will have :

$$\frac{1}{F} = \frac{-6,2844}{3\beta}, \quad F = -0,4774\beta, \quad \frac{1}{G} = \frac{-5,7843}{3\beta}, \quad G = 0,5186\beta.$$

Which two lenses joined together allow an aperture x, of which the radius $= 0,0452\beta$; since clearly the value x cannot exceed $x = my = 1$ dig., whereby β will have to be taken greater than 22,1 dig.

III. For the eyepiece lens, of which the focal length is $=\frac{-\beta}{m}=\frac{-\beta}{50}$, the radius of each face will be $=-\frac{2(n-1)\beta}{50}=-0,0212\beta$. But on taking $\pi'=\frac{1}{4}$ the radius of its aperture will be $x=+\frac{\beta}{200}=0,110$ dig., from which the radius of the apparent field of view becomes $\Phi=\frac{\pi'}{m-1}=\frac{1}{196}$ or an angle of $\Phi=17\frac{1}{2}$ minutes of arc. Finally, the length of this telescope will be $=\beta+r=21,658$ or $21\frac{2}{3}$ dig. [~inches.]

SCHOLIUM

141. In this example the construction of the second lens scarcely differs from the preceding; from which it is apparent, as with care the measures found should be observed, so that it may respond to the forbidden effect, and to arise most easily, so that, which objective lens may be resolved of a certain given magnification, that may serve along with other magnifications ; whereby also however great a skill the craftsman may have used, the magnification, to which it agrees, must be investigated, while evidently more and still more lenses may be added successively to that; then indeed for a certain multiple it will happen, that the telescope may produce an outstanding manner of working; on this account we may refrain from the other case mentioned above, so that for the first objective lens the first will have to be prepared from the crystal glass, the second from the crown glass, since these which we have established, may be seen to suffice, and much more may serve to arrange or a triple objective lens to be used and that such the first and the third lens may be made of crystal glass and the middle lens to be made from crown glass, since now above the most suitable perfect lens has been produced.

PROBLEM 4

142. *If the objective lens of the telescope shall be a triplicate, the first and third lenses of which shall be made from crystal glass, and the middle lens truly from crown glass, moreover the eyepiece lens also from crown glass, to describe the construction of this lens, so that it may be without all the confusion.*

SOLUTION

Therefore this telescope will be constructed generally from four lenses, for which there will be $n=1,58$, $n'=1,53$, $n''=n$ and $n'''=n'$, and since the three first lenses must be coalesced into one lens, there will be $\alpha+b=0$ and $\beta+c=0$, or

$$\frac{\alpha}{b}=-1 \text{ and } \frac{\beta}{c}=-1$$

whereby since the magnification shall be $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, there will become

$$m=\frac{-\gamma}{d} \text{ or } d=\frac{-\gamma}{m};$$

truly the remaining letters will be able to be expressed in a similar manner by γ, clearly

$$c=\frac{\gamma}{C},\ \ \beta=\frac{-\gamma}{C},\ \ b=\frac{-\gamma}{BC},\ \text{ and }\ \alpha=\frac{\gamma}{BC},$$

from which the focal lengths arise

$$p=\frac{\gamma}{BC},\ \ q=\frac{-\mathfrak{B}\gamma}{BC},\ \ r=\frac{\mathfrak{C}\gamma}{C},\ \ s=\frac{-\gamma}{m}.$$

From which premises for the confusion arising from the apertures of the lenses requiring to be removed, we will have this equation:

$$\mu\lambda-\frac{\mu'}{\mathfrak{B}}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''}{\mathfrak{C}\mathfrak{B}^3}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''\lambda'''}{B^3C^3m}=0,$$

which on account of $\mu''=\mu,\ \nu''=\nu$ and $\mu'''=\mu'$, will have the expansion:

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\frac{\mu\lambda''}{\mathfrak{B}^3\mathfrak{C}^3}-\frac{\mu'\lambda'''}{B^3C^3m}-\frac{\mu'\nu'}{B\mathfrak{B}}+\frac{\mu\nu}{B^3C\mathfrak{C}}$$

Lest we may adhere to the ratio $7:10$ too much, which we have used before, we may put in general

$$\frac{dn}{n-1}=\zeta\ \text{ and }\ \frac{dn'}{n'-1}=\eta,$$

so that there shall be approximately $\zeta:\eta=10:7$; then, since in our case there becomes $\pi=0$ and $\pi'=0,$ we will have for removing the colored fringe

$$0=\zeta-\frac{\eta(B+1)}{B}+\frac{\zeta(C+1)}{BC}\ \text{ or }\ \zeta(1+C+BC)=\eta(B+1)C,$$

from which successively we will see how they may act together. The third equation provided the total destruction of this confusion for us, evidently :

$$0=BC+C+\frac{\zeta m-\eta}{(\zeta-\eta)m}.$$

For the sake of brevity there may be put :

$$\frac{\zeta m-\eta}{(\zeta-\eta)m}=\theta,$$

there will be

$$0=BC+C+\theta,$$

from which there is produced:

$$C=\frac{-\theta}{B+1}\ \text{ or }\ B=-1-\frac{\theta}{C}.$$

But since the second equation may be changed into this form:

$$BC+\mathrm{C}+\frac{\zeta}{\zeta-\eta}=0,$$

both cannot be satisfied at the same time, unless there shall be $\theta=\frac{\zeta}{\zeta-\eta}$, that is, unless there shall be $\frac{\zeta m-\eta}{m}=\zeta$, or $\zeta m-\zeta m=\eta=0m$ or $m=\infty$, just as in the preceding case. Therefore we may regress to our first equation, in which we must substitute the due value in place of *B* or *C*. But since the ratio $\zeta:\eta$ is not to be known exactly, it is allowed for approximate values to be taken ; in the end in the third equation we may ignore this term divided by *m* and we will have:

$$0=BC+\mathrm{C}+\frac{\zeta}{\zeta-\eta} \quad \text{or} \quad 0=BC+\mathrm{C}+\frac{10}{3}$$

and hence

$$C=\frac{-10}{3(B+1)} \quad \text{and} \quad C+1=\frac{3B-7}{3(B+1)} \ ;$$

with which substituted and with division made by $(B+1)^3$ there is produced

$$\begin{aligned}0=&-1000\mu\lambda\mathfrak{B}^3+\ 1000\mu'\lambda'+\ \mu\lambda''(10\mathfrak{B}-7)^3-\frac{27\mu\lambda'''}{m}\\&+1000\mu'\nu'\mathfrak{B}\left(1-\mathfrak{B}\right)-30\mu\nu(10\mathfrak{B}-7)(1-\mathfrak{B}),\end{aligned}$$

which on taking $\lambda''=\lambda$ there becomes a quadratic equation, from which $\mathfrak{B}$ is defined.

But from this equation we understand a substitution of this kind also in general to succeed ; for since there shall be $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$, on account of $B+1 \ =\frac{1}{1-\mathfrak{B}}$ there will become

$$C=-\theta(1-\mathfrak{B}) \quad \text{and} \quad C+1=1-\theta+\theta\mathfrak{B}$$

and hence

$$\mathfrak{C}=\frac{-\theta(1-\mathfrak{B})}{1-\theta+\theta\mathfrak{B}},$$

and with th first equation reduced to this form, if clearly it may be multiplied by $\mathfrak{B}^3$:

$$0=\mu\lambda\mathfrak{B}^3-\mu'\lambda'-\mu\lambda''\left(\mathfrak{B}-1+\frac{1}{\theta}\right)^3+\frac{\mu'\lambda'''}{m\theta^3}-\mu'\nu'\mathfrak{B}(1-\mathfrak{B})+\frac{\mu\nu(1-\mathfrak{B})(\mathfrak{B}-1+\frac{1}{\theta})}{\theta},$$

with there being $\theta=\frac{\zeta m-\eta}{(\zeta-\eta)m}$; and if there may be put $\theta=\frac{10}{3}$, the preceding equation is produced at once.

Therefore we may put $\lambda''=\lambda$, and the setting out of this equation will produce the following quadratic equation set out according to the powers of the letter $\mathfrak{B}$:

$$\mathfrak{B}^2\left(3\mu\lambda\left(1-\tfrac{1}{\theta}\right)+\mu'\nu'-\tfrac{\mu\nu}{\theta}\right)+\mathfrak{B}\left(-3\mu\lambda\left(1-\tfrac{1}{\theta}\right)^2-\mu'\nu'+\tfrac{\mu\nu}{\theta}\left(2-\tfrac{1}{\theta}\right)\right)$$
$$+\mu\lambda\left(1-\tfrac{1}{\theta}\right)^3-\mu'\lambda'+\tfrac{\mu'\lambda'''}{m\theta^3}-\mu\nu\left(1-\tfrac{1}{\theta}\right)=0,$$

from which $\mathfrak{B}$ must be defined.

Therefore now we may put $\theta=\frac{10}{3}$; then truly $\lambda=\lambda'=\lambda''=1$, and for the eyepiece lens there shall be $\lambda'''=1,60006$; then truly

$$\mu=0,8724 \quad \text{and} \quad \nu=0,2529, \quad \mu'=0,9875, \quad \nu'=0,2196;$$
from which there becomes
$$l.\mu\nu=9,3436645, \quad l.\mu'\nu'=9,3361694.$$
For the term $\mathfrak{B}^2$:
$$\mathfrak{B}^2\left(\tfrac{21}{10}\mu+\mu'\nu'-\tfrac{3}{10}\mu\nu\right)$$
$$\mathfrak{B}^2\begin{pmatrix}+1,83204- & 0,06618\\ +0,21685 & \end{pmatrix}$$
$$\begin{array}{r} 2,04889\\ 0,06618\\ \hline 1,98271\end{array}$$
$$+\ 1,98271\mathfrak{B}^2-1,38675\mathfrak{B}\ 0,84271\ +\tfrac{0,04266}{2}=0.$$

which divided by 1,98271 will become
$$\mathfrak{B}^2=0,69942\mathfrak{B}+0,42503-\tfrac{0,02152}{m},$$

the resolution of which gives
$$\mathfrak{B}=0,34971\pm\sqrt{\left(0,54736-\tfrac{0,02152}{m}\right)}$$
or
$$\mathfrak{B}=0,34971\pm\left(0,73983-\tfrac{0,01454}{m}\right),$$

from which the two values of $\mathfrak{B}$ will be

$$\text{I.)}\mathfrak{B}=1,08954\ -\tfrac{0,01454}{m} \quad \text{II.)}\mathfrak{B}=-\ 0,39012\ +\ \tfrac{0,01454}{m}.$$

COROLLARY 1

143. Therefore with the first three lenses joined together a triple objective lens exists, the focal length of which is equal to γ, from which it will be required to define the

individual radii of the faces, between which the smallest must be notes, which shall be $=i\gamma$ *;* the quarter part of which $=\frac{1}{4}i\gamma$ will give the radius of the aperture, that this same objective lens allows.

