A list of the Definitions and Propositions in Book II of Euler's *Mechanica*.

This list should enable you to locate the sections of interest in the various chapter files.

CH. I: Concerning Motion Which In General Is Not Free.

PROPOSITION 10. Theorem.

74. *The force that a body moving on a surface under the action of no external forces exercises on the surface is made normally to this surface towards the convex side and has the ratio to the force of gravity as the height corresponding to the speed of the body is to half the radius of osculation of the curve described by the body.*

PROPOSITION 11.Problem.

79. *To determine the effect of any kind of force that acts upon a body on a given surface either in a vacuum or in a resisting medium.*

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CH. IIa: CONCERNING THE MOTION OF A POINT ON A GIVEN LINE IN A VACUUM.

PROPOSITION 12. Problem.

83. *A body which is moving on the curve AM is acted on everywhere by a force MF, the direction of which is parallel to the axis AP; to determine the speed of the body at individual points and the time taken for any part of the curve to be described by the body, with the force due to the curve acting on the body at individual points.*

PROPOSITION 13. Problem.

93. *If the force acting is uniform and acting downwards everywhere, to determine the descent of the body on a given curve AM, beginning from A at rest, and to find the force pressing on the curve at individual points M.*

PROPOSITION 14. Problem.

106. *If there is an infinite family of similar curves AM, AM, etc. beginning from the fixed starting point A, to find a curve CMM from these other curves cutting the arcs AM, AM, etc, which are traversed in equal times by a body descending along these arcs, as before, with a uniform force present acting downwards everywhere.*

PROPOSITION 15. Problem.

124. *If as before there is an infinite family of similar curves AM, AM, etc. and the straight line DE in the position given, to find that curve AMN, upon which the body arrives at the line DE in the shortest possible time from A.*

PROPOSITION 16. Theorem.

132. *The times of the descent, by which a body placed on the curves traverses the curves AM and Am, etc. similar and similarly from the point A, are in the ratio of the square root of the homologous sides.*

PROPOSITION 17. Problem.

137. *With the force present acting uniformly downwards, a body moves on some curve AM with an given initial speed at A; to determine the motion of the body on this curve and the force pressing the body to the curve sustained at individual points.*

DEFINITION 3.

147. *Oscillatory motion is reciprocal motion in which the body alternately approaches and recedes from the starting point of the motion M . Thus if the body is moving on a given curve MAN , first it descends on MA, then it ascends on AN, while it loses speed; then it descends from N again and ascends on the arc AM, with which done it again descends and this periodic motion continues. Such motion is called oscillatory.*

CH. IIb:

PROPOSITION 18. Problem.

161. *With a uniform force present acting in the downwards direction, to determine the time of the ascent or the descent through any arc of a circle EA, ending at the lowest point A.*

PROPOSITION 19. Problem.

177. *For a given force acting, to find the length of the pendulum making indefinitely small oscillations, which completes a to and fro motion in a time of one second.*

PROPOSITION 20. Problem.

187. *If the curve BAD, upon which oscillations are made, is a cycloid described by the circle with diameter AC on the horizontal base BD, to determine the time of the oscillation through each arc EAF, with a uniform force acting downwards.*

PROPOSITION 21. Problem.

195. *If a body is always drawn towards a fixed centre C by some force and it is moving on a given curve AM, to determine the motion of the body on this curve, and the force it exerts on individual points of the curve*

PROPOSITION 22. Problem.

205. *If a body is always drawn towards a centre of force by a centripetal force C and let the curve EAF be suited to oscillations, the determine the oscillatory motion of the body on this curve.*

PROPOSITION 23.Problem.

214. *If a body is acted on by any two forces, of which the direction of one is along the vertical MQ, and the other MP is horizontal ,to define the motion of the body from these forces acting on a given curve AMB.*

PROPOSITION 24. Problem.

218. *With a uniform force acting in a downwards direction, to determine the motion of a body on some curve AM not set up in the same plane.*

CH. IIc:

PROPOSITION 25. Problem.

224. *If a body is drawn downwards by some constant force, to find the curve AM , that a body descending on that curve presses upon equally everywhere.*

PROPOSITION 26. Problem.

