BOOK THREE.

GEOMETRICAL LOGISTICS.

CHAPTER I.

THE NOTATION AND NAMES OF GEOMETRICAL MAGNITUDES.

We have discussed arithmetic in the preceding book, here logistic geometry follows in order.
The geometry therefore is said to be the careful calculation of concrete [*i.e*. composite] magnitudes by means of concrete numbers.
A whole number is said to be concrete in as much as it refers to a concrete and continuous quantity.
So that 3*a*, if thus it may refer to the three lines each one inch long: —— —— ——, is a discrete number: But when it refers to a concrete and continuous line of three inches, in this manner : ——+——+——, it is called a concrete number, but here improperly, subject to a ratio being present.

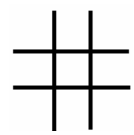
But properly, and by themselves, we call concrete numbers the roots of numbers which are unable to be measured by any number (either by an integer or fraction).

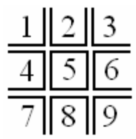
[We now consider concrete numbers to be discrete whole numbers, such as one might encounter in combinatorial analysis ; Napier in 1617 considers a concrete number as arising from an infinite number of finite fractions, concreted or compounded together, and not to be a whole or fractional number, and thus is an irrational number. The powers and roots of such numbers presented here by Napier is a precursor to the ideas of analysis : functions, continuity, inverses, etc.]

So that the bipartite or square root of seven, is greater than two but smaller than three, and it will not be found to be equal to, or commensurable with, any fraction in the whole essence of fractional numbers; therefore it is properly called a concrete number. Thus the tripartite or cube root of the number ten is not a discrete number, neither is it commensurable with a number, and so it is a concrete number; and thus all the roots of the infinitude of numbers, which are called surds or irrationals.

The generation of these concrete numbers is had by extracting the roots from these numbers not yet in place ; as we have instructed in Ch. 4 Book I, and in Ch. 9 Book II.
From which diversity of the roots, diverse notations and names of concrete roots arise.
So that the bipartite root of seven (that generally one calls the square root of seven) thus we write **⊔7**, and we may say 'the bipartite root of seven or the square root of seven '. Likewise, for the cube root of 10, we advance the tripartite root ten, and thus will write, **∟10**. Likewise, we write the quadrapartite root 11 thus, **⊐11**. Likewise, the quintupartite root of a number thus, □ ; sextupartite thus, ⊏ .

This variation of the characters of the roots with the index of their numbers may be supplied to us by this single shape (so that it may be remembered), divided into its separate parts.





As in the preceding examples, ⊔ L ⊐ □ ⊏ placed before the number, will denote the bipartite, tripartite, quadrupartite, quintupartite, sextupartite roots. And in the same way :

┐ septupartite [7th]
⊓ octupartite [8th]
┌ noncupartite [9th]; and likewise
┘° decupartite [10th]

┘┘ undecupartite [11th]
┘⊔ duodecupartite [12th]
┘└ tredecupartite [13th]
┘⊐ or ┘⊐ quadrudecupartite [14th]
┘□ quindecupartite [15th]
┘⊏ sedecupartite [16th]
┘┐ septemdecupartite [17th]
┘⊓ octodecupartite [18th]
┘┌ novemdecupartite[19th]
⊔° vigecupartite [20th]

⊔┘ 21st
⊔⊔ 22nd
⊔└ 23rd
⊔⊐ or ⊔⊐ 24th;etc.
Likewise └° 30 th
⊔° 40 th
□° 50 th
⊏° 60 th
┐° or ┐° 70 th
⊓° 80 th
┌° 90 th
┘°° 100 th
And thus indefinitely, in the manner of arithmetical figures.

Geometrical numbers therefore name rather than count magnitudes, therefore generally are called the names.

Some of the names are of a single term, as uninomials [Latin : *uninomia* : in the sense that only a single term is involved, with the root given by a distinct single name] ; others are of several named parts.

A power with a single named part is the same as a single concrete number, either properly or improperly called.

From which it follows that a single-named quantity is either a single simple number, or the root of some simple single number.

So that 10 is a simple number, and it is taken by geometers as a single quantity. Likewise ⊔10, L12, ⊐26 also in a like manner are the roots of numbers, and taken one by one truly as uninomial roots.

And since thus the radical of a single-valued quantity shall be the root of either a positive [abundant] or negative [deficient] number, its index being either even or odd, – in which case it comes about in a four-fold manner, that certain single-values quantities shall be positive, certain others negative, again certain others to be positive and negative, which we call twin valued ; and finally some others are neither positive nor negative , which we call foolish or trifling [meaning, that we should not worry about them ; here we will call hence call these imaginary cases.]

We have put the fundamentals of this great algebraic secret in place above in Ch. 6 pf Book I : because (just as it has not been made evident by anyone I know) yet after it became well-known, it would bring so much benefit to this art, and to the rest of mathematics.

In positive and negative monomials, there is little concern about placing the sign before or between the root symbol and number ; yet that is understood to be in place in front. But in twin and imaginary cases, the sign must always be put between the two.

In the first case ∟10 is an example, or (which is the same by Ch. 6 Book I.) **∟ +10** is a positive uninomial (by Ch. 6 Book I.). In the second case, the example **∟ –10**, is a negative uninomial (by the same chapter). In the third case the example **⊔10**, or **⊔+10** (which is the same as the above), signifies both a positive magnitude as when multiplied by itself makes +10, as well as a negative number which also multiplied by itself makes +10: Just as, for the sake of clarity, **⊔9**, or **⊔ + 9**, are both + 3 as well as –3; as we have shown in Ch. 6 of Book I above. In the final case, **⊔ – 9** is the example, because in the pure imaginary case, neither can it be signified as either positive or negative ; for negative nine does not have a square root, as Book I, Ch. 6 makes apparent.

In imaginary cases great care has to be taken so that the – sign of diminishing is put in place after the root symbol, rather than before.

So that, if for **⊔ –9** (which is the bipartite root of negative nine, both absurd and impossible to be obtained), you were to assume **–⊔ 9**, which signifies the negative magnitude for the bipartite root of the number nine, you will be greatly in error : for the bipartite root of nine (to wit **⊔ 9**) here is the positive twins , to wit + 3 and –3, that is, of positive three and of negative three ; and thus it will be the negative twin magnitude from these + 3 and – 3 ; and thus anyone who introduces **–⊔ 9** for **⊔–9**, offers a twin magnitude or with a two-fold significance, for a quantity both foolish and without significance ; therefore with regard to this beware, in which many are in error.

Evidently in signifying the rest of the uninomials the same sign is to be placed between the root symbol and the number, or each is to be put in front: Nor does it change value in these uninomials, by inserting a + sign in the first or middle empty position (by Ch. 6 of Book I.).

So that **⊔9, ⊔+9, + ⊔9**, and + **⊔**+9, in short signify the same, evidently both +3 as well as –3; likewise **∟27**, **∟ +27**, **+ ∟ 27**, or **+ ∟ +27**, prevail the same as only +3 ; likewise **∟ –27, + ∟ –27, – ∟ 27**, or **– ∟ +27**, likewise give rise to the same value –3 only; likewise with imaginaries **⊔ –9** is the same as +**⊔ –9**, evidently the imply the same impossibility; but be warned lest for these you should put in place **–⊔ 9** or **–⊔+9**, as we have cautioned in the preceding section.

And these are the ways in which monomials affect each other; the ways in which uninomials in turn are affected follow.

Thus there are two kinds of uninomials: either in turn commensurable, or incommensurable.

Commensurable numbers are those which act between each other as discrete or absolute numbers.

From which, any absolute number is commensurate with all absolute numbers. And likewise, two similar uninomials, of which the first radicand is a simple number, and the other, dividing the simple number, returns a number with a root of such a kind as the first number provided, indicated by the root sign, are said to be commensurable in turn, in the ratio that the root indicates.

So that 5 to 7, because they are absolute numbers or rationals, are commensurable ; likewise, the two similar uninomial roots , **⊔8** and **⊔2**, the simple number of which 8, divided by the simple number 2, returns 4; but it has the root of four for the sign **⊔**, to with the bipartite, and it is two ; therefore **⊔**8 and **⊔**2 are in turn commensurable in the ration of the root, evidently two.

All the other uninomials irreducible to this are agreed to be incommensurable.

So that └12 and **⊔**3, because they are dissimilar roots, are incommensurable; likewise, **⊔**6 and **⊔**2 (although they have similar roots) are incommensurable, because 6 divided by 2 produces 3, which without the root sign **⊔**, evidently the bipartite, extracted; but 12 and **⊔**4 are commensurable, because they give rise to the same as 12 and 2.

Etc.

[Thus Napier makes a valiant attempt to set up a system of names of roots for algebraic and arithmetical calculations, by using the above just workable but very unwieldy geometrical constructions; in the following we will see that he strays even further from the now familiar beaten path of indices, as he gives individual roots and powers special names, while leaving the unknown as a quantity without a name : the exact antithesis of modern algebra. Surely this text, incomplete as it is, and lacking the master's touch in its final production centuries later, is a testimony of how not to do something.]

THE ALGEBRA OF JOHN NAPIER
BARON OF MERCHISTON.

FIRST BOOK.

THE PART OF ALGEBRA CONCERNING NAMES.

CHAPTER I.

CONCERNING THE DEFINITIONS AND DIVISIONS OF THE PARTS , AND THE NAMES OF THE ARTS.

1. ALGEBRA is the science concerned with finding the solutions to questions of how great and of many times [something may be or occur].
2. And this is twofold : one part concerns the names, and the other is the positive part [see below.]
3. Those parts to be named are these which have a name [derived] from rational or irrational numbers.
4. Rationals are absolute numbers, or fractions ; which also may be treated by arithmetic.
5. Irrationals are the roots of numbers not having roots among the whole numbers.
6. And these (because they are magnitudes) are also observed in geometry.
7. The positive part of Algebra is the part, which produces quantities and hidden numbers through artificial substitutions ; which we shall talk about in Book II.
8. But here in Book I, in the first part of Algebra, we will discuss the names of magnitudes and numbers.
9. There are three kinds of names: uninomials or *uninomia* [*uninomia* : single names which for convenience here we Anglicize and call monomials, noting as above that no variable is present, only a root of some kind, of some number], plurinomials or several connected uninomials [*plurinomia*, several uninomials connected together by + or – signs], and *general* names, to be dealt with in that order.
10. Uninomials are single simple numbers, or the root of some simple number.
11. And uninomials are the roots of different numbers ; therefore they are expressed by propositioned different characters √ʓ, for the sake of the art and teaching ; and these characters are called radical signs.

So that the cube root of six is described thus, √𝔠6; likewise, the square root of five thus √ʓ5 ; and thus with the remainder, as follows :-

√ʓ	Square root.
√𝔠	Cube root.
√ʓʓ	Square root of the square root.
√ß	Supersolid root [*i.e.* 5th root].
√ʓ𝔠	Square root of the cube root[*i.e.* 6th root] .
√ʃß	The root two after the supersolid root [*i.e.* 7th root].
√ʓʓʓ	The square root of the square root of the square root [*i.e.* 8th root].
√𝔠𝔠	The cube root of the cube root. [*i.e.* 9th root]

And thus with the others indefinitely.
12. Certain of the roots and indices are simple : such as √q, √c, √β, √ſβ.
Certain others are multiples : such as √qq, √qc, √qqq, √cc, etc.
13. Again, certain of the roots and indices are squared names, which are named by the square, or otherwise.

So that the square root, the square root of the square root, the cube root of the square root, the supersolid root of the square root, etc.
Indeed certain are without the square, in the name of which without mention of the square, such as in the cube root, the cube root of the cube root, the supersolid root of cube root.
14. Two uninomials, affected by the same sign of the root, are called similar roots; which with diverse roots, are called dissimilar.
15. Any two rational numbers are commensurable ; and likewise similarly, two uninomial roots, the one simple number of which divided by the other simple number, returns a number with the root of such given such as may be shown, are said in turn to be commensurable.

So that 12 compared to 2, because they are rationals, also will be commensurable numbers ; and likewise the two roots √q**8** and √q**2** shall be similarly, the 8 of which divided by 2 returns the root of four with the root sign √q shown, which is 2; therefore √q**8** and √q**2** are commensurable numbers.

Corollary.

16. Hence it is plain that other uninomials irreducible to these are incommensurable.

So that √q6 and √q2 shall be incommensurable, because 6 divided by 2 returns 3, which is without a [whole number] square root; likewise, √c12 and √q3, are incommensurables; but 12 and √q4 are commensurables, because they become the same as 12 and 2.
[One might imagine that Cajori's : *A History of Mathematical Notations* would include reference to this work, but that is not the case. As it happens, Napier redefined some of the symbols available at the time for his own purposes. The origins of the square root sign may be of some interest, as discussed in this book.]

CAPUT II.

THE ADDITION OF UNINOMIALS.

1. If two uninomial propositions should be commensurable, divide the greater number by the lesser ; with the root of the quotient extracted (by Arithmetic) just as with the root shown ; add one to this root ; take the whole multiplied by itself as often as it is shown by the root ; hence multiply this into the absolute number of the lesser uninomial, and with the former root put in place before the product : this becomes equal to the uninomial for the first two uninomials.

So that the commensurable uninomias √q**12** and √q**3** are to be added; divide 12 by 3, making 4; from 4 extract √q, it becomes 2; add 1 to this, making 3; which is multiplied into itself as many times as the sign Q shows, to wit by squaring, making 9; this multiplied by 3 (to wit the number of the lesser uninomial) becomes 27; put in place the former sign with this, making √Q**27**, which is the sum of each of √Q**12** and √Q**3**. Likewise, √C**24** and √C**3** (by Ch. 1 prop. 14) are commensurable, and (by this method) added making √C**81**. Likewise, √Q $\frac{2}{3}$ added to √Q $2\frac{2}{3}$ produces √Q**6**.

2. If incommensurable uninomials were proposed, they cannot be connected otherwise than by the interposed sign **+** , which is called the sign of addition.
So that √C**5** and √Q**3** are to be added, they become √C**5** + √Q**3** ; which is to be pronounced thus , — the cube root of five increased by the square root of three. Likewise, √Q**6** and √Q**2** added becomes √Q**6 +** √Q**2**.

Corollary.

8. Hence it is apparent that positive binomials and multinomials arise from the addition of incommensurable uninomials, **—** said thus because with two or several uninomials are agreed to be connected by the **+** sign; with which in its place.

CHAPTER III.

THE SUBTRACTION OF UNINOMIALS.

1. If the proposed uninomials were commensurable, distinguish the number from the uninomials, from which a subtraction comes about, by subtracting the number one ; extract the root of the quotient as it may be denoted by the root sign ; from that take away one ; multiply the remainder into itself as many times as indicated by the root sign ; also multiply the product into number of the uninomial subtracted, and to this place in front the sign from the former root, and it becomes thence the left over or remainder of the subtraction of the first uninomials.

Examples.

Let √Q**12** be subtracted from √Q**27**; divide 27 by 12 arithmetically, it becomes $\frac{9}{4}$, the √Q of which is $\frac{3}{2}$; hence take away 1, it becomes $\frac{1}{2}$; which multiply into itself by the square becomes $\frac{1}{4}$ and multiply that into 12 becomes 3 ; and with which with the sign of its root put in front, becomes √Q3, for the remainder of the subtraction of √Q12 from √Q27. In a similar manner, √C24 subtracted from √C81 leaves √C3 . likewise, √Q $\frac{2}{3}$ taken from √Q6 leaves √Q $2\frac{2}{3}$.

2. If the proposed uninomias were incommensurable, write both together with the uninomial to be subtracted to be put after and place this sign – in between, which is called the sign of diminution.

Examples.

Let √ℭ5 be subtracted from √q3, √q3 – √ℭ5 remains, which is spoken about thus, ' the square root of two diminished by the cube root of five'. Likewise, √q2 from √q3 leaves √q3 – √q2.

Corollary.

3. Hence it is apparent, defective remainders or apotomes arise from the subtraction of incommensurable uninomials of binomials or plurinomials ; for an apotome is defined from the subtraction of uninomials or of several placed together with the subtraction sign.

CHAPTER IV.

THE EXTRACTION OF ROOTS FROM UNINOMIALS.

1. If the root to be taken were simple, and lies hidden in an absolute uninomial number, that may be extracted arithmetically with the retention of the former root sign.

Examples.

