

PARABOLA

PART SEVEN

Part seven presents the production of the parabola which arises from a variety of sources, such as lines, circles, ellipses, as well as arising from the parabola itself.

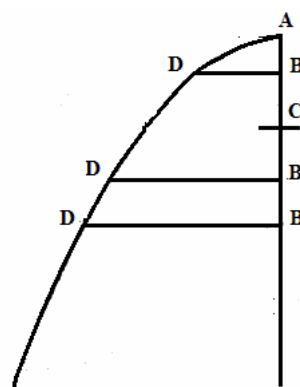
PROPOSITION CCXC.

Some right line AB shall be divided at C; moreover some number of right lines BD shall be put in place parallel to each other in order that the squares BD shall be equal to the rectangles CAB themselves.

I say the points A, D, D to belong to the parabola of which the latus rectum is AC.

Demonstration.

As the line AB is to the line AB, thus the rectangle CAB [i.e. CA.AB] is to the rectangle CAB: but the squares BD are put equal to the rectangles CAB, therefore the square BD is to the square BD, as the line AB is to the line AB: therefore the points A, D, D are for a parabola, of which the latus rectum is AC. Q.e.d.



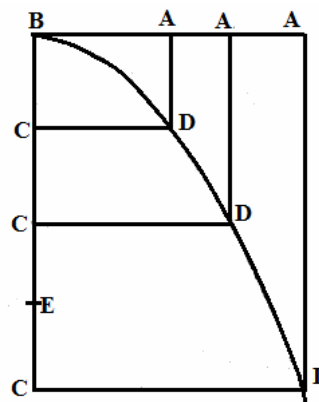
PROPOSITION CCXCI.

ABC shall be any angle: and the proportionals EB, BA, AD shall be prepared with some point E selected from the side BC; and indeed the lines AD shall be parallel to the side BC:

I say B, D, D to be for the parabola, of which the latus rectum is BE.

Demonstration.

The lines DC shall be drawn parallel to AB: therefore since CD shall be equal to AB, the right lines EB, CD, BC are in continued proportion; and with the rectangles EBC equal to the squares CD: whereby from the preceding, the points B, D, D, are for the parabola, of which the latus rectum is BE.



PROPOSITION CCXCII.

With the same figure in place: again the angle shall be ABC, and some number of sides AD shall be drawn parallel to BC: moreover it shall become so that as the square AB to the square AB, thus the line AD to the line AD.

Demonstration.

Again the lines DC shall be put in place parallel to AB; therefore as the line AD to the line AD, therefore the square DC to the square DC, that is BC to BC; therefore the points B, D, D lie on a parabola.

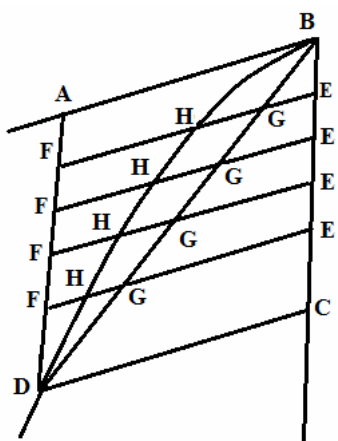
PROPOSITION CCXCIII.

BD shall be the diameter of the parallelogram ABCD, which some number of lines EF, parallel to the side AB, will cut at G, moreover FE, HE, GE will become proportionals.

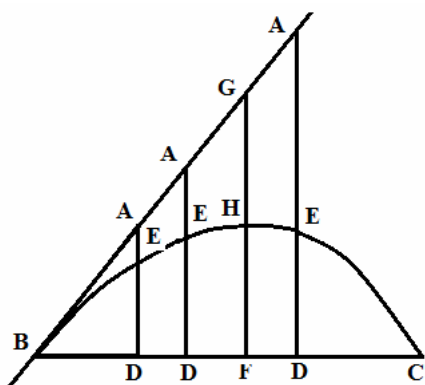
I say the points B, H, H lie on a parabola.

Demonstration.

Since the lines FE are equal, the rectangle FEG to the rectangle FEC is as the line GE to the line GE; that is as BE to BE, but the squares HE may be put equal to the rectangles FEG, therefore the square HE is to the square HE, as the line EB to the line EB: whereby B, H, H, lie on a parabola.



PROPOSITION CCXCIV.



Some number of parallel lines AD shall cut the sides of the angle ABC : which shall be divided at E, so that DE shall be to EA, just as CD is to DB. I say the points B, E, C to lie on a parabola.

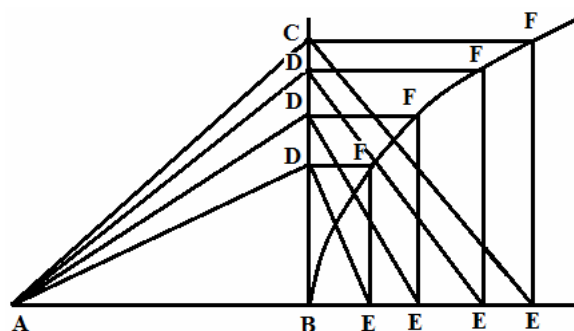
Demonstration.

With BC bisected at F, FG shall be erected parallel to AD; and FG shall be bisected at H ; then the parabola shall be described through B, H, C, to which the line AB shall be a tangent at B, that will divide the right lines AD at the points E, so that DE to EA, shall have the same ratio as CD to DB: whereby since they are divided thus, the points B E C pertain to a parabola.

PROPOSITION CCXCV.

ABC shall be a right angle triangle ; and a number of lines AD shall be put in place from A, so that with respect to D, indeed the lines DE will be normal to AD, crossing the side AB produces at E ; and the lines DF normal to BC crossing the lines DE at F.

I say the points B, F, F belong to the parabola, of which the latus rectum is AB.



Demonstration.

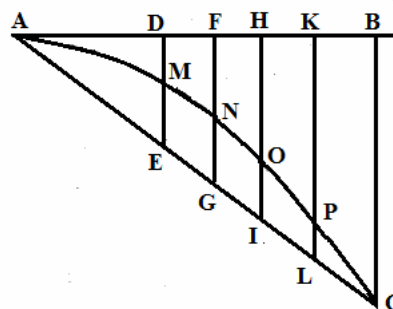
Indeed since the angles ADE shall be put to be right, and the line DB normal to the line AB, for the rectangles ABE are equal to the squares DB, that is FE : but those rectangles ABE are observed to be in proportion to each other, as the lines BE; therefore as BE to BE, thus the square EF to the square EF. From which B F F are points in a parabola, of which the latus rectum is AB. Q.e.d.

PROPOSITION CCXCVI.

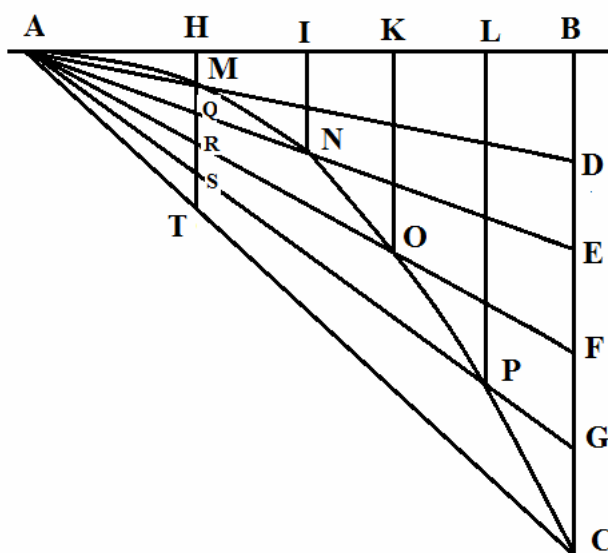
Some right lines DE, FG, HI, KL shall cut triangle ABC parallel to the side BC:
 moreover BC, KL, KP shall be in continued proportion: likewise BC, HI, HO: likewise
 BC, FG, FN : and finally BC, DE, DM.
 I say the points A, M, N, O, P, C belong to a parabola.

Demonstration.

Because the first line BC is common, from the
 series of the continuations BC, KL, KP: BC, HI, HO
 :BC, FG, FN, &c. the rectangles BCKP, BCHO,
 BCFN, BCDM are separated according to that ratio
 which the lines have : KP, HO, FN, DM. And
 therefore the squares KL, HI, FG, DE also preserve
 the same proportion: yet so that as the square KL to
 the square HI, thus the square KA is to the square
 HA, and as the square HI to the square FG, thus the
 square HA is to the square FA, &c.; therefore the squares AK, AH, AF, AD have that
 same ratio as the lines HP, HO, FN, DM; therefore A, M, N, O, P are points on a
 parabola.



PROPOSITION CCXCVII



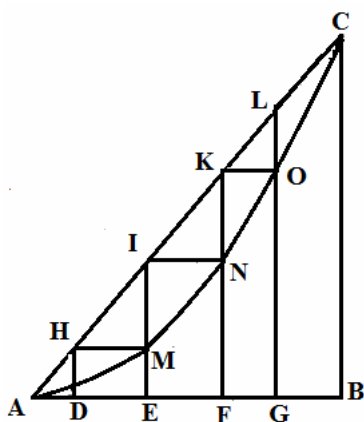
The side BC of the triangle ABC
 divided in some manner at the
 points D, E, F, G and with AD, AE,
 AF, AG joined, the side AB shall be
 divided at the points H, I, K, L just
 as BC has been divided at D, E,
 &c.; moreover the right lines HM,
 IN, KO, LP shall be dropped from
 H, I, K, L parallel to the side BC:
 and indeed HM shall cross AD at
 M, IN truly shall cross AE at N:
 then KO, with the right line FA at
 O, and LP, with AG itself at P.

I say A, M, N, O, P, C are points
 on a parabola.

Demonstration.

HM produced shall cross the lines AD, AE, AF, AG in Q, R, S, T, because from the hypothesis, AH is to AI, as BD to BE ; but as BD shall be to BE, thus HM shall be to HQ; HM is to HQ, as AH to AI, that is as HQ ad IN : therefore HM, HQ, IN are proportionals ; similarly it may be shown HM , HR, KO are proportionals: likewise HM, HS, LP; and finally HM, HT, BC: whereby the rectangles HMIN, HMKO, HMLP, &c. have that same proportion between each other, as the squares HQ, HR, HS: but as the square HQ to the square HR, thus the square BE to the square BF, that is by the hypothesis, the square AI to the square AK ; and as the square HR to the square HS, thus the square BF to the square BG, that is the square AK to the square AL: and therefore the rectangles HMIN, HMKO, HMLP maintain the same ratio between each other as the squares AI, AK, AL : but the rectangles HMIN, HMKO, HMLP are as the lines IN, KQ, LP; and therefore the squares AI, AK, AL have the same proportion between each other, as the lines IN, KO, LP; whereby A, M, N, O, P are points on a parabola. Q.e.d.

PROPOSITION CCXCVIII.



The side AB of the triangle ABC shall be divided at the points D, E, F, G so that the square of the ratio AD to AE, shall be as the square of the ratio AE to AF; and that ratio again shall be of AF to AG squared, &c. then the parallel lines DH, EI, FK, GL shall be erected from the points D, E, F, G parallel to the side CB: which will cut the lines HM, IN, KO parallel to the side AB at M, N, O.

I say that A,M, N, O, C to be points on a parabola.

Demonstration.

Because the square of ratio AD to AE, shall be as DH to EI ; that is EM to FN; just as the square of the ratio AE to AF shall be the line EM to the line FN. Similarly, it may be shown that as FN to GO, thus the square AF shall be to the square AG; therefore A, M, N, O are points on a parabola.

PROPOSITION CCXCIX.

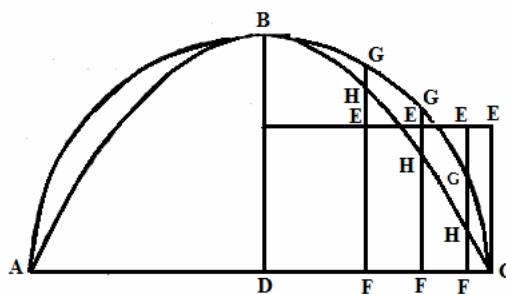
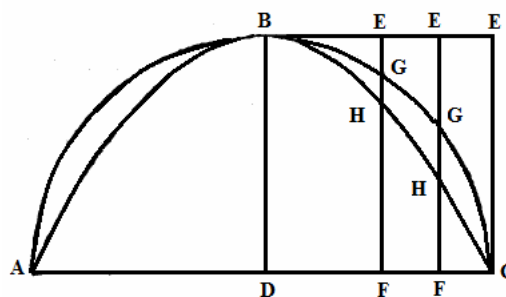
The two diameters AC, BD shall cut the semicircle ABC orthogonally at D; moreover the line EE shall be put in place parallel to AC : then with the points FF taken on AC, the normals FE may be erected, crossing the line EE at E, and the semicircle at GG .

Moreover EF, GF, HF shall be in proportion.

I say the points H, H, C to lie on a parabola.

Demonstration.

For since EF, GF, HF shall be proportionals, so that $EF^2 : GF^2 = GF^2 : HF^2$, thus the rectangle EFH to the rectangle EFH: but the rectangles EFH have that ratio between themselves as the lines HF, and therefore the lines HF are as the squares GF: that is as the rectangles AFC, therefore the points H,H,C are for a parabola.



Likewise it will apply to the ellipse, if the diameters AC, BD may be placed conjugate, and EF shall be parallel to BD.

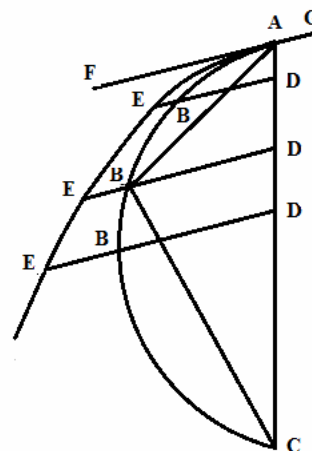
PROPOSITION CCC.

The right line AC shall subtend a segment of the circle ABC, which some number of lines parallel to the tangent FG drawn through A shall cut at D : moreover so that as AB shall be to AB, thus there will become DE to DE.

I say the points A, E, E to define a parabola.

Demonstration.

AB, BC shall be joined : since FG is a tangent, the angle CAG shall be equal to the angle ABC; but the angle CAG is equal to the angle ADB, since FG, DE are parallel ; therefore the angle ABC is equal to the angle ADB; and the triangles ABC, ABD are similar. Whereby, as AD to AB, thus AB is to AC and the rectangle DAC, is equal to the square AB. Therefore the square AB is to the square AB, as the rectangle DAC to the rectangle DAC, that is as the line DA to the line



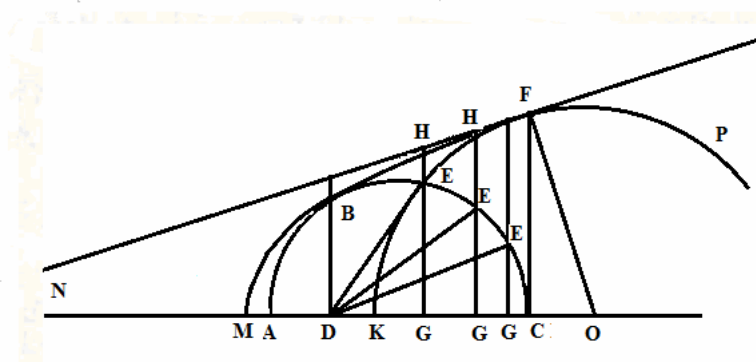
PROPOSITION CCCL.

I say the points GG define a parabola.

ordinate DB has been placed on the axis IC, and DI, IH are equal lines, the line HB will be a tangent to the parabola at B: truly since at the same point, the same right line also is a tangent to the circle at the same point, whereby the lines DE and FG are equal for the circle and the parabola. Therefore, since the lines FEG normal to the diameter AC may be put equal to the right line DE, the points G, G pertain to a parabola of which the apex shall designate the point which bisects HD.

PROPOSITION CCCII.

To extend the parabola proposed above to infinity by the same method.



