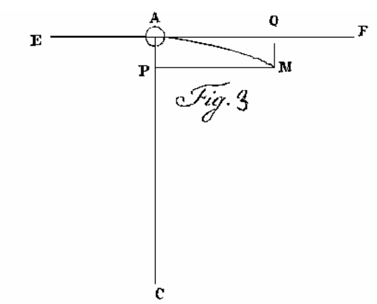
Chapter 8.

On the Effect of Forces on the Direction of Bodies.

62. If a moving body is pushed sideways by a force, the direction of which is perpendicular to the direction of the body, then the path described by the body becomes curved, without its speed suffering any change.

Since the force is acting neither forward nor backward on the body, its speed can be neither increased nor decreased, and the effect of the force will only be that the body is deflected from its rectilinear movement, and must consequently describe a curved line. But as soon as the path becomes curved, the direction of the force ceases to be perpendicular to it, if the force maintains its direction throughout, so that a change in speed can indeed soon arise. We must therefore consider this curving of the path only during an infinitesimally small interval of time, so that during this interval no significant change in speed can arise.



Let the body of mass M have until now moved along the straight line EA (Fig.3) with speed v, but let a force p begin to act at point A in direction AC, perpendicular to EA, and let this force maintain its direction throughout; this force will cause the body not to continue its movement along the straight line AF, but instead steer it along a certain curved line AM, the shape of which can be determined as follows. If the force were not present at all, the body would continue with its velocity v along the straight line AF, and after time t would traverse the path AQ, so that $AQ = vt$, since in uniform motion the path is found by multiplying the speed with the time. But if the body were at rest at A, and were pushed by the force p along the line AC, it would in time t be moved through the path $AP = nptt/2M$ (58). But if the body does both, i.e. it maintains its movement and at the same time is driven by the force p, it will after time t be neither at Q nor at P, but at M, if one draws the line QM parallel to AP and of the same length. Therefore in order to determine the true position of the body after time t, if one marks on the line AF the distance $AQ = vt$, and draws at Q the perpendicular line $QM = nptt/2M$, then point M will be the desired position of the body.

63.The curved path, described by a body under the action of a sideways directed force, can be regarded as a circular arc, the diameter of which varies as the mass of the body multiplied with the square of the speed, and divided by the force, i.e. as the so called living force divided by the acting force.

According to the foregoing, the curved line AM is such that after time t

$$AP = nptt/2M \quad \text{and} \quad PM = AQ = vt.$$

If we now put

$$AP = nptt/2M = x \quad \text{and} \quad PM = vt = \mathrm{y}$$

then we obtain for the time t two values, namely

$$tt = 2Mx/np \quad \text{and} \quad t = y/v \quad \text{or} \quad tt = yy/vv;$$

consequently the nature of the curved line is expressed by the equation

$$2Mx/np = yy/vv \quad \text{or} \quad yy = 2Mvvx/np,$$

which represents a parabola with parameter $2Mvv/np$. However since here we have to consider only an infinitesimally small part of the line, we can regards it as an infinitesimally small circular arc, and the diameter of the circle of which AM is an arc will be equal to the above parameter, i.e. it will be equal to $2Mvv/np$. Consequently half the diameter, or the radius of curvature of the path AM is Mvv/np. It thus varies as the mass of the body, multiplied with the square of the speed and divided by the force, since n represents a certain number that is determined by way in which the forces, masses and speeds are measured. If one has arbitrarily chosen such a way of measurement, and if one has for a single case determined the appropriate value for n, then one can even determine the actual magnitude of half the diameter of the curvature of AM; for if C is the centre of the arc AM, then $CA = Mvv/np$. In this way the curvature of a path due to a sideways acting force is best determined, and since time does not enter here, because in a longer or shorter time the body will only describe a larger or smaller arc of the same circle, one sees from this in the clearest possible way the effect which a force, directed at right angles to the motion, must exert on the state of the body.

64. If the movement of a body is curved due to a sideways acting force, then half the diameter of curvature is twice as large as the path, over which the body would loose its motion entirely, if the same force were to act on it in a reverse direction.

