

Chapter 16

On the various types of bodies.

119. We compare the liquid bodies with the solid and hard bodies, and a completely hard and solid body is such that no force is able either to compress it into a smaller space, or to alter its shape.

It is not the question here, whether there really exist bodies such that no force can either compress them or alter their shape, since we only imagine a type of extreme properties in order the better to define the others in terms of it.¹ But here we have to consider two points: the first concerns compression into a smaller space, and the other the change of shape. Since we have given to coarse matter the property that it can not be compressed into a smaller space, we must assign to it as its first distinguishing mark that a body that consists entirely of coarse matter can not by any force be compressed into a smaller space. As regards the change of shape, there is no doubt that, depending on how it is applied, a force would be able to remove from such a body parts by rubbing, piercing, carving, tearing or sawing, and so to alter its shape. But if one excludes such forces, and only considers those that act by mere pressure directed perpendicularly on the body, then it can still seem uncertain whether there could not be bodies that would resist all changes to their shape in this way. One can imagine a hard sphere that could not be flattened in the least either by a weight lying on it or by impact. Such a sphere could justifiably be considered completely hard. At the very least there are really such bodies on which a given force is unable to produce a change in shape, and in respect to this or to smaller forces such bodies can be regarded as completely hard, even if from larger forces they would suffer a change in shape.

120. Two types must be distinguished among the bodies that are not completely hard and solid. To the first type belong those that can not be compressed into a smaller space by any force, but can nevertheless suffer a change in shape; to the other type belong those that can also be compressed.

Amongst the bodies that can not be compressed into a smaller space we must count in particular the metals, but nevertheless their shape can be altered by a pressure or a knock. Thus a metallic sphere can through strong pressure or repeated knocks be expanded into a plate whilst maintaining the same density and filling an equally large space. Such bodies we count amongst the first type, but to the second we count such that can be compressed into a smaller space, through which the shape must also of necessity suffer a change. Excluding liquid matter, one finds amongst the bodies that, judging from appearance, are uniform, few in which a noticeable compression occurs. But this subdivision according to compressibility is applicable to liquid and solid bodies, and amongst both there are such that are incompressible and compressible to some extent. To this latter type of liquid

¹ From the correct “inert” of the manuscript the *Opera postuma* had made a senseless “immer” F.R. Here “inert derselben” has been rendered as “in terms of it”. Translator.

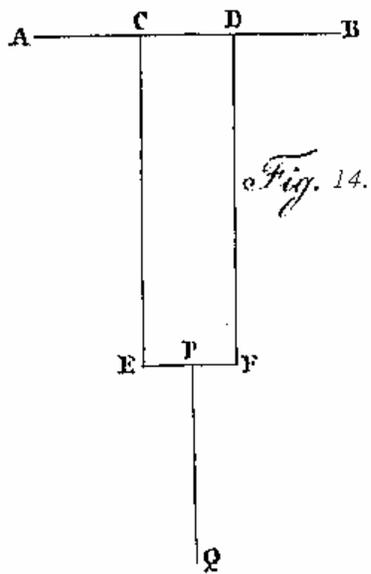
bodies belongs in particular the aether and also the air, of which the former, because of its essence, and the latter because of the particular mixing of subtle and coarse matter, are capable of very noticeable compression. But the other characteristic, that involves the changeability of shape, constitutes the prime difference between the liquid and the solid bodies. Liquid matter may have any desired shape, but the slightest force is able to change this according to the shape of the vessel that it is contained in; by contrast in solid bodies any given force is not able to effect a given change in its shape, or at least not all possible changes of shape. A piece of paper can by folding be made into infinitely many shapes, but it can not be expanded into a filament.

121. One calls soft bodies those whose shape can be altered by a small force, whilst those that require a large force are called hard. Here must also be mentioned flexible bodies, which can either be completely flexible or more or less flexible, depending on whether the smallest force is sufficient, or a smaller or larger one, to produce a given bending.

The designations soft and hard are regarded as opposites, but they differ only in degree; thus a less hard body can be regarded as soft and a less soft one as hard. Here it also depends on the magnitude of the force involved, for if the force is too small to change the shape of the body, then in relation to it the body is hard, although its shape can be changed by a larger force. Such soft bodies are glue and wax; for if we imagine spheres of such materials, then these can be pressed flat by a force; the larger the pressing force, the flatter can they be pressed by it. Here one can distinguish infinitely many degrees of softness, where the highest degree is equivalent to a liquid; thus glue and wax, when totally softened, entirely represent liquid matter, since the slightest force is able to press them into all possible shapes. But if the glue or the wax is less soft, then a sphere made from them can well be pressed flat by a given force, but the action ultimately ceases, so that if the sphere were to be pressed even flatter, a larger force would be needed. But even the smallest force appears to be able to change the shape of such a sphere a little, even if the change is barely noticeable. Such bodies are also flexible, but there are other kinds, to which flexibility is most appropriately assigned, such as a filament, a ribbon or a rope, amongst which those are called totally flexible, which even the smallest force can bend to the highest possible extent, such as may be the case with an extremely delicate filament. But others are such that they can not be bent beyond a certain degree by a given force, and the smaller the degree of bending is, the less flexible are such bodies. In this there are infinitely many kinds, which we are content to merely indicate.

122. As regards pulling forces, there are bodies which either can or can not be extended in length; in both cases, if the force is sufficiently great, the bodies are torn apart, and bodies that are able to withstand a very strong force are called tough.