COROLLARY 2

144. Truly again from the given magnification *m* and from the degree of the clarity *y* the radius of the aperture of the objective lens is defined $x=my$ and that in digits on taking for example $y=\frac{1}{50}$ dig., from which this equation will be had $my=\frac{1}{4}i\gamma$, from which for the actual measurement there is deduced $\gamma=\frac{4my}{i}$.

COROLLARY 3

145. But since both sides of the eyepiece lens must be equally concave, so that there shall be $\lambda'''=1,60006$, its focal length shall be $=d=\frac{-\gamma}{m}$, from which the radius of each face must be put in place $=-\frac{2(n'-1)}{m}\gamma=\frac{-1,06}{m}\gamma$; of which the radius of the aperture can be taken four times smaller, so that there shall be $x=\frac{\gamma}{4m}$.

EXAMPLE 1

146. With the magnification put to be $m=25$, to construct a telescope of this kind for the first value found for the letter $\mathfrak{B}$.

Therefore since there shall be $m=25$, there will become $\mathfrak{B}=+1,08896$, from which it follows

$$B=\frac{\mathfrak{B}}{1-\mathfrak{B}}=-12,24100 \quad \text{and} \quad \log(-B)=1,0878169.$$

Again $C=-\theta(1-\mathfrak{B})=0,2965,$ and hence on account of $BC+C+\theta=0$ we gather that $BC=-C-\theta=+\theta-\theta\mathfrak{B}-\theta=-\theta\mathfrak{B}=-3,6298$ and $\mathfrak{C}=\frac{C}{C+1}=0,22869.$

Now since the radii of the first faces of the lenses shall be F and G, of the second F' and G' and of the third F'' and G''; on account of the determinable distances

$$a=\infty,\ \ b=\frac{-\gamma}{BC},\ \ c=\frac{\gamma}{c},\ \ \alpha=\frac{\gamma}{BC},\ \ \beta=\frac{-\gamma}{c},\ \ \gamma=\gamma$$

and the numbers $\lambda=1,\ \ \lambda'=1,\ \ \lambda''=1$ there will become

$$F=\frac{\alpha}{\sigma}=\frac{\gamma}{BC\sigma},\qquad G=\frac{\alpha}{\rho}=\frac{\gamma}{BC\rho},$$
$$F'=\frac{b\beta}{\rho'\beta+\sigma'b}=\frac{-\gamma}{BC\rho'+C\sigma'},\qquad G'=\frac{b\beta}{\rho'b+\sigma'\beta}=\frac{-\gamma}{BC\sigma'+C\rho'},$$
$$F''=\frac{c\gamma}{\rho\gamma+\sigma c}=\frac{\gamma}{C\rho+\sigma},\qquad G''=\frac{c\gamma}{\sigma\gamma+\rho c}=\frac{\gamma}{C\sigma+\rho}.$$

Therefore since there shall be $\rho = 0,1414$, $\sigma = 1,5827$ and $\rho' = 0,2267$, $\sigma' = 1,6607$ with the calculation put in place we will obtain:

$$\begin{aligned} F &= -0,1741\gamma, & G &= -1,9484\gamma, \\ F' &= +\ 3,0243\gamma, & G' &= +0,1678\gamma, \\ F'' &= +\ 0,6155\gamma, & G'' &= +1,6376\gamma. \end{aligned}$$

But for the eyepiece lens the radius of each face will be $= -0,0424\gamma$. Moreover the minimum radius between these is $0,1678\gamma$; of which the fourth part is $0,0419\gamma$, and if the equation $x = my = 25y = \frac{1}{2}$ dig., there will be produced $r = 12$ dig.

The length of the telescope will be $= \gamma\left(1 - \frac{1}{m}\right) = 11,52$ dig. and the radius of the apparent field of view on account of $\pi = 0$ and $\pi' = 0$ will become $\Phi = -\frac{\pi''}{m-1}$, and on taking $\pi'' = -\frac{1}{4}$ there will become $\Phi = \frac{1}{96}$ in parts of the radius, or $\Phi = 35\frac{3}{4}$ minutes of arc, which the eye may observe at a single glance, if the radius of the pupil shall be equal to the radius of the aperture of the eyepiece lens, that is

$$= \frac{1}{4}\frac{\gamma}{m} = \frac{\gamma}{100} = \frac{3}{25}\text{ dig.};$$

otherwise if the pupil were smaller, the field of view must be diminished in the same ratio.

EXAMPLE 2

147. With the magnification put to be $m = 50$, to construct a telescope of this kind from the first value of $\mathfrak{B}$.

Since there shall be $m = 50$, there will become $\mathfrak{B} = +1,08925$, from which it follows:

$$B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -12,2045 \text{ and } \log(-B) = 1,0865194.$$

Again,

$$C = \left[= \frac{-10}{3(B+1)}\right] = 0,2975 \text{ and } BC = -3,6308.$$

Therefore since the preceding formulas even now may find a place, the radii of the individual faces are found to be expressed thus :

$$\begin{aligned} F &= -0,1740\gamma, & G &= -1,9478\gamma, \\ F' &= +3,0375\gamma, & G' &= +0,1678\gamma, \\ F'' &= +0,6155\gamma, & G'' &= +1,6333\gamma. \end{aligned}$$

The minimum of these radii is $0,1678\gamma$, of which the fourth part $0,0419\gamma$ must be put equal to the radius of the aperture $x = my = 1$ dig., from which there may be defined $\gamma = \frac{1}{0,0419} = 23,87$ dig., thus so that there may be put in place $\gamma = 24$ dig. But then the focal length of the eyepiece lens $= \frac{-\gamma}{m} = \frac{-12}{25}$ dig. and the radius of each face is $1,06 \cdot \frac{12}{25} = 0,509$ dig.

Therefore the length of this telescope will be $= \gamma\left(1 - \frac{1}{m}\right) = 23,52$ dig. and the radius of the apparent field of view $\Phi = -\frac{\pi''}{49} = \frac{1}{196}$ and $\Phi = 17\frac{1}{2}$ minutes of arc.

SCHOLIUM

148. I do not pursue this calculation for greater magnifications, since so small a difference may be produced, so that it may be seen scarcely possible to be carried out by the artisans; whereby we may set out the same example also from the other value found for $\mathfrak{B}$.

EXAMPLE 3

149. With the magnification to be put $m = 25$ to construct a telescope of this kind from the latter value of $\mathfrak{B}$.
Since there shall be $m = 25$, there will be $\mathfrak{B} = -0,38954$ and $1 - \mathfrak{B} = 1,38954$, from which there becomes

$$B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -0,280337 \text{ and } \log(-B) = 9,4476805;$$

then there will become

$$C = -\theta(1 - \mathfrak{B}) = -\,4,6318 \text{ and } BC = +1,29850.$$

Therefore since the formulas for the radii of the faces remains as above, we will find these as follows :

$$F = +\,0,486641\gamma, \quad G = +5,44632\gamma,$$
$$F' = +\,0,13523\ \gamma, \quad G' = -0,90450\gamma,$$
$$F'' = +1,07785\ \gamma, \quad G'' = -0,13719\gamma.$$

Among these radii the minimum is $0,13523\gamma$, of which the fourth part $0,03381\gamma$ must be equal to the radius of the aperture $x = my = \frac{1}{2}$ dig., from which $\gamma = \frac{1}{0,06762} = 15$ dig., thus so that the length of the telescope $=\gamma(1-\frac{1}{m})=14\frac{2}{5}$ dig.; moreover the focal length of the eyepiece lens will be $=-\frac{3}{5}$ dig., thus so that the radius of each face $=0,6360$ dig., and the radius of the apparent field of view will be as above $\Phi=35\frac{3}{4}$ minutes of arc, which can be seen by the eye at a single glance or perhaps a few in succession.

EXAMPLE 4

150. With the magnification put to be $m=50$, to construct a telescope of this kind from the latter value of $\mathfrak{B}$.

Since there shall be $m=50$, there will become $\mathfrak{B}=-\ 0,38983$ and $1-\mathfrak{B}=1,38983$; from which there is gathered $C=-\ 4,6328$ and $BC=+\ 1,2995.$
Therefore, since the formulas for the radii of the faces may remain the same, from these with the calculation made we will obtain:

$$F\ =+0,48624\gamma,\ \ G\ =+5,44218\gamma,$$
$$F'=+0,13520\gamma,\ \ G'=-0,90338\gamma,$$
$$F''=+1,07802\gamma,\ \ G''=-0,13906\gamma.$$

Among these the smallest radius is $0,13520\gamma$, of which the fourth part $0,03380\gamma$ must be equal to the radius of the aperture $x=my=1$ dig., from which $\gamma=29$ dig. and the focal length of the eyepiece lens $=-0,58$ dig. and the radius of each face $=0,6148$ dig. Therefore the length of the telescope will be $=28,42$ dig. and the radius of the field of view $\Phi=17\frac{1}{2}$ min. of arc.

SCHOLIUM

151. Even if this telescope actually may be constructed from four lenses, yet this is possible to be considered as if made from two lenses only, because the first three lenses will be coalesced into one lens, so that the objective will become a triplicate lens and with better success in place of the three perfect lenses required to be used above, since now we have seen by these perfect lenses not only the confusion of each kind to be removed, but thus so that also the confusion of the eyepiece lens now may be removed completely, which is why we have constructed the triplicate lens called into use for the given need, as they may not be perfect, but so that from these the confusion of the eyepiece lens also may be reduced to zero, which if they may be able to be perfected most precisely by the artisan, nothing further may be seen to be desired. Truly besides these telescopes are overwhelmed with these two difficulties ; the one difficulty being, that with three lenses joined together, thus the thickness must be made moderate, so that it shall no longer be able to be considered as vanishing, just as our calculation demands; so that, even if our artisan may prevail to carry out our instructions most exactly, yet at no time will it be able to hope for perfect agreement between theory and practice; the

other difficulty is put in place by in the narrowness of the apparent field of view, and one is required to be selected especially, that may agree with a greater extent ; therefore so that we may deliberate over this two-fold inconvenience, in the following chapter we may pursue this investigation further, while actually we will attribute more than two lenses to telescopes of this kind, which all shall be separated from each other in turn by certain intervals; where in the first place it will be required to inquire into this, whether in this manner also the confusion of each kind may be able to be removed equally happily, then truly whether in this manner the apparent field of view may be able to be made greater, and if besides the length of these telescopes may be produced smaller; while certainly from these it may be agreed the greatest degree of perfection shall be required to be considered.

