

SR, as falling on the concave lens: in this case it is certain they would assume the directions RQ, RQ; which, produced backward, would meet in the point F, which is the common focus of the convex and concave lenses. Now it is a general law, that in whatever manner rays are refracted in their passage from one place to another, they must always undergo the same refractions in returning from the last to the first. If, therefore, the refracted rays RQ, RQ, correspond to the incident rays SR, SR; then, reciprocally, the rays QR, QR, being the incident ones, the refracted rays will be RS and RS.

The matter will perhaps appear in a clearer light still, when I say that concave lenses have the power of rendering parallel those rays which, without the refraction, would proceed to their focus. You will please carefully to attend to the following laws of refraction, which apply to both convex and concave lenses.

1. By a convex lens (PLATE VI. *Fig. 31.*) parallel rays are rendered convergent.

Convergent rays become still more so (PLATE VII. *Fig. 1.*), and divergent less divergent.

2. By a concave lens parallel rays are rendered divergent. (PLATE VII. *Fig. 2.*)

Divergent rays become still more divergent, *Fig. 3.* and convergent rays less convergent.

All this is founded on the nature of refraction and the figure of the lenses, the discussion of which would require a very long detail; but the two rules which I have now laid down contain all that is essential. It is abundantly evident, then, that when the convex and the concave lenses are so combined that they acquire a common focus at F, they will distinctly represent distant objects, because the parallelism of the rays is restored by the concave lens after the convex lens had rendered them convergent. In other words, the rays of very distant objects, being nearly

parallel to each other, become convergent by a convex lens; and afterwards, the concave lens destroys this convergency, and again renders the rays parallel to each other.

Feb. 6th February 1762.

LETTER XC.—ON THE MAGNIFYING POWER OF POCKET-GLASSES.

THE principal article respecting telescopical instruments remains still to be explained, namely, their effect in magnifying objects. I hope to place this in so clear a light, as to remove every difficulty in which the subject may be involved; and for this purpose I shall comprise what I have to say in the following propositions.

1. Let Ee (PLATE VII. *Fig. 4.*) be the object, situated on the axis of the instrument which passes perpendicularly through both lenses in their centres. This object Ee must be considered as at an infinite distance.

2. If, then, the eye, placed at A, looks at this object, it will appear under the angle $E A e$, called its visual angle. It will, accordingly, be necessary to prove, that on looking at the same object through the glass, it will appear under a greater angle, and exactly as many times greater as the focal distance of the object-glass PAP exceeds that of the eye-glass QBQ.

3. As the effect of all lenses consists in representing the objects in another place, and with a certain magnitude, we have only to examine the images which shall be successively represented by the two lenses, the last of which is the immediate object of the sight of the person who looks through the instrument.

4. Now, the object Ee being infinitely distant from the convex lens PAP, its image will be repre-

sented behind the lens at Ff , so that AF shall be equal to the focal distance of the lens; and the magnitude of this image Ff is determined by the straight line fAe , drawn from the extremity of the object e , through the centre of the lens A , by which we see that this image is inverted, and as many times smaller than the object, as the distance AF is smaller than the distance AE .

5. Again, this image Ff holds the place of the object relatively to the eye-glass QBQ , as the rays which fall on this lens are precisely those which would almost form the image Ff , but are intercepted in their progress by the concave lens QBQ ; so that this image is only imaginary: the effect, however, is the same as if it were real.

6. This image Ff , which we are now considering as an object being at the focal distance of the lens QBQ , will be transported almost to infinity by the refraction of this lens. The preceding figure marks this new image at Gg , whose distance AG must be conceived as infinite, and the rays, refracted a second time by the lens QBQ , will pursue the same direction as if they actually proceeded from the image Gg .

7. This second image Gg being, then, the object of the person who looks through the instrument, its magnitude falls to be considered. To this effect, as it is produced by the first image Ff from the refraction of the lens QBQ , following the general rule, we have only to draw through the centre of the lens B a straight line, which shall pass through the point f of the first image, and that line will mark at g the extremity of the second image.

8. Let the spectator now apply his eye to B ; and as the rays which it receives pursue the same direction as if they actually proceeded from the image Gg , it will appear to him under the angle $G B g$,

which is greater than the angle $E A e$, under which the object $E e$ appears to the naked eye.

9. In order the better to compare these two angles, it is evident, first, that the angle $E A e$ is equal to the angle $F A f$, being vertical angles; for the same reason, the angle $G B g$ is equal to the angle $F B f$, being vertical and opposite at the point B . It remains to be proved, therefore, that the angle $F B f$ exceeds the angle $F A f$ as many times as the line AF exceeds the line Bf ; the former of which, AF , is the focal distance of the object-glass, and the other, BF , the focal distance of the eye-glass.

10. In order to demonstrate this, we must have recourse to certain geometrical propositions respecting the nature of sectors. You will recollect that the sector is part of a circle contained between two radii CM and CN (PLATE VII. *Fig. 5.*), and an arch or portion of the circumference MN . In a sector, then, there are three things to be considered; 1. The radius of the circle, CM or CN ; 2. The quantity of the arch MN ; 3. The angle MCN .

11. Let us now consider two sectors, MCN and $m c n$, whose radii CM and $c m$ are equal to each other; now it is demonstrated in the elements of geometry, that the angles C and c have the same proportion to each other that the arches MN and $m n$ have: in other words, the angle C is as many times greater than the angle c , as the arch MN is greater than the arch $m n$; but, instead of this awkward mode of expression, we say, that the angles C and c are proportional to the arches MN and $m n$, the radii being equal.

12. Let us likewise consider two sectors, MCN and $m c n$ (*Fig. 6.*), whose angles C and c are equal to each other, but the radii unequal; and it is demonstrated in geometry, that the arch MN is as many times greater than the arch $m n$, as the radius

CM is greater than the radius cm ; or, in geometrical language, the arches are in proportion to the radii, the angles being equal. The reason is obvious, for every arch contains as many degrees as its angle; and the degrees of a great circle exceed those of a small one as many times as the greater radius exceeds the smaller.