Construction & demonstration.

CD shall become equal to CF, parallel to BD: F will be a point on the parabola MBH, since all DE, shall be moved across in GEH to the parabola, then with CM made equal to MN, NF shall be put to be the tangent or the parabola, and FO shall be erected perpendicular to the tangent NF, and the circle with centre O and with the radius FO shall be described; and NF the line at F to be a tangent to that parabola: then the parabola shall be described, which arises from the circle KFP now described ; and NF, also this line at F, will be a tangent to the circle KFP, as has been shown in the first proposition; therefore the vertex of this same parabola is at M, since MC, CN shall be equal lines: therefore these two parabolas have the common vertex M, and together with the point F are one and the same parabola, which has been described by each circle: therefore the circle KFP produces the parabola described above in practice : and thus the same parabola will be continued indefinitely in practice. Q.e.d.

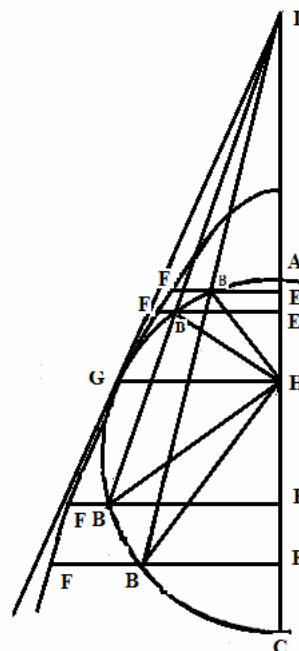
PROPOSITIO CCCIII.

AC shall be the diameter of the circle ABC, produced to some point D, from which the lines DB shall be sent into the circle; and the normals FE to AC shall be acting through B proportional to DB themselves ; the ends of which shall be F F.

I say the points F F lie on a parabola.

Demonstration.

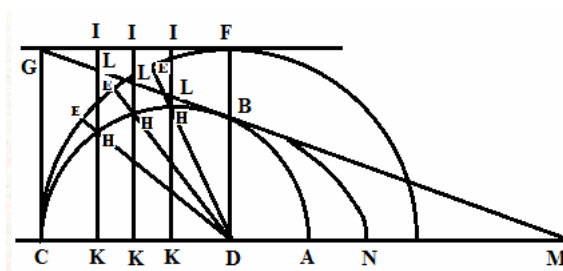
The line DG shall be drawn from D, tangent to the circle at G, and from G, the normal GH to AC shall be put in place, and the points HB shall be joined, therefore DB is to DB as BH to BH, but BH themselves moved to FE, shall produce the parabola [as above]; and therefore with DB to BH themselves moved, that is the proportionals in the same place are moved, also produce the parabola.



PROPOSITION CCCIV.

Some point D shall be taken in the circle ABC with diameter AC, beyond the centre; moreover, a circle is described with the radius DC ; and the square FDC shall be put in place, then some lines DE shall be drawn from D, crossing the circle ABC at H, and the square CEF at E: and the normals IK to AC are acting through H, and the lines HE shall become equal to the lines LI.

I say L, L to be points on a parabola.



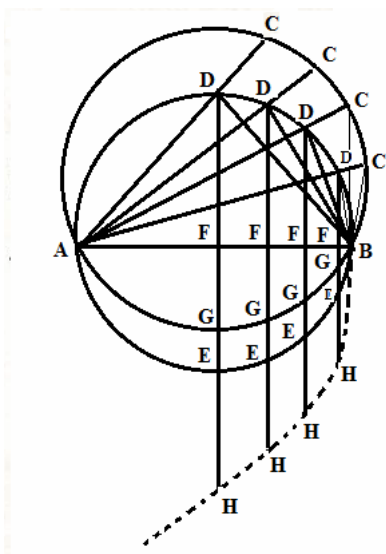
Demonstration.

Because FDC is a square, and D the centre of the circle CEF, the right lines DE are equal to the lines IK: moreover the lines HE themselves shall be put equal to the lines LI, therefore the remaining LK, are equal to the remaining HD. But HD are moved to KL for the parabola; therefore the points L, L now also belong to the parabola.

Corollary.

Again, the vertex of this parabola is found thus, from the normal DB erected from D; by the tangent BM is acting through B : then MD is bisected at N: it is evident N to be the vertex of the parabola.

PROPOSITION CCCV.



Some two circles ABC and ADB intersect each other at A and B : and with the lines ADC drawn from A, the right lines DEF dropped from D orthogonally cutting the diameter AB in F F. Moreover so that there will become: as DC to DC, thus FH to FH.

I say the points BHH to be for a parabola.

Demonstration.

DB, CB shall be joined: therefore the triangles DCB are similar: whereby so that as DC to DC, thus DB to DB transferred to FH produce a parabola, and therefore DC transferred to FH, produce a parabolic section.

PROPOSITION CCCVI.

The two circles ABC, ACD shall cross each other in turn at A and C, and indeed the centre of the circle ABC shall pass through the centre of the circle ACD, and from A the right lines AED shall be drawn, crossing the circle ABC at E , and the circle ACD at D: moreover the normals FG shall be put in place through EE to HB, which shall be equal or proportionals to the lines AD.

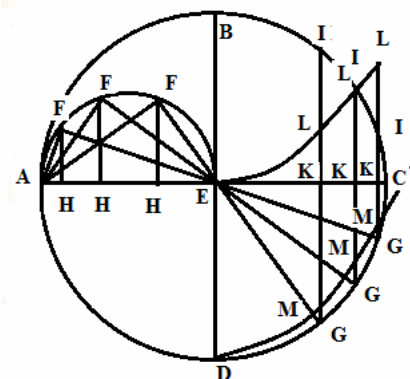
I say the points G, G belong to a parabola.

Corollary.

PROPOSITION CCCVII.

I say the points D, M, M, likewise E, L, L belong to parabolas.

AF shall be joined : Since FH, GK shall be parallel, the triangles FHE, EKG are similar; also the triangle FHE shall be similar to the triangle AFE ; and therefore the triangles AFE, EKG are similar, truly since AE is equal to EG, the triangles AFE are equal to the triangles EKG, and the sides AF, FE, equal to the sides GK, KE, so that the square AF shall be to the square AF, as the square GK to the square GK, thus as the line AH to the line AH, that is by the hypothesis as KM to KM; but as the square GK to the square GK, thus the rectangle AKC to the



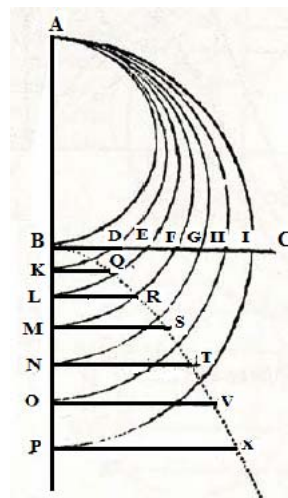
rectangle AKC; therefore as the rectangle AKC to the rectangle AKC, thus the line MK to the line MK: whereby the points D, M, M belong to a parabola [by §.47]. Which was the first part.

Again since EH shall be to EH, thus LK to LK, thus moreover EH to EH, thus the square EF to the square EF; that is the square EK to the square EK; and therefore the line LK to the line LK, as the square EK to the square EK: from which the points E L L belong to a parabola [by §.292].

PROPOSITION CCCVIII.

Some number of circles shall themselves be described tangent to the circle at the same point A for the diameter AB ; of which the diameters shall be AB, AK, AL, AM, &c. and with the line BC drawn from B, shall be a tangent to the minimum circle at B, moreover the remainder will cut BC at D, E, F, G, H; moreover the lines BD, BE, BF, BG, &c. shall be made equal to the lines KQ, LR, MN, NT, &c. which shall be tangents to the circles at L, M, N, O, &c.

I say the points B, Q, R, S, T, V belong to the parabola of which the latus rectum is AB.



Demonstration.

Because BD is normal to the common diameter AB, the square BD to the square BE shall be as the rectangle ABK to the rectangle ABL, that is (since the circles shall touch each other at A) as the line BK to the line BL, and therefore the square KQ is to the square LR, as the line BK to the line LB (since the lines LR, KQ shall be equal to the lines BD, BE); in the same manner it is shown the square MS to be to the square NT, as the line BM is to the line BN, etc. : therefore the points B, Q, R, S, T, &c. belong to the parabola; therefore the latus rectum is apparent to be AB, since the squares DK, RK, &c. always shall be equal to the rectangle on KB, BA, LB, BA, &c.

Corollary.

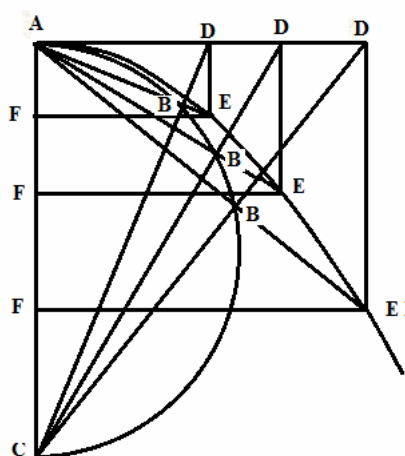
Hence the practice is readily apparent, for producing an indefinite parabola, arising from the first method: since indeed these tangential circles shall be able to be multiplied indefinitely, and for the line BC requiring to be extended, by which also the first parabola likewise will be able to be continued indefinitely; and thus the same will be able to be extended to infinity.

PROPOSITION CCCIX.

AC shall be the diameter of the semicircle ABC, and with the tangent AD acting through A, the lines CBD shall be drawn from C, crossing the tangent line at D, and intersecting the semicircle at B: then the normals DE shall be dropped from D which shall cut the lines AE at B.

I say the points A, E, E to lie on a parabola, of which the latus rectum is AC.

Demonstration.



The lines EF shall be put in place parallel to AD, since the tangent AD shall pass through A, the end of the diameter AC, the angle CAD is right : but also the angle ABC is the right angle in a semicircle, therefore the triangles ABD, ADC are similar: truly since ED is normal to AD, and DB normal to AE, the triangles ADB, ADE, are similar also ; but ADB similar to triangle CAD, therefore the triangles ADE, ADC also are similar; whereby ED to DA, as DA to AC: and thus the square AD, that is FE, is equal to the rectangle EDAC, that is FAC. From which the square FE is to the square FE as the rectangle FAC to the rectangle FAC, that is as the line FA to the line FA, therefore the points

A, E, E lie on a parabola; of which the latus rectum AC to be apparent from the demonstration.

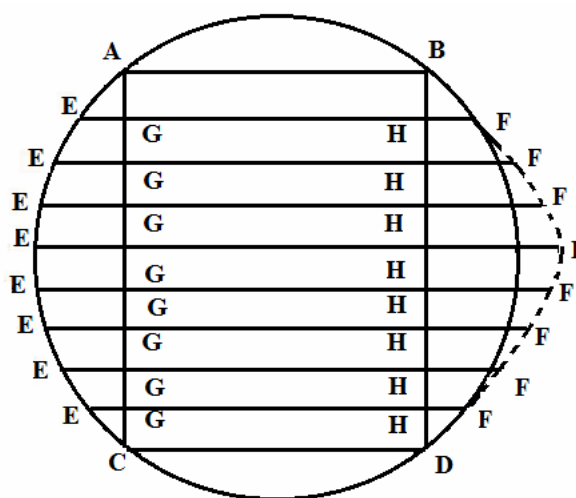
PROPOSITION CCCX.

The rectangle ABCD shall be inscribed in the circle ABC, and the lines EGHF shall be drawn parallel to one of the sides AB: moreover GH, HF, FI shall become proportionals.

I say the points B, I, D pertain to a parabola.

Demonstration.

Because GH is to HF, thus as HF is to FI, on being added together and inverted, so that as GF, that is EH, shall be to GH, thus as HI to HF; therefore the rectangle EGF, that is EHF, is equal to the



rectangle GHI : and the rectangle GHI to the rectangle GHI, so that the rectangle EHF shall be to the rectangle EHF, as the rectangle EHF to the rectangle EHF, that is as BHD to BHD, but the rectangle GHI is to the rectangle GHI as the line HI to the line HI; therefore so that as the rectangle BHD is to the rectangle BHD, thus the line HI to the line HI; therefore the points BID constitute a parabola.

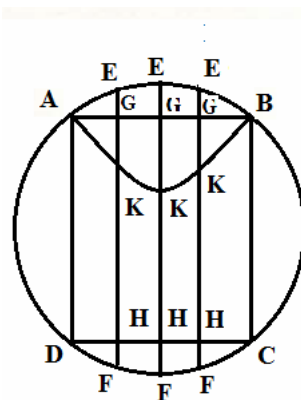
PROPOSITION CCCXI.

The rectangle ABCD shall be inscribed in the circle ABC, and with the parallel lines EGHF drawn to one side of DA, the rectangles HGK shall become equal to the rectangles EGF.

I say the points AKB belong to a parabola.

Demonstration.

As the rectangle EGF is to the rectangle EGF, thus the rectangle AGB is to the rectangle AGB: and therefore the rectangle HGK is to the rectangle HGK as the rectangle AGB to the rectangle AGB; but the rectangle HGK is to the rectangle HGK as the line GK to the line GK; and therefore the rectangle AGB is to the rectangle AGB, as the line GK to the line GK: therefore the points A, K, B are for a parabola. Q.e.d.



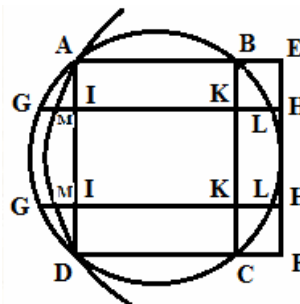
PROPOSITION CCCXII.

Again the rectangle ABCD shall be inscribed in the circle ABC, and with the line EF drawn parallel to the side BC, which shall be a tangent to the circle, and the right lines GH shall be put in place, parallel to the side AB: and the rectangles GIL shall be put equal to the rectangles HIM.

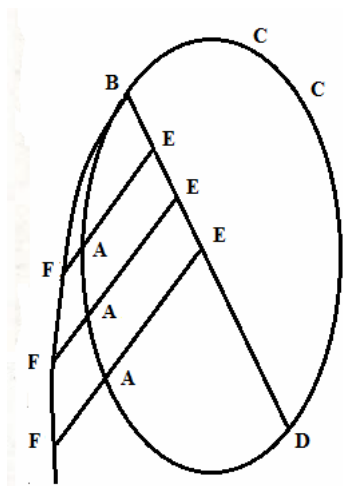
I say the points A, M, D to lie on a parabola

Demonstration.

Since the rectangles HIM shall be equal to the rectangles GIL, the rectangle HIM to the rectangle HIM is as GIL to GIL, that is the rectangle AID to the rectangle AID: but the rectangle HIM is to the rectangle HIM, as the line IM to the line IM; therefore as the rectangle AID to the rectangle AID, thus the line IM to the line IM: therefore the points M, M belong to a parabola.



PROPOSITIO CCCXIII.



a parabola.

One of the diameter BD shall be given in the ellipse ABC from the equal conjugate diameters, for which the ordinates AE shall be put in place. Moreover the squares AE, EB shall be made equal to the square EF.

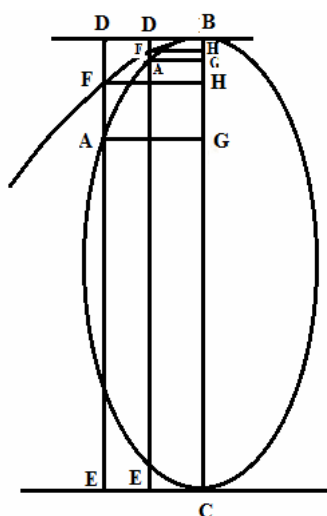
I say the points B, F, F to belong to a parabola.

Demonstration .

Since BD is one diameter from equal conjugate diameters, the squares AE are equal to the rectangles BED [§.176 Ellipse] : therefore with the square BE added, the two squares AE, EB are equal to the rectangle EBD; therefore the square FE is to the square FE, as the rectangle EBD to the rectangle EBD, that is as the line EB to the line EB. Therefore the points B, F, F belong to

Also this proposition agrees with the circle, and the demonstration is the same.

