Chapter One

Concerning Motion In General.
[p. 1]

DEFINITION 1.
1.

Motion is the translation of a body from the place it occupies to another place. Truly rest is a body remaining at the same place.

Corollary 1.
2. Therefore the ideas of the body remaining at rest and of moving to other places cannot be entertained together, except that they occupy a place. Whereby the place or position the body occupies shall be a property of the body, that can be said to be possible only for that body, and that the body is either moving or at rest.

Corollary 2.
3. And this idea of moving or of being at rest is a property of a body that relates to all bodies. For no body is able to exist that is neither moving nor at rest.

DEFINITION 2.
4. *The place* [occupied by a body] *is a part of the immense or boundless space which constitutes the whole world. In this sense the accepted place is accustomed to be called the absolute place* [p. 2]*, in order that it may be distinguished from a relative place, of which mention will soon be made.*

Corollary 1.
5. Therefore, when a body occupies successively one part and then another, of this immense space, then it is said to be moving ; but if it continues to be present at the same place always, then it is said to be at rest.

Corollary 2.
6. Moreover, it is customery to consider fixed boundaries of this space, to which bodies can be referred. And the relation with those is what is called the position [of the body]. Therefore bodies that maintain the same positions with respect to these boundaries are said to be at rest. On the other hand, those that change their positions are said to be moving.

Scholium 1.

7. If you are in agreement with these definitions, then the motion is usually called absolute, and the state of rest absolute. And these are taken to be the true and genuine definitions of these quantities, since they are compliant with the laws of motion, which will be explained in the following pages. Moreover, because of the immense nature of space and of its unbounded nature, mention of which has been made in the definitions given, we are unable to form a fixed idea of this. Thus, in place of this immense space and of the boundaries of this, we are accustomed to defining a finite space and the limits within which bodies can move, from which we can indicate the states of motion and of rest of bodies. Thus, we are accustomed to say that a body that keeps the same situation with respect to its boundaries is at rest, and truly that which changes with respect to the same, to be in a state of motion.[p. 3]

Scholium 2.

8. Concerning what has been said here regarding the boundaries of this immense and infinite space, such boundaries should be considered from a purely mathematical point of view. These boundaries can be summoned to establish our correct understanding, since there is little more that can be used, for metaphysical speculations seem to be in contradiction. Insomuch as, if we do not assert to give fixed and motionless boundaries to infinite space; but instead assert that space may have or may not have attending boundaries, then we can still postulate that such absolute motion and absolute rest may be represented in such a space, such as we are about to contemplate; and that such a finite space may indicate the state of motion or of rest of a body. For most conveniently, matters can be organized as follows: we can imagine the mind abstracting for us from the universe a void of infinite extend, and we can consider bodies to be arranged in that space so that, if they retain the same position, then they are in a state of absolute rest ; but if on the other hand they are passing from one part of space to another then this indicates that they are moving.

DEFINITION 3.

9. *Relative motion is the change in the position of a body that it is agreeable to assume within a space. And relative rest means that the body stays in the same place with respect to the same space. Thus taking the earth for this space, we say that those bodies are at rest that maintain their own unchanging places on the earth; and truly we are accustomed to say only those to be moving that proceed* [p. 4] *from one place to another with respect to the earth. In a similar manner for a ship's movement, those bodies that remain relatively at rest stay in the same place on the ship, while these are relatively moving that change their position on the ship.*
[The view of the universe from the 17th century onwards was thus one of an immense almost empty void in which there were fixed bodies or stars that acted as reference points. This was called absolute space. A body at rest in absolute space was absolutely at rest, and any body moving in absolute space was in absolute motion. There was thus a

preferred frame of reference. Other moving frames of reference associated with bodies such as the earth could be used for convenience, and the speeds of bodies in such frames related to their absolute counterparts.]

Corollary 1.

10. Relative motion and relative rest agree with absolute motion and rest, when the space and the body, with respect to which the motion and the state of rest are indicated, actually are at rest with respect to the immense and infinite space. For if the earth is actually considered at rest [in the absolute sense], these things which are at rest and these things which are moving, are also at rest or are moving in the absolute sense.

Corollary 2.

11. But relative motion and relative rest disagree [with their absolute counterparts] if that space is moving. For if the earth is not at rest with respect to the infinite space, neither those bodies which are at rest with respect to the earth are in a state of absolute rest, and also the absolute motions of bodies are different from their relative motion. For now it is indeed possible for a body that is in motion relative to the earth to be in a state of absolute rest.

Scholium.

12. It is evident that there are innumerable different states of relative motion or rest of bodies: indeed as one or another space is assumed, the relative motion and the state of rest are decided in that space, and all will produce relative motion from the state of rest of another. Thus the fixed stars are moving with respect to the earth, while any one of these is at rest with respect to the others. And as the fixed stars move with respect to the earth, so also do the planets [p. 5]. Moreover in the following derivations I wish both absolute motion and rest to be understood [to be the reference frame], unless I advise that relative motion is to be considered, and that will be brought to your attention.

PROPOSITION 1.
Theorem.

13. *Every body, that is carried from one place to another place by relative or absolute motion, passes through all the intermediate positions, and it is not able to suddenly arrive at the final place.*

Demonstration.

For absolute motion: if the body were suddenly able to arrive at the final place from the first place, it would be necessary for the body to be annihilated at the first place and suddenly produced in the final place, and which cannot be done according to the laws of nature, except by agreeing to a miracle. Therefore it proceeds from place to place until finally it arrives at the final place. *For relative motion :* if the body truly remains at rest on being substituted from the infinite space into the relative space, then the above reasoning prevails (10). But if it should move in the relative space, then it must pass through the individual intermediate places, and therefore the relative motion will be successive, and it will again be made through the intermediate positions. Q. E. D.

Corollary 1.

14. From these it follows that a motion cannot be immediate [in time], but to need time for the body to arrive at some place from some previous place [p. 6.]. Since indeed it has to pass through the individual intermediate places, this cannot be consistent with instantaneous motion.

Corollary 2.

15. Therefore it is necessary for the path to be assigned, by which the body moves, and with that known, there will be no point on that path from the first to the last that the body does not pass through in its progression. Moreover this is usually called the path that the body runs through or traverses.

Scholium.

16. This view is readily adapted too for bodies rotating about an axis. Since indeed the position of the body itself does not change: yet the motion belonging to the parts of this motion are to be understood, if the individual parts of the motion are considered to arise from just as many different parts. Indeed the individual parts with respect to infinite space are taken to change their position, and no parts are at rest except those which are placed on the axis itself. And in a like manner it is necessary for bodies to be considered in every respect, so that not only a whole body, but also the situation of the individual parts of the body, needs to be examined for change.

DEFINITION 4.

17. *A body is said to be moving equably or uniformly, that in equal intervals of time traverses equal distances. A motion is truly not equable that in equal times travels through unequal distances, or in traveling through equal distances are resolved by unequal intervals of time.* [p. 7; Euler uses the word 'equable' throughout his descriptions, where the modern equivalent word is 'uniform', which we use here for convenience, and which Euler also uses as a synonym.]

Corollary 1.

18. Therefore a body with a uniform motion extended by twice the time completes twice the distance, three times as long results in three times as far, and in general the distances traveled through are in proportion to the times, and in turn the times with the distances. As a ship at sea begins with an uniform motion, if in one hour it travels through two miles, then in two hours the distance completed is four miles, in three hours six miles, and in n hours, $2n$ miles.

Corollary 2.

19. On account of which, if a uniform motion is given, then from that there is obtained an accurate measurement of the time, [in circumstances where that] cannot be known except from the motion. For the ratio of the distances measured that the body traverses with a uniform motion, is noted to be the same as of the times in the same intervals.

Scholium.

20. Neither truly do we have from elsewhere the division of time into years, days, and hours other than from a motion that we consider as being regular. For with the earth taken as being at rest, the ancients believed that the sun was carried round in a regular motion, and the time it took to revolve around the earth they called a day. Again, they had taken the motion of the fixed stars around the earth to be regular too, and the time it took for the sunrise to return to its original position with respect to the fixed stars they called a year. Then these times were divided into equal parts, and in this way the hours, minutes, and seconds were arrived at. [p.8]. But it is readily apparent, if the uniform motion there, as they believed, was not really uniform, then there would be an error in the measurement of time too. And recently astronomers have detected an irregularity in these motions, and they have found that not all days are of the same length, on account of which they are accustomed to give a correction from other more regular motions; that they call the equalisation of times, from which the unequal lengths of the days are known.

DEFINITION 5.

21. *Every body that is moving is said to have a speed or velocity, and this is measured by the distance that body traverses in equal intervals of time, with a uniform motion. Clearly when body B travels twice the distance at a uniform speed that body A travels through in the same time, also moving uniformly, then body B is said to have a speed twice as great as body A.*

Corollary 1.

22. Therefore since a body in uniform motion will traverse equal distances in equal intervals of time (17), the body that has moved uniformly will have the same speed or velocity always. Truly in non-uniform motion, the body adopts one speed after another in succession.

Corollary 2.

23. Moreover the [variable] speed that a body has at some point in the distance traversed with a non-uniform motion, [p.9] is to be measured by the distance that the body would traverse in a given time with that uniform speed.

Corollary 3.

23 [*a*; (There are two sections 23)]**.** Again the speed of a body with absolute uniform motion can be measured from the distance that the body traverses, for example, in a single second. And the speed of the same perfectly well-known body is agreed upon, which prevails to be defined as the distance which that body travels through in a time of one second.

Scholium.

24. This ratio is also especially useful in the measurement of speed. For we see sailors finding the speed of the ship by measuring the distance that the ship travels in a given time. Commonly they take a quarter of an hour interval, and they find how many miles the ship has traveled in this time. From which is understood at the same time, how many feet the ship has traversed in the same time, if indeed the ship should progress with a uniform motion.

PROPOSITION 2.
Theorem.

25. ***For two bodies progressing with uniform motion, the speeds vary directly with the distances traveled, and inversely with the time with which these distances are traversed***. [p. 10]

Demonstration.

Let the two bodies be A and *a*, and the speeds of these are C and *c* ; that body A travels through a distance S in time T, truly the body *a* a distance *s* in time *t*. Now since they are in uniform motion, the distances are in proportion to the times (18), the distance that body *a* completes in time T can be determined from the proportion $t : \mathrm{T} = s : \frac{s\mathrm{T}}{t}$: therefore in a time T, the body *a* will move through a distance $\frac{s\mathrm{T}}{t}$. But body A in the same time T moves through a distance S. The speeds of the bodies ought to be measured from the distances the bodies travel in the same time (18). On account of which C : c = S : $\frac{s\mathrm{T}}{t}$, or C : c = $\frac{\mathrm{S}}{\mathrm{T}} : \frac{s}{t}$. From which it follows that the speeds are directly as the distances and inversely as the times, with which these are traversed. Q. E. D.

Corollary 1.

26. The equation $\frac{\mathrm{CT}}{\mathrm{S}} = \frac{ct}{s}$ is produced from the final ratio. In any uniform motion therefore the product of the speed and the time, if divided by the distance traveled in that time, always gives the same quotient.

Corollary 2.

27. Also there is the ratio $\mathrm{T} : t = \frac{\mathrm{S}}{\mathrm{C}} : \frac{\mathrm{s}}{\mathrm{c}}$. From which it follows that the times are in the ratio of the direct proportions of the distances and in the inverse ratio of the speeds, or to be as the distance divided by the speed.

Corollary 3.

28. Then the proportion found can also be changed into this S : *s* = CT : c*t*. From which the distances traversed with the uniform motion are gathered together to be in a ratio composed from the ratio of the speeds and from the ratio of the times. [p. 11]

Corollary 4.

29. Therefore with the speed of the body given, moving uniformly, together with some distance described, the time can be noted in which this distance has been traversed; clearly by dividing the distance by the speed. Since indeed we have shown that this amount is always in proportion to the time, we can take the same as a measure of time.

Corollary 5.

30. Similarly the speed can be expressed by the distance traveled through divided by the time, and the distance also by the product of the time and the speed.

Scholium I.

31. For if the speed is such that, in order that the moving body completes three feet in one second, and therefore we can call the speed by the number 3; we will try to find the time in which for example 60 feet in the same motion are completed. For 60 divided by 3 and the quotient is 20 will show that it takes 20 seconds for the motion to be traversed. And if the distance is sought for the time of 12 seconds to be traversed, the product will be 36 feet. And also for a body traveling 4.8 feet in 6 seconds, the speed that arises is 8, which shows that this body travels 8 in one second.

Scholium 2.

32. And hence the times, the distances, and the speeds are to be measured in the following ratios [i. e. units] that we will always adhere to.[p. 12] For we will always express the time in seconds, and the distance in feet, and these will be Rhenish feet. The speed truly, as it is now made, we will specify by the number of feet traversed per second. Below indeed it is more convenient for the speed to be determined from the units that it will encounter, which henceforth we will be using, but these units arises from those, and the former can easily be recalled.

PROPOSITION 3.
Theorem.

33. *In motion with any non-uniformity, the smallest elements of the distance are considered to be traversed by uniform motions.*

Demonstration.

As indeed in geometry the elements of curved lines are considered to be the elements of small straight lines, thus also in a similar way in mechanics non-uniform motion is resolved into an infinite number of uniform motions. For either the elements are actually traversed in a uniform motion, or with the change of the speed by an element of this kind is so small, that the increment or decrement can be ignored without error. In either case the truth of the proposition is therefore apparent. Q. E. D.

Corollary 1.

34. Therefore all the change of the speed of the non-uniform motion is considered to occur upon entering the individual elements, since the whole elements are placed to be traversed with a uniform motion. [p. 13]

Corollary 2.

35. Whereby after the manner of the analysis of the very small to be observed, if the speed in the first element were c, the speed in the following element will be $c + dc$, in the third $c + 2dc + ddc$, and thus henceforth. [Thus, each new speed is the old speed plus the increment of the old speed.]

