MEDITATIONS ON A NAUTICAL PROBLEM, WHICH THE MOST ILLUSTRIOUS KING PROPOSED TO THE PARISIAN ACADEMY OF SCIENCE

Indeed in everything we draw upon, we are led by the desire of acquiring knowledge and skill, and in which we consider beautiful to excell.
M. T. Cicero *de Officiis*

THE PROBEM : WHAT IS THE BEST WAY TO PLACE THE MASTS ON VESSELS, AS FAR AS THE RELATION BETWEEN THE POSITION AS WELL AS THE NUMBER AND HEIGHT OF THE MASTS IS CONCERNED.

1. Sailing generally depends on being driven forwards chiefly by sails alone, and not by oars, arising from the arrangement and placing together of the masts. Sails usually are connected to the masts by yardarms, and on being turned into the wind they drive the ship forwards on account of that force sustained. The implantation of the masts in the ship is to be investigated , so that a ship in general can proceed with the maximum speed, and in order that this can happen, the position, height, and number of masts is to be attended to in the most diligent manner. Because in the first place in this determination it is held that much study and labour be called upon so that, if the use of the rudder cannot be avoided, then the force of that must impede the motion of the ship as little as possible, as the action of the rudder always detracts from the speed of the ship. A line drawn in ships above the bilge water from the prow to the stern is called the keel of the ship, and in French, *la quille*, and into this the masts are inserted as in some manner to be in the middle of the ship. If the ship is moving along the direction of its keel, there is no need for the use of the rudder for the ship to be continuing in this situation, however the masts have been implanted in the keel. Now when the ship is not moving on a par with the keel, but the direction of motion of the ship makes an angle with the keel, and which angle of deviation is called in French, *l'angle de la derive*, [the *leeway* angle] then thus, that ship does not maintain the same angle of deviation or the same position, in whatever manner the masts are situated, but in order that the angle can be retained a special placing of the masts can be determined, which must then be the single position of the masts for any other angle of deviation. And thus with ships moving forwards in water, so that they arrive at the place of choice, now they are obliged to take one deviation and then another, to which some other position of the masts would have to be attributed. But since in ships with masts once put in place, as that cannot happen with the masts remaining immobile, this change is brought about with the aid of the rudder, so that the ship is maintained at the same angle of deviation.

2. But since the rudder must act, by which the resistance to the ship has to increase, and thus the speed of the ship is diminished, and there that is affected more and more by the rudder, therefore as it were by how much the position of the masts differs more from that position in which there is no need for the rudder. Hence lest the force of the rudder should increase excessively in the deviations of the ship that occur most often, such a position must be assigned to the masts in which the rudder is said not to be in use, so that

it is apparent that the speed does not decrease sensibly at any time arising from the action of the rudder.

3. Now however many masts are put in the ship, there is always a point in the keel of the ship where if the a mast is arranged with a single height which is equal to the sum of the heights of all the other masts and equiped with just as many sails, which produces the same effect, that point can be called the common centre of the forces propelling the ship. Now from the given position of the masts and from the forces exchanged from the wind with the aid of the sails, this centre is found easily, not in an unlike manner from that , in which the common centre of gravity of bodies placed on the same line is found, yet with this difference, because here the capacity of the sails of the masts is taken in place in which the weight of the body is taken in determining the centre of gravity of bodies ; and thus it is easier from the common centre of forces moving the ship to find position of the masts : and thus it what follows it suffices to determine a single centre, for this I observe, whatever number of masts should be inserted, they can be represented readily in the same way.

4. More masts are not inserted into ships, unless the height required is of so great a size that it cannot be had from a single mast, for then it is to be effected by more than should be made available from a single mast : hence since the height of the masts is sought, the height is not to be determined unless with a single mast, but rather by several equivalent masts acting together. For this height, when it becomes known, is to be distributed over all the parts, then those parts become so small or of such heights as masts of that kind can have ; and thus the number of masts may be found, and the position of these by the previous paragraph.
[This argument about the resistance of the rudder originated first from Johan Bernoulli and has been described in the book : *Essay d'Une nouvelle theorie de la manoeuvre des vaisseaux*. (Basle 1714). This work forms the first part of *Book II* of his *Opera Omnia*]

5. Now the height of the masts is to be determined by the capacity of the sails, which are the particular cause of the impelling force. Therefore a question has to be considered not only regarding the height of the masts, but concerning the height of the sails as well : indeed the height of the sails need not be contemplated, if only the force moving the ship along is considered, and indeed with the same force of propulsion remaining, we can consider whether that should be applied either altogether at a single point, or at several points separately, or in places with higher or lower masts. Now there a part of the force of the wind which inclines the ship forwards and immerses the prow deeper, which with higher masts in place increases the force that is applied to that effect: hence it is better with the sails made wider, in order that a sufficient amount of the force can be taken by the lower sails in place. Indeed if the higher sails are made shorter and with smaller widths then they extend themselves, and thus the force inclining the ship forwards increases; now since that is the case, the avoidance of this must be a proposition in the determination of the heights of the masts. Concerning how many and how high the masts should be made, there is a restriction put on this : the greatest attribute is the width, as long as the lower sails can be attached to the masts in place, unless here the force of the

wind is sensibly diminished, and some number of other circumstances regarding the sails allow that to happen.

6. Now this is to be observed lest the number of the sails be increased as you wish, for with too great an increase in the number of sails, it can happen that even if the ship is not capsized in the water, yet the prow is immersed further than the safety of the ship permits. In order that this is better understood, it is to be noted, whatever power of the wind applied to the sails, the force exerts a twofold effect on the ship, the one which propels the ship, and the other which inclines the ship, with the prow being immersed deeper ; one knows that this happens, as from the ship at rest being vertical, now while in motion it is inclined towards the prow, and thus with that the more with the greater wind, and in which the sails are placed at a higher point on the masts; from which it can come about that the propelling force of the wind be applied either higher or lower, so that the prow can be immersed further than is safe, or completely submerged.

7. Therefore lest the ship be excessively inclined, a limit has to be put in place how far the prow can be immersed without endangering the ship, and with which known, it is to be asked how great a force can be taken from the wind so that the ship can be inclined as far as this and no further, from which the greatest force is found by which the ship can be moved forwards, for is a greater is assumed, the ship might be at risk, because then is inclined further than is suitable: but if now the force is taken smaller, then the ship is able to move forwards at this stage without danger; hence in this way the maximum force propelling the ship is found, or the method of setting the masts in position is found, so that the ship can proceed with the greatest speed possible. And thus since I have this in place and with the height of the masts duly discarded, I can be persuaded to have satisfied the problem.

8. Hence I will establish my thoughts in these two chapters, and which by themselves are proposed to be solves, I assess carefully, and I will attempt a solution. Clearly in the first chapter I shall be concerned with the placing of masts, there I shall enquire about the position of the centre of the forces propelling the ship, where that can be assumed to be in the arrangement of the masts, so that it is most beneficial to the motion of the ship. But in the second chapter the height of the masts is treated, or perhaps concerning the height of a single mast, the equivalent for several ; certainly, not unless I consider a single mast to be erected, and I will seek that, and from its length easily found it can be estimated how many masts should be put in place, hence from the mast, or rather with the length of the sails, with the width of these given that can be foreseen by us, so that the ship without fear of mishap can proceed the fastest. And thus I advance a solution of this puzzle to the most illustrious and celebrated academy itself, thus as they are influential in all disciplines with erudition and sagacity, chiefly in physics and mechanics, but I ask and pray most humbly and abjectly that they may not disdain to read these small pages with care, and a judgement to be passed on them.

FIRST CHAPTER

CONCERNING GENERALLY THE PLACE WHERE IT IS ASSUMED THE CENTRE OF FORCES DRIVES THE SHIP

9. Since the ship proceeds in water propelled by the force of the wind, in order that the same position and deviation is conserved, and the ship is not rotated to the side on account of resistance on being carried through the water, it is required that the common centre of the forces propelling the ship be placed on a line in the mean direction of the resistive force, drawn from the water to the exact side of the ship, and since clearly this centre must be present on the keel of the ship also, it is to be assumed that this centre is the point on the keel, where it is cut by the line of the average direction of the resistance. Hence since that line of the mean resistance should be known, the centre of forces is known also, clearly the place where the masts must be arranged if only a single mast it to be erected.

10. If from the following chapter it is known that several masts are to be put in place on the ship, it happens now from what has been said, and thus the locations of these are found, initially they are to be gathered together on the keel and then with such distances from this centre, so that the sum of the products from the capacity of the wind on any mast by the distance of one of these to the centre, for one mast from the centre, is equal to the sum of the like factors from the other mast from the centre. Now since these sums of the factors are equal, the forces themselves remain in equilibrium , as the ship is not able to rotate about that centre. Hence on noting in this gathering together of the masts, that the ship always maintains the same deviation, thus so that there is no need for the aid of the rudder, as long as it is understood the same wind happens or at any rate as long as the wind, if the sails should be spread out exactly so that they constitute the surface of a plane, should hit the same surface of the sails clearly turned towards the stern of the ship ; but indeed the sails maintain the same position if they are completely spread out, and the ship also is directed towards the same location, however the wind should blow, but not with the line of the direction of the ship making an angle equal or greater than a right angle to the wind.

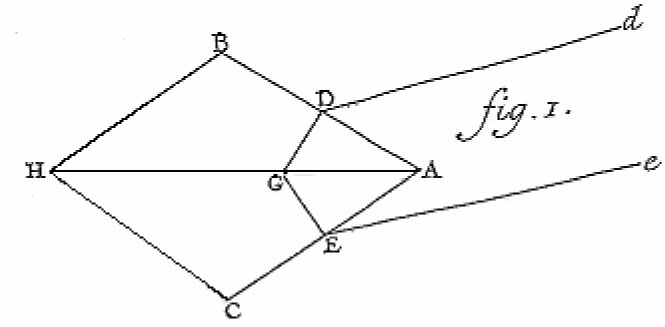
11. Now with the convenience of sailing it might be demanded that the ship be put in place in another deviation, since then the position of the mean line of the resistance changes, the position also of the centre of forces propelling the ship is assumed to be elsewhere, requiring to be applied nearer to the prow or to the stern, now I will examine how the change in the deviation of the ship changes the position of the centre of forces. Initially I put the angle of deviation in the first condition to be made larger, and the line of the mean directions of the resistance concurrent with the keel moves more towards the stern and then the centre of forces propelling the ship is assumed to be more towards the stern. Because if this cannot be done, nor can it be assisted by the rudder, then the ship does not remain in its present position, but on rotation the angle of deviation is increased, then the surfaces of the sails are withdrawn from the wind, now if that new deviation is

made smaller than the first, then the angle of deviation continually is diminished until it vanishes.

12. Now the rudder takes care of these impediments, because towards conserving the same deviation of the ship, for which there must be a greater impeding force, by which there must be a greater discrepancy between the common centre of forces taken with that which must be assumed . Now since thus the resistance is increased and the speed of the ship hence is diminished, it is possible for another remedy to this inconvenience to take place, by changing the location of the centre of forces itself, which can be done in two ways; firstly for the location of the masts themselves to be moved, and secondly moreover with the masts remaining and with the capacity of these to the wind changed with new sails now to be added above or with the expanse drawn together. But the first can be a remedy, if not all and at any rate a single mast may be rendered moveable ; which can happen and it may be tied in place with ropes in that place, so that the mast can move slowly towards the prow or the stern, for the smallest chance in position may suffice to transport the centre of forces sufficiently, especially if such a beginning were taken as the centre of forces, because the position is not a great distance from other centres which arise in other possible deviations of the ship. Hence since the angle of deviation is put in place greater and initially it should be, since then the centre of forces must approach the stern, this mobile mast is moving more towards the stern and then to this extent there is no more need for the rudder. If now the angle of deviation arises smaller, here the mast must be moved towards the prow.

13. If other circumstances do not permit the moving around of masts, it can be gone about in another way, clearly by the transportation of sails, or in the spreading out in one sail of new sails elsewhere, now so that the same total force remains the same with the masts shortened, for in this way also the centre of the forces is transferred to another place. And indeed since at first I might suppose the angle of deviation to increase, so that the centre of force approaches closer to the stern, the sails from the part of the centre towards the prow are to be diminished by contraction or at least by a lessening of the widths of certain sails and on the opposite direction from the other part of the centre towards the stern just as much new sails are to be extends or the height of the sails increased. Now in the other case with the decrease of the angle of deviation, the sales towards the stern are to be diminished and these towards the prow are to be increased. Now the rudder indicates how much is required to be removed or put in place ; for on adding or subtracting sails to the extent that the rudder need not be used any more. And then also the ship the ship is maintained in its position without the intervention of the rudder.

14. But whatever of these remedies it is pleasing to put to use, either in the first place by making moveable masts, or otherwise by a translation of the sails, or by neither of these but with the rudder, unless there should be a great need for the motion of mobile masts or the translation of the sails. In which case, if the third remedy is adhered to, where since that must be considered especially, then unless the rudder acts strongly and by which the speed of the ship may be diminished, such a position of the centre of the forces is to be selected in the construction of the masts, from which if the ship has other deviations, then the corresponding centres for these deviations do not differ very much. But for such a point as may be determined, it is necessary that the shape of the ship is drawn in the calculation, since the resistance of the water depends mainly on the shape of the sides which they thrust in the water.



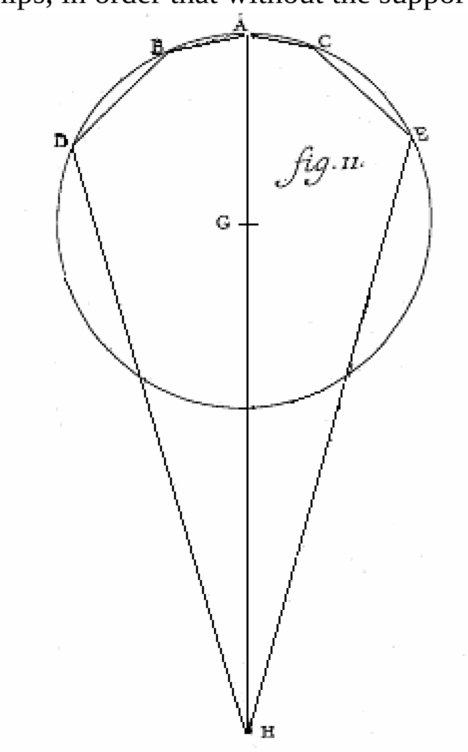
15. In order that we can reason from the simplest beginnings, let the bow of the ship be composed of two sides, straight lines, which supposition indeed clearly does not agree accurately with the ship, yet here for us where some non fixed point is sought, Fig. 1 may be able to shed a little light. Hence let *ABHC* (Fig. 1) be the shape of the ship, *A* is the prow of this and *H* the stern, *A* and *H* are the angles bisected by the spine *AH*, *AB* and *AC* are the equal sides and the sides of the stern are *BH* and *CH.* Let *AB* and *AC* be the sides of the ship exposed to the resistance, and these alone, because that always happens if the angle of deviation of the ship is less than half the angle of the stern *H.* Let *Dd* or *Ee* be the direction of motion of the ship, the ship is in contact with the water along this direction, or since the same thing comes about, and it is easier for the sake of understanding, I suppose the ship to be at rest and just as the water impinges on the ship, in the same direction *dD* or *eE* with the same speed, clearly on the sides *AB* and *AC*, neither of the sides *BH* or *CH* is able to be struck since the angle of deviation that *Dd* makes with the keel *HA,* is less than half the angle of the stern *H.*

16. It is to be observed from hydrostatics that the water resistance to be exercised normally to the same sides, and since the water on the same side *AB* and *AC* is incident everywhere at the same angle, then the centre of the impressed forces on the same sides *AB* or *AC* is at the middle *D* and *E* of these. Hence I consider the whole resistance as gathered together at these points, and the direction of the resistance is normal to the side *AB* with the line *DG,* and at *AC* with the line *EG,* which are in turn normal to the sides *AB* and *AC*. The common centre of the resistance is where the directions of these two directions mutually cut each other ; but they are concurrent as clearly it is on account of the equal sides *AC* and *AB* at the point of the keel *G* through which passes the line of equilibrium of the mean directions of the resistance ; but whichever position this line has, that cuts the keel *AH* at the point *G.* Hence the point *G* is that centre which is sought, for

which this is to be noted, that it is always constant, whatever the deviation of the ship shall be, with the size of this angle not exceeding half the angle of the stern *BHC*.

17. Hence if a shape of this kind is attributed to ships, from this the maximum convenience is obtained, since with the fixed location of the centre of forces remaining, the ship is able to remain in position at some angle of deviation without the aid of the rudder, with the sails once duly set, but as now it has been noted a number of times, the angle of deviation shall be less than half the angle the angle of the stern. And if from these things greater angles of deviation are taken, there also greater stern angles are able to be constructed, according to that, so that water never laps on the sides *BH* and *CH*. Now just as the point *G* is defined, it is readily deduced, clearly on each side bisecting the other of both sides composing the bow of the ship, and from the point of the bisection on the same side a perpendicular is erected, that is established which is sought ; for the point *G* is where that perpendicular cuts the keel of the ship.

18. If this shape is considered inconvenient on account of other causes which can be attributed to the ship, I am able in addition to indicate other shapes, which can be given to ships, in order that without the support of rudder, with fixed masts and sails, the ship can maintain the same deviation, or so that the common centre of forces remains located in the same place ; for nothing other is required for this to happen than that : for the shape of the ship arising ; the water bearing right lines [*i.e*. the shape of the prow or bows]; the perpendiculars from the mean points of the ship's prow; everything agrees on the same point on the keel on that same side, or so that all the sides themselves are chords of the same circle having the centre on the keel of the ship, for then at this centre all the perpendiculars agree on, for whatever water bearing side of the ship, from which that centre is the centre of forces sought. Let *ACEDB* (Fig. 2) be a circle, *G* the centre of this which is taken as the keel of the ship *AGH*. [Recall that *A* is the prow.] The chords are drawn from each part of the keel, however many it pleases such as *AB*, *BD* and *AC*, *CE*, and the lines are drawn constituting the prow *DH* and *EH*, having the shape of the ship asked for originally, and in order that the centre of forces remains in the same place, however with the change in the angle of deviation, but in deviating towards the angle *E* direction, it cannot exceed the angle *AHE* and in a deviation towards the angle *D* direction, the angle *AHD* cannot be exceeded; now the centre of the forces is at *G*.



19. Indeed this can be used in the construction of ships. but since the question is not concerned with this, we are more inclined to agree on an accepted shape of ships. I observe that after Johan Bernoulli, the most celebrated of men, Fig. 3, as being two circular segments on the same chord; now in this hypothesis for any angle of deviation the centre of forces is determined with much difficulty, since therefore because the sides of the ship perceiving the resistance are changeable into another angle of deviation, then because the shape is curvilinear, and thus the angle of incidence at any point is different, Here as there is no need for me to have a known centre of forces for any angle of deviation, by necessity it is not the way to treat the determination of the centre of forces under this hypothesis, for whatever angle of deviation, but it suffices if at least two centres at two deviations are taken of which one of the possible points is a maximum, and with the other now determined a minimum, which two centres are able to be as good as a boundary, between which that point is to be determined of the common centre of forces to be taken, which is being sought. Hence I assume these two deviations, that of the possible minimum or that the angle of which is zero or approaching zero, and the other the possible maximum for which I take a right angle or 90 degrees, beyond this angle the deviation of the ship cannot increase, since the stern is changed into the prow, and the prow into the stern. If I have determined the centre for each, I am sure that which is sought is contained, but it can be assumed to be more towards the centre from the first deviation, which is found to be nothing, than towards the latter, where the direction of motion of the ship makes a right angles with the keel, since the usual angles of the deviation of ships are always closer for a vanishing angle than for 90 degrees. And at once since it is free to assume between these two centres what is desired or what is the common centre of forces in the usual maximum deviations, such too has to be assumed, which easily and without much labour can be constructed.

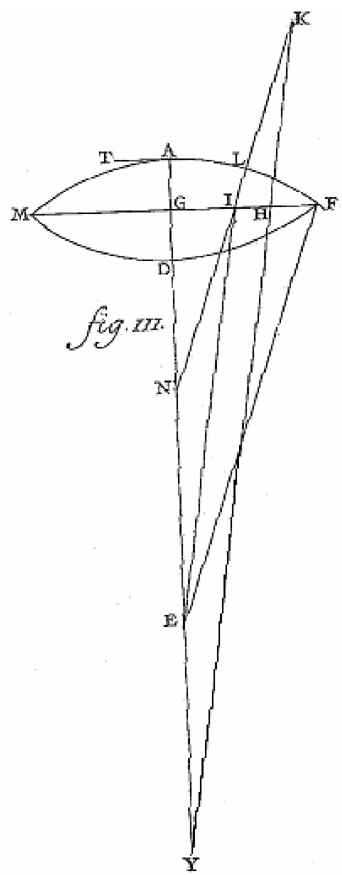


fig. III.