CAPUT IV

DE TELESCOPIIS PRIMI GENERIS QUAE SCILICET IMAGINE VERA DESTITUUNTUR ET OBIECTA SITU ERECTO REPRAESENTANT

PROBLEMA 1

1II. *Si telescopium primi generis ex duobus tantum lentibus constet, objectiva scilicet et oculari, eius constructionem evolvere et proprietates exponere.*

SOLUTIO

Cum hic sit $\frac{\alpha}{b}$ quantitas negativa et $\alpha + b$ positiva, si ratio multiplicationis ponatur $= m$, ob $m > 1$ distantia α, ut ante vidimus, debet esse positiva, altera vero b negativa, ut sit $b = \frac{-\alpha}{m}$ seu distantiis focalibus introductis $\alpha = p$ et $b = q = \frac{-p}{m}$ et intervallum binarum lentium $\alpha + b = (\frac{m-1}{m})p$, unde patet ex data multiplicatione m et distantia focali p omnia determinari. Verum haec distantia p tanta esse debet, ut lens obiectiva datam admittat aperturam, cuius, si claritatis gradus ponatur $= y$, semidiameter esse debet $x = my$, unde iam patet distantiam p maiorem esse debere quam $4my$ vel $5my$; unde, cum y in partibus digiti dari soleat vel uti $y = \frac{1}{50}$ dig., fit $x = \frac{m}{50}$, et $p > \frac{m}{10}$ dig .

Verum hic inprimis spectari debet aequatio pro semidiametro confusionis, quae dat

$$\frac{mx^3}{4p^3}\left(\mu\lambda-\frac{\mu'\lambda'}{m}\right)=\frac{1}{4k^3},$$

unde colligitur

$$p=kx\sqrt[3]{m\left(\mu\lambda-\frac{\mu'\lambda'}{m}\right)}=kmy\sqrt[3]{m\left(\mu\lambda-\frac{\mu'\lambda'}{m}\right)},$$

qui ergo valor, nisi forte minor sit quam $5my$, ipsi p tribui debet, ubi, ut supra notavimus, numerus k poni potest vel 80 vel 40 vel 50, prout maior vel minor distinctionis gradus desideratur; atque iam ex datis valoribus λ et λ' cum vitri specie, unde numeri μ et μ' pendent, ambae lentes construi hincque totum telescopium confici poterunt. Ad cuius proprietates cognoscendas quaeramus primo locum oculi eiusve distantiam a lente oculari invenimusque

$$O=\frac{m-1}{m}\cdot q$$

(§.30), quae cum ob $q<0$ sit negativa, oculum lenti oculari immediate applicari oportet; unde colligitur semidiameter campi ex §.37

$$\Phi=\frac{-\pi}{m-1}\ \text{et}\ \pi=\frac{+\omega}{q},$$

denotante ω semidiametrum pupillae; quare ob $q=\frac{-p}{m}$ fiet

$$\Phi=\frac{+m}{m-1}\cdot\frac{\omega}{p},$$

ubi inprimis notandum est lentem ocularem tantam sumi debere, ut aperturam admittat, cuius semidiameter sit $=\pi q=\omega$; ex quo necesse est, ut fiat $-q>4\omega$ vel 5ω hincque etiam $p>4m\omega$ vel $>5m\omega$, quae conditio iam in se complectitur primam ob $y<\omega$. Quod denique ad alteram confusionem attinet, cum destructio marginis colorati postulet, ut sit (§ 52)

$$0=\frac{dn}{n-1}B\pi\cdot p,$$

quod cum fieri nequeat, nisi lens obiectiva fuerit perfecta, evidens est marginem coloratum destrui non posse. Denique pro hac confusione penitus tollenda esse debet

$$0=\frac{dn}{n-1}\cdot p+\frac{dn'}{n'-1}\cdot q\ \text{sive}\ \frac{dn}{n-1}=\frac{1}{m}\cdot\frac{dn'}{n'-1},$$

cui casu adeo, quo lens obiectiva est perfecta, satisfieri nequit ob primum terminum evanescentem; quia autem m est numerus satis magnus, alterum membrum per se fit satis parvum, ut haec confusio non sit metuenda.

COROLLARIUM 1

112. Cum distantia focalis p maior esse debeat quam $5m\omega$, pro data multiplicatione m longitudo huius telescopii semper maior erit quam $5(m-1)\omega$, et, cum sit circiter $\omega = \frac{1}{20}$ dig., haec telescopii longitudo minor fieri non poterit quam $\frac{m-1}{4}$ dig., scilicet, si velimus, ut sit $m = 50$, longitudo telescopii minor esse nequit quam $12\frac{1}{4}$ dig., etiamsi formula

$$p = mky\sqrt[3]{m\mu\lambda - \mu'\lambda'}$$

multo minor reddi posset.

COROLLARIUM 2

113. Pro campo apparente invenimus eius semidiametrum $\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p}$, unde, cum sit $p > 5m\omega$, valor ipsius Φ semper certe minor erit quam $\frac{1}{5(m-1)}$, atque in minutis primis erit $\Phi < \frac{687}{m-1}$ minut., quo campo facile contenti esse possemus, nisi p deberet esse multo maius quam $5m\omega$.

COROLLARIUM 3

114. Quoniam margo coloratus tolli non potest, nisi lens obiectiva sit perfecta, hinc statim intelligimus, quanti sit momenti usus lentium perfectarum, quas supra descripsimus, ita ut earum beneficio his telescopiis insignis gradus perfectionis conciliari possit.

SCHOLION

115. Solutio huius problematis ita est generalis, ut ad omnes vitri species, ex quibus lentes parari possunt, pateat; quin etiam loco lentis obiectivae non solum lentes simplices, sed etiam duplicatae vel triplicatae atque ad eo perfectae substitui possunt; unde plurimae species huius telescopii, quod tantum ex duabus lentibus compositum spectamus, exhiberi possunt, quarum praecipuas in subiunctis exemplis contemplemur.

EXEMPLUM 1

116. Si ambae lentes fuerint simplices atque ex eadem vitri specie confectae, constructionem huius telescopii definire.

Pro hoc casu potissimum aequatio venit consideranda

$$p = mky\sqrt[3]{\mu(m\lambda - \lambda')},$$

quae distantiam focalem primae lentis determinat, siquidem valor hinc prodiens maior fuerit quam $5m\omega$. Videbimus autem, statim atque multiplicatio m fuerit notabilis, eius valorem multum esse superaturum istum limitem $5m\omega$ seu $\frac{1}{4}m$ dig., ita ut maximi sit momenti hanc formulam tam parvam reddere, quam fieri potest; quare statim faciamus $\lambda = 1$, ut lens obiectiva secundum § 59 elaborari debeat; quod vero ad lentem ocularem attinet, non convenit $\lambda' = 1$ ponere, sed potius e re erit ipsi huic litterae maiorem valorem tribuere; inprimis autem, ut haec lens maximae aperturae fiat capax, ea optime utrinque aeque concava redditur, ex quo numerus $\lambda' = 1,62991$ (§.61) pro ea vitri specie, qua $n = 1,55$ et qua artifices plerumque uti solent. Pro aliis autem speciebus tantum non differet, ut operae pretium sit differentiae rationem habere, praecipue cum litteras k et y tam adcurate definire non liceat. Sumamus ergo $y = \frac{1}{40}$ dig., ut satis magnam claritatem obtineamus, quae in hoc genere necessaria videtur, et $k = 40$, ut confusio satis reddatur exigua, eritque ob $\lambda = 1$, $\lambda' = 1\frac{5}{8}$:

$$p = m\sqrt[3]{\mu(m - 1\tfrac{5}{8})},$$

unde patet hic eas vitri species praeferri debere, quibus maior refractio n respondet, quia tum littera μ, minores nanciscitur valores. Cum autem perpetuo μ non multum differat ab unitate eiusque proptera radix cubica multo minus discrepet, quacunque vitri specie uti velimus, tuto sumere licebit

$$p = m\sqrt[3]{(m - 1\tfrac{5}{8})};$$

hoc autem casu circa marginem coloratum nihil efficere licet. Quare, si hinc distantiam focalem lentis obiectivae debite definiverimus atque n denotet refractionem vitri, ex quo ambae lentes sint parandae, constructio telescopii sequenti modo se habebit:

I. Lens obiectiva paranda est ex formulis §.59.
II. Lens ocularis utrinque aeque concava conficiatur sumendo radium utriusque faciei $= \frac{-2(n-1)p}{m}$ ob $q = \frac{-p}{m}$.
III. Hae duae lentes ad distantiam $AB = \frac{m-1}{m} \cdot p$ iungantur et tubo inserantur, ut oculus lenti concavae immediate adplicari possit.

IV. Hic tubus campum offeret, cuius semidiameter erit

$$\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p} = \frac{3437m}{m-1} \cdot \frac{\omega}{p} \text{ min.}$$

V. Hoc telescopium a vitio marginis colorati liberari nequit.

COROLLARIUM 1

117. Quodsi multiplicatio tanta sit, ut fiat $m=1\frac{5}{8}$, formula p definiens evanescit; nihilo vero minus sumi debet $p=5m\omega$ seu quasi $\frac{1}{4}m$ dig., hocque valore uti licet, etsi m aliquanto sit maius, dummodo illa formula non excedat $\frac{1}{4}m$ dig., quod evenit, quamdiu m non superat limitem $1\frac{41}{64}$, qui vix superat valorem $1\frac{5}{8}$; ex quo patet, statim atque multiplicatio m maior sit quam $1\frac{5}{8}$, distantiam focalem p maiorem capi debere quam $\frac{1}{4}m$ dig.

COROLLARIUM 2

118. Quare, si verbi gratia debeat esse $m=5$, capi oportet $p=7\frac{1}{2}$ dig. et $q=-\frac{3}{2}$, unde semidiameter campi apparentis prodit

$$\Phi=\frac{5}{4}\cdot\frac{2\omega}{15}=\frac{\omega}{6}=\frac{1}{120} \text{ ob } \omega=\frac{1}{20} \text{ sive } \Phi=\frac{3437}{120} \text{ min.} = 29 \text{ min.}$$

Longitudo autem telescopii erit 6 dig.

COROLLARIUM 3

119. Si multiplicatio desideretur $m=10$, reperitur $p=5\sqrt[3]{67}=20\frac{5}{16}$ dig. hincque $q=-2\frac{1}{32}$ dig., ita ut longitudo telescopii sit $18\frac{9}{32}$ dig.; tum vero semidiameter campi apparenis, quae est $\frac{\omega}{p+q}$, fit $\Phi=\frac{32\omega}{585}=\frac{8}{2925}$ et in minutis $\Phi=9'24''$, qui campus iam tam est exiguus, ut nullo modo tolerari possit, quare haec species telescopiorum ne quidem ad multiplicationem $m=10$ adplicari potest.