If a body of mass M moves with speed v, and if a sideways directed force acts on it, then, as shown in the previous section, its movement will follow a circular arc of half diameter Mvv/np. But if this body were now to be brought to rest by a force of the same magnitude, but acting in a direction opposite to its direction of motion, then it would have to traverse a path s such that $Mvv = 2nps$ (61); this path, over which the body would loose its entire movement because of the backwards acting force, would therefore be $s = Mvv/2np$, which is half as great as half the diameter of curvature, as found above. From this we conclude that half the diameter of curvature must be twice as large as the path over which the same force, acting backwards, would deprive the body of its entire motion. Comparing these two cases, were first we assume the force to act sideways, and then assume it to be directed backwards, leads us in the first case to a determination of half the diameter of curvature, that no longer depends on the number n or the way in which we express the various quantities by numbers, but that immediately yields us a line that is equal to half the diameter. For if s is the path along which the body is supposed to be deprived of its motion, then the half diameter that we seek is $2s$. Here one can also note that $2s$ also represents the path which the body would traverse with uniform speed v in just the time in which it can be brought to rest by the force p, acting backwards.

65. If the force maintains its magnitude throughout, but if it changes its direction continually so as to be always at right angles to the direction of motion, then the body will at constant speed describe a circle, the half diameter of which equals that we have just determined.

If a body of mass M initially moves with speed v, and is acted upon by a force p, that is directed perpendicular to the direction of the body, then its path will be bent into a circular arc of half diameter Mvv/np, and at least in an infinitely short time will not undergo any change in speed, which could only occur when the force no longer acts perpendicular to the direction of motion. But since we assume that the force p always remains perpendicular to the movement of the body, no change in the speed of the body can take place, and it must always travel with uniform speed on a circle the half diameter of which is given by Mvv/np or alternatively by $2s$, where, from the previous section, s represents the path along which the body would, due to the force p acting backwards, loose its entire motion. The body will therefore continually traverse the circle with a constant speed v; calling half the diameter of this circle r, we have $r = 2s$; from this one can see how large the speed of the body would have to be, for it to describe a given circle; for this to occur one would have that

$$Vv = npr \quad \text{or} \quad v = (npr/M)^{1/2}$$

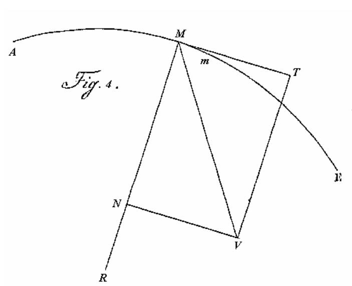
If however the speed v ,and also the circle on which the body is to circulate are given, then one can determine from this the magnitude of the force that must continually act perpendicularly on the body, which is given by $p = Mvv/nr$. Since the force must always act at right angles to the direction of the body, it is clear that the body must always be driven towards the centre of the circle.

66. For a body to move with a given speed in a circle, it must continuously be driven towards the centre of the circle with a force that varies with the mass of the body multiplied by the square of the speed and divided by half the diameter of the circle.

Putting the mass of the body = M, the velocity = v, the half diameter of the circle = r and the required force = p, we have found that $p = Mvv/nr$. If this force were to act backwards on the body, it would bring it to rest whilst it traverses a path that equals the half diameter r. If this path is called s, we found that $Mvv = 2nps$, from which follows that $p = 2nps/nr$ and also $s = r/2$. We can therefore describe the force necessary to move in a circle by saying that it must equal that force, which by acting outwards on the body, could bring it to rest whilst traversing a path equal to a quarter of the diameter. But should the force directed towards the centre suddenly cease, then the body would from this moment on move with its speed along a straight line that touches the circle. This is so because the body, due to its persistence, endeavors always to run with its current speed in its current direction, and as soon as the force directed towards the centre ceases, it follows this natural tendency. But if the body were attached to the centre by a thread, and thus kept to the circle by the thread, then the thread would take the place of the force, and through its tension keep the body to the circle. Since the thread will therefore have been put under tension with a force such as we have determined, namely $p = Mvv/nr$, one says that in this state the body has a force with which to put the thread under tension; and this is the force which otherwise is called *vis centrifuga*, that is identical with the force we have determined above as required for motion in a circle.