PROPOSITION CCCXIV.



Let BC be the diameter of the ellipse ABC, to which the lines DB, EC shall be tangents at B & C: and with DE put in place parallel to the diameter BC; the rectangles DAE shall become equal to the rectangle EDF.

I say the points B, F, F to be for a parabola.

Demonstration.

AG, FH shall be drawn parallel to DB, the ordinate AG to be put in place for BC; therefore the rectangle DAE is to the rectangle DAE, that is as BGC to BGC, as the square AG, to the square AG, that is the square FH to the square FH: and therefore the rectangle FDE is to the rectangle FDE, that is the rectangle HBC to the rectangle HBC as the square FH to the square FH; but the rectangle HBC is to the rectangle HBC, as the line HB to the line HB. Therefore as

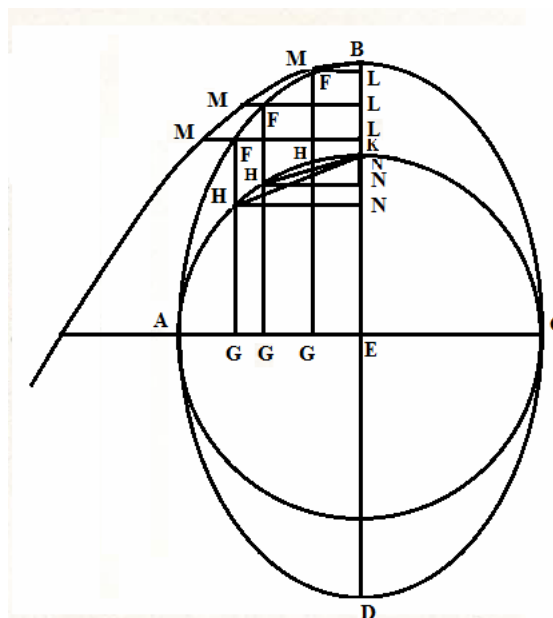
the square FH shall be to the square FH, thus the line HB shall be to the line HB: therefore the points B, F, F belong to the same parabola.

PROPOSITION CCCXV.

AC, BD shall be the axes of the ellipse ABC ; and on the minor axis AC as the diameter, the circle AKC shall be described AKC, which shall cut the line FG at H, parallel to the axis BD: and with the points HK joined, the lines FL shall be drawn normal to the axis BD, and LM shall become equal to HK. I say B, M, M to be points for a parabola.

Demonstration.

HN shall be put in place, parallel to FL: and E shall be the common centre for the circle and the ellipse; therefore so that as BE ad KE, thus FG to HG, that is LE to NE, and on permutating, so that as BE to LE, thus KE to NE; and whereby as KE to NK, is as BE to LB , and on permutating, so that as BE to KE, thus BL to KN, and BL is to BL, as KN is to KN: but as KN to KN, thus the square HK to the square HK, that is by the hypothesis as the square ML to the square ML; therefore so that as the line BL to the line BL, thus the square ML to the square ML; whereby the points B, M, M belong to a parabola.



PROPOSITION CCCXVI.

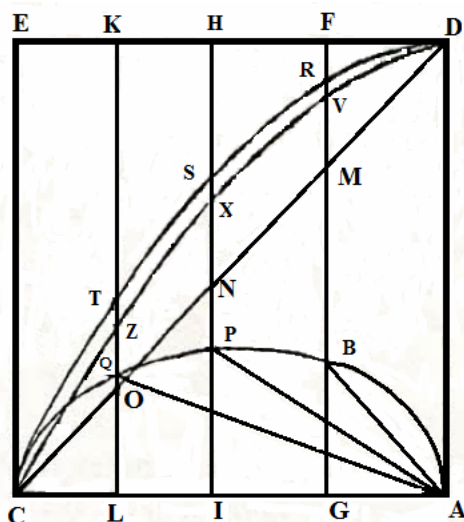
The square AE shall be put in place on the axis AC of the ellipse ABC, the diameter of which CD shall cross the remaining sides FG, HI, LK parallel to the side AB, at M, N, O; moreover the square AG, shall be equal to the rectangle GFR, and the square AI shall be equal to the rectangle SHI, and then the square AL shall be equal to the rectangle TKL.

I say the points D, R, S, T to belong to a parabola.

It shall be assumed in the proportion of the ellipse, on account of the following proportions.

Demonstration.

The rectangle RFG is to the rectangle SHI as RF to SH: and the rectangle SHI is to the rectangle TKL as the line SH to the line TK: and therefore the squares AG, AI, AL, that is DF, DH, DK, are held in the same ratio between each other, as the sections RF, SH, TK; whereby the points D, R, S, T belong to a parabola.



PROPOSITION CCCXVII.

With the same in place: the squares put in place BG, PI, QL shall become equal to the rectangles GFRV, HISX, KLTZ.

I say the points D, X, V, Z to be for a parabola.

Demonstration.

The rectangle GFRV to the rectangle HISX to be as the line RV to the line SX: and therefore the square GB is to the square PI, as the line RV to the line SX: but as the square BG is to the square PI, thus the rectangle AGC is to the rectangle AIC, therefore as the line RV is to the line SX, thus as the rectangle AGC is to the rectangle AIC: that is the rectangle DMC to the rectangle DNC, that is the line MR to the line SN, since RST has been shown to be a parabola; and therefore MV is to XN, as the rectangle DMC is to the rectangle DNC. Similarly to be shown: XN to be to ZO, as the rectangle DNC to the rectangle DOC; therefore the points V, X, Z are for a parabola.

PROPOSITION CCCXVIII.

With the same put in place : AB, AP, AQ shall be joined : and the rectangles GFV, IHX, LKZ shall be equal to the squares AB, AP, AQ.

I say the points V, X, Z again to belong to a parabola.

Demonstration.

Because BG, PI, QL are normals to AC, the square AB is equal to the squares AG, GB: but the square GB is put equal to the rectangle GFRV, and the square AG equal to the rectangle GFR: therefore the square AB, that is the rectangle GFV, is equal to the rectangles GFR, GFRV: similarly it is shown the rectangle IHX, to be equal to the rectangles IHS, IHSX, and the rectangle LKZ to be equal to the rectangles LKT, LKTZ, but since the rectangles GFR, GFRV shall be put equal to the squares AG, GB; and IHS, IHSX, equal to the squares AI, IP, &c., V, X, Z are shown to lie on a parabola ; and therefore now define a parabola.

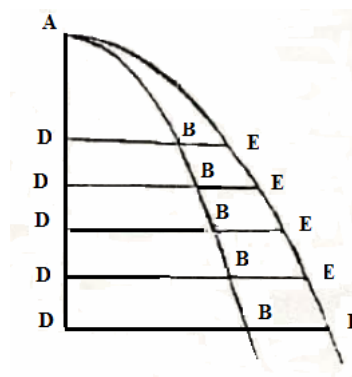
PROPOSITION CCCXIX.

AD shall be the diameter of the parabola ABC: and DB put to be the ordinate for that : moreover there shall become as DB to DB, thus DE to DE.

I say that the points A E E belong to a parabola.

Demonstration.

Indeed from the hypothesis, the square ED is to the square ED, as the square DB to the square DB, that is as the line AD to the line AD; therefore the points E, E belong to a parabola.

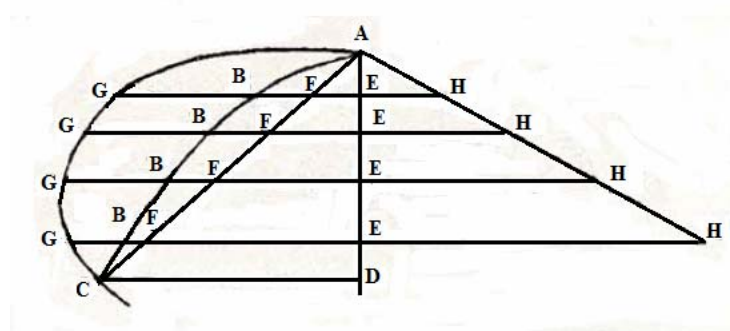


PROPOSITION CCCXX.

AD shall be the diameter of the parabola ABC, and the ordinates CD, BE put in place :
 and with the line AC drawn which shall cross the right line BE at FF, so that as FB to
 FB, thus BG to BG.

I say the points G, G to belong to a parabola.

Demonstration.



There shall become as BF to BG, thus FE to EH: because as FB is to FB, thus BG is to BG on permutating so that as FB to BG, thus there shall be FB to BG: but by the construction as FE is to EH, as FB is to BG, therefore as FE is to EH, thus FE is to EH; therefore since the points EE shall lie on a straight line, also the points HH shall be on a straight line, as agreed from the elements: then since FB shall be to BG, thus FE shall be to EH, on adding there will become on converting, GF shall be to BF, as FH to FE, and on interchanging GF to FH, as BF to EF, and by adding together, GH to be to FH, as BE to FE, and on interchanging again, GH to BE, as FH to FE, but since I have shown before to be FE to EH, as FE is to EH; also as there will be FH to FE, thus FH to FE, therefore so that as GH to BE, thus GH to BE, and on interchanging, as GH to GH, thus BE to BE ; and therefore the square GH is to the square GH, as the square BE to the square BE, that is, as EA to EA, that is, as AH to AH, therefore the points GG are defined by a parabola, of which the diameter is AH.

PROPOSITION CCCXXI.

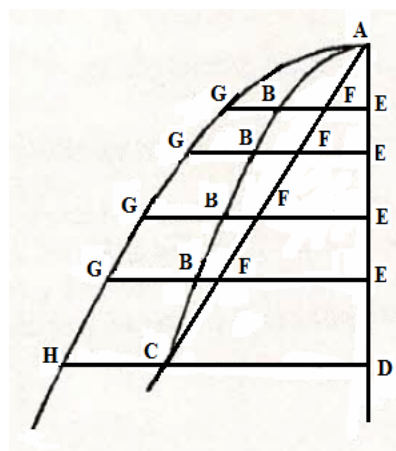
The same shall be put in place as at first, and the lines
 FE shall become equal to the right lines BG.

I say again A, G, G to belong to a parabola.

Demonstration.

Indeed since the lines FE shall be made equal to the
 lines GB, with the common lines BF added, the right
 lines FG are equal to FE, and the squares FG equal to
 the squares BE: therefore the square FG to the square
 FG is as the square BE to the square BE, that is as the
 line AE to the line AE, that is as AF to AF. Therefore
 the points A, G, G are for a parabola, of which the
 diameter is AFC.

Truly if the lines FG shall be proportionals to these lines FE, it is shown as in the
 preceding proposition, A, G, G to define a parabola.

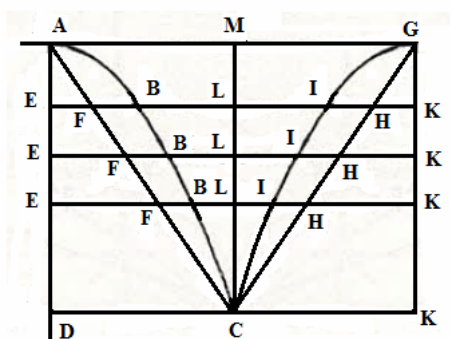


PROPOSITION CCCXXII.

The line AG shall be a tangent at D to the parabola ABC of which the diameter is AD :
 moreover the ordinate lines BE, CE shall be put in place ; and with AC joined BE will
 cross the lines at F, then with some tangent CG drawn to AG, by which the lines BE will
 cut the lines produces at HH: so that as FB shall be to FB: thus as HI to HI.

I say the points I, I to belong to a parabola.

HI shall become to HK, as BF to FE: from the elements it is apparent GKK to be a right line ; therefore HK shall be to HK, as FE to FE: and thus since IH shall be to H K, as BF



PROPOSITION CCCXXIII.

I say the points I, I to belong to a parabola.

AM shall become to MG, as LB to LI, and the right line GC shall be drawn crossing the lines BI in HH. Therefore so that as LF will be to LF, thus LH to LH. But from the hypothesis LI is to LI, as LB to LB ; and therefore the remainder HI is to HI, as the remainder FB to FB, therefore by the preceding the points I, I belong to a parabola.

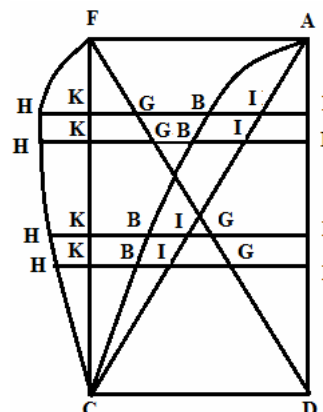
PROPOSITION CCCXXIV.

Let AD be the diameter of the parabola ABC, for which the ordinates CD, BE shall be put in place: and with the parallelogram DF completed, the line FD shall be drawn, cutting the right line EB at GG: moreover the lines GE shall be equal to the lines G, BH.

I say the points H H lie on a parabola.

Demonstration.

AC shall be joined, because DF is a parallelogram of which the diameters are AC, FD, and the lines KE are parallel to the lines KI, it follows that the lines EG are equal to the right lines KE, and the lines HB are equal to the lines KI. Therefore with the common lines KB taken away, the equal lines IB, HK shall remain: therefore the points H H are for a parabola.



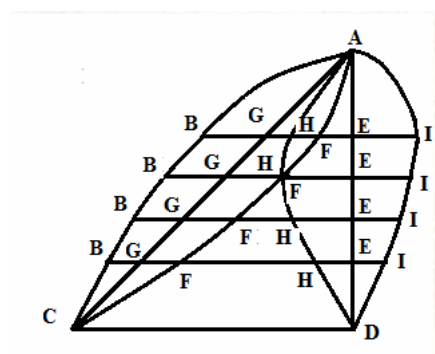
PROPOSITION CCCXXV.

Let AD be the diameter of the parabola ABC, to which the ordinates CD, BE shall be put in place, which the right line AC cuts at G, then with the parabola CFA described through A and CFA, for which the right line AD shall be a tangent at A, and FB shall be the diameter, the lines FG shall become proportional to EH, and the right lines BG proportional to EI.

I say both the points H,H as well as the points I, I to belong to a parabola.

Demonstration.

Indeed ad the rectangle AGC is to the rectangle AGC, thus the rectangle AED is to the rectangle AED ; but as the rectangle AGC is to the rectangle AGC, thus the line FG is to the line FG, that is as the line EH to the line EH: therefore as HE is to HE, thus the rectangle AED is to the rectangle AED : it is shown similarly, EI to be to EI, as the rectangle AED to the rectangle AED ; therefore the points H, H and I, I, are points on parabolas.



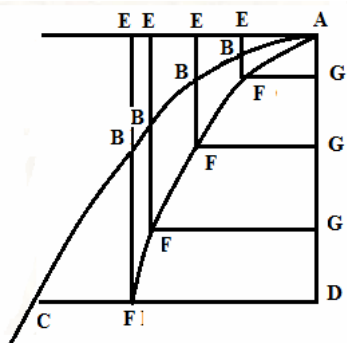
PROPOSITION CCCXXVI.

The line AE shall be a tangent to the parabola ABC at A : moreover AD shall be put as the diameter of the section, and EB to be parallel to that EB: then the lines BF shall become proportional to the lines EB.

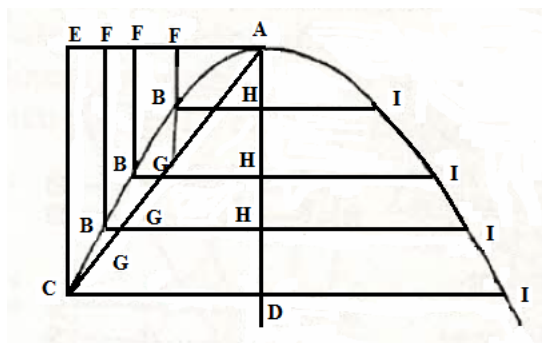
I say the points A, F, F to belong to a parabola.

Demonstration.