231. *With the curve AM* given, *and with the initial speed at A corresponding to the height b, to find the size of the force always acting downwards, which arises in order that a body descending along the curve AM exerts the same force everywhere on the curve.*

PROPOSITION 27. Problem.

239. *If a body is always drawn downwards by some force, to find the curve AM* (Fig. 32), *upon which the body is thus moving, in order that the total compression force sustained by the curve has a given ratio to the compression arising from the normal force.*

PROPOSITION 28. Problem.

253. A body is acted on by some force acting downwards *; to find the curve AM, on which the body descends in a uniform motion downwards as it is carried forwards or equally as it recedes to the horizontal AB.*

PROPOSITION 29. Problem.
262. *With a constant force everywhere pulling vertically downwards, to find the curve AM, upon which a body is progressing uniformly along a given direction AP.*
PROPOSITION 30. Problem.
270. *Under the hypothesis of a force acting uniformly and tending downwards, to find the curve AM, upon which a body descends uniformly receding from a given point Ct.*

CH. IId:
PROPOSITION 31. Problem.
282. *With a uniform force present acting downwards, to find the curve AM, upon which a body with a given initial speed moves thus, so that in equal times equal angles are completed about a fixed point C.*
PROPOSITION 32. Problem.
282. *If the body is attracted by some force to the centre of force C, to find the curve AM on which the body is carried with a speed equal to the uniform motion towards C.*
PROPOSITION 33.Problem.
300. *If a body is always attracted to the centre of force C, to determine the curve AM, upon which the body is moving with a uniform angular motion around the centre C.*
PROPOSITION 34.Problem.
306. *Let the uniform force acting be g pulling downwards everywhere and the given curve AT ; to find the curve AM, upon which a body thus descends, so that the time to pass through any arc AM is proportional to the square root of the corresponding applied line PT of the given curve AT.*
PROPOSITION 35.Problem.
316. *With a uniform force acting in the downwards direction, to find all the curves AMC; upon which a body beginning to descend from rest at A, arrives at the horizontal line BC in a given time.*

CH. IIe:
PROPOSITION 36. Problem.
328. *With a uniform force g acting downwards everywhere, to find all the curves AMC, upon which a body descends from rest at A in a given time to a straight line BC inclined at any angle to the horizontal.*
PROPOSITION 37. Problem.
337. *Let the body be always acted on by a constant uniform downwards force g ; to find the innumerable curves upon which the body, starting to move from rest at A, arrives at the vertical line EC in a given time.*
PROPOSITION 38. Problem.
341. *Let the body always be acted on by a uniform downwards force g and any curve is given BSC ; to find all the curves AMC, upon which the body by descending from A in a given time arrives at the given curve BSC.*

PROPOSITION 39. Problem.
358. *Let the body be acted on always by a uniform force g downwards and let the curve AMC be given, upon which the body released from A arrives at some given point C; to find all the curves ANC upon which the body in the same time descends in the same time to the point C from A.*
PROPOSITION 40. Problem.
361. *To find the general rule, following which a curve ought to be disposed, in order that a body descending on that curve arrives at any point on the curve in the shortest possible time.*

CH. IIf:
PROPOSITION 41. Problem.
367. *If a body is always drawn downwards by some force, the find the brachistochrone line AMC, upon which the body descends the most quickly from A to C.*
PROPOSITION 42. Theorem.
377. *Whatever the forces acting should be, that line is a brachistochrone, upon which the moving body presses with a force that is twice as great, as either the centrifugal force alone or the normal force alone.*
PROPOSITION 43. Problem.
385. *If the body is always drawn towards the centre of forces C, to find the brachistochrone line AM upon which the body reaches M from A the quickest.*
PROPOSITION 44. Theorem.
393. *A body from a given point A arrives the quickest at some given line BM on the brachistochrone AM, which crosses the given line BM at right angles, and this under the hypothesis of some force acting.*
PROPOSITION 45. Problem.
401. *Between all the curves joining the points A and C and equal in length, to determine that curve AMC, upon which a body arrives the quickest from A to C, according to the hypothesis that a uniform force g is acting directed downwards.*
PROPOSITION 46. Problem.
408. *To find the general law of tautochronous curves, upon which all descents to the point A are completed in the same time, with the descent taken to start from any point on the curve AM.*
PROPOSITION 47.Problem.
413. *If the body is acted on by some downwards force, to find the tautochrone line upon which all the descents are made in the same time.*
PROPOSITION 48.Problem.
422. *If the body is always drawn towards the centre of forces C by some force, to find the tautochrone line BMA, upon which the body completes all the descend as far as the point A in the same time.*

CH. IIg:

PROPOSITION 49. Problem.