The square root shall be extracted from this uninomial √ℭ4 *;* because a square root lies hidden in 4, with that extracted and with the former root sign retained, it will be √ℭ2. Likewise, with √ℭ extracted from √q $\frac{54}{16}$, or √q $\frac{27}{8}$ it will be √q $\frac{3}{2}$.

2. Truly if a simple root cannot be taken by extraction from the uninomial of an absolute number, then place before the absolute number both the sign of the root to be extracted and the former sign.
So that √qshall be extracted from this √q**3**, that will be √qq**3**; likewise, √ℭ of this √q**5**, becomes √qℭ **5**.

3. But if a multiple root may be extracted from a uninomial, in the first place extract one simple root from the multiple, thence another from this, and thus all one by one (by 1 and 2 of this chapter).
So that √qℭ may be extracted from this √ß **27**, hence extract (by 2) √q, that will be **√qß27** ; then extract √ℭ from this, (by 1), that will be **√qß3**. Likewise, √qq of this √ℭ**16** will be √ℭ**2**. Likewise, √qq of this √ℭ**10** will be **√qqℭ10**.

4. In root extractions from fractions, the same is to be put in place with the root symbol of the interposed line, and if that clearly is put in place before the numerator and the denominator.
So that if **√qß** is extracted from $\frac{2}{3}$, that will be $\frac{\sqrt{}q\mathfrak{C}2}{\sqrt{}q\mathfrak{C}3}$, or √q $\frac{\sqrt{}\mathfrak{C}2}{\sqrt{}\mathfrak{C}3}$, or better **√qß** $\frac{2}{3}$; for all these are completely the same.

CHAPTER V.

REDUCTION TO THE SAME ROOT.

If two uninomials were assigned dissimilarly roots, and you multiply the absolute number of each just as many times by itself, as is indicated by the associated dissimilar root of the other ; with the product for each put in place itself, and with the sign of each root put in front ; the uninomials will be reduced to the same root, with the former value saved.

Example.

√ℭ**3** and √♀**2** shall be two dissimilar roots, requiring to be reduced to the same root ; therefore multiply 3 into itself as a square, and 2 into itself as a cube, from the former 9 is made, and from the latter 8, which become √♀ℭ**9** and √♀ℭ**8**, with the roots for each placed in front, which are similar roots, also with the former value retained ; indeed√♀ℭ**9** prevails the same as √ℭ**3**, and √♀ℭ**8** prevails the same as √♀**2**, as is apparent by Ch. 4 Book I. Likewise, √♀♀**2** and √♀ℭ**5** are reduced thus : cube 2, and square 5 (for the square is similar in the first root), making √♀♀ℭ**8** and √♀♀ℭ**25**. Likewise, √♀**6** and 2 are √♀**6** and √♀**4**.

CHAPTER VI.

THE MULTIPLICATION AND DIVISION OF UNINOMIALS.

1. Every uninomial, with no sign written, is understood to have the positive sign written.
So that √♀**10** is taken for +√♀**10**.

2. The same sign multiplied or divided by the same gives rise to a positive sign [i.e. the sign of augmentation]; but contrary signs multiplied or divided in turn, give rise to a negative sign [*i.e.* the sign of diminution.].
The examples are below.

3. Therefore in the first place the uninomials occur with similar roots, if not through themselves, perhaps by reduction ; then multiply or divide one number by the other, with the root to be extracted as the product of the roots indicates, by Ch. 4 ; finally (by section 2 if this) multiply or divide the signs, and put in front of the root the sign produced, and thence it becomes produced by multiplication or division.

Let the uninomial √♀**12** be multiplied by √♀**3***;* multiply 12 by 3, making 36, of which the square root is 6, which is produced. Likewise, √ℭ**3** multiplied by **–** √ℭ**2** makes the product **–** √ℭ**6**, by section 2 of this. Likewise, **–** √ℭ**3** is multiplied by √♀**2**, first made by reduction to √♀ℭ**9** and √♀ℭ**8**, by Ch. 5; thence 9 multiplied into 8 makes 72, the root of which ♀ℭ (by Ch. 4 sect. 2) will be √♀ℭ**72**, to which the – sign must be placed in front, becoming **–** √♀ℭ**72**.

Examples of division.

Let √♀**l2** be divided by –√♀3, making the quotient produced by the above –2. Likewise, –√𝔠16 divided by √𝔠2 gives – 2. Likewise, √♀𝔠72 divided by √♀𝔠**9** makes √♀𝔠**8**, otherwise √♀**2**; as is apparent from Ch. 4, section 1.

Corollary.

4. Hence it is apparent from the square, cube of the uninomials, or to be multiplied to some power, with its absolute number by itself squared, cubed, or by being multiplied to that order , with the former root retained.

So that √♀ **2** cubed becomes √♀ **8**. Likewise, √𝔠**3** squared becomes √𝔠**9**, etc.

Corollary.

5. Hence it follows that some such root, on being multiplied into itself as often as indicated by its quadritinomial root [*i.e.* a 'squared' root sign, such as √♀𝔠], is taken away, along with the preceding sign and the root ; or not the quadritinomial, with only with one root taken away.

As –√𝔠**5** cubed makes –5. Likewise √♀**3** squared makes 3. Likewise, √♀𝔠**6** squared makes √𝔠**6**, cubed truly, √♀**6**. It happens in a similar manner with universal roots, and with the rules of operation, about which more later.

CHAPTER VII.

PLURINOMIALS.

1. Plurinomials exist, which are agreed to be composed from several uninomials with connecting signs.
2. Some plurinomials are called positive [abundant], (described in Ch. 2, sect. 8), others negative [deficient], generally from parts left over or apotomes, which are described in Ch.3, sect. 3.
3. The lowest plurinomials are these, all the uninomials of which are the squares roots of numbers, with or without a number attached.

So that √♀**3 +** √♀**5 –**√♀**2 + 5** is a plurinomial, likewise √♀**3 +** √♀**5–**√♀**2** is called the least plurinomial.

4. All the other plurinomials are called greater.
5. Others are called greater or lesser plurinomials from others, when there should be more or less uninomials.

So that the quadrinomial √♀**3 +** √♀**5 –**√♀**2 + 5** is greater than this plurinomial with three terms √♀**3 +** √♀**5 –** √♀**2**, as again the trinomial √♀**3 +** √♀**5 –** √♀**2** is more than a binomial, such as √♀**6 –** √♀**5**.

6. If you change certain given signs of plurinomials, but not all, from a positive you make its negative or apotome ; or on the contrary, from its negative form it becomes a positive form.

So that √q**5 + √q3 –√q2** shall be a positive trinomial, negative forms of this will be **√q5 – √q3 –√q2**, **√q5 – √q3 + √q2** , **√q3 – √q5 +√q2**, or **√q3 – √q5 – √q2**. Likewise, the same trinomial **√q5 + √q3 –√q2** may be negative, the positive of which will be **√q5 + √q3 +√q2**. From which example it is apparent that the same plurinomial can be either positive or negative, yet with diverse forms with respect to this form.

7. If the plurinomials were two given commensurable uninomials of the same sign, add these (by Ch.2 s.1), and affix the sign for that produced, and the diminished plurinomial becomes less.

Let this be the trinomial **√q12 + √q3 – 2**, **√q12** and **√q3** of which are commensurables, and of the same sign, added they make √q27, by which – 2 makes a diminution to a lesser plurinomial, viz. to √q27 –2, which is binomial.

8. If the given plurinomials were two commensurable uninomials of different signs, take the smaller away from the larger, (by Ch.3 s.1) and to the uninomial produced affix the sign of the greater uninomial, and it becomes a uninomial more diminished in the minus.

So that the trinomial shall be **√q10 + √q2 – √q8**, in which **√q2** and **– √q8** are commensurable, and they are observed with opposite signs ; therefore they make **√q2** taken away, to which the sign of the greater uninomial is affixed, and it becomes **– √q2**, which with **√q10** are made shorter to a binomial, viz. **√q10 – √q2**.

CHAPTER VIII.

THE ADDITION OF PLURINOMIALS.

1. All the additions of plurinomials are to resemble the additions of uninomials with the individual plurinomials connected by their signs ; then if which were commensurable, these (by the preceding 7 and 8) on shortening, and thence the sum is produced by addition.

So that **√q8 + √q8** shall be added to **4 – √q2**, initially through this they make **√q3 + √q8 + 4 – √q2**; then, because **+√q8** and **– √q2** are commensurables, thus by abbreviation it becomes (by Chap. 7 sect. 8) **√q3 + √q2 + 4**. Likewise, **√q5 + √q3** shall be added to **√q20 – √q12**, these initially by this becomes **√q5 + √q3 + √q20 – √q12**; then, by the given abbreviation, there will be **√q45 – √q3**. Likewise, **√c16 + √q18**, to **√c2 – √q2**, make **√c54 + √q8**. Likewise, **√c54 + √q18 – 1**, to **√q2 + √q3**, make **√c54 + √q32 + √q3 –1**.

Corollary.

2. Hence it is clear, in the addition of positives to their negatives, the positive and negative parts cancel each other, and indeed the remaining parts to be doubled.

So that the positive 12+√q3 added to its apotome **12 – √q3**, makes **12 + √q3 + 12 – √q3**, that likewise comes about as 24.

CHAPTER IX.

THE SUBSTRACTION OF PLURINOMIALS.

1. Plurinomials are to be subtracted by changing all the signs, then this conversion of the plurinomial is to be added (by the preceding chapter) to the plurinomial from which it was to be subtracted, and thence the remainder of the subtraction will be produced.

So that it shall be required to subtract **√q5+√q3** from the binomial **√q45 – √q3,** with the signs of this converted, thus, **– √q5 – √q3**, and with this added to **√q45 – √q3** (by the preceding chapter), it becomes **√q20 – √q12.** Likewise, **√c2 –√q2** shall be subtracted from **√c54 + √q3** , and **√c16 + √q18** will be the remainder**.** Likewise, from **√c54 + √q32 + √q3 –1** there shall be subtracted **√q2 + √q3,** and **√c54 + √q18 –1** will remain.

Corollary.

2. Hence it is apparent, in the subtraction of a defective from its positive [counterpart], the positives or defective parts are to be doubled, the rest truly in turn cancel each other.

As from the positive plurinomial **√q13 +7** let its defective **√q13 – 7** be subtracted, in the first place there becomes **√q13 + 7 – √q13 +7**, thence from this it becomes 14.

CHAPTER X.

THE MULTIPLICATION OF PLURINOMIALS.

1. Multiply the individual uninomial multiplicands by the individual multipliers, by Ch. 6 ; moreover the sum may be shortened (if these have commensurable terms), by sect. 7 and 8, Ch. 7.

So that **√q3 – √c2 + 6** shall be the multiplicand, **√q5 – 7** shall be multiplying :

√q15 – √qc 500 +√q180 – √q147 + √c686 – 42

will be produced, thus both incommensurable and unable to be shortened.

Likewise, let **√q8 + √q3 – 5** be the multiplicand, **√ql2 – √q2** the multiplier ; **√q96 + √q36** (otherwise 6) – **√q300 – √q16** (otherwise – 4) **–√q6 + √q50,** which sum produced abbreviated (by sect. 7 and 8, Ch. 7) makes **√q54 + 2 – √q300 + √q50.**

2. In multiplying a positive plurinomial by its negative counterpart, it suffices to multiply the positive part by the negative part, and to multiply each part in common itself ; for the remaining cross multiplications cancel out in turn.

So that the positive **√q7 + √q5** to be multiplied by its negative counterpart **√q7 – √q5**, becomes **7 – 5** (otherwise 2) for the total to be produced ; and indeed the transverse multiplications, **√q7** by **– √q5,** and **√q7** by **+ √q5**, are **– √q35** and **+ √q35**, because they remove each other, and so therefore are without use.

Corollary.

3. If the smallest of the positive [abundant] plurinomials may be multiplied into its negative counterpart [defective], a lesser plurinomial will be produced.

So that here the least positive trinomial **√Q11 –√Q3 + √Q2,** may be multiplied by its defective in some manner, viz. per **√Q1l – √Q3 – √Q2**, thence this binomial will be produced **12 – √Q132**, which also being multiplied by its abundant part **12 + √Q132** indeed makes a uninomial number, viz. 12.

Corollary.

4. If a positive smallest binomial is multiplied into its negative counterpart, an [absolute] number is produced.

So that, in what has been said, if you multiply the positive binomial **12 + √Q132** (by 2 of this chapter) into its negative **12 – √Q132**, a number is produced, viz. 12.

It should be noted, that an irrational binomial can be multiplied by such a plurinomial, so that thence a rational number may arise, in this way: Multiply two named cube [roots] into themselves in turn, and a positive trinomial is made from the positive binomial, or a negative trinomial from the negative one; this trinomial, if it shall be positive, can be multiplied by the negative binomial, or if negative, by the positive one, and a simple number will arise. Otherwise: By Prop. 2 Book viii. Euclid, find three magnitudes in that ratio which have in turn named cubes, or four quantities in the same ratio that have binomial biquadratics ; or five for supersolids [*i.e.* fifth powers] ; and then multiply as above.
[These results are related to the factorizations of the difference of two cubes, fourth powers, etc.]

Example.

From the positive binomial **√C6 + √C4** make the trinomial **√C36 + √C24 + √C16**, which multiplied by the negative **√C6 – √C4** makes 2. Likewise, from the negative **√C6 – √C4,** make the negative trinomial **√C36 – √C24 + √C16,** which multiplied by the positive binomial **√C6 + √C4**, makes 10.

Another example.

From **√QQ3 –√QQ2**, make the quadrinomial **√QQ27 – √QQ18 + √QQ12 – √QQ8**, which multiplied by **√QQ3 + √QQ2**, becomes 1.

CHAPTER XI.

THE DIVISION OF PLURINOMIALS.

1. If the divisor were a uninomial, divide that by a singular uninomial, by Ch. 6, and connect with the signs produced it will become the uninomial of the quotient.

So that **√q12+√q8** may be divided by **√q2**, it becomes **√q6 + 2.**
Likewise, **√q36300 + √q7200 – √q10800 + √q6600 + √q9900** may be divided by 12, the quotient becomes √q $\frac{3025}{12}$ + **√q50 – √q75** + √q $\frac{275}{6}$ + √q $\frac{275}{4}$.

2. If the divisor were a plurinomial, and that the least ; make a simple number from that plurinomial (by sect. 3 and 4 of Ch. 10); and by the same plurinomial by which you would multiply the divisor, also multiply the dividend ; divide by the simple number produced, and thence return the quotient of the first divisor and of the number to be divided.

Let 5 be required to be divided by this smallest trinomial **√ql1 – √q3 – √q2** so that if first you multiply into **√q11 – √q3+√q2**, thence it becomes **12 – √q132**; then you multiply this into **12 + √q132**, making 12 (by 3 and 4 of Ch.10) ; then multiply the dividend by the same trinomial **√q11 – √q3 + √q2**, viz. 5, becoming **√q275 – √q75 + √q50**; this again is multiplied by the afore given binomial **12 + √q132,** and it becomes **√q36300 + √q7200 – √q10800 + √q6600 + √q9900** for the new dividend, which divided by 12, provides the quotient √q $\frac{3025}{12}$ + **√q50 – √q75** + √q $\frac{275}{6}$ + √q $\frac{275}{4}$ agreeing with the 5 put in place divided by **√q11 – √q3 – √q2.**

3. If the divisor were from the above plurinomials, hardly ever will it be divided a whole number of times without a remainder, and therefore a line may be drawn between the dividend written above and the divisor written below the line, in the usual manner of dividing arithmetical fractions.

These are to be corrected : for the division can be made by **6+√c2**, as by every binomial, from the end of the preceding chapter.

So that **10 – √q3** may be divided by **6 + √c2**, becoming non other than $\frac{10-\sqrt{q}3}{6+\sqrt{c}2}$, with the dividing line in between, on this occasion, which may be called thus : **10 –√q3** divided by **6+√c2.**

Corollary.

4. Hence it is apparent, from the division by the above plurinomia, irrational fractional plurinomials arise.

CHAPTER XII.

THE EXTRACTION OF ROOTS FROM PLURINOMIALS.