Scholium.

36. This corollary depends on the basis of the demonstration of the given force, that the change of the speed possible, while an infinitely small element is traversed, ought to be infinitely small and that this force should vanish at the start of the interval, with the body having attained its speed ; except for this to be the case, a finite motion would have to be generated immediately, which would be absurd. From which it is seen that the proposition is admitted not to be possible, nevertheless if the motion and the speed itself is infinitely small, in which case the momentary increase or decrease can have a finite ratio to that initial speed. But we will say more about that below, when the motion to be produced will examined.

PROPOSITION 4.

PROBLEM.

37. ***A body moves with some kind of motion along the line AM*** (Fig. 1), ***with the speed of the body given at some point; it is necessary to determine the time in which the arc AM is completed.***

SOLUTION.

AM shall be the distance, either of the straight line or the curve equal to s, and the speed, that the body has at M shall be c, which will be some function of s. The element Mm is taken from M, that has been considered to be traversed with the uniform speed c [p.14]. With the element Mm to be called ds, the time in which the element is traversed is equal to $\frac{ds}{c}$ (29). Therefore by integration, the time is obtained in which the whole arc is completed, to equal $\int \frac{ds}{c}$. To the integral a constant of such a kind should be added, which restores this time to zero, when $s = 0$, following the known rules of integration. Q. E. I.

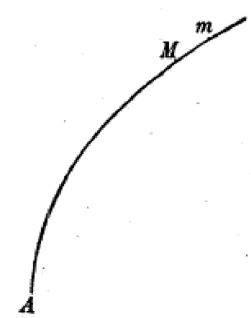

Fig. 1.

EXAMPLE 1.

38. Let the speed at M now be described in terms of some power of the distance AM, clearly $c = s^n$; then we have $\int \frac{ds}{c} = \frac{s^{1-n}}{1-n}$. To which there is no need to add on a constant, if $n < 1$ or n has a negative value : for the time in which the arc AM is traversed is itself given by $\frac{s^{1-n}}{1-n}$ [or, the constant of integration is zero]. But if the number $1 - n$ is negative, then there is given $\int \frac{ds}{c} = \frac{-1}{(n-1)s^{n-1}}$.
To which the infinite quantity $\frac{1}{(n-1)0^{n-1}}$ would have to be added for the time to go the distance AM. Therefore with the time in this case necessary to be infinite, for which the body remains at A , nor in any finite time to progress from that place. Truly this shall be the case, when n is a positive number greater than one. If truly $n = 1$, then the time cannot be shown by an algebraic expression ; for it comes about that $\int \frac{ds}{c} = ls$, [i. e. log s] to that also it is necessary to add an infinite amount, for the time spent in passing along AM.

Corollary 1.

[p. 15]

39. Therefore in the general case, bodies are unable to stop, except when the speeds of the body, even from the beginning, are traversed as powers of the distance of an exponent that is less than one.

Corollary 2.

40. The body is to progress along the straight line *AM* (Fig. 2),and the speed of the body at any point shall correspond to the line *MN* connected to the curve *AN,* which meets the straight line *AM* at *A*, thus, in order that the speed of the body at the beginning shall be zero. It is seen from the preceding, where the time to travel along *AM* is made finite, that it is necessary for the tangent *AB* at *A* to be perpendicular to *AM*. For with *M* coinciding with *A*, *MN* should be made equal to

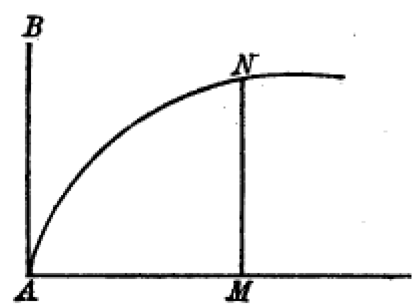

Fig. 2.

AM^n and with the number n smaller than one, a fraction as it were, from which the normality of the tangent follows. [For a small increment AM, the tangent to the curve is approx. MN/AM = AM^{n-1}, and as $n < 1$, this quantity diverges; hence AB is perpendicular to AM]. For if the tangent *AB* makes an acute angle or an angle infinitely small with *AM*, then the time to pass along will be made infinite.

EXAMPLE 2.

41. The body is to move along the straight line *AB* (Fig. 3) thus, in such a manner that, with the semi-circle *ANB* described upon this line, the speed at any point *M* is in proportion to the line *MN* attached to the circle at this place. It is understood that the speed at the point M is of such a size that the body can traverse the distance *m.MN* in one second. [i. e. the speed of the body at *M* is proportional to the length *MN*, where *m* is a constant of proportionality for a particular motion, and which has the dimensions of inverse time. Obviously, the speed is zero at A and B, while it is a maximum at C.] The radius of this semi-circle is put as AC = *a*, now the distance traversed will be AM = s; and

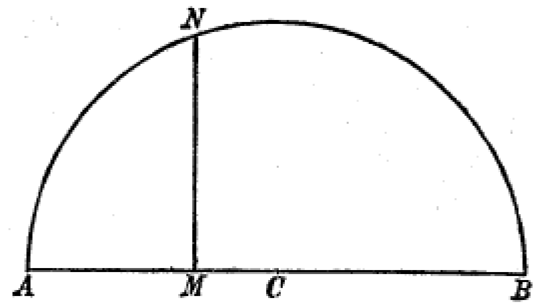

Fig. 3.

$MN=\sqrt{(2as-ss)}$. [From elementary geometry, the length *MN* is the geometric mean of the lengths *AM* or *s,* and MB or $2a-s$.] Therefore the speed at M, that we have put previously as *c*, in this case will be equal to

$m\sqrt{(2as-ss)}$. [p. 16] Therefore the time, in which the distance *AM* is traveled through, will be equal to

$$\int\frac{ds}{m\sqrt{(2as-ss)}}=\frac{1}{ma}\int\frac{ads}{\sqrt{(2as-ss)}}.$$ But

$\int\frac{ds}{\sqrt{(2as-ss)}}$ specifies the arc AN of the circle. [For according to Fig. 3'. due to the translator, the angle $d\theta$ is the increment in the arc length/radius, and sin θ = MN/*a* = *ds/NN'*, where *NN'* is an increment of the order of *ds*.] In accordance with this, the time in which the body travels through the distance *AM* will be $\frac{AN}{m.AC}$ seconds. Thus the time, in which the body moves from *A* to *B* will be $\frac{ANB}{m.AC}$ sec. Indeed since $\frac{ANB}{AC}=\frac{22}{7}$ approximately, the time will therefore be equal to $\frac{22}{7.m}$ seconds. From which it is understood, that whatever the size of the line *AB*, that it is always traversed in the same time. [Thus, we have the rudiments of simple harmonic motion, for which the period is independent of the amplitude, set out in this way; also, the constant *m* for a particular motion is seen to be the angular velocity ω.]

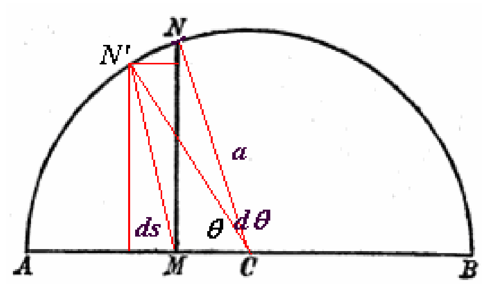

Fig. 3'

Corollary 3.

42. It appears from the solution of the problem that the body arrives at A from M in the same time (Fig. 1), from the backwards motion from M to A, but only if the motion has the same speeds at the same locations.

Corollary 4.

43. The applied line MN [that is, the y co-ordinate of a point on the curve for $n > 1$] of the curve AN represent the speed (Fig. 4), which the motion of the body has at the individual point M on the line AM, and moreover the curve has an angle less than a right angle at A with the straight line AM. From these arguments put in place already, the time has now been shown, in which the body arrives at M from A, to be of an infinite size. Whereby also with a retrograde motion of the body from M towards A , the length of time afterwards is finally infinite, i. e. the body never reaches A, even though at all points other than A the body has a finite speed. [p. 17]

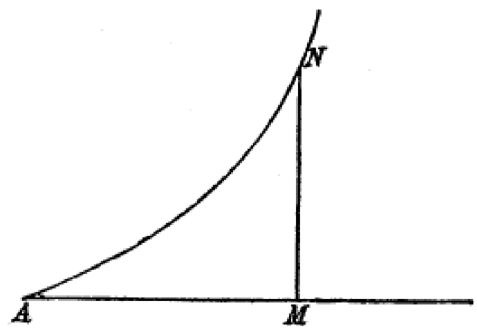

Fig. 4.

PROPOSITION 5.
Theorem.

44. ***Two bodies are moving on the right lines AM and am*** **(Fig. 5)*****, and the speeds of these bodies are to be expressed by the similar lines attached to the curves AN and an***[i. e. the y co-ordinates; Euler always refers to them in this way.] ***I say that these bodies travel homologous distances AM and am in the same time.*** [i. e. in the same ratio as the speeds.]

DEMONSTRATION.

Since AM and am are homologous distances, these therefore have the same ratio as the attached lines MN and mn; thus if the ratio is $m : n$, on putting $AM = s$ and $MN = c$, then $am = \frac{ns}{m}$ et $mn = \frac{nc}{m}$. The time to travel AM is given by $\int \frac{ds}{c}$ (37) ; moreover, the time to travel am will be obtained by putting $\frac{nds}{m}$ in place of ds and $\frac{nc}{m}$ in place of c in $\int \frac{ds}{c}$. From this product there is again

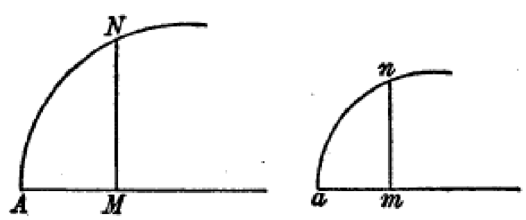

Fig. 5.

produced $\int \frac{ds}{c}$: whereby each time to travel through *AM* and *am* will be $\int \frac{ds}{c}$, and therefore these are equal. Q. E. D.

Corollary 1.

45. Hence it is also understood that (41) is included; for all the curves of circles are similar and the diameters homologous distances.

Corollary 2.

46. Let *a* be the [distance] parameter of the curve *AN*, which if taken either greater or smaller can change *AN* into another curve similar to itself. Truly for this to eventuate, the equation for the curve *AN* should be such that the applied line [for the speed] *c* is equal to a function of *a* and *s* of only one dimension. Moreover, for the different values of the distance *a* itself, *s* expresses the homologous distances, whether are taken to equal either *a* or *na*. Therefore according to what factor *c* is increased or decreased [p. 18] by an equation of this kind, so also are the proportional distances *na*, or how much larger or smaller a value is put in place for *a*, resulting in equal times for the distances traversed.

Scholium.

47. For this to be so, if *c* is equal to a function of *a* and *s* of one dimension, then the times for *a* or *na* are all equal, whatever the value of *a* : thus also if we were to consider *c* to be equal to a function of *a* and *s* which had *m* dimensions, then the times for *a* or *na*, for some *a*, would be in the ratio a^{1-m}. For $\frac{c}{a^{m-1}}$ will be the function of one dimension of *a* and *s*, which can be put as *k*. Hence, therefore, $c = a^{m-1}k$ and $\int \frac{ds}{c} = a^{1-m} \int \frac{ds}{k}$. But $\int \frac{ds}{k}$ will give, for the position $s = a$ or *na*, a constant quantity, in whatever manner *a* may be varied (46) . On account of this, $a^{1-m} \int \frac{ds}{k}$ will give some power of a^{1-m} . Consequently the time to traverse *na* will be as a^{1-m} .

DEFINITION 6.

48. *The extent* [or scale] *of the speed is represented by a curve, the attached lines* [*y* co-ordinates] *of which represent the speeds, which the body has at places corresponding to the distances that the body has traversed.* Thus of the body moving along the right line AM (Fig. 6) the scale of the motion is the curve AN, and of which the attached lines MN sets forth the speed of the body at some individual point M.

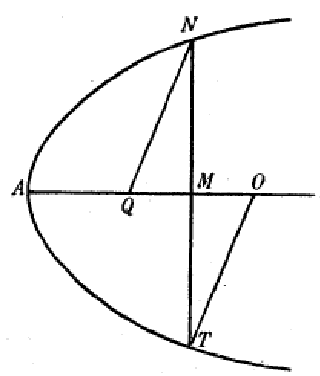

Fig. 6.

DEFINITION 7.

49. ***The scale of the times is a curve, of which the applied lines*** [y co-ordinates] ***represent the times at which the corresponding parts of the distance have been completed in the motion***. Thus if the curve *AT* (Fig. 6) were of this kind, [p. 19] so that any attached line of this MT [*y* co-ordinate] presents the time in which the distance AM is traversed, then AT will be the curve with the scale of the times.

Corollary.

50. Since the curve of the scale of the times ought to be found from the given scale of the speeds AN, it is now apparent from the preceding problem (37). Clearly if the given distance $AM = s$, the speed at *M*, i. e. *MN,* $= c$ and the time, in which *AM* is traversed i. e. $MT = t$, will be $t = \int \frac{ds}{c}$. Therefore from the given curve *AN*, with the quadrature permitted, the curve *AT* can be constructed.

PROPOSITION 6.

PROBLEM.

51. *From the given scale of the times AT* (Fig. 6) *to find and construct the scale of the speed AN.*

SOLUTION.

