An Essay Explaining
The Properties Of Air.

Author
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I.

It is not yet apparent how to approach the final innermost nature of things, and thus to gain an understanding of the ultimate structure of their parts, in order that an account may be given of the phenomenae which arise from them. Nevertheless, the truth of the matter is, as is generally the case in physics, that it is possible from our observations of the properties of natural bodies, to infer something of their structure. From which the perceived structure of bodies, by means of which many phenomena can be explained, will be made more perfect on this account. Indeed, if all the properties or matter can in short be derived from a theory, which cannot be understand otherwise, then there can be no doubt that this theory is true.

II. Since many phenomena involving air have been considered that have little dependency on each other, then it would surely be considered much better if a single theory could give a satisfactory explanation for everything. Moreover, the theory agreed upon here has largely been prepared in this manner, as in the first place it is thought out from a part of the structure [of the phenomina and experiments], and from that one part of which so many properies flow, since generally a theory can be developed in a number of ways : thus, can it then be examined successively, to establish if it is sufficient for other properties to be explained also? or, if more concepts of the original theory are then required, which in turn satisfy the greater part or indeed all of the phenomena.

III. Among other properties of air that we recognise, that property has seemed especially suitable, and from which the structure of air can be understood, by which air itself is known to expand immediately; and to expand itself further, if whatever impediments there are to further expansion are removed. However, before any other properties of air [need be examined], it seems that the elasticity of the air is the one outstanding property that can only be explained with the greatest difficulty, as its explanation must be derived easily from other [more fundamental] properties.

IV. We may wish either to ascribe this hidden property of air [*i. e.* its elasticity or pressure, a word not yet used by Euler] to a particular quality and ascribe it to an innate force, as there is no other way left to explain this property ; or an attempt can be made to derive it from the motion of a certain fine property of the matter [The idea of an underlying more subtle kind of matter is thus introduced, 'of a finer texture'; this is the first concept for which there is no hard evidence]. Moreover the attempt, or the dead force [as opposed to the idea of a living force, or a force produced by some obvious cause] so-called by Leibnitz, arises from the motion of the fine matter that it is able to pull, if the fine matter is moving in a circle ; by means of which any small particle is trying to escape from the centre, and thus the vortex can itself acquire a force from this kind of expansion. The Cel. Johan. Bernoulli has set out to explain all elastic forces from the use of this principle in a paper recently published in Paris *de communicatione motus*. [See, e. g. J. Bernoulli, *Opera Omnia,* Tom. III*,* p. 89, par. 15 *Discours Sur Le Mouvement*; *Addition*; the idea of the centrifugal force was developed by Huygens, in theorems first published at the end of his treatise *De Horologio Oscillatorio.*] From which the elastic force is asserted to arise from an application of the centrifugal force to fine matter.

V. Here I will follow this line of argument, as it appears to me that this shall be the most probable cause of the elasticity of the air, and which I submit for examination in this dissertation: [one may ask:] how many of the remaining properties of air are to be explained in a sufficient manner from this fundamental format [of Bernoulli], or how many less, as it may appear to be the case? Whether or not the air can be constituted in this way. If not, then what is a better set of circumstances that could be devised to determine the constitution of air?

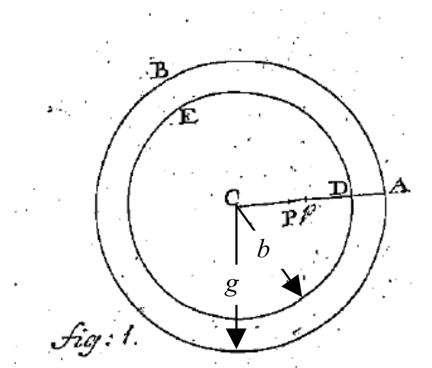
VI. Therefore I suppose that the air is constructed from a great quantity of infinitely small spherical vesicles [or minute bubbles surrounded by a permeable elastic skin or membrane], within which the fine matter is gyrating in the motion of a circle, and the centrifugal force is continuously trying to expand these vesicles and this arrangement [of vesicles] has itself spread to the most remote obstacles.
[Note: There is an attempt to explain pressure in a vesicle as due to a centrifugal force (which we now know to be a pseu-force); Euler and J. Bernoulli may have thought that everyday bubbles could get smaller and smaller and eventually encapsulate the air itself; very small bubbles in the atmosphere are of course very unstable due to the high internal pressure and simply burst or merge into larger stable bubbles. However, to give Euler his due, he considers the arrangement to be a hypothesis, and one that he examines in the following pages, though he becomes less critical rather than moreso, when his theory does not measure up against experimental evidence.]
I suppose again that the small vesicles are covered with a membrane [or skin], which indeed need not be necessary, since small vortices of this kind without membranes can be agreed upon, and yet the two kinds of particles cannot be mixed together. Indeed the one impedes the other that does not wander so far : and yet I suppose the small vesicles have membranes, since the air at no time is so pure as to be absolutely free from water vapour. Moreover it is highly probable that the vapours spread over the air particles like membranes.

VII. Thus it may be agreed upon that the air can consist of an infinite number of the smallest vesicles, the outer crust or shell of which is composed of water with more or less of this, according to the state of the air, and within this membrane the subtile matter gyrates with a certain velocity, which is acceleratedwith other fine matter constantly entering through the pores to the interior, lest the motion is finally used up and disappear. Indeed it can be agreed that heat once accepted is slowly lost, since indeed air becomes rarefied with heat, it follows that the fine matter is more agitated by the motion; and there is evidence that hence with the cessation of heat, the motion of the matter is slowed down. [Thus hot air appears to be associated with the fine matter moving faster]

VIII. It follows from these premises concerning the structure of air that it should expand to the final stage of rarefaction, when there is no known constraint. But with gravity added, matters are resolved otherwise, since gravity is opposed to the elastic force of the air. Indeed when the air above presses upon the air below with its own weight, that below is unable to expand further beyond the limit that its elastic force diminished by the continual expansion is equal to the compressing force of the incumbent air.

IX. Again it is apparent from the conceived constitution of air, that it is not possible to be compressed indefinitely, on account of the specific gravity of the air, which would be increased indefinitely. Since in any small vesicle taken with a certain determined amount of the more fine matter, and with that always always adhering to the surface on account of the centrufugal force, it is necessary that a vacuum is left in the centre of the circle ; which should become larger when the air becomes rarer : moreover with the continual compression of the opposed air, that vacuum space is continually diminished, until at last it has vanished completely, beyond which step it will be impossible to compress the air further.

X. Concerning how the vesicle restrains the velocity of the fine matter, it is necessary to assign the same velocity to the individual particles of this, and not [to assume] that the ones which are further from the centre have a greater velocity and likewise to assign a lesser speed for those closer to the centre. Indeed it may be the case, that from this assumption a theory is produced quite contrary to experience : on account of the larger centrifugal force necessary in larger vesicles, for from this theory it can be established that a small vesicle that has been either compressed or expanded should result in the same velocity of the fine matter, as there is nothing that can change it. At this stage there is no concern with retardation of the motion itself, since§ 6 is not concerned with the unchangeability of vesicles, but rather on account of certain resistance touched upon [*i. e.* the heat entering or leaving]. Whereby as the velocity of the fine material does not depend on the distance from the centre, it is necessary that stays constant everywhere. [Thus, a prediction contrary to Euler's experimental expectations is produced; we note however, that larger water bubbles or vesicles have smaller and not larger pressures than smaller ones in any case.]



XI. Let CAB be an air vesicle [Fig. 1: note that for convenience, figures in the translated part may have extra lengths marked.], as long as it can be compressed, which is hence replete inside with vortices of fine matter. Truly it is surrounded by a membrane of water ADEB, as thus the rest of the space CDE is filled with fine matter. Let AC $= g$, CD $= b$; 1 : π is taken for the ratio of the radius to the periphery [note that in this work, Euler's π is double our π.]; n is taken for the specific gravity of the fine matter ; and m for the specific gravity of the water or membrane. The capacity of the globule CAB = $\frac{2\pi g^3}{3}$, and the capacity of the globule CDE $=\frac{2\pi b^3}{3}$. Hence the volume of the membrane ADEB $=\frac{2\pi}{3}(g^3-b^3)$. On account of this, the mass of the fine matter filling the space CDE $= \frac{2\pi n b^3}{3}$, and the mass of the membrane = $\frac{2\pi m}{3}(\frac{g^3-b^3}{2})$. And these quantities of matter should remain the same in the expansion of the vesicles by whatever force.

XII. k can express the height through which a weight can fall to acquire a velocity equal to that of the fine matter. From which in the following way the centrifugal force, or or the force by which the surface of the globule CDE is being pressed, can be found. The variable CP $= x$ is taken from the centre of the circle, the differential of which P$p = dx$. There will be a spherical shell of thickness Pp and of radius CP equal to $2\pi xxdx$, which if taken with the density of the fine matter *n*, gives a mass or weight equal to $2\pi nxxdx$. When this matter is gyrating with a velocity acquired from the height *k*, [the following ratio:] is made following Huygens: as the radius *x* to twice the height $2k$ is thus equal to the weight of the gyrating matter $2\pi nxxdx$, to the weight of the centrifugal force on this membrane, which therefore will be equal to $4\pi nkxxdx$. [Thus, in an obvious notation,

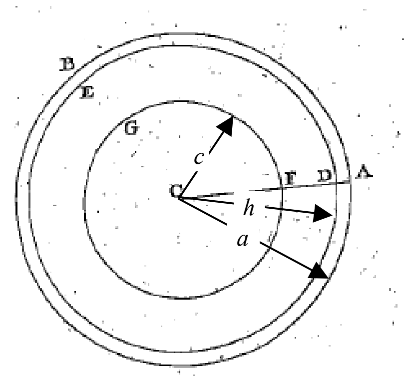


fig: 2.

$v^2 = 2gk; F_g = mg; F_c = \frac{mv^2}{r}; \frac{F_c}{F_g} = \frac{v^2}{rg};$

$F_c = mg.\frac{2k}{r} = \frac{4\pi r^2 dr.ng.2k}{r}$

$\to 4\pi xdx.nk$ in Euler's notation.]

Therefore the integral of this, $2\pi nkx^2$ will express the centrifugal force of the sphere of radius CP. Consequently the centrifugal force of the vessicle DE will be equal to $2\pi nkhh$.

XIII. Now we can consider a vessicle of air CAB spread out in some manner [Fig. 2]: The outer membrane of which ADEB can describe the water film, the middle region DFGE the fine matter gyrating around the centre, and the third or inner region CFG, the empty space or that which can be filled with the least amount of gyrating material. Calling AC = *a*, CD = *h*. There will be, by analysing as above, the following set up : the volume of the outer shell or water ADEB = $\frac{2\pi}{3}(a^3 - h^3)$. Then the volume of the middle shell of fine matter DFGE = $\frac{2\pi}{3}(h^3 - c^3)$. Moreover the third or total capacity of the whole vessicle will be $\frac{2\pi}{3}a^3$. The specific gravity of the air shall be *i*, or the weight of the whole vessicle is $\frac{2\pi i}{3}a^3$, which is equal to the sum of the weights of the parts, surely: $\frac{2\pi m}{3}(a^3 - h^3) + \frac{2\pi n}{3}(h^3 - c^3)$. Therefore: $ia^3 = ma^3 - mh^3 + nh^3 - nc^3$.