13. Finally, let us consider likewise the case when, as in the two sectors MCN and mcn (Fig. 7.), the arches MN and mn are equal; but the radii CM and cm unequal.

In this case, the angle C, which corresponds to the greater radius CM, is the smaller, and the angle c , which corresponds to the smaller radius cm , is the greater; and this in the same proportion as the radii. That is, the angle c is as many times greater than the angle C, as the radius CM is greater than the radius cm ; or, to speak geometrically, the angles are reciprocally proportional to the radii, the arches being equal.

14. This last proposition carries me forward to my conclusion, after I have subjoined this remark, that when the angles are very small, as in the case of pocket-glasses, there is no sensible difference in the chords of the arches MN and mn , that is, of the straight lines MN and mn .

15. Having made this remark, we return to Fig. 4. The triangles FAf and FBf may be considered as sectors, in which the arch Ff is the same in both. Consequently the angle FBf exceeds the angle FAf as often as the distance AF exceeds the distance BF. That is, the object Ee will appear through the instrument under an angle as many times greater as the focal distance of the object-glass AF exceeds the focal distance of the eye-glass BF, which was the thing to be demonstrated.

9th February 1762.

LETTER XCI.—DEFECTS OF POCKET-GLASSES. OF THE APPARENT FIELD.

You must be sensible that no great advantage is to be expected from such small instruments; and it has already been remarked, that they do not magnify objects above ten times. Were the effect to be carried farther, not only would the length become too great to admit of their being carried about in the pocket, but they would become subject to other and more essential defects. This has induced artists entirely to lay aside glasses of this sort, when superior effect is required.

The principal of these defects is the smallness of the apparent field; and this leads me to explain an important article relative to telescopes of every description. When a telescope is directed toward the heavens, or to very distant objects on the earth, the space discovered appears in the figure of a circle, and we see those objects only which are included in that space; so that if you wished to examine other objects, the position of the instrument must be altered. This circular space, presented to the eye of the spectator, is denominated the *apparent field*, or, in one word, *the field* of the instrument; and it is abundantly obvious, that it must be a great advantage to have a very large field, and that, on the contrary, a small field is a very great inconvenience in instruments of this sort. Let us suppose two telescopes directed toward the moon, by the one of which we can discover only the half of that luminary, whereas by the other we see her whole body, together with the neighbouring stars; the field of this last is therefore much greater than that of the other. That which presents the greater field relieves us not only from the trouble of frequently changing the position, but procures another very

great advantage; that of enabling us to compare, by viewing them at the same time, several parts of the object one with another.

It is therefore one of the greatest perfections of a telescope, to present a very ample field; and it is accordingly a matter of much importance to measure the field of every instrument. In this view, we are regulated by the heavens, and we determine the circular space seen through a telescope, by measuring its diameter in degrees and minutes. Thus, the apparent diameter of the full moon being about half a degree, if a telescope takes in the moon only, we say that the diameter of its field is half a degree; and if you could see at once only the half of the moon, the diameter of the field would be the quarter of a degree.

The measurement of angles, then, furnishes the means of measuring the apparent field; besides, the thing is sufficiently clear of itself. Supposing we could see through the instrument AB (*PLATE VII, Fig. 8.*) only the space POP, and the objects which it contains; this space being a circle, its diameter will be the line POP, whose middle point O is in the axis of the instrument. Drawing, therefore, from the extremities PP the straight lines PC, PC, the angle PCP will express the diameter of the apparent field; and the half of this angle, OCP, is denominated the semidiameter of the apparent field of such an instrument. You will perfectly comprehend the meaning, then, when it is said that the diameter of the apparent field of such an instrument is one degree, that of another two degrees, and so on; as also when it is marked by minutes, as 30 minutes, which make half a degree, or 15 minutes, which make the fourth part of a degree.

But in order to form a right judgment of the value of a telescope, with respect to the apparent field, we

must likewise attend to the magnifying power of the instrument. It may be remarked in general, that the more a telescope magnifies, the smaller, of necessity, must be the apparent field; these are the bounds which nature herself has prescribed. Let us suppose an instrument which should magnify 100 times; it is evident that the diameter of the field could not possibly be so much as two degrees; for, as this space would appear 100 times greater, it would resemble a space of two hundred degrees; greater, of consequence, than the whole visible heavens, which, from the one extremity to the other, contain only 180 degrees, and of which we can see but the half at most at once, that is a circular space of 90 degrees in diameter. From this you see, that a telescope which magnifies 100 times could not contain a field of so much as one degree; for this degree multiplied 100 times would give more than 90 degrees; and that accordingly a telescope which magnified 100 times would be excellent, if the diameter of its field were somewhat less than one degree; and the very nature of the instrument admits not of a greater effect.

But another telescope, which should magnify only 10 times, would be extremely defective, if it discovered a field of only one degree in diameter; as this field magnified 10 times would give a space of no more than ten degrees in the heavens, which would be a small matter, by setting too narrow bounds to our view. We should have good reason, then, to reject such an instrument altogether. Thus it would be very easy, with respect to the apparent field, to form a judgment of the excellence or defectiveness of instruments of this sort, when the effect is taken into consideration. For when it magnifies only 10 times, it may fairly be conjectured, that it discovers a field of 9 degrees; as 9 degrees taken

10 times give 90 degrees, a space which our sight is capable of embracing: and if the diameter of its field were only 5 degrees, or less, this would be an instrument very defective indeed. Now I shall be able to demonstrate, that if a telescope were to be constructed such as I have been describing, which should magnify more than 10 times, it would be liable to this defect: the apparent field multiplied by the magnifying power would be very considerably under 90 degrees, and would not even show the half. But when a small effect is aimed at, this defect is not so sensible; for if such an instrument magnifies only 5 times, the diameter of its field is about 4 degrees, which, magnified 5 times, contains a space of 20 degrees, with which we have reason to be satisfied: but if we wished to magnify 25 times, the diameter of the field would be only half a degree, which taken 25 times, would give little more than 12 degrees, which is too little. When, therefore, we would magnify very much, a different arrangement of lenses must be employed, which I shall afterwards explain.