As before, put $AM = s$, $MN = c$ and $MT = t$, it is required from the given equation between *s* and *t* to find the equation between *s* and *c*. Truly, this is easily effected from the above rule found : $t = \int \frac{ds}{c}$. For by differentiation it becomes $dt = \frac{ds}{c}$ and $c = \frac{ds}{dt}$. Therefore the normal *TO* to the curve is drawn at *T*, and it will be $\frac{ds}{dt} = \frac{MT}{MO}$. [For the ratio MT/MO is the tangent of the angle MOT the normal makes to the axis AO; this is the complement of the angle the gradient of the curve AT makes to the same axis, which is also *dt/ds*; from which the result follows.] Therefore as *MO* is to *MT*, thus a certain line is made to equal unity, which indicates the seconds for the fourth proportional, which is equal to *MN*. Therefore from *M* the interval is taken $MQ = 1$, and *QN* is drawn parallel to the normal *TO*, will be the point *N* on the scale of the speeds sought. Q. E. I. [p. 20]

EXAMPLE 1.

52. Let the scale of the times be a right line inclined at some angle to AM; then $t = ms$ and $dt = mds$. Therefore $c = \frac{ds}{dt} = \frac{1}{m}$ is produced. The scale of the speed is therefore a right line parallel to *AM*, and the body is carried forwards uniformly.

EXAMPLE 2.

53. The times are as some power of the distances described, or $t = s^m$ and thus $dt = ms^{m-1}ds$. From which $c = \frac{1}{ms^{m-1}} = \frac{1}{m}s^{1-m}$. Whereby, if the curve AT were the parabola of APOLLONIUS, i. e. $t = s^{\frac{1}{2}}$, then $m = \frac{1}{2}$ and $c = 2s^{\frac{1}{2}}$. From which it is apparent in this case that the scale of the speeds is of the same kind of parabola also.

Corollary.

54. It is also understood, if the equation is given between c and t, in the same way the distance traversed s and both the scales of the speeds and of the times can be found. For because $c = \frac{ds}{dt}$, then $ds = cdt$ et $s = \int cdt.$

Scholium.

55. Here it is reminded for these, that as far as scales of speeds and times have been described, not only are they seen to apply to absolute motion, but also to pertain to relative motion. For the nature of this motion has not yet been considered, neither has anything been assumed, that is a special property of absolute motion. Now indeed we will produce some propositions, that are peculiar to absolute motion, and from which in a certain way, the difference between absolute motion and relative motion will be made apparent. [p. 21]

PROPOSITION 7.

THEOREM.

56. *A body remains in a state of absolute rest, unless it is disturbed to move by some external cause.*

DEMONSTRATION.

We consider this body to exist in infinite and empty space, and it is seen that there is no reason why the body should be made to move from one place to another rather than stay at rest. Consequently on account of the lack of sufficient reason why it should move, it must remain at rest for ever. Nor indeed does this reason ever change in the universe; although it is possible to object that in the universe, there is sufficient reason that it might fall in one place or another. And indeed it cannot be believed that in that empty infinite space, the failure of sufficient reason for a single motion to occur [by this mechanism], is taken as the cause of the body remaining in a single place ; as there is no doubt that the nature of the body itself is the cause of this phenomenon [i. e., of staying at rest; Euler is touching on the idea of inertia : the innate tendency of a body to remain in its present state]. Clearly, the failure to move due to any insufficient reason, cannot give the true and essential cause of the event, but rather it rigorously demonstrates the true cause; and that likewise it shows that the hidden nature of the thing is the true essential cause, and this cause does not cease, from the failure of that other insufficient cause to move [the first explanation that Euler rejects]. Thus, the demonstration of ARCHIMEDES illustrates the principle with the equilibrium of two balances each similar to the other, that the truth of the matter can be shown not only in empty space, but also in the world. [Here two bodies cancel out each other's turning effects in the earth's gravity.] Moreover each is given a natural reason for this equilibrium [p. 22], and which is located on the earth also. In empty space a body should therefore be able to remain in a state of rest, since it is in the nature of the body for the reason given, and on that account for a body on the earth too, that once it is at rest, except by some other cause acting on it, it can be considered to remain at rest. Q. E. D.

Corollary 1.

57. It is therefore a law in the established nature of things, that every body shall remain in a state of rest, except that by some external cause it is disturbed to move.

Corollary 2.

58. As the fundamentals of this demonstration has been found from the nature of absolute rest, it is hence in error to extend this law to relative rest.

Scholium.

59. We are well taught from trials that this law does not extend to relative rest. For we see bodies on ships at relative rest, but if the ship is suddenly shaken violently, the bodies do not remain in a state of rest, but also are shaken and move from their own positions, and even if before they were at rest, nothing approached to cause this motion. [The modern physicist would take great exception to these assertions, as the bodies try to maintain their previous state.]

Corollary 3.

60. In like manner, from what we have asserted, that a body once at rest must continue to be at rest, except in the circumstance of being affected by some external cause, then the body can be shown, which is now in an state of absolute rest [p. 23], that previously it was always also in a state of rest; if indeed it were left to itself. Since there is no reason, whereby the state of its present position should have come about from one region or another, thus it is also concluded that the body was always present before in its present situation.

Corollary 4.

61. Therefore the body, since once it is at rest, if no external cause had neither acted on it or taken it away, then not only henceforth will it always remain at rest in the same place, but also previously it had to be for ever in a state of rest in that place.

Corollary 5.

62. It follows from this, that a body once set in a state of absolute motion is never able left to itself, to come to a state of absolute rest. For if finally it should come to rest, as before it is necessary that it was always at rest, which is contrary to hypothesis.

[One might regard this proposition and others of the same nature as Euler's musings. The idea of separating absolute and relative motion seems now to be rather naive, but as always, one must suspend judgment until the whole story unfolds in the future propositions. The ideas of absolute time and space were of course set out by Newton; but none of these agree completely with modern physics, which regards all inertial frames as equivalent.]

PROPOSITION 8.

THEOREM.

63. *A body having uniform absolute motion will always be moving, and with the same speed now that it had at an any earlier time, unless an external cause should act on it or have acted on it.*

DEMONSTRATION.

If indeed the speed with which a body moved was not kept the same always, then the speed might either increase or decrease. Moreover from this cause it might tend towards rest, which, since it is never allowed to do this as a consequence of (22), cannot happen. [p. 24] Likewise for the body to have emerged into its present state from rest is considered to be absurd. Besides if this body situated in infinite empty space should be considered in this way, which is that it has speeded up or slowed down, there is nor reason why it should have a greater or lesser speed than what it already has, on account of which it must always be moving with the same speed. Q. E. D.

Corollary.

64. Therefore how much we should see the speed of the moving body to have either increased or decreased, we must attribute the change to some external cause.

PROPOSITION 9.

THEOREM.

65. *The body with a given absolute motion shall progress in a straight line, or the distance that it describes shall be a straight line.*

DEMONSTRATION.

There is indeed no reason, if this body is considered to be placed in infinite and empty space, why it should depart from moving in a straight line from one region to another. And from which it is to be concluded, that it is from the nature of the body itself that it moves in a straight line. On account of which also in the world, where indeed with sufficient reason for this principle to cease being applicable, it is nevertheless observed that any body moving progresses along a straight line, unless obviously it is prevented. Q. E. D. [p. 25]

Corollary 1.

66. From these two propositions that universal law can be set out : every body provided with motion progresses uniformly in a straight line.

Corollary 2.

67. Therefore a body, that was compelled by some external cause to progress in a curve AM (Fig. 7), if, when it arrived at M, these causes suddenly ceased, then it would progress with the speed which it had at M, uniformly following the direction it would have from that time of being freed. Truly it is the tangent MT of no element of the curve other than the direction produced at M that is relinquished, on account of which the body at M progresses along the tangent MT with that uniform speed that the body had.

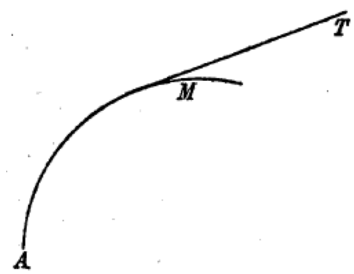

Fig. 7.

Scholium I.

68. These laws regarding absolute rest and motion have been gathered together under one authority. And this is the law NEWTON thus proposed in the *Principia*, as he has stated : Every body persists in its own state of being at rest or of moving uniformly in a direction, unless in as much as it is forced to change that state by impressed forces.

Corollary 3.

69. Moreover these laws are concerned with the continuation of motion as applied to absolute motion, and not these in relative motion that retain their force. As indeed it can be possible, that a body in a state of relative rest not to be maintained at rest, if also by no external cause it should be agitated (59), thus also the motions bodies had relative to others, are not always to be moved in the same direction uniformly. [Euler's agitated ship example; p. 26]

Corollary 4.

70. Therefore when a body has been disturbed, by no external cause, that may move in some non-uniform relative way, and yet either it is required by consideration to remain at rest or to move in some uniform manner. From this it is to be understood in some manner, how much the relative state differs from the absolute state.

Scholium 2.

71. In the foundations of theoretical astronomy, as have been set out by Newton, the sun and the fixed stars are established as being entirely non-affected by external causes, or the effect is so small as to be negligible. Though however we can neither see the sun to be in uniform motion nor to be progressing in a fixed direction with respect to the earth; , yet it is certain that the sun is either at rest or is moving uniformly in a fixed direction. It is necessary that irregularities in the motion of the sun should be observed from the position of the earth.

DEFINITION 8.

72. *The direction of the motion is to be determined by a straight line, along which the moving body is trying to progress, and according to this the body progresses, unless it is impeded by some external cause.*

Corollary.

73. Therefore a body having an absolute motion, unless it is affected by other causes, always keeps the same speed and moves in the same direction. [p. 27]

DEFINITION 9.

74. *The force of inertia is in all bodies is that in situ faculty of the body to maintain its state of rest or of continuing in its present state of motion in a straight line.* [This is another supposed Newtonian idea; by whom inertia was regarded as a force. See, e. g. Cohn's translation of the *Principia*, p. 96, § 4.7. However, as Euler points out, it had its origins with Kepler.]

Corollary.

75. Though indeed we have demonstrated with sufficient reasons that a body remains in a state of rest or of a continuation of uniform motion in a straight line, yet now we will note that this is not the effecting cause of the phenomenon, for that is situated in the nature of the body itself. This depends on the nature of the bodies themselves itself and is the reason for the conservation of the state of a body, and it is called the force of inertia.

Scholium.

76. KEPLER, who first formed this notion, and attributed to it a force, that all bodies have, all of these are resistant to that which is trying to disturb their state; and calling this inertia is better than calling it resistance, [Curiously enough, some engineering texts on physics still treat the inertial force on the same footing as other forces, so that the sum of the forces on any body is always zero; and likewise with the sources of e.m.f. in circuits.] agreeing with the idea that there is a perseverance, that we have connected to with our ideas. But it is easily understood these definitions are not different from what we have already asserted ; for it is the same force of continuing or of remaining at rest, and which offers resistance to hindrances to the state of the motion. Truly I have been inclined toward using this definition rather than the KEPLER one, since it has not yet been agreed, how bodies with disturbing forces resist changes in their motion. In addition this particular force of resistance has its own origin from the facility of remaining at rest or of continuing to move; and therefore should be explained from this basis.[p. 28]

PROPOSITION 10.

THEOREM.

77. *When the space* [our inertial frame of reference] *from which the relative motion is determined, is either absolutely at rest or moving uniformly in a direction, then the given laws prevail for a state of relative rest or of relative uniform motion.*

DEMONSTRATION.

If the space is from absolute rest that the relative motion is decided, then the proposition by itself is clear. For in this case the state of rest and relative motions are in agreement with the absolute motions, and thus any body also will be in a state of relative perpetual rest or uniformly moving in a fixed direction (10). For if indeed that space itself [our inertial frame of reference] should be moved uniformly in the same direction as these bodies, which are relatively at rest, then they will have the same absolute motion that the space itself has. Whereby with each thus progressing uniformly in the same direction by their own nature, thus they are able to continue : and in this case the law is observed (66). Truly a body, that is relatively moving uniformly in a direction, it also, if it has uniform rectilinear motion, proceeds absolutely with uniform rectilinear motion, as becomes apparent from the following proposition, where it is made transparent. Hence the relative motion here is in agreement with this law and therefore it can continue without an external force. Q. E. D. [p. 29]
[It appears that Euler has abandoned the idea of an inertial force as a working hypothesis in favour of inertial frames, which he calls 'spaces', and the rectilinear motion of a body has the same nature in all such spaces; he still maintains, however, that there is a reference inertial frame, which is the one defining absolute rest or motion, determined by the fixed stars. The model suggested by Newton and this one are of course in agreement with each other.]

Corollary 1.

78. Therefore a body not affected by some external cause, that is relatively either at rest, or moving uniformly in some direction, from the evidence the motion of this will be as that indicated, either absolutely at rest or absolutely to be moving uniformly in some direction.
[Thus, motion in any inertial frame can be compared to the absolute motion of the reference frame of the fixed stars.]

Corollary 2.

79. Likewise such relative motion will be preserved, each in its own state, for ever. For it is not only the body itself that moves, and the body is moved absolutely uniformly in some direction, but also that space itself, relative to which the body is considered, which progresses according to the same law. On account of which each will be continued on its own, and the relative motion will itself continue in this state without any approaching external cause.

Corollary 3.

80. Since the idea we have concerning all motion, is that it is relative (7), and these laws also are not sufficient to recognise how much of the motion the absolute motion any body shall be. For when we see a body progressing in uniform motion without being affected by an external cause, then we cannot conclude more than to say that this body is either in a state of absolute rest or to be moving with an absolute uniform motion in a given direction. Truly how much of this motion is absolute cannot be defined, and neither the direction that it has. [Thus, there is no unique reference frame in which the motion of the body can be defined; and this is especially the case if a special background reference frame of the fixed stars has to be included.]

Corollary 4.

81. Which therefore shall be deduced from the nature of the bodies, that remain in their own state of rest or of uniform motion [p. 30], not only will they pertain to absolute rest and motion, but also to their relative state of rest or motion, and in which space the body, from its own motion is considered to move uniformly in given direction.