XIV. Since the amounts of both water and fine matter should be equal to these, which were found above in the case of vesicles of the maximum compression, the following equatons are obtained:

$$\frac{2\pi}{3}(g^3 - b^3) = \frac{2\pi}{3}(a^3 - h^3) \text{ and } \frac{2\pi}{3}b^3 = \frac{2\pi}{3}(h^3 - c^3).$$

On account of which : $g^3 - b^3 = a^3 - h^3$ and $b^3 = h^3 - c^3$. Hence $h = \sqrt[3]{(a^3 - g^3 + b^3)}$ and $c = \sqrt[3]{(h^3 - b^3)} = \sqrt[3]{(a^3 - g^3)}$. If these are substituted in the last equaton of the previous §, it is found that $ia^3 = mg^3 - mb^3 + nb^3$. Hence, $b^3 = (ia^3 - mg^3) : (n - m)$. And again $h = \sqrt[3]{((i - m + n)a^3 - ng^3 : (n - m)}$ and $c = \sqrt[3]{(a^3 - g^3)}$. Hence in this way the letters *b, c,* and *h* are excluded from the calculation, denoting the distances of the inner parts from the centre.

[Note: There is almost total ignorance about the kinetic theory of gases at this time, which now provides the true explanation for the phenomena considered; instead an elaborate system of vortices of finer matter need to be set rotating within hypothetical vesicles at all angles to insure spherical symmetry - Euler seems to be using cylindrical symmetry to use a problem with spherical symmetry; from which there arises the problem of the central vacuum; and all presented with a total lack of experimental evidence for such structures. One wonders if Euler is trying to justify the theory; it was, after all the brainchild of his mentor, Johan. Bernoulli, set out with all the dignity and trappings of a true theory.]

XIV. The centrifugal force of the gyrating fine matter in the space DFGE with a velocity derived from the altitude *k* found in §11 can be found in this manner : The centrifugal force for a full globe of matter of radius *x* has been found to be equal to $2\pi nkxx$. Whereby should whole space CDE be filled with fine matter, the centrifugal force of this becomes equal to $2\pi nkhh$, from which if taken away, the centrifugal force of the matter in the space CFG , which is equal to $2\pi nkcc,$then there will remain the centrifugal force of the fine matter gyrating in the space FDEG, the amount of which will therefore be equal to $2\pi nk(hh - cc)$ and with the values found in §13 put in place instead of *h* and *c*, this will be equal to $2\pi nk[(\frac{(i-m+n)a^3-ng^3}{n-m})^{\frac{2}{3}}-(a^3-g^3)^{\frac{2}{3}}]$. By putting $h^3 = pg^3$, since $ia^3=mg^3-mh^3+nh^3$, then $ia^3=(m-mp+np)g^3$. Hence $g^3=ia^3:(m-mp+np)$. Thus, the weight equivalent to the centrifugal force is equal to :

$$2\pi nkaa[(\frac{m-i+pi-mp+pn}{m-pm+pn})^{\frac{2}{3}}-(\frac{m-pm+pn-i}{m-pm+pn})^{\frac{2}{3}}]=\frac{2\pi nkaa}{\sqrt[3]{(m-pm+pn)^2}}(\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-pm+pn-i)^2})$$

XVI. Since the formula for the centrifugal force is put into effect, as the air vesicles may themselves be trying to expand, this force will be equal to the elastic force of the air; therefore from the equation found, the size of the elastic force of the air can be found. Truly, with this formula in place at any rate, it may be appropriate to follow up the first law, by which the elastic force of the air of different fluid densities and for which steps in the speed may be changed, as the constant factor $2\pi n$ can be ignored, and the elastic force of air shall be as :
$\frac{kaa}{\sqrt[3]{(m-pm+pn)^2}}(\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-pm+pn-i)^2})$. Moreover, since $ia^3=(m-pm+pn)g^3$, I put *g* in place of *a* in the computation, in order that this constant can disappear; Therefore $aa=gg\sqrt[3]{(\frac{m-pm+pn}{i})^2}$, from which by substitution the elastic force will be as [where the constant term g^2 is ignored]:
$k(\sqrt[3]{(\frac{m-i+pi-pm+pn}{i})^2}-\sqrt[3]{(\frac{m-i-pm+pn}{i})^2})$. Therefore from the parts remaining, the elastic force of the air is as the altitude *k*, the velocity of the fine matter gyrating in the vesicles being compressed by the incumbent weight [of the atmosphere].

XVII. Truly when the elastic forces of the air are compared between themselves, it shall be by examining the expansive force acting on the same base. On account of which, in order that I can show the measure of the elastic force of the air in the customary way, it is necessary that I can investigate the pressing of the air on a given base. Now, up to this point, concerning the measurements that I have related, they have not been square, since the total force acting on the air globule has been computed, and which therefore acts on a much larger base, when the vesicle has been extended more. Moreover, these bases are as the squares of the radii of the vesicles. And also the elastic forces are in proportion to these. On account of which a constant radius e of a certain sphere may be taken, and thus the elastic force of the air found in the preceeding paragraph, to the force acting on the given base [area] shall be as a^2 to e^2. Hence it is required that the preceeding formula is multiplied by $e^2 : a^2$, that is [*i. e.* to reduce the force to the standard area] :

$a^2 = gg\sqrt[3]{(\frac{m-pm+pn}{i})^2}$. On account of which by absolute division, and by throwing away e^2 and g^2 as constants, the absolute elastic force of the air is obtained, which will be as : $k(\sqrt[3]{(\frac{m-i+pi-pm+pn}{m-pm+pn})^2} - \sqrt[3]{(\frac{m-i-pm+pn}{m-pm+pn})^2})$.

XVIII. The water film part of the vesicle can vanish ; then $g = b$ and therefore $p = 1$. On account of which the elastic force in this case will be $k(\sqrt[3]{1} - \sqrt[3]{(\frac{n-i}{n})^2})$ or on multiplication by the constant term $\sqrt[3]{n^2}$, it will be as $k(\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2})$. [Recall that n is the density of the fine matter, and i the density of the air; essentially, Euler has produced an equation of state for these variables, in which k is related to the velocity squared of the fine matter, corresponding to an absolute temperature.] The velocity or k is put constant, in order that the elasticity law for air alone is obtained for different condensations, then the elastic force will be as $(\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2})$. And hence I consider the following consequences. If the state of the air comes close to the maximum condensation, then $n - i$ is as near as not equal to zero, hence the elastic force in this case $\sqrt[3]{n^2}$ *i. e.* this will be constant. Hence for air now strongly compressed, the elasticity does not perceptibly change.

XIX. If [the air density] i is made very small with respect to n [density of the fine matter] itself, or if the density of the air to the density of the fine matter were to have a very small ratio, then $(n-i)^{\frac{2}{3}} = n^{\frac{2}{3}} - \frac{2}{3} n^{\frac{-1}{3}} i$. Hence with the air made very rare, the elasticity will be as the density of the air. Wherever we may observe the natural air around however this may be compressed, the elasticity increases in almost the same ratio, and there is no doubt that our air is much rarefied with respect to the fine matter, and that the ratio of the specific gravity of air to the specific gravity of the fine matter is extremely small.

XX. But yet when that absolutely cannot be disregarded, I say that it is necessary to take not only the first two terms, but three terms of the series $(n-i)^{\frac{2}{3}}$ are to be converted, which will show well enough the observed variations. Hence with this agreed upon, we have $(n-i)^{\frac{2}{3}} = n^{\frac{2}{3}} - \frac{2}{3} n^{\frac{-1}{3}} i - \frac{1}{9} n^{\frac{-4}{3}} i^2$. And hence the elastic force

A	B	A	B	A	B
12	0	8	$15\frac{1}{16}$	5	$41\frac{9}{16}$
$11\frac{1}{2}$	$1\frac{7}{16}$	$7\frac{1}{2}$	$17\frac{15}{16}$	$4\frac{3}{4}$	45
11	$2\frac{15}{16}$	7	$21\frac{3}{16}$	$4\frac{1}{2}$	$48\frac{12}{16}$
$10\frac{1}{2}$	$4\frac{6}{16}$	$6\frac{1}{2}$	$25\frac{3}{16}$	$4\frac{1}{4}$	$53\frac{11}{16}$
10	$6\frac{3}{16}$	6	$29\frac{11}{16}$	4	$58\frac{12}{16}$
$9\frac{1}{2}$	$7\frac{14}{16}$	$5\frac{3}{4}$	$32\frac{3}{16}$	$3\frac{3}{4}$	$63\frac{15}{16}$
9	$10\frac{2}{16}$	$5\frac{1}{2}$	$34\frac{15}{16}$	$3\frac{1}{2}$	$71\frac{5}{11}$
$8\frac{1}{2}$	$12\frac{8}{16}$	$5\frac{1}{4}$	$37\frac{15}{16}$	$3\frac{1}{4}$	$78\frac{11}{16}$
				3	$88\frac{7}{16}$

will be as $\frac{2}{3}n^{\frac{-1}{3}}i+\frac{1}{9}n^{\frac{-4}{3}}i^2$ or on multiplication by $9n^{\frac{4}{3}}$, as $6ni + i^2$. The elastic force is called v and it can become $fv = 6ni + ii$. [Where f is a constant of proportionality relating the density to the pressure v.] From this equation with the help of experiments, which have been investigated by Boyle, concerning the increment of the elastic force of air with the continual increase in compression, from which the ratio $n : i$ can be found. From which the limiting point and maximum density can be understood, to which the air can be compressed.

XXI. I have considered that the Boyle experiments here transcribed are consulted, since it may be permitted to infer the density or the specific gravity of the fine material from these, and what ratio to the air it might have. Initially air fills the volume of a tube 12 English inches long, and afterwards truly it is compressed by a column of mercury, the heights of mercury that has flowed into the tube and air are shown in the following table: the first column A indicates the interval of air in the tube, and the other B the height of the mercury compressing the air : these truly are expressed in English inches.

XXII. Therefore this little tube shows a column of mercury that reduces the volume of air into the given interval by its own weight. Truly not only this weight compresses the air, but also the weight of the atmosphere above must be added to it, which acts on the air along with the mercury. Therefore since the sum of the weight of the mercury and the atmosphere shall the that force by which the air is compressed, it is equal to the elastic force of the air. Hence, if the height of a column of mercury equal to the weight of the atmosphere is added to the the numbers in column B of the table, which Boyle himself observes to be $29\frac{1}{8}$ inches, then the relation between the density of the air and the elasticity is obtained. But since for this result to be accurate, the most exact height of mercury that is in equilibrium with the atmosphere must be observed, and this is a task fraught with much difficulty: I might prefer to chose $29\frac{1}{8}$ inches for the altitude from the numbers given in experiments themselves, when the number of these was sufficiently plenty, in order to deduce the weight of the atmosphere : but since to do that the most accurate experiments are required, (which for the present are unable to be had) I am obliged to retain the height $29\frac{1}{8}$.