EXEMPLUM 2

120. Si ambae lentes ex eadem vitri specie parentur, obiectiva vero statuatur duplicata secundum §. 65 construenda, ut sit $\lambda=\frac{1-\nu}{4}$, ac si vitro communi, pro quo est n = 1,55, utamur, erit $\lambda=0{,}1918$; sumtaque iterum unitate pro $\sqrt[3]{\mu}$ et posito ut ante $\lambda'=1\frac{5}{8}$, ut lens ocularis fiat aequaliter concava, erit

$$p = m\sqrt[3]{(0,1918m - 1\tfrac{5}{8})}$$

et ut ante $q = \frac{-p}{m}$ hincque distantia lentium $= \frac{m-1}{m}p$; quare, si inde pro data multiplicatione definiatur valor litterae *p,* constructio ita se habebit:

I. Lens obiectiva paranda est ex formulis §. 65 pro $n = 1,55$.
II. Lens ocularis utrinque fiat aequaliter concava, radio existente

$$= -2(n-1)\cdot\frac{p}{m} \quad = -\frac{11}{10}\cdot\frac{p}{m}.$$

III. Semidiameter campi apparentis erit ut ante

$$\Phi = \frac{m}{m-1}\cdot\frac{\omega}{p} = \frac{172m}{m-1}\cdot\frac{1}{p}\text{ min.}$$

IV. Aeque parum autem ac ante hoc casu margini colorato remedium afferri potest.

COROLLARIUM 1

121. Quando autem formula illa praebet $p < \frac{1}{4}m$ dig., nihilo minus statui debet $p = \frac{1}{4}m$ dig., quod inprimis evenit, si sit $m = 8\frac{1}{2}$ circiter; unde oritur $p = 0$; quare, nisi multiplicatio maior desideretur, sumi poterit $p = \frac{1}{4}m$ dig., unde fit $q = -\frac{1}{4}$ dig. et longitudo telescopii $\frac{1}{4}(m-1)$ dig. campique apparentis semidiameter $\Phi = \frac{688}{m-1}$ minut.

COROLLARIUM 2

122. Quodsi ergo multiplicatio proposita sit $m = 8\frac{1}{2}$, telescopium ita erit construendum:

I. Ob $p = \frac{17}{8}$ dig. $= 2\frac{1}{8}$ dig. lens obiectiva paretur secundum praecepta data.
ll. Ob $q = -\frac{1}{4}$ dig. radius utriusque faciei erit

$$= -\frac{1}{2}(n-1)\text{ dig.},$$

unde longitudo telescopii fit $= 1\frac{7}{8}$ dig., campi vero apparentis semidiameter $= 1^{\circ}\ 31'$. Quod telescopium omni attentione dignum videtur non obstante margine colorato.

COROLLARIUM 3

123. Si desideretur multiplicatio $m=15$, statim reperitur $p=16{,}15$ dig., hinc $q=-1{,}07$ dig., unde longitudo telescopii $=15{,}08$ dig., et semidiameter campi apparentis erit $=\frac{172}{15{,}08}$ minut. $=11'\ 24''$, unde patet hoc telescopium tam ob nimis exiguum campum quam ob nimis magnam longitudinem merito esse reiiciendum, dum contra casus praecedens maxime commendandus videtur.

EXEMPLUM 3

124. Si ambae lentes ex eadem vitri specie constant, obiectiva vero statuatur triplicata secundum §. 66 construenda, ut sit $\lambda = \frac{3-8v}{3\cdot 9}$, constructionem huius telescopii definire.

Utamur vitro communi, pro quo est $n=1{,}55$, eritque $\lambda=0{,}0422$, et maneat $\lambda'=1{,}629$; sumta iterum unitate pro $\sqrt[3]{\mu}$, erit

$$p = m\sqrt[3]{(0{,}0422m-1{,}629)},$$

unde reliqua ut in casibus praecedentibus determinantur.

Inprimis autem hic notari meretur casus, quo fit $0{,}0422m=1{,}629$ sive $m=38\frac{3}{5}$ pro qua sumi debet lentis obiectivae distantia focalis $p=9\frac{13}{20}$ dig. manente $q=-\frac{1}{4}$ dig. hincque longitudo tubi $=9\frac{2}{5}$ dig., ex qua semidiameter campi erit $=18'\ 17''$, qui quidem campus satis est parvus, sed ob tam notabilem multiplicationem facile tolerari potest, nisi forte margo coloratus offendat.

EXEMPLUM 4

125. Si pro lente obiectiva capiatur lens perfecta, ocularis autem maneat simplex atque adeo utrinque aequaliter concava, constructionem telescopii describere. Quoniam supra huiusmodi lentes perfectas descripsimus partim ex vitro coronario, partim ex vitro crystallino conficiendas, hic ante omnia attendendum est, quantae aperturae quaelibet sit capax; cum enim pro multiplicatione m hic esse debeat $x=\frac{m}{40}$ dig, ante omnia videndum est, an lens perfecta hic adhibenda tantam aperturam admittat; quae cautela sedulo esset observanda, si valor ipsius p quopiam casu prodiret $=0$, quo ut ante capi deberet $p=\frac{1}{4}m$, ita ut fieret $x=\frac{1}{10}p$, quod tantum in tertia lente triplicata locum habet. Verum non opus est, ut de hoc simus solliciti, quia ex formula radicali superiori pro hoc casu nunquam prodire potest $p=0$; quoniam enim lens est perfecta, erit per hypothesin $\lambda=0$, ita ut fiat

$$p=m\sqrt[3]{-1{,}629},$$

unde patet semper adeo fore $p > m$, scilicet $p = 1{,}17m$; quare statim sequitur hoc insigne incommodum, ut, mox ac multiplicationi m modicus valor tribuatur, campus apparens tam parvus sit proditurus, ut telescopium fere omni usu careat; cuius causa cum sit valor $\lambda = 0$, optandum hic esset, ut lens perfecta etiam nunc confusionem quandam exiguam pareret, ut illa formula pro quapiam multiplicatione praeberet $p = 0$. Secundum praecepta autem supra data tales lentes non difficulter inveniri possent, quae, dum nullam gignerent dispersionem, aliquam tamen confusionem producerent; verum eiusmodi investigatio commodius instituetur his telescopiis vel unam vel duas lentes novas adiungendo.

SCHOLION

126. Ratio huius insignia paradoxi, quod lentes perfectae hic minus utilitatis praestent quam lentes duplicatae et triplicatae praecedentes, in hoc manifesto est posita, quod hic non eiusmodi lente obiectiva egeamus, pro qua sit $\lambda = 0$, sed potius tali, ut $\lambda m - \lambda'$ redigi possit ad nihilum. Supra autem facile fuisset eiusmodi lentes compositas invenire, quae, dum confusioni colorum mederentur, pro priori confusione datum valorem numeri λ habuissent; verum hic non opus est, ut illum laborem repetamus, sed potius alio modo hanc investigationem ad praesens institutum accommodari conveniet; duas scilicet pluresve lentes, quae unitae lentem perfectam constituebant, hic tanquam disiunctas consideremus, quo pacto id commodi assequemur, ut non solum utraque confusio lentem etiam ocularem in calculo comprehendendo penitus tolli, sed etiam fortasse campus apparens ulterius extendi queat; quem in finem sequens problema praemitti oportet.

PROBLEMA 2

127. *Inter lentem obiectivam et ocularem aliam insuper lentem inserere, ut telescopium eidem primo generi maneat accensendum.*

SOLUTIO

Ponamus ergo telescopium constare tribus lentibus *PP, QQ, RR,* ac primo quidem requiritur, ut hae fractiones $\frac{\alpha}{b}$, $\frac{\beta}{c}$ sint negativae, tum vero, ut haec intervalla

$\alpha + b$, $\beta + c$ sint positiva, existente multiplicatione $m = \frac{\alpha}{b} \cdot \frac{\beta}{c}$ sive $m = \frac{\alpha}{c} \cdot B$ ob $B = \frac{\beta}{b}$, quae proinde quantitas erit positiva. Introducamus nunc altera elementa, quae supra litteris *B, C* et indicibus aperturae π, π' cum semidiametro campi Φ continebantur, ac pro priori conditione habebimus

$$\frac{\alpha}{b} = \frac{\mathfrak{B}\pi - \Phi}{\Phi} < 0,\ \frac{\beta}{c} = \frac{\mathfrak{C}\pi' - \pi + \Phi}{\mathfrak{B}\pi - \Phi} < 0,$$

unde, cum Φ ex rei natura semper sit positivum, debet esse $\mathfrak{B}\pi - \Phi$ negativum, at vero $\mathfrak{C}\pi' - \pi + \Phi$ positivum; et quia $\gamma = \infty$, ideoque $C = \infty$ et $\mathfrak{C} = 1$, unde posterior conditio dat $\pi' - \pi + \Phi > 0$. Pro campo autem apparente invenimus $\Phi = \frac{-\pi + \pi'}{m-1}$, unde, cum Φ et $m-1$ sint quantitates positivae, debet esse $-\pi + \pi'$ quantitas positiva, qua praecedens etiam conditio sponte continetur.

Ut autem praeterea intervalla lentium fiant positiva, has duas conditiones adipiscimur ex § 16:

1. $\frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi - \Phi} > 0$, unde cum denominator sit negativus, etiam numerator debet esse negativus seu $\mathfrak{B}\pi p < 0$; prouti ergo quantitas p fuerit vel positiva vel negativa, debet esse $\mathfrak{B}\pi$ vel negativum vel positivum.

2. $\frac{\mathfrak{B}\Phi p(\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\pi' - \pi + \Phi)} > 0$, ubi, cum Φ sit positivum, totus vero denominator negativus, etiam pro numeratore $\mathfrak{B}p(\pi' - (1-\mathfrak{B})\pi)$ debet esse < 0.

Ex his igitur conditionibus, si loco Φ valorem inventum substituamus, sequentes conclusiones consequemur:

1. $\pi' - \pi > 0$
2. ob $\mathfrak{B}\pi - \Phi < 0$ debet esse

$$(m-1)\mathfrak{B}\pi - \pi' + \pi < 0 \text{ seu}$$
$$\pi' - \pi > (m-1)\mathfrak{B}\pi \text{ sive } \pi' > ((m-1)\mathfrak{B} + 1)\pi$$

3. $\mathfrak{B}\pi p < 0$
4. $Bp(\pi' - (1-\mathfrak{B})\pi) < 0$;

quia hic igitur formulae 3 et 4 ambae sunt negativae, haec per illam divisa:

$$\frac{B(\pi' - (1-\mathfrak{B})\pi)}{\mathfrak{B}\pi} > 0;$$

unde, si denominator fuerit positivus, etiam numerator debet esse positivus et contra.

Consideremus nunc ambos casus extremos, alterum, quo media lens lenti obiectivae unitur, alterum, quo ea lenti oculari unitur. Priori casu, quo scilicet $\alpha + b = 0$, fit $\pi = 0$, quemadmodum supra iam notavimus pro lentibus quotcunque cum obiectiva lente coalescentibus. Posteriore casu, quo $\beta + c = 0$, debet esse $\pi' - (1-\mathfrak{B})\pi = 0$ seu $\pi' = (1-\mathfrak{B})\pi$, qui valor in conditione superiore secunda positus dabit $m\mathfrak{B}\pi < 0$ seu $\mathfrak{B}\pi < 0$. Cum autem campus apparens potissimum a lente oculari pendeat, cui respondet littera π', haec littera π necessario est positiva; quare, ut campus ob lentem mediam non minuatur, sed potius augeatur, numerum π negativum esse oportet, ex quo superiores conditiones propius hoc modo definientur:

1^{ma} scilicet $\pi' - \pi$ iam sponte fit > 0 ideoque omitti potest,
2^{da} est $\pi' > ((m-1)\mathfrak{B}+1)\pi$,
ex 3^{tia} autem sequitur $\mathfrak{B}p > 0$,
et 4^{ta} $\frac{B(\pi' - (1-\mathfrak{B})\pi)}{\mathfrak{B}\pi} > 0$.