1. Certain roots of plurinomials are evident, certain others are. We say that those are evident which are no more plurinomials than those of which they are the roots.
2. But we call roots hidden, which arise from many uninomials and with the roots of the plurinomials generally confused.
3. If the square root were to be extracted from binomials of the lowest order presented ; extract the square root from the difference of the squares of each uninomial, that you add

to and subtract from the same greater uninomial, evidently if they are commensurable (for otherwise the root sought will be obscure), and from the halves of these take the square roots (by Ch. 4), you connect these two roots as with the sign of the initial binomial, and this binomial will be the square root sought from the previous binomial.

Example.

Let the square root of this defective binomial **3 – √q5** be extracted; the squares of the uninomials are 9 and 5, the difference of which is 4, the square root of this difference is 2, commensurable to 3; therefore add these 3 and 2, making 5; and also subtract 2 from 3, 1 remaining ; from the two halves of 5 and 1 take the square roots, they become √q $\frac{5}{2}$ and √q $\frac{1}{2}$, which joined by the former sign, becomes √q $\frac{5}{2}$ – √q $\frac{1}{2}$ the square root of this binomial **3 – √q5**. Likewise, let the square root be extracted from **√q48 – 6**; the square root of the difference of the squares is **√q12**, and which added and subtracted to and from **√q48**, makes **√q108** and **√ql2**, of which the halves for the sign make the root sought **√qq27 – √qq3**. Likewise, **√q24+√q18** has this evident square root √qq $\frac{27}{2}$ **+**√qq $\frac{3}{2}$.

[Note that $\sqrt{(a\pm b)}=\sqrt{\frac{1+\sqrt{(a^2-b^2)}}{2}}\pm\sqrt{\frac{1-\sqrt{(a^2-b^2)}}{2}}$.]

4. The roots of all the remaining plurinomials of whatever kind are taken as hidden.

Example.

√q48 + √q28 is without an apparent root, because √q the difference of the squares, which is **√q20**, is not commensurable to **√q48,** since (as set out in 3) it must be commensurable. Likewise, the square or cube root of this **√c3 +1** is hidden; and thus with the others, except the lowest binomial now discussed.

5. But hidden roots cannot be extracted in any other way apart from having the root sign with the period placed in front of the given plurinomial, and that with the root, with the period following, is called the sign of a general [or universal] root. So that, if the square root of this **√q48 + √q28** were to be extracted ; affix to this binomial the root sign √q with this period, and it becomes thus **√q.√q48 + √q28,** which is pronounced thus : ' The whole [or universal] square root of the square root 48 added to the square root 28 ' ; for it is signified that **√q48** to be joined with the square root 28 into one sum ; and the square root to be taken is the sum of the total of this. Likewise, let the cube root of this be required to be extracted : **√c3 + √q2 – 1**, that will be **√c.√c3 + √q2 – 1**.

Corollary.

6. Hence it is apparent that general roots arise from the extraction of hidden roots,.

CHAPTER XIII.

IRRATIONAL FRACTIONS.

1. Those quantities with rational fractions which can be perfected by arithmetic, have to be brought about for by algebra irrational fractions.

Moreover with irrational fractional plurinomials, we are working with fractions in as much as they are fractions; and by algebra, in as much as they are plurinomials and irrationals.

So that, $\frac{\text{√♀3+2}}{\text{√℮3}}$ shall be required to be divided by $\frac{\text{√♀5}}{\text{√℮2−1}}$ because, with rationals by arithmetic, it is effected by the cross multiplication of each numerator by each denominator separately ; this therefore is made by multiplication algebraically and the wished quotient of the division is produced :

$$\frac{\text{√♀℮108 + √℮16 − √♀3 − 2}}{\text{√♀℮1125}}.$$

Likewise, let $\frac{\text{√♀3+2}}{\text{√℮3}}$ be added to $\frac{\text{√♀5}}{\text{√℮2−1}}$ and which by cross multiplication, and directly with the denominators, so that they shall have the same denominator, put in order by the rules of arithmetic; thus therefore multiplied algebraically, and it becomes in the first place with one denominator thus $\frac{\text{√♀℮108 + √℮16 − √♀3 − 2}}{\text{√℮6 − √℮3}}$ for one, and $\frac{\text{√♀℮1125}}{\text{√℮6 − √℮3}}$ for the other ; then by algebra, with the aid of arithmetic, add the numerators retained by that common denominator, and the product of the addition is made : $\frac{\text{√♀℮1125 + √♀℮108 + √℮16 − √♀3 − 2}}{\text{√℮6 − √℮3}}$.

CHAPTER XIV.

THE ADDITION AND SUBTRACTION OF UNIVERSAL ROOTS.

1. Universal roots are added with the sign of augmentation interposed, and subtraction with the sign of diminution.
√♀.10 + √♀2 shall be added to **√♀. 8 – √♀3**, with the + sign interposed, and it becomes **√♀.10 + √♀2 + √♀.8 – √♀3**. Likewise, **√♀.8 – √♀3** is subtracted from **√♀.10 + √♀2**, with the **–** sign interposed, and it becomes **√♀.10 + √♀2 – √♀.8 – √♀3.**

2. If the universal square root of the smallest negative binomial, may be added to its universal positive root, or taken from its universal root, or taken from its universal positive square root, by the signs **+** and **–**, an abbreviated amount will be produced (by Ch.16 follow sect.5).

So that from the addition **√♀.10 – √♀2** to **√♀.10 + √♀2** there will be produced, **√♀.10 + √♀2 + √♀.10 – √♀2** ; which by Ch. 16 sect. 5 can be abbreviated.

CHAPTER XV.

THE REDUCTION OF UNIVERSAL DIVISORS TO THE SAME SIGN.

1. Multiply each universal plurinomial into itself as often as indicated by the unlike universal of the associated plurinomial indicates, and you sign the product of each universal.

So that √ꝗꝗ**.2 – √𝔠3,** and **√ꝗ𝔠.7 + √ꝗ2** are to be reduced ; multiply the plurinomial **2 – √𝔠3** into itself as a cube, and **7 + √ꝗ2** into itself as a square, they make **5 + √𝔠1944 – √𝔠5184** and **51 + √ꝗ392,** from which with the common √ꝗ, together with the unlike ꝗ and 𝔠 , then they become **√ꝗꝗ𝔠. 5 + √𝔠1944 – √𝔠5184** and **√ꝗꝗ𝔠.51 + √ꝗ392** reduced to the same universal root, viz. **√ꝗꝗ𝔠.**

2. By the same account, the universals with the particulars are reduced to the same root. So that 3 and **√ꝗ.18 + √ꝗ243** become **√ꝗ.9** and **√ꝗ.18 + √ꝗ243.**
Likewise, **√ꝗ.13 + √ꝗ20** and **2 +√ꝗ3**, make **√ꝗ.13 + √ꝗ20** and **√ꝗ.7 + √ꝗ48.**

Corollary.

8. Hence it is apparent, that a uninomial be assigned a universal that is the same as the particular root.
So that **√ꝗ.9** and **√ꝗ9** are the same. Likewise, **√𝔠5** and **√𝔠*.5*** ; while these are said to be marked universally by this point.

CHAPTER XVI.

THE MULTIPLICATION AND DIVISION OF UNIVERSALS.

1. If a universal were multiplied or divided by a universal, first the universals are made of the same symbol (by Ch.15).
2. Then, with the universal symbols deleted (remembered in any case), the multiplication and division of the uninomials and plurinomials can be made in the usual manner.
3. Finally, affix to the product or quotient, the former universal symbol [or sign], with the preceding sign that must be given (by Ch.6 sect. 2).

√𝔠.5 + √ꝗ2 et **√𝔠.4 – √ꝗ3** shall be of the same universal sign to be multiplied in turn: Therefore take **5+√ꝗ2** by **4 – √ꝗ3**, and **20 + √ꝗ32 – √ꝗ75 – √ꝗ6** is produced, to which affix **√𝔠.** or **+ √𝔠.** becoming **√𝔠.20+√ꝗ32 – √ꝗ75 – √ꝗ6**. Likewise, **√ꝗ.4 + √𝔠2** shall be by multiplying by 3 or by **√ꝗ9**, or (by Ch.15, sect. 3) by **√ꝗ.9**: Therefore multiply **4 + √𝔠2** by 9, making **36 + √𝔠1458**, by Ch.6 and Ch.10. To these affix **√𝔠.** or **+√𝔠.** making **√𝔠.36 + √𝔠1458**. Likewise, **√𝔠.10 + √ꝗ2** multiplied into **– √ꝗ.10 – √ꝗ2** makes **– √ꝗ.98**, or **– √ꝗ98**.

4. If the universal square root, with the affixed sign **+** , shall be multiplied into the same universal, with the **–** attached ; delete the affixed sign and universal root, and change the signs of the remaining terms , and then the product of the multiplication will emerge.
So that **+√ꝗ.2 – √ꝗ3**, multiplied by **–√ꝗ.2 – √ꝗ3**, makes **+√ꝗ3 – 2.**

5. Truly, it the universal square be multiplied into itself, with the universal sign and plus or minus sign deleted, the product of the multiplication will arise.
So that **√ꝗ.10 + √ꝗ2** may be multiplied into itself, and **10 + √ꝗ2** will be produced.

6. Moreover if several universals were multiplied by several others, or divided, the whole that is produced from one by the other, will have that single universal sign affixed.
So that **√ℭ.10 + √ꝗ5 + √ꝗ.8 – √ꝗ3** may be multiplied by **√ꝗ.3 + √ꝗ6 – √ꝗ.4 – √ꝗ7**, in this way : Reduce **√ℭ.10 + √ꝗ5** with **√ꝗ.3 + √ꝗ6** to the same universal, the first becomes **√ꝗℭ.105 + √ꝗ2000**, and indeed the other, **√ꝗℭ.81 + √ꝗ6534**; multiply these in turn, and they become (by this chapter,) **√ꝗℭ.8505 + √ꝗ13068000 + √ꝗ13122000 + √ꝗ72037350**; which is produced from a single universal by a single multiplication. Therefore the total common product has that sign **√ꝗℭ** commonly affixed to that.
[Thus for the first product, we need to write down the terms as
$\left(10+\sqrt{5}\right)^2 = 105+20\sqrt{5} = 105+\sqrt{2000}$ and $\left(3+\sqrt{6}\right)^3 = 81+\sqrt{6534}$, etc.]
In a similar manner, multiply **3 + √ꝗ6** by **8 – √ꝗ3**, and with its universal affixed, viz. **+√ꝗ.**, and **+√ꝗ.24 – √ꝗ27 + √ꝗ384 – √ꝗ18** is made for the second part of the product. For the third, reduce **– √ꝗ.4 – √ꝗ7** with **√ℭ.10 + √ꝗ5** to the same universal, making **–√ꝗℭ.148 – √ꝗ21175**, and **√ꝗℭ.105 + √ꝗ2000** ; multiply these in turn, and **– √ꝗℭ.15540 – √ꝗ233454375 + √ꝗ43808000 – √ꝗ42350000** is made for the third part of the product. For the fourth, multiply **8 – √ꝗ3** by **4 – √ꝗ7**, and affix **–√ꝗ.** to the product, making **√ꝗ. 32 – √ꝗ48 – √ꝗ448 + √ꝗ21**, for the fourth part of the product. The four parts of which will be from the multiplication of one universal into another in some order ; whereby in some order it is taken under a single universal sign, and the whole product becomes:

√ꝗℭ.8505 + √ꝗ13068000 + √ꝗ13122000 + √ꝗ72037350 + √ꝗ.24 – √ꝗ27 + √ꝗ384 – √ꝗ18 – √ꝗℭ.15540 – √ꝗ233454375 + √ꝗ43808000 – √ꝗ42350000 – √ꝗ.32 –√ꝗ448 + √ꝗ21.

7. From which it comes about that with the universal quadratic root of the smallest binomial increased by its positive square root or diminished by its negative square root, to which you will prefixed **√ꝗ.** to the product from multiplying into themselves, thence it becomes a smaller abbreviated plurinomial of the same order.
So that if you multiply **√ꝗ.l0 + √ꝗ2 + √ꝗ.l0 – √ꝗ2** into itself, and you prefix **√ꝗ.**, it becomes **√ꝗ.20 + √ꝗ392** equal to **√ꝗ.10 + √ꝗ2 + √ꝗ.10 – √ꝗ2** and with that shorter. In the same manner **√ꝗ.20 – √ꝗ392** is made from **√ꝗ.10 + √ꝗ2 – √ꝗ.10 – √ꝗ2**.

[*i.e.* $\left(\sqrt{\left(10+\sqrt{2}\right)} \pm \sqrt{\left(10-\sqrt{2}\right)}\right)^2 = 20 \pm 2\sqrt{98}$, etc.]

8. The examples of division of universals are the same plurinomials which are described in the preceding Ch.11; but only if you prefix the universal sign to the divisors, the dividends, and the quotients.

So that from this and from Ch.11, sect.1, **√ℭ. √ϙ12 + √ϙ8** divided by **√ϙℭ 2**, or what is the same, divide by **√ℭ. √ϙ2**, the quotient is returned **√ℭ.√ϙ6 + 2**. Likewise, from this and Ch.11 sect. 2, **√ϙ.5** divided by **√ϙ.√ϙ11 – √ϙ3 – √ϙ2** returns the quotient **√ϙ.√ϙ $\frac{3025}{12}$ + √ϙ50 – √ϙ75 + √ϙ $\frac{275}{6}$ + √ϙ $\frac{275}{4}$**. And thus for the others.

CHAPTER XVII.

THE EXTRACTION OF UNIVERSAL ROOTS.

1. You can extract the root of a plurinomial presented without respect of a universal sign by Ch.12 ; and to this root you can prefix the sign of its universality.

So the square root required to be extracted from **√ℭ.3 – √ϙ5**, that will be (by Ch. 12, sect. 3) **√ϙ $\frac{5}{2}$ – √ϙ $\frac{1}{2}$**, to which by this you prefix its universal **√ℭ.** and the roots sought becomes **√ℭ.√ϙ $\frac{5}{2}$ – √ϙ $\frac{1}{2}$**. Likewise, the cube root of this **√ϙ.√ℭ3 +√ϙ2** will be (by this and Ch.12, sect. 5) **√ϙℭ .√ℭ3 + √ϙ2**. Likewise, the cube root of the square root of this **√β.7 – √ϙ48**, is required; first the square root of this will be **√β.2 – √ϙ3** ; then the cube root of this will be **√ℭβ.2—√ϙ3**, for the root sought.

2. If you should extract the root from several universals with signs, or with uninomials with signs, that will be said to be a universal of the universals ; and to the whole there must be prefixed the universal sign of the root extracted, and with a line drawn by the total.

So that the square root of this **5 + √ℭ2 – √ϙ.3 – √ϙ2** may be extracted, that root extracted by affixing the universal sign of the root, together with a line drawn in this manner :

√ϙ.<u>5 + √ℭ2 – √ϙ.3 – √ϙ2.</u>

Corollary.

3. Hence it follows that the effect with the universals extend only as far as the line is extended ; and if no line is drawn the effect of the universal sign on the following universal sign ceases from that cut off.

So that

√ϙ.<u>60 + √ϙ16 – √ϙ.6 – √ϙ4</u>

is signifying the square root of the whole **60 +√ϙ16 – √ϙ.6 – √ϙ4**, and that is

√q62 ; but if the line were missing, in this manner, **√q.60 +√q16 – √q.6 – √q4**, then the effect of the former **√q.** and the strength is extended only through **60 + √q16** , and of the latter **√q.** the strength is extended through the rest, viz. through **6 – √q4**. Likewise therefore

√q.$\underline{\mathbf{60 + \sqrt{q}16 - \sqrt{q}.6 - \sqrt{q}4}}$

appears as √q62; and **√q.60 + √q16 – √q.6 – √q4** is the same as 6:
And similarly with others of the same kind.

These things said about irrationals suffice, although there are other kinds of irrationals : As indeed by the extraction of roots from numbers not having uninomial roots arising (which we have discussed in the first part of this work), and from the addition and subtraction of incommensurable uninomials, plurinomials arise (concerning which we have treated in the second part of this work), and by the extraction of hidden roots from plurinomials universal roots arise (concerning which we have treated in this third and final part). Thus also from universal roots universals of universals arise, and from these again others universals arise indefinitely: The art of which if it falls little into use, because it is come upon most infrequently, you can deduce easily from the preceding.

END OF THE FIRST BOOK.

SECOND BOOK.

THE POSITIVE OR COSSIKE* PART OF ALGEBRA.