XXIII. But the densities are reciprocally as the volumes of the same mass of air; the volumes are truly shown by the first column A. Hence, the densities are as the reciprocals of the numbers of column A. Therefore if the density of air is put in its natural state as 1; the remaining densities will be obtained if the number 12 is divided by the corresponging numbers of column A. Then the elasticities, as we see, are as the numbers of the second column B increased by the number $29\frac{1}{8}$. Since truly $fv = 6ni + ii,$ and from observations produced in any case both v and i are given, the letters f and n can be determined; that can be best done from any two experiments. The first experiment is taken to determine the letter f; and $i= 1$ and $v =$ $29\frac{1}{8}$, hence $29\frac{1}{8}f = 6n+1$. Therefore $f=\frac{48n+8}{233}$. With this value substituted in the equaton we have : $48nv + 8v = 1398ni + 233ii$, consequently $n=\frac{8v-233ii}{1398i-480v}=\frac{233ii-8v}{480v-1398i}$.

XXIV. Hence, in order that n can be found, it is required to add some other experimental results. Therefore the final row is taken, and for this $i = 12:3 = 4$, and

$v = 88\frac{7}{16} + 29\frac{1}{8} = 117\frac{9}{16}$. Hence, $n = \frac{940\frac{1}{2} - 3728}{5592 - 5643} = \frac{2785}{51}$; hence n = 54.64. In order that it may be apparent, the extent to which experiments agree or disagree between themselves, that in which the air is reduced to a third of its space is taken. Hence, therefore, i = 3, and $v = 58\frac{12}{16} + 29\frac{2}{16} = 87\frac{7}{8}$. Hence we find $n = \frac{2097 - 703}{4218 - 4194} = \frac{1394}{24} = 58\frac{1}{12}$. But the experiment for which the air is only twice as dense is shown to have a value for n a little more than 17 than the value shown [it is in fact equal to 65.73]. From which huge discrepancies it can be understood how little accuracy these experiments have. By taking a number of steps, enough can be collected. [Rather than discarding the theory!]

XXV. Moreover I have observed that the calculation can be put in place from the remaining experiments, that where the air is less compressed, a smaller value of n is found. From which it can be understood, by ignoring the numbers in the rest of the table, either the height of the mercury equal to the height of the atmosphere was not taken with enough accuracy, or the tube was exceedingly narrow, so that it was not easy for the mercury to descend in it. The former can hardly be given credit : but the latter is more plausible since some deformation is involved in the measurements : Hence it can be concluded, that the mercury has not fallen successively, but appears to fall in jumps. There is the same unconformity with Boyle's experiments when we turn to the rarefaction of air, hence I have been unwilling to conclude anything : but I will defer a fuller judgement concerning the density of the finer matter for the time being, either until more accurate experiments come to hand, or there will be free time to set up the experiment itself.

XXVI. Moreover, so that it can be put clearer to the eye, by which law the elasticity of the air increases with density, the whole thing can be represented by a geometric figure. With the film of water ignored, it has been found that the elastic force of the air is proportional to $\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2}$: Hence it is apparent, that it can be set out by the following cubic parabola parabolam cubicalem secundam. Let AMC be the cubic parabola on the axes AB [Fig.3], in which the extended line PM will be taken to the two thirds power of the abscissa AP. Put AB = n and with the applied line BC erected, CD is drawn parallel to the axis. I say that if on that line, CQ is taken equal to i, then the corresponding applied line QM represents the elastic force of the air. For QM = BC - PM. But BC is as $\sqrt[3]{\mathrm{AB}^2}$ or $\sqrt[3]{n^2}$, and PM as $\sqrt[3]{\mathrm{AP}^2}$, or, as AP = AB - BP, (CQ) = n - i, then P is as $\sqrt[3]{(n-i)^2}$. Hence since QM is as $\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2}$ it is apparent that the elasticity of the air is proportional to this quantity itself.

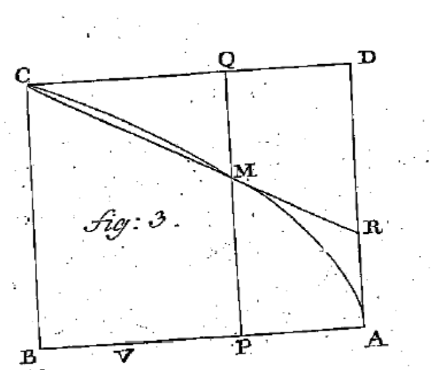
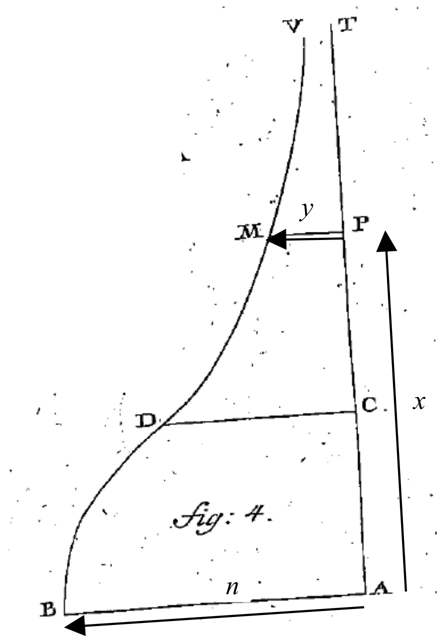


fig: 3.

XXVII. If that rule is accepted, by which the elastic forces of the air are put in the ratio of the densities; From this figure it will be apparent how much that departs from the truth, if this theorem is considered to be true as follows. The line CMR is drawn through the points C and M, cutting the perpedicular AD drawn from A on AB in R; this line expresses by its distances from CD the elastic forces following that rule of the air corresponding to the compression on the line CD. Therefore if QM denotes the natural elastic force of the air, this rule will show the elastic force for compressions that are just smaller or for rarefactions which are just larger, while either rules attribute nothing for air of infinite rarefaction. [The linear approximation.]

XXVIII. If certainly the ratio that *n* has to *i* can be agreed upon, it is possible to designate by how much this rule deviates in some case from the truth : With air present, the maximum elastic force is certainly AD, or the ratio is AD : QM. On account of this reduction I put the leap $n : i = q : 1$; and then $n = qi$, and hence the elastic force QM will be as $\sqrt[3]{qi^2} - \sqrt[3]{(qi-i)^2}$, as divided by the constant $\sqrt[3]{i^2}$, the elastic force of normal air will be as $\sqrt[3]{q^2} - \sqrt[3]{(q-1)^2}$. Some other level of compression is considered, for which the density shall be to normal air as *s* to 1. This density will be *si,* and thus the corresponding elastic force will be $\sqrt[3]{qi^2} - \sqrt[3]{(qi-si)^2}$, this will therefore be as $\sqrt[3]{q^2} - \sqrt[3]{(q-s)^2}$. Hence it follows, the elastic force of normal air is to the elastic force of air with densities around *s* as 1 to $\frac{\sqrt[3]{q^2}-\sqrt[3]{(q-s)^2}}{\sqrt[3]{q^2}-\sqrt[3]{(q-1)^2}}$, but following the common rule it is required to be as 1 to *s*, if $s = q$ then DR = q.QM, and AD = $\frac{\sqrt[3]{q^2}}{\sqrt[3]{q^2}-\sqrt[3]{(q-1)^2}}$.QM Whereby truly if *q* is much greater than 1, then $\sqrt[3]{q^2} - \sqrt[3]{(q-1)^2} = \frac{2}{3\sqrt[3]{q}}$; and thus AD = $\frac{3}{2}q$ .QM. This rule never departs from the truth by more than half.



XXIX. With the elastic force known for a given step for some compression, then the density of the air should be found for any given altitude. Since indeed the normal air is compressed by the weight of the incumbent air, it is necessary that, as higher altitudes are ascended, the air becomes rarer on account of the diminution of the weight of the air. For the air everywhere has expanded, since the force [due to the incumbent weight] compressing it is equal to the elasticity. Therefore the curve BMV shall be a scale of the density of air [Fig.4], the applied line PM of which expresses the density of air at the altitude P. A shall be that place, for which the density of air is a maximum, and thus where AB = *n*. Some place P is is taken, the height AP of which above A is called *x*; truly the density there is PM = *y*, there the elastic force of the air is as $\sqrt[3]{n^2} - \sqrt[3]{(n-y)^2}$, which is proportional to the compression arising from the air above PT. The pressing forces are moreover as the densities and the altitudes jointly: On account of which the pressing of the air above [*i. e.* the pressure] is as the area MPTV i.

e. as $-\int ydx$. And thus : $a\int ydx = \sqrt[3]{n^2} - \sqrt[3]{(n-y)^2}$, hence $aydx = \frac{2dy}{3\sqrt[3]{(n-y)}}$; thus $adx = \frac{2dy}{3y\sqrt[3]{(n-y)}}$ or by putting $a = \frac{2}{3}$, then $dx = \frac{dy}{y\sqrt[3]{(n-y)}}$, which can be integrated in this way, as by putting $x = 0$, y becomes equal to n.

XXX. If $n = y$ then dx is infinitely greater than dy, hence the tangent at B is parallel to the vertical axis AT. Therefore this curve is seen to have a point of inflection somewhere ; that which can be found in the way. Since $dx = \frac{dy}{y\sqrt[3]{(n-y)}}$; then

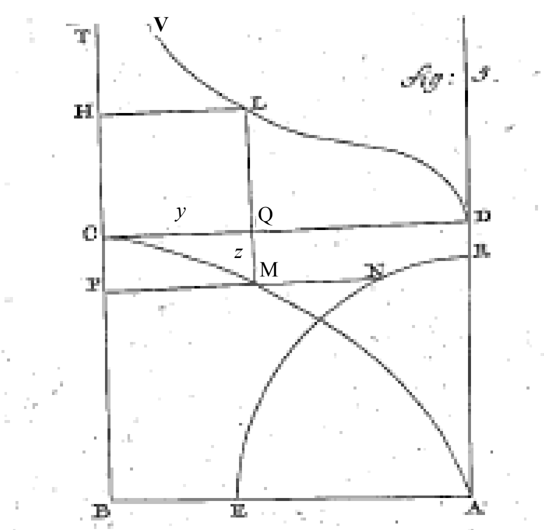
$dy = ydx\sqrt[3]{(n-y)}$. By taking dx as constant, this gives

$ddy = dydx\sqrt[3]{(n-y)} - \frac{1}{3}ydxdy(n-y)^{-\frac{2}{3}}$

$= 0.$

Thus $3n - 3y = y$. Consequenty $y = \frac{3}{4}n$.