In this section we consider how bodies react to the forces acting on them, and this depends primarily on the manner in which the forces are applied to the bodies. In the previous sections we have assumed forces that press on the bodies; but now we

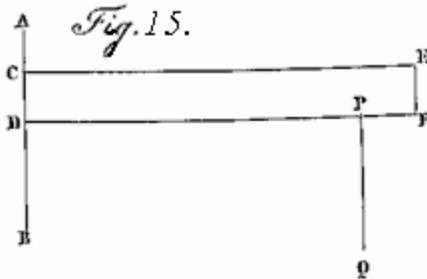


turn our attention to those that try to tear the bodies in two. Consider a body CDEF (Figure 14), one end CD of which is attached to an immovable wall AB, but at the other end EF of which a force PQ is pulling. However strong and tough the body may be, one can always increase the force so much that the body is eventually torn in two, provided its shape is such that a force sufficiently big can be applied. What matters here is how the parts of the body are attached to each other, and since, as we shall soon show, this attachment can not be infinitely strong, there must always be a force that is able to overcome it and to pull the parts apart, although it may frequently be impossible to apply such a big force. It will not be possible to pull a diamond apart, not because a sufficiently strong force is not available, but because one can not

attach such a big force to it. But before a body is torn apart, it either extends or it breaks suddenly. In the former case a distinction can be made whether or not through the extension the body is brought into a larger or smaller space, after which a smaller force will already produce a change in the shape of the body. But in the latter case, as long as the force is smaller than required for actual tearing, the shape of the body remains unchanged.

123. If a solid body is fixed at one end, but is pulled sideways at the other end by a force, then, provided the force is sufficiently strong, the body is either broken or bent; in the first case one says that the body is brittle, in the other case that it is flexible.

Let us imagine the body CDEF (Figure 15) again attached at one end to an immovable wall AB, and let it be pulled sideways at the other end by a force PQ.



Depending on the effect of such a force one can distinguish several kinds of bodies.

However strong the body might be, the force can be increased sufficiently to bring about a change in it. The body must either be broken or bent; or it can also happen that before it breaks it is bent. Bodies that ultimately are broken are called brittle, and a body is more brittle if it is broken by a smaller force. Here the thickness of the body must be taken into account, and also not only the force itself, but

the force multiplied by its distance from the wall. But if the body is only bent by the force, it is called flexible, a property already mentioned earlier. Completely flexible is a body that can be entirely bent around by even the smallest force; on the other hand it is the more or the less flexible, the more or the less the same force is able to

bend it. Sometimes the same force, provided it acts long enough, can produce an ever greater degree of bending, in which case time must also be taken into consideration; sometimes the body is bent by a certain force only to a certain degree, and is then in equilibrium with the force. All these particular circumstances can vary in infinitely many ways, giving rise to infinitely many kinds of bodies. One can however also apply the forces in other ways, and distinguish between bodies according to their effect; it would however be unnecessary to take this too far, before we are able to examine the reason for all these differences.

124. Some bodies are such that, if their shape has been altered by a force, they remain in this altered shape after the force has ceased to act; others are so that they assume again their earlier shape. These are called elastic, the others are called inelastic.

Since until now we have seen what changes in the shape of bodies can be brought about by forces acting on them, we must now discuss what takes place in the bodies after the forces have completely ceased to act on them. Here it is at once clear that once a body has really been torn apart or broken, it will remain in this destroyed state after the force has ceased to act. But if a solid body has only been bent to a certain degree by a force, or has had its shape altered in some other manner, then, after the force has ceased to act, there can be two different cases, depending on whether the body retains this altered shape, or reassumes its earlier shape; and since the latter occurs through a spring force or elasticity, these bodies are called elastic, the others inelastic. A sphere of glue is thus inelastic, because once it has been pressed flat, it will remain so; and a rod of lead, since once bent it will remain bent, is likewise inelastic. In contrast to this an ivory sphere is elastic because after a compression, although by an indiscernible amount, it will assume again its earlier shape, as is inferred from the fact that it bounces back; similarly, since a good sword blade, when bent, becomes straight again, it is elastic. One says of such bodies that they possess an elastic force; but one must not think that this force is in contradiction to the property of persistence; for it will be shown that this originates from the spring force of the aether. Since this can coexist with persistence, in the case of solid bodies a similar elastic force does not contradict persistence.

125. There are also bodies which are neither totally inelastic nor completely elastic, because following a change in their shape they resume their previous shape only to some extent, but not completely, for which reason they are assigned a larger or smaller elastic force.

We have seen that in bodies two kinds of change in shape can occur. One takes place keeping the extent constant, whilst the other involves compression into a smaller space, to which we may add the extension into a larger space. There are also bodies which, after having been extended, will contract again, the reason for which is however irrelevant. In this twofold change there can in solid bodies a force be present which is called perfect if the body completely reverts back to its previous shape; but it

**Euler E842: *An Introduction to Natural Science, Establishing the Fundamentals....*,
from his *Opera Postuma*. Translated from German by E. Hirsch.**

79

is called imperfect if the restoration is only partial. Liquid materials can also be more or less elastic or not be elastic at all; but because they can assume any shape, their elastic force can only be judged from whether liquid matter, after it has been compressed into a smaller space, will tend to expand again, a tendency that in particular the aether has got; there can also be no doubt that air is not perfectly elastic. But water is regarded by many Natural Scientists as inelastic liquid matter, since it can not by any means be compressed into a smaller space. But it is not the degree of compression that is relevant here, and if the water could only be, in a manner of speaking, compressed to a indiscernibly small extent, but then go back to its previous state, one would have to assign to it perfect elasticity. One can conclude with certainty from some experiments where a bladder filled with water re-bounced on impact, that water must be a perfectly elastic body. It also is only necessary to hit such a sphere with a hammer for elasticity to manifest itself.