[* This was the word used by Robert Recorde in *The Whetstone of Witte*, describing the form of algebra available in Elizabethian times. Note that the original was type set from a fair copy handwritten by Robert Napier, son of the author, soon after his demise, edited by Mark Napier, and printed in 1839. Thus the text was not corrected by Napier himself, so that misconceptions and typographical errors may be present, that have been printed in good faith.]

CHAPTER I.

THE DEFINITIONS AND DIVISIONS OF THE PARTS, AND WITH THE NAMES DESIGNATED.

1. The positive part of algebra, through imagined parts, we have said in Book I. cap. 1, sect. 7 , reveals a true magnitude and a true number sought.

2. Also the positions [or different terms], are certain small imagined marks [called 'notules'] noted with unity, which we will add, subtract, multiply or divide in turn and in place of unknown magnitudes and numbers.

3. But the investigation includes just as many diverse and different positions and notules of the positions, as there are different and unknown numbers or quantities.

The figures [*i.e.* symbols] and the names of which are 1℞, which, for example, with one put in the first place is called, 1*a*, *i.e.* one *a*, or with a second in place is called : 1*b*, *i.e.* one *b*, or with a third in place, 1*c*, *i.e.* one *c*, or with one in the fourth place, and thus through the alphabet.

[We have to undo our mathematical thinking to that of an earlier age to try to understand Napier here; for at this time mathematics was more or less synonymous with geometry; here the idea of a variable distance, or length along a line from some point perhaps, is suggested ; the generic name of such small noted quantities or 'notules' is 1℞, where the '1' indicates that 'one of ' a point is considered, and ℞ suggests that it is one unit of some 'thing' : 'Rem' in the masculine singular in Latin, denotes a 'thing' of some kind, and so can represent an unknown; the little stroke through the letter may be indicating that the symbol does not represent a point on a line, as in geometry. [This symbol had previously been used in Italy as the square root sign.]Thus below in the table, 1℞ can be three, so that perhaps generally we can thing of it as a precursor of our unknown, *x*, able to take any value, but unknown. Early mathematicians had a stumbling block about doing calculations with things of which they did not know the value ; thus the difficulty in writing down such a quantity: The un-informed have been known to ask : 'But what is *x* ?' Thus the unknown was left unwritten at the time, and calculations were done around it.

Specific values of quantities called *a*, *b*, etc. are sometimes called 1*a*, 1*b*, etc. See also the chapter on *De Arte Logistica*, by J.E.A. Steggall, from the *Napier Memorial Volume*, marking the tercentenary of the birth of logarithms in 1914, which I intend to include with this file. However, Steggall seems to be a little in the dark about Napier's meanings as well at times. More interpretive information is provided as we proceed.]

4. These notules [or notettes] of [values] put in place are commonly called by the name RES (because there they are put in place for all things with an unknown measure and number), and they are of first order.

5. The square is the product arising from any of these [unknown] things multiplied into itself, and is of the second order.

So that 1℞ multiplied into itself makes the first one squared, which is written thus 1♀. Likewise, 1***b***♀ multiplied into itself makes 1***b***♀, which is called one *b* squared. Likewise, **l*a*** multiplied by **1*a*** makes **l*a***♀, which is called one ***a*** squared; and thus with the others.
[Thus, a number followed by a square, cube, or higher order sign for a single position indicate the presence of an unknown : **3**♀, for example, means $\mathbf{3}x^2$, etc., depending on the context.]

6. The cube arises from some thing which is multiplied into its square ; and it is of the third order.

So that 1℞ multiplied by 1♀ makes one cubed, which is thus written as 1𝕮.
[This we believe includes the unknown *x*, so that it becomes $1.x \times 1.x^2 = 1.x^3$]
Likewise, **l*a*** multiplied by **l*a***♀ makes l***a***𝕮, which is pronounced thus ' one *a* cubed'. Likewise, 1***b*** multiplied by 1***b***♀ makes 1***b***𝕮, etc.

7. The square of the square is what comes about from multiplying any item into it cube, and it is of order four.

So that 1℞ multiplied by 1𝕮 produces the square of one squared, which is written thus 1♀♀. Likewise, 1***a*** by l***a***𝕮 makes l***a***♀♀, which is called ' The square of one *a* squared'. Thus l*b*♀♀, l*c*♀♀, etc. [Thus, again we have $1.x \times 1.x^3 = 1.x^4$, etc.]

8. The supersolid is what arises from the multiplication of something by the square of its square ; and it is with order five.

So that1℞ multiplied into l♀♀ makes 1*ß*, to wit one supersolid. [The 'solidus' was the shilling, a silver unit of currency in these days, and Napier has borrowed the word 'supersolidus', which could mean simply a gold coin, amongst other things; here it simply means something beyond a volume or of 3 dimensions.]
Likewise, l***a*** by **l*a***♀♀ makes 1***a****ß*, which is pronounced as 'one *a* supersolid'. Thus with 1***b****ß*, and 1*cß*, etc.

Corollary.

9. Hence others are apparent to arise from these orders in an infinite progression.

So that 1**R** multiplied by 1ß makes 1ʓℭ, which is the sixth in order [*i.e.* $1.x \times 1.x^5 = 1.x^{2.3}$, etc.]. Likewise, 1**R** by 1ʓℭ makes 1ſß, which is called the second supersolid, and it is the seventh in order. The rest can be considered from the following table, in which we substitute for example 1**R** with the value 3, 1***a*** with the value 2, and 1***b*** with the value 4, from which the values of the remaining orders follow by necessity, as below :

[In the following table, the characters of the first position, in the second column, are presumed to be evaluated with an unknown quantity *x* in place at this point, which is set as 3 ; thus the quantities on the left we would now consider as x, x^2, x^3,.... which become 3, 9, 27, etc. on the right side of this column. We note the reluctance to specify the unknown amount; thus, Napier's algebra has not quite lost its connection to arithmetic ; in the next two columns there is no trouble, as *a* and *b* are specified numbers, to be placed in the second and third positions. We would like to consider these as variables too, but fixed for the problem in hand at certain point on a line, while *x* is variable, and is the solution of some condition, such as the root of an equation. Note that expressions tend to be set out in the order of the columns, the unknown, followed perhaps by an *a* term, and by a *b* term, etc., finally perhaps, if more are present.]

number of the order 0	Characters and examples of orders of the first position.		Characters and examples of orders of the second position.		Characters and examples of orders of the third position.		&c.
1	1**R**	3	1***a***	2	1***b***	4	
2	1ʓ	9	1***a***ʓ	4	1***b***ʓ	16	
3	1ℭ	27	1***a***ℭ	8	1***b***ℭ	64	
4	1ʓʓ	81	1***a***ʓʓ	16	1***b***ʓʓ	256	&c.
5	1ß	243	1*a*ß	32	1***b***ß	1024	
6	1ʓℭ	729	1***a***ʓℭ	64	1***b***ʓℭ	4096	
7	1ſß	2187	1***a***ſß	128	1***b***ſß	16384	
8	1ʓʓʓ	6561	1***a***ʓʓʓ	256	1***b***ʓʓʓ	65536	&c.
9	1ℭℭ	19683	1***a***ℭℭ	512	1***b***ℭℭ	262144	
10	1ʓß	59049	1***a***ʓß	1024	1***b***ʓß	1048576	
11	1ſſß	177147	1***a***ſſß	2048	1***b***ſſß	4194304	
12	1ʓʓℭ	531441	1***a***ʓʓℭ	4096	1 ***b***ʓʓℭ	16777216	
13	1ſſſß	1594323	1***a***ſſſß	8192	1***b***ſſſß	67108864	
&c.	&c.		&c.		&c.		&c.

10. Any rational or irrational numbers with the signs of positive orders affixed, are said to be positive.

So that either **6℞**, **5*a***, ***7b*ℭ**, **√♀6*b***, or **√ℭ7*a*♀**, are said to be positive numbers. Meanwhile also the positive name is taken for any number [*i.e*. not indicated otherwise].

11. Any simple single positive number on its own is said to be positive, or assumed solitary.

So that **6*a*** is simple. Likewise, **√♀3ℭ**, and likewise, **√♀1*ab***.

12. A number are said to be composite which is put together from several simple numbers which are connected by plus or minus signs.

Such as **6*a* + √♀3ℭ**. Likewise, 5**℞ – 2√♀**, and again, **√♀30ℭ + 3*a* – 4℞*b*.**

13. A simple number is said to be pure which has only one sign of the [unknown] put in place after a single uninomial.

As **5a♀**. Likewise, **3ℭ**, and again **√♀2*c*ℭ**, etc., are said to be pure.
[Evidently, in modern terms in this context, we have : $5a^2, 3x^3$, and $\sqrt{2c^3}$]

14. A mixed [expression] is said to be simple, which has the signs of the positions written down after a single uninomial.

So that **5♀*ac*, 2℞*ac*, √♀1*ab*, √ℭ1*a*♀*b*ß*c***, and similar expressions indefinitely, are said to be mixed ; the origin of which will be treated below in Ch. 5, sections 2 and 3.

15. Simple rationals are rational numbers which have affixed signs of position and order. But irrationals are numbers which have prefixed irrational numbers.

So that **6*a***, and likewise **5♀*a*♀** and **2℞** are rationals; while **√♀6*a***, and likewise **√ℭ5♀*b*ß** , and **√♀ℭ7*b*ß** are irrationals.

16. Simple roots are said to be similar, the signs of which roots are either lacking or similar; but on the other hand the roots are dissimilar, if the signs of which roots are dissimilar.

So that **2♀** and 3***a***, likewise **√♀3℞** and **√♀5ℭ**, likewise **√ℭ6** and **√ℭ2*ab***, are similar roots ; and likewise both **√♀3ℭ** and **√ℭ3ℭ**, **√ß1*a*** and **5℞*b***, etc. are dissimilar roots.

17. Two simple [expressions] are of the same position, with characters or every kind of the same position, although not of the same order.

So that 2**℞** and **√♀5ß**, likewise **3℞*a*♀** and **√ℭ2♀*a*ℭ**, etc. are of the same position ; but **2℞** and **√♀**1*a*, likewise 3**℞***a*♀ and 2**℞**, are of different positions put in place. [Thus, an expression can have several terms formed from constants – fixed positions, or from variables in it – variable positions, and the operations of squaring, taking the root, etc., need not be the same. Thus, in the first two simple expressions, 2**℞** and **√♀5ß**, we would now write as $2x$ and $\sqrt{5x^5}$; these have an *x* in the first position only, on which different operations are performed. Again, **3℞*a*♀** and **√ℭ2♀*a*ℭ**, have an *x* and an *a* in the first and second positions, and on which various operations are performed : these we would now write as $3xa^2$ and $\sqrt[3]{2x^2a^3}$; the other examples clearly do not involve the same variables in place.]

18. Similarly simple roots are said to be of the same order acting, of which the sign and also the order shall be the same, although they shall not be of the same positions.

So that **3*a*** and ***2b***, likewise **√♀2ℭ** and **√♀5ℭ**, again **2℞*a*** and **5℞*b***, likewise **√♀2℞*a*** and **√♀3*bc***, are roots of the same order acting; but **2℞*a*♀** and **3*b***, again **√♀2℞*a*♀** and **√♀3*b***, likewise √♀2**℞*a*** and **√♀3℞*a*♀**, just as also others of the same kind are of different orders. Which truly are powers of the same order, and how powers may be reduced while acting, will be discussed below in Ch. 4 sect. 5.

19. Two simple terms of the same position and truly of the same order are commensurables, the uninomials of which (with the signs put in place and order disregarded) should be commensurables.

So that **3ℭ** and **2ℭ** [i.e. $3x^3$ and $2x^3$ at the position *x*] are commensurables, because 3 and 2 are commensurables, by Ch. 1 sect. 15 Book I; likewise **√♀12℞** and **√♀3℞** are commensurables, because **√♀12** and **√♀3** are commensurables, by Ch. 1 sect. 14 Book. I; thus **√♀2℞*a*♀** and **√♀3℞*a*♀**, all similar terms.

CHAPTER II.

THE ADDITION AND SUBSTRACTION OF POSITIVES.

1. If simple positive and commensurables[expressions] were to be added or diminished by subtraction, then add each uninomial, or take away the lesser from the greater, and affix the initial positive sign to what has been produced or remains.

So that **3℞** is to be added to **2℞**, making **5℞**; likewise, **4♀** to **3♀** becomes **7♀** [all of these we interpret as coefficients and powers of the hidden unknown *x* : thus,

$$3x + 2x = 5x \ ; \ \ 4x^2 + 3x^2 = 7x^2; \ \ 6a^3 + 9a^3 = 15a^3 \ ; \text{ etc. }];$$

likewise, **6*a*ℭ** added to **9*a*ℭ** becomes **15*a*ℭ**; likewise, **√♀2ℭ** added to **√♀8ℭ** makes **√♀18ℭ**, because **√♀2** added to **√♀8** makes **√♀18**, by Ch. 2 sect.1, Book I. Thus **√♀2℞*a*♀** added to **√♀8℞*a*♀** makes **√♀18℞*a*♀** ;

$$[i.e. \ \sqrt{2xa^2} + \sqrt{8xa^2} = \sqrt{18xa^2} \ ]$$

likewise, √♀2ℭ / 3 added to √♀8ℭ / 5 makes √♀242ℭ / 15, because the uninomials, by Ch. 2, sect.1, Book I, and by Ch.13, Book I, reduced to the same denominator become , √♀50 / 15 and √♀72 / 15, and added make √♀242 / 15. Likewise, 3**℞** may be subtracted from 5℞, leaving 2**℞**; again, **3*b*♀♀** from **8*b*♀♀** leaves **5*b*♀♀**; and **√ℭ3β** taken from **√ℭ192β** leaves **√ℭ81β**, by this section and Ch. 3, sect.1, Book I. Thus **√ℭ3♀*a*♀** taken from **√ℭ192♀*a*♀** leaves **√ℭ81♀*a*♀** .

[Thus, we interpret the last expression as $\sqrt[3]{192x^2a^2} - \sqrt[3]{3x^2a^2} = \sqrt[3]{81x^2a^2}$.]

2. If the simple terms were incommensurable, interpose the sign + into the addition, and the – sign into the subtraction.

So that if **3℞** is required to be added to **2♀**, it becomes **2♀ + 3℞**; likewise, **4♀** added to **2*a*♀** becomes **4♀ + 2*a*♀**; likewise, **√♀5ℭ** added to **√♀10ℭ** becomes **√♀10ℭ** + **√♀5ℭ**; again 5***a*♀*b*** added to **7*a*♀*b*** is **7*a*♀*b* + 5*a*♀*b***. Likewise, **3♀** is required to be taken from **2*a*♀**, **2*a*♀–3♀** remains; again, **2*a*** taken from **3*a*♀**, leaves **3*a*♀ – 2*a***; and, **√♀3♀** from **√ℭ12♀**, leaves **√ℭ12♀ – √♀3♀** ; thus **√♀2*a*** from **√♀2*ab*** becomes **√♀2*ab* – √♀2*a*.**

Corollary.

3. Hence it is apparent from the addition and subtraction of simple quantities that incommensurable composite quantities arise.

Just as it is agreed from the above examples that the outcomes of these are composite.

4. Moreover the composite quantities of these added or subtracted also can be abbreviated by the same rules as the plurinomials both in Ch. 8 and 9, and in Ch. 7, sect. 7 and 8, Book I. And what is said in that place concerning plurinomials and uninomials, here is understood about composite and simple [terms].

So that **√♀2ℭ*b*♀ +3♀ – 2℞ + 1** is required to be added to **5ℭ+√♀8ℭ*b*♀ – 4♀ + 3*a* – 6** : from that in the first place (by Ch. 8, sect.1, Book I.) by linking together the sum becomes **√♀2ℭ*b*♀ + 3♀ – 2℞ + 1 + 5ℭ +√♀8ℭ*b*♀ – 4♀ + 3*a* – 6**; then that cancelled down (by Ch. 7, sect. 7 and 8, Book I.) becomes **√♀18ℭ*b*♀ – 1♀ – 2℞ – 5 + 5ℭ + 3*a***.

[Thus, we interpret this expression as :
$\sqrt[2]{2x^3b^2} + 3x^2 - 2x + 1 + 5x^3 + \sqrt[2]{8x^3b^2} - 4x^2 + 3a - 6 = \sqrt[3]{18x^3b^2} + 5x^3 - x^2 - 2x + 3a - 5$.]

An example of subtraction.