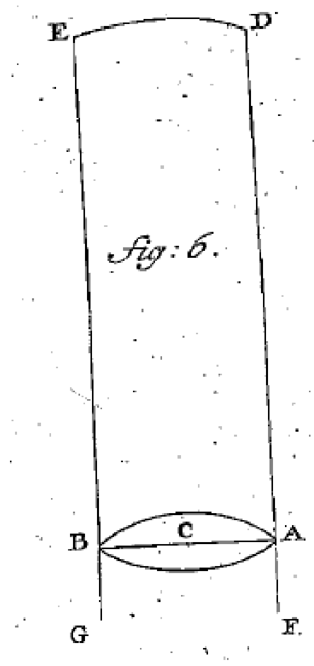
On account of this, the point of inflection is in that place for which the density of air is to the maximum density as 3 to 4. Therefore the line CD = $\frac{3}{4}$AB is applied, and the point D is the point of inflection [still on Fig. 4]. Hence the subtangent of this curve is $\frac{ydx}{dy} = \frac{1}{\sqrt[3]{(n-y)}}$. Thus it may be garnered that if [the density] y is very small with respect to n, then the subtangent is of constant length; as thus in this case the curve can be combined with the logarithmic curve.



XXXI. Indeed it is possible for this equation $dx = \frac{dy}{y\sqrt[3]{(n-y)}}$ to be reduced to the quadrature of the circle and the logarithm: but it is much more difficult to generate this curve by construction, than if it is performed by quadratures. [Euler now shows how to do this by construction.] Therefore the following cubic parabola AMC is designated, as in Fig. 5 and let CD = n. [The upper curve is to be generated, in which CT measures the height x from origin C corresponding to Fig. 4.] The density of the air is then taken somewhere on CD, consider CQ, and place CQ = y. The corresponding applied line of this [on the cubic parabola, giving the density at positon y] will be QM, equal to $\sqrt[3]{n^2} - \sqrt[3]{(n-y)^2}$, which is proportional to [the pressure or the weight of the air above this height] $-\int ydx$. QM is called z for brevity and hence the equation becomes : $-ydx = dz$ and $dx = \frac{-dz}{y}$, thus $x = \int\frac{-dz}{y}$. PM is drawn, which will be y, and on that line produced if necessary, PN is taken equal to $\frac{1}{y}$, the area PBEN = $\int\frac{-dz}{y}$. Wherefore in MQ extended QL is taken, which shall be as the area PBEN. The point L lies in the curve

sought. For that is indeed, with LH drawn, CH = LQ = $\int \frac{-dz}{y} = x$, and HL = CQ = y. Therefore in this manner the curve DLV will be determined.
[There is still the problem of evaluating the integral.]

XXXII. Thus far the properties of air which have been derived from the expounded theory, that contain nothing absolute except for the ratio they give, following which the elasticity of air for different densities, humidities and speeds of the fine matter ought to be considered. Now truly the height of a column of mercury can be completely determined, which as I will relate, is strong enough to sustain a given air globule . Thus AB shall be the horizontal diameter of an air vesicle, and what was found about these in § 14 must be understood. A column of mercury ABED of height AD = f will press against it, of such an extent that it remains in equilibrium with the force which the vesicle exerts due to its own expansion. Moreover this column acts on the individual points on the surface of the vesicle perpendicularly, and that force, which is as the altitude of a column f, and the base or surface of the vesicle, which it presses against, and the specific gravity jointly. Moreover, since the semidiameter AC shall equal a; the area of the largest is $\frac{\pi aa}{2}$, and thus the area of the semihemisphere of this is πaa, which is the base pressed on by the column of mercury. Again, the specific gravity of mercury can be expressed, with respect to the remaining specific gravities present, by the letter r, the pressing force which the column of mercury exerts on the vesicle will be equal to $\pi aarf$. [Recall that Euler's π is twice as large as the modern version; we would now consider the vertical forces acting on an element of surface, in which case the sum can be projected onto the horizontal diameter of the tube]



XXXIII. Moreover this pressing force should be destroyed by the centrifugal force arising from the fine matter, which also acts equally on the individual points of the surface. On account of which the force, which the centrifugal force exerts on the hemisphere, and that is striving to extend the surface, must be equal to the force due to the compressing column of mercury. Moreover that force is half the total elastic force of the whole spherical vesicle, which is equal to the weight found in §14 :
$\frac{2\pi nkaa}{\sqrt[3]{(m-pm+pn)}^2}(\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-pm+pn-i)^2})$; hence half of this force is equal to the weight of the column of mercury $\pi aarf$. Hence it follows that the equation arises :

$$rf\sqrt[3]{(m-pm+pn)}^2 =$$
$$nk\,[\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-i-pm+pn)^2}\,]$$

or

$$f=\frac{nk}{r}[\sqrt[3]{(\frac{m-i+pi-pm+pn}{m-pm+pn})^2}-\sqrt[3]{(\frac{m-i-pm+pn}{m-pm+pn})^2}\,].$$

XXXIV. In order that this equation shall avoid dealing even with the natural state of the air, for easily handling, I put *i* very small with respect to *n*; and hence the equation becomes :
$\sqrt[3]{(m-i+pi-pm+pn)^2} = \sqrt[3]{(m-pm+pn)^2} + \frac{2(pi-i)}{3\sqrt[3]{(m-pm+pn)}}]$. And in the same manner :
$\sqrt[3]{(m-i-pm+pn)^2} = \sqrt[3]{(m-pm+pn)^2} - \frac{2i}{3\sqrt[3]{(m-pm+pn)}}]$. From which with the values substituted, this equation arises :
$rf(m-pm+pn) = \frac{2pnik}{3r}$, hence $f = \frac{2p\pi ik}{3r(m-pn+pn)}$. But if the humidity of the air is made to be disappearing, then $p = 1$. Then it followa that $f = \frac{2ik}{3}$. If moreover the air had vapour present *p* in that case will be less than one, for where more vapour is lodged in the air, and thus in that case put $p = 1 - q$, and
$f = \frac{2p\pi ik(1-q)}{3r(qm+n(1-q))} = \frac{2ik}{3r} - \frac{2qmik}{3r(qm+n(1-q))} = \frac{2ik}{3r}(1 - \frac{qm}{(qm+n(1-q))})$.

XXXV. Moreover since *f* indicates the height of the mercury column consistent with equilibrium, the height of the mercury in the barometer can be expressed by the letter *f*. From which the equation is found, for the velocity of the fine matter gyrating in the vesicle, with the specific gravities of the air and of the fine matter, and the amount of water moving around in the air, will be able to be found from the height of mercury in the barometer. For *r* is the specific gravity of mercury, as also *m* is the specific gravity of water from elsewhere now agreed upon. And so I will run through the cases, for which the mecury rises, and for which it ought to fall; as thus it may be apparent, what has happened in the air from the rising or falling of the mercury in the barometer. Since it will of so much help to do this with the ratio, I ignore the factor $\frac{2}{3r}$, as constant, and *f* will be as $ik(1 - \frac{qm}{(qm+n(1-q))})$.

XXXVI. From hence it therefore follows from the remaining factor *ik* of the mercury in the barometre to rise or fall by a fraction $\frac{qm}{(qm+n(1-q))}$. Truly from the increase in this fraction, *q* also will increase, and for a decrease, truly *q* will also decrease : For with *q* increased by an element *dq*, the fraction increases by the element: $\frac{mndq}{(qm+n(1-q))^2}$. On account of which with the factor *ik* remaining, the mercury in the barometer rises with the decrease in the humidity of the air; that truly with the increase of the mercury will be required to fall. And I think that this is the reason why the ascent of mercury in the barometer is generally associated with a clear sky; and truly the fall, an announcement of rain and an unfavourable storm : For indeed in that case the air is free from vapours for the most part, and in the other case truly laden with them.

XXXVII. There can indeed be other concurrent reasons, on account of which the mercury is able to ascend or descend with an unchanged quantity of vapour : When the factor *ik* shall increase or decrease. But perhaps this factor is unable to either perceptibly increase or decrease; because if either letter is increased in some ratio the other letter is decreased by almost the same. For the velocity of the fine matter, of which *k* is the square, with more heat will accept an increase, but the same heat will rarefy the air, and it brings about a reduction in the quantity *i*, as if the factor *ik* remains the same always. From which is always understood without risk that the rise or fall of the mercury can be attributed to the increase or decrease in the variable amount water vapour in the air, nevertheless it is not be possible to deny that the factor *ik* affects the humidity and is able to increase or decrease the mercury.

XXXVIII. Nor indeed is it permitted hence to infer that it is necessary to keep a hygrometer also; since that shows the humidity of the air. But it needs to be considered, the barometer is affected by the whole mass of the air or depends on the state of the whole atmosphere; but the hygrometer only depends on the air around it. For that reason. an increase in the height of the mercury in the barometer will be accepted, if all the air in the atmosphere is free from vapour, and a decrease truly if the air is filled with vapour. Hence nearly all the dryness it is possible to show is collected by the hygrometer when the height of the mercury is at a minimum ; and similarly the hygrometer can indicate the humidity, when the mercury reaches its peak height. [Thus, according to Euler, the hygrometer is a fine tuning device for the humidity nearby.]

XXXIX. If the humidity of the air vanishes, this equation is obtained from §33 nearby $f = \frac{2ik}{3r}$, as *n* is not increasing. From this therefore, with the ratio $r : i$ from experiments, and as the height *f* is agreed on, it is possible to find the height *k*, from which a heavy weight falling can acquire that velocity, which is equal to the velocity of the fine matter gyrating in the vesicles ; Indeed it is $k = \frac{3rf}{2i}$. Concerning which I have observed the same expression except for the coefficient with the number $\frac{3}{2}$ to be the same for the velocity generated by the height, by which sound is moved through air, as hence the velocity of the fine matter keeps the same constant ratio to the speed of sound. But is is necessary to divert the mind from the humidity of the air, by means of which agreement *k* can be expressed in another way.

XL. The height of mercury in a barometer is observed to be from 22 as far as 24 inches of Rhenish feet, and beyond. Moreover, since I can suppose air free from vapour, I can assign the maximum value to the letter *f* which the height is able to have, truly 2460 scrup. of Rhenish feet. Then I put the ratio of *r* to *i* as 100000 to 1, as can be concluded from experiments on the weight of air. With which put in place, $k = \frac{30000.2460}{2} = 36900$ feet. Thus the fine matter is moving with such a velocity as a weight can acquire by falling from a height of 36900 pedibus in a vacuum. If therefore this matter could go on in its own direction for one second, it might make $1518\frac{1}{2}$ Rhenish feet. [~470 m/s. See E002, p. 1, in this series of translations. The true speed of sound is around 330 m/s.]

XLI. I finish this dissertation, since accurate experiments are lacking from which the remaining desired quantities could still be determined , and from which this theory could be more fully confirmed. The ratio *n* to *i*, or what ratio of the specific gravity of the fine matter has to the specific gravity of the air, is still uncertain. Truly this must be investigated by setting up and performing accurate experiments that I would use and study. Moreover for the quantity *n* if it can be found, then the formulae found can be easily applied to the working ; and with other suitable instruments that may be used according to the season, it may be possible to determine how much water is contained in the air. And perhaps we might be led by the hand as it were, to have a sufficient understanding of the many other things above.