Scholium.

82. And also we will not be very concerned with absolute motion, since the relative motion itself is continued by the laws. And therefore this relative motion we will more often change into other motions of the same kind, as the laws expounded are still observed : if it is allowed, we will contemplate the body in relation to that of another body progressing uniformly in this direction too. From which reason the body will not cease to move uniformly along a straight line, and that with innumerable different ways that this can be done, the most convenient of these can be selected.

PROPOSITION 11.

PROBLEM.

83. *A body can move uniformly with absolute motion along the line AL* (Fig. 8)*, and another body likewise moves uniformly too along the line AM. The size of the relative motion of the other body progressing along AM is sought in relation to the body progressing along AL.*

SOLUTION.

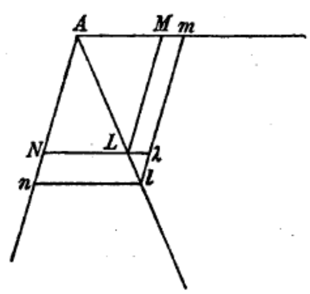

Fig. 8.

Let the speed of the body progressing along AL be a; and the speed of the other motion along AM be b : and these bodies set out at the same time from the point A. It is evident that, if the two distances AL and AM are taken in the ratio of the speeds a and b, [p. 31] both bodies arrive at the same time at L and M . Therefore draw the lines ML with the line AM making the angle AML, the sine of which is to the sine of the angle ALM, that AL makes, as AL to AM, i. e. as a to b, L will designate the place at which the body progressing along AL at the same instant, at which the

other will be present at *M*. Since truly the relative motion of any other body with respect to this body can be sought, this body actually moving along *AM* can be considered as being at rest at *A*. Therefore with the point *M* known from the translation of *A* to arrive at *L,* through *N* draw *AN* parallel and equal to *ML,* from *A*. In a similar manner when the body moving on *AM* arrives at the nearby place *m*, the other is found at *l*, and *ml* is parallel to *ML*, since
$Mm : Ll = b : a = AM : AL$. Indeed with the point *m* in a similar manner translated from *A* by taking $An = ml$ there comes about *l* in *n*, and *n* is on the same line AN. From these it follows that the body in absolute motion along AL is to be moving in relative motion along the line AN. Moreover the relative speed will be to the absolute speed as *Nn* to *Ll*, or as *ML* to *AL*. Which ratio when constant on account of the kind of the given triangle *ALM*, a body in absolute motion along *AL* is in uniform relative motion on progressing along the line *AN*. For with the position of the line *AN* found by taking the angle *LAN* so large, in order that the sine of this to the sine of the angle *NAM* is as *b* to *a*. Hence the absolute speed along *AL* will be as the relative speed along *AN* as the sine of the angle *MAN* to the sine of the angle *LAM*. Q. E. I. [p. 32]

Corollary 1.

84. Therefore a body progressing in absolute uniform motion along a line will also be progressing uniformly along [another] line in relative motion, if the manner of the body, from which the relative motion is indicated, should be progressing uniformly along some line. And that is what we have assumed in the preceding demonstration (77).

Corollary 2.

85. Moreover the construction of the line *AN* and of the relative speed has been most easily found by being set up in this manner. By assuming, as we have done, that *AL* and AM are in the ratio *a* to *b* and draw *ML*, then *AN* is drawn parallel to this line *ML* from *A*, and in this manner the relative motion will be described. Indeed the relative speed will be to the absolute speed as *ML* to *AL*.

Corollary 3.

86. Likewise it prevails from the same reasoning , that if with *AL* not the absolute motion, but travels relative and has the same relation to *AM* . Then truly another relative speed of the body will be produced by the body traveling along *AL* with respect to the size of the speed of the body traveling along *AM*.

Corollary 4.

87. It is therefore apparent, how the absolute motion can be changed into relative motions in an endless number of ways, which will always be uniform and made in along straight lines, but only if the absolute motion and the movements of the bodies from which the relative motions arose, were of this kind. [p. 33]

Scholium.

88. In the solution we have taken both bodies setting out from the same point *A* : but the solution is just as successful, if both bodies in the beginning start out from different points that were put at *A* and *B* (Fig. 9). For the body *A* progresses uniformly with an absolute motion along the right line *AL*, and indeed the other *B* similarly along the right line *BM*, thus so that the speeds are in the ratio *a* to *b*. *AL* and *BM* are taken in the same ratio *a* to *b*, and both bodies arrive at the same time at *L* and *M*. But since the relative motion of the body *A* with respect to that of *B* is required, body *B* must be considered to be at rest at *B*. Therefore, for this reason, body B is moved from *M* to *B*, as body *A* arrives from *L* at *N*, by drawing *BN* parallel and equal to *ML*: I say that the point *N* is on the line passing through *A*, thus as the body *A* is moved relatively along the line *AN*, in a uniform motion. For draw *NL* and it will be equal and parallel to *BM*. With the appearance of triangle *ANL* from this construction : whereby *NL* to *AL* will have the given ratio ; hence, since *NL* = *BM*, the ratio *AL* to *BM* is the given ratio, which hence, if both were taken once in the ratio *a* to *b*, will always be in the same ratio. From which it is apparent that the point *N* lies on the line *AN* and the relative speed along *AN* is to the absolute speed along *AL* as *AN* to *AL*, i. e. in the given ratio. Therefore the relative motion is made along the right line *AN* and it is uniform.

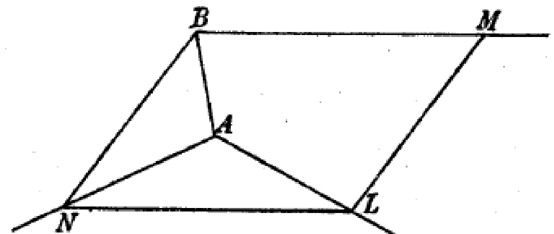

Fig. 9.

Corollary 5.

89. If therefore the absolute motion of the body *A* (Fig. 9) is given along the right line *AL,* and the uniform relative motion of this is given along *AN* [p. 34] with whatever speed. then the motion of the body *B* can be found that arises with respect to the relative motion of the body *A*. Indeed with the two distances *AL* and *AN* assumed that are traversed in the same given time, a line *BM* is drawn through any point *B* parallel to the line *NL*: and this will determine the path traversed by a given body *B*, and the speed of this will be to the absolute speed of the body *A* along *AL*, as *NL* is to *AL*. Truly the body *B* will be at the point *B* at the same time that body *A* is at *A*.

Corollary 6.

90. Therefore there are innumerable motions of the body *B*, since the point *B* could be assumed as we pleased, from which the same relative motion of the body *A* came about. But the speed of the body *B* was always the same and the direction of this followed the parallel line *NL*.

Corollary 7.

91. Also the uniform absolute motion stretching out along a line is able to be changed into any relative and likewise uniform motion made along a straight line. Indeed the line *AN* to be drawn can be chosen arbitrarily, and any speed assigned to that line. Indeed this same uniform motion is given to *B*, and the line of progression of the body *B*, from which here the relative motion arises.

Corollary 8.

92. In which case that relative motion alone without any external force will be able to continue. Indeed the absolute motions along *AL* and *BM*, since they have been made uniform along right lines, [p. 35] by themselves are to be continued. Indeed for as long as this motion lasts, so long also should the relative motion along *AN* also continue.

PROPOSITION 12.

PROBLEM.

93. *The body A* (Fig. 10) *moves in some manner along the line AL with absolute motion, and the body B along the line BM. The relative motion of body A is required with respect to body B.*

SOLUTION.

The arcs *AL* and *BM* are cut from the curves, which are traversed in equal times. Therefore the body *A* will be found at *L*, when *B* reaches *M* . But since the relative motion of the body *A* with respect to *B* is desired, the body *B* is taken as at rest. Whereby that known motion is transferred at *B* from *M* along the line *MB*, and the body *L* reaches *N*, drawn parallel to *LN* and equal to *MB*. Therefore the curve, on which the point N is found in this way, will be the path for the body *A* with the relative motion relative described. And by this relative motion the arc *AN* is traversed in the same time, by which the arcs *AL* and *BM* are completed. From which the relative speed at *N* is also to become known. Q. E. I.

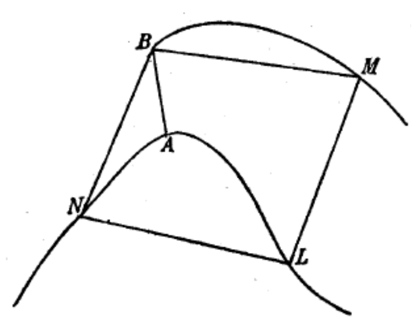

Fig. 10.

Corollary 1.

94. Therefore in this manner the relative motion of any body can be determined, with respect to the extent of the motion of some other moving body. [p. 36]

Corollary 2.

95. It is also understood from the solution, how from the given curves *AN* and *AL* together with the motions along these, it is possible to find a curve *BM* and the motion along that curve. And the curve AL is defined from the curves *BM* and *AN*.

Corollary 3.

96. It is also evident that the curve *BM* , on account of the arbitrary choice of the point *B,* can be taken in an endless number of other positions. However since the arc *BM*, which is described in the same time as *AL* and *AN*, always subtends *BM* equal and parallel to the line *LN*, which will itself always be similar and equal and parallel to itself, and the motion along that line will always be the same as along *BM*.

Scholium.

97. And these are the motions, which are concerned with the comparison of absolute and relative motion that I have judged worthy of reporting on. Moreover there is another way in which it is customary to described relative motion, as the motion along *AN* is called the motion of the body *A*, which actually is moving along the line *AL*, and with such for the motion observed from the body *B* moving along *BM*. Truly this motion will be observed with the body *B* placed in relative rest and the point *B* as considered at rest. Thus the relative motion of the stars with respect to the earth agrees with this motion, that we living on the earth consider as at rest. Indeed the earth from *B* has moved forwards to *M* and the star from *A* to *L*, we see that star from *M* following into the region *ML* and in the distance *ML*. Since indeed we are not able to apprehend motions from the place *B* any more, but still now we consider ourselves to be at *B*, then we will see the star [p. 37] from *B* and not at *L*, but at *N*, clearly in the same place and the same distance. Therefore the right line *BN* will be equal and parallel to the line *ML*, as we have found by considering our method.

GENERAL SCHOLIUM.

98. These laws of motion, which a body observes that is left to itself, either at rest or in continued motion, are seen particularly for these indefinitely small bodies, which can be consider as points. For in bodies of finite magnitudes, of which the individual parts have their own motions, the body will exert itself to observe these laws, but which will not always be possible to happen on account of the state of the body. The body therefore will follow a motion which is composed from the individual exertions of the parts of the body, and this hitherto on account of the insufficiency of the principles has not been possible to be defined, but this discussion is to be differed to the following. The different kinds of bodies will therefore supply the needs for the primary division of our work.

For in the first place, we will consider very small bodies or which can be considered as points. In the next case we will approach these bodies of finite magnitudes which are rigid and are not allowed to change their shape. In the third case we will consider flexible bodies. Fourthly, we are concerned with these which allow extension and contraction. Fifthly, we put under our examination the solution of the motion of many bodies, that are impeded by others, that their own motion may be completed as they exert themselves. Truly in the sixth case the motion of fluids will be the agenda.

For these bodies we will not only see, how the remainder of the motion is to be continued [p. 38]; but in addition we will inquire, how these are affected by the external causes or forces. Finally from all these inquiries the large scale variation of the whole body can be inferred, whether it is free or not. For a non-free state, I understand this :

when bodies are impeded, by which they are unable to progress in that direction, and which they try to overcome ; the motion of pendular bodies is of this kind which, since the are unable to descent directly, as they try, and so make oscillations. For the free state is to be understood : when bodies are progressing and which come upon no impediments to their motion anywhere, not only on account of their own force, or from disturbing forces pulling on them. Therefore it appears, from the things Mechanics will have as its agenda , and that there are many which have not even been touched upon. For besides the motion of points, which have been dealt with hitherto, nevertheless there are so few that it will be necessary to derive nearly all from first principles. I begin therefore with the motion of free points with any kinds of disturbing forces, because these left to themselves will follow the motion shown in this chapter. Hence on account of this I have resolved for the First Volume to be concerned with the motion of free points, and the following Volume truly set up to explore the motion of points which are not in free motion ; in both of which, and which will occur, as with these already dealt with, so likewise for these that follow, I shall derive the motions from first principles using the analytical method.

CAPUT PRIMUM

DE MOTU IN GENERE.

[p. 1]

DEFINITIO 1.

1.

Motus est translatio corporis ex loco, quem occupabat, in alium. Quies vero est permansio corporis in eadem loco.

Corollarium 1.

2. Motus igitur et quietis ideae in alias res cadere non possunt, nisi quae locum occupant. Quare cum hoc sit corporum proprium, locum occupare, de solo corpore dici potest, quod moveatur vel quiescat.

Corollarium 2.

3. Atque haec motus quietisque idea ita est propria corpori, ut ad omnis prorsus corpora pertineat. Nullum enim existere potest corpus, quod non vel moveatur vel quiescat.

DEFINITIO 2.

4. Locus est pars spatii immensi seu infiniti, in quo universus mundus consistit. Vocari hoc sensu acceptus [p. 2] locus solet absolutus, ut distinguatur a loco relatio, cuius mox fiet mentio.

Corollarium 1.

5. Quando igitur corpus successive aliam atque aliam huius immensi spatii partem occupat, movetur : at si perpetuo in eadem sede perseverat, tum quiescit.

Corollarium 2.

6. Concipi autem animo solent huius spatii termini fixi ad quos corpora referentur. Atque ista relatio est is, quod situs appelatur. Quae igitur corpora eundem servant situm respectu horum terminorum ea quiescere dicuntur. Contra vero, quae situm suum mutant, movere ducantur.

Scholion 1.