67. But if a body is to move with non-uniform speed on a curved line, then two forces are always required, one that pushes the body either forward or backward, thus achieving changes in speed, and another, acting sideways and determining the curvature of the line.

Suppose a body of mass M moves with variable speed along the curved line *AME* (Fig.4), and let its speed at point M be v. Since during an infinitely short time interval the motion can be regarded as uniform, as the increase or decrease in speed that occurs in this interval is infinitely small, just as the small path *Mm*, that is traversed during this interval, can be regarded as a circular arc with its centre at R and half diameter r, the force necessary to achieve this curvature must act sideways in the direction *MR*, and from the previous section it will be Mvv/nr.



Since subsequently the speed is to change, let us call dv the speed increase in the small time interval dt. For this a force is required that drives the body forward in its direction of motion, that is along the straight line *MT*, touching at M. Let this force be p, then we have from the above

$$Dv = npdt/M \quad \text{and therefore} \quad p = Mdv/ndt;$$

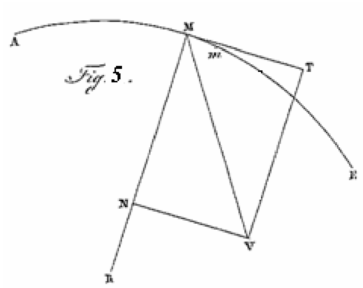
This is the force that, together with the previously found sideways acting force, should be able to produce the motion along the curved line *AME*. If the speed were to decrease, dv would be negative, and for the force p a negative value would also be found, in which case the force would have to be acting on the body backwards in the direction $M\Theta$.

If, instead of the infinitesimally small time interval dt, one wishes to bring into the calculation the infinitesimally small path *Mm*, then one puts the whole path, or the arc *AM* equal to s, so that $Mm = ds$, and since $v = ds/dt$, and the motion along *Mm* can be regarded as uniform, one has $1/dt = v/ds$, and the force acting along *MT* is found to be $p = Mvdv/nds$.

Of the two forces that are required, it is usual to call the first, that is directed along *MT*, the perpendicular force (*vis normalis*), and the second, that acts along *MT* or $M\Theta$, the touching force (*vis tangentiales*)

68.From this one can determine the change that must be produced by a force that acts at an angle to the motion of the body; for such a force can be decomposed into to other forces, one of which acts on the body either forwards or backwards, with the other force acting sideways, each producing in the state of the body the change previously determined.

We shall assume the force to be variable as regards both magnitude and direction, and when the body has reached M (Fig.5), if its direction is towards MT, and its speed is *v,* then a force *V* shall act on it in direction MV. If one draws the line MR perpendicular to MT, and constructs the right angled quadrangle VNMT, with diagonal MV, then it is known that the force *MV* is equivalent to the two forces that are represented by the sides MT and MN, of which one indicates the forward acting, and the other the sideward acting force. If one puts the forward acting force *MT* = *T*, and the sideward acting force *MN* = *N*, one obtains from the relations of the sides MT and MN to the diagonal MV the equations



$$T = \mathrm{MT/MV} \times V \quad \text{and} \quad N = \mathrm{MN/MV} \times \mathrm{V}.$$

From the first the speed of the body will gain an increase that in the infinitely small time interval dt will amount to $dv = nTdt/M$, where M represents the mass of the body; alternatively, if ds is the infinitesimally small path Mm that has meanwhile been traversed, then, since $ds = vdt$, one finds $vdv = nTds/M$. The other force N will cause the body to curve its path such that, putting half the diameter of curvature $MR = r$, one finds $r = Mvv/nN$.

However here it is not the intention to show for the general case how to find the complete movement of the body, that is to say the line *AM* and the speed at every point, since this belongs into the Special Theory of Motion; but in the next chapter a simpler method will be proposed.