TENTAMEN EXPLICATIONIS
PHAENOMENORUM AERIS.

Auctore
Leonh. Euler.

I.
Quanquam ad intima rerum penetralia et cognitionem ultimae partium structurae aditus non ita patet, ut phaenomenorum, quae inde oriuntur ratio reddi queat : Tamen, ut plerumque a Physicis factum est, a corporum naturalium proprietatibus, quas observavimus, quodammodo ad ipsam eorum structura, concludere licet. Ex qua percepta corporum structura, quo plura phaenomena explicari possunt, eo perfectior ea est; Et, si ex qua Theoria omnes prorsus proprietates, quas quidem cognoscere impossibile est, derivari possent, dubium non est, quin ea vera sit, et re ipsa existat.

II. Aeris quam plurima nota sunt phaenomena, eaque parum a se invicem dependentia; ut is profecto multum praestitisse censendus sit; qui eadem theoria omnibus satisfacere posset. Sed theoriam ita maxime conficere convenit, ut primum excogitetur partium structura, ex qua una tantum proprietas fluat; id quod plerumque pluribus modis fieri potest : et deinceps inquiratur, num ea caeteris quoque proprietatibus explicandis sufficiat? et, si plures primum theoriae conceptae sunt, tum quaeratur, quae maximae parti vel quae omnibus satisfaciat.

III. Inter alias aeris, quas cognoscimus, proprietates ea inprimis idonae visa est, secundum quam structora aeris adornetur, qua aer sese continuo expandere conatur, et reipsa se expandit, si quae impedimento fuerant, removenatur. Haec enim aeris elasticitas prae caeteris proprietatibus maxime explicatu difficilis videtur, ut eius ratione cognita facile fluere videantur.

IV. Nisi velimus hoc aeris phaenomenon occultae cuidam particularum proprietati et vi insitae asscribere, alia via non superest, nisi ut conatus iste a motu materiae cuiusdam subtilis derivetur. Conatus autem seu vis mortua, uti a Leibnitio vocatur, a materia mota ortum trahere potest, si ea in gyrum moveatur; quo sit ut quaevis particula in centro aufugere conetur, atque ita huiusmodi vortex vim sese expandendi acquirat. Hoc usus principio Cel. Ioh. Bernoulli omnem vim elasticam explicare instituit in schediasmate *de communicatione motus* Lutetiae nuper impresso. Quo vim elasticam a vi centrifuga materiae subtilis oriri asserit.

V. Sequor itaque hac in re, istum, uti mihi videntur; maxime probabilem elasticitatis aeris causam, atque in hac dissertatione examini subiiciam, quantum aeris structura ex hoc fundamento formata reliquis aeris proprietatibus explicandis sufficiat, quantumve minus, ut appareat, utrum aer hanc partium structuram habere possit, an vero non? Quo in casu meliorem oporteret excogitare aeris constitutionem.

VI. Suppono igitur aerem constare acervo infinitarum minimarum bullularum, in quibus materia subtilis motu circulari gyratur et vi centrifuga bullulas continuo expandere conatur, easque reipsa semotis obstaculis, expandit. Suppono porro bullulas esse pellicula obductas; quid quidem opus non esset, cum huiusmodi vorticuli sine pelliculis constare possent et tamen mutuo non permiscerentur. Unus enim alterum impedit, quominus extravagetur: Attamen propterea bullulas pelliculis obductas suppono, quod aer nunquam tam purus sit, ut prorsus a vaporibus liber sit. Vapores autem particulas aeris ad instar pellicularum obducere valde probabile est.

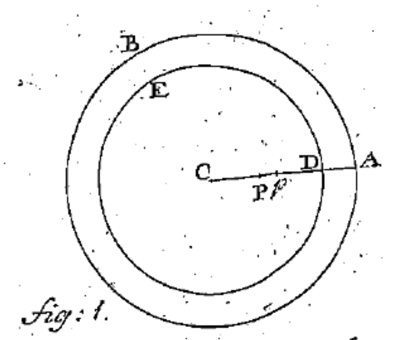
VII. Constet itaque aer infinito bullularum minimarum numero, quarum crusta exteror sit aquea pro diverso aeris statu maior minorve; intra hanc crustam gyretur materia subtilis certa cum velocitate, quae subinde ab alia subtiliori adhuc materia omnes poros penetrante accelerationes nanciscitur, ne motus tandem consumatur et evanescat. Constat enim aerem calorem semel acceptum sensim amittere, cum autem aer calore rarefiat, sequitur materiam subtilem motu vehememiore agitari; cessante ergo calore, indicio id est, motum materiae esse retardatum.

VIII. Ex hisce de structura aeris praemissis consequitur, eum in infinitum se expandere debere atque extremum raritatis gradum accipere, quando nihil est quod eius conatum compescat. Sed accedente gravitate, aliter se res habebit, eritque, quod vi aeris elasticae se opponet. Quum enim aer superor inferiorem premat pondere suo, inferior ulterius se expandere nequit, quam quoad eius vis elastica, quae expansione continuo diminuitur, aequalis sit vi incumbentis aeris comprimenti.

IX. Patet porro ex concepta aeris constitutione, eum in infinitum comprimi non posse, propter gravitatem specificam, quae in infinitum augeretur. Nam cum in qualibet bullula certa et determinata materiae subtilis quantitas comprehendatur, eaque semper superficiei adhaereat ob vim centrifugam, necesse est ut circa centrum spatium vacuum relinquatur; id quod eo maius esse debet, quo magis aer rarus fuerit : Contra autem continuata aeris compressone, id spatium vacuum continuo diminuetur, donec tandem prorsus evanescat, ultra quem densitatis gradum aerem comprimere impossibile erit.

X. Quod ad velocitatem materiae subtilis attinet, oportet singulis eius particulis eandem attribuere velocitatem, neque quae a centro remotiores sunt, iis maiorem et proprioribus minorem adscribere velocitatem. Praeterea enim, quod hinc theoria nascatur experientiae penitus contraria, ob vim centrifugam in maioribus bullulis maiorem, ex hoc elucere potest, quod bullulam condensando vel expansioni relinquendo velocitas materiae subtilis eadem manere debeat, cum nihil sit, quod eam immutet ; Huc enim non pertinet retardatio, de qua § 6 quae non propter immutationem bullulae, sed propter resistentiam quandam contingit. Quare cum velocitas materiae subtilis non a distantia a centro pendere queat necesse est eam ubique constantem statuere.

XI. Sit CAB bullula aerea, quoad fieri potest compressa, quae proin est materia subtili vorticosa penitus repleta. Circumdata vero sit crusta aquae ADEB, ut ergo reliquum spatium CDE materia subtili impleatur. Sit AC = g, CD = b. Sumatur pro ratione radii ad periphereim, 1 : π, pro gravitate specifica materiae subtilis, n et pro gravitate specifica aquae seu crustae m. Erit capacitas globuli CAB = $\frac{2\pi g^3}{3}$, et capacitas



globuli CDE = $\frac{2\pi b^3}{3}$. Ergo soliditas crustae ADEB = $\frac{2\pi}{3}(g^3-b^3)$. Quamobrem erit massa materia subtilis spatium CDE implentis = $\frac{2\pi n b^3}{3}$, et massae crustae = $\frac{2\pi m}{3}(\frac{g^3-b^3}{2})$. Et hae massarum quantitates in quantum vis expansis bullulis eaedem manere debent.

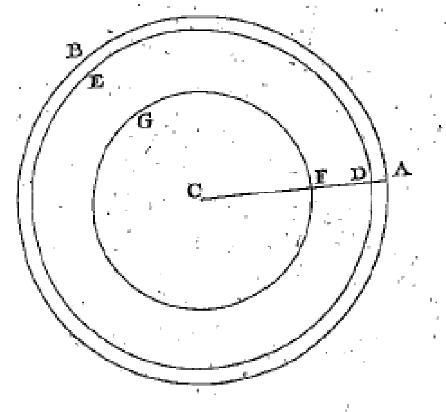


fig: 2.

XII. Exprimat k altitudinem, ex qua grave cadendo velocitatem acquirit, materiae subtilis velocitati aequalem; Unde sequenti modo vis centrifuga, seu vis, qua superficies globuli CDE premitur, invenietur. Sumatur a centro indeterminata CP = x, cuius differentiale Pp = dx. Erit crusta sphaerica crassitiei Pp et radii CP = $2\pi xxdx$, quae si ducatur in densitatem materiae subtilis, dat massam $2\pi nxxdx$, seu pondus. Quum haec materia gyretur velocitate ex altitudine k acquisita, fiat secundum Hugenium, ut radius x ad duplam altitudinem $2k$ ita pondus materiae gyrantis $2\pi nxxdx$, ad pondus vi centrifugae huius crustae aequale, quod ergo erit = $4\pi nkxxdx$. Huius ergo integrale $2\pi nkx^2$ exprimit vim centrifugam sphaerae radii CP. Consequenter vis centrifuga bullulae DE erit = $2\pi nkbb$.

XIII. Consideremus nunc bullulam aeream quomodocunque expansam CAB : Cuius extrema crusta ADEB designet materiam aqueam, media DFGE materiam subtilem circa centrum gyrantem, et tertia seu intima CFG, spatium vacuum, vel quod id minimum materia gravitatis experti sit repletum. Dicantur AC = a, CD = c. Erit, computo ut supra, instituto, soliditas crustae extremae seu aquae ADEB = $\frac{2\pi}{3}(a^3 - b^3)$. Dein soliditas crustae mediae seu quantitas materiae subtilis DFGE = $\frac{2\pi}{3}(b^3 - c^3)$. Tertio autem capacitas totius bullulae erit $\frac{2\pi}{3}a^3$. Sit gravitas specifica aeris seu totius bullulae, i erit pondus eius $\frac{2\pi i}{3}a^3$, id quod aequale est summae ponderum partium, nempe $\frac{2\pi m}{3}(a^3 - h^3) + \frac{2\pi n}{3}(h^3 - c^3)$. Est igitur $ia^3 = ma^3 - mh^3 + nh^3 - nc^3$.

XIV. Cum et quantitates materia aquae, it quantitas materiae subtilis aequales esse debeant iis, quae supra erant inventae in casu bullulae maxime compressae, sequentes obtinebuntur aequationes

$$\frac{2\pi}{3}(g^3 - b^3) = \frac{2\pi}{3}(a^3 - h^3) \text{ et } \frac{2\pi}{3}b^3 = \frac{2\pi}{3}(h^3 - c^3).$$

Quamobrem $g^3 - b^3 = a^3 - h^3$ et $b^3 = h^3 - c^3$. Unde $h = \sqrt[3]{(a^3 - g^3 + b^3)}$ et $c = \sqrt[3]{(h^3 - b^3)} = \sqrt[3]{(a^3 - g^3)}$. Si haec substitunatur in superioris § ultima aequatione, reperietur $ia^3 = mg^3 - mb^3 + nb^3$. Unde $b^3 = (ia^3 - mg^3):(n - m)$. Et porro $b = \sqrt[3]{((i - m + n)a^3 - ng^3 : (n - m)}$ ac $c = \sqrt[3]{(a^3 - g^3)}$. Hoc ergo modo ex calculo exluduntur litterae b, c, et h denotantes interiorum bullulae partium a centro distantias.