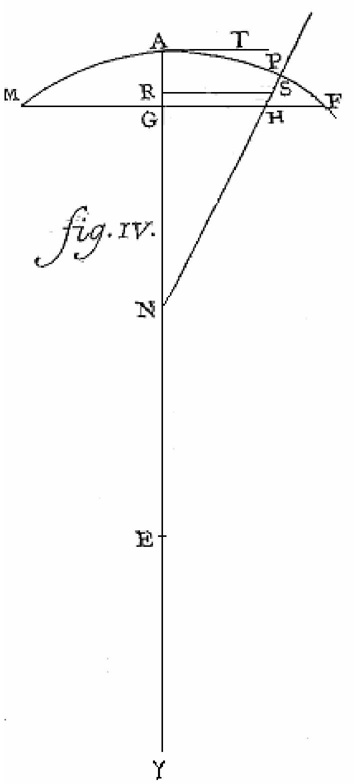
20. Thus I will investigate the first centre, when the deviation is 90 degrees. Let *FAMD* (Fig. 3) be the ship, *F* the prow, *FM* the keel, *N* the centre of the arc *FAM*, from the centre *N* the line *NGA* is drawn bisecting the keel at *G*, that also bisects the arc *FAM;* and it is normal to the keel. Hence the ship is moving along the direction *NA* in water, thus so that the angle of deviation plainly is 90°, because the arc *AM* is similar and equal to the arc *AF,* and as much resistance is apparent here, the line *AN* is the line of equilibrium resistance, and thus the point *G* where the keel *F M* is cut by *NA* to be the common centre of the forces, in that deviation of the ship. And thus I have now the common centre of the forces of the ship when the motion of this makes an angle of 90 degrees with the

angle of the keel; moreover for the vanishing deviation to define that centre is a much more arduous task, from which I do not add here my analytical construction that I give, lest I should be exceedingly prolix, but I will adjoin the demonstration of this by the most celebrated Johan Bernoulli from his *Manoeuvre des Vaisseaux*.
[Ch. X. This is a fairly long analytical derivation more tedious than difficult which I have not examined in detail: perhaps it is analogous to the derivation of the centre of gravity of an arc, where weight replaces water resistance along the tangent direction, and the position of the centre is found by integration. The problem is presented in two dimensions only.]

21. Thus we place the ship to be moving in the water along the direction of the keel *MF*, now indeed wherever it lies, the centre of forces is always taken on the keel, this zero deviation of the keel is going to remain the same. But that point on the keel *FM* is sought in which in the keel is cut by the line of the mean equilibrium direction of the resistance of the arc *FA* only, which is offering resistance on the part of the keel *FM* alone ; for at *A* the direction of the ship is along the tangent *AT*, through which the resistance is borne ; and indeed at the same point of the keel *FM* by which the arc *AF* is cut by the line of the equilibrium resistance, it is also cut by the line of the mean direction or of the equilibrium resistance as the arc *DF* is carried along, because these two arcs *AF* and *DF* are similar and both experience the resistance of the water equally. And hence that point, at which the keel *FM* is cut by the line of equilibrium resistance of the arc *AF*, now is the centre of forces of the ship, since the deviation has vanished. And this point also is the boundary of all the centres in all the deviations of the ship ; towards the prow or that centre before all others approaches nearest to the prow.

22. Moreover I determine that centre as follows. From the centre *N* the line *NL* is drawn bisecting the arc *AF* at *L*, and the keel *FM* at *I;* that is produced as far as *K* in order that *IK* = *IN*, also the radius *AN* is produced, and on that the points *E* and *Y* are taken, so that *EY* = *NE* = *AN*. The points *E* and *I* are joined by the line *EI:* and to this the parallel line *KH* is drawn from *K*, which produced passes through the point *Y*, for since *KI* = *IN* it crosses that line produced to some point that is as far from *E*, as the amount *E* is distant from *N*, on account of *NI* = *I K;* hence this point is itself the point *Y*. But the point *H* on the keel of the ship FM, where that is cut by the line *KY*, is the common centre of forces, since clearly the ship is moving along the direction of the keel.

23. The desired account of this construction is taken from the most celebrated Johan Bernoulli's *Manoeuvre des Vaisseaux,* from Chapter XIII. paragraph 4, where the centre of the mean resistance is determined for whatever arc comes to mind. Which paragraph, lest the most illustrious judges have a need, so that they have a description of this, (a construction of this demonstration is provided by me below), thus I have added the words of the most celebrated author together with the diagram of this : " Let the arc *APF* of some circle be given (Fig. 4) moved in the water along the tangent *AT, N* is the centre of this arc; *NA* the line to the point of contact, *F G* the perpendicular on *NA* ; *AE* the diameter of the same arc *APF*. Extend *AE* to *Y* in such a way that *EY* is equal to the line. Take *NR* equal to three quarters of the third proportional from *YG* to *EG.* Raise the perpendicular *RS* and make that equal to three quarters of *GF.* Finally draw *NS.* I say that the point *S* is the centre of the average resistance, and *N S* the axis of the average resistance."



24. Hence that line of the mean equilibrium resistance *NS* where that cuts the keel *FG,* clearly there at *H* will be the common centre of the resistive forces. But from my construction the same point *H* can be found from that since the line *GH* in each construction can be determined equally, as I demonstrate thus. In Bernoulli's construction, there is

$$GH = \frac{RS \cdot NG}{RN}.$$

on account of the triangles *NRS, NGH* ; moreover

$$RS = \frac{3}{4}GF \text{ and } NR = \frac{3}{4}\frac{EG^2}{YG}.$$

From which with these put in place, there is found

$$GH = \frac{GF \cdot NG \cdot YG}{EG^2}$$

25. Now from my own construction (Fig. 3) established from the work of Bernoulli, then

$$GH = \frac{GI \cdot YG}{EG}$$

on account of the similar triangles *EGI* and *YGH;* for the lines *EI* and *YH* are parallel. *EF* is drawn, that is parallel to the line *NL,* for *LN* bisects the arc *AF,* from which since *N* is the centre of this arc, hence the arc *AL* is a measure of the angle *ANL;* now since then *NA* = *NE,* the point *E* is on the periphery of the same circle and thus with the measure of the angle *AEF* is half the arc *AF,* that is, the arc *AL;* hence the angle *ANL* is equal to the angle *AEF*, and thus the line *NI* is parallel to the line *EF;* hence the triangles *NGI* and *EGF* are similar, on account of which then

$$GI = \frac{GF \cdot NG}{EG}$$

which substituted into the above equation in place of *GI*, there comes about

$$GH = \frac{GF \cdot NG \cdot YG}{EG^2}$$

And thus since in figures 3 and 4 the letters are placed adjacent to the same corresponding points, then *GH* in figure 3 is the same as *GH* in figure 4 and thus the same point *H* also is in each figure. From which that is concluded to have been determined correctly by me.

26. Hence the two limits of the centres have been determined, clearly the points *G* and *H*, between that centre which is sought can be assumed, at which the masts on ships are to be arranged in place. Now that point is to be taken closer to the point *H* than to *G*, since the deviations of the ship more often are within the angle 45 degrees as that they can overcome. Moreover the point *I* between the points *G* and *H* now is to be determined (Fig. 3), because I note the point is always closer to the point *H* than to the point *G;* for the distance *HI* as itself to the distance *GI* the ratio *EY* to *EG,* that is, as *EY* is equal to *EN*, that ratio is as *EN* to *E G* which is always a smaller inequality. From which I say if that centre sought is assumed around the point *I*, not to have strayed far from the goal ; for besides because it is nearer to the point *H* rather than the point *G*, the same can be taken with that which is being found, if the sides *AF* and *DF* are considered as straight lines, the centre is to be determined also : for the point *I* here is determined from the bisection of either side *AF* and from the point of bisection *L* in *AF* a normal is erected, for the point at which there is the concurrence of the line *LN* and the keel *FM*, that is the point *I* itself. Hence that point is easily found in the future for obtaining the centre.

27. Hence it is evident, not without a reminder from me that the force of the sails is to be greater towards the prow, than the stern, since the centre *I* always is to be found at the prow of the ship. And thus if on a ship with only a single mast erected, that must be put at that point *I*. If two masts, the one at one side of the point *I*, the other from the other side, at such distances from *I* which are reciprocally as the forces which they receive from the wind. The same thing is found if more masts are to be erected. And thus the optimal position of the masts and the most useful has been pointed out. It remains plainly in ending this chapter, that I can add the masts at such an angle to the horizontal.

28. Since vertical masts receive the wind at right angles, if without doubt the line of the wind is perpendicular to the plane of the sails, which is the maximum force of the wind, as which increases in the squared ratio of the sine of the incident angle with all else being equal, and as the sails are happy to receive the maximum force of the wind to be propelling the ship, therefore without a long inquiry the masts are to be set up thus, in order that when the ship should be in full motion, then the masts are vertical. And thus since the angle is given at which the ship must be inclined, the masts from the beginning

towards the stern must be inclined at this angle in order that the ship can be moving in full motion, and the prow is being submerged at a given angle, then the masts are now made vertical by ropes with a force towards the stern, since they must be extended fully to sustain a greater force from the wind, from which it happens that the masts immediately are inclined towards the prow, but intelligent shipwrights are able to find solutions to this hindrance as well as to others that can arise.

SECOND CHAPTER

CONCERNING THE HEIGHT OF THE MASTS, OR THE SIZE OF THE FORCE PROPELLING THE SHIP.

29. If the ship is propelled by the wind blowing into the sails, it is agreed that the force exercised be experienced in a two-fold manner. One by which the ship is moved forwards, now the other by which the ship is inclined towards the prow or by which the prow is immersed deeper. The influence of sails are employed to bring about the first, unless the ship be propelled by labourous rowing. Now the latter effect is inconvenient in sailing, since on that account the impelling force cannot be increased as it pleases, as the prow can only be immersed so far without danger.

30. But with the removal of this obstructing inconvenience, and the ship placed beyond all danger, how great an expanse of sail is to be admitted which may be made which enables the ship to be inclined at a certain fixed position, and in which situation it can persevere without any change ; since hence that inclination of the ship not only depends on the amount of sail, but also and particularly on the place of application and on the width of the sails, is to be determined between all those cases in which the ship at a given degree or is inclined at a given angle of inclination [to the vertical], with that which moves the ship forwards the fastest, or which admits the maximum abundance of sail ; for this case, plainly to be in order that how much faster the ship can be moved in the absence of danger.

31. Since accordingly an angle of inclination is proposed or that angle, that these must constitute on the ship with a vertical line, which with the ship at rest itself becomes vertical, is required in order that the amount of sail applied to the masts, for the ship to be precisely equal to the proposed angle of inclination. Now towards determining the size of this force itself, since whatever the force of the wind it lays bare a two–fold effect on the ship, it is necessary that at first we inquire how great a portion of the force of the wind is designated to send the ship forwards and how much to the inclination of the ship. But how I may find this, I argue in the following manner.

32. In the first place, since I foresee that the resistance of the water confers a great deal to that effect, I consider the water to offer no resistance but the ship is to be moved most freely, yet with the same weight of the water. It is apparent in this hypothesis that no portion of the wind is taken up in inclining the ship, but the whole force of the wind is in service to be propelling the ship ; for we can put the ship to be inclined only slightly,

clearly from the ordinary situation in which the centre of gravity has descended to the lowest point to which it is possible, being turned slightly, it is apparent that the ship cannot remain in this position as the speed of the ship is reduced ; for since the ship cannot persist in this unnatural position, again it tries to revert to the natural position ; because in a two–fold manner this can happen, either if the masts go back and that raises the prow again from the water, then the natural position is obtained ; or moreover if the ship itself by progressing faster as the masts are freed from the more restricted position thus restores itself; the form proposition cannot happen since the wind does not permit the masts to go back, the latter the ship can carry out easily, since no resistance is found, which is able to stand in the way of that restoration, and thus the ship in this manner plainly is not inclined in the water as it progresses without resistance by the force of the wind brought to bear in any amount and thus the total force the wind exerts on the sails is taken up in moving the ship forwards, and none in the inclination of the ship.

33. Now I pass to the other extreme and I suppose the water produces an infinite resistance on the ship, clearly it is possible to consider the water changed into the hardest ice, but the ship to persevere in the smoothest hollow to this, for in this manner it comes about that the ship is unable to move on account of the infinite resistance of the water, but yet the ship is able to be inclined ; for there is no resistance to the motion on account of the perfectly smooth surface of the ice. And thus the whole expanse of the sails is kept busy on inclining the ship.

34. With these two extremes considered, I arrive at ordinary water, which is as a mean between these two extremes ; for plainly neither no resistance is directed towards the ship nor an infinite resistance, from which now it is now able clearly, since the water participates to some extent from each of the extremes, the force of the wind must both propel the ship and also incline the ship. Hence it is to be considered carefully what proportion of the wind goes into moving the ship forwards, and how much is occupied in the inclination of the ship, which two proportions must be equal to the total force of the wind, since their effects may be produced along the same directions. Accordingly the force of the wind propelling the ship has been increased by the force of the wind in inclining the ship to equal the total force of the wind.

35. If the effect of the wind is considered otherwise, it is apparent that the power of the wind is used up in overcoming the resistance of the water, and part in moving the ship forwards; which two parts, since they also produce their effects along the same direction, likewise are taken to equal the whole force of the wind. Hence on comparing the same distribution with that we have established in the previous paragraph, we find the sum of the forces of that wind which inclines the ship and of that which moves the ship along, to be equal to the sum of the forces of that wind which overcomes the resistance of the water and of that which moves the ship along; on taking away the force of the propelling wind from each equation, there arises the force of the wind overcoming the resistance of the water to be equal to the force of the wind inclining the ship. And thus it is apparent how great a force may be devoted to the inclination of the ship, clearly of such a size as the amount that is equal to overcoming the resistance of the water. Hence since the resistance of the ship is in the square ratio of the speed of this, also the force dedicated to

overcoming the resistance of the water is in the square ratio of the speed of the ship ; hence from which the faster the ship proceeds, there more also is the ship inclined, and in the initial motion the speed of the ship is infinitely small, and also the inclining force is infinitely small, and with the speed of the ship increasing the angle of inclination becomes greater.

[There are many difficulties in applying Latin to describe ideas not prevalent in Roman times, although new words were gradually introduced; thus, the word power *potentia* really meant originally the sort of political power or influence held by a ruler, while the word force *vis* designated forces in the sense of armed soldiers to do the ruler's bidding. Euler is highly anthropomorphic in his choice of words describing the actions of inanimate objects, so we have the word potential as a sort of general power or influence, while the word vis is the actual mechanism that carries out the action. A complication is the modern use of the word *power* in physics as the rate of supply of energy, which is not meant at all in the above paragraph.]

36. Just as falling bodies little by little acquire a greater speed from the force of gravity continually acting on those descending nor suddenly on those is the final speed acquired to be received and in the same way a piece of wood thrown into the torrent indeed initially has an infinitely small speed, now that is continually increased, thus also with the wind blowing into the sails initially the speed of the ship is infinitely small, but that increases continually, until finally it as acquires such a speed that it cannot be increased further, for if the water offers no resistance to the ship, at last the ship acquires a speed equal to that of the wind, but with resistance from the water the speed finally after an infinite time acquires a speed less than the speed of the wind, clearly so much less so that the remaining speed of the wind attacking the sails is exactly equal to that required to overcome the resistance. I say that after an infinite time has passed, but now only after a short interval of time, the speed of the ship only acquires a speed which sensibly cannot be increased further.

37. Hence since the ship proceeds with an accelerated motion, the resistance also increases and then the force dedicated to overcoming the resistance also increases ; and hence also the force inclining the ship, as thus the angle of inclination continually increases until finally since the ship remains with the same speed, it continues unchanged ; but with the ship preceding uniformly, the whole force propelling the sails is taken up in overcoming the resistance of the water, and then also the whole force of the wind, since the ship should be with the maximum speed, is used up in inclining the ship.

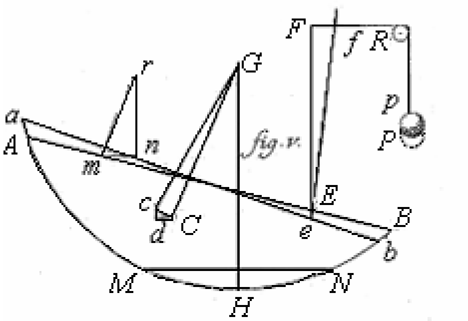
38. But since the angle is proposed at which the ship must be inclined, without doubt at a distance this maximum angle must be the maximum of these at which the ship can be inclined, or it must be the angle of inclination with the ship in full motion ; for if that angle should be equal to the angle of inclination soon after the start of the motion, then the angle of inclination without pause would increase further, and finally be much greater and equal to that proposed ; hence we have taken to recognise the maximum angle of inclination as the final angle, clearly with that given we investigate the size of the force exchanged by the wind which shall only be equal to the proposed angle of inclination

finally, or since that angle then remains constant, the force is required which the ship is able to maintain as far as this angle of inclination.

39. So that I can reveal this more easily, I assume that only a single mast of the ship to be fastened in place, and with the point of this in some place, around which wherever the sails are turned and hence the forces of the wind are equally spread, I consider the whole force of the wind admitted to be gathered together, which point hence is the equivalent of the common centre of the sails, just as it is called later also. So that moreover I can find the force more easily on the ship at the proposed angle, in place of the wind in the computation I draw a weight that I consider applied at the same common centre of the sails, and the mast horizontally, which can happen with the aid of a pulley exerting the force, and thus the weight is to be determined, which is equal to the force on the ship at the given angle of inclination, with which in place afterwards I draw on the comparison of the force of the wind with weights, as in place of the weight found, I introduce the wind again into the calculation, and thus I can determine what size of a force is to be taken from the wind in order that the ship is inclined at the proposed angle.

40. But as now it is to be noted how great the forces shall be at the inclination of the ship. hence I am able to consider the ship as at rest, or what amounts to the same thing, I consider the water as if frozen into ice, thus still slippery so that the ship can be reclined freely in its hollow without any restriction of the inclination ; for in this manner the ship can be considered as set up in a medium of infinite resistance, and thus by that force alone, for which inclination of the ship serves to be applied at the centre of the sails and inclines the ship in the same way, as if the ship were proceeding ordinarily in water. Hence here also, where I lead the weight in place of the wind in the calculation, I consider the ship arranged in ice in the same way, and I find the weight that the ship at the proposed angle of inclination can have.

41. But it is not sufficient in finding the weight sought to put in place the angle of inclination ; for in addition it is required that the shape of the ship be known, at the weight and the centre of gravity of this. So that the weight of the ship and the position of the centre of gravity can be attained, I generally treat these according to whatever special cases they can be applied to; by the weight of the ship I do not understand the weight of the empty ship but of the laden ship, and in the same manner I understand the centre of gravity of the laden ship. But with regard to the shape of the ship, I consider the keel as curved in the arc of as circle, in that manner it is equal to the arc of a circle, which reaches into the water; it is sufficient that the radius of curvature is taken in the calculation, or rather the distance of the centre of curvature of the keel from the centre of gravity of the ship. If the curvature of the keel is not exactly circular not much matter much, but for that curvature a circular curvature is to be assumed that lies close to that.



42. With these in place let *AMHNB* (Fig. 5) be the ship or rather the keel of that, *B* the prow and *A* the stern, *MN* the surface of the water : and let the ship thus be inclined so that the line *mr*, which in the horizontal state of rest of the ship is perpendicular with the vertical *rn*, now the angle *mrn* is made. Let *C* be the centre of gravity of the whole ship, and *G* the centre of the arc *AMNB*, or if the arc *AMN B* should not be exactly circular, *G* is the centre of the circular arc of curvature approximately equal to the curvature of the keel or such an arc which passes through the points *M* and *N*, and the segment under the chord *MN* is taken, equal to *MHN; GH* is the vertical line in place on the ship which is normal to *MN* and thus also so that it bisects the arc *MHN* . *GC* is the distance of the centre of gravity *C* from the centre of curvature *G*. *EF* is the vertical mast in which *F* is the common centre of the sails, at that point in place of the wind there is applied the weight *P*, since around the pulley *R* the mast is drawn along the horizontal direction *FR*, how great the weight *P* ought to be is sought as the ship is able to be kept in that position.

43. In the natural position of the ship the centre of gravity *C* descends to the place as low as possible. But it is apparent since an equal arc *MHN* is held always under *MN* or maintained by the surface of the water, that the centre of gravity *C* is unable to descend more than it is in the vertical position *GH;* for since the distance *GC* always remains the same and the point *G* also is unchanged, the whole mass of the ship being considered to be congregated at *C*, it is evident that the pendulum *GC* cannot remain at rest unless the point *C* is on the vertical line *GH.* Hence the line *GC* would be vertical in a state of rest, from which the angle *CGH* is the angle of inclination of the ship and thus equal to the angle *mrn.*

44. But in order that I can find the size of the weight *P* because with the ship remaining in equilibrium in that unnatural state, I put the weight *P* to descent a little amount along the infinitely small line element *Pp;* since the ship is supposed to be unable to progress on account of the water changed into ice, it turns a little in its hollow about the centre of the cavity *G* in order that from the position *AMHNB* it arrives at the position *aMHNb*, and the mast *EF* at the position *ef;* thus in order that $Ff = Pp$. The centre of gravity *C* arrives at *e*, thus in order that with *Gc* drawn the angle *CGc* is equal to the angle *FEf.* From *c* there is sent the vertical *cd*, crossing the horizontal through *C* at *d* , and the centre of gravity of the ship rises through the height *cd*, but the triangle *Ccd* is similar to the triangle *rmn*, for since the line *cd* is parallel to the line *GH*, then the sum of the angles *Gcd* and *HGc* is equal to two right angles ; now the angle *CcG* is right , hence the angle *Ccd* plus the angle *cGH* constitutes a straight line; but since the triangle *Ccd* is right angled at *d*, then the sum of the angle *Ccd* et *cCd* is also equal to a right line, from which the angle *cCd* is equal to the angle *HGc,* or since they differ only by an infinitesimal part from the angle *CGH*, or from the angle *mrn,* in addition the angles *d* and *n* are equal, because each is a right angle, from which the triangles *rmn* and *Ccd* are similar.