The right lines FG are drawn parallel to the tangent AE: therefore so that as EB shall be to EB, thus BF shall be to BF, also EF shall be to EF, as EB to EB. But the ratio EB to EB, is the square of the ratio EA to EA, that is FG to FG; and therefore the ratio EF to EF, that is AG to AG, is the square of the ratio GF to GF: therefore the points A, F, F lie on a parabola.



PROPOSITION CCCXXVII.



Let AD be the diameter of the parabola ABC : with the tangent AE acting through A, the diameters FB shall be dropped crossing the parabola at B, and the right line AC at G: moreover the ordinate lines HB shall be put in place : so that there shall become as FG to FG, thus BH to HI, BH to HI.

I say the points A, I, I to lie on the parabola.

Demonstration.

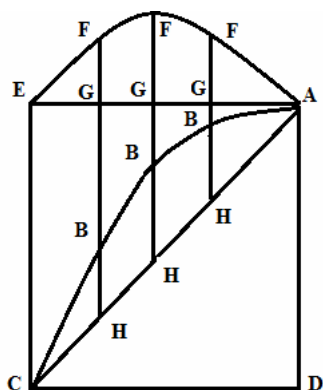
As FG to FG, thus as AF to AF, that is HB to HB: and therefore as HI is to HI, as BH to BH: and the square HI to the square HI, as the square HB to the square HB, that is, as the line AH to the line AH : therefore the points A, I, I are for a parabola.

PROPOSITION CCCXXVIII.

AD shall be the diameter of the parabola ABC equal to the latus rectum : and with the ordinate CD drawn the square CA shall be put in place : and with the points B, B taken on the periphery, the diameters BF shall be erected from B: crossing the lines AE in G: and the right lines AG shall be made equal to the lines BF.

I say the points A, F, E to lie on a parabola.

Demonstration.



The right line AC shall be drawn crossing the lines FB at H H: because AD is equal to the latus rectum, and the ordinate CD put in place for AD, the right lines AD, CD are equal: and from which the right lines AG, GH also are equal: and the lines HG equal to the lines FB themselves : therefore with the common lines BG removed, the lines HB, FG are equal. Whereby FG is to FG, as BH to BH; that is as the rectangle AHC is to the rectangle AHC, that is as the rectangle AGB to the rectangle AGE; therefore the points A, F, E are for a parabola.

PROPOSITION CCCXXIX.

The right line AC shall subtend the parabola ABC, which shall cut some number of diameters at D : moreover the lines BD shall be proportional to the lines DE.

I say the points A, E, C to represent a parabola.

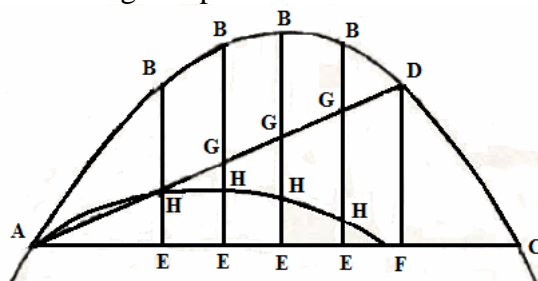
Demonstration.

As BD is to BD, thus ED is to ED, but as BD is to BD, thus the rectangle ADC is to the rectangle ADC: and therefore as ED is to ED, as the rectangle ADC is to the rectangle ADC, whereby the points A E C belong to a parabola.

PROPOSITION CCCXXX.

The right lines AC, AD shall subtend the parabola ABC, and with the diameter EF dropped from D, which shall cross the right line AC at F, some right lines BE shall be drawn parallel to DF, cutting the line AD at GG: and the right lines BG shall be proportionals to the line EH.

I say the points A, H, F to belong to a parabola.



Demonstration.

As the rectangle AGD is to the rectangle AGD thus the line BH is to the line BG : but as BG is to BG, thus EH is put in place for the line EH, and therefore EH is to EH, as the rectangle AGD is to the rectangle AGD, that is as the rectangle AEF is to the rectangle AEF; whereby the points A, H, F belong to a parabola.

PROPOSITION CCCXXXI.

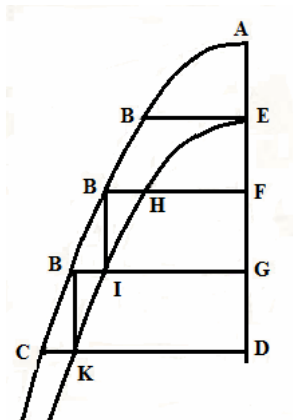
Let the diameter AD of the parabola ABC be divided into equal parts by the points E, F, G, D: and with the ordinate lines drawn EB, FB, GB, DC, the diameters BH, BI, BK shall be dropped from the points B, meeting the ordinates put in place at H, I, K.

I say the points E, H, I, K to lie on a parabola.

Demonstration.

Since the parts of the diameter AE, EF, FG, &c. are put equal, EF is to EG, as AE to AF; that is as the square EB to the square FB, that is, the square FH to the square GI. It is to be shown in the same manner that as EG to ED, thus the square GI to the square DK.

Therefore the points E, H, I, K lie on a parabola. Q.e.d.

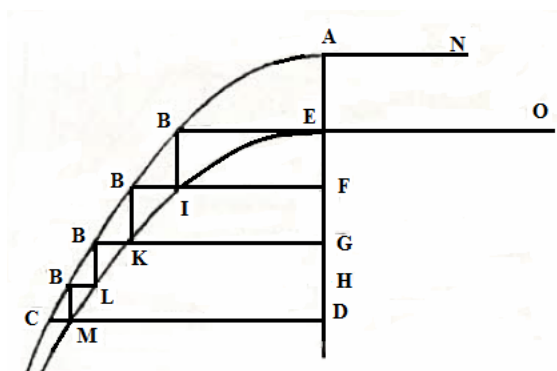


PROPOSITION CCCXXXII.

Let AD be the diameter of the parabola ABC, divided at the points E, F, G, H, D, so that AE, EF, FG, GH, &c. shall be continued proportionals: moreover the ordinate lines EB, FG, GB, &c. shall be drawn, and the diameters shall be dropped from B, agreeing with the ordinates put in place.

I say the points E, I, K, &c. to belong to the same parabola: and if the series of continued proportionals AE, EF, FG, &c. were of greater inequalities; I say the parabolas to cross each other at some other point; truly if the series were of smaller inequalities I say the sections never intersect.

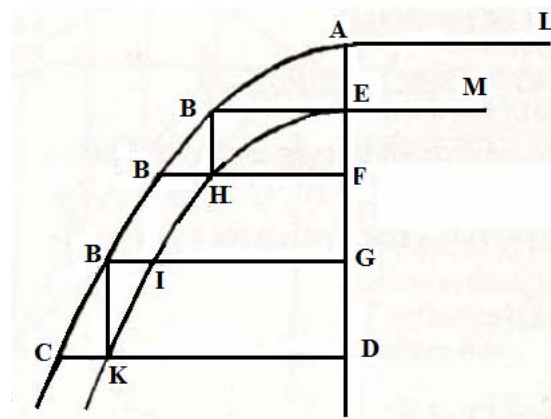
Demonstration.



Since AE, EF, FG, &c. are proportionals, EF is to EG, as AE to AF; that is as the square EB is to the square FB; that is as the square FI to the square GK. Similarly the square FI is shown to be to the square HL as the line EF, to the line EM: therefore the points E, I, K, L, &c. belong to a parabola. Which establishes the first part.

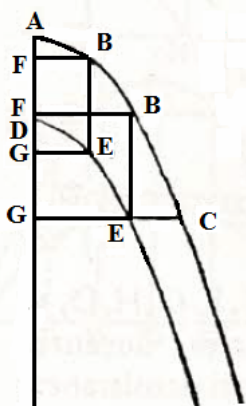
But the series AE, EF, FG; &c. may be put in place with greater inequalities: I say the parabolas to intersect at some point,

indeed the latus rectum of the parabola ABC shall be AN, and EO the latus rectum of the parabola EIK: therefore the square EB shall be equal to the rectangle EAN, and the



square FI equal to the rectangle FEO: from which since the squares EB, FI also shall be equal to the rectangles EAN, FEO: whereby the right line AE shall be put greater than EF, the latus rectum AN shall be smaller than the latus rectum EO: and thus the parabolas shall meet at some point.

Now AE, EF, FG, &c. shall be composed from the continuation of terms of smaller inequalities : I say the parabolas never cross each other : indeed LA may be put to be the latus rectum of the parabola ABC, and ME, the latus rectum of the parabola EHI, therefore since the squares EB, FH are equal, also the rectangles LAE, MEF are equal: and since AE shall be put smaller than EF, LA will be greater than ME; so that at no point shall the sections meet.



PROPOSITION CCCXXXIII.

Let AD be the axis of the parabola ABC, of which some part AD shall be dropped, taken equal to the diameters BE.

I say the points D, E, E to be points on a parabola equal to the parabola ABC.

Demonstration.

Some lines EG shall be put in place, parallel to the ordinate BF drawn. Therefore since the line AD is equal to the line BE, that is equal to FG; therefore with the common line FD either taken or added, the right line AF is equal to DG. So that DG is to DG, as AF to AF, that is, as the square FB to the square FB, that is, as the square EG to the square EG. Therefore the points D, E, E, belong to a parabola, because truly both the lines AF, DG, as well as the lines FB, GE are equal to each other, and therefore the square FB is equal to the square GE; also the latus rectum AD is equal to the latus rectum DG, so that the parabolas are equal.

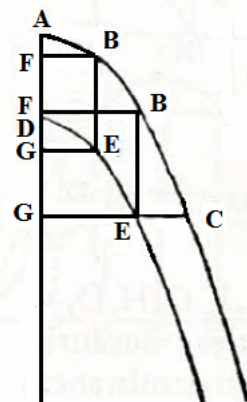
PROPOSITION CCCXXXIV.

With the same in place :

I say that the parabolas never meet each other.

Demonstration .

In the parabola DEE, the ordinate line GE to the axis DG, is put in place crossing to the parabola ABC at C; therefore since the latus rectum of each parabola are equal, and the line DG is smaller than AG, also the rectangle under DG and with its latus rectum is smaller than the rectangle under AG with the latus rectum of the same length: and therefore the square EG is less than the square CG : and the point E falls within the parabola ABC: since the same shall be able to be shown for all the points of the same parabola, it is evident these sections never come together.



Moreover sections of this kind are called parallel parabolas, or asymptotes: the remaining properties of which, and the wonderful symbolification with a hyperbola placed between the asymptotes, we will demonstrate in the eighth part of this book.

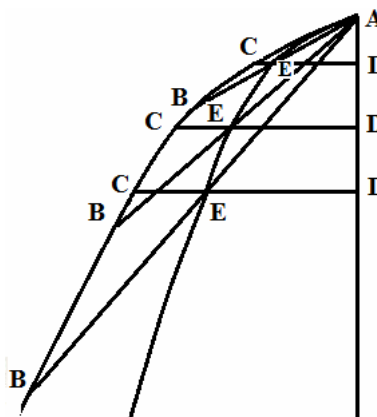
PROPOSITION CCCXXXV

Some number of lines AB sent from the vertex A of the parabola ABC, of which the diameter is AD, shall be divided proportionally at E.

I say the points A, E, E to lie on a parabola.

Demonstration.

The lines DC shall be drawn through E, the ordinates to the diameter AD: therefore the lines DC also will be divided at E in proportion; and so that as DC to DC, thus DE to DE; therefore the points A, E, E belong to a parabola.



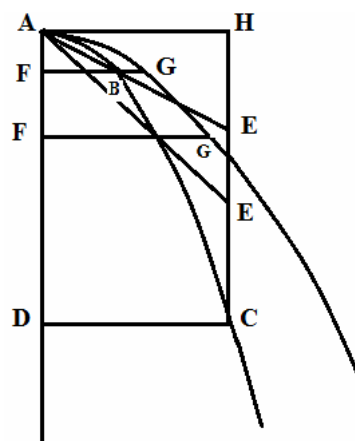
PROPOSITION CCCXXXVI.

Let AD be the diameter of the parabola ABC, to which the ordinate DC shall be put in place ; moreover the line AB shall be sent down from A, crossing the diameter CE at E; which shall cut the tangent acting through A at H, then with the ordinate FB drawn through B, so that there shall be as HE to HE, thus FG to FG.

I say AG to belong to a parabola.

Demonstration.

Since both FB, AH as well as FA, HE are parallel, the triangles AFB, AHE are similar ; whereby as HE is to HE, thus FB is to FB: but as HE to HE, thus FG is put to FG; therefore FG is to FG, as FB to FB, from which the points A, G, G belong to a parabola; [see §.319 above.]

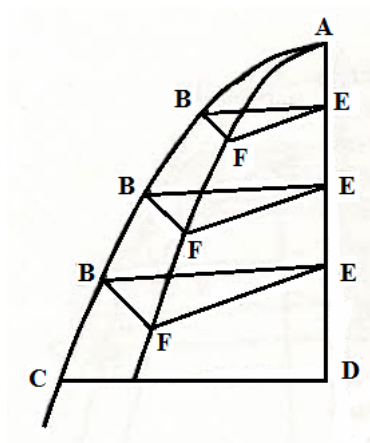


PROPOSITION CCCXXXVII.

Let AD be the diameter of the parabola ABC, for which the ordinate lines shall be BE; moreover the lines BF shall be drawn from B, parallel to each other, and the lines EB in proportion to each other EB.

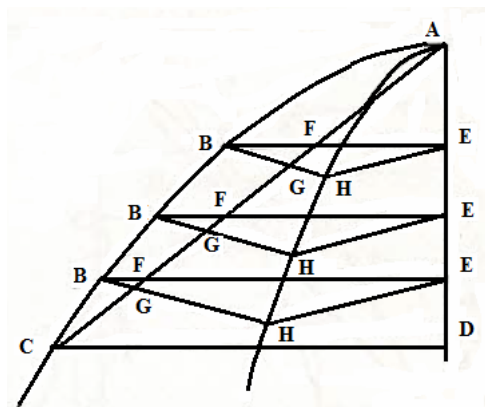
I say the points A, F, F to belong to a parabola.

Demonstration.



The points EF shall be joined : Since both the lines FB as well as the lines EB shall be parallel amongst themselves, the angles EBF are equal; from which since the sides EB, BF shall be in proportion to each other; and thus the lines EF in turn are parallel to each other; and the lines EB in proportion to each other: therefore the square EF is to the square EF, as the square EB to the square EB, that is, as the line AE to the line AE; from which the points A, F, F belong to a parabola.

PROPOSITION CCCXXXVIII.

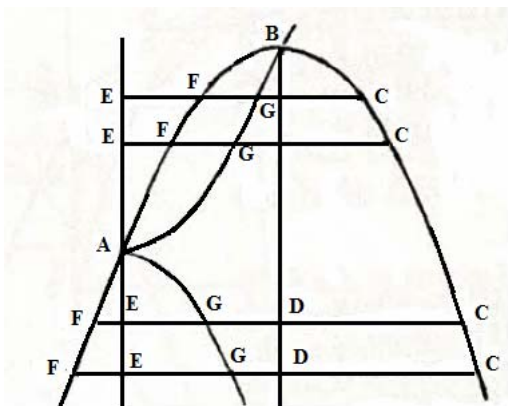


Let AD be the diameter of the parabola ABC, and CD, BE the ordinates put in place for that; and indeed with AC joined, BE shall be divided at F: moreover with the parallel lines BG drawn, which cross the line AC at G: there shall become as FB to BG, thus EB to BH.

I say the points AH belong to a parabola.

Demonstration.

Indeed the points EH shall be joined: Because EB, BH are proportional lines, with the lines FB, BG; and with the common angle FBG, the triangles FBG are similar to the triangle EBH: but the triangles FBG also are similar since both the lines FB amongst themselves, as well as the right lines BG shall be parallel: and therefore the triangles EBH are similar. Whereby as EB to EB, thus EH to EH, and thus from the preceding, the points AH are for the same parabola.



PROPOSITION CCCXXXIX.

Let BD be the axis of the parabola ABC, to which the parallel line AE is put in place, cutting the parabola at A: moreover the ordinate lines EC are drawn to the axis, and between FE, EC the means EG are put in place, & inter FE, EC the means EG are put in place.