XIV. Vis centrifuga materiae subtilis in spatio DFGE gyrantis velocitate ex altitudine k producta ex §11 inveniri potest hoc modo : Vis centrifuga materiae globum radii x implentis inventa est = $2\pi nkxx$. Quare se materia subtilis totum spatium CDE impleret, soret eius vis centrifuga = $2\pi nkbb$, a qua si auferatur, vis centrifuga materiae spatii CFG = $2\pi nkcc$, restabit vis centrifuga materiae subtilis in spatio FDEG gyrantis, cuius quantitas proin erit = $2\pi nk(bb - cc)$ et subrogatis loco b et c valoribus §13 inventis, erit ea = $2\pi nk[(\frac{(i-m+n)a^3-ng^3}{n-m})^{\frac{2}{3}} - (a^3 - g^3)^{\frac{2}{3}}]$. Ponatur $h^3 = pg^3$, ob $ia^3 = mg^3 - mh^3 + nh^3$, $ia^3 = (m - mp + np)g^3$. Unde $g^3 = ia^3 : (m - mp + np)$. Erit ergo pondus vi centrifugae aequavalens =

$$2\pi nkaa[(\tfrac{m-i+pi-mp+pn}{m-pm+pn})^{\frac{2}{3}} - (\tfrac{m-pm+pn-i}{m-pm+pn})^{\frac{2}{3}}] = \frac{2\pi nkaa}{\sqrt[3]{(m-pm+pn)^2}}(\sqrt[3]{(m-i+pi-pm+pn)^2} - \sqrt[3]{(m-pm+pn-i)^2})$$

XVI. Cum vi centrifuga efficiatur, ut bullulae aereae sese continuo extendere conentur, erit ea aequalis vi elasticae aeris; ex inventa igitur aequatione, quanta sit aeris elasticitasm inveniri poterit. Verum cum hoc loco primum legem duntaxat, qua aeris vis elastica pro diversis densitatis, humoris et celeritatis gradibus immutetur, persequi conveniat, factor $2\pi n$ utpote constans negligi, eritque vis aeris elastica ut $\frac{kaa}{\sqrt[3]{(m-pm+pn)^2}}(\sqrt[3]{(m-i+pi-pm+pn)^2} - \sqrt[3]{(m-pm+pn-i)^2})$. Cum autem sit $ia^3 = (m - pm + pn)g^3$, loco *a* in computum duco *g*, ut ea tanquam constans abiici possit; Erit ergo $aa = gg\sqrt[3]{(\frac{m-pm+pn}{i})^2}$, quo substituto erit vis elastica aeris ut $k(\sqrt[3]{(\frac{m-i+pi-pm+pn}{i})^2} - \sqrt[3]{(\frac{m-i-pm+pn}{i})^2})$. Coeteris igitur paribus est aeris vis elastica ut altitudo *k*, velocitatem materiae subtilis in bullulis gyrantis gravi descendenti impremens.

XVII. Verum cum vires aeris elasticae inter se comparantur, id sit aeris vim expansiuam in eandem basin agentem explorando. Quamobrem, ut mensura, aeris vis elasticae, ut consuetum estm exhibeam, necesse est, ut pressionem aeris in datam basin investigem. Nam, quae hucusque de ista mensura tradidi, huc non quadrant, quia vis tota globuli aeris elastica est supputata, quae propterea in tanto maiorem basin agit, quando magis bullula est extensa. Sunt autem hae bases ut quadrata radiorum bullularum; Et iis etiam vires elasticae proportionantur. Quocirca assumatur constans quidam sphaerae radius *e*, fiatque ut a^2 ad e^2 ita vis aeris elastica inventa paragr. praeced. ad vim in datam basin agentem. Multiplicetur ergo oportet formula praecedens per $e^2 : a^2$ at vero est $a^2 = gg\sqrt[3]{(\frac{m-pm+pn}{t})^2}$. Quamobrem absoluta divisione, abiectisque e^2 et g^2 tanquam constantibus obtinebitur vis aeris elastica absoluta, quae erit vt $k(\sqrt[3]{(\frac{m-i+pi-pm+pn}{m-pm+pn})^2} - \sqrt[3]{(\frac{m-i-pm+pn}{m-pm+pn})^2})$.

XVIII. Evanescat pars bullulae aquea; erit $g = b$ et ideo $p = 1$. Quamobrem vis aeris elastica hoc casu erit $k(\sqrt[3]{1} - \sqrt[3]{(\frac{n-1}{n})^2})$ seu multiplicato per constantem $\sqrt[3]{n^2}$, erit ea ut $k(\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2})$. Ponatur *k* seu velocitas constans, ut obtineatur lex elasticitatum pro solis aeris diversis condensationibus, erit tum vis elastica, ut $(\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2})$. Et hinc sequentes duco consequentias. Si status aeris quam proxime ad maximam condensationem accedat, erit $n - i$ tantum non = 0, ergo vis elastica hoc in casu erit ut $\sqrt[3]{n^2}$ *i. e.* ea erit constans. Aere ergo iam vehementer compresso, vis elastica amplius sensibiliter non immutatur.

XIX. Deinde si *i* respectu ipsius *n* valde paruum sit, seu si densitas aeris ad densitatem materiae subtilis admodum exiguam habuerit rationem erit $(n-1)^{\frac{2}{3}} = n^{\frac{2}{3}} - \frac{2}{3} n^{\frac{-1}{3}} i$. Aere ergo valde rarefacto elasticates erunt vt densitates aeris. Quare cum circa aerem naturalem observemus quantumvis is comprimatur elasticitatem propemodum in eadem ratione crescere, dubium non est, quin aer noster admodum sit dilatatus respectu materiae subtilis, atque rationem specificam aeris ad gravitatem specificam materiae subtilis perquam esse exiguam.

XX. Attamen cum ea prorsus negligi nequeat, Oportet seriei, inquam $(n-i)^{\frac{2}{3}}$ convertitur, non tantum duos primos, sed tres accipere terminos, qui variationes obseruatas satis exacte monstrabunt. Hoc ergo pacto erit $(n-1)^{\frac{2}{3}} = n^{\frac{2}{3}} - \frac{2}{3} n^{\frac{-1}{3}} i - \frac{1}{9} n^{\frac{-4}{3}} i^2$. Atque hinc vis elastica erit ut $\frac{2}{3} n^{\frac{-1}{3}} i + \frac{1}{9} n^{\frac{-4}{3}} i^2$ seu multiplicando per $9n^{\frac{4}{3}}$, ut $6ni + i^2$. Dicatur vis elastica *v* fiatque $fv = 6ni + ii$. Ex hac igitur aequatione ope experimentorum, qua circa aeris incrementum vis elasticae eo continuo magis condensato, instituta sunt a Boyleo, invenietur ratio $n : i$. Ex quo intelligetur extremus et maximus densitatis gradus, ad quem aerem comprimere possibile est.

XXI. Consultum ergo esse duxi experimanta Boyleana huc transcribere, ut ex iis de densitate seu gravitate specifica materiae subtilis concludere liceat, et quodnam ad aerem rationem habeat. Aer primo in tubo spatium 12 digit. Angl. replebat postea vero cum columna mercuriali comprimebatur altitudines aeris et mercurii superaffusi in sequinti tabula exhibentur, cuius prior columna A indicat spatium aeris in tubo, it altera B altitudinem mercurii comprimentis aerem : hae vero in digitis Anglic. exprimuntur.

A	B	A	B	A	B
12	0	8	$15\frac{1}{16}$	5	$41\frac{9}{16}$
$11\frac{1}{2}$	$1\frac{7}{16}$	$7\frac{1}{2}$	$17\frac{15}{16}$	$4\frac{3}{4}$	45
11	$2\frac{15}{16}$	7	$21\frac{3}{16}$	$4\frac{1}{2}$	$48\frac{12}{16}$
$10\frac{1}{2}$	$4\frac{6}{16}$	$6\frac{1}{2}$	$25\frac{3}{16}$	$4\frac{1}{4}$	$53\frac{11}{16}$
10	$6\frac{3}{16}$	6	$29\frac{11}{16}$	4	$58\frac{12}{16}$
$9\frac{1}{2}$	$7\frac{14}{16}$	$5\frac{3}{4}$	$32\frac{3}{16}$	$3\frac{3}{4}$	$63\frac{15}{16}$
9	$10\frac{2}{16}$	$5\frac{1}{2}$	$34\frac{15}{16}$	$3\frac{1}{2}$	$71\frac{5}{16}$
$8\frac{1}{2}$	$12\frac{8}{16}$	$5\frac{1}{4}$	$37\frac{15}{16}$	$3\frac{1}{4}$	$78\frac{11}{16}$
				3	$88\frac{7}{16}$

XXII. Exhibet igitur haec tubula columnam mercurialem, quae pondere suo aerem in datum spatium redigit. Hoc vero pondus non solum aerem comprimit, sed ei insuper adiici debet pondus atmosphaerae, quod simul cum mercurio in aerem agit. Cum ergo summa ponderis mercurii et atmosphaerae ea sit, vis qua aer comprimitur, erit ea aequalis vi aeris elasticae. Unde, si numeris tabulae B addatur altitudo mercurii ponderi atmosphaerae aequalis, quam Boyleus se $29\frac{1}{8}$ dig. obseruasse scribit; habebitur relatio inter densitates aeris et elasticitates. Sed cum ad istud accurate praestandum, exactissime altitudinem mercurii atmosphaeram aequilibrantis observasse necesse sit; idque multis difficultatibus perturbetur : Mallem relicta hac altitudine $29\frac{1}{8}$ dig. ex experimentis ipsis, quum numerus eorum abunde sufficiat, deducere pondus atmosphaerae : Sed quia ad hoc accuratissima requiruntur experimenta, (in quibus praesentia haberi nequeunt) altitudinem $29\frac{1}{8}$ retinere cogor.

XXIII. Sed densitates erunt reciproce ut volumina eiusdem massae aereae; volumina vero columnae A exhibeantur: Ergo densitatis erunt reciproce ut numeri columnae A. Si igitur densitas aeris in statu naturali ponatur, 1; reliquae densitates habebuntur si numerus 12 per reliquos respondentes numeros columnae A dividatur. Deinde esasticitate, ut vidimus, sunt ut numeri secundae columnae B aucti numero $29\frac{1}{8}$. Cum vero sit $fv = 6ni + ii$, atque ex pbservationibus allatis habeantur in quolibet casu et v et i, duae hae literae f et n determinari debent; Id quod duobus quibusuis experimentis praestabitur. Sumatur ad literam $v = 29\frac{1}{8}$, unde $29\frac{1}{8}f = 6n+1$. Ergo $f = \frac{48n+8}{233}$. Quo valore in aequatione substituto habitur $48nv + 8v = 1398ni + 233ii$, consequenter $n = \frac{8v-233ii}{1398i-480v} = \frac{23ii-8v}{480v-1398i}$.