45. But is to be observed from mechanics that two weights, themselves placed in some manner, maintain equilibrium as even with a small change in their position, the ascent of the centre of gravity of one itself has a reciprocal ratio to the descent of the centre of gravity of the other, as the first weight to the second weight, or directly in the ration of the second weight to the first weight. With this applied to our example, since the ship and the weight *P* must maintain themselves in equilibrium also, then the weight of the ship that will be called *Q*, is to the weight *P* as the descent of this *Pp,* is to the ascent of the centre of gravity of the ship *cd,* from which $p \cdot Pp = Q \cdot cd$, or on account of *Pp* = *Ff* then $p. Ff = Q \cdot cd$.

46. But because the angle *FEf* is equal to the angle *CGc*, and the angle *EFf* is right on account of the vertical *EF* and the horizontal *FR* , the triangles *GCc* et *EFf* are similar and thus

$$Ff : EF = Cc : CG, \text{from which } Ff = \frac{EF \cdot Cc}{CG}$$

consequently

$$P \cdot EF \cdot Cc = Q \cdot CG \cdot cd, \text{or } P = \frac{Q \cdot CG \cdot cd}{CG \cdot Cc} \;;$$

now on account of the similar triangles *rmn*, and *Ccd* , then *Cc:cd* = *rm:mn*, that is, as the whole sine to the sine of the angle of inclination, which ratio since that is proposed, is put as 1: *s*, then

$$P = \frac{Q \cdot CG \cdot s}{EF}.$$

Let the distance of the centre of gravity *C* from the centre of curvature of the keel *G*, surely be *CG* = *b*. *E F* is half the height of the mast since *F* is the centre of the sails, and the sails everywhere are supposed to be of the same width ; but the whole height of the mast is put in place (clearly of one, for that must be equivalent to several if more are inserted in the ship) which is put by us equal to *z*, hence $EF = \frac{1}{2}z$, and there is had

$$P = \frac{2Qbs}{z}.$$

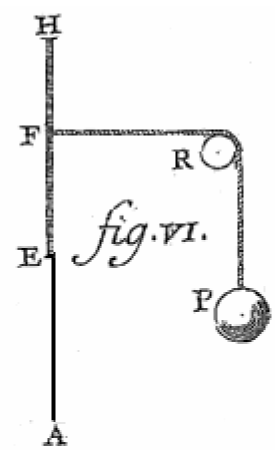
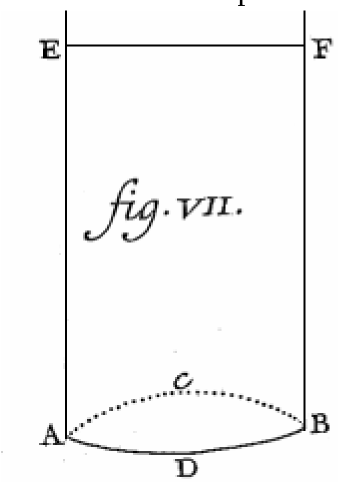


fig. VI.

47. Hence the weight *P* has been determined that the ship must maintain at the given angle of inclination ; for this weight an equivalent force must be extracted from the wind : hence a definition is necessary for that too so that first I examine what ratio the force of the wind has to the weight, or in order that I can express the force of the wind in terms of terms of weight. Indeed this can be put in place from experiments, now also I will show that it is possible to deduce the proportion from theory. The experiment can be done in the following way. Let some short mast be made *AH* (Fig. 6) moveable around the point *A*, and to this is tied a plane sail *EH*, that is set out in the wind, which strikes on that along the direction *RF*, and the mast tries to rotate

around the pole *A*; but a small rope *FR* is attached at the point *F* at the centre of the sail , which is drawn by the weight *P* around the pulley *R* thus in order that the mast is pulled back by this force, and moreover the weight *P* to be added or subtracted from that can be determined from experiment and then the sail remains in a vertical position, and then *P* is the weight equivalent to that wind blowing into the sail *EH*, and since the capacity of the sail and the speed of the wind have been noted, from that easily the comparison can be put in place for other winds and sails, either greater or smaller.

48. But generally the ratio between the force of the wind and the weight can become known in the first place from theory, as the I will embrace the argument more generally, I move away from the wind or the air and in place of that I consider a liquid of some kind, and I set out to compare the percussions of this with weights. Let *EADBF* (Fig. 7) be a cylindrical vessel fills with this liquid as far as *EF*, moreover the base *ACBD* is horizontal, it is apparent in the first place that the bottom from the pressing liquid, thus so that with this bottom pierced somewhere, the liquid can escape with such a speed as it has acquired by falling from the height *FB* . Just as the celebrated Herman made available in the appendix of his *Phoronomia*, and was first demonstrated publicly by the most celebrated Bernoulli, the bottom hence sustains the same pressing force of the liquid bearing down, and it should strike with the same speed [as a body that had fallen through the same height] if the liquid should flow out through the opening.



49. But the citation of the most acute Bernoulli has been pointed out recently in the book of Michelotte : *De separatione fluidorum* that the liquid flowing through the opening is agreed to have perhaps half the density of that present in the vessel ; indeed between two globules or atoms of the liquids emerging there is nothing but empty space, thus so that the globules which were in contact within the vessel, on passing out are moved apart, thus so that in equal intervals perhaps half of the fluid emerging from the opening is contained, compared to that in the vessel, from which reasoning is returned the celebrated phenomenon of the contraction of the thread of liquid erupting from the vessel. Hence on applying this idea to our case, it may be said that the base of the vessel by bearing the pressing force of the liquid contained in the vessel, and for the same to be sustained equal to that weight if a liquid twice as rare [*i. e*. half as dense] erupted from that opening with a speed which it could acquire on falling from that height *FB*.
[The phenomenon not understood at the time was surface tension, responsible for shrinking the diameter of the jet ; the argument about half the density is thus erroneous. However, there was a need by Euler to amend the formula for the force of the wind to include the effect of density, for which a factor of 2 was introduced, in order that the formula agreed numerically with the experimental results of the time. This has been done in an ad hoc manner, although the final formula to be presented is basically correct; meanwhile, we will rest with the assertion that the density of the flowing liquid must be doubled to find the pressing force on the base of the vessel.]

50. Hence I have the ratio or proportion between the weights and the force of percussion of the liquids ; and from these indeed it can be concluded, since some liquid with some speed has erupted directly or perpendicularly on a plane, the plane sustains the same force as if it were placed and should experience the pressing force of a liquid twice as dense and of such a height, with the weight falling from which it is able to acquire a speed equal to the speed of the liquid flowing : hence since the weight of this liquid becomes known with twice the density, for the said bases on equal planes and height, the equivalent weight of the liquid flowing is obtained.

51. This may be applied to the wind, and it is apparent that the sail is supported directly also by the air being removed, as if in the horizontal position, the sails might bear the pressing force of a liquid with twice the density of air, and from a height for which the weight falling should be able to acquire a speed equal to the speed of the wind. Let v be the speed of the wind, clearly that which the sail requires or with the relative speed. Now from experiment it is agreed that a weight on descending from a height of 15 Rhenish feet gathers a speed by which it is able to traverse a distance of 30 feet in a time of one second ; as the speed of the wind is v, with this being accomplished, or from the distance traversed in the given time that we can measure, v designates the number of Rhenish feet that it can traverse in a time of one second. [A Rhenish foot is approx. 3% longer than an Imperial foot.]

52. Since the heights in the descent of the bodies are as the squares of the speeds acquired, and the body on descending from a height of 15 feet acquires a speed as 30, this becoming as 900 the square of 30, to vv, the square of the speed of the relative wind, thus as 15 feet is to [the required height h, or $\frac{h}{15}=\frac{vv}{900}$ or $h=\frac{vv}{60}$] becoming $\frac{15vv}{900}=\frac{vv}{60}$ feet, which is the height from which the body falling is able to acquire a speed equal to the speed of the wind v.
[Thus, the matter of the speed relating to the height is attended to easily.]

53. Thus I have the height of this liquid which by its weight is equivalent to the force of the wind. The base is the surface of the sails ; but now the length of this is the same as the height of the mast, now put equal to z. In addition let the width of the sails be equal to a, hence that base is equal to az. But a and z are also to be expressed in Rhenish feet since v now is expressed thus, hence the bulk [volume] of this with its weight equivalent to the force of the wind is equal to $\frac{azvv}{60}$ cubic feet.

54. Hence there remains to be found the weight equal to the striking force of the wind, as we look for the weight of this liquid; but since that is put twice as dense as air, also it has twice the weight, from which since a cubic foot of air weighs as nearly $\frac{1}{12}$ of a pound, a cubic feet of that fluid weighs $\frac{1}{6}$ of a pound, from which the weight of $\frac{azvv}{360}$ cubic feet of air is equivalent to $\frac{azvv}{360}$, and this is the weight, which by drawing prevails to perform

the same effect and the wind with speed v strikes the sails; hence this weight can be put equal to the weight P, that also was put in place of the force of the wind, and then $P = \frac{azvv}{360}$.

[The whole volume of the imaginary vessel containing the wind strikes the sail area each second. Thus, we can imagine a continuous collision process between the sail and the wind, which may be inelastic, in which case the momentum imparted per second is ρv^2 per unit area, elastic, in which case the pressure becomes $2\rho v^2$, or more likely somewhere in between. The snag in such simple analysis being of course that the expended wind has to go somewhere, and air cannot be travelling in two opposite directions through itself at the same time; hence the difficulty. Other mechanisms arise, vortices are generated, etc, that get round this difficulty.

A sailor's manual gives the pressure on a sail by the formula $P = 0.00431v^2$, where the speed v is in knots; which tells me that a wind of 12 knots produces a pressure on the sail of approx. 0.6 lb per sq.ft. If we take 12 nautical miles per hour as around 72000/3600 or 20 ft/sec, then Euler's formula with $a = z = 1$ ft. gives P just over 1 lb, or twice what it should be. If this force or pressure difference was accounted for by a simple compression as in Boyle's Law, then a doubling of the density would mean a doubling of the pressure, which is not the case at all, as in this case the increase in density would be around 1/20 to account for the 0.6 lb. There cannot of course be an accurate determination of this effect, as there are too many factors to consider, such resistive forces are usually represented by a formula such as $P = k\rho v^2$, where k is found experimentally for a given situation. Thus it appears that Euler's formula works better for modern data, using the actual air density for an inelastic collision, or where $k = 1$.]

55. But in §46 it was found that $P = \frac{2Qbs}{z}$. From which there becomes

$$\frac{2Qbs}{z} = \frac{azvv}{360}, \text{ or } azzvv = 720Qbs.$$

But as must be found with perfect uniformity, b must be expressed in Rhenish feet also, and Q in pounds. Clearly the distance of the centre of gravity from the centre of gravity in feet, and the weight of the ship in pounds, in order that everything can be referred to the same unity, the equation moreover can be reduced to this in taking the square root of each,

$$zv = 12\sqrt{\frac{5Qbs}{a}}$$

from which there is found

$$z = \frac{12}{v}\sqrt{\frac{5Qbs}{a}} \quad .$$

56. Hence now behold the equation from which the height z of the masts sought can be determined. In the first place from the given weight of the ship Q in pounds; secondly fro the distance b of the centre of curvature of the keel from the centre of gravity of the ship in feet; thirdly from the width of the sails or the length a of the yardarms by which the same is supported everywhere, also in feet; and fourthly from the relative speed of the wind, clearly that which attacks the ship ; for since the ship also has a speed, the air is unable to strike the ship with its own speed, but the sail is attacked by the speed, by which the speed of the wind exceeds the speed of the ship ; moreover this speed v must also be expressed in the same Rhenish feet, clearly indicating how many feet may be measured out by the wind in one second with respect to the wind, in addition clearly the sine of this s on taking the whole sine equal to 1 according to that given. And thus the height of the mast z can be determined.

57. It is to be observed in the expression for the height of the mast z that the resistance of the water does not come into the calculation, and hence from that the height of the mast will be easier to compute. But since the force of the wind is required with the ship now in full motion, the speed of the ship must be taken from the speed of the wind, and the speed v is obtained; and hence it is no wonder that the resistance of the water is not present in the calculation ; for in place of that the relative speed v has been introduced. For in the determination of this resistance for given speed of the wind, the speed of the ship is required, and towards an understanding of this certainly the resistance of the water and the parts of the ship on which the water strikes must be introduced into the calculation.

58. But since with the given speed of the wind it is with difficulty that the speed of the ship can be foreseen so that it is possible to have the relative speed of the ship, which must be known in the expression for the heights of the masts, it is necessary that I present a method of treating the speed of the ship in whatever interval completed it is to be found. Indeed it is sufficient for the maximum speed of the ship to be indicated, or that which it has acquired in traversing an infinite interval, since v is the speed of the ship relative to the wind, since the ship has now acquired the maximum speed. Now since here the opportunity is offered for convenience, and the maximum speed of the ship is able to be found from this easily, I offer a way of finding the speeds of the ship in whatever interval it has completed here in the middle; for from that the law of the acceleration of the ship is to be seen, and since ships are not able to traverse an infinite interval, in order that a uniform speed be produced, but in a somewhat askew interval now it they acquire so much speed which will not be increased further later, and it is apparent too how great a distance must be traversed by the ship, in order that it proceeds sensibly with a uniform motion.

59. Now in finding this it is necessary that the resistance of the water be taken into the calculation. However since the shape of such ships in moving water does not interact directly on the water, but is experienced obliquely and more slanting in one place than another. Hence on account of the water grazing off the surface of the ship, it is not possible to measure the resistance, since that water too is at another angle of deviation; towards meeting this inconvenience I assume some plane which is struck normally by

that water with the speed of the ship, and undergoes the same resistance as the ship. For in this way the resistance of the ship is easier to consider, since the angle of incidence is supposed to be right always, and the water striking a constant interval, but hence not unless attention is paid to the speed with which it is struck by the water.

60. But for this plane with the same resistance suffered as the ship without sensible error I consider it possible to assume the maximum transverse section of the ship, clearly of that part of the ship that endures in the water, indeed this is moving along the direction of the keel of the ship on striking the water, bears much more resistance than the ship, and hence and hence that section I assume for the plane receives more punishment, now with the ship moving at an angle, the resistance of this is increased also and with the prow of the ship more deeply submerged, the surface of the ship takes an increase of the dividing water, from which the resistance also is increased, especially with the rudder they are using. On account of which the resistance which that transverse section on striking the water normally could become more lively, unless plainly it is equal to or a little smaller than the resistance of the ship. And hence that maximum transverse section of the ship is not of the whole ship but rather of the part of this immersed in the water, for it is possible to take without sensible error the same plane experiencing the resistance as the ship.

61. Therefore let this section of the ship be equal to ff, moreover ff is to be expressed in cubic feet, and in addition let the height of the parallelepiped be equal to h with the base ff with which capacity or bulk a part of the ship is immersed, which height also must be expressed in feet, since on being compared with the height or width of the sails which are also expressed in feet. Hence the bulk part of the ship immersed in the water is equal to hff cubic feet, for hff is the volume of this parallelepiped that equals the part of the ship immersed in water.

62. The wooden material of the ship and its load are put equally dispersed throughout the ship, in order that the ship can be considered as a homogeneous body, certainly of the same density everywhere, yet with the density of this changed by the load, the ratio of the [mean] density of the ship to the density of the water shall be as K to m, and to the density of the air as K to n. Hence a part of the ship is submerged until the [submerged] mass is to the [same volume of] water as $Khff$ [to $mhff$]. Now since the whole mass of the ship can be considered as homogeneous, the volume of the ship itself has the ratio of the part of the ship submerged as the density of water m to the density of the ship K; hence to the mass of the whole ship as $mhff$. [Archimedes Principle of flotation.] And with these in place thus I can come to understanding the speed of the ship.

[Thus, the mass of the ship is equal to $mhff$, *i.e.* the upthrust of the water; and

$$K \times \text{volume of ship} = \text{mass of ship} = mdff \text{ ,or,}$$
$$m/K = \text{volume of ship}/dff = \text{whole volume of ship}/\text{submerged volume.}]$$

63. Now let the ship be in motion, and let it traverse a distance of *y* feet; let the speed then acquired be equal to *v*, clearly *v* indicates the number of feet that a body with a uniform speed *v* is able to traverse in a time of one second, let the speed of the wind be *c* and in the same way *c* can be expressed by the number of feet completed by the wind in a time of one second, from which the relative [*i. e.* with respect to the ship] speed of the wind is equal to $c - v$. But the capacity [*i.e.* area] of the sails is equal to *az* and the space or plane [*i.e.* area] that it strikes in water and offering resistance is equal to *ff* .

64. Let the ship be moved through an infinitely small distance, now for the element of distance described *y*, clearly by *dy*, and the acceleration is sought while the ship is moved forwards through *dy*. But within that interval the ship experiences an impulse from the wind, by which the ship is accelerated, and it is to be retarded also by the resistance of the water. Hence from the increment of the speed generated by the wind there is to be taken the decrement of the speed produced by the resistance of the water. And there remains the element or increment of the speed of the ship while it goes through the interval *dy*.

65. Because the air with the speed *c,* which is greater than the speed of ship, is moving forwards, there shall be a force from the air in the sails and thus the speed of the ship is increasing; now that increment of the speed can be found from the law of the sharing of motions in the collision of bodies, when the bodies are elastic, for the air and the sails to be used and hence the water and the parts of the ship intruding into water are to be considered as elastic bodies, if not the whole yet the small parts of these which have been disturbed, for since with the ship once moving ; the sails are always supposed to be equally spread out, and the shape of the ship also remains unchanged; it is required that the sails and the surface of the ship, if the shape of these may be changed in some way by the air striking and the water resisting, then yet they are restored at once, and thus because of that they can be taken as elastic bodies.

66. I consider air in this manner as a collection of infinitely small globules the diameter of which is equal to the element by which the ship is moved forwards, surely equal to *dy* itself, hence with such an abundance of globules of this kind, just as many are able to be captured by the sail with the speed *c*, that strike the sail with the speed *v* continually. Hence from the given mass of the ship and the mass of the air rushing into the sails, the speed of the ship after the encounter can be found, if clearly while the ship is carried through *dy* the resistance of the water can be removed with what value it had for the initial speed, or that which the ship had while ready to be advanced through *dy*, there remains the element of the speed which the ship acquired in the interval *dy*, with the resistance of the water removed.

67. But it is agreed from the rules of the sharing of the motion , that if the body *A* with a speed such as α runs into the body *B* with a speed of moving *b*, then after the collision the speed of the body *B* is equal to $\frac{2A\alpha+(B-A)b}{A+B}$

[assuming an elastic collision with both bodies moving initially in the same direction with $\alpha > b$; the equation results from equating the initial and final quantities of motion or momentum : $A\alpha + Bb = A\alpha' + Bb'$, as well as the initial and finial vix viva, clearly related to the kinetic energy, (an unknown idea at the time) : $A\alpha^2 + Bb^2 = A\alpha'^2 + Bb'^2$];

and as I can apply this in our case with the mass of air incident *A*, but this mass is as the volume taken by the density of air as I have put as *n*, and moreover the volume of air incident is the thickness of the air equal to *dy* and as large a quantity of sail it is sufficient to fill, the surface of the sails removing the wind is equal to *az* and hence the volume of air striking the sail is *azdy*, consequently the mass of air striking is *nazdy*, this value is to be substituted in place of *A*.

68. But for the speed α of the body *A* there can be put *c*, the speed of the wind, and for the body *B* there is required to put the whole mass of the ship which is being propelled by the wind, hence *B* = *mhff*, and indeed in § 62 it was found that the mass of the ship to be equal to *mhff*, but in place of the speed *b* there must be put the speed of the ship *v*. With these values substituted the speed of the ship after the collision is found :

$$\frac{2naczdy + mhffv - nazvdy}{nazdy + mhff}$$

from this if the speed from before the collision is taken away, namely *v*, then there is found the increment of the speed during the interval *dy*, produced from the impulse of the wind, clearly

$$\frac{2naczdy - 2navzdy}{nazdy + mhff}$$

But since in the denominator *nazdy* is infinitely small relative to *mhff*, that vanishes and the denominator is *mhff* alone; hence the increment in the speed arising from the wind is equal to

$$\frac{(c-v)2nazdy}{mhff}.$$

69. Hence this is the increment of the speed produced by the force of the wind ; there remains to be found the decrement of the speed effected from the force of the resistance of the water. This can be found also from the same argument, I suppose without doubt that the water consists of globules, the diameter of which is equal to *dy*, it is apparent since the ship is moving through *dy*, as many globules impinge on the ship, and that normal to the direction of motion of the ship, as the plane *ff* is able to capture; for I suppose, as now since it is to be shown that the same is returned, that the ship suffers the same resistance , as the plane *ff* bears the water on striking with the same speed. Hence the volume of water impinging on that ship is equal to *ffdy*, which taken with the density of water *m*, gives the mass of that water ; surely that is equal to *mffdy*.

70. Now since the water is supposed at rest, now the ship with a speed *v*, proceeding from that lemma, the speed of the ship after the interaction can be elucidated on placing that through the interval *dy*, the ship receiving nothing from the wind. If the body *A* with speed α strikes body *B* at rest, then after the collision the speed of the body *A* remaining is equal to $\frac{(A-B)\alpha}{A+B}$. Here the mass of the ship *mhff* is to be compared with *A*, now the speed of this *v* with α , truly the mass of the water resisting *mffdy* is to be compared with *B* and hence the remaining speed of the ship after the interaction is equal to

$$\frac{mhffv-mffvdy}{mhff+mffdy}$$

which if it is taken from the speed of the ship *v*, before the interaction is had, by which the speed of the ship in going through the element *dy* is diminished by the resistance of the water, if no new increment is taken from the wind, then clearly the speed lost through *dy*, is equal to

$$\frac{2mffvdy}{mhff+mffdy}=\frac{2mffvdy}{mhff}=\frac{2vdy}{h}$$

with *mffdy* vanishing relative to *mhff* .