13th February 1762.

LETTER XCII.—DETERMINATION OF THE APPARENT FIELD FOR POCKET-GLASSES.

To ascertain the apparent field being of very great importance in the construction of telescopes, I proceed to the application of it to the small glasses which I have been describing.

The lens P A P (PLATE VII. *Fig. 4.*) is the object-glass, Q B Q the eye-glass, and the straight line E F the axis of the instrument, in which is seen, at a very great distance, through the instrument, the object E e, under the angle E A e, which represents the semidiameter of the apparent field, for it extends as

far on the other side downward. The point E, then, is the centre of the space seen through the instrument, the radius of which, E A, as it passes perpendicularly through both lenses, undergoes no refraction; and in order that this ray may have admission into the eye, the eye must be fixed somewhere on the axis of the instrument B F, behind the eye-glass, so that the centre of the pupil shall be in the line B F; and this is a general rule for every species of telescope. Let us now consider the visible extremity of the object e, whose rays exactly fill the whole opening of the object-glass P A P; but it will be sufficient to attend only to the ray E A, which passes through the centre of the object-glass A, as the others surround, and little more than strengthen this ray; so that if it is admitted into the eye, the others, or at least a considerable part of them, find admission likewise; and if this ray is not admitted into the eye, though perhaps some of the others may enter, they are too feeble to excite an impression sufficiently powerful. Hence this may be laid down as a rule, that the extremity e of the object is seen only so far as the ray e A, after having passed through the two lenses, is admitted into the eye.

We must therefore carefully examine the direction of this ray e A. Now, as it passes through the centre of the object-glass A, it undergoes no refraction; conformably to the rule laid down from the beginning. That rays passing through the centre of any lens whatever are not diverted from their direction, that is, undergo no refraction. This ray, e A, therefore, after having passed through the object-glass, would continue in the same direction, to meet the other rays issuing from the same point e, to the point f of the image represented by the object-glass at F f; the point f being the image of the point e of the object; but the ray meeting, at m, the concave lens,

but not in its centre, will be diverted from that direction; and instead of terminating in f , will assume the direction mn , more divergent from BF , it being the natural effect of concave lenses to render rays always more divergent. In order to ascertain this new direction mn , you will please to recollect that the object-glass represents the object Ee in an inverted position at Ff , so that AF is equal to the focal distance of this lens, which transports the object Ee to Ff . Then this image Ff occupies the place of the object with respect to the eye-glass QBQ , which, in its turn, transports that image to Gg , whose distance BG must be as great as that of the object itself; and for this effect, it is necessary to place the eye-glass in such a manner that the interval BF shall be equal to its focal distance.

As to the magnitude of these images, the first Ff is determined by the straight line eAf drawn from e through the centre A of the first lens; and the other Gg by the straight line fBg drawn from the point f through the centre B of the second lens. This being laid down, the ray Am directed toward the point f is refracted, and proceeds in the direction mn ; and this line mn being produced backward, will pass through the point g , for mn has the same effect in the eye as it if actually proceeded from the point g . Now, as this line mn retires farther and farther from the axis BF , where the centre of the pupil is, it cannot enter into the eye, unless the opening of the pupil extends so far; and if the opening of the pupil were reduced to nothing, the ray mn would be excluded from the eye, and the point e of the object could not be visible, nor even any other point of the object out of the axis AF . There would, therefore, be no apparent field, and nothing would be seen through such an instrument, except the single point E of the object, which is in its axis.

It is evident, then, that a telescope of this sort discovers no field, but as far as the pupil expands; so that in proportion as the expansion of the pupil is greater or less, so likewise the apparent field is great or small. In this case the point e will therefore be still visible to the eye, if the small interval Bm does not exceed half the diameter of the eye, that the ray mn may find admission into it; but in this case, likewise, the eye must be brought as close as possible to the eye-glass: for as the ray mn removes from the axis FB , it would escape the pupil at a greater distance.

Now it is easy to determine the apparent field which such an instrument would discover on the eye-glass: you have only to take the interval Bm equal to the semidiameter of the pupil, and to draw through that point m , and the centre of the object-glass A , the straight line mae ; then this line will mark on the object the extremity e , which will be still visible through the instrument, and the angle Eae will give the semidiameter of the apparent field. Hence you will easily judge, that whenever the distance of the lenses AB exceeds some inches, the angle BAm must become extremely small, as the line or the distance Bm is but about the twentieth part of an inch. Now if it were intended to magnify very much, the distance of the lenses must become considerable, and the consequence would be, that the apparent field must become extremely small. The structure of the human eye, then, sets bounds to telescopes of this description, and obliges us to have recourse to others of a different construction whenever we want to produce very considerable effects.

16th February 1762.

LETTER XCIII.—ASTRONOMICAL TELESCOPES, AND
THEIR MAGNIFYING POWER.

I PROCEED to the second species of telescopes, called astronomical, and remark, that they consist of only two lenses, like those of the first species; with this difference, that in the construction of astronomical telescopes, instead of a concave eye-glass, we employ a convex one.

The object-glass PAP (PLATE VII. *Fig. 9.*) is, as in the other species, convex, whose focus being at F, we place, on the same axis, a smaller convex lens QQ, in such a manner that its focus shall likewise fall on the same point F. Then placing the eye at O, so that the distance BO shall be nearly equal to the focal distance of the eye-glass QQ, you will see objects distinctly, and magnified as many times as the focal distance of the object-glass AF shall exceed that of the eye-glass BF: but it is to be remarked, that every object will appear in an inverted position; so that if the instrument were to be pointed toward a house, the roof would appear undermost, and the ground-floor uppermost. As this circumstance would be awkward in viewing terrestrial objects, which we never see in an inverted situation, the use of this species of telescopes is confined to the heavenly bodies, it being a matter of indifference in what direction they appear; it is sufficient to the astronomer to know that what he sees uppermost is really undermost, and reciprocally. Nothing, however, forbids the application of such telescopes to terrestrial objects; the eye soon becomes accustomed to the inverted position, provided the object is seen distinctly, and very much magnified.