7. Si hac significatione expositae voces accipiantur, vocari solent motus absolutus, quiesque absoluta. Atque hae sunt verae et genuinae istarum vocum definitiones, sunt enim accommodatae ad leges motus, quae in sequentibus explicabantur. Quoniam autem immensi illius spatii cuiusque terminorum, quorum in datis definitionibus mentio est facta, nullam nobis certam formare possimus ideam; loco huius immensi spatii eiusque terminorum considerare solemus spatium finitum, limitesque corporeos, ex quibus de corporum motu et quiete indicamus. Sic dicere solemus, corpus, quod respectu [p. 3] horum limitum situm eundem conservat, quiescere, id vero, quod situm eodem respectu mutat, moveri.

Scholion 2.

8. Quae hic de immenso et infinito spatio cuiusque terminis dicti sunt, considerari debent ut conceptus pure mathematici. Qui, quanquam metaphysicis speculationibus videntur contrarii, nihilo tamen minus ad institutum nostrum recte adhibentur. Namque non asserimus, dare huiusmodi spatium infinitum quod habeat limites fixos et immobiles; sed sive sit, sive non sit, non curantes, postulamus tantum, ut motum absolutum et absolutam quietam contemplaturus sibi tale spatium repraesentet, ex eoque de corporum statu vel quietus vel motus indicet. Ratiocinium enim commodissime hoc modo instituetur, ut animum a mundo abstrahentes imaginemur nobis spatium infinitum atque vacuum, et in eo corpora collocata esse concipiamus, quae si in hoc spatio situm suum retinent, absolute quiescere, sin autem ex alia huius spatii parte in aliam transeunt, absolute moveri indicanta sunt.

DEFINITIO 3.

9. *Motus relativus est situs mutatio respectu cuiuseam spatii pro lubitu assumti. Atque quies relativa est permansio in eodem situ respectu eiusdem spatii. Ita terram pro hoc spatio accipientes, ea quiescere dicimus, quae in terra situm suum immutatum tenent; ea vero moveri, quae ex alio situ respectu terrae* [p. 4]*in alium progrediuntur, hocque sensu solem moveri dicimus. Simili modo in navi propulsa relative quiescunt, quae eundem in navi tenent locum, et relative moventur, quae in navi locum suum mutant.*

Corollarium 1.

10. Conveniunt motus relativus et quies relativa cum absolutis, quando spatium corpusve, cuius respectu motus et quies indicantur, revera quiescit respectu spatii illius immensi et infiniti. Si enim terra revera quiescit, quae huius respectu moventur et quiescunt, etiam absolute moventur et quiescunt.

Corollarium 2.

11. Discrepant autem relativus motus et quies ab absolutis, si spatium illud movetur. Nam si terra respectu spatii infiniti non quiescit, neque quae eius respectu quiescunt, absolute quiescunt; atque etiam motus absolutus differet a relativo. Quin imo fieri potest, ut corpus, quod relative movetur, idem absolute quiescat.

Scholion.

12. Perspicuum est statum hunc corporum relativum vel motus vel quietis innumerabilibus modis posse esse diversum : prout enim aliud atque aliud assumitur spatium, cuius respectu motus et quies diiudicantur, alii prodibunt motus relativi aliaque quies. Sic stellae fixae respectu telluris moventur, quaelibet vero respectu reliquarum quiescit. Atque planetae [p. 5] tam respectu terrae, quam stellarum fixarum moventur. In sequentibus autem semper et motum et quietem absolutam intelligi volo, nisi expresse monuero etiam ad relativa pertinere, quae afferentur.

PROPOSITIO 1.
Theorema.

13. *Omne corpus, quod sive motu absoluto sive relativo in alium locum transfertur, per omnia loca media transit, neque subito ex primo in ultimum potest pervenire.*

Demonstratio.

Pro motu absoluto si corpus subito ex loco primo in extremum perveniret, necesse esset, ut in primo fuisset annihilatum, statimque in ultimo de novo productum, id quod per leges naturae, nisi accedat miraculum, fieri nont potest. Procedet igitur in sequentem, donec tandem in extremum perveniat. *Pro motu relativo* si corpus, quod in locum spatii in infiniti substituitur, re vera quiescat, superius valet ratiocinium (10). At si moveatur, ipsum quoque per singula loca media transire debet, et propterea etiam motus relativus erit successivus, fietque per singula media loca. Q. E. D.

Corollarium 1.

14. Sequitur ex his etiam motum non posse fieri in instanti, sed tempore opus esse, quo ex alio [p. 6.] loco in alium perveniat corpus. Quia enim per singula loca media debet transire, hoc cum motu instanteo consistere non potest.

Corollarium 2.

15. Poterit igitur etiam via assignari, per quam corpus transit, atque ea cognita, nullam in ea erit punctum, quod corpus ex primo loco in ultimum progressum non attigerit. Vocari autem solet haec via spatium percursum.

Scholion.

16. Facile quoque est hae ad corpora circa axem rotata accommodare. Quoniam enim ipsum corpus situm non mutat : tamen motus inest in eius partibus, qui cognoscetur, si singulae partes ut totidem diversa corpora seorsim consideruntur. Singulae enim respectu spatii infiniti situm suum mutare deprehendentur, neque ullae quiescent nisi quae ipso axe sunt positae. Atque simili modo omnia corpora contemplari oportet, ut non solum ipsius totius, sed singularum etiam partium situs eiusque mutatio inspiciatur.

DEFINITIO 4.

17. *Corpus aequabiliter vel uniformiter moveri dicitur, quod aequalibus temporibus per aequalia spatia currit. Motus vero inaequabilis est, qui aequalibus temporibus sit per spatia inaequalis, seu qui aequalia spatia inaequalibus temporis intervallis absolvit.* [p. 7.]

Corollarium 1.

18. Corpus igitur motu aequabili latum duplo tempore absoluit spatium duplum, triplo triplum, atque in genere spatia percursa sunt temporibus proportionalia, temporaque spatiis vicissim. Ut navis super mari aequabili motu incedens, si una hora duo percurrit milliaria, eadem duabus horis quatuor absolvet milliaria, tribus sex, et n horis $2n$ millaria.

Corollarium 2.

19. Quamobrem si datur motus aequabilis, habebitur ex eo accurata temporis mensura, quae nisi ex motu cognosci non potest. Metiendis enim spatiis, quae corpus aequabiliter motum percurrit, innotescet simul temporum, quibus ea erant percursu, ratio.

Scholion.

20. Neque vero aliunde habemus temporis in annos, dies, et horas divisionem, nisi ex motu, quem tanquam aequabilem spectamus. Posita enim terra quiescente crediderunt veteres solem motu aequabili ferri, tempusque, quo circa terram revolvitur, diem appelaverunt. Porro sumserunt stellarum fixarum circa terram motum quoque esse aequabilem, atque tempus, quo sol eundem respectu stellarum fixarum locum, revertitur, annum posuerunt. Denique haec tempora in partes aequales diviserunt, hocque modo horas, et minuta sunt adepti [p.8]. Facile autem patet, si motus isti non sint, ut creduntur, aequabiles, hanc quoque temporum mensuram esse erroneam. Atque re ipsa recentiores astronomi in his motibus inaequalitatem detexerunt, et invenerunt dies omnes non esse aequales inter se, quamobrem correctionem etiam adhibere solent ex aliis magis aequabilibus motibus; quam termporis aequationem vocant, ex qua inaequalitas dierum cognoscitur.

DEFINITIO 5.

21. *Omne corpus, quod movetur, celeritatem seu velocitatem habere dicitur, eaque mensuratur spatio, quod id corpus aequaliter motum dato tempore percurrit. Scilicet quando corpus B eodem tempore duplum spatium motu aequabili absolvit, quo corpus A etiam aequalibiter motum simplum percurrit, corpus B duplo maiorem habere dicitur celitatem, quam corpus A.*

Corollarium 1.

22. Quia igitur in motu aequabili corpus aequabilibus temporibus aequalia percurrit spatia (17.), habebit corpus aequabiliter motum perpetuo eandem celeratem, seu velocitatem. In motu vero inaequabili corpus successive aliam atque aliam induit celeritatem.

Corollarium 2.

23. Celeritas autem, quam corpus inaequabiliter motum in quovis spatii percursi puncto habet, [p.9] mensuranda est ex spatio, quod ea celeritate aequabiliter motum dato tempore percurrere posset.

Corollarium 3.

23. Celeritas porro corporis aequabiliter moti absolute mensurari potest spatio, quod dato tempore verbi gratia uno minuto secundo percurritur. Atque is celeritatem corporis cuiuspiam perfecte cognoscere censendus est, qui spatium definire valet, quod corpus ea celeritate motum tempore minuti secundi percurrit.

Scholion.

24. Maxime etiam est in usu haec celeritatis metiendae ratio. Nautas enim navis celeritatem exploraturos videmus spatium mensurare, quod navis dato tempore percurrit. Vulgo autem accipiunt intervallum quatuor horarum, et investigant, quot milliaria navis hoc tempore absolvat. Ex quo simul intelligitur, quot pedes navis tempore minuti secundi percurrat, siquidem motu aequabili progrediatur.

PROPOSITIO 2.
Theorema.

25. *Duorum corporum aequabili motu progredientium celeritates sunt directe ut spatia quaecunque percursa et inverse ut tempore, quibus ea spatia erunt percursa.* [p. 10]

Demonstratio.

Sint duo corpora A et *a*, eorumque celeritates C et *c* ; percurrat illud A spatium S tempore T, hoc vero *a* spatium s tempore *t*. Iam quia in motu aequabili spatia sunt temporibus proportionalia (18), determinabitur spatium, quod corpus *a* tempore T absolvit, ex hac proportione $t : \mathrm{T} = s : \frac{s\mathrm{T}}{t}$: movebitur ergo corpus *a* tempore T per spatium $\frac{s\mathrm{T}}{t}$. At corpus A movetur eodem tempore T per spatium S. Celeritates vero corporum mensurari debent spatiis eodem tempore percursis (18). Quocirca erit $\mathrm{C} : \mathrm{c} = \mathrm{S} : \frac{s\mathrm{T}}{t}$, seu $\mathrm{C} : \mathrm{c} = \frac{\mathrm{S}}{\mathrm{T}} : \frac{s}{t}$. Ex quo sequitur celeritates esse directe ut spatia et inverse ut tempora, quibus ea sunt percursa. Q. E. D.

Corollarium 1.

26. Ex ultima analogia prodit haec aequatio $\frac{\mathrm{CT}}{\mathrm{S}} = \frac{ct}{s}$. In quovis igitur motu aequabili factum ex celeritate in tempus, si dividatur per spatium eo tempore percursum, dabit semper eundem quotum.

Corollarium 2.

27. Erit etiam $\mathrm{T} : t = \frac{\mathrm{S}}{\mathrm{C}} : \frac{\mathrm{s}}{\mathrm{c}}$. Ex quo sequitur tempore esse in ratione composita ex directa spatiorum et inversa celeritatum, seu esse ut spatia per celeritates divisa.

Corollarium 3.

28. Deinde inventa proportio transmutatur etiam in hanc $\mathrm{S} : s = \mathrm{CT} : \mathrm{c}t$. Ex qua colligatur spatia motu aequabili percursa esse in ratione composita ex ratione celeratatum et ratione temporum. [p. 11]

Corollarium 4.

29. Data igitur celeritate corporis aequabiliter motu una cum spatio quovis descripto, innotescet tempus, quo hoc spatium est percursum; dividendo scilicet spatium per celeritatem. Cum enim hunc quotum tempori semper proportionalem esse ostenderimus, poterimus eundem pro temporis mensura usurpare.

Corollarium 5.

30. Similiter celeritas poterit expremi per spatium percursum divisum per tempus, atque spatium etiam ipsum per factum ex tempore in celeritatem.

Scholion I.

31. Si enim celeritas tanta sit, ut corpus ea motum tempore minuti secundi absolvat spatium trium pedum, et propterea celeritatem exponamus numero 3; poterimus invenire tempus, quo 60 pedes v. gr. eodem motu absolvuntur. Dividatur enim 60 per 3 quotus 20 indicabit hos 60 pedes 20 minutis secundis percurri. Et si quaeratur spatium tempore 12 minutis secundis percursum, prodibit id 36 pedum. Atque etiam corporis 6 minutis secundis 4.8 ped. percurrentis proveniet celeritas 8, quae indicat hoc corpus minuto secundo 8 ped. percurrere.

Scholion 2.

32. Atque hinc tempora, spatia et celeritates mensurandi rationem in sequentibus semper adhibimus [p. 12]. Tempore nempe in minutis secundis perpetuo exprimemus, et spatia pedibus, iisque Rhenanis. Celeritates vero, ut iam est factum, denotabimus numero pedum, qui minuto secundo percurrunt. Infra quidem commodior celeritates determinandi ratio occurret, qua deinceps sumus usuri, sed ea tamen ex hac nascitur, ad eamque facile revocatur.

PROPOSITIO 3.
Theorema.

33. *In motu quantumvis inaequabili, mimima spatii elementa motu aequabili percurri concipi possunt.*

DEMONSTRATIO.

Quemadmodum enim in geometria curvarum linearum elementa ut lineolae rectae considerantur, ita etiam simili modo in mechanica motus inaequalbilis in infinitos aequalbiles resolvitur. Vel enim revera elementa aequabili motu percurruntur, vel mutatio celeratis per huiusmodi elementa est tantilla, ut incrementum aut decrementum sine errore negligi possit. In utroque casu ergo apparet proportionis veritas. Q. E. D.

Corollarium 1.

34. Omnis ergo celeritatis mutatio in motu inaequabili in singulorum elementorum initis fieri concipienda est, quia integra elementa aequabili motu percurri ponuntur. [p. 13]

Corollarium 2.