71. And thus the ship in going through the element *dy*, takes an element of speed $\frac{(c-v)2nazdy}{mhff}$ from the wind. But it loses from its speed $\frac{2vdy}{h}$ in overcoming the resistance. From which on subtracting the element of retardation of the motion of the ship from the element of acceleration, there is found the increment of the speed of the ship *v*, while it is brought through *dy* , clearly

$$dv=\frac{(c-v)2nazdy}{mhff}-\frac{2vdy}{h}=\frac{(c-v)2nazdy-2mffvdy}{mhff}$$

72. Hence it is apparent that the increment of the speed with *dy* remaining constant , to be as [*i. e.* y is the independent variable with the second derivative zero.]

$$(c-v)\cdot naz-m\cdot ffv\,,$$

or as

$$c\cdot naz-v\cdot(naz+mff\ ).$$

Hence the more the speed of the ship *v* increases, with that more the element of the speed decreases, until if it should be

$$v=\frac{nacz}{naz+mff}$$

then the speed of the ship does not increase further, but remains the same ; hence this is the maximum speed that the ship is able to acquire, for the same speed of the wind, following from the capacity of the sails and the resistance of the area of water ; from which we conclude that the maximum speed of the ship with all else being equal to be as the speed of the wind, and that speed is itself to the speed of the wind as *naz* to *naz* + *mff*. From which hence the more the capacity of the sails is increased, from that too the more the maximum speed is increased [towards] the speed of the wind, with the area or plane *ff* the same; hence it is concluded, on *az* remaining as the same capacity of the sails as that and *ff* remain the same, in order that the speed of the ship to be the same, then either the sails are to become wider and shorter, in the manner that they have the same capacity.

73. Hence thus the maximum speed of the ship has been found to be equal to

$$\frac{nacz}{naz+mff}.$$

Now towards determining the speed of the ship at whatever the interval traversed the equation found in paragraph 71 must be integrated. In order that this can be effected that can be reduced to this :

$$\frac{2\cdot dy}{mhff} = \frac{dv}{c\cdot naz-v(naz+mff)}$$
$$= \frac{-1}{naz+mff} \cdot \frac{-dv(naz+mff)}{c\cdot naz-v(naz+mff)},$$

the integral of this equation can be had from logarithms :

$$\frac{2y}{mhff} = \frac{-1}{naz+mf} \cdot l(c\cdot naz - v(naz+mff)) - l\,\text{const.},$$

or on being reduced,

$$\frac{2naz\cdot y+2mff\cdot y}{mhff} = l\ \text{const}\ - l(c\cdot naz - v(naz+mff)).$$

To determine the constant there is put *y* = 0, and *v* must be equal to zero, from which

$$l\,\text{const.} = l\,cnaz.$$

Hence there becomes

$$\frac{2naz\cdot y+2mff\cdot y}{mhff} = l\,cnaz\ - l(c\cdot naz - v(naz+mff)).$$

74. The speed is called *u*, by which the speed *v* differs from the maximum speed that the can acquire, then we have

$$v = \frac{nacz}{naz+mff} - u.$$

With this value substituted in place of *v* there becomes

$$\frac{2naz\cdot y+2mff\cdot y}{mhff}= l\,cnaz\ \ -l(u(naz+mff\,))$$
$$= lc-lu+l\,naz-l(naz+mff\,).$$

And hence the distance y can be found, which absolute speed the body can acquire whatever the amount different the speed is from the maximum speed, as it is possible to have the maximum speed with which the ship traversed the interval without sensible error; now with the letters determined in numbers, not only the logarithms of Vlacq and Briggs must be assumed, but hyperbolic logarithms which are obtained, if the Vlacq logarithms are multiplied by 2.302585093 as an approximation [*i. e. ln*(10)].

75. But if we return to the equation expressing the height of the mast *z*, since there the quantity *v* is found, which indicates the relative speed of the wind, when the ship is moving with the maximum speed ; hence there can be found the speed *v*, if from the speed of the wind *c* there is taken away the maximum speed of the ship, namely $\frac{nacz}{naz+mff}$ hence there becomes $v=\frac{mcff}{naz+mff}$.

76. But here *c* indicates the number of Rhenish feet which the wind can run through in a time of one second, surely since ships in the most violent of winds, obviously in which blowing the ships are to be in danger, they must be constructed, for *c* to be put from 80 as far as 100 feet, just as from experiments with the forces of various winds in place it is allowed to conclude, as clearly the most strong winds complete as distance of 80 as far as to 100 feet in a time of one second.

77. On putting the value found in place of *v* found in equation § 55 :

$$zv=\ 12\sqrt{\frac{5Qbs}{a}}$$

and there obtained

$$\frac{mcffz}{naz+mff}=\ 12\sqrt{\frac{5Qbs}{a}}$$

from which the height of the mast sought

$$z=\ \frac{12mff\sqrt{\frac{5Qbs}{a}}}{cmff-12na\sqrt{\frac{5Qbs}{a}}}$$

Therefore here we have the most perfect equation, from which it is known how to determine the height *z* in a pure form in feet.

78. The equation found can be reduced still further on getting rid of *m* and *n;* for since *m* to *n* is the ratio of the density of water to the density of air, *i. e.* which is approximately as 800 to 1, there is put 800in place of *m*, and one in place of *n*, and there is found this equation :

$$z = \frac{9600\,ff\sqrt{\frac{5Qbs}{a}}}{800cff - 12a\sqrt{\frac{5Qbs}{a}}} = \frac{2400\,ff\sqrt{\frac{5Qbs}{a}}}{200cff - 3a\sqrt{\frac{5Qbs}{a}}},$$

clearly in feet.

79. Hence in a ship in the first place with the given maximum transverse section of the ship immersed in water *ff* in square Rhenish feet. In the second place with the distance of the centre of curvature of the keel from the centre of gravity of the whole ship *b* in feet. In the third place with the width of the sails or of the yardarms *a* likewise in Rhenish feet. In the fourth place with the whole weight of the ship *Q* in pounds and as in the fifth place the interval *c* which the wind in a one second can traverse too, for which it is possible to assume from 80 to as far as 100, here I put $36\sqrt{5}$ for *c* in order that the numerator and the denominator are able to divided by $\sqrt{5}$.

80. With this in place there is found for the heights of the mast :

$$z = \frac{800\,ff\sqrt{\frac{Qbs}{a}}}{2400\,ff - a\sqrt{\frac{Qbs}{a}}} = \frac{800\,ff\sqrt{Qabs}}{2400aff - a\sqrt{Qabs}}$$

on multiplication both the numerator and the denominator by *a*, from that the equation *z* can be determined for the height sought in Rhenish feet ; which height when it has been found, if it is greater than such as can be made from a single mast, that is to be shared out in so many parts while such a number of masts can be had to which the parts are equal respectively. And thus from this equation the number of masts is determined also. Now these masts thus determined the inclination of the ship at such an angle the sine of which has the ratio to the total sine as *s* to 1. Moreover this ratio has previously had to be assumed and indeed such in order that the angle is between all the angles that the ship can be inclined the most without danger, as also the maximum propelling force can be found.

81. From that equation defining the heights of the mast these conclusion can be deduced, which have the greatest use in the fabrication and in the loading of the ship and as in the compilation of the sails, or hence it can be concluded how ships are to be formed and for the loading what width of the sails are to be given, in order that the force can be found, as great as it can become, propelling the ship at the proposed angle of inclination.

82. Therefore in the first place it is apparent at once that the greater the distance *b* shall be, of the centre of curvature of the keel of the ship from the centre of gravity of the same, there the height of the masts are able to be assumed greater also, or there a greater force can be extracted from the wind. Hence in the loading of the ship, it has to be attended to that the centre of gravity is put in place so that it is as low as it can become, which can be brought about if the particular heavier merchandise is gathered together in a place of the ship as low as possible, and as according to custom, with the keel loaded with heavy gravel, from which it happens that the common centre of gravity descends to the lowest place, and thus the distance of this from the centre of curvature is increased and hence also the force of the wind that is allowed.

83. Now in the construction of ships it follows that it is most useful for the keel to be curved less, but hence it must not be thought from this that it would be best if the keel should be a straight line, or the section of the ship along the length should be rectangular, for the keel which is held in place under water must be the continuous arc of a circle, moreover this conclusion that it be composed from three straight lines cannot be deduced from this conclusion: thus when I say the most useful for moving the ship along, for which the keel is less curves, that is to be understood thus that the longer the ship shall be so also the keel shall be longer, with the depth of the parts of the ship submerged the same, indeed thus the distance of the centre of curvature is lengthened more, and thus the distance of this from the centre of gravity.

84. If on the other hand ships are made shorter, with the same parts of the ship immersed remaining the same , or the keel is curved in the arc of a small circle so that the centre of gravity and the centre of curvature coincide, it is apparent from the equation, clearly as no force can be extracted from the wind; for in short the minimum force is equal to that required to overturn the ship.

85. And hence also it can be concluded, since the transverse curvature of the ship is very large, or since the transverse section of the ship is the segment of a very small circle relative to the part that is the section of the keel of the ship, there to go beyond the fixed limit of the inclination of the ship, so that the angle of deviation is greater. Which curvatures indeed have been said to be greater than the curvature of the keel, unless that curvature prevails as when the ship is progressing in the direction of the keel ; but since the angle of deviation of the keel has been given, in place of the curvature of the keel there is required to place the curvature of the line drawn on the bottom of the ship along the direction of the motion of the ship and bisecting the ship, which line can be called an imaginary keel.

86. Since the deviation of the ship is thus in place, b indicates the distance of the centre of gravity from the centre of curvature of the imaginary keel, and since these imaginary keels are the arcs of small circles where the deviation of the ship is greater, then also the centre of curvature of the imaginary keel is closer to the centre of gravity, in order that thus the line b decreases also, and hence the height of the masts or the force propelling the ship there is required to be diminished more, from which the deviation of the ship becomes greater; hence it is especially dangerous for ships to be attributed a large deviation, for if the propelling force remains the same, the ship is inclined at a much greater angle than proposed.

87. Hence towards removing this inconvenience, and in order that neither the height of the masts or the among of the sails is required to be diminished with the deviations of the ship, ships are to be constructed with some width, in order that the difference between the curvature of the keel and of the imaginary keel, when the deviation of the ship is 90 degrees, should not be very large; so that hence the imaginary keels do not differ from the customary deviations of ships as long as the curvature does not differ, and hence the

distance *b* of the centre of gravity of the ship is not changed sensibly when the ship moves with deviation.

88. Then I observe, as if the ship should be made of such a length, or the keel is the arc of such a circle, that the distance *b*, of the centre of gravity from the centre of curvature of the keel is equal to $\frac{5760000 f^4}{Qas}$ feet, then an infinite number of masts must be constructed or one mast of infinite length for this so that the ship can be inclined at the given angle, and if *b* were greater than $\frac{5760000 f^4}{Qas}$ feet, and not even an infinite force to be equal for the angle of inclination.

89. For if $b = \frac{5760000 f^4}{Qas}$, in the given equation in§ 80, surely

$$z = \frac{800\, ff \sqrt{Qabs}}{2400aff - a\sqrt{Qabs}}$$

the denominator of the fraction equal to *z* vanishes and hence *z* becomes infinitely long. Hence it is therefore apparent that the amount I ask for have longer ships rather than shorter; for if such a length or *b* is equal to $\frac{5760000 f^4}{Qas}$, then the number of the masts or sails can be multiplied arbitrarily without danger to the ship.

90. Then because from the equation the width of the sails *a* can be deduced, which indeed is seen to be a puzzle, but nevertheless it is definitely true, because the more the height is increased of the sails, there also the greater the height *z* of the masts must be increased without danger to the ship, since the ship is still not beyond the proposed angle of inclination. For it is apparent with *a* increasing, indeed the numerator of the fraction expressing the height *z* is to be diminished; for that is the fraction [§ 78]

$$\frac{2400\, ff \sqrt{\frac{5Qbs}{a}}}{200cff - 3a\sqrt{\frac{5Qbs}{a}}}.$$

Now it is to be observed, the other part of the denominator $3a\sqrt{\frac{5Qabs}{a}}$ or $3\sqrt{5Qabs}$ affected by the – sign increases in the same ratio, and since the part of the denominator affected by the positive sign *200cff* remains the same, the whole denominator decreases in a greater ratio than the numerator, from which fraction both the height itself *z*, with the increase in the width of the sails, or the width of the yardarms, is increased.

91. Hence it is therefore apparent how great the benefits to the yardarms shall be, how many can be made, the lengths to be had, in order that hence the magnitude of the forces propelling the ship can be increased also. If with the width of the sails increased, the masts can be left with the same heights, this can be a great convenience to increasing the speed of the ship ; now with the increase in the width of the sails, not only the height of the masts can remain the same, but in addition that can be increased, from which increase

with the increase of the width of the sails the force propelling the ship will be increased much more, and hence also the speed of the ship without danger to the ship.

92. But the width of the sails a becomes equal to $\frac{5760000f^4}{Qbs}$ feet, the length of the masts z on account of the vanishing denominator becomes infinite, and hence the height of the masts and the number can be multiplied at will without danger to the ship ; for whatever the increase in the height and in the number of masts the ship is still not inclined at the proposed angle, since at last an infinite force shall be equal at that angle of inclination, clearly if the width of the sails is equal to $\frac{5760000f^4}{Qbs}$, but if that were made larger, nor does an infinite force suffice to keep the ship inclined at an angle, if the sine of this is to the total sine as s to 1.

93. I arrive finally at the angle of inclination, and I note that the greater that is assumed, the more force can be taken from the wind, therefore in order that is can be assumed to be a little larger, it is necessary that the ship is in no danger, it is allowed for the prow to be immersed deeper; therefore in order that this can be done, as clearly the angle of inclination can be assumed to be large in the absence of danger to the ship it is useful if the prow of the ship become raised more than the remaining part of the ship, thus indeed the ship will not try be put to the test, even if the prow is immersed a little further, and hence the angle of inclination can be assumed to have a certain size.

94. To obtain the same thing also, the greatest and heaviest load with which the ship can be loaded must be sent to the stern of the ship ; for in this way the stern is depressed and the prow raised, in order that thus a greater part of the prow remains out of the water, which without danger to the ship can be immersed in the water, and in this manner the angle of inclination can also be assumed to be greater. And from these conclusions hence it is apparent, which are to be observed in the construction and loading of the ship, then in making ready the sails so that the ship free from danger can be moved with the greatest speed, and which without doubt are able to have the greatest use in practice if they should be observed. And that from my proposed theory both the height and the number of the masts can be found for any ship without much labour, in order that the ship is not in danger and yet can be conveyed with the maximum speed.

95. And so since the height of the masts z can be determined, it is easy to see beforehand what the maximum speed of the ship shall be. For that is as found in [§ 75], equal to

$$\frac{nacz}{mff+naz},$$

or since there is $m = 800$ and $n = 1$ then that is equal to $\frac{acz}{800ff+az}$

But there is also the equation

$$z=\frac{2400ff\sqrt{\frac{5Qbs}{a}}}{200cff-3a\sqrt{\frac{5Qbs}{a}}}$$

as found in § 78; if that value is substituted in place of z , the maximum speed of the ship is found :

$$\frac{2400acff\sqrt{\frac{5Qbs}{a}}}{160000cf^4} \text{ or } \frac{3\sqrt{5Qabs}}{200\,ff};$$

or the speed of the ship is of such a size, that in the time of one second it is able to traverse a distance of $\frac{3\sqrt{5Qabs}}{200\,ff}$ feet.

96. Since the speed of the wind does not enter into the expression for the maximum speed of the ship, [The speed of the wind does enter the other formula, however,]it is apparent that the ship will travel along with this speed whatever the speed of the wind should be blowing, but only if the ship should be at the proposed angle of inclination. It is apparent in turn that the maximum speed of the ship is in the square root ratio of the width of the sails, evidently if that width is made to quadruple, then the ship will travel with twice the speed, and in the same manner the speed of the ship is also as the square root of the distance of the centre of gravity from the centre of curvature of the keel, and in the square root ratio of the sine of the angle of inclination of the ship. Then also if several ships should be perfectly similar, but of different sizes, since the weights of these are in the three on two ratio of the surfaces, and hence Q becomes as f^3, then the speeds of these ships, with all else being equal, are in the reciprocal ratio as the square roots of the lengths of the same ships, from which therefore the smaller ships are made, also the faster they are propelled with all else being equal, clearly if all were similar.

97. Now I have mentioned a number of times, if the height z should be found so great that there cannot be one mast of such a height, then several are to be taken of which the heights taken together are equal to z and several masts then produce the same effect, and a single mast of length z, if this is possible, clearly that is equal to a itself, if the width of the sails are the same.

98. But since these extra ones produce the same effect, then it is apparent because with these remaining made with the same width and height or length, if with both the remaining capacity of the sails and the width of these, the force both propelling as well as inclining the ship also remains the same, just as now it is possible to deduce from these produced, therefore either more or less masts are to be put in place, but if the same quantity of sails or spread and the same width remains, that factor from the length and width of the sails remains the same, and thus in the same manner as long as both the speed and the inclination remain the same, is moved forwards.

99. Now I suppose here that the sails are to be fixed to the masts at the lowest possible place, but since that cannot happen, on account of the force of the wind there in the lower parts of the masts clearly either hindered or especially deprived of power, then the height of the masts is greater than the width of the sails, but which was considered equal in the theory; thus since the centre of the sails falls above the mean point of the masts, it is necessary then that if the capacity of the sails is equal to az, that the ship is inclined beyond the angle of inclination : now since the width of the sails is less than z, the capacity of the sails is also less than az, from which it happens to be just about the

compensation present, so that the ship is not yet inclined beyond the proposed angle of inclination, but thus since the width of the sails is less than the height of the masts, the force propelling the ship is less than was put in the theory. And that force is smaller when several masts are erected on the ship, than if masts were put in place higher than can be taken, thus so that the number of masts is restricted.

100. Here at last I put an end to my meditations, since it is seen that I have carefully assessed the matter proposed in the problem, and I may have satisfied the problem. I do not think that it is necessary to confirm my theory by experiment, since the whole has been deduced from both the surest and the most irrefutable principles of mechanics, and thus concerning that there cannot be the least doubt or the truth can be put to the test in practice. But if that theory should be applied to some specific example, I would investigate the lengths of the masts for the ship, and it would become apparent at once that it would not fail. If perhaps the most illustrious academy should deem these small pages worthy of the proposed reward, the name of the author and the place where he abides can be found from the attached page.

MEDITATIONES SUPER PROBLEMATE NAUTICO, QUOD ILLUSTRISSIMA REGIA PARISIENSIS ACADEMIA SCIENTIARUM PROPOSUIT

Omnes enim trahimur, et ducimur ad cognitionis et scientiae cupiditatem, in qua excellere pulchrum putamus.
M. T. Cicero de Officiis

PROBLEME QUELLE EST LA MEILLEURE MANIERE DE MATER LES VAISSEAUX TANT PAR RAPPORT A LA SITUATION QU'AU NOMBRE ET A LA HAUTEUR DES MATS

1. A Constitutione et collocatione malorum, potissimum universa navigatio dependet in navibus quae non a remis sed solis velis propelluntur. Vela scilicet antennis alligata malis applicantur, et vento obversa, eius impetum sustinendo navem promovent. In implantatione malorum in hoc est incumbendum, ut navis, qua absque discrimine potest maxima, velocitate incedat, quod ut obtineatur, ad locum, altitudinem, et numerum malorum, diligentissime est attendendum. Quod ad locum primo attinet, in eius determinatione opera atque studium summum est adhibendum, ut gubernaculum, cuius actione de navis celeritate semper quicquam detrahitur, si eius usus plane evitari nequeat, minimam, quam possibile est vim, impendere debeat. Vocatur linea in navibus super sentina a prora ad puppim ducta, spina navis, et Gallice *la quille,* in hac inseruntur mali ut quilibet sit in medio navis. Si navis secundum directionem spinae istius movetur, gubernaculo opus non erit ad navem in isto situ continendam, ubicumque mali, modo in spina, sint plantati. Verum cum navis non iuxta spinam promovetur, sed directio motus navis cum spina angulum constituit, qui angulus, deviationis angulus, et Gallice *l'angle de la derive* appellatur, tum non ita, ubicumque siti sint mali in spina, navis istum deviationis angulum conservabit, seu eandem positionem, sed ad hanc retinendam peculiaris malorum locus est determinandus, qui malorum locus alius esse deberet, in quolibet alio angulo deviationis. Et ita cum naves in aqua progrediendo, ut ad optatum perveniant locum, modo hanc, modo aliam deviationem recipere debeant, pro quovis angulo alius malis tribuendus esset locus. Quod autem in navibus rnalis semel erectis cum fieri nequeat, malis immotis manentibus, ope gubernaculi efficiendum est, ut navis in eodem deviationis angulo conservetur.