Having given this description, three things fall to be demonstrated: first, that by this arrangement of the lenses objects must appear distinctly; secondly,

that they must appear magnified as many times as the focal distance of the object-glass exceeds that of the eye-glass, and in an inverted position; and thirdly, that the eye must not be applied close to the eye-glass, as in the first species, but must be removed to nearly the focal distance of the ocular.

1. As to the first, it is demonstrated in the same manner as in the preceding case: the rays eP , eP , which are parallel before they enter into the object-glass, meet by refraction in the focus of this lens at F; the eye-glass must, of course, restore the parallelism of these rays, and distinct vision requires that the rays proceeding from every point should be nearly parallel to each other when they enter the eye. Now, the eye-glass, having its focus at F, is placed in such a manner as to render the rays FM, FM, by the refraction, parallel, and consequently the eye will receive the rays No, No, parallel to each other.

2. With respect to the second article, let us consider the object at Ee (PLATE VII. *Fig. 10.*), but so that the distance EA shall be almost infinite. The image of this object, represented by the object-glass, will therefore be Ff, situated at the focal distance of that lens AF, and determined by the straight line eAf , drawn through the centre of the lens. This image Ff, which is inverted, occupies the place of the object with respect to the eye-glass, and being in its focus, the second image will be again removed to an infinite distance by the refraction of this lens, and will fall, for example, at Gg, the distance AG being considered as infinite, like that of AE. Now, in order to determine the magnitude of this image, you have only to draw through the centre B of the lens, and the extremity f of the first image, the straight line Bfg. Now this second image Gg being the immediate object of vision to

the person who looks through the telescope, it is evident at once that this representation is inverted; and, as it is infinitely distant, will appear under an angle $G B g$. But the object itself $E e$ will appear to the naked eye under the angle $E A e$: now you are sensible, without being reminded, that it is indifferent to take the points A and B , in order to have the visual angles $E A e$ and $G B g$, on account of the infinite distance of the object. You now see here, as in the preceding case, that the triangles $F A f$ and $F B f$ may be considered as circular sectors, the line $F f$ measuring the arch of both; and the angles themselves being so very small, no sensible mistake can be committed in taking the chord for the arch. As, then, the radii of these two sectors are the lines $A E$ and $B F$, the arches being equal to each other, it follows, as was formerly demonstrated, that the angles $F A f$ (or, which is the same thing, $E A e$) and $F B f$ (or, which is the same thing, $G B g$) have the same proportion to each other that the radii $B F$ and $A F$ have. Therefore, the angle $G B g$, under which the object is seen through the telescope, as many times exceeds the angle $E A e$, under which the object is seen by the naked eye, as the line $A F$ exceeds the line $B F$; which was the second point to be demonstrated. I am under the necessity of deferring the demonstration of my third proposition till next post.

20th February 1762.

LETTER XCIV.—OF THE APPARENT FIELD, AND
THE PLACE OF THE EYE.

In fulfilling my engagement respecting the third particular proposed, namely, to determine the place of the eye behind the telescope, I remark that this subject is most intimately connected with the appa-

rent field, and that it is precisely the field which obliges us to keep the eye fixed at the proper distance; for if it were to be brought closer, or removed farther off, we should no longer discover so large a field.

The extent of the field being an article of such importance, indeed so essential, in all telescopes, it must be of equal importance to determine exactly the place of the eye from which the largest field is discoverable. If the eye were to be applied close to the eye-glass, we should have nearly the same field as we have with the pocket-glass, which becomes insufferably small whenever the magnifying power is considerable. It is therefore a vast advantage to astronomical telescopes, that by withdrawing the eye from the eye-glass, the apparent field increases to a certain extent; and it is precisely this which renders such telescopes susceptible of prodigious magnifying powers, whereas those of the first species are in this respect extremely limited. You know that with the astronomical telescope, the magnifying power has been carried beyond two hundred times, which gives them an inconceivable superiority over those of the first species, which can scarcely magnify ten times; and the trifling inconvenience of the inverted position is infinitely overbalanced by an advantage so very great.

I will endeavour to put this important article in the clearest light possible.

1. The object $E e$ (PLATE VII. Fig. 11.) being infinitely distant, let e be its extremity, still visible through the telescope, whose lenses are $P A P$ and $Q B Q$, fitted on the common axis $E A B O$;—it falls to be attentively considered what direction will be pursued by the single ray which passes from the extremity e of the object, through the centre A of the object-glass. You will recollect that the other rays,

which fall from the point e on the object-glass, only accompany and strengthen the ray in question eA , which is the principal with respect to vision.

2. Now this ray eA , passing through the centre of the lens PP , will undergo no refraction, but will pursue its direction in the straight line Afm , and passing through the extremity of the image Ff , will fall on the eye-glass at the point m ; and here it is to be observed, that if the size of the eye-glass had not extended so far as the point m , this ray would never have reached the eye, and the point e would have been invisible. That is to say, it would be necessary to take the extremity e nearer to the axis, in order that the ray Afm may meet the eye-glass.

3. Now this ray A will be refracted by the eye-glass in a way which it is very easy to discover. We have only to consider the second image Gg ; though infinitely distant, it is sufficient to know that the straight line Bf produced will pass through the extremity g of the second image Gg , which is the immediate object of vision. Having remarked this, the refracted ray must assume the direction nO , and this produced passes through g .

4. As, therefore, the two lines On and Bf meet at an infinite distance at g , they may be considered as parallel to each other; and hence we acquire an easier method to determine the position of the refracted ray nO : you have only to draw it parallel to the line Bf .

5. Hence it is clearly evident that the ray nO will somewhere meet the axis of the telescope at O , and as usually, when the magnifying power is great, the point F is much nearer to the lens Q than to the lens PP , the distance Bm will be somewhat greater than the image Ff ; and as the line nO is parallel to fB , the line BO will be nearly equal to Bf , that is, to the focal distance of the eye-glass.