From the latest produced **√♀18ℭ*b*♀ – 1♀ – 2℞ – 5 + 5ℭ + 3*a***
this quantity **√♀2ℭ*b*♀ + 3♀ – 2℞ + 1** can be subtracted : initially (by Ch.9 sect. 1) with the signs converted and with the simple terms linked together, and it becomes **√♀18ℭ*b*♀ – 1♀ – 2℞ – 5 + 5ℭ + 3*a* – √♀2ℭ*b*♀ – 3♀ +2℞ + 1**; finally, with this shortened down, and it becomes **√♀8ℭ*b*♀ – 4♀ – 6+ 5ℭ+ 3*a***, as above.

CHAPTER III.

THE EXTRACTION OF ROOTS FROM SIMPLE NUMBERS.

1. Every such pure sign has all its roots in place, whatever and how many the characters of these shall be, and there is nothing besides these.

So that **0♀𝕮** [in expressions like this, we will usually consider $0 \equiv x^0 = 1$; another interpretation being that the initial zero stands for any number; both assumptions will usually be valid; occasionally, zero actually means 0, but the context will make this clear;] has within itself the square root, likewise the cube root, and finally the cube of the square, and nothing besides that.

2. Every such [expression] of mixed sign has just as many roots in place, of whatever kind and however many there shall be, to be found in the characters commonly repeated in the different individual positions, and nothing besides that.

So that **0♀𝕮*ßa*♀♀𝕮** has in place square roots, because the [square] sign ♀ is found both in the first as well as in the second position, and likewise the cube by the same account, and finally likewise the cube of the square ; and besides zero, just as neither the supersolid, because *ß* is not found between the sign of the second position of this example, nor the square of the square, because ♀♀ is not found among the signs of the first position of the same example.

[We interpret this expression as $(0.x)^{2.3.5} \times a^{2.2.3} \equiv (0)^{2.3.5} \times a^{2.2.3}$, as the variable *x* is not shown; the point of this note being that certain roots, *viz.* , the square and cube can be taken of both positions, while others can not.]

3. To extract the root in place from a pure sign, is the number of the order of the pure sign divided by the number of the order of the kind of the root, and to note the sign of the quotient of the order.

So that the cube root may be extracted from **0♀𝕮**, the number of the order is ♀𝕮 is 6, which divided by the number of the cubic order, viz., by 3, the quotient becomes 2, the sign of the order of which is ♀; therefore **0♀** becomes the cube root of this **0♀𝕮**; thus the square root of the same **0♀𝕮** is **0𝕮**; likewise the cube root of the square root is **0R**.

4. Some root in place is extracted from a mixed sign when (by the preceding) such a root can be extracted from its individual different positions.

So that the cube root shall be extracted from **0♀𝕮*ßa*♀♀𝕮**: initially, (by the preceding) the cube root may be extracted from **0♀𝕮*ß***, and that will be **0♀*ß***; then (by the same) the cube root will be extracted from ***a*♀♀𝕮**, and that will be ***a*♀♀**; from which also the total cube root of this **0♀𝕮*ßa*♀♀𝕮** will be **0♀*ßa*♀♀**. Thus the square root of the same example will be this : **0𝕮*ßa*♀𝕮**; likewise the cube root of the square root will be **0*ßa*♀**.

5. If some root of a simple whole number is to be extracted, not only does such a root of this simple number enfold an absolute number, but also the positive sign of this has to be itself put in place (by sect.1 and 2, of this chapter) ; then extract that root from the number, and ascribe to that root the first of the signs of the root to be retained (if it were that).

So that the cube root shall be extracted from **64♀ℭ**: in the first place, the cube root of the absolute number shall be 4, then the cube root of the sign ♀ℭ will be ♀, by sect. 1 of this; therefore the whole cube root of these **64♀ℭ** will be **4♀** : likewise the square root of the same **64♀ℭ** will be 8ℭ: likewise the cube root of the square root will be 2**R**: in a similar manner also the square root of this √ℭ**9♀*a*♀*β*** is √ℭ**3R***aβ*.

Corollary.

6. Hence it comes about that such simple numbers having in place both a number and such a positive signed root, such that either its whole or part is indicated by the root, that can be abbreviated by deleting the root, and by extracting the root from the remainder (by the preceding).

So that, let that simple number be √♀ℭ**4♀*β***, which can be abbreviated thus, with the particular root ♀ deleted, √ℭ4♀*β* remains, extract such a root (by the preceding), viz., the quadratic, that will be √ℭ**2*β*** for the abbreviation produced, likewise with the same as √♀ℭ**4♀*β***. Likewise, √♀ℭ**64♀ℭ** thus is shortened ; with the whole root √♀ℭ deleted, **64♀ℭ** remains, extract such a root of this (by the preceding), viz., the cubic of the square, and that will be 2**R**, which provided the same value as √♀ℭ**64♀ℭ**

7. If the root of such a simple number (of which some root shall be extracted) and with neither the absolute number nor the positive sign in place, then affix the root sign to the simple whole root denoting that.

So that the square root of this **4ℭ** shall be extracted, that shall be √♀**4ℭ** ; likewise, the cube root of 4ℭ shall be √ℭ**4ℭ**; likewise, the cube root of this √♀**3R** will be √♀ℭ**3R**; likewise, the square root of this **4♀*a*** will be √♀**4♀*a***.

CHAPTER IV.

THE MULTIPLICATION OF SIMPLE NUMBERS BY THEMSELVES, AND REDUCTION.

1. To multiply a pure signed number into itself by squaring or cubing, or to some other order, is to multiply in turn the numbers of each order, and to note down the sign of the order of the product.

So that **0♀** (of which a number of the order 2) may be multiplied into itself cubically (whose cube is a number of order 3 :) therefore multiply 2 into 3, 6 is produced, whose sign is of order ♀ℭ, therefore **0♀ℭ** is the cube of this **0♀**; likewise, **0♀** multiplied by the supersolid into itself makes **0♀ß**; likewise, **0℞** multiplied into itself by the cube of the square makes **0♀ℭ**.

[Thus, $\left((0.x)^2\right)^3 = (0.x)^6 \equiv (0)^6$, as x is not retained, etc.]

2. To multiply a number of mixed signs into itself to some order, is to multiply the signs of the individual positions into themselves to that order (as given previously).

So that **0ß*a*♀** may be multiplied into itself in the cube of the square: initially (by the preceding 1) multiply **1ß** into itself in the cube of the square, making **0♀ℭß**, then multiply ***a*♀** into itself in the cube of the square, becoming ***a*♀♀ℭ**, and as a consequence the total **0♀ℭß*a*♀♀ℭ** will be the cube of the square of this **0ß*a*♀**.

3. Therefore if you wish to multiply a whole simple number into itself quadratic ally, or cubically, or to some order, initially multiply both the absolute number of this arithmetically, as well as the sign of this (by 1 and 2 of this chapter) into themselves to that order, then such a root of the product may be extracted (by Ch. 3, sect. 5 and 7), such as its root sign denotes (whatever it shall be).

So that **3ℭ** may be multiplied quadratic ally into itself: therefore multiply 3 into itself, by arithmetic, and **ℭ** into itself quadratic ally (by 1 of this chapter), becoming **9♀ℭ** for the true square of the cube of three.

[Thus, for a whole simple number $\left(3^3\right)^2 = 3^6$, and so on for the others following].

Likewise, **2♀♀** multiplied into itself cubically makes **8♀♀ℭ** . Likewise, **√ℭ3℞** multiplied into itself quadratic ally makes **√ℭ9♀** . Likewise, let **√♀2℞*a*♀*c*** be multiplied into itself quadratic ally ; initially 2 and **℞*a*♀*c*** may be squared, and it becomes (by 2 of this chapter) **4♀*a*♀♀*c*♀**, the root of which may be extracted as indicated, viz., here with the square root extracted, that will be (by Ch.3, sect. 5) 2℞*a*♀*c* for the true square of this √♀2℞*a*♀*c*; √♀2℞*a*♀*c* which indeed is squared more easily (by Ch. 6 sect. 6, Book I.) with the deletion of the root. [*i.e.* $\left(\sqrt[2]{2xa^2c}\right)^2 = \left(\sqrt[2]{2^2x^2a^4c^2}\right) = 2xa^2c$.]

4. If simple dissimilar roots are to be reduced to similar roots, multiply each rational part into itself as many times as all the remaining dissimilar roots indicate (by the preceding), and to each produced by itself put in place, prefix all the dissimilar roots with single and common roots.

So that **√ℭ3℞** and **√♀2℞** are reduced thus : multiply **3℞** into itself square wise, and **2℞** into itself cube wise, and they become **9♀** and **8ℭ**; and affix both with the dissimilar root, and they become **√♀ℭ9♀** and **√♀ℭ8ℭ**, evidently of the same root, but of the same values of this **√ℭ3℞** and **√♀2℞** : [*i.e.* $\sqrt[3]{3x}$ and $\sqrt[2]{2x}$ become $\sqrt[6]{(3x)^2}$ and $\sqrt[6]{(2x)^3}$.]

Likewise, **√♀6♀** and **2℞** are reduced thus ; multiply **2℞** into itself square wise, because the first has the root **√♀**, but **6♀** is not multiplied, because 2℞ is without roots ; therefore they become **6♀** and **4♀**, for which affix that one with the single root, and the terms become **√♀6♀** and **√♀4♀**, with the roots of this the same and besides actually of the same order : Likewise, **√♀♀2*b*♀** and **√♀ℭ3*ac*** thus are reduced ; multiply **2♀*b*** into itself cubically, and **3***ac* into itself square wise, (because the roots of these differ in the cube and in the squares in the one, and in the other there is agreement of the squares), the terms become **8♀ℭ*b*ℭ** and **9*a*♀*c*♀**, with which prefix that one with the common root √♀, together with the dissimilar roots ♀ and ℭ, making **√♀♀ℭ8♀ℭ*b*ℭ** and **√♀♀ℭ9*a*♀*c*♀**, with the same roots and values as previously.

Example of the reduction of several roots into two roots.

These three **√♀2♀**, **√ℭ3R**, and **√♀♀1*a*** shall be reduced to the same root: initially multiply **2♀** into itself by the cube of the square, but not by the square of the square of the cube (because in one the first of the squares agrees with the final, but not in the remaining square), therefore the first becomes, with its said roots, **√♀♀ℭ64♀♀ℭ**; but the second with the rational multiplied in the square of the square, makes the product of these, with the roots owed, **√♀♀ℭ81♀♀**; and finally only multiplied by the cube, and with its roots summoned becomes **√♀♀ℭ1*a*ℭ**; which indeed are three terms now with the same root; and thus with the rest.

Corollary.

5. Hence it follows that certain simple dissimilar powers of a said root of the same order, by reduction become powers of the same order acting ; but indeed not certain others, for it is apparent that one of these becomes of greater order, and the other to become of lesser order.

As in the above examples **√♀6♀**, and **2R** are powers of the same order, because reduced they constitute **√♀6♀** and **√♀4♀**, which (by Ch.1 sect.18, Book II.) are actually of the same order ; but **√ℭ3R** and **√♀2R** reduced constitute **√♀ℭ9♀** and **√♀ℭ8ℭ**, of which the one, viz., **√♀ℭ8ℭ** is of higher or of greater order, and truly the other **√♀ℭ9♀** is of lesser order.

CHAPTER V.

THE GENERAL MULIPLICATION OF POSITIVES.

1. To multiply the signs of the same position in turn is to add the numbers of the same signs, and to note down the numbers of the order of the product.

So that **0*a*♀** shall be multiplied by **0*a*ℭ**, the orders of the numbers ***a*♀** and **aℭ** are 2 and 3, which added make 5, of which the order is ***a*ß** ; therefore **0*a*ß** is the product of the multiplication **0*a*♀** into **0*a*ℭ**. Thus **0*b*** multiplied by **0*b*ℭ**, makes **0*b*♀♀**. Likewise **0♀** by **0ß** makes **0ßß**. [In these cases, a simple zero is retained.]

2. To multiply the pure signs of different positions in turn, that is to write the connected signs together, always affix with the sign of the first position (if that there shall be) before the rest.

So that **0*a*♀** may be multiplied by **0*b*𝕮**, **0*a*♀*b*𝕮** is produced, the pronouncement of which, when it becomes a number such as **6*a*♀*b*𝕮**, is this : **6*a*♀** multiplied by **1*b*𝕮**. Likewise, **0*a*♀** by **0𝕮** does not produce **0*a*♀𝕮** but **0𝕮*a*♀**, with the affixed sign of the first position ; that indeed is thus called **0𝕮*a*♀**, or the whole of nothing cubed [of *x*] of the first position multiplied into one [*a*] squared of the second position. But on the other hand, **0*a*♀𝕮** is from the whole of the second position [or term] only (as is apparent from the table of this Ch.1), and thus it is called the cube of the square of one *a*.

[*i.e.* we have $0a^2 \times 0b^3 = 0a^2b^3$ for the first product ;

and we have $0.a^2 \times (0.x)^3 = 0^3 a^2$ for the final product, as the *x* has been suppressed.]

3. Hence it follows, that mixed products are produced from the pure signs of different positions multiplied in turn.

So that in the above example, the pure number **0*a*♀** multiplied by the pure number **0*b*𝕮** produces **0*a*♀ *b*𝕮**, by the preceding ; which is mixed (by Ch.1, sect. 14).

4. For mixed quantities, as long as the positions are shared in turn, are multiplied according to the first section of this ; but as long as they are of different positions, they are multiplied by the second section of this chapter.

So that **0R*a*♀** multiplied by **0𝕮*a*** makes **0♀♀*a*𝕮** ; evidently with the similar positions multiplied in turn (by section 1 of this), viz. **0R** by **0𝕮**, and ***a*♀** by ***a***, becoming **0♀♀** and **0𝕮**; and again **0♀♀** multiplied by ***a*𝕮** becomes (per section 2 of this) **0♀♀*a*𝕮**. Likewise, **0R*a*♀** multiplied by **0𝕮*b*** makes **0♀♀*a*♀*b*** ; because **0R** by **0𝕮** makes **0♀♀**, and **0♀♀** by ***a*♀** makes (by section 2 of this) **0♀♀*a*♀**, and **0♀♀*a*♀** multiplied by **0*b*** makes (by the same section 2) **0♀♀*a*♀*b***.

[*i.e.* we have for the first product:

$(0\times x)\times(1\times a^2)\times(0\times x^3)\times(1\times a) = (0\times x^4)\times(1\times a^3)$ or $(0\times x^4)\times a^3 \equiv (0)^4 \times a^3$; while for the final product, we have :

$(0\times x)\times(1\times a^2)\times(0\times x^3)\times(1\times b) = (0\times x^4)\times a^2 \times b \equiv (0)^4 \times a^2 \times b$.]

5. If simple numbers shall be multiplied in turn : in the first place, they have the same roots, perhaps by reduction (Ch. 4, sect. 4); then, multiply the absolute numbers in turn ; in the third place, multiply in turn the positive sign by that now given ; in the fourth place, extract such a root of the number and sign produced such as indicated by it root, (by Ch. 3) and to this finally the sign must be affixed, (by Ch. 6, Book. I.)

So that **2*a*♀** multiplied by **5*a*𝕮** makes **10*a*ß**, because 2 multiplied by 5 makes 10, and ***a*♀** by ***a*𝕮** makes *a*ß. Thus **2*a*♀** multiplied by **5𝕮** makes **10𝕮*a*♀**. Likewise, √♀ **2R*a*♀** multiplied by √♀ **8R** makes the product **4R*a*** ; because with the numbers multiplied in turn, and with the signs in turn, they make **16♀*a*♀**, the root of which as indicated by the root, viz. the square root, is **4R*a***. Likewise, **2R*a*** may be multiplied by **– √♀3*ab*** ;

first by reduction they become **√℥ 4℥*a*℥,** and **– √℥3*ab***; then with the numbers multiplied in turn, and with a positive sign in turn, they become **12℥*a*ℭ*b***, the square root of which with the desired sign is **– √℥12℥*a*ℭ*b***, for the product of the multiplication.

6. If composites shall be multiplied in turn; the individual simple numbers are to be multiplied into the individual numbers ; moreover the abbreviated product by the addition and subtraction of commensurables, in the manner by which plurinomials are multiplied and abbreviated, by Ch.7, sect.7 and 8; and Ch.10, sect.1 Book I.