2. Cum autem gubernaculum agere debet, resistentia qua navi resistitur augetur, et ita celeritas navis minuitur, idque eo magis quo major a gubernacula effectus efficiendus est, scilicet igitur quo magis situs malorum ab eo situ differt, quem habere deberent ad id, ut gubernaculo plane opus non sit. Ne ergo nimium excrescat vis gubernaculi, talis malis assignandus est locus,

qui in illis navis deviationibus, quas navis crebrius habet, ab illo loco, quo gubernaculum non in usum vocandum esset, non multum discrepet, quo fiet ut gubernaculi actione celeritas navis nunquam sensibiliter decrementum patietur.

3. Verum quotquot in nave positi sint mali, semper erit punctum in spina navis ubi si collocetur malus unicus altitudinis quae aequalis est summae altitudinum illorum plurium totidemque velis instructus, qui eundem efIectum edat, istud punctum vocare licet centrum commune virium navem propellentium. Datis vero loco malorum et eorum viribus ope velorum a vento mutuatis, centrum istud facile reperietur, non absimili modo ei, quo centrum commune gravitatis plurium corporum in eadem recta iacentium reperitur, hoc tantum discrimine, quod ibi capacitas velorum malorum eo loco sumatur, quo in determinatione centri gravitatis pondus corporum consideratur; et ita facilius erit dato centro communi virium promoventium navem locum malorum invenire: in posterum itaque sufficiet unicum istud centrum determinasse, hoc enim noto, quotcunque mali sint navi inserendi, eorum loci facile reperientur.

4. Plures mali navibus non inseruntur, nisi tantae altitudinis, quanta requiritur unicus malus haberi nequit, tum enim pluribus efficiendum est quod unicus praestare debuisset: cum ergo altitudo malorum desideratur, altitudo nonnisi unici mali, pluribus aequipollentis determinanda est. Haec enim, cum cognita fuerit, in tot partes est distribuenda, donec partes illae tantillae fiant seu tantae altitudinis, cuius mali haberi possunt; et sic invenietur numerus malorum et per paragraphum praecedentem quoque eorum locus.

5. Altitudo vero malorum determinanda est quatenus ea capax est velorum, quae sunt praecipua causa vis impulsivae; non igitur tam de altitudine malorum, quam de altitudine velorum quaestio est interpretanda: esset quidem nec altitudo velorum contemplanda, si vis navem promovens sola respiciatur, etenim eadem manente vi propulsiva, ubicumque ea applicetur sive in unico puncto tota sive in pluribus divisim, sive in locis malorum sublimioribus sive humilioribus; verum ea portio vis venti quae navem inclinat scilicet proram profundius immergit, crescit quo in altioribus malorum locis vis ea sit applicata: praestat ergo quo latiora fiant vela, ut sufficiens virium quantitas in locis malorum inferioribus possit comprehendi; si enim arctiora fiant et minoris latitudinis in sublimius sese extenderent vela, et ita vis navem inclinans cresceret, quod vero id ipsum est, quod effugiendum in determinatione altitudinis malorum propositum esse debet: quo circa cum altitudo malorum quantum fieri potest, circumscribenda sit, vela malis in locis quoad fieri potest humillimis applicari debent, nisi venti vis ibi sensibiliter diminuta sit, atque velis quantum aliae circumstantiae id permittunt, maxima tribuenda est latitudo.

6. Verum nec haec observando numerus velorum pro lubitu multiplicari potest, nimis enim aucto velorum numero contingere posset ut navis si non prorsus in aquam prosternatur, tamen proram ulterius quam securitas navis permittit, immergat. Quod ut melius concipiatur, notandum est, quamlibet venti potentiam in velis applicatam,

duplicem in navem exercere vim, unam quae navem propellit, alteram quae navem inclinat, proram profundius immergendo; facit scilicet, ut quae quiescente nave verticalia fuere, nunc dum sit in motu versus proram inclinentur, idque eo magis quo major est venti vis, et quo in sublimiori loco malorum sit applicata; unde fieri potest vi propellente vel nimium aucta vel nimis sublime applicata, ut prora ulterius, quam tutum est, immergatur vel penitus submergatur.

7. Ne igitur navis nimium inclinetur, terminus est constituendus quousque prora immergi possit absque navis periculo, quo cognito, quaerendum est quantum virium a vento sit excipiendum ut navis eousque praecise et non ulterius inclinetur, unde habebitur vis qua navis promoveri potest maxima, si enim maior assumeretur, navis periclitaretur, quia tum navis ulterius quam par est, inclinaretur: sin vero minor sumatur vis, navis celerius adhuc absque periculo promoveri posset; maxima ergo hoc modo invenietur vis navem propellens, seu invenietur modus malos implantandi, ut navis, quam possibile est celerrime procedat. Cum itaque haec de loco atque altitudine malorum rite excussero, Problemati me satisfecisse persuasus esse potero.

8. Meditationes ergo meas in duo ista capita figam, et quae in ipsis solvenda proponuntur, perpendam, solutionemque tentabo. In primo scilicet Capite de loco malorum mihi agendum erit, ibi in locum centri virium navem propellentium inquiram, ubi illud in collocatione malorum assumendum sit, ut navis motui maxime sit proficuum. In secundo autem Capite tractandum erit de altitudine malorum, seu saltem de altitudine unici mali, pluribus aequipollentis; concipiam nempe nonnisi unicum malum erigendum esse, eumque quaeram, ex cuius longitudine inventa facile erit iudicare, quot mali sint inserendi, de altitudine ergo mali, seu potius de longitudine velorum, data eorum latitudine nobis prospiciendum erit, ut navis quam absque periculo potest celerrime procedat. Accedo itaque ad ipsam huius aenigmatis solutionem atque ILLUSTRISSIMAM AC CELEBERRIMAM ACADEMIAM, ut pro sua pollent, uti in omnibus disciplinis, ita potissimum in scientiis Physico-Mechanicis, eruditione atque sagacitate, hasce exiles pagellas attente legere, suumque de eis iudicium ferre, haud dedignari velint, humillime atque demisse rogo atque oro.

CAPUT PRIMUM
DE LOCO UBI ASSUMI DEBET COMMUNE CENTRUM VIRIUM NAVEM PROPELLENTIUM

9. Cum navis in aqua procedit propulsa a vi venti, ut in eodem situ, eademque deviatione conservetur, et navis non in latera rotetur propter resistentiam ab aqua perferendam, oportet ut centrum commune virium navem propellentium situm sit in linea mediae directionis vis resistentiae, ab aqua in navis latera exactae, scilicet cum hoc centrum in spina navis quoque existere debeat, assumendum erit hoc centrum in puncto spinae, ubi a linea mediarum directionum resistentiae secatur. Cum ergo linea ista mediarum directionum cognita fuerit, innotescet quoque centrum virium, locus scilicet ubi collocari debet malus si unicus tantum sit erigendus.

10. Si ex Capite sequente innotuerit plures malos esse implantandos navi, id ex dictis iam ita fiet, sicque eorum loci invenientur, primum in spina sunt collocandi et dein in talibus ab isto centro distantiis, ut summa factorum ex capacitate venti uniuscuiusvis mali in distantiam eius a centro ab una parte istius centri sit aequalis summae similium factorum ex altera parte. Cum enim istae summae factorum aequales fuerint, vires sese in aequilibrio conservabunt, ut navis circa centrum illud gyrari nequeat. Hoc ergo in collocatione malorum observato, navis perpetuo eandem deviationem conservabit, ita ut opus non sit gubernaculi adminiculo, quamdiu scilicet idem fiat ventus vel saltem quamdiu ventus, si vela exacte sint expansa ut planam superficiem constituant, eandem velorum superficiem scilicet eam puppi obversam ferit; modo enim vela eundem conservent situm si sint exacte expansa, navis quoque versus eundem locum dirigitur, quisquis ventus flaverit, modo non cum linea directionis navis angulum recto aequalem vel maiorem constituat.

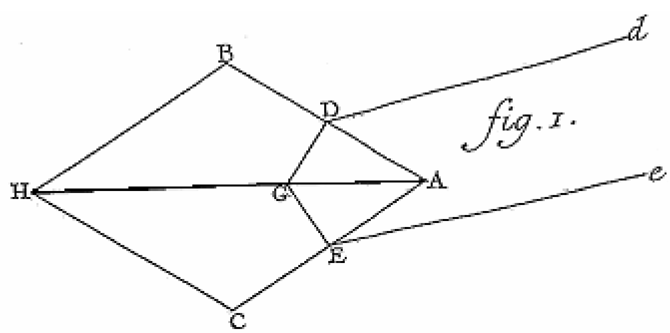
11. Verum cum commoditas navigandi postulaverit ut navis in aliam deviationem collocetur, quia tum positio lineae mediarum directionum resistentiae mutatur, quoque locus centri virium navem propellentium alibi assumendus esset, vel prorae propius vel vero puppi admovendo, quomodo vero mutata deviatione navis locus centri virium mutandus sit investigabo. Ponam primo angulum deviationis pristino maiorem fieri, et linea mediarum directionum resistentiae versus puppim magis cum spina concurret et inde centrum virium navem propellentium ad puppim magis assumendum esset. Quod si non fiat, nee gubernaculo succurratur, navis in sua positione non permanebit, sed rotando angulum deviationis augmentabit, donec velorum superficies a vento avertantur, sin vero nova illa deviatio priore minor ponatur angulus deviationis diminuetur continuo donec evanescat.

12. Hisce vero impediendis inservit gubernaculum, quod ad conservandam eandem navis deviationem, eo maiorem vim impendere debet, quo centrum commune virium assumtum magis ab illo quod assumtum esse deberet discrepat. Verum cum sic resistentia augeatur et proinde celeritas navis diminuatur, alio remedio huic incommodo occurri poterit, mutando reipsa locum centri virium, quod duplici modo fieri potest; primo ipsos malos de loco movendo, secundo autem manentibus malis immotis eorum capacitatem venti mutando vela nova vel super addendo vel iam expansa contrahendo. Priori modo mederi posset, si non omnes saltem unicus malus mobilis redderetur; quod fieri posset et locum ubi locatur et ea loca quibus funibus alligatur ita fabricando, ut aliquantulum malus de loco reptare possit vel ad proram, vel ad puppim, minima enim loci mutatio sufficiet ad centrum virium sufficienter transvehendum, praesertim si ab initio tale assumtum fuerit centrum virium, quod ab aliis centris quae in aliis possibilibus navis deviationibus locum habent non multum distat. Cum ergo angulus deviationis maior statuatur ac in initio fuerat, cum tum centrum virium puppi accedere deberet, malus iste mobilis ad puppim magis movebitur eousque donec gubernaculo opus non amplius sit. Sin vero angulus deviationis minor evadat, malus hic versus proram promovendus erit.

13. Si aliae circumstantiae non permittunt ut mali mobiles reddantur, altero modo abviam iri poterit, scilicet transportatione velorum, seu expansione in uno malo, novorum velorum in alio vero ut eadem vis conservetur totidem malorum contractione, hoc enim

modo quoque centrum virium in alium transferetur locum. Et quidem cum primo supposuerim angulum deviationis crescere, ut centrum virium ad puppim magis accedat, vela ex parte centri versus proram diminuenda sunt contractione vel saltem diminutione latitudinis quorumdam velorum et contra ex altera centri parte versus puppim tantundem velorum de novo extendendo vel latitudinem velorum augendo. In altero vero casu decrescentis anguli deviationis, vela versus puppim diminuenda et ea versus proram augmentanda erunt. Quantum vero demendum sit adponendumve, gubernaculum indicabit; eousque enim addendum detrahendumve est velis donec gubernaculum nil amplius agere debeat. Atque tum quoque navis in suo situ absque interventu gubernaculi conservabitur.

14. Quodcumque autem istorum remediorum adhibere lubuerit, sive primo fabricatione mali mobilis, sive altero translatione velorum, sive horum neutro sed gubernaculo, ne multum opus sit motione mali mobilis aut translatione velorum, aut si tertium remedium adhibeatur, ubi ad hoc quam maxime respiciendum est, ne gubernaculum valide agere debeat, unde celeritas navis diminueretur, talis est in constitutione malorum locus centri virium eligendus a quo si navis alias deviationes habeat, centra illis deviationibus competentia non multum differant. Tale autem punctum ut determinetur, necesse est, ut figura navis in computum ducatur, cum resistentia aquae dependeat potissimum a laterum figura, quae in aquam impingunt.



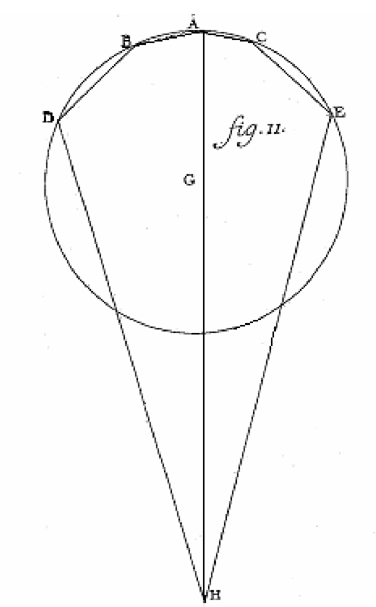
15. Ut a simplicissimis initium ducamus, sint duo navis latera rostrum componentia, lineae rectae, quae quidem suppositio licet navi accurate non competat, tamen hic nobis ubi non fixum aliquod punctum quaeritur, aliquam Fig. 1 lucem foenerari poterit. Sit ergo *ABHC* (Fig. 1) navis figura, *A* eius prora, *H* autem puppis, *AH* spina angulos *A* et *H* bisecans, erunt et latera *AB*, *AC*aequalia et latera puppis *B H* et *CH.* Sint *AB* et *AC* partes navis resistentiae expositae, eaeque solae, quod semper continget si angulus deviationis navis minor erit quam dimidius angulus puppis *H.* Sit *Dd* vel *Ee* directio motus navis, impinget navis secundum hanc directionem in aquam, seu cum res eodem redeat, facilioris conceptus gratia supponam navem quiescere et aquam iuxta eandem directionem *dD* vel *eE* eadem celeritate quam habebat navis, in navem impingere, scilicet in latera *AB* et *AC*, neutrum laterum *BH* vel *CH* ferire poterit cum sit angulus deviationis quem *Dd,* cum spina *HA,* constituit minor quam angulus dimidius puppis *H.*

16. Notum est ex hydrostatica aquam in haec latera resistentiam suam normaliter in eadem latera exercituram, et cum aqua in idem latus *AB* et *AC* illidem ubique eodem angulo incidat, erit centrum virium eidem lateri *AB* vel *AC* impressarum in earum medio *D* et *E.* In his ergo punctis totam resistentiam tanquam congregatam concipiam, eritque

directio resistentiae cum sit in latera normalis in latere *AB* linea *DG* et in *AC* linea *EG* quae sunt sigillatim normales in latera *AB* et *AC*. Hae duae directiones ubi sese mutuo secant, erit centrum commune virium resistentiae; concurrunt autem ut palam est ob latera *AC* et *AB* aequalia in puncto spinae *G* per quod transit linea aequilibrii mediarum directionum resistentiae; quamcumque autem haec linea habeat positionem, secabit ea spinam *AH* in puncto *G*. Erit ergo punctum *G* id ipsum centrum quod quaeritur, de quo hoc notandum est, quod sit semper constans, quaecumque sit navis deviatio, modo eius angulus angulum puppis *BHC* dimidium non excedat.

17. Si ergo navibus huiusmodi figura tribueretur, maximum hoc commodum obtineretur quod, loco centri virium manente fixo, navis absque gubernaculi ope in quolibet deviationis angulo, malis semel rite constitutis conservari posset, modo, ut iam aliquoties notavi, angulus deviationis minor sit quam angulus puppis dimidius. Atque si ex re erit maiores deviationis angulos usurpare, eo maiores quoque puppis anguli construi possent, ad id, ut aqua latera *BH* atque *OH* nunquam lambat. Punctum vero *G* quomodo definiatur, facile colligi potest, scilicet bisecando alterutrum laterum rostrum navis componentium, et ex bisectionis puncto in idem latus perpendicularem erigendo, erit factum quod quaeritur; punctum enim *G* erit ubi ista perpendicularis spinam navis secat.

18. Si haec figura ob alias causas incommoda videretur quae navi tribuatur, possum insuper alias figuras indigitare, quae navibus dari possent ut absque gubernaculi



adminiculo immotis malis et velis, navis eandem deviationem obtineat, seu ut centrum commune virium in eodem loco maneat; nil aliud enim ad hoc requiritur quam ut, existente figura navis aquam ferientis ex lineis rectis conflata, perpendiculares ex punctis mediis singulorum navis aquam ferientium laterum, in eadem latera, conveniant omnes in eodem spinae puncto, seu ut omnia ista latera sint chordae eiusdem circuli centrum in spina navis habentis, tum enim in hoc centro convenient omnes perpendiculares in medium cuiusvis lateris navis in aquam impingentis, unde centrum istud circuli ipsum erit centrum virium quaesitum. Sit *ACEDB* (Fig. 2) circulus, centrum eius *G* et diameter quae pro spina navis accipietur, *AGH*. Ducantur chordae ex utraque parte spinae quot quaecumque lubuerit ut *AB*, *BD* et *AC*, *CE*, ducanturque lineae proram constituentes *DH* et *EH*, habebitur figura navis hanc praerogativam habens ut centrum virium in eodem maneat loco, utcumque mutato deviationis angulo, modo deviationis versus plagam *E* angulus, angulum *AHE* non excedat et deviationis versus plagam *D* angulus, angulum *A H D* non excedat; centrum vero virium erit in *G*.

19. Hoc usum quidem habere posset in constructione navium, sed cum de hoc non sit quaestio, propius ad figuram navium receptam accedendum est. Contemplabor eam post Virum celeberrimum JOANNEM BERNOULLIUM Fig. 3 tanquam duo segmenta circularia aequalia super eadem chorda ; in hac vero hypothesi multo difficilius pro quovis deviationis angulo centrum virium determinatur, cum ideo quod latera navis resistentiam sentientia sint mutabilia in alio deviationis angulo, tum quod figura sit curvilinea, adeoque incidentiae angulus in quovis puncto alius est. Hic mihi quia non pro qualibet deviatione centrum virium cognitum habere opus est, necesse non erit modum tradere centrum virium in ista hypothesi pro quovis deviationis angulo determinandi, sed sufficiet si duo saltern centra in duabus deviationibus quarum una possibilium maxima est, altera minima determinavero, quae duo centra limitum adinstar esse possunt, quos inter determinandum est punctum illud loco centri communis virium accipiendum, quod quaeritur. Assumo ergo hasce duas deviationes minimam illam possibilium seu illam cuius angulus est aequalis nihilo seu evanescit, et alteram possibilium maximam pro qua accipiam angulum rectum seu 90 graduum, ultra hunc angulum deviatio navis crescere nequit, cum puppis in proram et prora in puppim converteretur. Pro utroque si determinavero centro, certus sum, inter ea id quod quaeritur contineri, magis autem versus centrum pro priori deviatione, quae nulla est, inventum, assumendum est, quam versus posterius, ubi directio motus navis cum spina constituit angulum rectum, cum anguli deviationum navis magis consuetarum propius semper sint angulo evanescenti quam 90 gradubus. Ac subinde cum sit liberum assumere inter ista duo centra illud quod desideratur seu quod sit centrum commune virium in maxime consuetis deviationibus, tale quoque assumendum est, quod facile et sine multo labore construi possit.

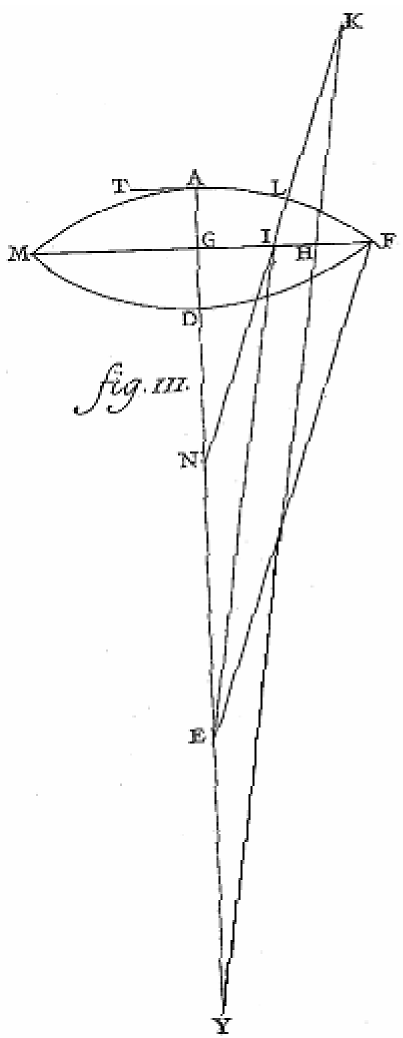
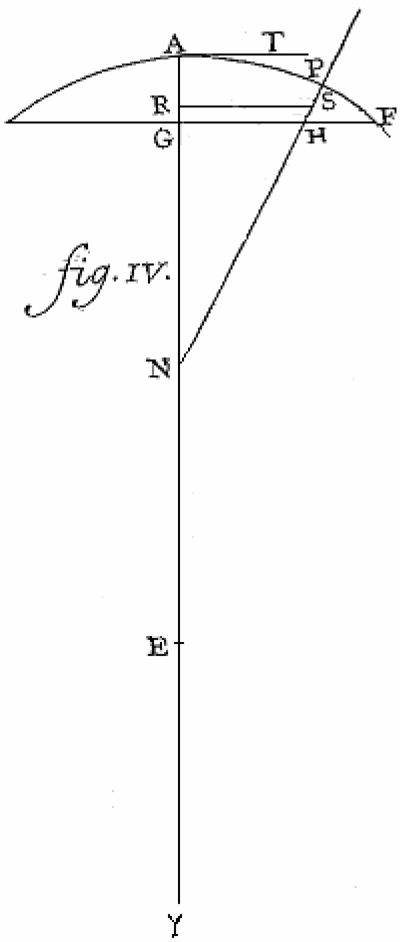


fig. III.