Consideretur adhuc locus oculi, cuius distantia a lente oculari fit $O = \frac{\pi'}{m\Phi} r$, quae ob $\frac{\pi'}{m\Phi}$ positivum fieret positiva, si modo r esset positivum; at cum sit $r = c$ ob $C = \infty$ et $\mathfrak{C} = 1$, erit $r = \frac{Bp\Phi}{\pi' - \pi + \Phi}$; cuius denominator cum sit positivus, examinandum est, utrum Bp sit positivum an negativum; at si Bp esset positivum, distantia O quoque foret positiva, sin autem Bp esset negativum, foret quoque distantia O negativa., oculusque lenti tertiae immediate applicari deberet, de quo casu praecepta supra data sunt observanda.

COROLLARIUM 1

128. Quia, statim ac multiplicatio m fit modicae quantitatis, Φ multo minus est quam π, cum $\mathfrak{B}\pi - \Phi$ sit negativum, quantitas $\mathfrak{B}\pi$ fiet quoque negativa, et ob $\pi < 0$ erit $\mathfrak{B}$ positivum. Hinc pro tertia conditione $\mathfrak{B}\pi p < 0$ debebit esse p positivum (excepto scilicet casu, quo π quam minimum habet valorem ideoque p etiam negativum esse posset), et per tertiam et quartam conditionem coniunctim erit ob denominatorem negativum etiam numerator $B(\pi' - \pi + \mathfrak{B}\pi)$ negativus; si ergo fuerit $\pi' - \pi + \mathfrak{B}\pi > 0$, erit $B < 0$, contra vero $B > 0$.

COROLLARIUM 2

129. Hae igitur conditiones impleri possunt pluribus modis, dum plura elementa manent indeterminata; statim enim patet quantitatem α seu p tam affirmativum quam negativum valorem accipere posse; at quia $\mathfrak{B}p > 0$ ob $\pi < 0$, si p statuamus positivum, etiam $\mathfrak{B}$ debet esse positivum; sin autem p sumatur negative, etiam $\mathfrak{B}$ debet esse negativum; interim tamen, cum sit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$, etiamsi sit $\mathfrak{B}$ positivum, littera B etiam nunc esse potest tam positiva quam negativa; altero vero casu, quo $\mathfrak{B}$ est negativum, semper etiam B fit negativum.

SCHOLION 1

130. Eodem modo, quo hoc problema resolvimus, conditiones etiam invenire possunt, quando duae pluresve lentes inter obiectivam et ocularem inseruntur seu quando huiusmodi telescopium ex quatuor pluribusve lentibus est compositum; ponamus enim quatuor id lentibus constare atque sequentes sex conditiones erunt adimplendae:

$$1.\ \frac{\alpha}{b} < 0, \qquad 2.\ \frac{\beta}{c} < 0, \qquad 3.\ \frac{\gamma}{d} < 0,$$

$$4.\ \alpha + b > 0, \ \ 5.\ \beta + c > 0, \ \ 6.\ \gamma + d > 0,$$

existente $\delta = \infty$ ideoque $D = \infty$ et $\mathfrak{D} = 1$; unde, si loco harum litterarum valores supra dati introducantur, hae sex conditiones praebebunt sequentes formulas, in quibus Φ semper ut positivum ponitur:

$$1.\ \mathfrak{B}\pi - \Phi < 0, \ \ 2.\ \frac{\mathfrak{C}\pi' - \pi + \Phi}{\mathfrak{B}\pi - \Phi} < 0, \quad 3.\ \frac{\pi'' - \pi' + \pi - \Phi}{\mathfrak{C}\pi' - \pi + \Phi} < 0,$$

quae tres conditiones commodius ita referuntur:

$$1.\ \mathfrak{B}\pi - \Phi < 0, \ \ 2.\ \mathfrak{C}\pi' - \pi + \Phi > 0, \quad 3.\ \pi'' - \pi' + \pi - \Phi < 0.$$

Pro tribus reliquis conditionibus, quia in singulis denominatores sunt negativi, etiam numeratores oportet esse negativos, unde sequentes conditiones erunt adimplendae:

4. $\mathfrak{B}\pi p < 0$,
5. $Bp(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi) < 0$,
6. $BCp(\pi'' - (1-\mathfrak{C})\pi') < 0$,

quae, prout p fuerit vel positivum vel negativum, duplici modo considerari poterunt; in hoc negotio autem inprimis consideranda est expressio pro campo apparente, quae est $\Phi = \frac{-\pi + \pi' - \pi''}{m-1}$; quae quia tam magna desiderari solet, quam fieri potest, curandum est, ut fractiones π et π'' obtineant valores negativos eosque maximos, qui tamen $\frac{1}{4}$ vel $\frac{1}{6}$ superare nequeunt, ac si forte hoc fieri nequeat et alteruter debeat esse positivus, tum, ut is fiat quam minimus, erit efficiendum.

SCHOLION 2

131. His iam praemissis videamus, quo modo superiori incommodo, quo lentes perfectae pro hoc telescopiorum genere ineptae sunt deprehensae, remedium afferri possit. Considerabimus igitur telescopium ut tribus lentibus compositum ac duas priores prorsus uniamus, ut intervallum $\alpha + b$ evanescat sicque lens obiectiva fiat duplicata; verum nunc singula elementa ita definiamus, ut non pro sola obiectiva utraque confusio destruatur, sed pro toto telescopio. Quoniam vero ad hoc duplici vitri specie opus est,

adhibere cogimur binas illas species anglicas, scilicet vitrum coronarium et crystallinum. Unde duo potissimum problemata nascuntur, prout vel prima lens ex coronario, secunda vero ex crystallino, vel contra prior ex crystallino, secunda vero ex coronario fuerit paranda; de tertia autem lente oculari perinde fere erit, sive eam ex vitro coronario sive ex crystallino conficere velimus, dummodo ea utrinque aeque concava reddatur, quandoquidem ea hoc modo maximam aperturam admittit, a qua campus apparens dependet.

PROBLEMA 3

132. *Si telescopii lens obiectiva sit duplicata ac prior quidem ex vitro coronario, posterior vero ex crystallino parata, lens autem ocularis etiam ex vitro coronario, constructionem huius telescopii pro quavis multiplicatione m describere.*

SOLUTIO

Cum igitur hic sit $\alpha + b = 0$ sive $\alpha = -b$ et $\frac{\alpha}{b} = -1$, erit multiplicatio $m = -\frac{\beta}{c}$ seu $c = -\frac{\beta}{m}$, ubi littera β exprimit distantiam focalem ipsius lentis obiectivae duplicatae ideoque, ut ex problemate 1 patet, debet esse positiva; unde lens ocularis erit concava. Cum igitur sit $b = \frac{\beta}{B}$, $q = \mathfrak{B}b = \frac{\mathfrak{B}\beta}{B} = \frac{\beta}{B+1}$, erit $\alpha = \frac{-\beta}{B}$, et litterae μ et ν una cum μ'' ex refractione $n = 1,53$, litterae vero μ' et ν' ex refractione $n = 1,58$ sunt sumendae; unde pro confusione ex apertura lentium destruenda habebimus hanc aequationem:

$$\mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu\lambda''}{mB^3} - \frac{\mu'\lambda'}{\mathfrak{B}B} = 0.$$

Cum autem sit $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 7:10$ atque $n'' = n$, ob valorem distantiae O negativum pro margine colorato tollendo nanciscimur hanc aequationem:

$$\pi'(3B + 10) = 10\pi;$$

deinde vero pro hac confusione penitus tollenda satisfieri oportet huic aequationi:

$$0 = -7 + \frac{10(B+1)}{B} - \frac{7}{mB} \quad \text{seu} \quad 0 = -7B + 10(B+1) - \frac{7}{m};$$

unde reperitur $B = \frac{7-10m}{3m}$, $\mathfrak{B} = \frac{10m-7}{7m-7}$, ex qua littera B perfecte determinatur, ita ut ex prima aequatione tantum litterae λ et λ' definiendae restent, quia ob lentem ocularem utrinque aequalem λ'' iam definitur. Inde igitur commodissime definitur numerus λ' :

$$\lambda' = \frac{\mu\mathfrak{B}^3\lambda}{\mu'} + \frac{\mu\mathfrak{B}^3\lambda''}{m\mu' B^3} - \frac{\nu'\mathfrak{B}^2}{B},$$

in qua quidem aequatione λ pro lubitu accipi posset; sed ne λ' unitatem nimis superet, conveniet sumi $\lambda = 1$, sicque omnia iam erunt determinata, ita ut nihil amplius supersit, quod ex aequatione media posset determinari, quia ratio litterarum π et π' ex praemissis iam datur. Cum enim sit

$$b = \frac{\beta}{B} = \frac{p\Phi}{\mathfrak{B}\pi - \Phi} = \frac{-\beta\Phi}{B(\mathfrak{B}\pi - \Phi)}$$

hincque $\pi = 0$ et cum pro campo apparente sit $\Phi = \frac{-\pi+\pi'}{m-1}$, erit $\pi' = (m-1)\Phi$, unde pro secunda aequatione prodit

$$0 = (m-1)\Phi(3B + 10);$$

quod cum fieri nequeat praeter casum $3B + 10 = 0$ seu $\frac{7-10m}{m} + 10 = 0$ hincque $m = \infty$, margo coloratus tolli nequit, nisi multiplicatio sit maxima ideoque pro maioribus multiplicationibus erit insensibilis; ad quem casum cum haec telescopia accommodari conveniat, margo coloratus non erit metuendus sufficietque, si primae et tertiae aequationi satisfecerimus. Inventis igitur quantitatibus B, λ et λ' pro data multiplicatione m gradus claritatis y assumatur, quo contenti esse voluerimus, indeque habebitur semidiameter aperturae primae lentis x. Si deinde distantiam focalem totius lentis obiectivae, quae est aequalis β, ut indefinitam spectemus, habebimus inde distantiam focalem prioris lentis $\alpha = \frac{-\beta}{B}$; et pro posteriore distantias determinatrices $b = \frac{\beta}{B}$ et β, ex quibus cum numeris λ et λ' utramque lentem poterimus construere; in qua constructione notetur minimus radius sive convexitatis sive concavitatis eiusque parti quintae vel etiam quartae aequetur $x = my$; unde ipsa quantitas β in digitis determinabitur. Hinc porro colligimus distantiam focalem lentis ocularis $= c = \frac{-\beta}{m}$; ex qua si huic lenti utrinque figura aequalis tribuatur, ut scilicet maximae aperturae fiat capax, radius istius curvaturae erit $= -\frac{2(n-1)\beta}{m}$, uti supra iam ostendimus § 61, ubi etiam invenimus pro hac lente fore $\sqrt{(\lambda'' - 1)} = \frac{\sigma-\rho}{2\tau}$, unde valor ipsius λ'' definitur.