So that **2*a*℥ +√℥3℞ – 2℞*a* – 4** is to be multiplied by **√℥12ℭ + 2℥**, multiply the simple singles into the singles, and they make **√℥48ℭ*a*℥℥ +6℥ – √℥48ß *a*℥ – √℥192ℭ + 4℥*a*℥ + √℥12ß – 4ℭ*a* – 8℥**, which thence by abbreviation become :

√℥48ℭ*a*℥ – 2℥ – √℥48ß *a*℥ – √℥192ℭ + 4℥*a*℥ + √℥12ß – 4ℭ*a* .

CHAPTER VI.

THE SITUATION AND PLACING TOGETHER OF SIMPLE COMPOSITES.

1. The interval of orders is the difference between the simple orders of the same root, by which the number of the greater order exceeds the number of the lesser closest order following.

So that in this composition **√ℭ3℞ – √ℭ2℥**, the interval is 1; because the number of the order **℞** is 1, and the number of the order **℥** is 2, the difference of which is 1. Likewise in this with the like root **√℥3ℭ + √℥2℞**, the interval between **√℥3ℭ** and **√℥2℞** is 2, because the difference between the numbers of these orders is 2. Thus the interval between **√℥2℞** and **√℥5** is 1, because indeed the number of the order **℞** is 1, and the number of the order of the simple number is 0; therefore the difference of these, or the interval, will be 1.

[Thus, the unknown x is considered to present for **℞**, of first order, while the last term is just a number, of zero order or power.]

2. The simple number 'made up' part [*i.e*. that part containing the unknown x : this can be interpreted as we have defined above] of some order is nothing or 0, equipped with the signs and roots also of that positive order.

So that the simple 'made up' part of the cubic order is **0ℭ**. Likewise the simple unknown part of the square is **0℥**. Likewise the simple made up part for the square root of any cube is **√℥0ℭ**. Likewise the simple made up part to be considered for the cube root of the supersolid is **√ℭ0ß**.

3. The same intervals are returned, when (by 1, 2, and 3 Book. VII. of Euclid,) the maximum common measure dividing that is taken, and with the interval of this measure the simple true or fictional numbers, are progressing from the maximum order to the minimum, (when truly they are lacking).

As in this above example, **√♀3ℭ + √♀2R – √♀5,** [*i.e.* $\sqrt{3x^3}+\sqrt{2x}-\sqrt{5}$] there are (by section 1 of this,) two different intervals [of the square root], viz. 2 and 1, of which 2 and 1, the maximum common measure, from Euclid Book, VII. Prop. 1, is one : therefore as with this unit interval from the maximum order, *viz*. from √♀3**ℭ** to progress to the minimum, *viz*. √♀**5**: in this manner, subtract one from the number of this order √♀3**ℭ**, √♀**0**♀ becomes the simple fictitious number [*i.e.* our x^2], because the true value is missing. Likewise, take one from the number of this order √♀0♀ , √♀**2R** is made with the order of this; finally take 1 from the number of this order; the minimum order of this is made, *viz*. of this √♀**5**. Therefore thus you will gather from the example presented : **√♀3ℭ + √♀0♀ + √♀2R – √♀5**, and all the intervals will be the same. Likewise in this [example] **1ℭ – 3R – 6** the maximum common measure is 1, through which this progression becomes **1ℭ + 0♀ – 3R – 6**, and the intervals will be the same. Likewise in this : **√ℭ1♀ℭ – √ℭ3R + √ℭ8** there will be different intervals, *viz*. 5 and 1, of which one will be finally the common measure. Therefore thus you will gather (made by the successive subtraction of one,) the orders :

√ℭ1♀ℭ + √ℭ0ß + √ℭ0♀♀ + √ℭ0ℭ + √ℭ0♀ – √ℭ3R + √ℭ8.

Likewise with this example : **√ℭ2ʃʃß + √ℭ3ß + √ℭ10R** the intervals will be 6 and 4, and the common measure of 6 and 4 is two; therefore with the progression of two's this [composition will be in place] : **√ℭ2ʃʃß + √ℭ0ℭℭ + √ℭ0ʃß + √ℭ3ß + √ℭ0ℭ – √ℭ10R**, the intervals of which are the same, *viz*. they agree with two.

4. So that therefore simple [terms] of compositions may be correctly gathered together ; initially, all the roots are made similarly, by Ch. 4, sect. 4; then, the ones of the same order are put together, and the ones with greater order are put before those with smaller order; thirdly, all the intervals are made the same (by the preceding) : finally, you will be able to abbreviate the simple abbreviable numbers (if allowed,) by Ch.8, sect.6.

So that **2– √ℭ3R +1♀ +√ℭ2♀ℭ** may be made from the same root, the [terms] become **√ℭ8 – √ℭ3R + √ℭ1♀ℭ + √ℭ2♀ℭ** ; then, with the greater orders put first, and likewise with the positions of those of the same order together, [the expression] becomes **√ℭ2♀ℭ + √ℭ1♀ℭ – √ℭ3R + √ℭ8**; thirdly, with the same intervals it becomes :
√ℭ2♀ℭ + √ℭ1♀ℭ + √ℭ0ß + √ℭ0♀♀ + √ℭ0ℭ + √ℭ0♀ – √ℭ3R + √ℭ8;
finally, you will abbreviate this (if it is allowed), and it becomes **√ℭ2♀ℭ + 1♀ + √ℭ0ß + √ℭ0♀♀ + 0R + √ℭ0♀ – √ℭ3R + 2**. Likewise **1♀ + √♀2R – 3** with the same ratios thus correctly located together, **1♀ + √♀0ℭ + 0R + √♀2R – 3**: and thus with the rest.

5. Hence it arises that the simple terms of a composition collated correctly by the preceding, maintain the proportional order ; to wit, the intermediate individual squares of this will be of the same order (powers at any rate,) the product of which will be from the preceding multiplied by the nearest subsequently.

As in this final example, from the preceding i ii iii iiii v

1♀ + √♀0𝕮 + 0R + √♀2R – 3, the square of the second [term] is **0𝕮**, and multiply **1♀** into **0R**, it becomes also **0𝕮**. Likewise the square of the third will be **0♀**, and multiply **√♀0𝕮** into **√♀2R**, it also becomes **0♀**, or (what is the same) **√♀0♀♀**. Likewise the square of the fourth term will be **2R**, and multiply **0R** into **3**, makes **0R**; and both **2R** and **0R** are of the same order : thence the proportionals are always said to be in similar orders.

6. Hence it follows also (when the lesser orders are located later) that a simple and absolute number (because it is of no positive order) is to be located finally, of all the orders to be gathered in place.

7. In mixed compositions or of several positions, for as many diverse positions as there were, so many characters of the sum of the positions are put in place for the unknown thing, or the order of the unknown things in order, and just as many characters of things also into themselves, and multiplied together and to be summed, make the order of the whole squares, and these individual characters of the things multiplied into those of the squares, and to be summed, make the whole order of the cubes, and thus thenceforth to infinity.

As in **1♀ – 1R + 1*a*♀ + 1*a* – 18**, there are two different positions, *viz*. **R** and ***a*** ; [*i.e*. the expression is equivalent to $x^2 - x + a^2 + a - 18$] therefore **– 1R + l*a*** is put in place for the unknown, with order one [*i.e.* here we would choose the new variable to be $-x + a$, or **– 1R + l*a*** ; in order to get a square term, but Napier merely considers **1♀ – 1R** to be augmented by the term **– 0R*a*** to give another square term – rather than making a substitution, where the cross term **– 2R*a*** arises ; clearly this is a step in the right direction, but inadequate really.]

Likewise multiply (by Ch. 5,) **– R + *a*** into itself, the terms become **+♀ – 0R*a* + ♀,** and therefore **1♀ – 0R*a* + 1*a*♀,** clearly shall be three simple terms, that effect a single order only, surely the square. Therefore in this place the correct situation is **1♀ – 0R*a* + 1*a*♀ + 1*a* – 18**. In a similar manner the cubic order of this shall be this.......

[This example is not given, but it is not hard to see what the form must be.]

CHAPTER VII.

DIVISION.

1. A pure number with a greater sign divided by a pure number of a lesser sign of the same position, is the sign of the order of the interval of the same after a simple number, or after a number of these gathered together.

As **0*a*𝕮** may be divided by **0*a*♀** the sign of the interval is ***a*R**, which at least is located after a collection of simple numbers, or after the numerator of this, as it may happen, and it becomes **0*a*R**, or $\frac{0aR}{0}$, for the quotient of the division. Likewise **0ß** by **0𝕮** becomes the quotient **0♀**, or $\frac{0♀}{0}$. [In these cases, the zero appears to act as any number before the cube of *a*, and which is not itself to be cubed; thus they are not actually zeros, and the term ***a*R** indicates that the number is just *x*.]

2. Divide the pure smaller sign by the greater of the same position [*i.e.* ***a*** has the same value], the sign put in place is that of the same interval after the denominator of the simple number of the fraction.

So that **0*a*♀** may be divided by **0*a*𝕮**; the sign of the interval is ***a***, which is put in place at any rate after the place of the simple number of the denominator, and for the quotient of the fraction it becomes $\frac{0}{0a℞}$. Likewise the division **0𝕮** by **0β** makes the quotient $\frac{0}{0♀}$. [The modern reader may raise an eyebrow at these results ; the problem arises from dealing with numbers that appear essentially to be zero from the start; such expressions seem to indicate the presence of terms not acted on by the power, rather than actual zeros, in cases like this. In addition, such expressions will become part of larger expressions, where such untidy parts are absorbed into something meaningful.]

3. To divide a pure sign by the sign of another position, is to write the sign to be divided above, (*viz*. after the numerator of the simple fraction of the numerator,) and to write the divining sign below, (*viz.* after its denominator,) and the quotient will always be a fraction.

So that **0*a*𝕮** shall require to be divided by **0♀**, the quotient shall be $\frac{0a𝕮}{0♀}$. Likewise **0♀** being divided by **0*a*𝕮**, the quotient becomes $\frac{0♀}{0a𝕮}$.

Corollary.

4. Hence it follows, in a division with mixed terms it must be made by sections 1 and 2, just as they share positions ; but just as they are of different positions, by section 3.

So that **0𝕮*a*♀** may be divided by **0♀*a***; divide **0𝕮** per **0♀** (by sect. 1), it becomes **0℞**. Likewise ***a*♀** divided by ***a***, becomes ***a*** *;* therefore the whole quotient is **0℞*a***. Likewise **0𝕮*a*♀** divided by **0β*b*** in this manner, **0♀** is divided by **0β** and it becomes (by sect. 2,) and it becomes $\frac{0}{0♀}$. Likewise, ***a*♀** is divided by ***b*** (by section 3) and it becomes $\frac{a♀}{b}$; therefore the division becomes for the whole quotient $\frac{0a♀}{0♀b}$. Likewise **0*b*♀*c*** being divided by **0*bc*♀** gives the quotient $\frac{0b}{0c}$.

5. From every integer there is made a fraction of the same value by writing the number one below, and by putting a line in place between them.

As 5 is a whole number, and from that the fraction is made $\frac{5}{1}$. Likewise √𝕮**7** is a whole number, and from that there is made the fraction $\frac{√𝕮7}{1}$.

6. Therefore if a simple number were divided by a simple number, in the first place they make similar roots, (by Ch. 4, section 4;) following this, divide the simple number of the division required by the simple number of the divisor; in the third place, (by what has been said now,) divide the positive sign to be divided by the sign of the divisor ; in the fourth place, extract such a root of the number and sign for the quotient produced, such as indicates its root, (by Ch. 3,) and finally prefix to this the due sign, by Ch. 6 Book 1.

So that **12*b*𝓒** shall be divided by 3***b*♀** divide 12 by 3, making 4 ; then divide ***b*𝓒** by ***b*♀**, making ***b;*** therefore the whole quotient will be **4*b***. Likewise **√♀20β** may be divided by **√♀8𝓒**; divide number by number, becoming $\frac{5}{2}$, and divide the sign by the sign, and it becomes ♀. Therefore extract the root from these $\frac{5}{2}$♀, or $\frac{5♀}{2}$, or from $2\frac{1}{2}$♀, (which by section 1 of this are the same,) extract the square root ; that is $\frac{\sqrt{♀}5♀}{\sqrt{♀}2}$, or √♀$\frac{5}{2}$♀, or √♀$2\frac{1}{2}$♀ for the quotient [which is simply $\frac{5}{2}$]; and these shall be the same by Ch. 4, section 4 Book I. Likewise, from the converse, **√♀8𝓒** may be divided by **√♀20β**, the quotient will be $\frac{\sqrt{♀}2}{\sqrt{♀}5♀}$, or more correctly √♀$\frac{2}{5}$♀. Likewise, **– √♀12♀*a*𝓒*b*** may be divided by **√♀3*ab*** : divide the number and sign as has been said, and it becomes **4♀*a*♀**, the root of which as indicated by the root, *viz.* the square root, is **– 2℞*a*** for the quotient, with its due sign. Likewise, **4℞*a*** shall be divided by **– √♀2℞*a*♀** : first, they are made of the same signs, **√♀16♀*a*♀** and **– √♀2℞*a*♀**; then, divide both the number and the sign of this, and it becomes **8℞**, the square root of this for the quotient, with its due sign, is **– √♀8℞**.

7. Here it is required to observe, that if the quotient of positive signs (by 2 and 3 of this) were a fraction, and the quotient of the absolute number were an integer, from this fraction it must become a fraction (by 5 of this).

So that if **12𝓒** shall be divided by **3β** ; divide number by number, making 4, a whole number; and divide sign by sign, evidently it becomes the fraction $\frac{0}{0♀}$, or $\overline{♀}$; and therefore from the whole number 4, or $\frac{4}{1}$ and the whole quotient becomes $\frac{4}{1♀}$. Likewise **15*b*♀*c*** may be divided by **5*bc*♀**, the quotient of the whole numbers divided becoming the integer 3, and the quotient of the signs $\frac{0b}{0c}$, or $\frac{b}{c}$ which is the fraction, and therefore from 3 the fraction is made $\frac{3}{1}$ and therefore the whole quotient $\frac{3b}{1c}$, and neither $3\frac{b}{c}$ nor $\frac{3b}{0♀}$.

8. If a composite [expression] were divided by a simple one, divide (as now has been said) some simple part of the of the composite by this simple divisor, and connect the simple quotients with the due signs.

So that **12♀ – √♀2♀ + 6** may be divided by **2℞** ; divide **12♀** by **2℞**, making **6℞**; likewise divide **– √♀2♀** by **2℞**, making **– √♀$\frac{1}{2}$** ; finally divide **+ 6** by **2℞**, making $\frac{+3}{1℞}$; from which joined together the total quotient it becomes **6℞– √♀$\frac{1}{2}$ + $\frac{3}{1℞}$**.

(If a composite of one order shall be divided by a certain composite, etc., so that it shall be divided by **6℞ – √♀3♀** , do as in Ch.11, section 2, Book I.)

9. If a composite were to be divided by a composite of several orders : initially, each term of the simple composites are put into the correct place (by Ch. 6, sect. 4) ; then divide the simple maximum of the order to be divided by the simple maximum of the dividing order, (by 6 of this,) and a simple first quotient will be produced; multiply the whole of the divisor by this, take the product away from the total to be divided, note down the remainder, with the rest from the division deleted. From these remaining put in place make another [expression] to be divided, from which, in the same manner as before,

another simple quotient, perhaps will be produced, then finally either nothing will be left to be divided, or at least they will be agreed to be from smaller proportional orders than the divisor; with which completed, collect and connect the individual said parts of the quotient with its plus and minus signs, and make the quotient of the whole, with the final remainders noted, if which there shall be.