I say A, G, B to be points on a parabola.

Demonstration.

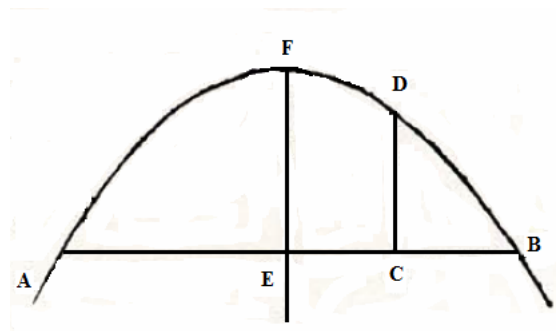
Indeed as the line AE to the line AE, thus the rectangle FEC to the rectangle FEC. And therefore the square EG is to the square EG, as the line AE to the line AE : whereby A, G, B are points on a parabola.

PROPOSITION CCCXL.

With the right lines AB, CD given mutually perpendicular; to find the parabola of which DC shall be a given diameter DC.

Construction & demonstration.

AB shall be bisected at E, EF shall be drawn parallel to CD: and so that the rectangle AEB thus shall be to the rectangle ACB, as the line EF to the line DC. It is apparent that A, F, D, B to be the points for the parabola sought, of which a diameter is DC.

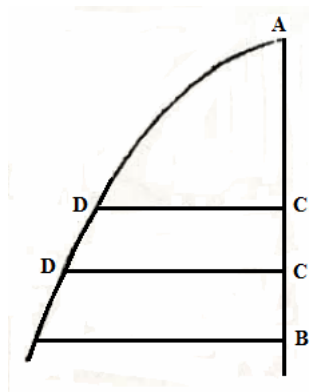


PROPOSITION CCCXLI.

With the right side AB given, to show the parabola of this.

Construction & demonstration.

AB shall be cut in some manner at CC, and the line CD shall be erected from C, so that the squares CD shall be equal to the rectangles CAB ; each with its own rectangle : it is evident the points A, D, D to lie on the parabola sought.

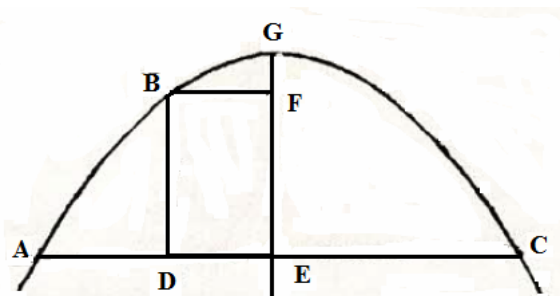


PROPOSITION CCCXLII.

With the three points A, B, C given, and not in a straight line, and the line BD, which shall pass through some of the points given, it will be required to describe a parabola through A,B,C of which some will be the diameter BD.

Construction & demonstration.

AC joined shall be bisected at E, and EF shall be placed parallel to BD, and BF parallel to the right line AC: moreover it shall become so that as the square AE shall be to the square BF, thus the line EG shall be to the line FG, and a parabola shall be described through the points B, G, C parabola, of which the diameter shall be BD, it is evident that also to pass through A. Therefore, &c. Q.e.d.



PARABOLAE

PARS SEPTIMA

*Varias exhibet geneses, quae tum è lineis, circulis, ellipsis,
 tum ex ipsa oriuntur parabola.*

PROPOSITIO CCXC.

Esto AB linea utcumque divisa in C; ponantur autem ad AB rectae quotcumque BD inter se aequidistantes : ut CAB rectangulis aequalia sint quadrata BD.

Dico puncta A, D, D esse ad parabolam cuius latus rectum est AC.

Demonstratio.

Ut AB linea ad AB, sic CAB rectangulum ad rectangulum CAB: sed CAB rectangulis aequalia ponantur quadrata BD, quadratum igitur BD est ad quadratum BD, ut AB linea ad lineam AB : puncta igitur A, D, D sunt ad parabolam, cuius latus rectum AC, Quod fuit demonstrandum.

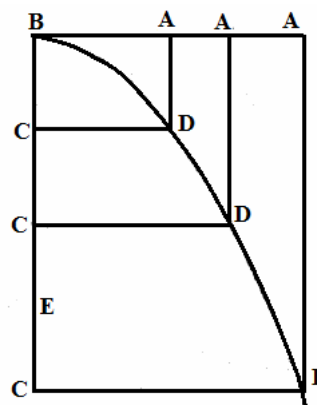
PROPOSITIO CCXCI.

Esto angulus quicumque ABC: sumptoque in BC latere, quovis puncto E fiant proportionales EB, BA, AD; & AD quidem aequidistant lateri BC:

Dico B, D, D esse ad parabolam cuius latus rectum est BE.

Demonstratio.

Ducantur DC parallelae AB: quoniam igitur CD aequatur AB, rectae EB, CD, BC in continua sunt analogia; & EBC rectangulis aequalia quadrata CD : quare per praeceditem, puncta B, D, D, ad parabolam sunt, cuius latus rectum est BE.



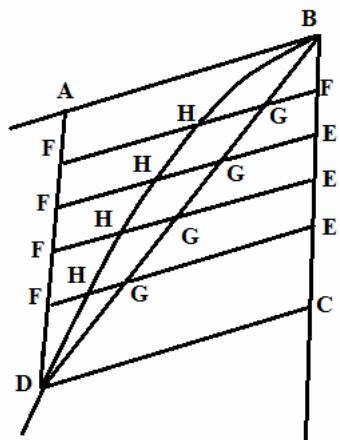
PROPOSITIO CCXCII.

Eadem manente figura: sit iterum angulus ABC, & BC lateri quotcumque eductae parallelae AD: fiat autem ut AB quadratum ad quadratum AB, sic AD linea ad lineam AD.

Demonstratio.

Ponantur iterum DC parallelae AB. erit igitur DC quadratum ad quadratum DC, ut AD
 linea ad lineam AD, id est BC ad BC; puncta igitur B, D, D, ad parabolam sunt.

PROPOSITIO CCXCIII.



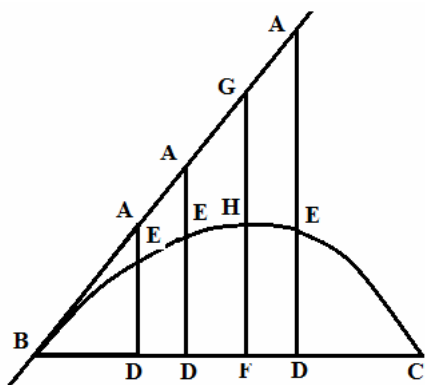
Esto ABCD parallelogrammi diameter BD, quam in G,
 secant lineae quotcunque EF, parallelae lateri AB, fiant
 autem proportionales FE, HE, GE.

Dico B, H, H puncta esse ad parabolam.

Demonstratio.

Quoniam FE lineae aequales sunt, rectangulum FEG ad
 FEC rectangulum est ut GE linea ad lineam GE; id est ut
 BE ad BE, sed FEG rectangulis aequalia ponantur
 quadrata HE, quadratum igitur HE est ad quadratum HE,
 ut EB linea ad lineam EB: quare B, H, H, ad parabolam
 sunt.

PROPOSITIO CCXCIV.



Latera anguli ABC secant quotcunque parallae AD:
 quae dividantur in E, ut DE sit ad
 EA, sicut CD est ad DB.

Dico B, E, C, puncta esse ad parabolam.

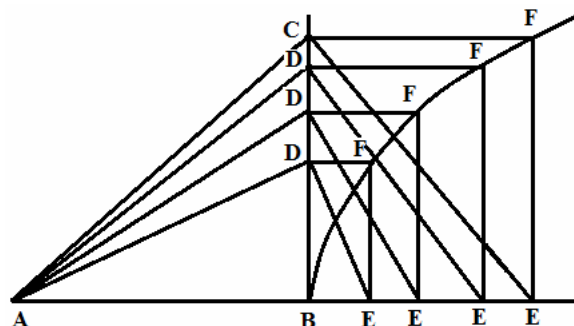
Demonstratio.

Divisa BC bifariam in F, erigatur FG parallela AD;
 seceturque FG bifarium in H; dein per B, H, C
 describatur parabola quae in B contingat AB lineam,
 dividet illa rectas AD in E punctis, ut DE ad EA,
 eam habeant rationem quam CD ad DB: quare cum sic divisae ponantur, erunt puncta
 BEC ad parabolam.

PROPOSITIO CCXCV.

Sit ABC triangulum rectangulum;
 ductisque ex A lineis quocunque AD,
 ponant ut ad D, lineae DE, DF : & DE
 quidem normales ad , occurrentes AB
 lateri producto in E : DF vero normales
 ad CB: occurrant autem DF lineis in F,
 rectae EF parallelae lateri CB.

Dico B, F, esse ad parabolam
 cuius latus rectam est AB.



Demonstratio.

Cum enim anguli ADE ponantur recti, & DB linea normalis ad AB, rectangulis ABE
 aequalia sunt quadrata DB, id est FE : sed ABE rectangula illa inter se servant
 proportionem, quam linea BE; igitur ut BE ad BE, sic EF quadratum ad quadratum EF.
 Unde BFF sunt ad parabolam cuius latus rectum AB. Quod erat demonstrandum. ~

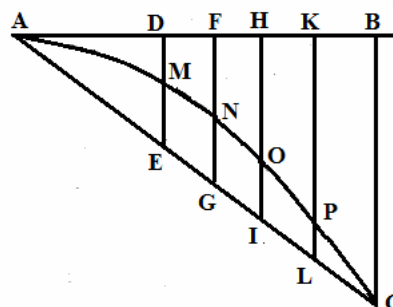
PROPOSITIO CCXCVI.

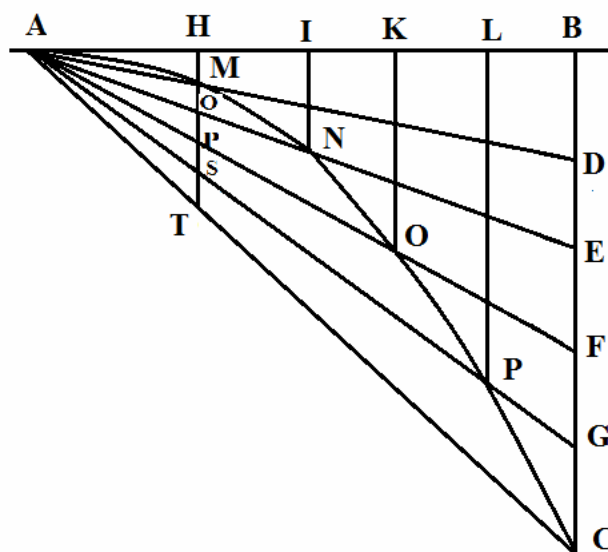
Secent ABC triangulum rectae quocunque DE, FG, HI, KL: lateri BC parallelae: fiant
 autem continuae proportionales BC, KL, KP: item BC, HI, HO: item BC, FG, FN :
 denique BC, DE, DM.

Dico puncta A, M, N, O, P, C, esse ad parabolam.

Demonstratio.

Quoniam BC linea communis prima est, seriebus
 continuarum BC, KL, KP: BC, HI, HO :BC, FG, FN,
 &c. rectangula BCKP, BCHO, BCFN, BCMD illam
 sortiuntur rationem quam habent lineae KP, HO, FN,
 DM. Igitur & quadrata KL, HI, FG, DE eandem
 quoque servant proportionem: sed ut KL quadratum,
 ad quadratum HI, sic KA quadratum est ad
 quadratum HA, & ut HI quadratum ad quadratum
 FG, sic HA quadratum est ad quadratum FA, &c.;
 igitur quadrata AK, AH, AF, AD illam habent
 rationem quam linea HP, HO, FN, DM; puncta igitur
 A, M, N, O, P ad parabolam sunt.





PROPOSITIO CCXCVII

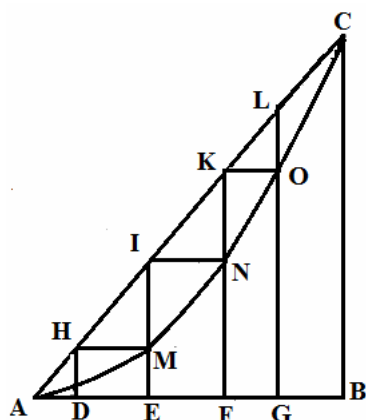
Esto ABC trianguli latus BC
 utcunque divisum in punctis
 divisum D, E, F, G iunctusque
 AD, AE, AF, AG, dividatur latus
 AB in punctis H, I, K, L sicut BC
 est divisum in D, E, &c.
 demittantur autem ex H, I, K, L
 rectae HM, IN, KO, LP
 parallelae lateri BC: & HM
 quidem occurrat AD in M; IN
 vero ipsi AE in N: dein KO,
 rectae FA in O, & LP, ipsi AG in
 P.

Dico A, M, N, O, P, C esse ad

parabolam.

Demonstratio.

Producta HM, occurrat lineis AD, AE, AF, AG in Q, R, S, T, quoniam est AH ad AI,
 ut BD ad BE, ex hypothesi; sit autem ut BD ad BE, sic HM ad HQ, HM est ad HQ, ut AH
 ad AI, id est ut HQ ad IN proportionales; igitur sunt HM, HQ, IN similiter ostenduntur
 proportionales HM, HSR, KO: item HM, HS, LP; denique HM, HT, BC quare rectangula
 HMIN, HMKO, HMLP, &c. illam inter se proportionem habent, quam HQ, HR, HS
 quadrata: sed est ut HQ quadratum ad quadratum HR, sic BE quadratum ad quadratum
 BF, id est per hypothesim AI quadratum ad quadratum AK; & ut HR quadratum ad
 quadratum HS, sic quadratum BF ad quadratum BG, id est quadratum AK ad quadratum
 AL: igitur & rectangula HMIN, HMKO, HMLP eam inter se servant rationem quam AI,
 AK, AL quadrata: sed rectangula HMIN, HMKO, HMLP sunt ut lineae IN, KQ, LP;
 igitur & quadrata AI, AK, AL eam inter se habent proportionem, quam lineae
 IN, KO, LP; quare A, M, N, O, P sunt ad parabolam. Quod erat demonstrandum.



PROPOSITIO CCXCVIII.

Esto ABC trianguli latus AB divisum in D, E, F, G punctis ut ratio Ad AE, duplicata sit rationis AE ad AF; & illa rursum duplicata sit rationis AF ad AG, &c. dein ex D, E, F, G punctis rectae erigantur DH, EI, FK, GL parallelae lateri CB: quas in M, N, O secant lineae HM, IN, KO aequidistantes lateri AB.

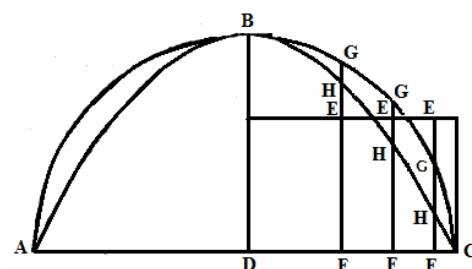
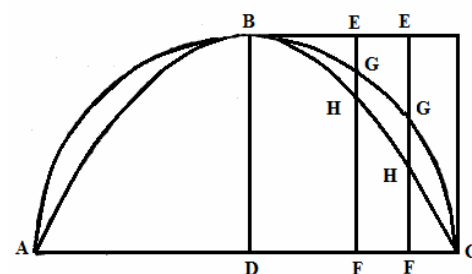
Dico A, M, N, O, C puncta esse ad parabolam.

Demonstratio.

Quoniam ratio Ad AE, id est DH ad EI; id est EM ad FN, duplicata est rationis AE ad AF, quadratum AE ad quadratum AF, ut EM linea ad lineam FN. Similiter ostendetur esse ut FN ad GO, sic AF quadratum ad quadratum AG; puncta igitur A, M, N, O sunt ad parabolam.

PROPOSITIO CCXCIX.