6. If, then, the eye is placed at O , it will receive not only the rays which proceed from the middle of the object E , but those likewise which proceed from the extremity e , and consequently those also which proceed from every point of the object; the eye would even receive at once the rays BO and nO , even supposing the pupil infinitely contracted. In this case, therefore, the apparent field does not depend on the largeness of the aperture of the pupil, provided the eye be placed at O ; but the moment it recedes from this point, it must lose considerably in the apparent field.

7. If the point m were not in the extremity of the eye-glass, it would transmit rays still more remote from the axis, and the telescope would, of course, discover a larger field. In order, then, to determine the real apparent field which the telescope is capable of discovering, let there be drawn, from the centre A of the object-glass, to the extremity m of the eye-glass, the straight line Am , which, produced to the object, will mark at e the visible extremity; and consequently the angle $E Ae$, or, which is the same thing, the angle BAm , will give the semidiameter of the apparent field, which is consequently greater in proportion as the extent of the eye-glass is greater.

8. As, then, in the first species of telescopes, the apparent field depended entirely on the aperture of the pupil, and as in this case it depends entirely on the aperture of the eye-glass, there is an essential difference between these two species of instruments, greatly in favour of the latter. The figure which I have employed in demonstrating this last article respecting the place of the eye and the apparent field, may greatly assist us in the elucidation of the preceding articles.

If you will be so good as to reflect, that the object-glass transports the object Ee to Ff , and that the

eye-glass transports it from Ff to Gg , this image Gg being very distant from the immediate object of vision, ought to be seen distinctly, as a good eye requires a great distance in order to see thus. This was the first article.

As to the second, it is evident at first sight, that as instead of the real image Ee , we see through the telescope the image Gg , it must be inverted. Finally, this image is seen by the eye placed at O under the angle GOg , or BO_n , whereas the object itself Ee appears to the naked eye under the angle $E Ae$: the telescope, therefore, magnifies as many times as the angle BO_n is greater than the angle $E Ae$. Now, as the line no is parallel to Bf ; the angle BO_n is equal to the angle $F Bf$, and the angle $E Ae$ is equal to its opposite and vertical angle $F A f$; hence the magnifying power must be estimated from the proportion between the angles $F Bf$ and $F A$; accordingly, as the angle $F Bf$ contains the angle $F A f$ as often as the line AF , that is the focal distance of the object-glass, contains the line BF , that is the focal distance of the eye-glass, the magnifying power will be therefore expressed by the proportion of these two distances. This is proof sufficient that the elements of geometry may be successfully employed in researches of quite a different nature—a reflection not unpleasing to the mathematician.

23d February 1762.

LETTER XCV.—DETERMINATION OF THE MAGNIFYING POWER OF ASTRONOMICAL TELESCOPES, AND THE CONSTRUCTION OF A TELESCOPE WHICH SHALL MAGNIFY OBJECTS A GIVEN NUMBER OF TIMES.

YOU now have it clearly ascertained, not only how many times a proposed instrument will magnify, but what is the mode of constructing a telescope

which shall magnify as many times as may be wished. In the first case, you have only to measure the focal distance of both lenses, the object-glass as well as the eye-glass, in order to discover how much the one exceeds the other. This is performed by division, and the quotient indicates the magnifying power.

Having, then, a telescope, the focal distance of whose object-glass is two feet, and that of the eye-glass one inch, it is only necessary to inquire how often one inch is contained in two feet. Every one knows that a foot contains twelve inches; two feet accordingly contain twenty-four inches, which are to be divided by one. But whatever number we divide by one, the quotient is always equal to the dividend; if, then, it is asked, how often one inch is contained in twenty-four inches, the answer, without hesitation, is, twenty-four times; consequently, such a telescope magnifies twenty-four times, that is, represents distant objects in the same manner as if they were twenty-four times greater than they really are; in other words, you would see them through such a telescope under an angle *twenty-four* times greater than by the naked eye.

Let us suppose another astronomical telescope, the focal distance of whose object-glass is thirty-two feet, and that of the eye-glass three inches. You see at once that these two lenses must be placed at the distance of thirty-two feet and three inches from each other; for, in all astronomical telescopes, the distance of the lenses must be equal to the sum of the two focal distances, as has been already demonstrated.

To find, then, how many times a telescope of the above description magnifies, we must divide thirty-two feet by three inches; and, in order to this, reduce these thirty-two feet into inches, by multiplying them by twelve.

32 this produces 384 inches; and these again
 12 divided by three, the focal distance, in inches;
 3³⁸⁴ of the eye-glass, gives a quotient of 128,
 128 which indicates that the proposed telescope
 magnifies 128 times, which must be allowed to be
 very considerable.

Reciprocally, therefore, in order to construct a telescope which shall magnify a given number of times, say 100, we must employ two convex lenses; the focal distance of the one of which shall be 100 times greater than that of the other; in this case the one will give the object-glass, and the other the eye-glass. These must afterwards be fitted on the same axis, so that their distance shall be equal to the sum of the two focal distances; that is, they must be fixed in a tube of this length, and then the eye being placed behind the eye-glass, at its focal distance, will see objects magnified 100 times.

This arrangement may be varied without end, by assuming an eye-glass at pleasure, and adapting to it an object-glass, whose focal distance shall be 100 times greater. Thus, taking an eye-glass of one inch focus, the object-glass must be of 100 inches focus, and the distance of the lenses 101 inches. Or, taking an eye-glass of 2 inches focus, the object-glass must have its focus at the distance of 200 inches, and the distance of the lenses will be 202 inches. If you were to take an eye-glass of 3 inches focus, the focal distance of the object-glass must be 300 inches, and the distance of the lenses from each other 303 inches. And if you were to take an eye-glass of 4 inches focus, the object-glass must have a focal distance of 400 inches, and the distance of the two lenses 404 inches, and so on, the instrument always increasing in length. If, on the contrary, you were to assume an eye-glass of only half an inch focus, the object-glass must have a focal dis-

tance of 100 half inches, that is, of fifty inches, and the distance between the lenses would only be 50 inches and a half, which is little more than four feet. And if an eye-glass of a quarter of an inch focus were to be employed, the object-glass would require a focal distance of only 100 quarters of an inch, or 25 inches, and the distance between the two lenses 25 inches and a quarter, that is little more than two feet.