35. Quare secundum notandi modum analyseos infinite parvorum, si celeritas in primo elemento fuerit c, erit celeritas in secundo $c + dc$, in tertio $c + 2dc + ddc$, et ita porro.

Scholion.

36. Demonstrationis datae vis hoc nititur fundamento, quod celeritatis mutatio, quae fieri potest, dum elementum infinite parvum percurritur, debeat esse infinite exigua et evanescere prae celeritate, quam corpus iam habet, hoc enim nisi esset, generaretur motus finitus in instanti, quod esset absurdam. Interim tamen videtur haec propositio admitto non posse, si ipse motus et celeritas est infinite parva, quo casu incrementum vel decrementum momentaneum habere potest rationem finitam ad illam. Sed de hoc infra videbimus, ubi motus generatio considerabitur.

PROPOSITIO 4.

PROBLEMA.

37. *Moveatur corpus motu utcunque inaequabili per lineam AM* (Fig. 1), *data vero sit celeris corporis in quovis loco; oportet determinare tempus, quo arcus AM absolvitur.*

SOLUTIO.

Sit spatium *AM*, sive nit linea recta sive curva, = s, et celeritas, quam corpus habet in *M*, sit c, quae erit functio quaedam ipsius s. Ab *M* accipiatur elementum *Mm*, quod igitur motu aequalbili idque celerate c percurri [p.14] concipiendum est. Vocato elemento *Mm*, ds, erit tempus, quo hoc elementum percurritur, $=\frac{ds}{c}$ (29). Integrando ergo habebitur tempus, quo totus arcus *AM* absolvitur, $=\int\frac{ds}{c}$. Ad integrale vero talis adiici debebit constans, quae reddat hoc tempus = 0, si ponitur $s = 0$, secundum notas integrationis regulas. Q. E. I.

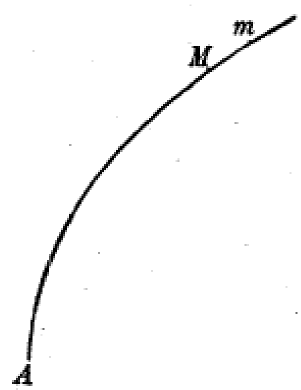

Fig. 1.

EXEMPLUM 1.

38. Sit celeritas in *M* ut potestas quaecunque spatii iam descripti AM, scilicet $c=s^{n}$; erit $\int\frac{ds}{c}=\frac{s^{1-n}}{1-n}$. Ad quod constantem non opus adiicere, si n < 1 vel negativum habeat valorem : dabit enim ipsum $\frac{s^{1-n}}{1-n}$ tempus, quo arcus AM percurritur. At si fuerit 1 – n numerus negativus, habebitur $\int\frac{ds}{c}=\frac{-1}{(n-1)s^{n-1}}$.

Ad quod constans $\frac{1}{(n-1)0^{n-1}}$, i. e. infinita quantitas, debet addi, quo totum habeatur tempus per AM. Tempore ergo in his casibus opus est infinito, quo corpus ex A persistet neque unquam inde egrediatur. Fit hoc vero, quoties est n numerus positivus unitate maior. Si vero est n = 1, tempus nequidem algebracie potest exhiberi; provenit enim

$\int \frac{ds}{c} = ls,$ ad quod etiam quantitatem infinitam adde oporteret, quo tempus per AM haberetur.

Corollarium 1.

[p. 15]

39. In mundo ergo alii casus subsistere nequeunt, nisi in quibus celeritates motus saltem principio sint ut spatiorum percursorum potestates exponentis minoris, quam est unitas.

Corollarium 2.

40. Progrediatur corpus in recta *AM* (Fig. 2), sitque in quovis loco celeris eius ut applicat *MN* curvae *AN,* quae cum recta *AM* in *A* concurrat, ita ut celeritas corporis in principio *A* sit nulla. Perspicuum est ex praecedentibus, quo tempus per *AM* fiat finitum, oportere tangentem *AB* in *A* esse ad *AM* perpendicularem. Coincidente enim *M* in *A* debet *MN* fieri $= AM^n$ et n numerus unitate minor, scilicet fractio, ex quo normalitas tangentis sequitur. Sin vero tangens *AB* angulum constituat acurum vel infinite parvum cum *AM*, tempus per *AM* fiet infinitum.

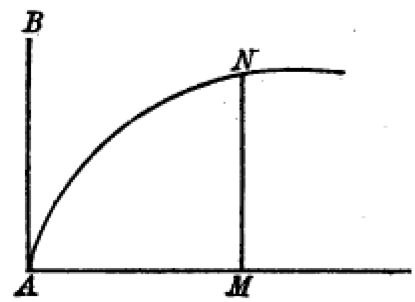

Fig. 2.

EXEMPLUM 2.

41. Moveatur corpus per rectam *AB* (Fig. 3) ita, ut descripto super ea semicirculo *ANB* celeritas in quovis puncto *M* sit ut applicata circuli in eo loco *MN*. Id quod ita potest intelligi, celeritatem in *M* tantam esse, qua corpus minuto secundo possit percurrere spatium = *m.MN*. Ponatur huius semi-circuli radius AC = a, spatium iam percursum AM = s; erit $MN = \sqrt{(2as - ss)}$. Celeritas ergo in M, quam posuimus c, erit hoc casu $= m\sqrt{(2as - ss)}$. [p. 16] Idcirco tempus, quo spatium AM percurritur, erit $= \int \frac{ds}{m\sqrt{(2as-ss)}} = \frac{1}{ma}\int \frac{ds}{\sqrt{(2as-ss)}}$. At $\int \frac{ads}{\sqrt{(2as-ss)}}$ denotat ipsum circuli arcum *AN*. Quamobrem tempus, quo spatium *AM* percurritur, erit $= \frac{AN}{m.AC}$ minut. secundis. Atque tempus, quo corpus ab *A* ad *B* movetur, erit $= \frac{ANB}{m.AC}$ min.sec. Est vero quam proxime $\frac{ANB}{AC} = \frac{22}{7}$. Ergo tempus hoc erit $= \frac{22}{7.m}$ minutis secundis. Ex quo intelligitur, quantacunque sit linea *AB*, eam perpetuo eodem tempore percurri.

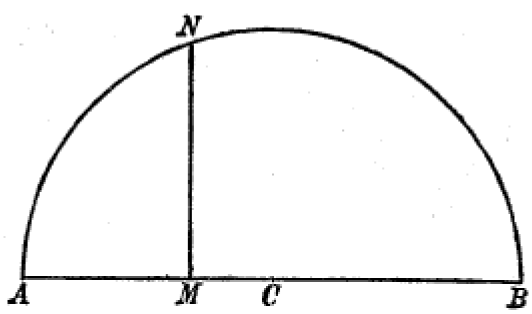

Fig. 3.

Corollarium 3.

42. Ex solutione problematis apparet etiam eodem tempore, quo corpus ex A in M pervenit (Fig. 1), idem motu retrogrado ex M in A perventurum, si modo in utroque motu in iidsem locis aequales habeat celeritates.

Corollarium 4.

43. Repraesentent curvae AN (Fig. 4) applicatae MN celeritates, quas corpus in recta AM celeritates, quas corpus in recta AM motum habet in singulis punctis M, constituat autem curva in A cum recta AM angulum recto minorem. His positis iam est ostensum tempus, quo corpus ex A in M perveniet, fore infinite magnum. Quare etiam motu retrogrado corpus ex M versus A latum post tempus demum infinitum, i. e. nunquam, in A pertingit, quamvis ubique nisi in A habeat celeritatem finitam. [p. 17]

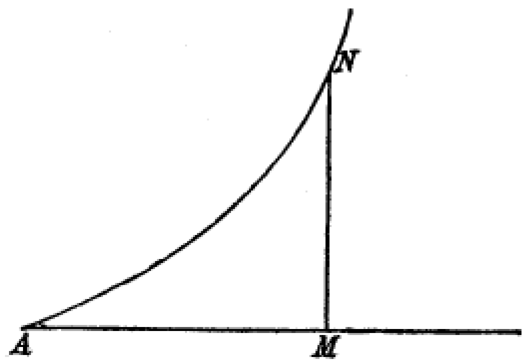

Fig. 4.

PROPOSITIO 5.
Theorema.

44. *Moveantur duo corpora in rectis AM et am* (Fig. 5) *exprimanturque eorum celeritates applicatis curvarum AN et an similium. Dico haec corpora percursura spatia homologa AM et am eodem tempore.*

DEMONSTRATIO.

Sint igitur *AM* et *am* spatia homologa, habebunt ea eandem rationem quam applicatiae *MN* et *mn*; sit ista ratio $m : n$, erit, positis $AM = s$ et $MN = c$, $am = \frac{ns}{m}$ et $mn = \frac{nc}{m}$.
Est vero tempus per $AM = \int \frac{ds}{c}$ (37) ; tempus autem per am habebitur ponendo

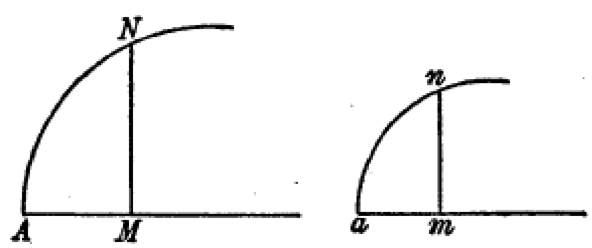

Fig. 5.

$\frac{nds}{m}$ loco ds et $\frac{nc}{m}$ loco c in $\int \frac{ds}{c}$. Hoc vero facto iterum prodit $\int \frac{ds}{c}$: quare utrumque tempus per AM et am erit $\int \frac{ds}{c}$, sunt igitur ea aequalia. Q. E. D.

Corollarium 1.

45. Intelligitur hinc quoque ratio eius, quod (41) est dictum; sunt enim circuli omnes curvae similes et diametri spatia homologa.

Corollarium 2.

46. Sit curvae *AN* parameter *a*, quae sive maior sive minor accipiatur, curva *AN* mutetur in aliam sui similem. Hoc vero ut eveniat, huiusmodi debet esse aequatio pro curva *AN*, ut applicata *c* aequatur functioni ipsarum *a* et *s* unius tantum dimensionis. Pro varis autem valoribus ipsius *a* *s* exprimet spatia homologa, si accipiatur = *a* vel *na*. Quoties [p. 18] igiter *c* huiusmodi definitur aequatione, spatia *na*, sive magnum sive parvum ponatur *a*, aequalibus percurrentur temporibus.

Scholion.

47. Quemadmodum, si *c* aequatur functioni ipsarum *a* et *s* unius dimendionis, tempora per *a* vel *na* sunt omnia aequalia, quicquid sit *a* : ita etiam si fuerit *c* aequal functioni ipsarum *a* et *s*, quae habeat *m* dimensiones, tempora per *a* vel *na*, quicquid sit *a*, tenebunt rationem a^{1-m}. Nam $\frac{c}{a^{m-1}}$ erit functio unius dimensionis ipsarum *a* et *s*, quae ponatur *k*. Erit ergo $c = a^{m-1}k$ et $\int \frac{ds}{c} = a^{1-m}\int \frac{ds}{k}$. At $\int \frac{ds}{k}$ dabit, posito $s = a$ vel *na*, quantitatem constantem, utcunque varietur a (46) . Quamobrem $a^{1-m}\int \frac{ds}{k}$ dabit multiplum quoddam posestatis a^{1-m}. Erit consequenter tempus per *na* ut a^{1-m}.

DEFINITIO 6.

48. *Scala celeritatum est curva, cuius applicatae repraesentant celeritates, quas corpus motum habet in locis respondentibus spatii, quod percurrit.* Ita corporis in recta AM (Fig. 6) moti scala celeritatum est curva AN, cuius applicatae MN exponunt celeritatem corporis in singulis punctis M.

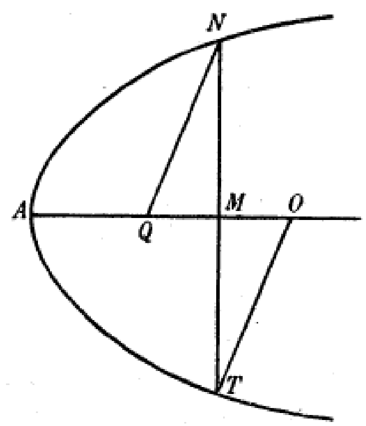

Fig. 6.

DEFINITIO 7.

49. *Scala temporum est curva, cuius applicatae repraesentant tempora, quibus partes spatii percursi respondentes absolvuntur.* Ita si curva AT (Fig. 6) fuerit eiusmodi, [p. 19] ut eius applicata quaevis MT exhibeat tempus, quo spatium AM percurritur, curva AT scala temporum.

Corollarium.

50. Quemadmodum ex data scala celeritatum AN inveniri debeat scala temporum, iam ex praecedente problemate (37) apparet. Scilicet si dicatur spatium $AM = s$, celeritas in M, i. e. $MN, = c$ et tempus, quo AM percurritur i. e. $MT = t$, erit $t = \int \frac{ds}{c}$. Ex data igitur curva AN concessis quadraturis construi potest curva AT.

PROPOSITIO 6.

PROBLEMA.

51. *Data scala temporum AT* (Fig. 6) *invenire et construere scalam celeritatum AN.*

SOLUTIO.

Ponatur ut ante $AM = s$, $MN = c$ et $MT = t$, oportebit ex data aequatione inter s et t inveniri aequationem inter s et c. Facile vero hoc efficietur ex supra invento canone $t = \int \frac{ds}{c}$. Fit enim differentiando $dt = \frac{ds}{c}$ atque $c = \frac{ds}{dt}$. Ducatur ergo ad curvam AT in T normalis TO, erit $\frac{ds}{dt} = \frac{MT}{MO}$. Fiat ergo ut MO ad MT, ita linea quaedam unitate expressa, qua minutum secundum indicatur, ad quartam proportionalem, quae erit $= MN$. Sumantur igitur ab M intervallum $MQ = 1$, et ducatur QN parallela normali TO, erit punctum N in scale celeritatum quaesita. Q. E. I. [p. 20]

EXEMPLUM 1.