So that **1♀♀ + 71♀ + 120 –154R –14ℭ** shall be required to be divided by **6 +** 1♀ **– 5R** : initially, (by Ch. 6,) correctly arrange these in place :—

1♀♀ –14ℭ +71♀ –154R +120 (1♀
1♀ – 5R +6

Then divide **1♀♀** by **1♀**, the quotient becomes **1♀**, that is noted at the half bracket, as above ; multiply the whole divisor by this quotient, thence it becomes **1♀♀ –5ℭ +6♀**, which you subtract from the whole dividend ; **–9ℭ +65♀ –154R +120** is left ; therefore note this down in this form, with the rest deleted :—

	–9ℭ	**+65♀**			
1♀♀	**– 14ℭ**	**+ 71♀**	**– 154R**	**+ 120**	**(1♀**
1♀	**– 5R**	**+ 6**			
	1♀	**– 5R**	**+6**		

Divide these remaining by the same divisor in the same manner as before, and this situation arises :—

		+20♀			
	– 9ℭ	**+65♀**	**– 100R**		
1♀♀	**– 14ℭ**	**+71♀**	**– 154R**	**+ 120**	**(1♀– 9R**
1♀	**– 5R**	**+ 6**	**+ 6**	**+ 6**	
	1♀	**– 5R**	**+ 5R**		
		1♀			

Finally divide these remaining in the same manner as at first, and this situation arises :—

		+20♀			
	– 9ℭ	**+65♀**	**– 100R**		
1♀♀	**– 14ℭ**	**+71♀**	**– 154R**	**+ 120**	**(1♀– 9R + 20**
1♀	**– 5R**	**+ 6**	**+ 6**	**+ 6**	
	1♀	**– 5R**	**+ 5R**		
		1♀			

Therefore the total quotient is **1♀– 9R + 20**, and the final remainder become zero.
Another example is expressed by a single type, in which **1ℭ – 11♀ + 40R– 36** shall be divided by **1R – 4 :**

```
                          +12
                  +12℞
          – 7♀
1ℭ       –11♀    – 40℞    –36    (1♀–7℞ + 12 for the whole quotient.
1℞       – 4
          1℞       – 4
          1℞       – 4
```

And then finally 12 emerges, having a smaller order than the divisor **1℞– 4.**

10. If the dividend were not wholly or completely divisible by the divisor (*viz.* without a total remainder), then by writing the divisor under the remainder a fraction is made, that may be located near to the quotient, or if you prefer the divisor being written under the whole dividend.

As in the above example, the true quotient will be **1♀–7℞ + 12** 12/(1℞ - 4) or if you like (1ℭ –11♀+ 40℞ –36)/(1℞ - 4) ; and it becomes this because the divisor cannot divide that wholly, or to divide without a remainder.

CHAPTER VIII.

THE EXTRACTION OF ROOTS FROM COMPOSITE [EXPRESSIONS].

1. If the square root of a composite expression were required to be extracted, that composite will first be put into correct order (by Ch. 6, section 4) ; then, extract the root of the maximum order from the simple [term] or from simple [terms] (by Ch. 3, section 5 and 7), that you put in place at the semi-bracket for the quotient, and delete that simple term of the maximum order, with no remainder if you are able. Secondly, divide the remaining the first part of the composite expression remaining by twice the whole quotient (by Ch. 7, section 9), with no remainder left, if you are able. Write a new quotient of this division after the quotient already made, take the square of this new quadratic from the said part present above, with the remainders noted down. And repeat this second task by three, four or more times, until finally either no remainders shall be left, and then the whole quotient with its joining plus and minus signs will be the true root in place for the question sought, or if some very small parts shall remain above, then the said quotient is said to be an approximate root, and not the true root.

So that from this composite [expression] **√♀4ℭ +1♀ – √♀576℞ + 144 – 23℞** the square root may be extracted : in the first place, the simple terms are put in order correctly, in this situation :—

1♀ + √♀4ℭ – 23℞ – √♀576℞ + 144

[i.e. $x^2 + 2x^{\frac{3}{2}} - 23x - 4x^{\frac{1}{2}} + 144$]

[the original has some typographical errors present] then from the first, *viz*. from **1♀** extract the true root, which will be **1℞** for the quotient, and the rest will remain, in this situation : —

1♀ + √♀4ℭ – 23℞ – √♀576℞ + 144 (1℞ ;

in the second place, divide the first part of the remainder by twice the quotient, *viz*. by **2℞** or **√♀4♀** , *viz*. **√♀4ℭ** is divided, the new quotient becomes the quotient **+ √♀1℞** and the remainder becomes **– 23℞ – √♀576℞ + 144**, from which take the square of this new quotient, which is **1℞**, and the quotient and remainder will become in this situation :—

– 24℞
1♀ + √♀4ℭ – 23℞ – √♀576℞ + 144 (1℞ + √♀1℞
+√♀4♀

At this point repeat this second work, evidently divide the first part of the above remainder, viz. **– 24℞ – √♀576℞**, by twice the quotient, *viz*. by **2℞ + √♀4℞** , and the new quotient becomes **–12**, and the new remainder **+ 144**, from which take away the most recent square of the quotient, *viz.* **–144**, and nothing is left over, so that it becomes apparent in this situation :—

– 24℞
1♀ + √♀4ℭ – 23℞ – √♀576℞ + 144 (1℞ + √♀1℞–12
+√♀4♀ + 2℞ + √♀1℞

From which it is apparent that **1℞ + √♀1℞–12** [i.e. $x+\sqrt{x}-12$] is the true square root in place of the composite expression written above, because nothing remains after the extraction.

Another example [the 2nd.].

The square root shall be extracted from **√ℭ4♀ – 8 – √ℭ16℞** [i.e. $\sqrt[3]{4x^2}-8-\sqrt[3]{16x}$] :

first, this is put in place :–

√ℭ4♀ – √ℭ16℞ – 8 (√ℭ2℞

Secondly, this may be put in place :–

– 9
√ℭ4♀ – √ℭ16℞ – 8 (√ℭ2℞ – 1

– √ℭ16℞

From which it is apparent that **√ℭ2℞ – 1** is an approximation and not the true square root of this expression **√ℭ4♀ – 8 – √ℭ16℞**, because a remainder is left over, *viz*. **– 9**.

[In modern terms we have, (noting that in Napier's scheme, the remainder is placed *above* the main expression) :

$$\text{√ℭ4♀} - \text{√ℭ16℞} - 8 = (2x)^{\frac{2}{3}} - 2(2x)^{\frac{1}{3}} - 8 = \left((2x)^{\frac{1}{3}} - 1\right)^2 - 9 \textbf{.]}$$

An example of several positions [3rd.].

The square root shall be extracted from **1♀ + 2℞*a* +1*a*♀** + **1℞ + 1a –110**.

[*i.e.* $x^2 + 2ax + x^2 + x + a - 110$]

Initially, put this in place, (by Ch.6, Prop. 7) :—

1♀ + 2℞*a* +1*a*♀ + 1℞ + 1a –110 (1℞

Secondly, this is put in place :—

1♀ + 2℞*a* +1*a*♀ + 1℞ + 1a –110 (1℞+1*a*
+ 2℞

Thirdly, this is put in place :—

–110 $\frac{1}{4}$
1♀ + 2℞*a* + 1*a*♀ + 1℞ + 1a –110 (1℞+1*a* + $\frac{1}{2}$ approx. for the root.
+ 2℞ 2℞ + 1a

[*i.e.* $x^2 + 2ax + a^2 + x + a - 110 = (x+a)^2 + \frac{1}{2}.2(x+a) + \frac{1}{4} - 110\frac{1}{4} \cong (x + a + \frac{1}{2})^2 - 110\frac{1}{4}$.]

Another example of several positions [4th].

The square root shall be extracted from **1♀ + 6℞*a* – 7**.
Initially, this is put in place (by Ch.6, Prop. 7) :—

–9*a*♀ – 7
1♀ + 6℞*a* +0*a*♀ – 7 (1℞

Secondly, this is put in place :—

–9*a*♀
1♀ + 6℞*a* +0*a*♀ – 7 (1℞ + 3a for the root approximately.
2℞

[i.e. $x^2 + 6ax - 7 = (x + 3a)^2 - 9a^2 - 7$.]

A more difficult example of one position [5th].

The square root shall be extracted from **1♀ – √♀8♀ – 6℞ + 8 + √♀32**.
Initially, this becomes the situation :—

1♀ – √♀8♀ – 6℞ + 8 + √♀32 (1℞

Secondly, this becomes the situation :—

– 3 – √♀8
1♀ – √♀8♀ – 6℞ + 8 + √♀32 (1℞– √♀2 – 3 for the root approx.
+ 4√♀4♀ + 6℞

[i.e. $x^2 - 2\sqrt{2}x - 6x + 8 + \sqrt{32} = \left(\left(x - \sqrt{2}\right) - 3\right)^2 - 3 - 2\sqrt{2}$.]

Sixth example.

The square root shall be extracted from **1♀ – 0℞*a* + 1*a*♀ – 1℞ + 1*a* – 18**.
Firstly, this is put in place, by Ch. 6, Prop. 7 :—

1♀ – 0℞*a* +1*a*♀ – 1℞ + 1*a* – 18 (1℞

Secondly :—

+2℞*a*
1♀ – 0℞*a* +1*a*♀ – 1℞ + 1*a* –18 (1℞–1*a*
+2℞

Thirdly, this becomes the situation :—

+2℞*a* –18 $\frac{1}{4}$
1♀ – 0℞*a* 1*a*♀ – 1℞ + 1*a* –18 (1℞–1*a*–$\frac{1}{2}$
+2℞ +2℞ – 2*a*

$$[i.e.\ x^2 + a^2 - x + a - 18 = \left(\left(x-a\right)-\tfrac{1}{2}\right)^2 + 2ax - 18\tfrac{1}{4}\ .]$$

2. If the cube root of some composite expression were to be extracted, that composite may be arranged in order first (by Ch.6, section 4); then from the maximum order simply extract the cube root (by Chap. 8, sections 5 and 7), which you put in place near the bracket for the quotient, and delete the simple number of the maximum order. Secondly, divide the first part of the composite expression not deleted by three times the square of the whole quotient (by Ch. 7, section 9), with the divided number deleted, and the rest noted down. Write the new quotient of this division after the first quotient, and take from the said remainders, three times the square of the new multiplied into the first preceding quotient, and from the same take the new cube, with the remainders noted. And repeat this second task again and again, while at last either nothing is left over, and then the whole quotient with the connecting plus and minus signs will be the true root sought in place ; or if as some most small parts shall remain, then the quotient is said to be the approximate root, and not the true root.

So that, let the cube root of the following composite expression be extracted, which is arranged correctly in the first place thus :—

1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192R + 64 (1♀

viz. the cube root is extracted from **1♀ℭ**, which is **1♀**, which is put in place for the quotient. Secondly, divide **12ß** by three times the square of the quotient, *viz*. by **3♀♀**, the new quotient becomes **+ 4R** with **+ 12ß** deleted, in this situation :—

1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192R + 64 (1♀ + 4R
+ 3♀♀

Then, multiply three times the square of this new quotient **+ 4R**, *viz*. **48♀**, into the first quotient, *viz*. into **1♀**, making **48♀♀**, which you take from **60♀♀** &c., **+ 12♀♀ + 96ℭ + 240♀ + 192R + 64** remains, from which parts remaining also take the cube of these +4**R**, which is **64ℭ**, leaving **+12♀♀ + 96ℭ + 240♀ + 192R + 64**, in this case :—

+ 12♀♀ + 96ℭ
1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192R + 64 (1♀ + 4R
+ 3♀♀ + 48♀♀ + 64ℭ

In the third place, repeat the second task, *viz.* divide the first part of the remainder by three times the square the square of the quotient, which is **3♀♀ + 24ℭ + 48♀**, divide the said part of the said remainder, and the quotient and remainder become as below:—

+ 12♀♀ + 96ℭ + 48♀
1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192R + 64 (1♀ + 4R + 4

	+ 3♀♀	+ 48♀♀	+ 64𝕮	
		+ 3♀♀	+ 24𝕮	+ 48♀

Then, multiply three times the square of this new quotient, *viz*. 48, into the whole of the preceding quotient, *viz.* into **1♀ + 4℞**, making **48♀ + 192℞**, which taken from these remaining, *viz*. from **48♀ + 192℞ + 64** , leaves **+ 64**, from which take also the cube of the latest quotient, which is **+ 64**, and nothing remains, as follows :—

		+ 12♀♀	+ 96𝕮	+ 48♀			
1♀𝕮	+ 12ß	+ 60♀♀	+ 160𝕮	+ 240♀	+ 192℞	+ 64	(1♀ + 4℞ + 4
	+ 3♀♀	+ 48♀♀	+ 64𝕮				for the true
		+ 3♀♀	+ 24𝕮	+ 48♀			cube root.
				+ 48♀	+ 192℞	+ 64	

Another example [2nd].

Let the cube root of the following composite number be extracted, which may be arranged thus :—

1𝕮 – 10♀ + 31℞ – 30 (1℞

Secondly, it may be set out thus :—

		$-\frac{7}{3}$ ℞	$+\frac{190}{27}$	
1𝕮	– 10♀	+ 31℞	– 30	(1℞ – $3\frac{1}{3}$ for the approx. cube root, but
	+ 3♀	$+\frac{100}{3}$ ℞	$-\frac{1000}{27}$	not of the true root, from the remainders extant.

$$[i.e.\ \ x^3-10x^2+31x-30=\left(x-3\tfrac{1}{3}\right)^3-\tfrac{7}{3}x+\tfrac{190}{27}\ .]$$

3. If you should wish to extract the true root of a composite expression, yet which may not have the true root in place, but only an approximation, to that composite expression affix with the universal sign, and it becomes thence the true hidden root.

As from the second example of the square root presented,
viz. from **√𝕮4♀ – √𝕮16℞ – 8** the true square root may be extracted, that will be **√♀.√𝕮4♀ – √𝕮16℞ – 8**

Likewise, let the true cubic root be extracted from **1𝕮–10♀ + 31℞ – 30**, that will be **√𝕮.1𝕮–10♀ + 31℞ – 30.**

4. But we omit the square roots of square roots, supersolids, and the other higher orders, both because they are most rare in use, as well as because they can be considered from what has been said,.

So that if the square root of a square root shall be extracted, that can be extracted by amending the rule of the cube root extraction thus. In the first place, ' for the extraction of the cube root ' read , ' for the extraction of the fourth root.' Secondly, for ' three squares,' read, 'four cubes.' Thirdly, for 'three squares multiplied into the first preceding quotient,' read, ' six squares multiplied into the first square of the preceding quotient and four of the new cubes multiplied into the first antecedent quotient,' etc. Fourthly, for ' the cube of the new,' read, ' the square of the new square.' And thus the emended rule will be of service to extracting the square root of the square root.

But truly if you wish to amend the rule for extracting the fifth root, for ' cube,' read, 'supersolid,' and for ' three squares,' read, 'five squares or squares,' and for' three squares multiplied into the first preceding quotient,' read, 'ten squares multiplied into the cube of the first antecedent quotient, and ten of the new cubes multiplied into the preceding squares, and five squares of new squares into the first preceding quotient, ' etc.
And for ' cube of the new,' read, ' supersolid of the new; ' and in a similar manner, for all the higher roots being extracted, rules will be able to be put in place.

Example of the rule of the square root of the square root.

l*a*♀♀	**+ 4*a*ℭ*b***	**+ 6*a*♀*b*♀**	**+ 4*b*ℭ*a***	**+ l*b*♀♀**	(**l*a* + l*b*** for the true
	+ 4*a*ℭ	**+ 6*b*♀*a*♀**	**+ 4*b*ℭ*a***	**+ l*b*♀♀**	fourth root.

Example of the rule of the supersolid.

l*a*ß	**+ 5*a*♀♀*b***	**+ l0*a*ℭ*b*♀**	**+ 10*a*♀*b*ℭ**	**+ 5*b*♀♀*a***	**+ l*b*ß**	**(1*a*+l*b*** true root
	+ 5*a*♀♀	**+ l0*b*♀*a*ℭ**	**+ 10 *b* ℭ*a*♀**	**+ 5*b*♀♀*a***	**+ l*b*ß**	supersolid.

5. Thus it is apparent from the previous examples that some of the remainders extracted have no positive signs, and these whole remainders are said to have a set form [or to be formed]; other remainders have positive signs, and these are said not to have a set form [or not to be formed, or to be called unformed]. [Thus, if the original expression is equated to zero, the exercise becomes one of extracting a root, or an approximate root, or showing that no real root can be extracted, depending on the sign of the final remainder, if such can be found.]

As in the extractions of the square root in examples 2,3, and 4, [of section 1] all the remainders of the first operation are unformed, but the final remainders of the same examples are numbers, and thus are said to be formed. [The remainders are, –9, $-110\frac{1}{4}$, and $-9a^2-7$, respectively.]
Truly in the examples 5 and 6 of the squares, and example 2 of the cubes [section 1], all the remainders, both the first as well as the final, emerge from that positive, are said not to be not formed or unformed. [The 5th example has remainder $-3-2\sqrt{2}$, and so would be formed, while the 6th example has the remainder $+2ax-18\frac{1}{4}$, and so is indeed

unformed, as it can be positive or negative; the second cube root has the remainder $-\frac{7}{3}x+\frac{190}{27}$, and so also is unformed.]