Semicirculum ABC secet orthogonaliter in D diametri duae AC, BD ponatur autem EE linea parallela AC: dein sumptis in AC punctis FF, erigantur normales FE,



occurrentes EE lineae in E, semicirculo in GG. Fiant autem proportionales EF, GF, HF.

Dico H, H, C puncta esse ad parabolam.

Demonstratio.

Cum enim proportionales sint EF,GF, HF, erit ut quadratum FG ad quadratum FG, sic A EFH rectangulum ad rectangulum EFH: sed EFH rectangula illam inter se habent rationem, quam lineae HF, igitur & HF lineae sunt ut quadrata GF: id est ut AFC rectangula, puncta igitur H,H,C ad parabolam sunt.

Idem continget in ellipsi, si AC, BD diametri ponantur coniugatae, & EF equidistantes ipsi BD.

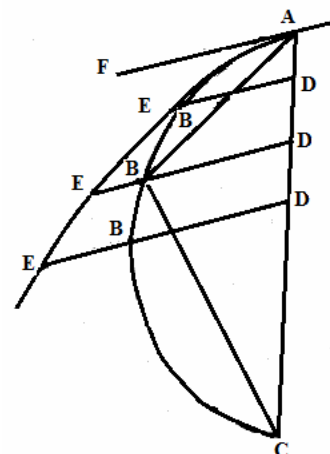
PROPOSITIO CCC.

Segmentum circuli ABC subtendat recta AC, quam in D secent quotcunque DE parallelae contingenti FG per A ductae: fiat autem ut AB ad AB, sic DE ad DE.

Dico A, E, E puncta esse ad parabolam.

Demonstratio.

Iungantur AB, BC: quoniam FG contingens est, angulus CAG aequatur angulo ABC; sed angulo CAG aequalis est angulus ABC, quia FG, DE aequidistant; angulo igitur ABC aequalis est angulus ADB; & triangula ABC, ABD similia sunt. Quare, ut Ad AB, sic AB ad AC & DAC rectangulo, aequale est quadratum AB. Quadratum igitur AB est ad quadratum AB, ut DAC rectangulum as rectangulum DAC, id est ut DA linea ad lineam DA: sed AB quadratis aequalia sunt quadrata DE, quadrata igitur eam habent rationem, quam obtinent lineae DA; quare A, E, E puncta ad parabolam sunt.



Fiat CD, aequalis CF, parallela BD: erit F ad parabolam MBH, cum omnes DE, translatae in GEH ad parabolam sint, facta deinde CM, aequali MN, ponantur NF contingens parabolam, & erigatur FO perpendicularis ad NF contingentem centroque O intervallo FO circulus describatur; continget ille parabolam, & NF lineam in F: tum

parabola describatur, quae ex descripto iam circulo KFP, oritur; haec quoque continget circum KFP, oritur; haec quoque continget circum KFP & NF, lineam in F, ut in priori propositione ostensum est; vertex igitur eiusdem est in M, cum MC, CN lineae aequales sint: ergo parabolae illae duae communem habent verticem M, & punctum F; una cum igitur eademque sunt parabola, quae per utrumque circum est descripta: ergo circum KFP parabolam priori praxi descriptam producit: atque ita eadem parabola eadem praxi in infinitum continuabitur. Quad erat propositum.

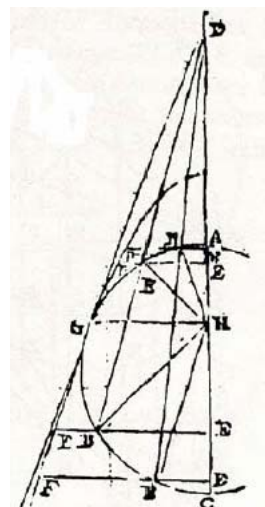
PROPOSITIO CCCIII.

Esto circuli ABC diameter AC, utcunque producta in D, ex quo, in circum immittantur lineae DB; aganturque ad AC per B normales FE proportionales ipsis DB; quarum termini sint FF.

Dico FF puncta esse ad parabolam.

Demonstratio.

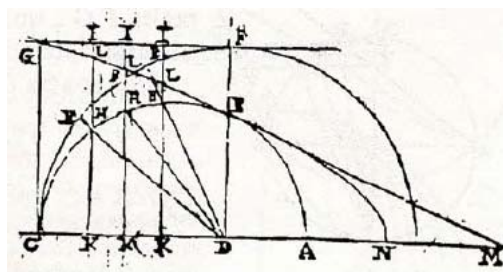
Ducatur ex D linea DG, contingens circum in G, & ex G, ponatur GH normalis ad AC, iunganturque HB, igitur DB ad DB est ut BH ad BH, sed BH translatae in FE, producant parabolam; igitur & ipsis DB id est BH, proportionales in eundem locum translatae, producant quoque parabolam.



PROPOSITIO CCCIV.

Assumptum sit in ABC circuli diametro AC, punctum quodcunque D, extra centrum; radio autem DC, quadrans describatur circuli CEF; perficiaturque quadratum FDC, tum ex D rectae ducantur quotcunque DE, occurrentes circulo ABC in H, & CEF quadranti in E: & per H agantur normales IK ad AC, fiantque HE lineis, aequales LI.

Dico L, L puncta esse ad parabolam.



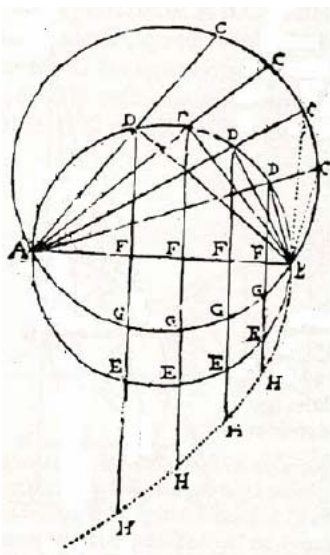
Demonstratio.

Quoniam FDC quadratum est, & D, centrum circuli CEF, rectae DE, lineis IK aequales sunt: ponantur autem ipsis HE, aequales LI, reliquae igitur LK, reliquis HD translatae in KL ad parabolam sunt ; igitur L, L puncta iam quoque sunt ad parabolam.

Corollarium.

Porro vertex huius parabolae sic invenitur, erecta ex D normali DB; per B agatur contingens BM : dein MD dividatur bifariam in N: patet N esse verticem parabolae.

PROPOSITIO CCCV.



Intersecant sese in A & B, circuli duo quivis ABC, ADB: ductisque ex A lineis, ADC, demittantur ex D rectae DEF orthogonaliter secantes AB diametrum in FF. Fiat autem ut DC ad DC, sic FH ad FH.

Dico BHH puncta esse ad parabolam.

Demonstratio.

Iungantur DB,CB: erunt igitur triangula DCB similia : quare ut DC ad DC, sic DB ad DB translatae in FH producant parabolam, igitur & DC translatae in FH, sectionem producant parabolicam.

PROPOSITIO CCCVI.

Occurrant sibi invicem circuli duo ABC, ACD in A & C, & ABC quidem circulus transeat per centrum circuli ACD, ducanturque ex A rectae AED, occurrentes circulo ABC in E , & ACD in D: ponantur autem per EE ad H B, normales FG, quae aequales vel proportionales sint lineis AD.

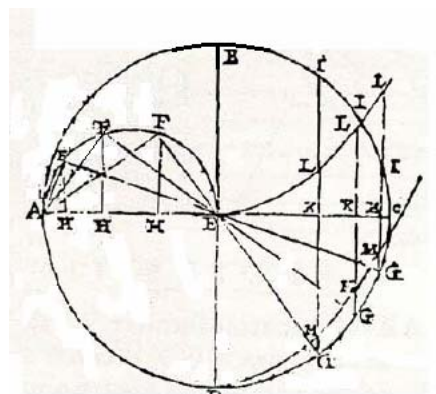
Dico puncta G, G esse ad parabolam.

Corollarium.

PROPOSITIO CCCVII.

Dico puncta D, M, M, item E, L, L esse ad parabolas.

Lungantur A F: Quoniam FH, GK aequidistant, triangula FHE, EKG sunt similia ; est autem & FHE triangulum similia ttriangle AFE ; igitur & AFE, EKG triangula similia sunt, quia vero AE aequalis est EG, triangula AFE aequalia sunt triangulis EGK, & AF, FE latera , aequalia lateribus GK, KE, unde ut AF quadratum ad quadratum AF, id est GK quadratum ad quadratum GK, sic AH linea ad lineam AH, id est per hypothesim KM ad KM; sed est ut GK quadratum ad quadratum GK, sic AKC rectangulum ad rectangulum AKC; igitur AKC rectangulum ad rectangulum AKC, sic MK linea est ad lineam MK: sunt. Quod erat primum.



Rursum cum sit ut EH ad EH, sic LK ad LK, sic autem ut EH ad EH, sic EF, quadratum ad quadratum EF, id est EK quadratum ad quadratum EK ; erit & LK linea ad lineam LK, ut FK quadratum ad quadratum EK: unde ELL puncta sunt ad parabolani.

PROPOSITIO CCCVIII.

Sint ad AB diametralem quotcunque descripti circuli contingentes sese in eodem puncto A; quorum diametri AB, AK, AL, AM, &c. ductaque ex B linea BC, cuntingat circulum minimum in B, reliquos autem secet in D, E, F, G, H fiant autem lineis BD, BE, BF, BG, &c. aequales rectae KQ, LR, MN, NT, &c. quae circulos contingant in LM, NO, &c.

Dico B, O, R, S, T, V puncta esse ad parabolam cuius AB, latus rectum.

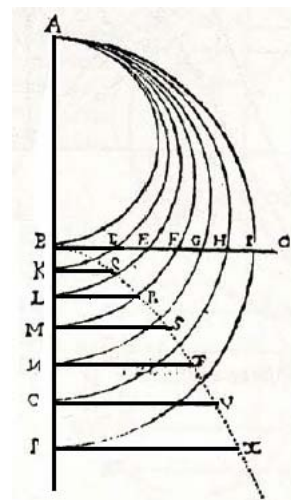
Demonstratio.

Quoniam BD normalis est ad communem diametralem AB, quadratum BD, & ad quadratum ABK rectangulum ad rectangulum ABL, id est (quia circuli contingunt sese in

A) ut BK linea est ad lineam BL, igitur & quadratum KQ est ad quadratum LR, ut BK linea ad lineam LB (cum LR, KQ lineae aequales sint BD, BE) eodem modo ostenditur MS quadratum esse ad quadratum NT, ut BM linea est ad lineam BN, & sic de caeteris : puncta igitur B, Q, R, S, T, &c. sunt ad parabolam AB ergo latus rectum esse patet, cum semper quadrata DK, RK, &c. aequalia fint rectangulis super KB, BA, LB, BA, &c.

Corollarium.

Hinc facilis patet praxis, producendi in infinitum parabolam, priori methodo ortam: cum enim in infinitum multiplicari, possint circuli illi contingentes, & BC lineae protendi, poterit quoque in infinitum continuari praxis qua prius parabolam produxi; adeoque eadem poterit in infinitum extendi.



PROPOSITIO CCCIX.

Esto ABC semicirculi diameter AC, actaque per A contingence AD, ducantur ex C lineae CBD, occurrentes contingenti in D, semicirculo in B: dein ex D normales demittantur DE quas in E secant lineac B, AE.

Dico puncta A, E, E esse ad parabolam cuius latus rectum AC.

Corollarium.

PROPOSITIO CCCX.

Demonstratio.

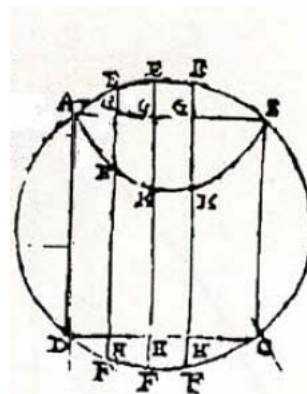
PROPOSITIO CCCXI.

Inscriptum sit circulo ABC rectangulum ABCD,
 ductisque unilaterum DA parallelis EGHF diant EGF
 rectangulis aequalia rectangula HGK.

Dico puncta AKB esse ad parabolam.

Demonstratio.

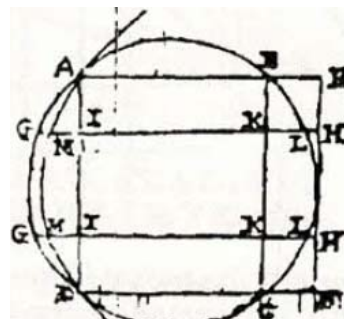
Ut EGF rectangulum est ad rectangulum EGF, sic AGB
 rectangulum est ad rectangulum AGB: igitur & HGK
 rectangulum est ad rectangulum HGK ut rectangulum
 AGB ad AGB ; sed HGK ut GK linea ad lineam GK;
 igitur & AGB rectangulum est ad rectangulum AGB, ut GK linea ad lineam GK: puncta
 igitur A, K, B ad parabolam sunt. Q.e.d.



PROPOSITIO CCCXII.

Inscriptum sit iterum rectangulum ABCD, circulo ABC,
 ductaque lateri BC parallela EF, quae circulum contingat,
 ponantur rectae GH, aequidistantes lateri AB: fiantque
 GIL rectangulus, aequalia rectangula HIM.

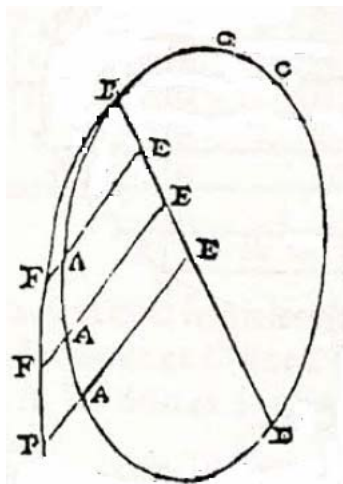
Dico A, M, D puncta esse ad parabolam.



Demonstratio.

Cum aequalia sint rectangula HIM, rectangulis GIL, HIM rectangulum ad HIM
 rectangulum est ut GIL ad GIL, id est AID rectangulum ad rectangulum AID: sed HIM
 rectangulum est ad rectangulum HIM, ut IM linea ad lineam IM; igitur ut AID
 rectangulum ad rectangulum AID, sic IM linea ad lineam IM : puncta igitur M, M sunt ad
 parabolam.

PROPOSITIO CCCXIII.



Sit in ABC ellipsi una ex diametris coniugatis aequalibus, diameter BD, ad quam ordinatim ponantur AE. Fiant autem quadratis AE, EB aequalia quadrata EF. Dico puncta B, F, F esse ad parabolam.

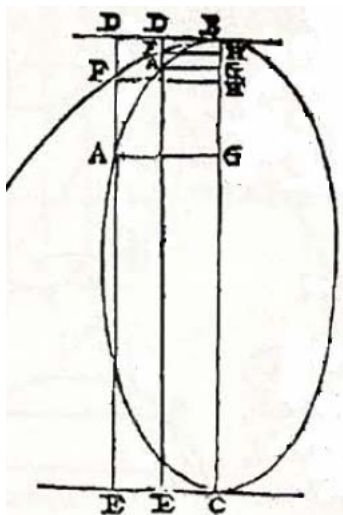
Demonstratio.

Quoniam BD una est diametris coniugatis aequalibus, quadrata AE aequalia sunt rectangulis BED: addito igitur quadrato BE, quadrato duo AE, EB aequalia sunt rectangulis EBD; igitur FE quadratum est ad quadratum FE, ut EBD rectangulum ad rectangulum EBD, id est ut EB linea ad lineam EB. Igitur B, F, F puncta sunt ad parabolam.

Circulo quoque convenit hac propositio, eademque est

demonstratio.

PROPOSITIO CCCXIV.