52. Sit scala temporum linea recta ad AM utcunque inclinata; erit $t = ms$ et $dt = mds$. Prodibit igitur $c = \frac{ds}{dt} = \frac{1}{m}$. Scala celeritatum ergo erit linae recta ipsi AM parallela, atque corpus motu feretur aequabili.

EXEMPLUM 2.

53. Sint tempora ut potestates quaecunque spatiorum descriptorum, seu $t = s^m$ ideoque $dt = ms^{m-1}ds$. Ex quo erit $c = \frac{1}{ms^{m-1}} = \frac{1}{m}s^{1-m}$. Quare si curva AT fuerit parabola APOLLONIANA, i. e. $t = s^{\frac{1}{2}}$, erit $m = \frac{1}{2}$ atque $c = 2s^{\frac{1}{2}}$. Ex quo apparet hoc casu scalam celeritatum quoque esse huiusmodi parabolam.

Corollarium.

54. Intelligitur etiam, si detur aequatio inter c et t, quomodo inveniendum sit spatium percursum s atque utraque scala celeritatum et temporum. Quia emim est $c = \frac{ds}{dt}$, erit $ds = cdt$ et $s = \int cdt$.

Scholion.

55. Monendum hic est ista, quae hactenus de scalis celeritatem et temporum sunt tradita, non solum ad motum absolutum spectare, sed etiam ad relativum pertinere. Nondum enim ipsa motus natura est considerata, neque quicquam est assumtum, quod motui absoluto esset proprium. Nunc vero afferemus quasdam propositiones, quae motui absoluto sunt peculiares, [p. 21] ex quibusque quodammodo intera inter motus absolutos et relativos differentia poterit perspici.

PROPOSITIO 7.

THEOREMA.

56. *Corpus absolute quiescens perpetuo in quiete perseverare debet, nisi a causa externa ad motum sollicitetur.*

DEMONSTRATIO.

Concipiamus corpus hoc existere in spatio infinito atque vacuo, perspicuum est nullam esse rationem, quare potius in hanc vel illam plagam moveatur. Consequenter ob defectum sufficientis rationis, cur moveatur, perpetuo quiescere debebit. Neque vero haec ratio in mundo cessat; quamvis obiici posset esse in mundo sufficientem rationem, quare in hanc potius quam illam plagam cedat. Etenim non est credendum in spatio infinito illo et vacuo defectum sufficientis rationis ad motum unicam esse causam permansionis in quiete; sed nullum est dubium, quin in ipsa corporis natura sita sit cause huius phaenomeni. Defectus scilicet sufficientis rationis non potest pro vera et essentiali cuiusquam eventus causa haberi, sed tantum veritatem idque rigide demonstrat. Quin et simul indicat in ipsa rei natura occultam esse causam veram essentialem, quae non cessat, cessante illo sufficientis rationis defectu. Ita ARCHIMEDIS demonstratio de aequilibrio bilancis utrinque sibi similis non solum in vacuo, sed etiam in mundo rei veritatem evincit. Alia autem eaque [p. 22] genuina datur huius aequalibrii ratio, quae etiam in mundo locum habet. Cum igitur in vacuo spatio verum sit corpus quiescens in quiete permanere debere, erit in ipsa corporis natura etiam huius rei ratio posita propter quam in mundo quoque corpus, quod semel quiescit, nisi ab alia causa urgeatur, in quiete persistere cogitur. Q. E. D.

Corollarium 1.

57. Est igitur lex in ipsa rerum natura fundata, quod omne corpus quiescens, nisi ab alia causa externa ad motum sollicitetur, in quiete debeat perseverare.

Corollarium 2.

58. Quemmadmodum fundamentum huius demonstrationis ex ipsa quietis absolutae naturae est petitum, perperam ista lex ad quietem relativam extenditur.

Scholion.

59. Experientia autem ipsa edocemur hanc legem in quiete relativa non valere. Videmus enim corpora in navi relative quiescentia, si navis subito concutiatur, in quiete non permanere, sed simul concuti et de loco suo moveri, etiamsi ante quievissent, nullaque accessisset causa ea commovens.

Corollarium 3.

60. Simili modo, quo evicimus corpus semel quiescens perpetuo quiescere debere, nisi a causa externa afficiatur, potest ostendi, corpus, quod nunc quivescit absolute [p. 23], ante hac semper quoque quievisse, siquidem sibi ipsi fuerit relictum. Ut enim nulla est ratio, quare potius ex hac quam illa plaga in eum, quo nunc stat, locum pervenerit, ita concludendum est etiam in eo loco antea semper constitisse.

Corollarium 4.

61. Corpus igitur, quod semel quiescit, si ulla causa externa in id neque agat neque egerit, id non solum in posterum quiescet semper, sed etiam ante perpetuo quievisse statuendum est.

Corollarium 5.

62. Sequitur ex hoc corpus semel absolute motum in quietem pervenire nunquam posse sibi relictum. Nam si tandem quiesceret, idem oporteret antea quoque semper quievisse, quod est contra hypothesin.

PROPOSITIO 8.

THEOREMA.

63. *Corpus absolutum habens motum aequabiliter perpetuo movebitur, et eadem celeritate iam antea quovis tempore fuit motum, nisi causa externa in id agat aut egerit.*

DEMONSTRATIO.

Si enim corpus motum celeritatem non conservaret semper eandem, tum vel augeri deberet vel diminui eius celeritas. Hoc autem casu ad quietem inclinaret, quod, quia nunquam quietem consequi potest (22), accidere nequit. [p. 24] Illo casu vero ex quiete provenisse censendum esset, quod aeque foret absurdum. Praeterea si hoc corpus in spatio infinito et vacuo positum concipiatur eiusque via, qua est ingressum et ingrediatur, consideretur, nulla est ratio, quare potius in hoc maiorem minoremve habeat celeritatem quam in illo loco, quocirca perpetuo eadem moveri debebit celeritate. Q. E. D.

Corollarium.

64. Quoties igitur corpus motum vel celerius vel tardius ingredi videmus, causae externae hanc mutationem adscribere debemus.

PROPOSITIO 9.

THEOREMA.

65. *Corpus absoluto motu praeditum progredietur in linea recta, seu spatium, quod describit, erit linea recta.*

DEMONSTRATIO.

Nulla est enim ratio, si corpus hoc in spatio infinito et vacuo positum concipiatur, quare in hanc potius quam aliam regionem a linea recta declinaret. Et quo concludendum est, hoc ab ipsa corporis natura pendere, ut motum in linea recta progrediatur. Quamobrem in mundo etiam, ubi quidem hoc sufficientis rationis principium cessat, nihilominus statuendum est omne corpus motum in directum progredi debere, nisi scilicet impediatur. Q. E. D. [p. 25]

Corollarium 1.

66. Ex his duabus propositionibus consicitur ista lex universalis : omne corpus motu praeditum aequabiliter in linea recta progredi.

Corollarium 2.

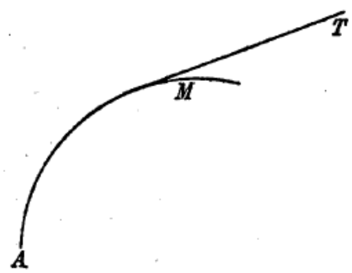

Fig. 7.

67. Corpus ergo, quod a causis externis coactum fuit in linea curva AM (Fig. 7) progredi, si, cum in M pervenerit, causae hae externae subito cessent, tum ea celeritate, quam habebat in M, aequabiliter secundum directionem, quam ipso tempore liberationis habuit, in recta progredietur. Est vero tangens MT nil aliud nisi curvae elementum in M in directum productum, quamobrem corpus in M sibi relictum in tangente MT aequabiliter ea celeritate, quam in M habuit, progredietur.

Scholion I.

68. Has de absoluta quiete et motu leges auctores in una sunt complexi. Hancque NEUTONUS in *Principiis Phil.* ita proponit, ut dicat : Omne corpus perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Corollarium 3.

69. Pertinent autem hae leges de motus continuatione ad motus tantum absolutos, neque eae in motibus relativis vim suam retinent. Quemadmodum enim fieri potest, ut corpus relative quiescens non perseveret in quiete, etiamsi a nullis causis externis agitetur (59), [p. 26] ita etiam corpora motum habentia relativum non semper aequabiliter in directum relative movebuntur.

Corollarium 4.

70. Quando igitur corpus a nullis causis externis est sollicitatum, id, quomodocunque inaequabiliter relative moveatur, tamen vel absolute quiescere vel uniformiter in directionem moveri consendum est. Ex hocque quodammodo potest intelligi, quantum status relativus ab absoluto differat.

Scholion 2.

71. In Astronomiae principiis mechanicis, prout a Neutono sunt tradita, statuuntur sol et stellae fixae a causis externis vel omnino non affici, vel tam parum, ut effectus sit insensibilis. Quanquam igitur solem neque aequabiliter neque in directum progredi videmus respectu terrae, tamen eum absolute vel quiescere vel uniformiter in directum moveri certum erit. Difformitates ergo illae in motu solis observatae in ipsa terra positae sint necesse est.

DEFINITIO 8.

72. *Motus directo sive determinato est linea recta, in qua corpus motum uniformiter progredi conatur, et re ipsa progreditur, nisi a causis externis impediatur.*

Corollarium.

73. Corpus igitur habens motum absolutum, nisi ab aliis causis afficiatur, perpetuo eandem motus directionem eandemque celeritatem conservabit. [p. 27]

DEFINITIO 9.

74. *Vis inertiae est illa in omnibus corporibus insita facultas vel in quiete permanendi vel motum uniformiter in directum continuandi.*

Corollarium.

75. Quanquam enim permansionem in quiete motusque uniformem continuationem in directum ex principio sufficientis rationis demonstravimus, tamen iam notavimus hanc non esse causam phaenomeni efficientem, sed eam in ipsa corporis natura esse sitam. Haec igitur ex corporum natura pendens causa status sui conservationis est id, quod vis inertiae appellatur.

Scholion.

76. KEPLERUS, qui primus hanc vocem formavit, tribuit eam ei vi, quam omnia habent corpora, resistendi omni illi, quod ea de statu suo deturbare conatur; atque haec vox inertiae melius cum hac resistentiae ideae congruit quam illa perseverantiae, cum qua nos coniunximus. Sed facile intelligitur has definitiones re a se invicem non differre ; eadem enim est vis motum vel quietem continuans, et quae impedimentis resistit. Malui vero hac uti definitione quam KEPLERIANA, quia nondum constat, quomodo corpora viribus sollicitantibus resistant. Praeterea vero haec ipsa resistendi vis originem habet suam ab hac quietem motumve continuandi facultate; ideoque ex hac debet explicari. [p. 28]

PROPOSITIO 10.

THEOREMA.

77. *Quando spatium, ex quo motus relativus determinatur, absolute vel quiescit vel movetur uniformiter in directum, tum leges datae de motu et quiete etiam in statu corporum relativo valebunt.*

DEMONSTRATIO.

Si spatium, ex quo motus relativus diiudicatur, absolute quiescit, propositum per se est clarum. Nam hoc casu quies et motus relativi cum absolutis congruent, adeoque omne corpus etiam relative vel perpetuo quiescet vel uniformiter movibitur in directum (10). Sin vero illud spatium ipsum moveatur uniformiter in directum, tum ea corpora, quae relative quiescunt, eundem habebunt motum absolutum, quem habet ipsum spatium. Quare ea quoque uniformiter in directum progrediuntur huncque motum ex suo natura poterunt continuare : ut igitur et hoc casu lex (66) observetur. Corpus vero, quod relative movetur uniformiter in directum, id etiam, si ipsum spatium uniformem habet motum rectilineum, aequabiliter in recta progredietur absolute, quemadmodum tum ex sequente apparebit propositione, tum per se perspicuum est. Motus ergo hic relativus quoque legi est consentaneus et propterea sine vi externa continuari poterit. Q. E. D. [p. 29]

Corollarium 1.

78. Corpus igitur a nulla causa externa affectum, quod relative vel quiescit vel uniformiter movetur in directum, indicio erit ipsum spatium ad quod eius motus indicatur, absolute vel quiescere, vel in directum aequabiliter moveri.

Corollarium 2.

79. Talis quoque motus relativus in suo statu per se ipsum perpetuo conservabitur. Non solum enim ipsum, quod movetur, corpus absolute movetur uniformiter in directum, sed etiam spatium illud, quo relatio aestimatur, iuxta eandem legem progreditur. Quamobrem uterque per se continuabitur, atque motus relativus iste in hoc statu nulla accedente causa externa perseverabit.

Corollarium 3.

80. Quia omnis idea, quam de motu habemus, est relativa (7), hae quoque leges non sufficiunt ad cognoscendum, qualis sit cuiuspiam corporis motus absolutus. Quando enim corpus a nulla cause externa affectum aequabiliter in recta progredi videmus, plus inde concludere non possumus, quam hoc corpus etiam absolute vel uniformiter in directum moveri vel quiescere. Quantus vero sit eius motus absolutus, definire non licet, neque quam habeat directionem.

Corollarium 4.

81. Quae igitur ex hac corporum natura, quod in statu suo vel quietis vel motus uniformis in [p. 30] rectum permaneant, deducentur, non solum ad motum et quietem absolutam pertinebunt, sed etiam ad eum statum relativum, quo spatium corpusve, ex quo motus aestimatur, uniformite in directum progreditur.

Scholion.

82. Atque etiam non admodum erimus solliciti de motu absoluto, cum iste relativus iisdem contineatur legibus. Et propteria motum hunc relativum ipsum saepius mutabimus in alios huiusmodi, ita tamen, ut traditae leges observentur : si scilicet eum, relatione ad aliud corpus uniformiter in directum quoque progrediens facta, contemplabimur. Qua ratione non cessabit aequabiliter in recta progredi, idque innumerabilibus modis fieri potest, ex quibus, commodissimus erit, seligi poterit.