20. Indagabo itaque primo centrum, cum deviatio est graduum 90. Sit *FAMD* (Fig. 3) navis, *F* prora, *FM* spina, *N* centrum arcus *FAM,* ex centro *N* ducatur *NGA* spinam bisecans in *G,* bisecabit ea quoque arcum *FAM;* eritque in spinam normalis. Moveatur ergo navis iuxta directionem *N A* in aqua, ita ut angulus deviationis sit 90° palam est, quia arcus *AM* similis est et aequalis arcui *AF,* atque tantam quantam hic resistentiam patitur, fore ipsam *AN* lineam aequilibrii resistentiae, adeoque punctum *G* ubi spina *F M* ab *N A* secatur fore centrum commune virium, in isthac navis deviatione. Habeo itaque iam centrum commune virium navis cum eius motus directio cum spina angulum 90

graduum constituit; pro deviatione autem evanescente magis erit arduum istud centrum definire, unde meam quam dabo constructionis analysin hic non subiungam, ne nimium sim prolixus, sed eius demonstrationem ex Cel. BERNOULLII *Manoeuvre des Vaisseaux* depromam.



21. Ponamus itaque navem secundum directionem spinae *M F* moveri in aqua, verum quidem est ubicumque centrum virium in spina accipiatur, hanc navis deviationem, quae nulla est, conservatam iri. Quaeritur autem illud punctum in spina *F M* in quo secatur spina a linea aequilibrii mediarum directionum resistentiae arcus *FA* tantum, qui hac in parte spinae *FM* solus resistentiam patitur; nam in *A* erit navis directio tangens *AT*, secundum quam resistentiam perfert; etenim in eodem puncto spinae *F M* quo a linea aequilibrii resistentiae arcus *AF* secatur, secabitur quoque a linea mediarum directionum seu aequilibrii resistentiae quam arcus *D F* perfert, quia hi duo arcus *AF* et *DF* similes sunt et aequales et aquae resistentiam aequaliter sufferunt. Et hinc punctum illud, quo spina *F M* a linea aequilibrii mediae resistentiae arcus *A F* secatur, verum erit centrum virium navis, cum deviatio evanescit. Et hoc punctum proinde etiam erit terminus centrorum in omnibus navis deviationibus; versus proram seu istud centrum prae omnibus aliis proxime accedit ad proram.

22. Sic autem istud centrum determino. Ex centro *N* ducatur recta *NL* arcum *AF* bifariam secans in *L*, spinamque *FM* in *I;* producatur ea in *K* usque ut sit *I K* = *IN*, producatur quoque radius *AN*, in eaque sumantur puncta *E* et *Y*, ut sit *EY* = *NE* = *AN*. Iungantur puncta *E* et *I* recta *EI:* huicque parallela ducatur ex *K* linea *KH* , quae producta transibit per punctum *Y*, nam quia *KI* = *IN* occurret illa linea producta in aliquo puncto quod tantum distat ab *E*, quantum *E* distat ab *N*, ob *NI* = *I K;* hoc punctum ergo ipsum erit punctum *Y*. Punctum autem *H* in spina navis *FM*, ubi ea a linea *KY* secatur, erit centrum commune virium, cum nempe navis secundum directionem spinae movetur.

23. Rationem huius constructionis petere est ex Cel. BERNOULLII *Manoeuvre des Vaisseaux,* ex Capitis XIII. paragrapho 4, ubi centrum mediae resistentiae, quam quilibet arcus circularis subit, determinat. Quem paragraphum, ne Illustrissimi Iudices opus habeant, aliunde demonstrationis meae constructionem quaerere, ipsissimis celeb. Auctoris verbis una cum eius figura hic adiungo, sic se habent eius verba: " Soit donné un arc de cercle quelconque *APF* (Fig. 4) mû dans l'eau suivant la tangente *AT, N* est le centre de cet arc; *NA* le rayon au point d'attouchement, *F G* perpendiculaire sur *NA* ; *AE* le diametre du meme arc *AP*F. Prolongez *AE* en *Y* en sorte

que *EY* est égale au rayon. Prenez *NR* egal aux trois quarts de la troisieme proportionelle de *YG* à *EG*. Elevez la perpendiculaire *RS* et la faites egale aux trois quarts de *GF*. Tirez enfin *NS*. Je dis que le point *S* sera le centre de la resistence moyenne, et *NS* l'axe de l'equilibre de la resistence moyenne. "

24. Linea ergo ista aequilibrii mediae resistentiae *NS* ubi ea secat spinam *FG*, ibi nempe in *H* erit centrum commune virium resistentiae. Ex mea autem constructione idem reperiri punctum *H* ex eo patere potest quod linea *GH* in utraque constructione aequaliter determinetur, quod ita demonstro. In constructione BERNOULLIANA est

$$GH = \frac{RS \cdot NG}{RN}.$$

ob trianguala *NRS*, *NGH* ; est autem

$$RS = \frac{3}{4} GF \text{ et } NR = \frac{3}{4} \frac{EG^2}{YG}$$

Unde his valoribus substitutis erit

$$GH = \frac{GF \cdot NG \cdot YG}{EG^2}$$

25. Ex mea vero constructione (Fig. 3) fundata in BERNOULLIANA, est

$$GH = \frac{GI \cdot YG}{EG}$$

ob triangula similia *EGI* et *YGH;* lineae enim *EI* et *YH* sunt parallelae. Ducatur *EF*, erit ea parallela lineae *NL*, bisecat enim *LN* arcum *AF*, unde cum *N* sit centrum illius arcus, erit arcus *AL* mensura anguli *ANL;* cum vero sit *NA* = *NE* erit punctum *E* in peripheria eiusdem circuli et inde anguli *AEF* mensura erit dimidius arcus *AF*, id est, arcus *AL;* est ergo angulus *ANL* aequalis angulo *AEF* , adeoque linea *NI* parallela lineae *EF;* sunt ergo triangula *NGI* et *EGF* similia, quocirca erit

$$GI = \frac{GF \cdot NG}{EG}$$

quod substitutum in superiore aequatione loco *GI*, proveniet

$$GH = \frac{GF \cdot NG \cdot YG}{EG^2}$$

Cum itaque in figuris 3 et 4 punctis respondentibus eaedem appositae sint literae, erit *GH* in figura 3 eadem cum *GH* in figura 4 ideoque punctum *H* idem quoque erit in utraque figura. Unde concluditur illud a me recte esse determinatum.

26. Determinati ergo sunt duo centrorum limites, nempe puncta *G* et *H*, inter quae assumendum est illud quod quaeritur centrum cuius respectu mali in navibus collocentur. Propius vero versus punctum *H* quam versus *G* sumendum illud est, cum deviationes navium saepius sint infra angulum 45 graduum, quam eum superent. Est autem inter puncta *G* et *H* punctum 1 (Fig. 3) iam determinatum, quod observo semper propius esse puncto *H* quam puncto *G;* distantia enim *HI* se habet ad distantiam *GI* ut *EY* ad *EG*, id est, cum *EY* sit aequalis *EN*, erit ilIa ratio ut *EN* ad *E G* quae est semper minoris inaequalitatis. Unde autumo si illud centrum quaesitum circa in puncto *I* assumatur, haud

multum a scopo aberratum iri; nam praeterquam quod puncto *H* propius sit quam puncto *G,* idem deprehenditur cum eo quod inveniretur, si latera *AF* et *DF* tanquam lineae rectae considerentur, quodque centrum iam determinatum est: punctum enim 1 hic determinabitur bisecando latus alterutrum *AF* et ex bisectionis puncto *L* in *AF* normalem erigendo, punctum enim in quo est concursus linearum *LN* et spinae *FM* , erit istud punctum *I.* Facillime ergo inveniri poterit punctum istud in posterum pro centro habendum.

27. Manifestum ergo est, me non monente vim velorum versus proram multo maiorem fore, quam ad puppim, cum centrum *I* semper in prora navis reperiatur. Si itaque in nave unicus tantum erigendus sit malus, ille ponetur in puncto isto *I.* Si duo mali, unus ex una parte puncti *I,* alter ex altera parte, in talibus distantiis ab *I* quae sint reciproce ut vires quas a vento excipiunt. Eodem modo se res habebit si plures mali in nave sint erigendi. Atque sic locus malorum optimus et utilissimus est indigitatus. Restat ad hoc Caput plane absolvendum, ut addam qualem angulum cum horisonte mali constituere debeant.

28. Cum mali verticales ventum ad angulos rectos excipiant, si nimirum linea venti in planum velorum perpendicularis est, quae est vis maxima venti, utpote quae crescit in duplicata ratione sinus anguli incidentiae caeteris paribus, utique mali maxima vi navem propellendi gaudebunt, absque longa igitur disquisitione mali ita sunt constituendi, ut cum navis in pleno motu fuerit, mali tum sint verticales. Cum itaque detur angulus ad quem navis inclinari debeat, mali ab initio versus puppim angulo isto inclinari debent ut cum navis plene moveatur, proraque ad datum angulum submergatur, mali tum fiant verticales, verum cum funes versus puppim a vi quam a vento sustinere debent extendantur magis, unde fit ut mali protinus ad proram inclinent, cui autem facile, ut et aliis quae hic impedimentum quoddam creare possint, intelligentes Naupegi mederi poterunt.

CAPUT ALTERUM

DE ALTITUDINE MALORUM, SEU QUANTITATE VIRIUM NAVEM PROPELLENTIUM

29. Si navis a vento vela infLante propellitur, duplicem in navem exerceri vim experientia constat. Unam qua navis promoveatur, alteram vero qua navis inclinetur versus proram seu qua prora profundius immergitur. Prioris effectus gratia vela adhibentur, ne operoso remigando navis propelli debeat. Posterior effectus verum est incommodum in navigationibus, cum propter illum vis impelIens non pro lubitu augeri queat, ne prora prorsus aut saltern tantum quam sine periculo nequit immergatur.

30. Huic autem incommodo obviam eundo, et navem extra omne periculum ponendo, tanta velorum copia est admittenda quae faciat ut navis ad certum aliquem et fixum gradum inclinetur, quo situ et perseverare possit sine uIlo discrimine; cum proinde ista navis inclinatio non solum a velorum quantitate, verum etiam et praecipue a loco applicationis et latitudine velorum dependeat, determinandus est inter omnes illos casus

quibus navis ad datum gradum seu ad datum inclinationis angulum inclinetur, ille qui navem celerrime promovet, seu qui velorum maximam admittit copiam; hoc enim casu, palam est fore ut navis quantum absque periculo potest celerrime promoveatur.

31. Cum itaque proponatur angulus inclinationis seu ille angulus, quem constituere debent ea in nave cum linea verticali, quae nave quiescente in ipsa verticali fuere, oportet ut determinetur quantitas velorum quae malis applicata, navi ad propositum angulum inclinandae praecise par sit. Verum ad vis istius quantitatem determinandam, quum quaelibet venti vis duplicem in navem exerat effectum, necesse est ut primum inquiramus quanta vis venti portio navi promovendae destinata sit et quanta navi inclinandae. Hoc autem ut inveniam, sequenti modo ratiocinor.

32. Primo, cum praevideam resistentiam aquae ad istum effectum multum conferre, ponam aquam navi plane nullam resistentiam opponere sed navem liberrime transmittere, manente tamen eadem aquae gravitate. Patet in hac hypothesi nuIlam venti portionem in nave inclinanda consumi, sed totam venti vim navi propellendae inservire; ponamus enim navem aliquantulum tantum inclinari, scilicet ex ordinario situ quo centrum gravitatis ad infima quae potest descendit, detorqueri, patet navem hoc in situ permanere non posse utcumque celeriter navis deferatur ; navis enim cum in situ isto non naturali perseverare nequeat, rursus in naturalem reverti conabitur; quod duplici modo fieri poterit, vel si mali retrocedant et ita proram rursus ex aqua extollent, donec situs naturalis obtineatur, vel autem si navis ipsa celerius quam mali progrediendo ex situ coacto erumpat et ita sese restituat; prius fieri nequit cum ventus malos regredi non permittat, posterius navis facillime peraget, cum nullam inveniat resistentiam, quae restitutionem istam impedire posset, et ita navis hoc modo in aqua non resistente progrediendo plane non inclinabitur quantacumque venti vis adhibeatur adeoque tota vis, quam ventus in vela exerit, in nave promovenda insumetur, et nulla in nave inclinanda.

33. Transeo iam ad alterum extremum et suppono aquam navi infinitam resistentiam facere, scilicet concipi potest aqua in glaciem durissimam conversa, cavitas autem cui insistit navis politissima, hoc modo enim fiet ut navis promoveri nequeat ob resistentiam respectu aquae resistentiam infinitam, attamen inclinari poterit navis; motui enim inclinationis non resistetur ob superficiem glaciei perfecte laevigatam. Expansis itaque velis patet totam venti vim in nave inclinanda occupatam fore.

34. Bisce duobus extremis consideratis, pervenio ad aquam naturaliter consistentem, quae est tanquam medium inter duo extrema ista; nec enim plane nullam obvertit navi resistentiam nec infinitam, unde iam palam esse potest, cum ab utroque extremorum aqua aliquid participet, venti vim et navem propellere debere et navem quoque inclinare. Perpendendum ergo est quanta vis venti portio in promovenda, et quanta in inclinanda nave occupetur, quae duae portiones totam vim venti adaequare debent, cum effectus suos secundum easdem directiones edant. Est itaque vis venti navem propellens aucta vi venti navem inclinante aequalis totae venti vi.

35. Si effectus venti aliter consideretur, patet partem potentiae venti consumi in superanda resistentia aquae, atque partem in promovenda nave; quae duae partes, cum

effectus suos quoque secundum eandem directionem edant, simul sumptae totam venti vim adaequant. Comparando ergo istam distributionem cum ea quam in paragrapho praecedente instituimus, inveniemus, summam virium venti eius quae navem inclinat et eius quae navem promovet, aequalem esse summae virium venti eius quae aquae resistentiam superat et eius quae navem promovet; demta ex hac aequatione utrinque vi navem propellente, emerget vim venti resistentiam aquae superantis aequalem esse vi venti navem inclinantis. Atque ita patet quanta vis ad inclinandam navem impendatur, nempe tanta, quanta superandae resistentiae aquae par est. Cum ergo sit resistentia navis in duplicata ratione celeritatis eius, erit quoque vis superandae resistentiae destinata, et hinc quoque vis navem inclinans erit in duplicata ratione celeritatis navis; quo celerius ergo navis procedit, eo magis quoque navis inclinabitur, et in ipso motus initio cum celeritas navis adhuc est infinite parva, erit quoque vis navem inclinans infinite parva, et crescente navis celeritate angulus inclinationis augmentabitur.

36. Quemadmodum corpora cadentia paulatim maiorem acquirant celeritatem a vi gravitatis continuo ea ad descensum sollicitante nec illis subito celeritas ea quam tandem acquirunt communicatur et sicut lignum torrenti iniectum ab initio infinite parvam quidem habet celeritatem, ea vero continuo augetur, sic quoque vento vela impellente ab initio navis celeritas est infinite parva, crescit autem ea continuo, donec tandem tantam acquirat celeritatem quae ulterius augeri nequit, si enim aqua nullam opponeret navi resistentiam, tandem navis acquireret celeritatem aequalem celeritati venti, resistente autem aqua celeritatem tandem post tempus infinitum quidem acquiret navis minorem venti celeritate, tanto scilicet minorem ut ventus celeritate residua vela petens praecise superandae resistentiae par sit. Dico post tempus demum infinitum, sed iam post aliquantum temporis spatium, tantam acquirit navis celeritatem quae sensibiliter ulterius non crescit.

37. Cum ergo navis motu accelerato procedat, resistentia quoque crescit et tunc vis superandae resistentiae destinata etiam crescit; et proinde quoque vis navem inclinans, ut adeo angulus inclinationis continuo crescat donec tandem cum navis celeritas eadem permanserit, immutatus remaneat; nave autem uniformiter procedente, tota vis vela propellens in superanda aquae resistentia consumitur, et tunc quoque tota venti vis, cum navis celeritas maxima fuerit, in inclinanda nave consumetur.

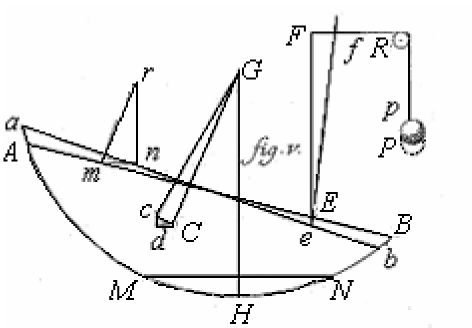
38. Cum autem proponatur angulus ad quem navis inclinari debet, procul dubio hic angulus maximus esse debet eorum ad quos navis inclinatur, seu debet esse angulus inclinationis cum navis fuerit in pleno motu; si enim isti angulo aequalis fieret inclinationis angulus mox ab initio motus, tum angulus inclinationis protinus cresceret, et tandem multo fieret maior ac erat propositum; maximum ergo inclinationis angulum in posterum pro cognito habebimus, nempe eo dato investigabimus quantitatem vis a vento mutuandae quae navi tandem ad propositum angulum inclinandum par sit, seu cum iste angulus dein idem permaneat, requiritur vis quae navem ad hunc usque angulum inclinatam conservare possit.

39. Ut istud commodius detegam, unicum tantum malum navi infixum supponam, et in eius puncto aliquo, circa quod quoquo- versum vela et proinde vis venti aequaliter sunt

dispersa, totam venti vim admittendam congregatam considerabo, quod punctum ergo instar centri communis velorum, quemadmodum in posterum quoque vocabitur, erit. Quo autem facilius vim ad navem ad propositum angulum inclinandum requisitam inveniam loco venti pondus in computum ducam, quod in eodem centro communi velorum applicatum ponam, atque malum horisontaliter, quod ope trochleae fieri poterit, trahens, atque sic determinandum est pondus, quod navi ad datum angulum inclinandum par sit, quo facto postmodum tradam methodum vim venti cum ponderibus comparandi, ut loco ponderis inventi, ventum rursus in computum introducam, atque sic determinem quantum virium a vento excipiendum sit ut navis ad propositum angulum inclinetur.

40. Cum autem iam notum sit quantum virium inclinationi navis destinatum sit, proinde navem tanquam quiescentem considerare potero, seu quod eodem redit, aquam tanquam in glaciem congelatam considerabo, ita tamen laevigatam ut navis in cavitate sua liberrime absque ulla strictione inclinari et reclinari possit; hoc enim modo navis tanquam in medio infinite resistente constituta erit considerata, et proinde ea vis sola, quae inclinandae navi inservit in centro velorum applicata navem eodem modo inclinabit, ac si navis in aqua naturali processerit. Hic ergo quoque, ubi loco venti pondus in computum duco, navem eodem modo collocatam in glacie contemplabor, et indagabo pondus quod navem ad propositum angulum inclinare possit.

41. Non sufficit autem ad pondus quaesitum inveniendum proponere angulum inclinationis; sed praeterea requiritur ut cognoscatur figura navis, pondus atque locus centri gravitatis eius. Quod ad pondus navis et locum centri gravitatis attinet, ea generaliter tractabo ut ad quoslibet speciales casus applicari possint; per pondus navis autem non intelligo pondus navis vacuae sed oneratae, et eodem modo centrum gravitatis oneratae navis intelligo. Quod autem ad figuram navis [attinet], spinam eius tanquam in arcum circularem curvatam concipio, modo ea eius pars sit arcus circuli, quae in aquam intrat; sufficit [ut] huius curvaturae radius in computum ducetur, seu potius distantia centri curvaturae spinae a centro navis gravitatis. Si spinae curvedo non exacte sit circularis non multum refert, sed pro ea curvatura assumenda est curvatura circularis ad eam quam proxime accedens.



42. His positis sit *AMHNB* (Fig. 5) navis seu potius eius spina, *B* prora et *A* puppis, *M N* superficies aquae: sitque navis ita inclinata ut linea *mr,* quae in statu quietis navis in horisontem perpendicularis fuerat cum verticali *rn,* nunc faciat angulum *mrn.* Sit *C* centrum gravitatis totius navis, et *G* centrum arcus *AMNB,* seu si arcus *AMN B* non fuerit exacte circularis, *G* est centrum arcus circularis curvaturae spinae proxime aequalis seu talis arcus qui transit per puncta *M* et *N*, et segmentum sub chorda *MN* comprehendit, aequale ipsi *MHN; GH* est linea verticalis in isto navis situ quae erit in *MN* normalis et proinde [talis] quoque ut et arcum *MHN* bisecat. *GC* est distantia centri gravitatis *C* a centro curvaturae *G. EF* est malus verticalis in quo sit *F* centrum commune velorum, in isto puncto loco venti sit applicatum pondus *P,* quod circa trochleam *R* malum secundum

directionem horisontalem *FR* trahit, quaerendum est quantum debeat esse pondus *P* quod navem in ista positione conservare possit.