Here, then, are several methods of producing the same effect, that of magnifying 100 times; and if every thing else were equal, we should not hesitate about giving the preference to the last, as being the shortest: for here the telescope, being reduced to little more than two feet, would be more manageable than one much longer.

No one, then, would hesitate about preferring the shortest telescopes, provided all other circumstances were the same, and all the different species represented objects in the same degree of perfection. But though they all possess the same magnifying power, the representation is by no means equally clear and distinct. That of two feet in length certainly magnifies 100 times, as well as the others; but on looking through such a telescope, objects will appear not only dark, but blunt and confused, which is undoubtedly a very great defect. The last telescope but one, whose object-glass is 50 inches focus, is less subject to these defects: but the dimness and confusion are still insupportable; and these defects diminish in proportion as we employ greater object-glasses, and are reduced to almost nothing on employing an object-glass of 300 inches, with an eye-glass of 3 inches focus. On increasing these measurements, the representation becomes still clearer and more distinct; so that in this respect long telescopes are preferable to short, though otherwise

less commodious. This circumstance imposes on me a new task, that of farther explaining two very essential articles in the theory of telescopes: the one respects the clearness, or degree of light in which objects are seen; and the other the distinctness and accuracy of expression with which they are represented. Without these two qualities, all magnifying power, however great, procures no advantage for the contemplation of objects.

27th February 1762.

LETTER XCVI.—DEGREE OF CLEARNESS.

IN order to form a judgment of the degree of clearness in which objects are represented by the telescope, I shall recur to the same principles which I endeavoured to elucidate in treating the same subject with reference to the microscope.

And, first, it must be considered, that in this research it is not proposed to determine the degree of light resident in objects themselves, and which may be very different, not only in different bodies, as being in their nature more or less luminous, but in the same body, according as circumstances vary. The same bodies, when illuminated by the sun, have undoubtedly more light than when the sky is overcast, and in the night their light is wholly extinguished; but different bodies illuminated may differ greatly in point of brightness, according as their colours are more or less lively. We are not inquiring, then, into that light or brightness which resides in objects themselves; but, be it strong or faint, we say that a telescope represents the object in perfect clearness, when it is seen through the instrument as clearly as by the naked eye; so that if the object be dim, we are not to expect that the telescope should represent it as clear.

Accordingly, in respect of clearness, a telescope is perfect, when it represents the object as clearly as it appears to the naked eye. This takes place, as in the microscope, when the whole opening of the pupil is filled with the rays which proceed from every point of the object, after being transmitted through the telescope. If a telescope furnishes rays sufficient to fill the whole opening of the pupil, no greater degree of clearness need be desired; and supposing it could supply rays in greater profusion, this would be entirely useless, as the same quantity precisely, and no more, could find admission into the eye.

Here, then, attention must be paid chiefly to the aperture of the pupil, which, being variable, prevents our laying down a fixed rule, unless we regulate ourselves according to a certain given aperture, which is sufficient, when the pupil, in a state of the greatest contraction, is filled with rays; and for this purpose the diameter of the pupil is usually supposed to be one line, twelve of which make an inch; we sometimes satisfy ourselves with even the half of this, although to the diameter of the pupil only half a line, and in some cases still less.

If you will please to consider, that the light of the sun exceeds that of the moon 300,000 times, though even that of the moon is by no means inconsiderable, you will be sensible that a small diminution in point of clearness can be of no great consequence in the contemplation of objects. Having premised this, all that remains is to examine the rays which the telescope transmits into the eye, and to compare them with the pupil; and it will be sufficient to consider the rays which proceed from a single point of the object, that, for example, which is in the axis of the telescope.

1. The object being infinitely distant, the rays which fall from it on the surface of the object-glass

P A P (PLATE VII. *Fig. 12.*), are parallel to each other: all the rays, then, which come from the centre of the object, will be contained within the lines L P, L P, parallel to the axis E A. All these rays taken together are denominated the *pencil* of rays which fall on the object-glass, and the breadth of this pencil is equal to the extent or aperture of the object-glass, the diameter of which is P A P.

2. This pencil of rays is changed, by the refraction of the object-glass, into a conical or pointed figure P F P, and having crossed at the focus F, it forms a new cone *m F m*, terminated by the eye-glass; hence it is evident that the base of this cone *m m* is as many times smaller than the breadth of the pencil P P, as the distance F B is shorter than the distance A F.

3. Now these rays F *m*, F *m*, on passing through the eye-glass Q B Q, become again parallel to each other, and form the pencil of rays *n o*, *n o*, which enter into the eye, and there depict the image of the point of the object whence they originally proceeded.

4. The question, then, resolves itself into the breadth of this pencil of rays *n o*, *n o*, which enter into the eye; for if this breadth *n n* or *o o* is equal to, or greater than, the opening of the pupil, it will be filled with them, and the eye will enjoy all possible clearness; that is, the object will seem as clear as if you were to look at it with the unassisted eye.

5. But if this pencil *n n o o*, were of much less breadth than the diameter of the pupil, it is evident that the representation must become so much more obscure; which would be a great defect in the telescope. In order to remedy it, the pencil must therefore be at least half a line in breadth; and it would be still better to have it a whole line in breadth, this being the usual aperture of the pupil.

6. It is evident that the breadth of this second pencil has a certain relation to that of the first, which it is very easy to determine. You have only to settle how many times the distance *n n* or *m m* is less than the distance P P, which is the aperture of the object-glass. But the distance P P is in the same proportion to the distance *m m*, as the distance A F to the distance B F, on which the magnifying power depends; accordingly, the magnifying power itself discovers how many times the pencil L P, L P, is broader than the pencil *n o*, *n o*, which enters into the eye.