430. *If a body is acted on by any forces, to find the curve AM , upon which all the descents are made in equal times as far as to the point A.*

PROPOSITION 50. Problem.

432. *According to the hypothesis of gravity acting uniformly downwards, if the curve ANB is given, to find the curve BMF adjoined to that curve, so that all the descents upon the composite curve as far as A are completed in equal times, the descent starting from any point of the curve BMF.*

PROPOSITION 51. Problem.

446. *According to the hypothesis of gravity acting uniformly downwards, if the curve AM is given, to find a curve AN of this kind, such that the oscillations which are performed on the composite curve MAN are all isochronous to each other.*

PROPOSITION 52. Problem.

455. *According to hypothesis of uniform gravity acting uniformly downwards, to find the continuous curve MAN, upon which all the semi-oscillations can be completed in equal times.*

CH. IIIa: CONCERNING THE MOTION OF A POINT ON A GIVEN LINE IN A MEDIUM WITH RESISTANCE.

PROPOSITION 53. Problem.

465. *If a body is acted on by a given force g in a medium with whatever resistance, to determine the motion of the body descending on a given curve AM, and the compressive force sustained by the curve at particular points.*

PROPOSITION 54. Problem.

475. *If the body is always acted on by a uniform force g downwards in a medium with some resistance, to determine the motion of the ascending body on the given curve AM and the force pressing on the curve sustained at individual points M.*

PROPOSITION 55. Problem.

481. *In a medium with some kind of uniform resistance and under the hypothesis of uniform gravity g acting, to determine the motion of the body descending on the given straight line AMB inclined in some manner to the horizontal.*

PROPOSITION 56. Problem.

489. *A medium resists in some multiple ratio of the speeds, and the point A is given, from which an infinite number of straight lines AM can be drawn; to determine the curve CMD of this kind, so that a body descending along any line AM has the same speed at M.*

PROPOSITION 57. Problem.

498. *If the resistance were in some multiple ratio of the speed, to find the curve, which has this property, that a body descending on any chord AM arrives at M from A in a given constant time.*

CH. IIIb:

PROPOSITION 58. Problem.

505. *According to the hypothesis of uniform gravity g, and in a medium with some uniform resistance, to determine the motion of the body with the initial speed of ascent given at A on the straight line AB inclined at some angle to the horizontal.*

PROPOSITION 59. Problem.

518. *According to the hypothesis of uniform gravity g and with the resistance proportional to the square of the speed, the body descends on some curve AMB; to determine the motion of this body, and the force sustained by the curve at individual points.*

PROPOSITION 60. Problem.

526. *With the force arising acting uniformly and with a uniform medium with resistance in the square ratio of the speed, to determine the motion of the body ascending on the given curve AMD and the force pressing on the curve sustained at individual points M.*

PROPOSITION 61. Problem.

532. *Let whatever curves MA and NA be joined at the lowest point A and the body descends on the curve [MA and ascends on the curve] AN in a medium with uniform resistance following the square of the speeds; to be compares between themselves are the descends on the curve MA and the ascents on the curve AN.*

PROPOSITION 62. Problem.

540. *Let the given curve be the cycloid ACB described by the circle of diameter CD rolling along upon the given horizontal base AB, and the body descends on that cycloid from A in a medium with the resistance proportional to the square of the speed; to determine the motion of the descending body.*

PROPOSITION 63. Problem.

549. *Let the given curve ACB be a cycloid described on the horizontal base AB and considering the body to complete downwards oscillations on that curve in a medium with resistance in the square ratio of the speeds; to determine the motion of the oscillations.*

CH. IIIc:

PROPOSITION 64.Problem.

565. *If the resistance of the medium is partially constant and partially proportional to the square of the velocity, to determine the motion of a body oscillating on the cycloid MCB, even in the case in which the resistance is very small.*

PROPOSITION 65. Problem.