PROPOSITIO 11.

PROBLEMA.

83. *Moveatur corpus absolute aequabiliter in recta AL* (Fig. 8) *aliudque corpus aequabiliter quoque in recta AM. Quaeritur corporis in AL absoluto motu lati motus relativus respectu corporis alterius in AM progredientis.*

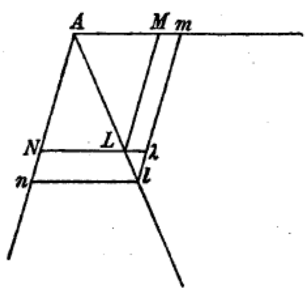

Fig. 8.

SOLUTIO.

Sit celeritas corporis in AL progredientis *a*; et celeritas alterius in *AM* moti *b* : simulque egrediantur haec corpora ex puncto A. Perspicuum est, si sumantur duo spatia *AL, AM* in ratione celeritatum *a* et *b*, [p. 31] ambo corpora eodem momento in L et *M* pervernire. Ducta igitur recta *ML* cum recta *AM* angulum faciente *AML*, cuius sinus est ad sinum anguli *ALM*, quem cum *AL* efficiet, ut *AL* ad *AM*, i. e. ut *a* ad *b*, designabit *L* locum in quo reperietur corpus in *AL* progrediens eodem momento, quo alterum in *M* existit. Quia vero corporis illius motus relativus respectu huius desideratur, hoc, quod in *AM* revera movetur, ut quiescens in *A* debet considerari. Puncto igitur *M* cogitatione in *A* translato perveniet *L* in *N* ducta *AN* parallela et aequali ipsi *ML* ex *A*. Simili modo quando corpus in *AM* motum pervenit in locum proximum *m*, alterum in *l* reperietur, eritque *ml* parallela ipsi *ML*, quia $Mm : Ll = b : a = AM : AL$. Puncto vero *m* simili modo in *A* translato sumendo $An = ml$ perveniet *l* in *n*, eritque *n* in eadem recta AN. Ex quo sequitur corpus absolute in AL motum relative in recta AN moveri. Celeritas autem relativa erit ad absolutam ut *Nn* ad *Ll*, seu ut *ML* ad *AL*. Quae ratio cum sit constans ob triangulum *ALM* specie datum, corpus absolute in *AL* aequibiliter motum relative quoque aequabiliter in recta *AN* progredietur. Positio vero rectae *AN* invenietur sumendo angulo *LAN* tanto, ut eius sinus sit ad sinum anguli *NAM* ut *b* ad *a*. Celeritas denique absoluta per *AL* erit ad celeritatem relativam per *AN* ut sinus anguli *MAN* ad sinum anguli *LAM*. Q. E. I. [p. 32]

Corollarium 1.

84. Corpus igitur absolute aequabiliter in directum progrediens quoque relative aequabiliter in directum promovebitur, si modo corpus, ex quo relatio iudicatur, quaque aequabiliter in directum progrediatur. Atque hoc est, quod in praecedente demonstratione (77) assumsimus.

Corollarium 2.

85. Constructio ceterum lineae *AN* et celeritatis relativae inventio facillime hoc modo institui potest. Sumtis, ut iam fecimus, *AL* et AM in ratione *a* ad *b* ductaque *ML*, ducatur huic *ML* parallela *AN* ex *A*, erit haec via motu relativo descripta. Celeritas vero relativa erit ad absolutam ut *ML* ad *AL*.

Corollarium 3.

86. Idem ratiocinium valet, si *AL* non motu absoluto, sed relativo percurratur et *AM* eadem relatione. Tum vero prodibit corporis per *AL* moti alius motus relativus respectu corporis *AM* lati.

Corollarium 4.

87. Patet igitur, quomodo motus absolutus in infinitos relativos possit transmutari, qui semper erunt aequabilis et in directum fient, si modo motus absolutus fuerit huiusmodi et motus eorum corporum, ex quibus relativi oriuntur. [p. 33]

Scholion.

88. Assumsimus in solutione ambo corpora ex aedem loco *A* egredi : sed solutio non minus succedit, si ambo corpora in principio in diversis punctis *A* et *B* (Fig. 9) fuerint posita. Nam progrediatur corpus *A* motu absoluto aequabiliter in recta *AL*, alterum vero *B* similiter in recta *BM*, ita ut celeritates sint ut *a* ad *b*. Sumantur *AL* et *BM* in eadem ratione *a* ad *b*, pervenient ambo corpora simul in *L* et *M*. At quia corporis *A* motus relativus respectu corporis *B* requiritur, corpus *B* ut quiescens in *B* debet considerari. Transferatur ergo cogitatione corpus *B* ex *M* in *B*, perveniet corpus *A* ex *L* in *N*, ducendo *BN* parallelam et aequale ipsi *ML*: dico punctum *N* fore in recte per *A* transeunte, ita ut corpus *A* relative moveatur in recta *AN*, idque aequabiliter. Ducta enim *NL* aequalis erit et parallela ipsi *BM*. His factis specie datur triangulum *ANL* : quare *NL* ad *AL* habebit rationem datam; ergo, ob *NL* = *BM*, erit ratio *AL* ad *BM* data, quae ergo, si semel sumta fuerit in ratione *a* ad *b*, semper erit eadem. Ex quo apparet punctum *N* esse in recte *AN* et celeritatem relativam per *AN* esse ad absolutam per *AL* ut *AN* ad *AL*, i. e. in ratione data. Motus igitur relativus per *AN* fiet in recta eritque aequabilis.

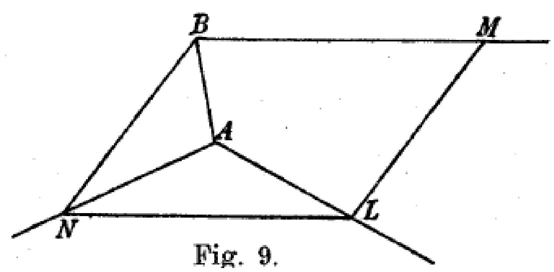

Fig. 9.

Corollarium 5.

89. Si igitur detur corporis *A* (Fig. 9) motus absolutus per rectam *AL* eiusque relativus aequabilis per *AN* [p. 34] quacunque celeratate, poterit inveniri motus corporis *B*, cuius respectu motus relativus corporis *A* oritur. Sumtis enim duobus spatiis *AL* et *AN* eodem tempore percursis ducatur per punctum quodvis arbitrarium *B* recta *BM* parallela ipsi *NL*: determinabit haec viam a corpore *B* percursam, eiusque celeritas erit ad celeritatem corporis *A* absolutam per *AL*, ut est *NL* ad *AL*. Erit vero corpus *B* in *B* eodem tempore, quo est *A* in *A*.

Corollarium 6.

90. Dantur ergo innumerabiles motus corporis *B*, quia punctum *B* pro lubitu potest assumi, ex quibus motus relativus corporis *A* idem provenit. At corporis *B* celeritas semper erit eadem eiusque directo secundum parallelam ipsi *NL*.

Corollarium 7.

91. Intelligitur etiam motum absolutum aequabilem in directum tendentem transmutari posse in relativum quemcunque itidem aequabilem et in recta factum. Potest enim recta *AN* pro arbitrio duci et celeritas per eam poni quaecunque. Semper enim datur motus aequabilis quoque et recta progrediens corporis *B*, ex quo hic motus relativus existit.

Corollarium 8.

92. Motus deinde iste relativus per se sine ulla vi externa poterit continuari. Motus enim absoluti per *AL* et per *BM*, quia fiunt aequabiliter in lineis rectis, [p. 35] per se continuantur. Quamdiu vero isti motus durant, tamdiu etiam relativus per *AN* continuare debet.

PROPOSITIO 12.

PROBLEMA.

93. *Moveatur corpus A* (Fig. 10) *absolute quomodocunque in lina AL et corpus B in linea BM. Requiritur motus relativus corporis A respectu corporis B.*

SOLUTIO.

Abscindantur in curvis *AL* et *BM* arcus, qui aequalibus temporibus percurruntur, *AL* et *BM*. Reperietur ergo corpus *A* in *L*, quando *B* in *M* pertingit. Sed quia corporis *A* motus relativus respectu *B* desideratur, corpus *B* ut quiescens in *B* debet considerari. Quare transferatur id cogitatione ex *M* per rectam *MB* in *B*, pervenietque corpus *L* in *N*, ducta *LN* parallela et aequali ipsi *MB*. Curva igitur, in qua est punctum N hoc modo inventum, erit via a corpore *A* motu relativo descripta. Atque hoc motu relativo arcus *AN* eodem tempore percurretur, quo arcus *AL* et *BM* absolvuntur. Ex quo celeratas relativa in *N* quoque innotescit. Q. E. I.

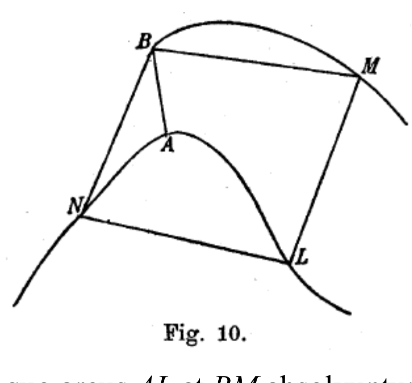

Fig. 10.

Corollary 1.

94. Determinatur igitur hoc modo poterit motus relativus corporis quocunque motu lati respectu corporis quomodocunque etiam moti. [p. 36]

Corollary 2.

95. Intelligatur etiam ex solutione, quomodo datis curvis *AN* et *AL* una cum motibus per eas inveniri possit curva *BM* et motus per eam. Nec non definetur curva AL ex curvis *BM* et *AN*.

Corollary 3.

96. Perspicuum quoque est curvam *BM* ob punctum *B* arbitrarium infinitis modis posse aliter esse positam. Quia tamen arcus *BM*, qui eodem tempore, quo *AL* et *AN*, describitur, subtensa *BM* semper est aequalis et parallela lineae *LN*, ea semper erit sibi similis et aequalis et parallela, atque motus per eam perpetuo idem.

Scholium.

97. Et haec sunt, quae de comparatione motuum absolutorum et relativorum afferenda iudicavi. Motus autem relativus alias hoc modo solet describi, ut motus per *AN* dicatur motus corporis *A*, quod revera in linea *AL* movetur, qualis ex corpore *B* in *BM*, moto spectatur. Spectator vero in corpore *B* relative quiescens ponitur et ipsum *B* ut quiescens considerans. Ita motus stellarum relativus respectu telluris congruit cum eo motu, quem nos in terra degentes eamque tanquam quiescentem considerantes intuemur. Terra enim ex *B* in *M* promota et stella ex *A* in *L*, videmus eam ex *M* secundum plagam *ML* et in distantia *ML*. Quia vero nos de loco *B* motos esse nos apprehendimus, sed etiam nunc in *B* consistere putamus, videbimus stellam [p. 37] ex *B* non in *L*, sed in *N*, eadem scilicet plaga eademque distantia. Propterea recta *BN* aequalis erit et parallela rectae *ML*, ut nostro considerandi modo invenimus.

SCHOLIUM GENERALE.

98. Istae motus leges, quas corpus sibi relictum vel quietem vel motum continuando observat, spectant proprie ad corpora infinite parva, quae ut puncta possunt considerari. In corporibus enim finitae magnitudinis, quorum singulae partes alios habent motus insitos, quaelibet pars quidem has leges observare conibitur, quod autem non semper propter corporis statum fieri potest. Corpus igitur ipsum eum sequetur motum, qui ex singularum partium conatibus componitur, isque adhuc ob insufficientiam principiorum non potest definiri, sed haec tractatio ad sequentia est differenda. Diversitas igitur corporum suppeditabit nobis operis divisionem primariam. Primo enim contemplabimur corpora infinite parva seu quae tanquam puncta spectari possunt. Deinde corpora finitae magnitudinis aggrediemur ea, quae sunt rigida neque figuram suam mutari patiuntur. Tertio agemus de corporibus flexibilibus. Quarto de iis, quae extensionem ex contractionem admittunt. Quinto plurium corporum solutorum motus examine subiiciemus, quorum alia impediunt, quin motus suos possint, ut conantur, absolvere. Sexto vero de motu fluidorum erit agendum. De his vero corporibus non solum videbimus, quomodo sibi [p. 38] relicta motus continuent; sed praeterea inquiremus, quomodo ea a causis externis scilicet potentiis afficiantur. Denique in his omnibus disquisitionibus magnam inferet varietatem status corporum vel liber vel non liber. Per statum non liberum hic intelligo, quando corpora impediuntur, quo minus in ea directione progrediantur, qua conantur; cuiusmodi est motus corporum pendulorum, quae, quia non possunt directe, uti conantur, descendere, oscillationes efficiunt. Ex quo intelligitur statum liberum esse, quando corpora nullum inveniung impedimentum in quamvis plagam progrediendi, in quam tum ex propria vi, tum a potentiis sollicitata tendunt. Apparet igitur, quibus de rebus in Mechanica sit agendum, et quam sint multa, quae etiam nunc nequidem sunt libata. Nam praeter motum punctorum, quae adhuc sunt tractata, tam pauca sunt, ut fere omnia demum invenire et ex principiis derivare necesse sit. Incipio igitur a motu punctorum liberorum a potentiis quibuscunque sollicitatorum, quia, quos sibi ipsa relicta sequantur motus, hoc capite iam est ostensum. Hanc ob rem primum istum Tomum motui punctorum libero destinavi, in sequente vero punctorum motum non liberum pertractare constitui; in quorum utroque, quae occurrent, cum ex his iam traditis, tum ex sequentibus principiis methodo analytica sum derivaturus.