7. Since, then, the breadth *n n* or *o o* must be one line, at least half a line, the aperture of the object-glass P P must at least contain as many half lines as the magnifying power indicates; thus, when the telescope is to magnify 100 times, the aperture of its object-glass must have a diameter of 100 half lines, or 50 lines, which make 4 inches and 2 lines.

8. You see, then, that in order to avoid obscurity, the aperture of the object-glass must be greater, in proportion as the magnifying power is greater. And, consequently, if the object-glass employed is not susceptible of such an aperture, the telescope will be defective in respect of clearness of representation.

Hence it is abundantly evident, that in order to magnify very greatly, it is impossible to employ small object-glasses, whose focal distance is too short, as a lens formed by the arches of small circles cannot have a great aperture.

1st March 1762.

LETTER XCVII.—APERTURE OF OBJECT-GLASSES.

You have now seen that the magnifying power determines the size or extent of the object-glass, in order that objects may appear with a sufficient

degree of clearness. This determination respects only the size or aperture of the object-glass; however, the focal distance is affected by it likewise, for the larger the lens is, the greater must be its focal distance.

The reason of this is evident, as in order to form a lens, whose focal distance is, for example, two inches, its two surfaces must be arches of a circle whose radius is likewise about two inches. I have therefore represented (PLATE VII. Fig. 18.) two lenses P and Q, the arches of which are described with a radius of two inches. The lens P, being the thicker, is much greater than the lens Q; but I shall demonstrate afterwards that thick lenses are subject to other inconveniences, and these so great as to oblige us to lay them altogether aside. The lens Q, then, will be found more adapted for use, being composed of smaller arches of the same circle; and as its focal distance is two inches, its extent or aperture $m n$ may scarcely exceed one inch. Hence this may be laid down as a general rule, that the focal distance of a lens must always be twice greater than the diameter of its aperture $m n$; that is, the aperture of a lens must of necessity be smaller than half the focal distance.

Having remarked, then, that in order to magnify 100 times, the aperture of the object-glass must exceed 4 inches, it follows, that the focal distance must exceed 8 inches; I shall presently demonstrate that the double is not sufficient, and that the focal distance of this lens must be increased beyond 300 inches. The distinctness of the expression of the image requires this great increase, as shall afterwards be shown: I satisfy myself with remarking, at present, that with regard to the geometrical figure of the lens, the aperture cannot be greater than half its focal distance.

* Here, therefore, I shall go somewhat more into the detail respecting the aperture of the object-glass, which every magnifying power requires; and I remark, first, that though a sufficient degree of clearness requires an aperture of four inches, when the telescope is to magnify 100 times, we satisfy ourselves, in astronomical instruments, with one of three inches, the diminution of clearness being scarcely perceptible. Hence artists have laid it down as a rule, that, in order to magnify 100 times, the aperture of the object-glass must be three inches; and for other magnifying powers, in that proportion. Thus, in order to magnify 50 times, it is sufficient that the aperture of the object-glass be an inch and a half; to magnify 25 times, three quarters of an inch suffice, and so of other powers.

Hence we see that for small magnifying powers a very small aperture of the object-glass is sufficient, and that, consequently, a moderate focal distance may answer. But if you wished to magnify 200 times, the aperture of the object-glass must be six inches, or half a foot, which requires a very large lens, whose focal distance must exceed even 100 feet, in order to obtain a distinct and exact expression. For this reason, great magnifying powers require very long telescopes, at least according to the usual arrangement of lenses which I have explained. But, for some time past, artists have been successfully employing themselves in diminishing this excessive length. The aperture of the object-glass, however, must follow the rule laid down, as clearness necessarily depends on it.

Were you desirous, therefore, of constructing a telescope which should magnify 400 times, the aperture of the object-glass must be twelve inches, or a foot, let the focal distance be rendered as small as you will: and if you wished to magnify 4000 times,

the aperture of the object-glass must be ten feet, a very great size indeed, and too much so for any artist to execute; and this is the principal reason why we can never hope to carry the magnifying power so far, unless some great prince would be at the expense of providing and executing lenses of such magnitude; and, after all, perhaps they would not succeed.

A telescope, however, which should magnify 4000 times, would discover many wonderful things in the heavens. The moon would appear 4000 times larger than to the naked eye; in other words, we should see her as if she were 4000 times nearer to us than she is. Let us inquire, then, to what a degree we might be able to distinguish the different bodies which she may contain. The distance of the moon from the earth is calculated to be 240,000 English miles, the 4000th part of which is 60 miles: such a telescope would accordingly show us the moon as if she were only 60 miles distant; and, consequently, we should be enabled to discover in her the same things which we distinguish in objects removed to the same distance. Now, from the top of a mountain, we can easily discern other mountains more than 60 miles distant. There can be no doubt, then, that with such an instrument, we should discover on the surface of the moon many things to fill us with surprise. But in order to determine whether the moon is inhabited by creatures similar to those of the earth, a distance of 60 miles is still too great; we must have, in order to this effect, a telescope which should magnify ten times more, that is 40,000 times, and this would require an object-glass of 100 feet aperture, an enterprise which human art will never be able to execute. But, with such an instrument, we should see the moon as if she were no farther distant than from Berlin to Spandau, and

good eyes might easily discern men at this distance, if any there were, but too indistinctly, it must be allowed, to be completely assured of the fact.

As we must rest satisfied with wishing on this subject, mine should be to have at once a telescope which should magnify 100,000 times;* the moon would then appear as if she were only half a mile distant.

The aperture of the object-glass of this telescope must be 250 feet, and we should see, at least, the larger animals which may be in the moon.

6th March 1762.

LETTER XCVIII.—ON DISTINCTNESS IN THE EXPRESSION: ON THE SPACE OF DIFFUSION OCCASIONED BY THE APERTURE OF OBJECT-GLASSES, AND CONSIDERED AS THE FIRST SOURCE OF WANT OF DISTINCTNESS IN THE REPRESENTATION.