COROLLARIUM 1

133. Cum hic distantia oculi post ultimam lentem O fiat negativa ideoque oculus huic lenti immediate adplicari debeat, in formula campum apparentem declarante $\Phi = \frac{-\pi+\pi'}{m-1}$ fractio π' sumi debet $= \frac{\omega}{c}$, ut scilicet campum inveniamus, quem uno obtutu conspicimus; expediet autem aperturam istius lentis tantam fieri, quantam curvatura facierum admittit, sicque nihil obstat, quominus ipsi π' valor $= \frac{1}{4}$ vel $= \frac{1}{5}$ tribuatur.

COROLLARIUM 2

134. Quod hic de valore ultimae litterarum π, π', π'' etc. notavimus, latissime patet, ut scilicet ei semper valor $\frac{1}{4}$ vel $\frac{1}{5}$ tribui possit, dummodo in computo campi apparentis eius valor ad ; imminuatur, si quidem hic fuerit minor; quippe quo modo campus uno obtutu conspectus definitur. Quando autem apertura lentis ocularis maior fuerit pupilla, tum pupilla eam quasi peragrando successive totum campum conspiciet, quem verus valor ipsius π' definit, sicque in posterum hanc limitationem a pupilla petitam penitus omittare poterimus, dummodo notetur casu, quo π' maius quam $\frac{\omega}{c}$, hunc campum non uno obtutu apparere.

COROLLARIUM 3

135. Hoc igitur pacto telescopium adipiscimur primi generis, quod obiecta sine ulla confusione sive ab apertura lentium sive a diversa radiorum natura oriunda repraesentabit, ita ut in illo nihil amplius possit desiderari, nisi quod campus apparens nimis sit exiguus; quo tamen defectu omnia telescopia tam NEUTONIANA quam GREGORIANA aeque laborant.

SCHOLION 1

136. Si haec ad praxin accommodare velimus, inchoandum erit a valore litterae *B*, quem tertia aequatio suppeditat, scilicet $B=\frac{7-10m}{3m}$, qui, statim atque *m* sit numerus modice magnus, abit in $B=-\frac{10}{3}$; quia autem hic valor $-\frac{7}{3}$ derivatus est ex DOLLONDI experimentis, unde rationem $\frac{dn}{n-1}:\frac{dn'}{n'-1}=7:10$ deduximus, nemo certe arbitrabitur hanc rationem tam exacte veritati respondere, ut non satis notabiliter ab ea discrepare possit; quam ob causam ridiculum plane foret, si circa valorem huius litterae *B* nimis scrupulosi esse vellemus; neque etiam res ipsa tantam praecisionem exigere videtur, cum iam plurimum praestitisse is sit censendus, qui hanc confusionis speciem, quae hactenus nullo plane modo imminui posse est credita, plurimum imminuere potuerit, etiamsi ad nihilum non reduxerit; audacter igitur statuere poterimus $B=-\frac{10}{3}$ pro quacunque multiplicatione indeque tantum superest, ut formula pro λ' inventa evolvatur; in quo nihil omnino negligere licebit, quoniam, ut supra iam invenimus, solus terminus $\frac{\mu\mathfrak{B}\lambda''}{m\mu' B^3}$, tanti erat momenti, ut a lente obiectiva perfecta optatus effectus exspectari non potuerit.

SCHOLION 2

137. Quoniam in sequentibus plurimum intererit, ut lentibus ocularibus eiusmodi figura tribuatur, quae maximae aperturae sit capax, hocque manifesto eveniat, si ambae huius lentis facies reddantur aequales; pro huiusmodi lente valor litterae λ ita definietur, ut fiat $\sqrt{\lambda-1}=\frac{\sigma-\rho}{2\tau}$, quem igitur pro praecipuis vitri speciebus hic exhibeamus:

n	$\sqrt{\lambda-1}$	λ
1,53	0,77464	1,60006
1,55	0,79367	1,62991
1,58	0,82125	1,67445.

Cum igitur nunc habeamus valorem $\lambda''=1,60006$, per ea, quae in problemate sunt constituta, habebimus $\mu=0,9875,\ \ \mu'=0,8724,\ \ \nu'=\ 0,2529$; sumto $B=-\frac{10}{3}$ et $\mathfrak{B}=+\frac{10}{3}$ aequatio prima resolvenda induet hanc formam:

$$\lambda'=3,3001\lambda-\frac{0,1426}{m}+0,1548,$$

ex qua, ne valor ipsius λ' praeter necessitatem nimis magnus prodeat, statuamus $\lambda=1$ fietque

$$\lambda'=3,4549-\frac{0,1426}{m},$$

cuius aequationis usum in aliquot exemplis ostendamus.

EXEMPLUM 1

138. Huiusmodi telescopium construere, quod obiecta vicies quinquies aucta repraesentet; seu sit $m=25$.
Cum sit $\lambda=1,$ erit $\lambda'=3,4492$ et $\lambda'-1=2,4492$ et $\sqrt{(\lambda'-1)}=1,5649$;
atque hinc sequens singularum lentium constructio colligetur:

I. Pro lente prima ex vitro coronario facta ob eius distantiam focalem $p=\alpha=+\ \frac{3\beta}{10}$ et $\sqrt{(\lambda-1)}=0$ fiet

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,1807\beta\\ \text{posterioris} &= 1,3233\beta.\end{cases}$$

II. Pro secunda lente ex vitro crystallino, cum sint distantiae determinatrices $b=\frac{\beta}{B}=-\frac{3}{10}\beta$ et litterae

$$\rho=0,1414,\ \ \sigma=1,5827,\ \ \tau=0,8775 \text{ et } \sqrt{(\lambda'-1)}=1,5649,$$

si pro radiis anterioris et posterioris faciei ponamus litteras *F* et *G,* habebimus

$$F = \frac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}, \qquad G = \frac{b\beta}{\rho\beta+\sigma b\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$$

atque hinc

$$\frac{1}{F} = \frac{3\sigma-10\rho\mp7\tau\sqrt{(\lambda'-1)}}{3\beta}, \qquad \frac{1}{G} = \frac{3\rho-10\sigma\pm7\tau\sqrt{(\lambda'-1)}}{3\beta}$$

quibus evolutis prodit

$$\frac{1}{F} = \frac{3,3341\mp9,6124}{3\beta}, \qquad \frac{1}{G} = \frac{-15,4028 \pm 9,6124}{3\beta}$$

ut igitur radii non nimis fiant parvi, uti oportet signis superioribus, unde obtinebimus

$$\frac{1}{F} = \frac{-6,2783}{3\beta}, \quad F = -0,4779\beta, \quad \frac{1}{G} = \frac{-5,7904}{3\beta}, \quad G = -0,5181\beta.$$

III. Pro tertia lente oculari ex vitro coronario paranda constructio est facillima, dum utriusque faciei radius esse debet

$$= 2(n-1)r = +\ 1,06r = -0,0424\beta.$$

Binae priores lentes sibi invicem immediate iunguntur, ut unam quasi lentem constituant, cuius aperturae semidiameter maior esse nequit quam quarta circiter pars radii minimi, quae est $= 0,0452\beta$, et habebimus $x = 0,0452\beta$. Debet autem esse $x = my$ denotante *y* gradum claritatis, atque iam notavimus statui posse $y = \frac{1}{50}$ dig., ita ut hoc casu habeamus $x = \frac{1}{2}$ dig., quocirca valor ipsius β ita determinabitur, ut sit $\beta = 11,1$ dig.; saltim β hoc limite non debet capi minus, unde superiores mensurae absolute innotescunt. Campi autem apparentis semidiameter ob $\pi = 0$ erit $\Phi = \frac{\pi'}{m-1} = \frac{\pi'}{24}$;
sumtoque $\pi' = \frac{1}{4}$ erit
in minutis primis $35\frac{3}{4}$ min., quem campum oculus uno obtutu cerneret, si semi diameter pupillae esset $\pi' r = 0,1110$. Quanto autem est minor, tanto minorem quoque campum uno obtutu videbit. Longitudo autem huius telescopii erit $= 10\frac{3}{4}$ digit.

SCHOLION

139. Hoc ergo telescopium ad praxin satis accommodatum videtur, cum eius longitudo minor sit undecim digitis et, tamen vicies quinquies obiecta augeat, campo apparente non adeo exiguo existente; hincque etiam patet, quantum lens perfecta hic immutari debuerit,

ut etiam confusionem a lente oculari oriundam tolleret. Verum hic notandum est constructionem huius instrumenti summam artificis sollertiam requirere minimumque errorem commissum totum opus irritum reddere, quare non nisi post plura tentamina successus sperari poterit. Multo maiore autem sollertia erit opus, si maiorem quoque multiplicationem desideremus, uti ex sequenti exemplo erit manifestum.

EXEMPLUM 2

140. Huiusmodi telescopium conficere, quod obiecta quinquagies multiplicet; seu sit $m = 50$.

Erit pro hoc casu $\lambda' = 3,4521$ et $\sqrt{(\lambda' - 1)} = 1,5659$, qui valor praecedentem superat $\frac{1}{1000}$, hoc est sui parte $\frac{1}{1565}$, ita ut superior formula $\sqrt{(\lambda' - 1)}$ per $1 + \frac{1}{1565}$ multiplicata praebeat praesentem valorem, et cum reliqua elementa maneant ut ante, erit:

I. Pro prima lente radius faciei $\begin{cases} \text{anterioris} = 0,1807\beta \\ \text{posterioris} = 1,8233\beta. \end{cases}$

II. Pro secunda lente habebimus

$$\frac{1}{F} = \frac{3,3341 \mp 9,6185}{3\beta}, \quad \frac{1}{G} = \frac{-15,4028 \pm 9,6185}{3\beta}$$

sumtisque signis superioribus habebimus:

$$\frac{1}{F} = \frac{-6,2844}{3\beta}, \quad F = -0,4774\beta, \quad \frac{1}{G} = \frac{-5,7843}{3\beta}, \quad G = 0,5186\beta.$$

Quae duae lentes iunctae aperturam admittent, cuius semidiameter $= 0,0452\beta$; quo scilicet maior non debet esse valor $x = my = 1$ dig., quare capi debebit β maius quam 22,1 dig.