43. In situ navis naturali descendit centrum gravitatis *C* ad locum, quam possibile est infimum. Patet autem cum semper aequalis arcus *MHN*sub linea *MN* seu superficie aquae contineatur, centrum *C* gravitatis magis descendere non posse quam cum sit in ipsa verticali *GH;* cum enim distantia *GC* semper eadem maneat et punctum *G* immutatum quoque sit, totam navis molem in *C* congregatam concipiendo, manifestum est pendulum *GC* quiescere non posse nisi sit punctum *C* in linea verticali *GH.* Linea ergo *GC* fuit in statu quietis verticalis, unde angulus *CGH* erit angulus inclinationis navis et proinde aequalis angulo *mrn.*

44. Ut autem inveniam quantitatem ponderis *P* quod cum nave in isto situ non naturali in aequilibrio consistat, pono pondus *P* aliquantulum descendere per lineolam infinite parvam *Pp;* cum navis progredi non posse supponitur ob aquam in glaciem mutatam, in sua cavitate circa centrum cavitatis *G* aliquantulum vertetur ut ex situ *AMHNB* in situm *aMH Nb* veniat, et malus *EF* in *ef;* ita ut sit *Ff* = *Pp.* Centrum gravitatis *C* perveniet in *e,* ita ut ducta *Ge* angulus *CGe* aequalis sit angulo *FEf.* Ex *c* demittatur verticalis, *cd*, horisontali per *C* transeunti in *d* occurrens, ascendit centrum gravitatis navis per altitudinem *cd,* triangulum autem *Ccd* simile erit triangulo *rmn,* nam quia linea *cd* parallela est lineae *GH,* erit summa angulorum *Gcd* et *HGc* aequalis duobus rectis; angulus vero *CcG* est rectus, ergo angulus *Ccd* plus angulo *cGH* constituit unum rectum; cum autem triangulum *Ccd* in *d* sit rectangulum, erit summa angulorum *Ccd* et *cCd* quoque recto aequalis, unde erit angulus *cCd* aequalis angulo *HGc,* seu cum nonnisi infinitesima parte differant angulo *CGH,* seu angulo *mrn,* praeterea anguli *d* et *n* aequales sunt, quia uterque rectus est, unde triangula *rmn* et *Ccd* sunt similia.

45. Sed notum est ex Mechanica, duo pondera utcunque sita sese in aequilibrio conservare cum vel tantillum mutata eorum positione, ascensus centri gravitatis unius se habeat ad descensum centri gravitatis alterius reciproce, ut pondus prioris ad pondus posterioris, seu directe, ut pondus posterioris ad pondus prioris. Hoc applicando in nostro exemplo, cum navis et pondus *P* se quoque in aequilibrio servare debeant, erit pondus navis quod *Q* vocabitur, ad pondus *P* ut descensus huius *Pp,* ad ascensum centri gravitatis navis *cd,* unde erit $p \cdot Pp = Q \cdot cd$, seu ob *Pp* = *Ff* erit $p. Ff = Q \cdot cd$.

46. Quia autem angulus *FEf* aequalis est angulo *CGc,* et angulus *EFf* est rectus ob *EF* verticalem et *FR* horizontalem, erunt triangula *GCc* et *EFf* similia adeoque

$$Ff : EF = Cc : CG, \text{unde } Ff = \frac{EF \cdot Cc}{CG}$$

consequenter

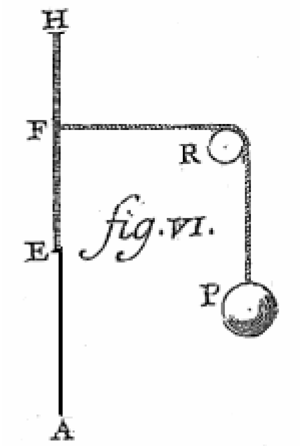
$$P \cdot EF \cdot Cc = Q \cdot CG \cdot cd, \text{seu } P = \frac{Q \cdot CG \cdot cd}{CG \cdot Cc} ;$$

verum ob triangula *rmn*, *Ccd* similia, est *c:cd* = *rm:mn*, id est, ut sinus totus ad sinum anguli inclinationis, quae ratio cum sit proposita, ponatur ea ut 1: *s*, erit

$$P = \frac{Q \cdot CG \cdot s}{EF}.$$

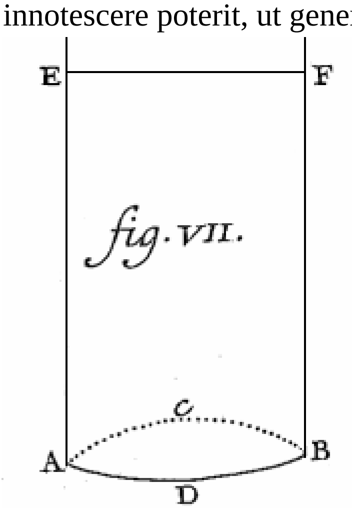
Sit distantia centri gravitatis *C* a centro curvaturae spinae *G*, nempe *CG* = *b*.
E F est dimidia mali altitudo cum sit *F* centrum velorum, et vela supponantur ubique eiusdem latitudinis; ponatur autem tota mali altitudo (mali scilicet unius, cui, si plures sint navi inserendi, aequipollere debent) quae hic nobis determinanda proponitur, aequalis *z*, erit ergo $EF = \frac{1}{2}z$, et habebitur

$$P = \frac{2Qbs}{z}.$$



47. Determinatum ergo est pondus *P*, quod navem in dato inclinationis angulo conservare potest; huic ponderi aequivalere debet vis a vento excipienda: ad hanc ergo quoque definitionem necesse est ut primum inquiram in rationem quam vis venti ad pondera habeat, seu ut vim venti in ponderibus exprimam. Hoc quidem experientia institui posset, verum etiam a priori ex theoria proportionem deduci posse monstrabo. Experientia hoc sequenti modo fieri potest. Fiat malus utcunque brevis *AH* (Fig. 6) circa punctum *A* mobilis, huic sit alligatum velum planum *EH*, quod vento exponatur, qui secundum directionem *RF* in illud impingat, malumque circa polum *A* rotare conetur; applicetur autem in puncto *F* centro veli, funiculus *FR* qui circa trochleam *R* trahatur a pondere *P*ita ut malus ab isto pondere retrahatur, determinetur autem experientia pondus *P* ei addendo vel subtrahendo donec malus in situ verticali conservetur, et tum erit pondus *P* quod vento istud velum *EH* inflanti aequipollet, et cum innotuerit capacitas veli et celeritas venti, ex inde facile comparatio in aliis venti celeritatibus et aliis velis vel maioribus vel minoribus institui poterit.

48. Generaliter autem ratio inter vim venti et pondera a priori ex theoria hoc modo innotescere poterit, ut generalius rem complectar, abstraham a vento seu aere et eius loco quodlibet fluidum contemplabor, eiusque percussiones cum ponderibus comparare tentabo. Sit vas cylindricum *EADBF* (Fig. 7), isto fluido usque in *EF* repletum, basis autem *ACBD* sit horisontalis, patet, fundum istud premi a fluido incumbente, ita ut perforato ubivis hoc fundo, fluidum tanta celeritate efflueret quantam acquirere potest corpus cadendo ex altitudine *F B* . Quemadmodum Claro HERMANNUS in suis annexis ad Phoronomiam, Celeberrimo BERNOULLIO suppeditante, primus publice demonstravit, fundum ergo sustinet pressionem fluidi ferendo idem ac si idem fluidum ea celeritate qua efflueret per foramen, in illud impingeret.



49. Demonstravit autem modo citatus acutissimus BERNOULLI apud MICHELOTTUM in Libro *De separatione fluidorum* fluidum per foramen effluens dimidiae saltem densitatis censendum esse, eius

quam in vase habebat; inter duos enim globulos seu atomos fluidi effluentis contineri tantundem vacui, ita ut globuli quae in vase contigui fuerant; in egressu separentur, ita ut in aequali spatio saltem dimidium contineatur fluidi in exitu ex foramine, quam eius in vase, unde rationem reddit celebris phaenomeni de contractione fili fluidi ex vase erumpente. Hoc ergo in nostro casu applicato, dicendum est fundum vasis ferendo pressionem fiuidi in vase contenti, idem sustinere ac si fluidum duplo rarius celeritate, aequali ei quam grave ex altitudine *FB* descendendo acquirere potest, in id irrueret.

50. Habeo ergo rationem seu proportionem inter pondera et vim percussionis fluidorum; ex hisce enim concluditur, cum fluidum quodvis celeritate quacumque in planum directe seu perpendiculariter irruit, planum idem sustinere ac si in situ horisontali positum sufferret pressionem fluidi duplo densioris et altitudinis tantae, ex qua grave cadendo celeritatem aequalem celeritati fluidi allabentis acquirere potest: cum ergo innotuerit pondus huius fluidi duplo densioris baseos aequalis plano dato et altitudinis dictae, habebitur pondus vi fluidi illius allabentis aequivalens.

51. Applicetur hoc ad ventum, et patebit vela ventum directe excipiendo idem sustinere ac si in situ horisontali posita perferrent pressionem fluidi quod aere duplo densius est, et altitudinis ex qua grave cadendo acquirere potest celeritatem aequalem celeritati venti. Sit v celeritas venti, ea scilicet qua vela petit seu celeritas respectiva. Experientia autem constat grave ex altitudine 15 pedum Rhenanorum descendendo celeritatem adipisci qua cum tempore unius minuti secundi percurrere possit 30 pedes; ut celeritatem venti v, exeffectu seu spatio percurso dato tempore metiamur, designet v numerum pedum Rhenanorum quos tempore unius minuti secundi percurrere potest.

52. Cum altitudines in descensu corporum sint ut quadrata celeritatum acquisitarum, et corpus ex altitudine 15 pedum descendendo acquirat celeritatem ut 30, fiat ut 900 quadratum ipsius 30 ad vv quadratum celeritatis venti respectivae, ita 15 pedes ad $\frac{15vv}{900}=\frac{vv}{60}$ pedes, quae est altitudo ex qua corpus cadendo acquirere potest celeritatem aequalem celeritati venti v.

53. Habeo itaque altitudinem illius fluidi quod suo pondere aequivalet vi venti. Basis erit superficies velorum; est autem eorum longitudo quae eadem est cum altitudine mali, iam posita aequalis z. Sit praeterea latitudo velorum = a, erit ergo basis illa aequalis az. Sunt autem a et z etiam in pedibus Rhenanis exprimenda cum v iam sit ita expressa, erit ergo moles fluidi illius suo pondere aequivalentis vi venti = $\frac{azvv}{60}$ pedibus cubicis.

54. Restat ergo ad pondus vi percussivae venti aequipollens inveniendum, ut gravitatem fluidi illius inquiramus; quia autem fiuidum illud duplo densius ponitur quam aer, erit etiam duplo gravius, unde cum pes cubicus aeris ponderet quam proxime $\frac{1}{12}$ librae, ponderabit pes cubicus illius fluidi $\frac{1}{6}$ librae, unde $\frac{azvv}{360}$ pedes cubici pondere aequabunt $\frac{azvv}{360}$ libras, et hoc est pondus, quod trahendo eundem effectum praestare valet ac ventus

celeritate ut v vela impellente; hoc ergo pondus aequale ponendum est ponderi P, quod quoque loco vis venti positum fuit, et erit $P = \frac{azvv}{360}$.

55. Inventum autem fuerat §46 $P = \frac{2Qbs}{z}$. Unde erit

$$\frac{2Qbs}{z} = \frac{azvv}{360}, \text{seu } azzvv = 720Qbs.$$

Ut autem perfecta reperiatur uniformitas, b in pedibus quoque Rhenanis et Q in libris exprimenda sunt. Nempe distantia centri gravitatis a centro curvaturae in pedibus, et pondus navis in libris, ut omnia ad eandem referantur unitatem, aequatio autem ad hanc reducetur extrahendo utrinque radicem quadratam,

$$zv = 12\sqrt{\frac{5Qbs}{a}}$$

unde invenitur

$$z = \frac{12}{v}\sqrt{\frac{5Qbs}{a}}.$$

56. En ergo iam aequationem, ex qua altitudo quaesita malorum z determinari potest. Datis primo pondere navis Q in libris; secundo distantia b centri curvaturae spinae a centro gravitatis navis in pedibus; tertio latitudine velorum seu longitudine antennarum quae ubique eadem supponitur a,in pedibus quoque; et quarto celeritate venti relativa, nempe ea qua navem petit; cum enim navis quoque celeritatem habeat, aer sua celeritate in navem impingere nequit, sed vela petit celeritate, qua celeritas venti celeritatem navis excedit; haec autem velocitas v exprimenda est in pedibus itidem Rhenanis, scilicet indigitat ea quot pedes ventus uno minuto secundo emetiatur celeritate respectiva, praeterea angulus inclinationis nempe sinus eius s existente sinu toto = 1 per se datus est. Et sic altitudo mali z determinari poterit.

57. Notandum est in expressione [altitudinis] mali z resistentiam aquae non in computum venire, et hinc eo facilius erit altitudinem mali supputare. Cum autem requiratur vis venti cum navis iam fuerit in pleno motu, a celeritate venti detrahenda est celeritas navis et habebitur celeritas v; et hinc mirum non est quod resistentia aquae non in computum ineat; eius enim loco introducta est celeritas respectiva v. Ad hanc enim determinandam data venti celeritate, requiritur navis celeritas, ad cuius cognitionem utique resistentia aquae et partes navis in quas aqua impingit in computum duci debent.

58. Cum autem difficile sit data venti celeritate navis celeritatem praevidere ut celeritas venti respectiva haberi possit, quae in expressione altitudinis mali cognita esse debet, necesse est ut methodum tradam navis celeritatem quovis peracto spatio inveniendi. Sufficeret equidem celeritatem navis maximam seu eam quam acquirit spatio infinito percurso indicasse, cum v sit celeritas venti respectiva, cum navis maximam iam acquisierit celeritatem. Verum cum hic commoda offeratur occasio, et celeritas navis maxima exinde facillime inveniri queat, modum inveniendi navis celeritatem quovis peracto spatio, hic in medium proferam; ex eo enim legem accelerationis navis videre

erit, et cum naves non quidem infinitum spatium percurrere debeant, ut uniformiter procedant, sed aliquanto spatio perverso iam tantam acquirunt celeritatem quae sensibiliter postmodum non crescit, patebit quoque quantum spatium navis percurrere debeat, ut sensibiliter uniformi motu procedat.

59. Ad hoc vero inveniendum necesse est ut resistentia aquae in computum ducatur. Quia autem navium figura talis non est quae nave in aqua mota aquam normaliter percutiat, sed oblique et in uno loco obliquius quam alio, aquae resistentiam patiatur. Non ergo pro ratione superficiei navis aquam stringentis resistentiam metiri licet, cum ea quoque in alio deviationis angulo alia sit, ad huic inconvenienti occurrendum assumam aliquod planum quod aquam ea qua navis movetur celeritate, normaliter feriendo, eandem cum nave resistentiam subeat. Hoc modo enim facilius erit resistentiam navis contemplari, cum angulus incidentiae supponatur semper rectus, et spatium aquam feriens constans, nonnisi ergo ad celeritatem qua in aquam impingit attendum erit.

60. Pro hoc autem plano eandem cum nave resistentiam patiente absque sensibili errore assumi posse video sectionem navis transversalem maximam, eius scilicet navis partis quae in aqua degit, haec quidem cum navis secundum spinae directionem movetur aquam normaliter feriendo, multo maiorem sufferret resistentiam quam navis, et hinc istam sectionem pro illo plano assumendo in excessu peccaretur, verum nave oblique mota, resistentia eius quoque augetur atque cum prora navis profundius submergitur superficies navis aquam findens incrementum accipit, unde resistentia quoque augebitur, praecipue cum gubernaculo utuntur. Quocirca resistentia, quam sectio illa transversalis aquam normaliter feriendo maior vixerit, nisi plane sit aequalis aut aliquantulum minor, quam resistentia navis. Et proinde sectio ilIa transversalis maxima non totius navis sed saltem partis eius aquae immersae, pro plano eandem cum nave resistentiam patiente absque sensibili errore accipi poterit.

61. Sit itaque ista sectio aequalis ff, est autem ff exprimenda in pedibus quadratis, sit praeterea altitudo parallelepipedi cuius basis est ff quod capacitate seu mole partem navis sub aqua mersam adaequat $= h$, quae altitudo etiam in pedibus est exprimenda, cum comparanda sit cum latitudine velorum et altitudine eorundem quae in pedibus exprimuntur. Erit ergo moles partis navis aquae immersae aequalis hff pedibus cubicis, erit enim hff moles parallelepipedi illius quod partem navis aquae mersam adaequat.

62. Ponatur materia navis eiusque onus per omnes partes navis aequaliter dispersa, ut navis tanquam corpus homogeneum considerari possit, eiusdem nempe ubique densitatis, immutato tamen ejus pondere sit ratio istius navis densitatis ad densitatem aquae ut K ad m, et ad densitatem aeris ut K ad n. Erit ergo pars navis aquae immersa quoad massam ut $Khff$. Totius vero navis massa cum ut homogenea consideretur, se habet ad partem navis submersam ut densitas aquae m ad densitatem navis K; erit ergo massa totius navis ut $mhff$. Hisce positis sic ad cognitionem celeritatis navis pervenio.

63. Sit navis iam in motu, et percurrerit spatium y pedum; sit eius celeritas tum acquisita $= v$, indicat nempe v numerum pedum quos corpus celeritate v motu uniformi minuto secundo percurre potest, sit celeritas venti = c eodem modo c exprimetur per numerum pedum quos ventus uno minuto secundo absolvere potest, unde venti celeritas respectiva erit $= c - v$. Est autem capacitas velorum $= az$ et spatium seu planum quod in aquam impingit et resistentiam excipit $= ff$.

64. Promoveatur navis per distantiam infinite parvam, nempe per elementum spatii descripti y, scilicet per dy, et quaeratur acceleratio dum navis per dy promovetur. Patitur autem inter ea navis impulsum a vento, quo navis acceleretur, retardatur vero etiam a resistentia aquae. Est ergo ab incremento celeritatis a vento generato subtrahendum decrementum celeritatis a resistentia aquae productum. Et habebitur elementum seu incrementum celeritatis navis dum per spatiolum dy pergit.

65. Quia aer celeritate c, quae maior est navis celeritate, promovetur, impetus sit ab aere in vela et inde navis celeritas augetur, istud vero incrementum celeritatis ex lege communicationis motus in collisione corporum inveniri potest, cum corpora sunt elastica, aer enim et vela uti et deinceps aqua et partes navis in aquam irruentes tanquam corpora elastica sunt consideranda, si non integra tamen particulae eorum minimae ex quibus sunt conflata, cum enim nave semel mota, vela aequaliter semper expansa supponantur, et navis figura immutata quoque maneat, necesse est ut vela et superficies navis si eorum figura ab aere impingente et aqua resistente aliquo modo immutetur, tamen sese statim restituant, et ita pro elasticis haberi queant.

66. Aerem ad hoc contemplor ut congeriem globulorum infinite parvorum quorum diameter aequalis sit elemento quo navis promovetur nempe ipsi dy, tanta ergo copia huiusmodi globulorum, quantam vela capere possunt celeritate c, impinget in vela celeritate v pergentia. Datis ergo mole navis et mole aeris in vela irruentis, celeritas navis post conflictum reperietur, si scilicet dum navis per dy fertur resistentia aquae tolleretur abs qua si dematur pristina celeritas seu ea quam habebat dum esset in procinctu per dy promoveri, remanebit elementum celeritatis, quod per spatiolum dy navis acquireret, demta resistentia aquae.

67. Constat autem ex regulis communicationis motus, si corpus A incurrat celeritate ut α in corpus B celeritate b motum, tum fore post conflictum ceIeritatem corporis B aequalem $\frac{2A\alpha+(B-A)b}{A+B}$; ut hoc ad nostrum casum applicem et A Massa aeris incidentis, haec autem Massa est ut volumen ductum in densitatem aeris quam posueram ut n, volumen autem aeris incidentis: erit aerea lamina crassitiei $= dy$ et tanta quanta velis implendis sufficit, velorum superficies ventum excipiens est $= az$ et inde volumen aeris impingentis erit $azdy$, consequenter massa aeris impingentis est $nazdy$, hic valor loco A est substituendus.

68. Pro α autem celeritate corporis A ponetur c, celeritas venti, et pro corpore B ponenda erit totius navis Massa quippe quae a vento propellitur, erit ergo $B = mhff$, etenim § 62

inventum fuit massam navis aequari *mhff*, loco autem celeritatis *b* poni debet *v* celeritas navis. His valoribus substitutis reperietur celeritas navis post conflictum

$$\frac{2naczdy+mhffv-nazvdy}{nazdy+mhff}$$

ab hac celeritate si detrahatur ea ante conflictum, nempe *v*, reperietur incrementum celeritatis per spatiolum *dy*, ab impulsu venti productum, nempe

$$\frac{2naczdy-2navzdy}{nazdy+mhff}$$

Cum autem sit in denominatore *nazdy* respectu *mhff* infinite parvum, evanescet illud et denominator erit solum *mhff;* erit ergo incrementum celeritatis a vento ortum

$$=\frac{(c-v)2nazdy}{mhff}.$$

69. Hoc est ergo incrementum celeritatis a vi venti productum; inveniendum restat decrementum celeritatis a vi resistentiae aquae effectum. Hoc eodem quoque modo arguendo innotescet, supponam nimirum aquam consistere ex globulis, quorum diameter sit = *dy*, patet cum navis per *dy* movetur, in tot navem impingere globulos, idque normaliter ad directionem motus navis, quot planum *ff* capere potest; suppono enim, cum ut iam ostensum est eodem redeat, navem eandem pati resistentiam, quam suffert planum *ff* directe aquam eadem celeritate percutiendo. Erit ergo volumen aquae in quod navis impingit = *ffdy*, quod ductum in densitatem aquae *m*, dabit massam illius aquae; erit nempe ea = *mffdy*.