573. *In a medium that resists in the simple ratio of the speeds, to determine the oscillatory motion of the body on the cycloid ACB with the medium present as with the force acting to be uniform.*

PROPOSITION 66. Theorem.

573. *In a uniform medium, that offers resistance in the simple ratio of the speeds, all the descends on the cycloid AMC are made in equal times and likewise also all the ascents on the cycloid CNB are completed in equal times, if a certain force is acting and it is uniform and directed downwards.*

PROPOSITION 67. Problem.

593. *In a uniform medium, in which the resistance is in a ratio of the multiple of the speeds of which the exponent is 2m, to determine the motion of the body on the curve CMA , in which any arc CM is proportional to a power of the abscissa CP, of which the exponent is 1 – m.*

CH. IIId:

PROPOSITIO 68.Problem.

602. *In a uniform medium, which resists in the ratio of the quadruple of the speeds, so to determine any ascent or descent of the body on the cycloid ACB.*

PROPOSITION 69. Problem.

607. *In a medium that resists in the ratio of the quadruple power of the ratio of the speeds, if the speed of the descent of the body on the curve AMC from a given point A is given at individual points, to find the speed of the same body beginning at some other point E.*

PROPOSITION 70. Problem.

613. *If the resistance were taken as very small with respect to the force acting and proportional to some power of the speeds, to determine the motion of the body on some curve AM.*

PROPOSITION 71.Problem.

621. *In a medium which resists in the ratio of some power of the speeds, to find the curve AM , upon which the body thus descends, so that at the individual points M it has a speed corresponding to the height, which is equal to the corresponding applied line PL of the given curve BL.*

PROPOSITION 72. Problem.

634. *If the curve AM is given, upon which the body is moving in vacuo, to find the curve am, upon which the body moves in a medium with resistance thus descends, so that the speed at a is equal to the speed at A and with equal arcs AM and am taken, so that the speeds at individual points M and m are also equal.*

CH. IIIe:

PROPOSITION 73.Problem.

649. *If the force is uniform and acting downwards and the medium resists according to some power of the ratio of the speed, to determine the curve AM, upon which the body by descending progresses along the horizontal AH at a steady rate.*

PROPOSITION 74. Problem.

655. *To find the curve AM, upon which the body is descending uniformly downwards in a medium with some kind of resistance, with a uniform absolute force directed downwards acting.*

PROPOSITION 75. Problem.

663. *Between all the curves that join the points A and C, to determine that curve AMC, upon which the body descending from A to C acquires the maximum speed with the resistance present as some power of the ratio of the speeds, and with the uniform force acting downwards.*

PROPOSITION 76. Theorem.

673. *In a medium with any kind of resistance and under the hypothesis of absolute forces of any kind, that curve AMC is a brachistochrone or that produces the shortest time of descent between A and C; in which the centrifugal force is equal to the normal force, and directed in the same plane.*

PROPOSITION 77. Problem.

679. *In a uniform medium, which resists in some ratio of the powers of the speeds, and with the absolute force present uniform and directed downwards, to determine the brachistochrone AM upon which the descending body arrives at A from M in the shortest possible time.*

CH. IIIf:

PROPOSITION 78. Problem.

691. *In a medium with some kind of resistance and with some kind of forces acting to find the brachistochrone AM, upon which a descending body arrives the fastest from A to M.*

PROPOSITION 79. Problem.

699. *According to the hypothesis of uniform gravity and a uniform medium, which resists according to some ratio of the speeds, to determine the curve of constant pressure AM , which sustains the same pressing force everywhere by a body descending on it.*

PROPOSITION 80.Problem.

707. *In a uniform medium, which resists in the ratio of the squares of the speeds, and with an absolute force acting downwards, to compare the speeds between themselves at the point A, which are acquired in different descents of the body on the curve MA.*

PROPOSITION 81.Problem.

719. *According to the hypothesis of a uniform force acting in the downwards direction and with a uniform medium, which resists in the ratio of the square of the speeds, to find the tautochrone curve AM, upon which all the descents as far as the point A are completed in equal times.*

CH. IIIg:

PROPOSITION 82. Problem.