III. Pro lente oculari, cuius distantia focalis est $= \frac{-\beta}{m} = \frac{-\beta}{50}$, radius utriusque faciei erit $= -\frac{2(n-1)\beta}{50} = -0,0212\beta$. Sumto autem $\pi' = \frac{1}{4}$ erit aperturae eius semidiameter $x = +\frac{\beta}{200} = 0,110$ dig., unde semidiameter campi apparentis fit $\Phi = \frac{\pi'}{m-1} = \frac{1}{196}$ sive angulus $\Phi = 17\frac{1}{2}$ min. prim. Longitudo denique huius telescopii erit $= \beta + r = 21,658$ sive $21\frac{2}{3}$ dig.

SCHOLION

141. In hoc exemplo constructio lentis secundae vix discrepat a praecedente; unde patet, quam adcurate mensurae inventae observari debeant, ut effectus voto respondeat,

facillimeque evenire posse, ut, quae lens obiectiva datae cuidam multiplicationi destinatur, ea longe alii multiplicationi inserviat; quare quantamcunque etiam sollertiam artifex adhibuerit, multiplicatio, cui convenit, explorari debet, dum scilicet ei successive aliae atque aliae lentes oculares adiunguntur; tum enim pro certa quadam multiplicatione fieri poterit, ut telescopium egregium affectum producat; hanc ob causam supersedeamus altero casu supra memorato, quo pro lente obiectiva lens prior ex vitro crystallino, posterior ex coronario parari debebat, quoniam haec, quae evolvimus, sufficere videntur, et multo magis expediet pro lente obiectiva lentem triplicatam exhibere eamque talem, cuius prima et tertia lens ex vitro crystallino, media ex coronario sit confecta, quia iam supra hinc aptissima lens perfecta est nata.

PROBLEMA 4

142. *Si lens obiectiva telescopii sit triplicata, cuius prima et tertia lens ex vitro crystallino, media vero ex coronario sit conficienda, lens autem ocularis etiam ex vitro coronario, huius telescopii constructionem describere, ut omni confusione careat.*

SOLUTIO

Hoc igitur telescopium ex quatuor omnino lentibus constabit, pro quibus erit $n=1,58$, $n'=1,53$, $n''=n$ et $n'''=n'$, et quia tres priores lentes in unam quasi coalescere debent, erit $\alpha+b=0$ et $\beta+c=0$, sive

$$\frac{\alpha}{b}=-1 \text{ et } \frac{\beta}{c}=-1$$

quare cum sit multiplicatio $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, erit

$$m=\frac{-\gamma}{d} \text{ seu } d=\frac{-\gamma}{m};$$

reliquae vero litterae simili modo per γ exprimi poterunt, scilicet

$$c=\frac{\gamma}{C},\ \beta=\frac{-\gamma}{C},\ b=\frac{-\gamma}{BC}, \text{ et } \alpha=\frac{\gamma}{BC},$$

ex quibus distantiae focales oriuntur

$$p=\frac{\gamma}{BC},\ q=\frac{-\mathfrak{B}\gamma}{BC},\ r=\frac{\mathfrak{C}\gamma}{C},\ s=\frac{-\gamma}{m}.$$

Quibus praemissis pro confusione ex apertura lentium orta destruenda habebimus hanc aequationem:

$$\mu\lambda-\frac{\mu'}{\mathfrak{B}}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''}{\mathfrak{C}\mathfrak{B}^3}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right)-\frac{\mu'''\lambda'''}{B^3C^3m}=0,$$

quae ob $\mu''=\mu$, $\nu''=\nu$ et $\mu'''=\mu'$, evoluta dabit :

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\frac{\mu\lambda''}{\mathfrak{B}^3\mathfrak{C}^3}-\frac{\mu'\lambda'''}{B^3C^3m}-\frac{\mu'\nu'}{B\mathfrak{B}}+\frac{\mu\nu}{B^3C\mathfrak{C}}$$

Ne nimis rationi $7:10$, qua ante usi sumus, inhaereamus, ponamus in genere

$$\frac{dn}{n-1}=\zeta \text{ et } \frac{dn'}{n'-1}=\eta,$$

ut sit circiter $\zeta:\eta=10:7$; deinde, quia nostro casu fit $\pi=0$ et $\pi'=0$, pro margine colorato abolendo habebimus

$$0=\zeta-\frac{\eta(B+1)}{B}+\frac{\zeta(C+1)}{BC} \text{ sive } \zeta(1+C+BC)=\eta(B+1)C,$$

ex qua quid conludere liceat, deinceps videbimus. Tertiam aequationem nobis praebet destructio tota huius confusionis, scilicet istam:

$$0=BC+C+\frac{\zeta m-\eta}{(\zeta-\eta)m}.$$

Ponatur brevitatis gratia

$$\frac{\zeta m-\eta}{(\zeta-\eta)m}=\theta,$$

eritque

$$0=BC+C+\theta,$$

unde prodit

$$C=\frac{-\theta}{B+1} \text{ vel } B=-1-\frac{\theta}{C}.$$

Cum autem secunda aequatio abeat in hanc formam:

$$BC+\mathrm{C}+\frac{\zeta}{\zeta-\eta}=0,$$

ambabus simul satisfieri nequit, nisi sit $\theta=\frac{\zeta}{\zeta-\eta}$, hoc est, nisi sit $\frac{\zeta m-\eta}{m}=\zeta$, sive $\zeta m-\zeta m=\eta=0m$ sive $m=\infty$, prorsus ut in casu praecedente. Regrediemur igitur ad nostram aequationem primam, in qua sive loco *B* sive loco *C* valorem debitum substituamus. Cum autem rationem $\zeta:\eta$ non tam exacte nosse , licet, sufficiet valores proximos sumsisse; hunc in finem in tertia aequatione terminum per *m* divisum negligamus et habebimus:

$$0=BC+\mathrm{C}+\frac{\zeta}{\zeta-\eta} \quad\text{sive}\quad 0=BC+\mathrm{C}+\frac{10}{3}$$

hincque

$$C=\frac{-10}{3(B+1)} \quad\text{et}\quad C+1=\frac{3B-7}{3(B+1)} \;;$$

quibus substitutis et divisione facta per $(B+1)^3$ prodit

$$0=-1000\mu\lambda\mathfrak{B}^3+\ 1000\mu'\lambda'+\ \mu\lambda''(10\mathfrak{B}-7)^3-\tfrac{27\mu\lambda'''}{m}$$
$$+1000\mu'\nu'\mathfrak{B}\left(1-\mathfrak{B}\right)-30\mu\nu(10\mathfrak{B}-7)(1-\mathfrak{B}),$$

quae sumto $\lambda''=\lambda$ fit aequatio quadratica, ex qua $\mathfrak{B}$ definitur.

Ex hac autem aequatione cognoscimus huiusmodi substitutionem etiam in genere succedere; cum enim sit $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$, ob $B+1\ =\frac{1}{1-\mathfrak{B}}$ fiet

$$C=-\theta(1-\mathfrak{B})\quad\text{et}\quad C+1=1-\theta+\theta\mathfrak{B}$$

hincque

$$\mathfrak{C}=\tfrac{-\theta(1-\mathfrak{B})}{1-\theta+\theta\mathfrak{B}},$$

et ipsa aequatio prima reducetur ad hanc formam, si scilicet per $\mathfrak{B}^3$ multiplicetur:

$$0=\mu\lambda\mathfrak{B}^3-\mu'\lambda'-\mu\lambda''\left(\mathfrak{B}-1+\tfrac{1}{\theta}\right)^3+\tfrac{\mu'\lambda'''}{m\theta^3}-\mu'\nu'\mathfrak{B}(1-\mathfrak{B})+\tfrac{\mu\nu(1-\mathfrak{B})(\mathfrak{B}-1+\frac{1}{\theta})}{\theta},$$

existente $\theta=\frac{\zeta m-\eta}{(\zeta-\eta)m}$; ac si ponatur $\theta=\frac{10}{3}$, praecedens aequatio sponte prodit.

Statuamus igitur $\lambda''=\lambda$, et evolutio huius aequationis sequentem praebebit aequationem quadraticam secundum potestates litterae $\mathfrak{B}$ dispositam:

$$\mathfrak{B}^2\left(3\mu\lambda\left(1-\tfrac{1}{\theta}\right)+\mu'\nu'-\tfrac{\mu\nu}{\theta}\right)+\mathfrak{B}\left(-3\mu\lambda\left(1-\tfrac{1}{\theta}\right)^2-\mu'\nu'+\tfrac{\mu\nu}{\theta}\left(2-\tfrac{1}{\theta}\right)\right)$$
$$+\mu\lambda\left(1-\tfrac{1}{\theta}\right)^3-\mu'\lambda'+\tfrac{\mu'\lambda'''}{m\theta^3}-\mu\nu\left(1-\tfrac{1}{\theta}\right)=0,$$

ex qua $\mathfrak{B}$ definiri debet.

Nunc igitur statuamus $\theta=\frac{10}{3}$; tum vero $\lambda=\lambda'=\lambda''=1$, et pro lente oculari sit $\lambda'''=1,60006$; tum vero

$$\mu=0,8724\quad\text{et}\quad\nu=0,2529,\quad\mu'=0,9875,\quad\nu'=0,2196;$$

unde fit

$$l.\mu\nu=9,3436645,\quad l.\mu'\nu'=9,3361694.$$

Pro termino $\mathfrak{B}^2$:

$$\mathfrak{B}^2\left(\tfrac{21}{10}\mu+\mu'\nu'-\tfrac{3}{10}\mu\nu\right)$$

$$\mathfrak{B}^2\begin{pmatrix}+1,83204- & 0,06618\\ +0,21685 & \end{pmatrix}$$

$$\begin{array}{r} 2,04889\\ 0,06618\\ \hline 1,98271\end{array}$$

$$+\ 1,98271\mathfrak{B}^2-1,38675\mathfrak{B}\ 0,84271\ +\tfrac{0,04266}{2}=0.$$

qua divisa per 1,98271 fiet

$$\mathfrak{B}^2=0,69942\mathfrak{B}+0,42503-\tfrac{0,02152}{m},$$

cuius resolutio suppeditat

$$\mathfrak{B}=0,34971\pm\sqrt{\left(0,54736-\tfrac{0,02152}{m}\right)}$$

vel

$$\mathfrak{B}=0,34971\pm\left(0,73983-\tfrac{0,01454}{m}\right),$$

unde bini ipsius $\mathfrak{B}$ valores erunt

$$\text{I.)}\mathfrak{B}=1,08954\ -\tfrac{0,01454}{m}\quad \text{II.)}\mathfrak{B}=-\ 0,39012\ +\ \tfrac{0,01454}{m}.$$

COROLLARIUM 1

143. Tribus igitur prioribus lentibus immediate coniunctis existit lens obiectiva triplicata, cuius distantia focalis erit aequalis γ, ex qua radios singularum facierum definire oportet, inter quos notetur minimus, qui sit $=i\gamma$; cuius pars quarta $=\frac{1}{4}i\gamma$ dabit semidiametrum aperturae, quam ista lens obiectiva admittit.