XXIV. Ut hinc inveniatur n, oportet experimentorum allatorum aliquod adiungure. Sumatur igitur ultimum, erit $i = 12:3 = 4$, et $v = 88\frac{7}{16} + 29\frac{1}{8} = 117\frac{9}{16}$. Unde $n = \frac{940\frac{1}{2}-3728}{5592-5643} = \frac{2785}{51}$ hinc erit $n = 54.64$. Ut pateat, quantum experimenta inter se conveniant vel disconveniant, accipiatur id quod aer in triplo minus spatium est redactum; Erit ergo $i = 3$, et $v = 58\frac{12}{16} + 29\frac{2}{16} = 87\frac{7}{8}$. Unde habetur $n = \frac{2097-703}{4218-4194} = \frac{1394}{24} = 58\frac{1}{12}$. At experimentum quo aer duplo tantum densior exhibetur paulo plus quam 17 pro valore ipsius n exhibet. Ex qua ingenti discrepantia intelligi potest, quam parum accurata haec sint experimenta: Id quod praeterea ex saltibus, qui in iis deprehenduntur, satis colligi potest.

XXV. Id autem ex reliquis experimentis calculum instituens observavi, inde quo minus aer erat compressus, eo minorem ipsius n valorem inventum. Ex quo intellgi potest, reliquis in numeris saltibus neglectis, vel altitudinem mercurii atmosphaerae aequiponderantis non satis accurate esse assumtam, vel tubum nimis fuisse angustum, ut ne facillime quidem mercurius in eo descendere potuerit. Prius quidem vix credi potest : Sed posterius eo magis verisimile est, quod tanta insit difformitas experimentis : Unde concludi debet, mercurium non successive, sed quasi per saltus descendisse. Eandem difformitatem in Boylei experimentis circa rarefactionem aeris advertens, inde quicquam concludere nolui : sed plenius de densitate materiae subtilis iudicium tamdiu differam, donec vel accuratiora experimenta in manus veniant, vel ipsi instituere vacauerit.

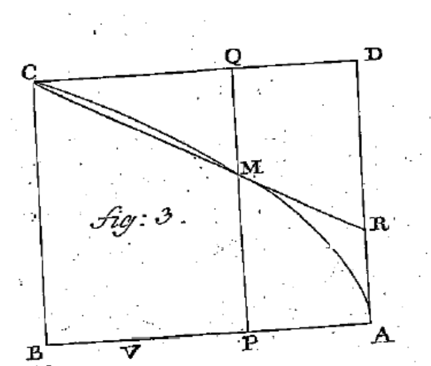
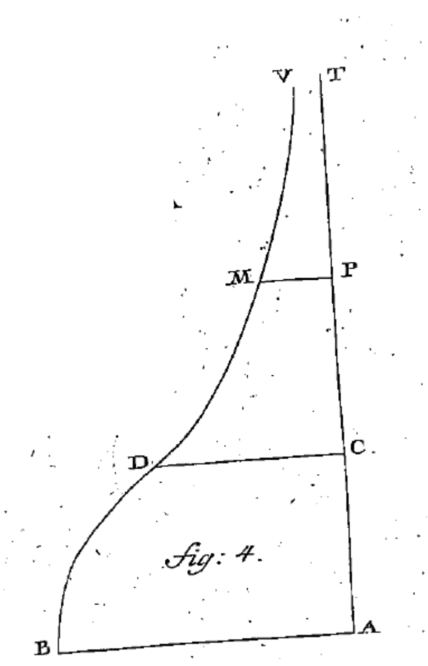


Fig: 3.

XXVI. Ut autem clarius ob oculos ponatur, qua lege elasticitates aeris pro diversis densitatibus crescant, tota res figura geometrica repraesentari potest. Neglectis pelliculis aqueis inventa est aeris vis elastica proportionalis $\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2}$: Unde patet, id per parabolam cubicalem secundam praestari posse. Sit AMC parabola cubicula super axe AB, in qua applicatae PM ferit in ratione subsesquiplicata abscissarum AP. Capiatur AB = n et erecta applicata BC, ducatur axi parallela CD. Dico si in ea capiatur CQ = i, applicatam correspondentem QM repraesentare vim aeris elasticam. Nam est QM = BC - PM. Sed BC est ut $\sqrt[3]{AB^2}$ seu $\sqrt[3]{n^2}$, et PM ut $\sqrt[3]{AP^2}$, seu, ob AP = AB - BP, (CQ) = $n - i$, erit P, ut $\sqrt[3]{(n-i)^2}$. Ut ergo sit QM ut $\sqrt[3]{n^2} - \sqrt[3]{(n-i)^2}$ cui quantitati etiam, ut patet, proportionalis est aeris elastica.

XXVII. Si ea accipiatur regula, qua vires aeris elasticae in ratione densitatum ponuntur; Ex hac figura patebit quantum ea a vero,si modo hanc theoriam veram appellare licet, aberret. Ducatur per puncta C et M recta CMR perpedicularem AD ex A in AB ductam secans in R; exprimet haec recta distantiis suis a CD vires elasticas secundum istam regulam aeri iuxta abscissas in linea CD condensato respondentes. Si igitur QM naturalem aeris vim elasticam denotet, regula ista in condensationibus iusto minorem exhibebit vim elasticam, at in rarefactionibus iusto maiorem, donec utraque regula aeri infinite rarefactio elasticitatem nullam attribuat.

XXVIII. Si certo constaret ratio quam n ad i habet, quantum haec regula in quovis casu a vero aberret, assignari posset : Nec non aeris vis elastica maxima AD, seu ratio AD : QM. Ob hunc defectum pono saltem $n : i = q : 1$. eritque $n = qi$, adeoque vis elastica QM erit ut $\sqrt[3]{qi^2} - \sqrt[3]{(qi-i)^2}$, dividatur per $\sqrt[3]{i^2}$ utpote constantem, erit vis elastica aeris naturalis ut $\sqrt[3]{q^2} - \sqrt[3]{(q-1)^2}$. Assumatur quivis alius condensationis gradus, quo densitas sit ad naturalem ut s ad 1. Erit ea densitas si, adeoque vis elastica respondents erit $\sqrt[3]{qi^2} - \sqrt[3]{(qi-si)^2}$, erit ea igitur ut $\sqrt[3]{q^2} - \sqrt[3]{(q-s)^2}$. Unde sequitur, elasticitatem aeris naturalis esse ad elasticitatem aeris s vicibus densioris ut 1 ad $\frac{\sqrt[3]{q^2}-\sqrt[3]{(q-s)^2}}{\sqrt[3]{q^2}-\sqrt[3]{(q-1)^2}}$ sed secundum regulam vulgarem oporteret esse ut 1 ad s, si $s = q$ tum erit DR = q.QM, et AD = $\frac{\sqrt[3]{q^2}}{\sqrt[3]{q^2}-\sqrt[3]{(q-1)^2}}$.QM Quia vero q valde est magnum respectu 1, erit $\sqrt[3]{q^2} - \sqrt[3]{(q-1)^2} = \frac{2}{3\sqrt[3]{q}}$; erit itaque AD = $\frac{3}{2}q$ .QM. Regula ergo ea plua dimidio nunquam a vero aberrare potest.

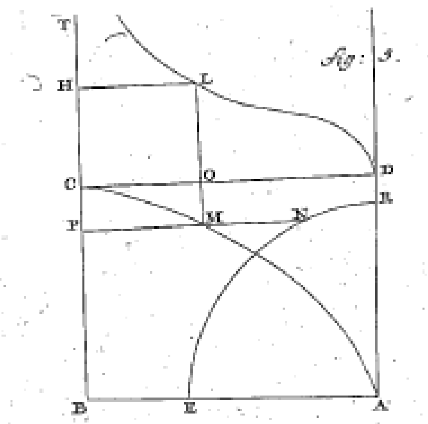


XXIX. Cognita pro quovis condensationis gradu aeris elasticitate, poterit inde inveniri quanta esse debeat aeris densitas in data quacunque altitudine. Cum enim aer naturalis comprimatur a pondere aeris superincumbentis, necesse est, ut, quo altius ascendatur, aer ob imminutum ibi atmosphaerae pondus rarior fiat. Nam ubique eiusque aer dilatatur, quoad pressio aequalis sit eius elasticitati. Sit igitur curva BMV scala densitatum aeris, cuius nimirum applicatae PM exprimant aeris densitates in altitudinibus P. Sit A is locus, quo densitas aeris est maxima, adeoque ubi AB = n. Accipiatur locus quicunque P, cuius altitudo AP super A dicatur x; densitas vero ibi seu PM = y, erit ibi aeris vis elastica ut $\sqrt[3]{n^2}-\sqrt[3]{(n-y)^2}$, cui proportionalis esse debet pressio ab aere superiore PT orta. Pressiones autem sunt ut densitates et altitudines coniunctim: Quamobrem erit pressio aeris superioris ut area MPTV i. e. ut $-\int ydx.$ Est iaque $a\int ydx=\sqrt[3]{n^2}-\sqrt[3]{(n-y)^2}$, adeoque $aydx=\frac{2dy}{3\sqrt[3]{(n-y)}}$; unde $adx=\frac{2dy}{3y\sqrt[3]{(n-y)}}$ seu posito $a=\frac{2}{3}$, erit $dx=\frac{dy}{y\sqrt[3]{(n-y)}}$ quae hoc modo integrari debet, ut posito $x = 0$, y fiat $= n$.

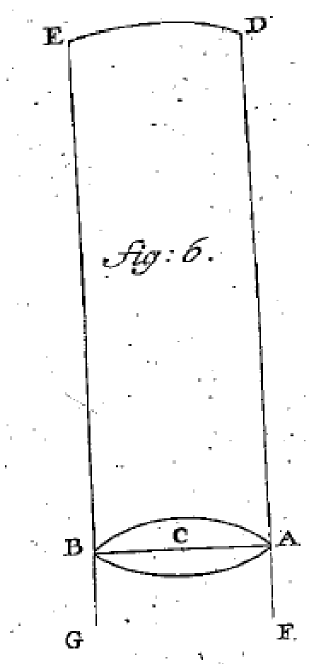
XXX. Si fiat n = y erit tum dx infinities maius quam dy, ergo tangens in B parallela erit axi verticali AT. Propterea haec curva alicubi punctum flexus contrarii habere videtur; id quod hoc modo invenietur. Quia est $dx=\frac{dy}{y\sqrt[3]{(n-y)}}$; erit $dy=ydx\sqrt[3]{(n-y)}$. Assumto dx pro constante, erit

$$ddy=dydx\sqrt[3]{(n-y)}-\tfrac{1}{3}ydxdy(n-y)^{\frac{-2}{3}}=0.$$

Unde $3n - 3y = y$. Consequenter $y=\frac{3}{4}n.$ Quam ob rem punctum flexus contrarii eo erit loco, quo densitatis aeris est ad maximam ut 3 ad 4. Applicetur igitur CD = $\frac{3}{4}$AB, erit in puncto D punctu flexus contrarii. Est deinde subtangens huius curvae $\frac{ydx}{dy}=\frac{1}{\sqrt[3]{(n-y)}}$. Unde colligitur si y fuerit respectu ipsius n valde paruum, tum esse subtangentem constantem; Ut adeo hoc in casu haec cum curva cum logarithmica confundatur.