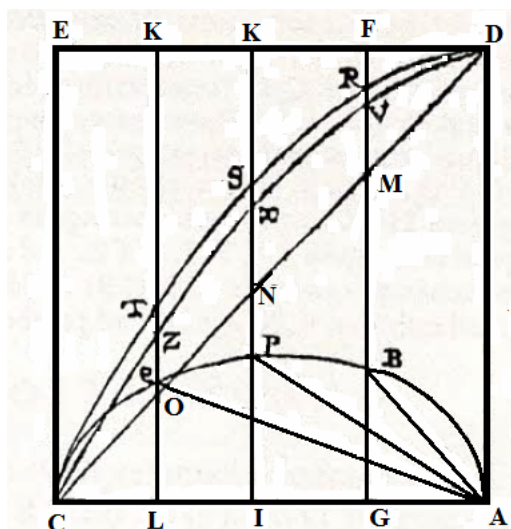
Esto ABC ellipseos diameter BC, quam in B & C contingant rectae DB, EC: positisque DE parallelis diametro BC; fiant DAE rectangulis aequalia rectangula EDF.

Dico B, F, F puncta esse ad parabolam.

Demonstratio.

Ducantur AG, FH parallelae DB, patet AG ordinatim esse positas ad BC; igitur DAE rectangulum est ad rectangulum DAE, id est BGC ad BGC, ut AG, quadratum ad quadratum AG, id est FH quadratum ad quadratum FH: igitur & FDE rectangulum est ad rectangulum FDE, id est HBC ad HBC rectangulum, ut

FH quadratum ad quadratum FH; sed HBC rectangulum est ad rectangulum HBC, ut HB linea ad lineam HB. Igitur ut FH quadratum ad quadratum FH, sic HB linea ad lineam HB: puncta igitur B, F, F puncta ad eandem sunt parabolam.



PROPOSITIO CCCXVII.

Iisdem positis fiant BG, PI, QL quadratis, aequalia rectangula GFRV, HISX, KLTZ.
Dico puncta D, X, V, Z esse ad parabolam.

Demonstratio.

Rectangulum GFRV ad rectangulum HISX est ut RV linea ad lineam SX : igitur & quadratum GB est ad quadratum PI, ut RV linea ad lineam SX: sed ut BG quadratum ad quadratum PI, sic AGC rectangulum ad rectangulum AIC, igitur ut RV ad SX lineam, sic AGC rectangulum ad rectangulum AIC, id est DMC rectangulum ad rectangulum DNC, id est MR linea ad lineam SN, quia RST ostensa est parabola; igitur & MV est ad XN, ut DMC rectangulum ad rectangulum DNC. Similiter ostendam esse XN ad ZO , ut DNC rectangulum ad rectangulum DOC; punctae igitur V, X, Z ad parabolam sunt.

PROPOSITIO CCCXVIII.

Iisdem positis: iungantur AB, AP, AQ : fiantque rectangula GFV, IHX, LKZ aequalia quadratis AB, AP, AQ.
Dico V, X, Z puncta rursum esse ad parabolam.

Demonstratio.

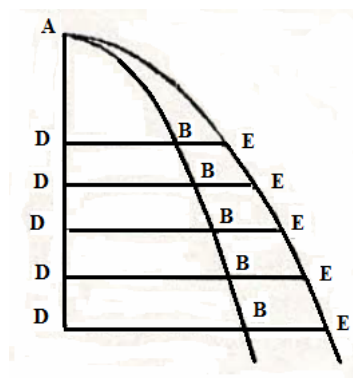
Quoniam BG, PI, QL, normales sunt ad AC, the square AB is equal to the squares AG, GB: but the square GB put in place is equal to the rectangle GFRV, and the square AG is equal to the rectangle GFR: therefore the square AB that is the rectangle GFV, is equal to the rectangles GFR, GFRV: similarly it is shown the rectangle IHX, to be equal to the rectangles IHS, IHSX, and the rectangle LKZ to be equal to the rectangles LKT, LKTZ, but since the rectangles GFR, GFRV may be put equal to the squares AG, GB; and IHS, IHSX, equal to the squares AI, IP, and V, X, Z have been shown to lie on a parabola; and therefore now are for a parabola.

PROPOSITION CCCXIX.

AD shall be the diameter of the parabola ABC: & ordinatim ad illam positae DB: fiat autem ut DB ad DB, sic DE ad DE.
 Dico AEE esse ad parabolam.

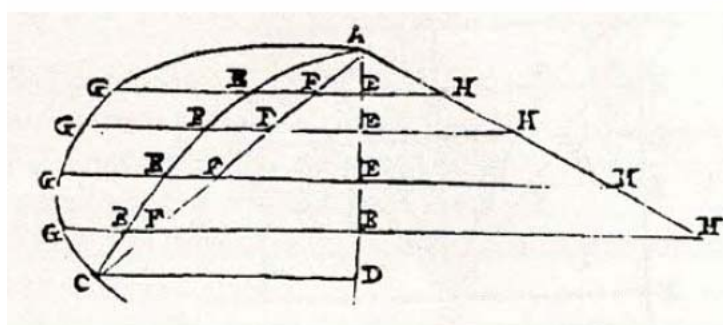
Demonstratio.

Est enim ED quadratum ad quadratum ED ut DB quadratum ad quadratum DB ex hypothesi, id est ut AD linea ad lineam AD; puncta igitur E, E ad parabolam sunt.



PROPOSITIO CCCXX.

Sit ABC parabolae diameter AD, & ordinatim positae CD, BE: ductaque linea AC quae BE rectis occurrat in FF, fiat ut FB ad FB, sic BG ad BG.
 Dico G, G puncta esse ad parabolam.



Demonstratio.

Fiat ut BF ad BG, sic FE ad EH: quoniam est ut FB ad FB, ita BG ad BG erit permutando ut FB ad BG, sic FB ad BG: sed per constructi: FE est ad EH, ut FB ad BG, ergo FB est ad EH, ut FB ad EH; igitur cum puncta EE in directum sint, puncta quoque

HH in directum sunt, ut patet ex elementis: deinde quia est ut FB ad BG. sic FE ad EH, erit componendo, convertendo GF ad BF, ut FH ad FE, & permutando GF ad FH, ut BF ad EF, & componendo GH ad FH, ut BE ad FE, iterumque permutando GH ad BE, ut FH ad FE, sed cum ante ostenderim esse FE ad EH, ut FE est ad EH; erit quoque ut FH ad FE, sic FH ad FE, ergo ut GH ad BE, sic GH ad BE, & permutando GH ad GH, ut BE ad BE; igitur & quadratum GH id quadratum BE est, ut quadratum BE ad quadratum BE, hoc est ut EA ad EA, hoc est, ut AH ad AH, puncta igitur GG ad parabolam sunt, cuius diameter AH.

PROPOSITIO CCCXXI.

Ponantur eadem quae prius, & fiant FE lineis aequales rectae BG.

Dico iterum A, G, G esse ad parabolam.

Demonstratio.

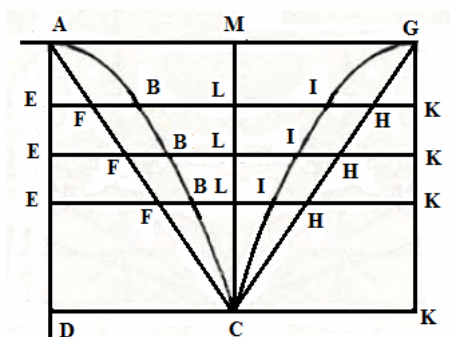
Cum enim FE lineis aequales ponantur GB, additis communibus BF, rectae FG aequales sunt FE, & FG quadrata aequalia quadratis BE: igitur quadratum FG ad quadratum BE est ut BE quadratum BE, id est ut AE linea ad lineam AE, id est AF ad AF. Puncta igitur A, G, G ad parabolam sunt, cuius diameter AFC.

Si vero FG lineae proportionales sint ipsis FE, ostenditur ut in praecedenti propositione, A, G, G esse parabolam.

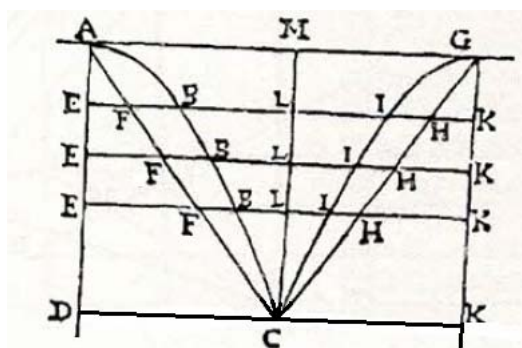
PROPOSITIO CCCXXII.

Parabolam ABC cuius diameter AD, contingat in D linea AG: ponantur autem ordinatim lineae BE, CD; iunctaque AC occurrat BE lineis in F, dein ducta quacunque CG ad AG, contingentem, qua BE lineas productas secet in HH: fiat ut FB ad FB: sic HI ad HI.

Dico puncta I, I esse ad parabolam.



Demonstratio.



Fiant HI ad HK, ut BF ad FE: patet ex elementis GKK esse in directum; erit igitur HK ad HK, ut FE ad FE: & cum sic IH ad HK, ut BF ad FE, erit componendo, ut BE ad FE, sic IK ad HK, & ut BE ad BE, sic IK ad IK: unde quadratum IK est ad quadratum IK, ut BE quadratum ad quadratum BE, id est ut AE linea ad lineam AE: id est ut GK ad GK. Puncta igitur G, I, I sunt ad parabolam.

PROPOSITIO CCCXXIII.

Eadem manente figura : perficiatur parallelogrammum ADC, & MC, latus occurrat lineis EB in L : fiat autem ut LB ad LB, sic LI ad LI.
 Dico puncta I, I esse ad parabolam.

Demonstratio.

Fiat AM ad MG, ut LB ad LI, & recta ducatur GC occurrens BI, lineis in HH. Erit igitur ut LF ad LF, sic LH ad LH. Sed ex hypothesi est LI ad LI, ut LB ad LB ; igitur & HI est ad HI, residuum ut FB ad FB, residuum unde per praecedentem puncta I, I sunt ad parabolam.

PROPOSITIO CCCXXIV.

Esto ABC parabolae diameter AD, ad quam ordinatim ponantur CD, BE: perfectoque parallelogrammo DF, ducatur linea FD, secans EB rectas in GG: fiant autem GE lineis aequales G, BH.

Dico HH puncta esse ad parabolam.

Demonstratio.

Iungantur AC, quoniam DF parallelogrammum cuius diametri AC, FD, & KE lineae aequidistant lateri AF, lineis EG aequales sunt rectae KI; igitur & HB lineis aequales sunt KI. Dempitis igitur communibus KB, manent IB, HK lineae aequales: puncta igitur HH ad parabolam sunt.

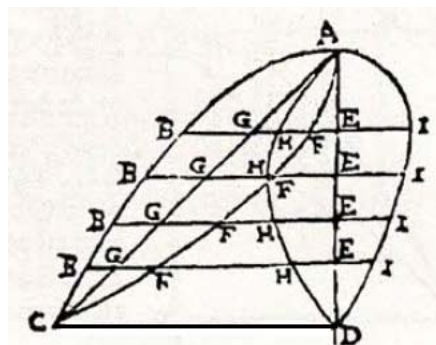
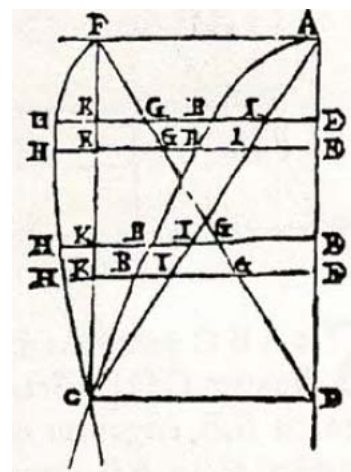
PROPOSITIO CCCXXV.

Esto ABC parabolae diameter AD, ad quam ordinatim ponantur lineae CD, BE, quas in G secet recta AC, descripta dein per A & C, parabola CFA quam in A recta contingat AD, & FB diameter sit, fiant FG lineis, proportionales EH, & rectis BG proportionales EI.

Dico tam H,H puncta quam I, I esse ad parabolam.

Demonstratio.

Est enim ut AGC rectangulum ad rectangulum AGC, sic AED rectagulum ad rectangulum AED ; sed ut AGC rectangulum est: ad rectangulum AGC, sic FG linea est ad lineam FG, id est EH linea ad lineam EH: igitur & HE est ad HE, ut AED rectangulum ad rectangulum AED : similiter ostenditur, esse EI ad EI, ut AED rectangulum ad rectangulum AED ; puncta igitur H, H & I, I, puncta ad parabolas sunt.

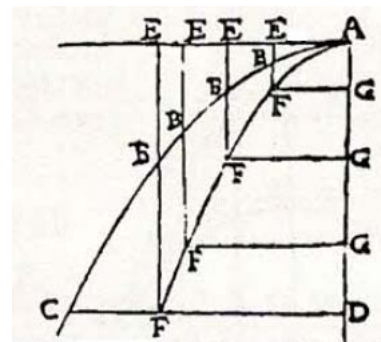


PROPOSITIO CCCXXVI.

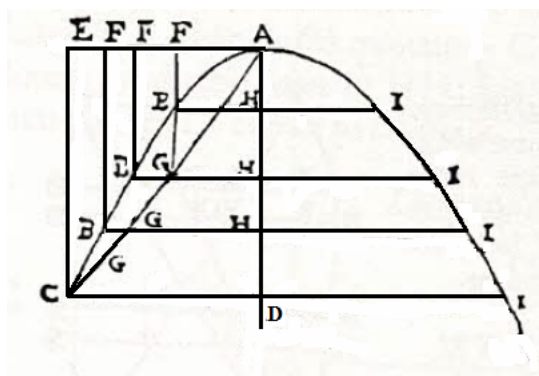
Parabolam ABC contingat in A linea AE : ponatur autem sectionis diameter AD, & illi parallelae EB: dein rectis EB proportionales fiant lineae BF.
 Dico A, F, F puncta esse ad parabolam.

Demonstratio.

Ducantur FG parallelae contingenti AE: cum igitur sit ut EB ad EB, sic BF ad BF, erit quoque EF ad EF, ut EB ad EB. Sed ratio EB ad EB, duplicata est rationis EA ad EA, id est FG ad FG; igitur & ratio EF ad EF, id est AG ad AG, duplicata est rationis GF ad GF: igitur A, F, F puncta sunt parabolam.



PROPOSITIO CCCXXVII.



Esto ABC parabolae diameter AD: actaque per A contingente AE, demittantur diametri FB occurrentes parabola: in B, & AC in G: ponantur autem ordinatim lineae HB: fiatque ut FG ad FG, sic B, HI ad B, HI.

Dico puncta A, I, I esse ad parabolam.

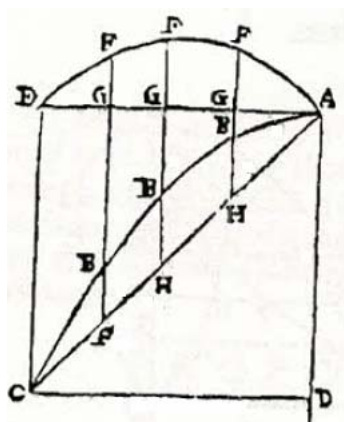
Demonstratio.

Ut FG ad FG, sic AF est ad AF, id est HB ad HB: igitur & HI est ad HI, ut BH ad BH: & HI quadratum ad quadratum HI, ut HB quadratum ad quadratum HB, id est ut AH linea ad lineam AH : puncta igitur A, I, I ad parabolam sunt.

PROPOSITIO CCCXXVIII.

Sit ABC parabola diameter AD aequalis lateri recto: ductaque ordinatim CD perficiatur quadratum CA: sumptis que in peripheria punctis B, B erigantur ex B diametri BF: occurrentes AE lineae in G: fiantque rectis AG aequales lineae BF.
 Dico A, F, E puncta esse ad parabolam.

Demonstratio.



Ducatur recta AC occurrens FB lineis in HH: quoniam AD lateri recto aequalis est, & CD ordinatim posita ad , rectae AD, CD , aequales sunt: unde & recte AG, GH quoque aequales sunt : & HG lineae aequales ipsis FB : demptis igitur communibus BG, manent HB, FG lineae aequales. Quare FG est ad FG, ut BH ad BH; id est ut AHC rectangulum ad rectangulum AHC, id est AGB rectangulum ad rectangulum AGE. puncta igitur A, F, E parabolam sunt.