70. Cum vero aqua quiescens supponatur, navis vero celeritate *v* procedens ex isto lemmate celeritas navis post conflictum elucescet posito quod per spatiolum *dy*, nihil a vento excipiat navis. Si corpus *A* celeritate α in corpus *B* quiescens impingat, erit post conflictum celeritas corporis *A* residua = $\frac{(A-B)\alpha}{A+B}$. Hic massa navis *mhff* cum *A* est comparanda, eius celeritas vero *v* cum α, massa vero aquae resistens *mffdy* cum *B* comparanda est erit ergo celeritas navis residua post conflictum

$$=\frac{mhffv-mffvdy}{mhff+mffdy}$$

quae si auferatur a celeritate navis *v*, ante conflictum habebitur decrementum celeritatis, quo navis celeritas per spatiolum *dy* pergendo a resistentia aquae imminueretur, si non novum incrementum a vento acciperet, erit nempe celeritas amissa per *dy*, aequalis

$$\frac{2mffvdy}{mhff+mffdy}=\frac{2mffvdy}{mhff}=\frac{2vdy}{h}$$

evanescente *mffdy* respectu *mhff* .

71. Navis itaque pergendo per elementum *dy*, a vento accipit celeritatis elementum $\frac{(c-v)2nazdy}{mhff}$. Amittit autem de sua celeritate in superatione resistentiae $\frac{2vdy}{h}$. Unde subtrahendo elementum retardationis motus navis ab elemento accelerationis, reperietur incrementum celeritatis navis *v*, dum per *dy* fertur, nempe

$$dv = \frac{(c-v)2nazdy}{mhff} - \frac{2vdy}{h} = \frac{(c-v)2nazdy - 2mffvdy}{mhff}$$

72. Patet hinc incrementum celeritatis esse manente *dy* constante, ut

$(c-v)\cdot naz - m\cdot ffv$,
seu ut

$$c\cdot naz - v\cdot (naz + mff).$$

Quo magis ergo crescit celeritas navis *v*, eo magis decrescit elementum celeritatis, donec si fuerit

$$v = \frac{nacz}{naz+mff}$$

tum celeritas ulterius non crescat, sed eadem maneat; est ergo haec celeritas quam navis acquirere potest maxima, iisdem manentibus celeritate venti, capacitate velorum et spatio resistentiam aquae excipiente; unde concludimus celeritatem navis maximam caeteris paribus esse ut celeritatem venti, eamque se habere ad venti celeritatem ut
naz ad *naz* + *mff*. Quo magis ergo capacitas velorum augetur, eo magis quoque eeleritas venti augebitur manente spatio seu plano *ff* eodem, et manente *az* capacitate velorum eadem ut et *ff*, celeritatem navis fore eandem, sive vela sint latiora, sive arctiora, modo eiusdem sint capacitatis, hinc concluditur.

73. Sic ergo inventa est celeritas navis maxima aequalis

$$\frac{nacz}{naz+mff}.$$

Ad determinandum vero celeritatem navis quovis percurso spatio aequatio paragrapho 71 inventa integranda est. Ad hoc efficiendum eam ad hanc reduco:

$$\frac{2\cdot dy}{mhff} = \frac{dv}{c\cdot naz - v(naz+mff)}$$
$$= \frac{-1}{naz+mff} \cdot \frac{-dv(naz+mff)}{c\cdot naz - v(naz+mff)},$$

huius aequationis integrale per logarithmos habetur

$$\frac{2y}{mhff} = \frac{-1}{naz+mf}\cdot l(c\cdot naz - v(naz+mff)) - l\,\text{const.},$$

seu reducendo

$$\frac{2naz\cdot y+2mff\cdot y}{mhff}= l\text{ const } -l(c\cdot naz-v(naz+mff)) .$$

Ad determinationem constantis ponatur $y = 0$, et debet esse v aequari nihilo, unde erit

$$l\,\text{const.} = l\,cnaz .$$

Erit ergo

$$\frac{2naz\cdot y+2mff\cdot y}{mhff}= l\,cnaz -l(c\cdot naz-v(naz+mff)).$$

74. Dicatur celeritas, qua celeritas v a celeritate, quam navis acquirere potest maxima, differt, u, erit

$$v=\frac{nacz}{naz+mff}- u .$$

Hoc valore substituto loco v erit

$$\frac{2naz\cdot y+2mff\cdot y}{mhff}= l\,cnaz -l(u(naz+mff))$$
$$= lc-lu+l\,naz-l(naz+mff).$$

Et hinc inveniri poterit distantia y, qua absoluta corpus acquiescerit velocitatem utcunque parum a celeritate maxima differentem, ut haberi possit spatium quo percurso celeritas navis absque sensibili errore pro maxima haberi queat; determinatis vero literis in numeris, logarithmi eorum non VLAQUIANI aut BRIGGIANI assumi debent, sed logarithmi hyperbolici qui habentur, si logarithmi VLAQUII ducantur in 2.302585093 quam proxime.

75. Sed revertamur ad aequationem altitudinem mali z exprimentem, cum ibi reperiatur quantitas v, quae indicat celeritatem venti respectivam, cum navis promovetur celeritate maxima; invenietur ergo celeritas v, si a celeritate venti c subtrahatur celeritas navis maxima nempe $\frac{nacz}{naz+ mff}$ erit ergo $v=\frac{mcff}{naz+ mff}$.

76. Indigitat autem hic c numerum pedum Rhenanorum quos ventus uno minuto secundo percurrere potest, nempe cum naves pro vehementioribus ventis, quippe quibus spirantibus naves in periculo esse possunt, instrui debeant, pro c poni potest spatium 80 usque ad 100 pedum, quemadmodum experimentis a variis celebribus viris institutis concludere licet, quod nempe venti vehementissimi tempore unius minuti secundi spatium 80 usque ad 100 pedum absolvant.

77. Ponatur autem valor loco v inventus in aequatione § 55 inventa $zv = 12\sqrt{\frac{5Qbs}{a}}$ et habebitur

$$\frac{mcffz}{naz+ mff}= 12\sqrt{\frac{5Qbs}{a}}$$

ex qua reperietur altitudo mali quaesita

$$z = \frac{12mff\sqrt{\frac{5Qbs}{a}}}{cmff - 12na\sqrt{\frac{5Qbs}{a}}}$$

Hic ergo habemus aequationem perfectissimam, ex qua altitudo z in meris cognitis determinari potest, scilicet in pedibus.

78. Ulterius adhuc aequatio inventa reduci potest exterminando m et n; cum enim sit m ad n ut densitas aquae ad densitatem aeris, i. e. quam proxime ut 800 ad 1, ponatur loco m 800, et loco n unitas, et reperietur ista aequatio

$$z = \frac{9600\,ff\sqrt{\frac{5Qbs}{a}}}{800cff - 12a\sqrt{\frac{5Qbs}{a}}} = \frac{2400\,ff\sqrt{\frac{5Qbs}{a}}}{200cff - 3a\sqrt{\frac{5Qbs}{a}}}$$

sc. pedibus.

79. Datis ergo in nave primo sectione maxima transversa portionis navis aquae immersae ff in pedibus quadratis Rhenanis. Secundo distantia centri curvaturae spinae a centro gravitatis navis totius b in pedibus. Tertio latitudine velorum seu longitudine antennarum a itidem in pedibus Rhenanis. Quarto pondere totius navis Q in libris ut et quinto spatio c quod ventus minuto secundo percurrere potest in pedibus quoque, pro quo ab 80 ad 100 usque pedes assumi possunt, hic ego pro c ponam $36\sqrt{5}$ ut et numerator et denominator per $\sqrt{5}$ dividi queat.

80. Hoc posito habebitur altitudo mali

$$z = \frac{800\,ff\sqrt{\frac{Qbs}{a}}}{2400\,ff - a\sqrt{\frac{Qbs}{a}}} = \frac{800\,ff\sqrt{Qabs}}{2400aff - a\sqrt{Qabs}}$$

multiplicato et numeratore et denominatore per a, ex hac aequatione determinabitur altitudo quaesita z in pedibus Rhenanis; quae altitudo cum inventa fuerit, si sit maior quam ut unicus malus tantus construi possit, distribuenda ea erit in tot partes donec mali tanti haberi queant qui aequales sunt illis partibus respective. Et sic ex hac aequatione determinatur quoque numerus malorum. Hi vero mali sic determinati navem inclinabunt ad tantum angulum cuius sinus se habet ad sinum totum ut s ad 1. Haec ratio autem antea est assumenda et quidem talis ut angulus iste sit inter omnes illos angulos ad quos navis absque periculo inclinari possit maximus, ut maxima quoque inveniatur vis propellens.

81. Ex ista aequatione altitudinem mali definiente haec consectaria deducere licet, quae in fabricatione atque oneratione navium ut et confectione velorum magnum usum habere possunt, seu exinde conc1udi potest quomodo sint naves formandae atque onerandae quaecunque velis sit latitudo danda, ut maxima quam fieri potest, reperiatur vis ad navem ad propositum angulum inclinandam.

82. Patet igitur primo statim quo maior sit *b* distantia centri curvaturae spinae navis a centro gravitatis eiusdem, eo maiorem quoque posse assumi altitudinem malorum, sive eo maiorem a vento excipi posse vim. In oneratione ergo navium in id est attendendum ut centrum gravitatis in loco quo fieri potest infimo sit positum, quod obtinebitur, si merces specifice graviores in loco navis quoad fieri potest infimo collocentur, atque ut in usu est, carina gravi oneretur sabulo, unde fiet ut centrum commune gravitatis ad infimum locum descendat, adeoque distantia eius a centro curvaturae augeatur et proinde quoque vis venti admittenda.

83. Pro navibus vero fabricandis sequitur utilissimum esse quo spina minus incurvetur, ne quis autem putet hinc sequi optimum fore si spina fieret linea recta seu sectio navis secundum longitudinem rectangulum, spina enim quae sub aqua continetur, continuus debet esse arcus circuli, sic autem esset composita ex tribus lineis rectis, unde haec conclusio deduci nequit: cum itaque dico utilissimum esse promotioni navis, quo spina minus incurvetur, id ita est intelligendum quo longior sit navis seu quo longior sit spina, manente altitudine partis navis submersae eadem, sic enim distantia centri curvaturae elongabitur magis, et proinde eius distantia a centro gravitatis.

84. Si contra naves ita breves fiant, manente altitudine partis navis immersae eadem, seu spina in arcum tam exigui circuli incurvetur ut centrum gravitatis et centrum curvaturae coincidant, patet ex aequatione, plane tum nullam a vento excipi posse vim; vis enim minima navi subvertendae prorsus par erit.

85. Et hinc quoque concludi potest, cum curvatura transversalis navis valde magna sit, seu cum sectio navis transversalis sit segmentum circuli valde parvi respectu circuli cuius portio est sectio navis secundum spinam, eo magis ultra fixum terminum navem inclinatum iri quo maior sit angulus deviationis. Quae enim supra de curvatura spinae dicta sunt nonnisi valent quam cum navis secundum spinae directionem promovetur; cum autem angulus deviationis navi datus fuerit, loco curvaturae spinae ponenda erit curvatura lineae in fundo navis ductae secundum directionem motus navis et navem bisecantis, quam lineam, spinam imaginariam nuncupare licet.

86. Cum navis itaque habuerit deviationem, *b* significat distantiam centri gravitatis a centro curvaturae spinae imaginariae, et cum spinae istae imaginariae sint arcus eo minorum circulorum quo deviatio navis maior est, erit quoque tum centrum curvaturae spinae imaginariae centro gravitatis propius, ut inde linea *b* quoque decrescat, et igitur altitudo malorum seu vis navem propellens eo magis erit diminuenda, quo deviatio navis fiat maior; maxime ergo erit periculosum navibus magnam tribuere deviationem, si enim manserit vis impellens, navis valde ultra angulum propositum inclinabitur.

87. Huic incommodo [ergo obviam eundum est], et ne altitudo malorum aut velorum copia in deviationibus navis minuenda sit, naves aliquantum magnae latitudinis construi possent, ut differentia inter curvaturam spinae verae et spinae imaginariae, cum navis

deviatio fuerit 90 graduum, non sit valde magna, ut proinde spinae imaginariae in solitis navis deviationibus a spina vera quoad curvaturam non differant, et proinde distantia *b* centri gravitatis navis a centro curvaturae spinae, sensibiliter non imminuatur cum navis in deviatione promota fuerit.

88. Observo deinde, quod si navis tantae longitudinis fabricetur, seu spina sit arcus tanti circuli, ut distantia *b,* centri gravitatis a centro curvaturae spinae sit aequalis $\frac{5760000 f^4}{Qas}$ ped., tum infiniti mali constituti debeant aut unus infinitae altitudinis ad hoc ut navis ad datum angulum inclinetur, et si *b* fuerit major quam $\frac{5760000 f^4}{Qas}$ pedes, nec infinitum vim fore parem navi ad angulum propositum inclinandae.

89. Si cum fuerit $b = \frac{5760000 f^4}{Qas}$ in aequatione § 80 data, nempe

$$z = \frac{800 ff \sqrt{Qabs}}{2400aff - a\sqrt{Qabs}}$$

denominator fractionis cui *z* aequalis evanescit et inde *z* fiet infinite longa.
Hinc ergo patet quantam praerogativam habeant naves longiores prae brevioribus; si enim longitudo tanta fuerit at *b* sit aequalis $\frac{5760000 f^4}{Qas}$ mali seu numerus velorum pro arbitrio multiplicari poterit absque periculo navis.

90. Dein quod ad latitudinem velorum *a* ex aequatione deducitur, quod quidem paradoxum videtur, sed nihilominus verissimum est, quo magis augeatur velorum altitudo, eo magis quoque altitudinem malorum *z* augeri absque navis periculo, cum tamen navis non ultra propositum angulum inclinetur. Patet enim cum *a* crescat, numeratorem quidem fractionis altitudinem *z* exprimentis diminui; est enim ilIa fractio [§ 78]

$$\frac{2400 ff \sqrt{\frac{5Qbs}{a}}}{200cff - 3a\sqrt{\frac{5Qbs}{a}}}$$

Verum notandum, alteram denominatoris partem $3a\sqrt{\frac{5Qabs}{a}}$ seu $3\sqrt{5Qabs}$ signo – affectam in eadem ratione crescere, et cum denominatoris pars *200cff* signo + affecta maneat, denominator totus in maiore ratione decrescit quam numerator, unde fractio ipsa et eo ipso altitudo *z,* aucta latitudine velorum seu longitudine antennarum augebitur.

91. Hinc ergo patet quanti sit emolumenti antennas, quantum fieri potest, longas adhibere, cum inde quantitas virium navem propellentium quoque augeri possit. Si latitudine velorum aucta, mali eiusdem altitudinis reliqui possent, magnum hoc esset commodum ad augendam navis celeritatem; verum aucta latitudine velorum, non solum altitudo malorum eadem manere potest, sed ea praeterea augeri poterit, unde aucta latitudine velorum vis propellens navem multo magis augebitur, et proinde quoque celeritas navis, absque periculo navis.

92. Quin imo si latitudo ve1orum a fiat = $\frac{5760000 f^4}{Qbs}$ pedum reperietur longitudo malorum z ob denominatorem evanescentem infinita, et hinc altitudo malorum atque numerus pro lubitu multiplicari poterit absque navis periculo; utcunque enim augeatur altitudo et numerus malorum navis tamen non ad propositum angulum inclinabitur, cum demum vis infinita navi ad istum angulum inclinandae par sit, si nempe fuertl latitudo velorum = $\frac{5760000 f^4}{Qbs}$, sin autem ea maior insuper fuerit, nec vis infinita sufficiet ad navem ad angulum cuius sinus est ad sinum totum ut s ad 1 inclinandam.

93. Pervenio tandem ad angulum inclinationis, et noto quo major ille assumatur, eo maiorem posse a vento accipi vim; ut igitur aliquantulum ingens assumi posset, oportet ut navis in nullo sit periculo, licet prora profundius immergatur; ad hoc igitur efficiendum, ut scilicet angulus inclinationis magnus assumi possit absque navis periculo utile esse potest si prora navis magis elevata fiat quam reliqua navis pars, sic enim navis non periclitabitur, etsi prora aliquo usque immergatur, et hinc angulus inclinationis aliquantus assumi poterit.

94. Vel etiam ad idem obtinendum, maxima et gravissima quibus navis onerari debet, onera puppi sunt immittenda; hoc enim modo puppis deprimetur et prora elevabitur, ut adeo maior restet prorae pars extra aquam, quae sine navis periculo aquae immergi potest, et hoc modo angulus inclinationis major quoque assumi poterit. Ex hisce ergo consectariis patet, quaenam observanda sint cum in fabricatione et oneratione navium, tum in confectione velorum ut navis qua absque periculo potest maxima promoveatur celeritate, et non dubito quin ista in praxi magnum usum habere queant si observentur. Atque ex ista mea theoria proposita quavis nave, inveniri poterit absque multo labore et altitudo et numerus malorum, ut navis non sit in periculo et tamen maxima celeritate deferatur.

95. Cum itaque determinata sit altitudo malorum z, praevideri facile poterit navis celeritas maxima. Est enim ea ut inventum est [§ 75], aequalis $\frac{nacz}{mff+naz}$, seu cum sit m = 800 et n = 1 erit ea
$= \frac{acz}{800ff+az}$
Est autem

$$z = \frac{2400ff\sqrt{\frac{5Qbs}{a}}}{200cff - 3a\sqrt{\frac{5Qbs}{a}}}$$

quemadmodum § 78 reperi; si iste valor loco z substituatur, reperietur celeritas navis maxima

$$\frac{2400acff\sqrt{\frac{5Qbs}{a}}}{160000cf^4} \text{ seu } \frac{3\sqrt{5Qabs}}{200ff};$$

seu navis celeritas tanta erit, ut tempore unius minuti secundi percurrere possi spatium pedum $\frac{3\sqrt{5Qabs}}{200\,ff}$.

96. Cum venti celeritas non ingrediatur expressionem celeritatis navis maximae, patet navem hac celeritate processuram quacumque celeritate ventus flaverit, modo navem ad angulum propositum inclinandam par fuerit. Patet denuo exinde celeritatem navis maximam esse in ratione subduplicata latitudinis velorum, nempe si ea quadruplae latitudinis conficiantur, tum navem duplo celerius processuram, eodem modo celeritas navis est quoque in subduplicata ratione distantiae centri gravitatis totius navis a centro curvaturae spinae, atque etiam in subduplicata ratione sinus anguli inclinationis navis. Dein quoque si plures sint naves perfecte similes, sed diversae magnitudinis, cum pondera earum sint in ratione sesquiplicata superficierum et proinde erit Q ut f^3, erunt earum navium celeritates caeteris paribus in ratione reciproca subduplicata longitudinum navium earundem, quo minores ergo conficiuntur naves, quoque velocius propelluntur caeteris paribus, scilicet si fuerint per omnia similes.

97. Iam aliquoties memoravi, si altitudo z tanta reperiatur ut unus malus tantae altitudinis haberi nequeat, tum plures esse sumendos quorum altitudines junctim sumtae inventae z aequales sint qui plures mali tum eundem effectum edent, ac unicus longitudinis z, si haberi potuisset, si nempe latitudo velorum ubique fuerit eadem, nempe aequalis ipsi a.

98. Quod autem illi plures eundem edant effectum, exinde patet quod manente facto ex latitudine velorum in altitudinem seu longitudinem eodem, sive manente capacitate velorum ut et latitudine eadem, vis cum propellens tum inclinans navem eadem quoque permaneat, quemadmodum ex iam allatis colligere licet, sive ergo plures sive pauciores constituantur mali, modo eadem velorum magnitudo seu copia eademque latitudo maneat factum illud ex longitudine et latitudine velorum idem permanebit adeoque navis eadem modo tum quoad celeritatem tum quoad inclinationem promovebitur.

99. Suppono vero hic vela malis ad infimum usque locum applicari, quod vero cum fieri nequeat, ob venti vim vel ibi in inferioribus scilicet partibus malorum vel plane impeditam vel maxime debilitatam, altitudo malorum major erit quam longitudo velorum, quae autem in theoria aequales consideratae fuerant; cum itaque centrum velorum supra punctum malorum medium cadat, necesse est tum fore si capacitas velorum esset aequalis az, ut navis ultra propositum angulum inclinetur: verum cum longitudo velorum minor sit quam z, capacitas velorum quoque minor erit quam az, unde propemodum compensationem fieri existimandum est ut navis tamen non ultra propositum angulum inclinetur, sed sic cum longitudo velorum minor fuerit quam altitudo malorum, vis navem propellens minor erit ac in theoria positum fuerit. Eoque minor erit quo plures fuerint mali in nave erecti, mali ergo si plures fuerint inserendi altissimi quam fieri potest sumantur, ut ita numerus malorum restringatur.

100. Hic tandem hisce meis meditationibus finem impono, cum uti videtur materiam in problemate propositam satis perpenderim, problematique satisfecerim. Haud opus esse existimavi istam meam theoriam experientia confirmare, cum integra et ex certissimis et irrepugnabilibus principiis Mechanicis deducta, atque adeo de illa dubitari, an vera sit ac

an in praxi locum habere queat, minime possit. Si autem ea applicaretur ad exemplum aliquod speciale, longitudinem malorum pro nave proposita investigando, statim apariturum foret, eam haud fallere. Si forte ILLUSTRISSIMA ACADEMIA istas pagellas dignaretur pretio proposito nomen Autoris et locum ubi degit, ex apposita schedula cognoscere erit.