738. *In the rarest medium, which resists in some ratio of the powers of the speeds, and according to the hypothesis of a uniform force acting downwards, to determine the tautochrone curve AM, upon which either all the descents or all the ascents are completed in equal times.*

PROPOSITION 83. Problem.

751. *According to the hypothesis of a uniform force acting downwards and a uniform medium that resists in the square ratio of the speeds, for the given curve MA to find the other curve AN of this kind joined to that at A, in order that the body descending on some arc MA on the given curve by ascending on the curve sought completes the arc AN which is equal to the arc MA.*

PROPOSITION 84. Problem.
767. *With these things put in place as before, to find the continued curve of this kind MAN, in order that in some semi-oscillation, which always begins on the arc MA, upon that the arc of the descent MA made is equal to the arc of the following ascent AN.*
PROPOSITION 85. Problem.
771. *According to the preceding hypothesis of gravity and resistance if the curve MA is given, upon which the descent is completed, to find the curve for the ascents with this property, that the time of each ascent is equal to the time of the preceding descent.*

CH. IIIh:
PROPOSITIO 86. Problem.
782. *With everything put in place as before, to find the case in which the two curves MA and AN constitute a single continuous curve, upon which the descent and the following ascent are completed in equal times.*
PROPOSITION 87. Problem.
788. *According to the hypothesis of uniform gravity acting downwards and for a uniform medium with resistance in the ratio of the square of the speeds, if some curve is given MA, upon which the descending body completes the descent, to find a suitable curve AN joined to that for the ascent, such that all the semi-oscillations which are made on the curve MAN are completed in equal times.*
PROPOSITION 88. Problem.
799. *If the curves MA and AN have that property, that all the semi–oscillations which begin on the curve MA, are between themselves isochrones in a medium that resists in the square ratio of the speeds, to determine the case in which these two curves MA and AN joined together constitute one continuous curve.*
PROPOSITION 89. Problem.
811. *According to the hypothesis of gravity acting uniformly downwards g with some given curve am for the descents in vacuo to find the curve AM for the descents in a medium of this innate character with uniform resistance in the ratio of the square of the speeds, in order that all the descents on MA are isochrones with respect to all the descents on ma, if the speeds at the bottom points a and A are equal.*

CH. IVa: CONCERNING THE MOTION OF POINTS ON A GIVEN SURFACE.
PROPOSITION 90. Problem.
821. *For a given path $Mm\mu$ on some surface to find the position of this path with respect to a given plane APQ, and of the radius of osculation of this path at M, as long as neither the position nor the length of the radius lies on the surface.*
PROPOSITION 91. Problem.
833. *In any given surface to determine the line, that the body describes in that motion, acted on by no forces either in a vacuum or in a medium with some kind of resistance.*
PROPOSITIO 92. Problem.
845. *To determine the effect of the pressing force on the body moving on any surface, that is not acted on by any additional forces.*

PROPOSITION 93. Problem.
849. *To determine the effect on the motion of the body of the tangential force that pulls the body along the tangent line MT on some surface.*
PROPOSITION 94. Problem.
851. *To determine the effect of the normal deflecting force N on a body moving on any surface.*

CH. IVb:
PROPOSITION 95. Problem.
856. *If a body moving on a surface is acted on by any forces, to define the normal forces, evidently the normal force pressing* [*on the surface*], *the deflecting force* [*in the tangent plane normal to the curve*], *and the force along the tangent, with all arising from resolution.*
PROPOSITION 96. Problem.
864. *According to the hypothesis of gravity g acting downwards uniformly, to determine the line that a body projected on some surface in vacuo describes.*
PROPOSITION 97. Problem.
869. *According to the hypothesis of uniform gravity g acting downwards, to determine the motion of a body on the surface of cylinder of any kind, the axis of which is vertical.*
PROPOSITION 98. Problem.
876. *If a body is moving on the surface of a solid of revolution, of which the axis is the vertical line AL, in vacuo with gravity g acting uniformly, to determine the motion of the body on a surface of this kind.*
PROPOSITION 99. Problem.
896. *Pendulums are set in motion in rotational motion ; to determine the motion and the curved line described on the surface of a sphere.*