XXXI. Potest quidem aequatio pro ista curva $dx=\frac{dy}{y\sqrt[3]{(n-y)}}$ ad quadraturam circuli et logarithmos reduci : sed inde multo difficilior inascitur eius curvae constructio, quam si quadraturus construatur. Designet igitur AMC parabolam cubicalem secundam, ut in fig. 3 sitque CD = n. Assumatur aeris densitas quaevis in CD, puta CQ, ponaturque CQ = y. Cuius applicaa respondens QM erit $\sqrt[3]{n^2}-\sqrt[3]{(n-y)^2}$, cui proportionalis accipi $-\int ydx$ debet. Dicatur QM, z, brevitatis ergo : eritque $-ydx=dz$ et $dx=\frac{-dx}{y}$, atque $x=\int\frac{-dz}{y}.$ Ducatur PM, quae erit y, et in ea producta, si opus est,capiatur PN $=\frac{1}{y}$, erit area PBEN = $\int\frac{-dz}{y}.$ Quapropter in MQ prolongata accipiatur QL, quae sit ut area PBEN. Erit



punctum L in curva quaesita. Est enim ea, ducta LH, CH = LQ = $\int \frac{-dz}{y} = x$, et HL = CQ = y. Hoc igitur modo curva DLB determinabitur.

XXXII. Quae hucusque aeris proprietates ex theoria exposita derivatae sunt, eae nihil absoluti in se continent, sed tantum rationem dant, secundum quam elasticitas aeris pro diversis densitatibus, humiditatibus et materiae subtilis celeritatibus existimari debeat. Verum nunc absoluti quid tradam altitudinem columnae mercurialis determinaturus, quam datus aereus globulus sustinere valet. Sit itaque AB diameter horizontalis bullulae aereae, de qua intelligi debent, quae § 14 inventa sunt. Incumbat ei columna mercurialis ABED altitudinis AD = f, quae tanta sit, ut in aequilibrio consistat cum vi, quam bullula habet, sese expandendi. Haec autem columna in singulis bullulae punctis perpendiculariter agit in eius superficiem, idque vi, quae est ut altitudo columnae f, et basis seu superficies bullulae, quam premit, atque gravitas specifica coniunctim. Cum autem semidiameter AC sit = a; erit circulus maximus bullulae $\frac{\pi aa}{2}$, adeoque semisuperficies eius = πaa, quae est basis, quae a columna mercuriali premitur. Exprimatur porro gravitatis specifica mercuri, respectu habito ad reliquis gravitates specificas, litera r, erit pressio, quam column mercurialis in bullulam exercet = $\pi aarf$.

XXXIII. Haec autem pressio destrui debet pressione a vi centrifuga materiae subtilis orta, quae etiam in singula superficiei puncta aequaliter agit. Quamobrem vis, qua vis centrifuga in haemisphaerium agit, idque extendere annititur, aequalis esse debet vi comprimenti columnae mercurialis. Vis autem ea est dimidium vis elasticae totius bullulae, cuius aequale pondus §14 inventum est,
$\frac{2\pi nkaa}{\sqrt[3]{(m-pm+pn)}^2}(\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-pm+pn-i)^2})$; huius ergo dimidio aequari debet pondus columnae mercurialis $\pi aarf$. Unde sequens enascitur aequatio :
$rf\sqrt[3]{(m-pm+pn)}^2 = nk[\sqrt[3]{(m-i+pi-pm+pn)^2}-\sqrt[3]{(m-i-pm+pn)^2}]$ seu
$f = \frac{nk}{r}[\sqrt[3]{(\frac{m-i+pi-pm+pn}{m-pm+pn})^2}-\sqrt[3]{(\frac{m-i-pm+pn}{m-pm+pn})^2}]$.

XXXIV. Ut haec aequatio tractatu facilior evadat saltem pro naturali aeris statu, pono i admodum paruum respectu n; et propterea erit
$\sqrt[3]{(m-i+pi-pm+pn)^2} = \sqrt[3]{(m-pm+pn)^2}+\frac{2(pi-i)}{3\sqrt[3]{(m-pm+pn)}}]$. Atque eodem modo
$\sqrt[3]{(m-i-pm+pn)^2} = \sqrt[3]{(m-pm+pn)^2}+\frac{2i}{3\sqrt[3]{(m-pm+pn)}}]$. Quibus valoribus substitutis, orietur haec aequatio $rf(m-pm+pn) = \frac{2pnfk}{3r}$, unde $f = \frac{2p\pi ik}{3r(m-pn+pn)}$. Sed si ponatur humiditas in aere evanescens, erit p = 1. Tum igitur erit $f = \frac{2ik}{3}$. Si autem aer vaporibus fuerit infectus p eo minor erit unitate, quo plus vaporum in aere hospitatur; ponatur itaque hoc in casu p = 1 - q, erit
$f = \frac{2p\pi ik(1-q)}{3r(qm+n(1-q))} = \frac{2ik}{3r}-\frac{2qmik}{3r(qm+n(1-q))} = \frac{2ik}{3r}(1-\frac{qm}{(qm+n(1-q))})$.

XXXV. Cum autem f indicet altitudinem columnae mercurialis in aequilibrio consistens, exprimet eadem litera f altitudinem mercurii in barometro. Ex inventa igitur aequatione, datis velocitate materiae subtilis in bullulis gyrantis, aeris et materiae subtilis gravitatibus specificis, atque quantitate aquae in aere versantis, inveniri poterit altitudo mercurii in barometro. Nam *r* gravitas specifica mercurii, ut et m gravitas specifica aquae aliunde iam constant. Percurram itaque casus, quibus mercurius ascendere, et quibus descendere debet; ut inde pateat, quid in aere acciderit et ascendente et descendente mercurio in barometro. Ad hoc cum tantum ratione opus sit, negligo factorem $\frac{2}{3r}$, tanquam constantem, eritque *f* ut $ik(1-\frac{qm}{(qm+n(1-q))})$.

XXXVI. Hinc igitur consequitur manente facto, *ik* mercurium in barometro ascendre decrescente fractione $\frac{qm}{(qm+n(1-q))}$. Haec vero fractio crescente *q* etiam crescit, decrescente vero *q* decrescit : Nam crescente *q* elemento *dq*, fractio crescet elemento $\frac{mndq}{(qm+n(1-q))^2}$. Quamobrem manente facto *ik* mercurius in barometro ascendere decrescente aeris humiditate; ea vero aucta mercurius descendere debebit. Atque hanc puto esse rationem, cur ascensus mercurii in barometro plerumque coelum serenum, descensus vero pluviam adversumque tempestatem praenunciet : Illo enim casu aer maximam partem a vaporibus vacuus est, hoc vero iis magis infectus.

XXXVII. Possunt quidem aliae concurrere rationes, ob quas mercurius ascendere vel descendere queat immutata vaporum quantitate : Quando scilicet factum ik crescit vel decrescit. Sed fortasse hoc factum sendibiliter neque crescere neque decrescere potest. proptera alterutram literam eadem fere ratione auctam, qua altera diminuitur. Nam velocitas materiae subtilis, cuius quadratum est ut *k*, augmenta accipit aucto calore, sed idem calor aerem rarefacit, et quantitatem *i* minorem efficit, ut ergo factum *ik* quasi semper permaneat. Ex quo intelligitur tute semper ascensum vel descensum mercurii diminutae vel auctae vaporum in aere versantium quantitati attribui posse, quanquam negari non possit et factum *ik* quodammodo effectum humiditatis et augere et diminuere posse.

XXXVIII. Neque vero hinc inferre licet, barometrum idem ac hydrometrum praestare oportere; cum et hoc humiditatem aeris monstret. Sed id considerandum est, barometri effectus a tota aeris massa seu totius atmosphaerae statu pendere; hygrometri autem a solo aere id ambiente. Quamobrem altitudo mercurii in barometro incrementa accipit, si universus aer a vaporibus liberatur, decrementa vero si is vaporibus impraegnatur. Unde colligitur hygrometrum sere summam siccitatem ostendere posse, cum altitudo mercurii minima sit; et similiter hygrometrum humiditatem indicare posse, cum mercuriis summam altitudinem attigerit.

XXXIX. Si humiditas aeris evanescat, habetur iuxta § 33 haec aequatio $f=\frac{2ik}{3r}$, quam n non ingreditur. Ex ea igitur cum experimentis ratio *r : i*, ut et altitudo *f* constet, inveni potest altitudo *k*, ex qua grave cadendo velocitatem acquirit ei, qua materia subtilis in bullulis aeris gyratur, aequalem; Est enim $k=\frac{3rf}{2i}$. Circa quam expressionem observo eam excepto coefficiente numero $\frac{3}{2}$ eandem esse cum altitudine generante velocitatem, qua sonus per aerem promovetur, ut ergo velocitas materiae subtilis constantem habeat rationem ad velocitatem soni. Hic autem animum abducaere oportet humiditate aeris, qua accedente *k* alio modo exprimetur.

XL. Observatur altitudo mercurii in barometro a 22 usque ad 24 et ultra dig. Pedis Rhenani. Cum autem aerem a vaporibus vacuum supponam, attribuo literae f maximam, quam habere potest altitudinem , nempe 2460 scrup. Ped. Rhenani. Dein rationem r ad I pono ut 100000 ad 1. Quemadmodum ex experimentis de gravitate aeris concluditur. Quibus positis erit $k = \frac{30000.2360}{2} = 36900$ pedibus. Adeoque materia subtilis velocitate movetur tanta, quanta a gravi ex altitudine 36900 pedibus in vacuo descendente acquiritur. Si ergo haec materia sua velocitate in directum pergeret, conficeret uno minuto secundo $1518\frac{1}{2}$ ped. Rhenanos.

XLI. Isto hanc dissertationem finio, cum desint accurata experimenta ex quibus reliqua adhuc desiderata determinentur, et quibus Theoria haec plenius confirmetur. Incerta est adhuc rato *n* ad *i*, seu quam habet gravitas specifica materiae subtilis ad gravitatem specificam aeris. Ad hanc vero investigandam accuratis experimentis ad id facientibus instituendis operam studiumque adhibebo. Quantitas autem *n* si haberetur, facile formulae inventae ad praxin applicarentur; atque aliis idoneis instrumentis adhibendis quovis tempore, quantum aquae in aere contineatur, assignari posset. Et forsitan multa insuper alia, ad quae, iis quae sufficiunt cognitis, quasi manu duceremur.