PROPOSITIO CCCXXIX.

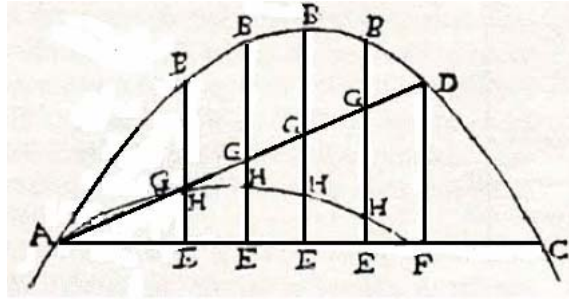
Parabolam ABC subtendat recta AC, quam in D secent quotcunque diametri BD : fiant autem BD lineis proportionales DE.
 Dico puncta A, E, C esse ad parabolam.

Demonstratio.

Ut BD ad BD, sic ED est ad ED, sed est ut BD ad BD, sic ADC rectangulum ad rectangulum ADC: igitur & ED est ad ED; ut ADC rectangulum ad rectangulum ADC quare AEC puncta ad parabolam sunt.

PROPOSITIO CCCXXX.

Parabolam ABC subtendant rectae AC, AD, demissaque ex D diametro FD, quae AC lineae occurrat in F, ducantur quotvis rectae BE aequidistantes DF, secantes AD lineam in GG: fiantque rectis BG proportionales lineae EH.
 Dico puncta A, H, F esse ad parabolam.



Demonstratio.

Ut AGD rectangulum est ad AGD rectangulum sic BH linea ad lineam BG : sed ut BG ad BG, sic EH ponitur ad lineam EH, igitur & EH est ad EH, ut AGD rectangulum ad rectangulum AGD, id est AEF rectangulum ad rectangulum AEF; quare A, H, F puncta sunt ad parabolam

PROPOSITIO CCCXXXI.

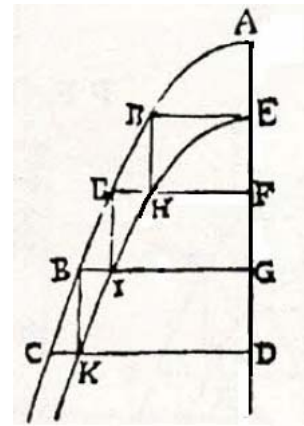
Esto ABC parabolae diameter AD, divisa in partes aequales, punctis E, F, G, D: ductisque ordinatim lineis EB, FB, GB, DC, demittantur ex B punctis diametri BH, BI, BK, convenientes cum ordinatim positis in H, I, K.

Dico E, H, I, K puncta esse ad parabolam.

Demonstratio.

Quoniam diametri partes AE, EF, FG, &c. ponuntur aequales, EF est ad EG, ut AE ad AF; id id est ut EB quadratum ad quadratum FB, id est FH quadratum ad quadratum GI. Eodem modo ostenditur esse ut EG ad ED, sic GI quadratum ad quadratum DK.

Puncta igitur E, H, I, K sunt ad parabolam. Quod erat demonstrandum.

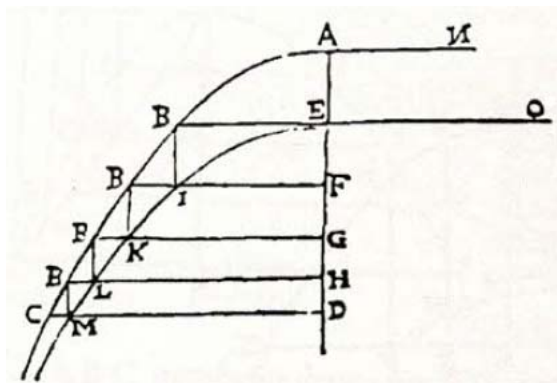


PROPOSITIO CCCXXXII.

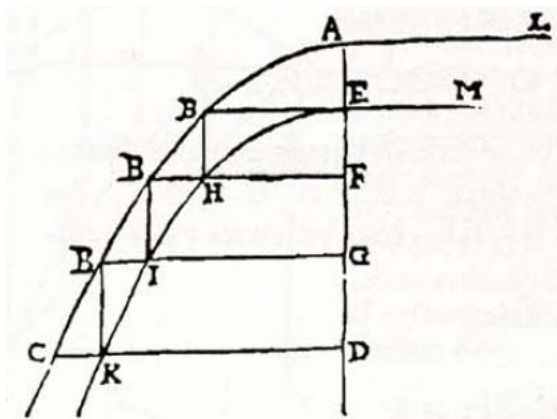
Esto ABC parabolae diameter AD, divisa punctis E, F, G, H, D, ut AE, EF, FG, GH, &c. sint continuae proportionales: ducantur autem ordinatim lineae EB, FG, GB, &c. & ex B, diametri demittantur convenientes cum ordinatim positis.

Dico puncta E, I, K, &c. ad eandem esse parabolam: & si series continuarum AE, EF, FG, &c. maioris fuerit inaequalitatis; dico parabolas in aliquo puncto convenire; si vero minoris fuerit inaequalitatis dico nulquam sectiones convenire.

Demonstratia.



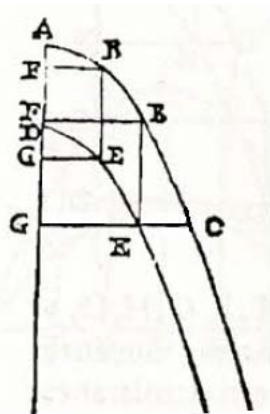
Quoniam AE, EF, FG, &c. proportionales sunt, EF est ad EG, ut AE ad AF; id est ut EB quadratum ad quadratum FB; id est FI quadratum ad quadratum GK. Similiter oftenditur esse FI quadratum ad quadratum HL ut EF, linea ad lineam EM: puncta igitur E, I, K, L, &c. ad parabolam sunt. Quod erat primum.



Ponantur autem series AE, EF, FG; &c. maioris inaequalitatis: dico parabolas convenire in aliquo puncto, sit enim ABC parabolae latus rectum AN, & EIK parabolae latus rectum EO: erit igitur quadrato EB aequale rectangulum EAN, & FI quadrato aequale retangulum FEO: unde cum EB, FI quadrata aequalia sint rectangula quoque EAN, FEO: sunt aequalia: quare rectum AE maior ponatur EF, erit AN latus rectum minus latere recto EO: adeoque parabolae in aliquo puncto

convenientis: Sit iam AE, EF, FG: &c. terminorum continuatio minoris inaequalitatis: dico parabolas nusquam sibi occurrere: ponatur enim LA latus rectum parabolae ABC, & ME, latus rectum parabolae EHI, quia igitur EB, FH quadrata aequalia sunt, rectangula quoque LAE, MEF aequalia sunt: & cum AE minor ponatur EF, erit LA maior quam ME; unde nusquam convenient sectiones.

PROPOSITIO CCCXXXIII.



Esto ABC parabolae axis AD, in quo sumpta cuivis parti
 AD, demittantur aequales diameter BE.
 Dico D, E, E puncta esse ad parabolam aequalem parabola
 ABC.

Demonstratio.

Ductis ordinatim BF, ponantur EG, parallelae. Quoniam
 igitur AD lineae, aequalis est BE id est FG, dempta vel
 addita communi FD, rectea AF aequalis est DG. Unde DG
 est ad DG, ut AF ad AF, id est ut FB quadratum ad
 quadratum FB, id est EG quadratum ad quadratum EG.

Puncta igitur D, E, E, ad. parabolam sunt, quia vero tam AF, DG, quam
 FB, GE lineae inter se aequales sunt, adeoque FB quadratum aequale quadrato GE; latera
 quoque recta axium AD, D G aequalia sunt, unde & parabolae aequales.

PROPOSITIO CCCXXXIV.

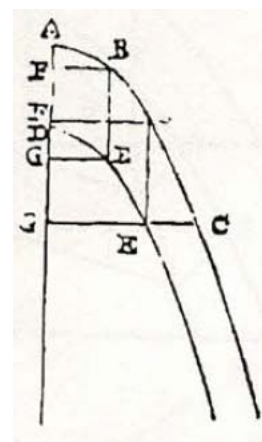
Iisdem positis:

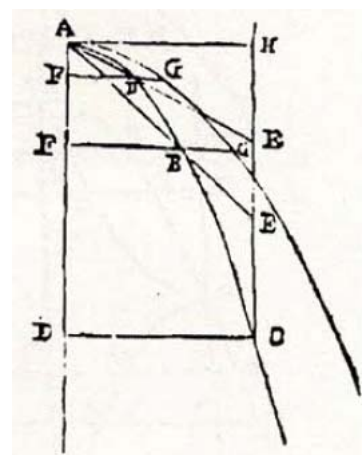
Dico parabolas illas nusquam convenire.

Demonstratio .

Ponatur in parabola DEE ordinatim ad axem DG, linea GE
 occurrens ABC parabolae in C; quoniam igitur latera recta
 utriusque parabolae aequalia sunt, & DG linea minor AG,
 rectangulum quoque sub DG & latere illius recto minus est
 rectangulo sub AG & latere recto: igitur & EG quadratum minus
 est quadratum CG : & C punctum cadit intra parabolam ABC:
 idem cum de omnibus punctis eiusdem parabolae ostendi possit,
 patet sectiones illas nusquam convenire .

Vocentur autem sectiones eiusmodi, parabolae parallelae, sive
 asymptoticae: quarum proprietates reliquas, & miram cum
 hyperbola inter asymptotos posita, symbolifationem, octavan
 parte huius libri, exhibebimus.

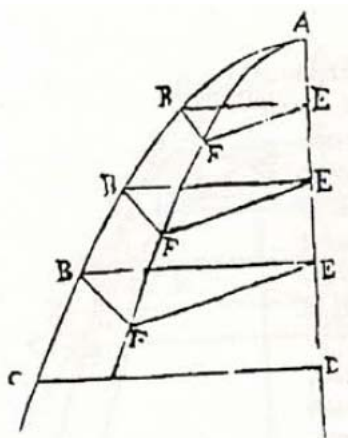




PROPOSITIO CCCXXXVII.

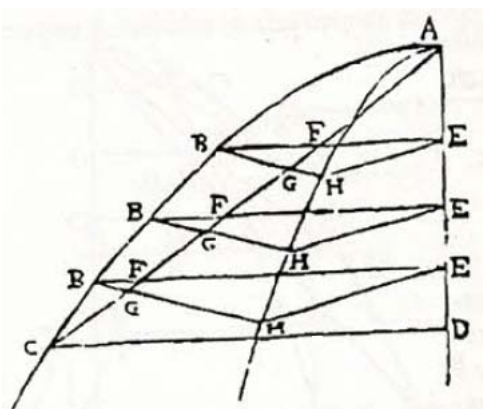
Esto ABC parabola diameter AD, ad quam ordinatim lineae BE; ducantur autem ex B lineae BF, inter se parallelae, & proportionales lineis EB.
 Dico puncta A, F, F esse ad parabolam.

Demonstratio.



Iungantur EF: Quoniam tam FB lineae inter se, quam EB aequidistant, anguli EBF aequales sunt; unde cum proportionalia sint latera EB, BF, aequales angulos EBF inter se similia sunt; adeoque & EF lineae aequidistantes sunt ad invicem; & EB lineis proportionales: quadratum igitur EF est ad quadratum EF, ut EB quadratum ad quadratum EB, id est ut AE linea ad lineam AE; unde puncta A, F, F sunt ad parabolam

PROPOSITIO CCCXXXVIII.



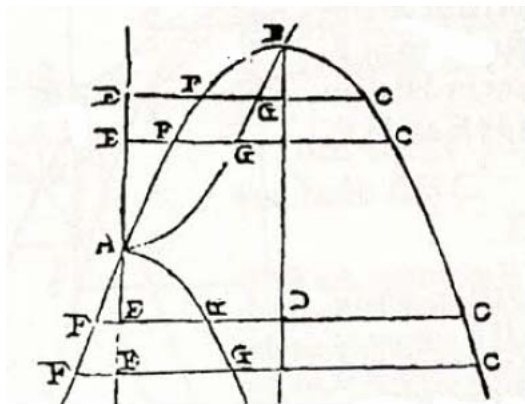
Esto ABC parabola diameter AD, & ordinatim ad illam positae CD, BE: & EB quidem iuncta AC, dividat in F: ductis autem parallelis BG, quae AC lineae occurrant in G: fiat ut FB ad BG, sic EB ad BH.

Dico AH puncta esse ad parabolam.

Demonstratio.

Iungantur enim EH: Quoniam EB, BH lineae proportionales sunt, lineis FB, BG; & angulus FBG communis, triangulis FBG similia sunt trianglea EBH: sunt autem & FBG triangu-
 quoque similia, cum tam FB lineae inter se, quam BG rectae aequidistant: igitur & EBH triangu-
 la similia sunt. Quare ut EB ad EB, sic EH ad EH, adeoque per praecedentem puncta AH ad eandem sunt parabolam.

PROPOSITIO CCCXXXIX.



Esto ABC parabolae axis BD, cui parallela ponatur AE, secans parabolam in A: ducantur autem ad axem ordinatim lineae EC, & inter FE, EC mediae ponantur EC, & inter FE, EC mediae ponantur EG.

Dico A, G, B puncta esse ad parabolam.

Demonstratio.

Est enim ut AE linea ad lineam AE, sic FEC rectangulum ad rectangulum FEC. Igitur & EG quadratum est ad quadratum EG, ut AE linea

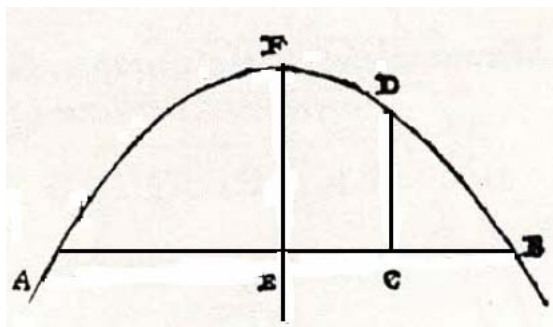
ad lineam AE : quare A, G, B puncta ad parabolam sunt.

PROPOSITIO CCCXL.

Datis rectis AB, CD se mutuo decussantibus; invenire parabolam cuius DC data sit diameter.

Constructio & demonstratio.

Divisa AB bifariam in E, ducatur EF parallel CD: fiatque ut AEB rectangulum ad rectangulum ACB, sic EF linea ad lineam DC. Patet A, F, D, B puncta esse ad esse ad parabolam quaesitam, cuius diameter est DC.

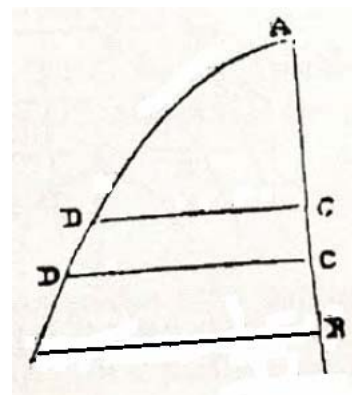


PROPOSITIO CCCXLI.

Dato AB latere recto, exhibere illius parabolam.

Constructio & demonstratio.

Secetur AB utcumque in CC, & ex C lineae erigantur CD, ut CD quadrata aequalia sint rectangulis CAB; singula singulis: patet A, D, D puncta esse ad parabolam quaesitam.



PROPOSITIO CCCXLII.

Datis tribus punctis A, B, C non in directum positis, & linea BD, quae per aliquod ex datis punctis transeat, oportet parabolam describere per A, B, C puncta cuius aliqua sit diameter BD.

Constructio & demonstratio.

Iuncta AC secetur bifariam in E, ponaturque EF parallela BD, & BF rectae AC: fiat autem ut AE quadratum, ad quadratum BF, sic EG linea ad lineam FG, describaturque per B, G, C parabolam, cuius diameter sit BD, patet illam quoque per A transire. Igitur, &c. Quod erat faciendum.