DISTINCTNESS of expression is a quality of so much importance in the construction of telescopes, that it seems to take precedence of all the others which I have been endeavouring to explain; for it must be allowed, that a telescope which does not represent distinctly the images of objects, must be very defective. I must, therefore, unfold the reasons of this want of distinctness, that we may apply more successfully to the means of remedying it.

They appear so much the more abstruse, that the principles hitherto laid down do not discover the source: in fact, this defect is thus to be accounted for—one of the principles on which I have hitherto proceeded is not strictly true, though not far from the truth.

* Dr. Herschel has been able to apply a magnifying power of 6500 times to the fixed stars.—ED.

You will recollect that it has been laid down as a principle, that a convex lens collects into one point of the image all the rays which come from one point of the object. Were this strictly true, images represented by lenses would be as distinctly expressed as the object itself, and we should be under no apprehension of defect in regard to this.

Here, then, lies the defectiveness of this principle; lenses have the property now ascribed to them only around their centre; the rays which pass through the extremities of a lens collect in a different point from those which pass toward the centre, though all proceed from the same point of the object; hence are produced two different images, which occasion indistinctness.

In order to set this in the clearest light, let us consider the convex lens PP, (PLATE VII. Fig. 14.) on the axis of which is placed the object E e, of which the point E, situated upon the axis, emits the rays EN, EM, EA, EM, EN, to the surface of the lens. To the direction of these rays, as changed by refraction, we must now pay attention.

1. The ray EA, which passes through the centre A of the lens, undergoes no refraction, but proceeds forward in the same direction, on the straight line ABF.

2. The rays EM and EM, which are nearest to the first, undergo a small refraction, by which they will meet with the axis somewhere at F, which is the place of the image F f, as has been explained in some of my preceding letters on this subject.

3. The rays EN and EN, which are more remote from the axis EA, and which pass toward the extremities NN of the lens, undergo a refraction somewhat different, which collects them, not at the point F, but at another point G, nearer the lens; and these rays represent another image G g, different from the first F f.

4. Let us now carefully attend to this particular circumstance, not hitherto remarked; it is this, that the rays passing through the lens, toward its extremities, represent another image G g, than what is represented by those passing near the centre MAM.

5. If the rays EN, EN, were to retire still farther from the centre A, and to pass through the points P; P of the lens, their point of re-union would be still nearer to the lens, and would form a new image, nearer than even G g.

6. Hence you will easily perceive, that the first image F f, which is named the principal image, is formed only by the rays which are almost infinitely near the centre; and that according as the rays retire from it, toward the extremities of the lens, a particular image is formed nearer the lens, till those passing close to the extremities form the last, G g.

7. All the rays, therefore, which pass through the lens PP represent an infinity of images disposed between F f and G g; and at every distance from the axis the refraction of the lens produces a particular image, so that the whole space between F and G is filled with a series of images.

8. This series of images is accordingly denominated the diffusion of the image; and when all these rays afterwards enter into any eye, it is natural that the vision should be so much disturbed, as the space FG, through which the image is diffused, is more considerable. If this space FG could be reduced to nothing, no confusion or indistinctness need be apprehended.

9. The greater portions of their respective circles that the arches PAP and PBP are, the greater likewise is FG the space of diffusion. You see a good reason, then, for rejecting all lenses of too great thickness, or in which the arches which form the surfaces of the lens are considerable segments of their

circles (as in PLATE VII. *Fig. 15.*); of which the arches PAP and PBP are the fourth part of the whole circumference, so that each contains 90 degrees; this would, consequently, produce an insufferable confusion.

10. The arches, then, which form the surfaces of a lens, must contain much less than 90 degrees: if they contained so much as 60, the diffusion of the image would be even then insupportable. Authors who have treated the subject, admit of 30 degrees at most: and some fix the boundary at 20 degrees. A lens of this last description is represented by *Fig. 16.* of Plate VII. in which the arches PAP and PBP contain only 20 degrees, each being but the eighteenth part of the whole circumference of its respective circle.

11. But if this lens were to supply the place of the object-glass in a telescope, the arches PAP and PBP must contain still many degrees less. For though the diffusion of the image be perceptible of itself, the magnifying power multiplies it as many times as it does the object. Therefore, the greater the magnifying power proposed, the fewer must be the number of degrees which the surfaces of the lens contain.

12. When the telescope is intended to magnify 100 times, you will recollect that the aperture of the object-glass must be 3 inches, and its focal distance 360 inches, which is equal to the radii within which the two arches PAP and PBP are described; hence it follows that each of these two arches contains but half a degree; and it is distinctness of expression which requires an arch so small. If it were intended to magnify 200 times, half a degree would be still too much, and the measure of the arch, in that case, ought not to exceed the third part of a degree. This arch, however, must receive an extent of 6 inches; the radius of the circle must therefore

be so much greater, and consequently also the focal distance. This is the true reason why great magnifying powers require telescopes of such considerable length.

9th March 1762.

LETTER XCIX.—DIMINUTION OF THE APERTURE OF LENSES, AND OTHER MEANS OF LESSENING THE SPACE OF DIFFUSION, TILL IT IS REDUCED TO NOTHING.

WHEN the space of an object-glass is too great to admit of distinctness of expression, it may be very easily remedied: you have only to cover the lens with a circle of pasteboard, leaving an opening in the centre, so that the lens may transmit no other rays but those which fall upon it through the opening, and that those which before passed through the extremities of the lens may be excluded; for as no rays are transmitted but through the middle of the lens, the smaller the opening is, the smaller likewise will be the space of diffusion. Accordingly, by a gradual diminution of the opening, the space of diffusion may be reduced at pleasure.

Here the case is the same as if the lens were no larger than the opening in the pasteboard, thus the covered part becomes useless, and the opening determines the size of the lens; this then is the remedy employed to give object-glasses any given extent.

PP is the object-glass (PLATE VII. *Fig. 17.*), before which is placed the pasteboard NN, having the opening MM, which is now the extent of the lens. This opening MM is here nearly the half of what it would be were the pasteboard removed; the space of diffusion is therefore much smaller. It is remarked, that the space of diffusion, in this case, is only the fourth part of what it was before. An