COROLLARIUM 2

144. Porro vero ex multiplicatione m data et gradu claritatis y definitur semidiameter aperturae lentis obiectivae $x=my$ idque in digitis sumendo v. gr. $y=\frac{1}{50}$ dig., unde habebitur ista aequatio $my=\frac{1}{4}i\gamma$, ex qua per mensuram absolutam colligitur $\gamma=\frac{4my}{i}$.

COROLLARIUM 3

145. Cum autem lens ocularis debeat esse utrinque aeque concava, ut sit $\lambda'''=1,60006$, erit eius distantia focalis $=d=\frac{-\gamma}{m}$, unde radius utriusque faciei statui debet

$=-\frac{2(n'-1)}{m}\gamma=\frac{-1,06}{m}\gamma$; cuius aperturae semidiameter sumi potest quater minor, ut sit $x=\frac{\gamma}{4m}$.

EXEMPLUM 1

146. Posita multiplicatione $m=25$ construere huiusmodi telescopium ex valore priore pro littera $\mathfrak{B}$ invento.

Cum igitur sit $m=25$, erit $\mathfrak{B}=+1,08896$, ex quo sequitur

$$B=\frac{\mathfrak{B}}{1-\mathfrak{B}}=-12,24100 \quad \text{et} \quad \log(-B)=1,0878169.$$

Porro $C=-\theta(1-\mathfrak{B})=0,2965$, hincque ob $BC+C+\theta=0$ colligimus $BC=-C-\theta=+\theta-\theta\mathfrak{B}-\theta=-\theta\mathfrak{B}=-3,6298$ et $\mathfrak{C}=\frac{C}{C+1}=0,22869.$

Sint nunc radii facierum primae lentis F et G, secundae F' et G' ac tertiae F'' et G''; ob distantias determinatrices

$$a=\infty,\quad b=\frac{-\gamma}{BC},\quad c=\frac{\gamma}{c},\quad \alpha=\frac{\gamma}{BC},\quad \beta=\frac{-\gamma}{c},\quad \gamma=\gamma$$

et numeros $\lambda=1,\quad \lambda'=1,\quad \lambda''=1$ erit

$$F=\frac{\alpha}{\sigma}=\frac{\gamma}{BC\sigma},\qquad G=\frac{\alpha}{\rho}=\frac{\gamma}{BC\rho},$$
$$F'=\frac{b\beta}{\rho'\beta+\sigma'b}=\frac{-\gamma}{BC\rho'+C\sigma'},\qquad G'=\frac{b\beta}{\rho'b+\sigma'\beta}=\frac{-\gamma}{BC\sigma'+C\rho'},$$
$$F''=\frac{c\gamma}{\rho\gamma+\sigma c}=\frac{\gamma}{C\rho+\sigma},\qquad G''=\frac{c\gamma}{\sigma\gamma+\rho c}=\frac{\gamma}{C\sigma+\rho}.$$

Cum igitur sit $\rho=0,1414,\quad \sigma=1,5827$ et $\rho'=0,2267,\quad \sigma'=1,6607$ calculo instituto obtinebimus:

$$F=-0,1741\gamma,\qquad G=-1,9484\gamma,$$
$$F'=+3,0243\gamma,\qquad G'=+0,1678\gamma,$$
$$F''=+0,6155\gamma,\qquad G''=+1,6376\gamma.$$

At pro lente oculari radius utriusque faciei erit $=-0,0424\gamma$. Inter illos autem radios minimus est $0,1678\gamma$; cuius parti quartae $0,0419\gamma$ si aequetur $x=my=25y=\frac{1}{2}$ dig., prodibit $r=12$ dig.

Longitudo telescopii erit $=\gamma\left(1-\frac{1}{m}\right)=11,52$ dig. et semidiameter campi apparentis ob $\pi=0$ et $\pi'=0$ fiet $\Phi=-\frac{\pi''}{m-1}$, et sumto $\pi''=-\frac{1}{4}$ erit $\Phi=\frac{1}{96}$ in partibus radii vel $\Phi=35\frac{3}{4}$ min. prim., quem oculus uno obtutu conspiceret, si semidiameter pupillae aequalis esset semidiametro aperturae lentis ocularis, hoc est

$$=\frac{1}{4}\frac{\gamma}{m}=\frac{\gamma}{100}=\frac{3}{25}\text{ dig.};$$

alioquin si pupilla minor esset, in eadem ratione campus deberet imminui.

EXEMPLUM 2

147. Posita multiplicatione $m=50$ construere huiusmodi telescopium ex valore priore ipsius $\mathfrak{B}$.
Cum sit $m=50$, erit $\mathfrak{B}=+1,08925$, ex quo sequitur

$$B=\frac{\mathfrak{B}}{1-\mathfrak{B}}=-12,2045 \text{ et } \log(-B)=1,0865194.$$

Porro

$$C=0,2975 \text{ et } BC=-3,6308.$$

Cum igitur praecedentes formulae etiam nunc locum habeant, radii singularum facierum ita reperiuntur expressi:

$$F=-0,1740\gamma, \qquad G=-1,9478\gamma,$$
$$F'=+3,0375\gamma, \qquad G'=+0,1678\gamma,$$
$$F''=+0,6155\gamma, \qquad G''=+1,6333\gamma.$$

Horum radiorum minimus est $0,1678\gamma$, cuius parti quartae $0,0419\gamma$ aequalis statui debet semidiameter aperturae $x=my=1$ dig., ex quo definitur $\gamma=\frac{1}{0,0419}=23,87$ dig., ita ut statui possit $\gamma=24$ dig. Tum autem erit distantia focalis lentis ocularis $=\frac{-\gamma}{m}=\frac{-12}{25}$ dig. radiusque utriusque faciei $1,06\cdot\frac{12}{25}=0,509$ dig.

Longitudo ergo huius telescopii erit $=\gamma\left(1-\frac{1}{m}\right)=23,52$ dig. et semi diameter campi apparentis $\Phi=-\frac{\pi''}{49}=\frac{1}{196}$ et in minutis primis $\Phi=17\frac{1}{2}$ minut.

SCHOLION

148. Ad maiorem multiplicationem hunc calculum non prosequor, quia differentia prodiret tam exigua, ut ab artificibus vix videatur exsequenda; quare eadem exempla etiam ab altero valore pro $\mathfrak{B}$ invento evolvamus.

EXEMPLUM 3

149. Posita multiplicatione $m=25$ construere huiusmodi telescopium ex valore posteriore ipsius $\mathfrak{B}$.
Cum sit $m=25$, erit $\mathfrak{B}=-0,38954$ et $1-\mathfrak{B}=1,38954$, unde fit

$$B=\frac{\mathfrak{B}}{1-\mathfrak{B}}=-0,280337 \text{ et } \log(-B)=9,4476805;$$

deinde fiet

$$C = -\theta(1-\mathfrak{B}) = -\,4,6318 \text{ et } BC = +1,29850.$$

Quia igitur formulae pro radiis facierum manent ut supra, inveniemus eos, ut sequitur:

$$F \;= +\,0,486641\gamma, \quad G \;= +5,44632\gamma,$$
$$F' \;= +\,0,13523\gamma, \quad G' = -0,90450\gamma,$$
$$F'' \;= +1,07785\gamma, \quad G'' = -0,13719\gamma.$$

Inter hoc radios minimus est $0,13523\gamma$, cuius parti quartae $0,03381\gamma$ aequari debet semidiameter aperturae $x = my = \frac{1}{2}$ dig., unde $\gamma = \frac{1}{0,06762} = 15$ dig., ita ut telescopii longitudo $= \gamma(1-\frac{1}{m}) = 14\frac{2}{5}$ dig.; distantia autem focalis lentis ocularis erit $= -\frac{3}{5}$ dig., ita ut radius faciei utriusque $= 0,6360$ dig., et semidiameter campi apparentis erit ut supra $\Phi = 35\frac{3}{4}$ min., qui ab oculo uno obtutu vel saltim successive conspici poterit.

EXEMPLUM 4

150. Posita multiplicatione $m = 50$ construere huiusmodi telescopium ex valore posteriore ipsius $\mathfrak{B}$.

Cum sit $m = 50$, erit $\mathfrak{B} = -\,0,38983$ et $1-\mathfrak{B} = 1,38983$; unde colligitur $C = -\,4,6328$ et $BC = +\,1,2995.$

Cum igitur formulae pro radiis facierum maneant eaedem, ex iis facto calculo nanciscemur:

$$F = +0,48624\gamma, \; G = +5,44218\gamma,$$
$$F' = +0,13520\gamma, \; G' = -0,90338\gamma,$$
$$F'' = +1,07802\gamma, \; G'' = -0,13906\gamma.$$

Inter quos radios minimus est $0,13520\gamma$, cuius parti quartae $0,03380\gamma$ aequari debet semidiameter aperturae $x = my = 1$ dig., unde $\gamma = 29$ dig. et distantia focalis lentis ocularis $= -0,58$ dig. et radius utriusque faciei $= 0,6148$ dig. Longitudo ergo telescopii erit $= 28,42$ dig. et semidiameter campi $\Phi = 17\frac{1}{2}$ min.

SCHOLION

151. Etsi haec telescopia quatuor lentibus revera constant, ea tamen quasi tantum ex duabus lentibus composita spectare licet, propterea quod tres priores lentes in unam coaluerunt, ut lens obiectiva fieret triplicata et meliore successu loco lentium. triplicatarum perfectarum supra traditarum usurpanda, quandoquidem iam vidimus lentibus illis perfectis solam ipsarum confusionem utriusque generis annihilari, ita ut confusio lentis ocularis etiam nunc tota subsisteret; quamobrem lentes triplicatas hic in

usum vocatas data opera ita instruximus, ut non essent perfectae, sed ut iis etiam confusio lentis ocularis ad nihilum redigeretur, quae si modo artifex exactissime perficere posset, nihil amplius desiderari posse videretur. Verum duabus adhuc difficultatibus haec telescopia premuntur; altera est, quod tribus huiusmodi lentibus coniungendis crassities ita fiat modica, ut non amplius tanquam evanescens spectari possit, quemadmodum calculus noster postulat; unde, etiamsi artifex nostras mensuras exactissime exsequi valeret, neutiquam tamen perfectus consensus inter theoriam et praxin sperari posset; altera difficultas in angustia campi apparentis est posita, maximeque est optandum, ut campo maior amplitudo concilietur; quo igitur huic duplici incommodo consulamus, in sequenti capite hanc investigationem ulterius prosequamur, dum huius generis telescopiis revera plures duabus lentes tribuemus, quae omnes a se invicem certis intervallis sint disiunctae; ubi inprimis in hoc erit inquirendum, num hoc modo etiam utriusque generis confusio aeque feliciter tolli possit, deinde vera num hoc modo campus apparens magis amplificari possit, ac si praeterea longitudo horum telescopiorum minor prodiret; tum certe iis summus perfectionis gradus conciliatus esset censendus.