Certain types of the remainders are formable, certain reformable, and certain in short are lacking in form and unable to be reformed.

As will become apparent from the examples following next.

6. Remainders are formable when the second part of the rule of extraction may be exercised [that is, the remainder at some stage of extracting can be reduced towards a pure number, or at least the power of *x* is diminished], by which zero remainders are thence returned, or smaller remainders are returned from the previous unformed expression. And the operation of the second part of the rule of extraction is said to be its conformation.

As in all the examples above, both the squares and the cubes, all the remainders besides the last are said to be formable, because they are correctly formed through the second part of the rule of extraction alone, and the most recent remainders thence emerge less unformed.

7. Remainders are reformable which, if divided by some composite, become equal to zero (or to an equation equal to 0), and hence you may divide, if there is a need, the most recent remainders extant by the other, and the other equation is equated to 0 ; the remainders finally are either zero, or are reformable, and these equations are called Reforming Equations, and the task of dividing is called the Reformation.

Example.

The approximate square root may be extracted from **1ℨ – 0℞*a* + l*a*ℨ – 1℞ + l*a* – 18**, (which occurs in example 6 above,) and both the root **1℞ – l*a* –** $\mathbf{\frac{1}{2}}$, as well as the remainder *viz.* **+ 2℞*a* –18** $\mathbf{\frac{1}{4}}$, are unformed ; but this composite expression may be given, as an example **1℞*a* + 1℞– l*a* –10**, which is equal to zero ; divide that remainder by this one, and the remainder **–2℞ + 2*a*** + **1** $\mathbf{\frac{3}{4}}$ emerges, which, because it is formable by Prop. 6 of this Chapter [*i.e*. recall that $x^2+a^2-x+a-18=\left((x-a)-\frac{1}{2}\right)^2+2ax-18\frac{1}{4}$; and hence, since $\frac{2xa-18\frac{1}{4}}{xa+x-a-10}=2+\frac{-2x+2a+1\frac{3}{4}}{xa+x-a-10}$, we have $x^2+a^2-x+a-18=\left((x-a)-\frac{1}{2}\right)^2+2(xa+x-a-10)-2(x-a)+1\frac{3}{4}$] ; and thus the remainder **+ 2℞*a* – 18** $\mathbf{\frac{1}{4}}$ is said to be reformable, and the composite expression **1℞*a* + 1℞– l*a* –10** is said to be the reformer, and its operation is called the reformation.

Another example.

Likewise the square root may be extracted from :

1ℨ + 4℞*a* + l*a*ℨ – 4℞*b* – 4*ab*+4*b*ℨ+4℞ + 4*a* – 8*b* – 61, and that will be **1℞+1*a* – 2*b* + 2**, and the remainder **+ 2℞*a* – 65** will be unformed.

$$[i.e.\ x^2+4xa+a^2-4xb-4ab+4b^2+4x+4a-8b-61$$
$$=\left(\left(x+a-2b\right)+2\right)^2+2ax-65.]$$

But an equation may be given equal to 0, which shall be **1℞*a* – 1*ab* – 1*b* – 5**, by which you divide that remainder, and the remainder emerges **+2*ab* + 2*b* – 55** $\frac{2ax-65}{ax-ab-b-5}=2+\frac{2ab+2b-55}{ax-ab-b-5}$; which, because it is neither formed nor informed, that must be divided by another equation to 0, e.g., by **2*ab* – 3℞ – 3*a* + 8*b* – 21**, and the remainder emerges **3℞+3*a* – 6*b* – 34** ;[*i.e.* $\frac{2ab+2b-55}{2ab-3x-3a+8b-21}=1+\frac{3x+3a-6b-34}{2ab-3x-3a+8b-21}$,] which, because they are formable (with respect to the aforemade approximate root *viz*. **1℞ + 1*a* –2*b* +2**), thus both the remainder **2℞*a* – 65**, and the remainder **2*ab*+2*b* – 55** are said to be reformable, and both the composite expression **1℞ – 1*ab* – 1*b* – 5**, and the composite **2*ab* – 3℞ – 3*a* + 8*b* – 21** are said to be reforming equations.

8. Therefore in order that unformed remainders become formed, you will conform the conformables (by 6 Prop. of this Chapter); and you will reform the reformables (by 7,), and you will note all the latest remainders, and if which became zero or formed, all is well, then indeed all the conforming quotients are coupled by their signs and abbreviated, and the approximate root will be reformed; truly the quotients of the reformation are of no value, and are to be discarded.

As the square root of the penultimate example was **1℞ – 1*a* – $\frac{1}{2}$**, and the remainder **+ 2℞*a* – 18$\frac{1}{4}$**, which, because it is reformable (by Prop. 7) by it reformer **1℞*a* + 1℞– l*a* –10**, and with the quotient deleted the remainder emerges **– 2℞ + 2*a* –** 1$\frac{3}{4}$, which because by conforming they are formable (by 7), and the noted formals emerge, *viz*. **+$\frac{3}{4}$** and the quotient of the conformation **–1** with the aforementioned root joined by its plus and minus signs and abbreviated, becomes **1℞ – 1*a* – 1$\frac{1}{2}$** for the approximate reformed root.

Likewise the remainder of the final example was initially **2℞*a* – 65**, which with the reformation by its reformer, *viz*. **1℞ – 1*ab* – 1*b* – 5**, and with the quotient deleted, the remainder **2*ab* + 2*b* – 55** emerges as we have said in Prop. 7 ; which again on reformation by another reformer (as we have advised in the same place,) *viz*. by **2*ab* - 3℞ – 3*a* + 8*b* – 21**, with the quotient deleted the remainder arises **3℞ + 3*a* – 6*b* – 34** which, because it is formable (by Prop. 6) to its approximate root, *viz*. to **1℞ +1*a* – 2*b* + 2**, you will conform, and the whole reformed approximate root will be **1℞ +1*a* – 2*b* +** 3$\frac{1}{2}$, and the last formal remainder to be noted is $-\frac{169}{4}$ or **42$\frac{1}{4}$**.

9. But if with the failure of reformation of remainders, they are present after the final conformation, these are called deforms or irreformables.

So that, the last remainders of examples 5 and 6 of the extraction of the square, and 2 of the extraction of the cube (if no reformators occur,) are called deforms and irreformables.

10. There are two kinds of deformed remainders and roots, singular and plural, the singular of which are these deforms which have some one single and pure positive root, or a mixture of which the singular ones are these deforms which have only one pure and simple positive, or of a mixture of positive parts in one root, or for that quotient there was not another similar or of the same position, neither in the quotient or root nor among the remainders.

As the square root of this **1♀+ 6℞*a* – 7** is **1℞ + 3*a***, and the remainder is **– 9*a*♀ – 7**, which thus are said to be singular because in these there is no more than one positive [value of *x*] in the first place, which is **1℞**. Likewise the square root of this **1♀*a*♀ – 6℞a – 1*a* + 8 = 0** is **1℞*a* – 3**, and the remainder is **–l*a* – 1**, in which the sign **℞** is not found more often than once.

11. Roots and their remainders are said to be plurals, when several simple roots are found at some one position or among the remainders.

So that the approximate cube root of this expression **1ℭ – 9♀ + 36℞ – 80 = 0** is **1℞ – 3**, and the remainder will be **+9℞ – 53**. In which the sign of the first position **℞** is found twice.

Likewise the approximate square root of this expression **1♀ + 1*a*♀ – 1℞ + 1*a* – 18** (by lacking a reformator,) will be **1℞*a* –1*a* –** $\frac{1}{2}$, and the remainder will be **2℞*a* – 18**$\frac{1}{4}$, as we have discussed in the above example 6, in which the first two positions are simple, *viz*. **1℞** and 2**℞*a*;** likewise the same number of the second position, *viz*. **1*a*** and 2**℞*a***.

12. Thus there are four forms of roots. The first form is of true roots, the second is of formed roots, the third is of singular roots, and the fourth is of several roots; the use of the extraction of which we will teach below.

CHAPTER IX.

EQUATIONS AND THEIR EXPLANATION.

1. An equation is a collection of unknown positive values with other values equal to each other [*i.e.* constants], from which the value of the position is sought.

So that if putting **1℞** for which number or magnitude sought, the value of which being unknown, afterwards through the hypothesis of the question taking **3℞** to be equal to 21, three things may be joined together with its equality 21, that joining together of equality is called an equation ; and hence the [unknown] thing is introduced as a single position with the value 7.

2. In turn two equal lines are placed between the parts of an equation, which is called the sign of the equation.

So that **3℞ = 21**, which is pronounced thus: three things equals twenty one. Likewise **℞ = 7**, which is pronounced : one thing equals seven.

3. Some equations are of one position only, others of several positions.

An equation such as **l*a*♀+3*a* = 10** is of one position only ; an equation such as **2♀ – l*a* = 6** is of several positions.

4. Likewise some of the equations are undeveloped, which can be reduced to lesser terms and made more clear and succinct, while others are said to be most perfect, which on the other hand are maximally clear and succinct.

So that, **3℞ = 21** is an undeveloped equation, because it can be reduced to perfection, *viz*. into **℞ = 7**. Likewise, **5*a*♀ = 20** is an undeveloped equation, because it can be perfected, *viz*. it can be reduced into **1*a*♀ = 4**. But also **1a♀ = 4** is undeveloped, because at this point it can be made more perfect, indeed most perfect, *viz*. it can be reduced into **l*a*** = **2**, by an art which we shall treat below. Likewise **12♀ +3*a* = 6** is an undeveloped equation, because it can be reduced into the more perfect **4♀ + la = 2**.

5. Likewise some equations are simple, others quadratic, others cubic, others higher : the simple equations of which are agreed to be from two ordinary numbers only.

So that **3℞** = **27**, or **1℞ = 9**; likewise **5*b*♀ = 20**, are called simple equations.

6. Some of the simple equations are real, which are of things equal to a number [*i.e.* can be solved to give actual whole numbers or fractions]; others are equal to roots, which are of certain squares, cubes, or of other equations for higher numbers.

Reals, such as **3℞ = 21**, or **1℞ = 7**. Likewise **l*a* =3**. Likewise **2℞ =√♀3 –** 1. Roots, such as **2♀ = 8**. Likewise **3ℭ** = 24. Likewise **1*a*ß** = **√ℭ9**, etc.

7. An equation is quadratic which depends on three proportional orders.
So that **2♀ + 3℞ = 4**, or **3℞ = 2♀ – 4**. Likewise **l*a*♀ℭ– l0 = 3*a*♀**. [This is in error.] Likewise **12 – √♀1℞ = 1℞**.

8. A cubic equation is one which depends on four orders in proportion.
Such as **1ℭ – 9♀ = 24 – 26℞**. Likewise **l ℭ+ 0♀ – ℞2 = 4**. Likewise **l*a*♀ℭ – 2*a*♀ = 4** is a cubic equation, because (by Prop. 4 Ch. 6) thus on being put in order **l*a*♀ℭ** + **0*a*♀♀ – 2*a*♀ = 4**, it depends on four orders.

9. A square of the square equation is one that depends on five proportional orders ; supersolid, on six ; the cube of the square, seven: and thus with the remaining orders to infinity.

The square of the square, as **2♀♀ – 28ℭ + 142♀ = 308℞ – 240**. The supersolid [squared], as **1*b*♀ß – 4*b*♀♀♀ +l*b*♀ℭ – 3*b*♀♀ – l*b*♀ = 12**. The cube of the square, as **l*a*♀ℭ – 8*a*ß + 2*a*♀♀ – 6*a*ℭ + l*a*♀ = l*a* + 6**.

10. An illusive [or silly] equation is that which it is impossible to solve, and if a certain impossible quantity is sought, the answer to that falls on an illusive equation.

So that **1℞ = 3℞** is an illuding equation, if indeed it is impossible that some quantity shall be equal to its triple. Likewise **1♀ = 4℞ – 5** is an illuding equation, if indeed no squared thing can be equal to four things or to four times its root with five taken ; as will be shown below.

11. The exposition is the reduction of an undeveloped equation to the most perfect and real equation, and the part of a real equation which is equal to one thing is called the exposition, and that solves the question.

As when this undeveloped equation is reduced **3℞ = 21** to this most perfect **1℞ = 7**, the exposition of each equation will be 7, because it is equal to one thing (*viz*. to **1℞**). Likewise this undeveloped equation **5♀ = 20** is reduced to this more perfect form **1♀ = 4**, then to the most perfect and real **1℞ = 2**, which indeed with the aid of the reduction is called the exposition, and 2 being the exponent, because it is equal to one thing : later we will discuss a question to be solved by a real exponent.

12. Every equation besides the illusive ones have at least a single exponent, valid or invalid.

This we will expound on later, here it suffices to be forewarned.

13. Valid exponents are those which put in place are noted with a **+** sign, and always are greater than nothing. Indeed invalid exponents are those which put in place by themselves are noted with a negative sign **–**, and these are less than nothing.

As in this equation **1℞ = 7**, seven is a valid exponent, because (by Prop. 1, Ch. 6, Book I.) the sign **+** is understood to be noted. But in this real equation **1℞ = – 7**, the exponent is said to be invalid by opposing reason, because the **–** sign is noted thus, **– 7**, and it is less than nothing.

14. Some of the exponents also are able to be expressed by a number and magnitude only, still others by a number only, yet others by a magnitude only, others partially from that, and others in no manner.

These will be discussed with their examples, extended more by the order, in Chapters 11, 12, 13.

15. The part of the one foremost power of the first subordinate of any equation is called a minimal part, however many signs and terms it may have ; and the foremost and predominating sign is called the ductrix or leader [of this part]; the rest of the signs are called intermediate.

As in this equation **1𝕮 – 3 + √♀2 +** (3℞ – 4)/(1♀ + 1) **– √♀.6 + √♀1℞ = 0**,
in which **1𝕮** is called a minimal part, and **+** is called its leader sign.
Likewise 3 is called a minimal part, and the **–** its leader. Likewise **√♀2** a minimal, and **+** its leader. Likewise (3℞ – 4)/(1♀ + 1) is called a minimal part, and **+** its leader, because its action is extended into the whole fraction. Truly the rest of the signs of this fraction are said to be intermediates. Likewise **√♀.6+ √♀1℞** is called a minimal part, and the **–** sign its leader, because the value of the whole universal roots extends its action in the aggregate, and the remaining **+** sign is called intermediate.

CHAPTER X.

THE PREPARATION OF GENERAL EQUATIONS.

1. Preparation is the reduction of undeveloped equations into more perfect ones, which afterwards is reduced to the most perfect real exposition.

So that **5*a*♀ = 20** first is prepared , and it becomes **1*a*♀=4**: then it is set out and it becomes **1*a* = 2**. The manners in which they are prepared will now be stated, and in which truly they are set out will be made clear later.

2. Undeveloped equations are prepared and made clear in five ways : by transposition, abbreviation, division, multiplication, and by [root] extraction.

The rules and examples of which ways follow.

3. If you carry across a minimal part from one side of an equation into the opposite side, and you put in place a contrary leading sign to that, the parts will be equal (as before), and the part is said to be transposed.

So that, from the latter part of this equation **4R – 6 = 5R – 20**, if you transpose – 20 into the first part of the equation, change its sign, in this case, **4R – 6 + 20 = 5R** : Likewise at this stage if you transpose **4R**, it becomes **– 4R**, and in this case, **– 6 + 20** = 5**R** – **4R**. Likewise if you transpose **–√♀.3♀ – 2** of this equation, **1♀ – √♀.3♀ – 2 = 3*a***, it will be **+ √♀.3♀ – 2**, and in this case, **1♀ = 3*a* + √♀.3♀ – 2** ; and if also now you transpose **3*a***, it will be – **3*a***, in this case, **+ 1♀ – 3*a* = √♀.3♀ – 2** and the parts of the opposite sides are equal, as they were before.

4. If you were to transpose all the other smallest terms of the equation (by the foregoing) into the opposite side, the whole composition will equal to zero, and the equation is said to zero ; and this equation must be abbreviated (by Prop. 4, Ch. 2 of this).

As in the example written above, **4R – 6** = 5**R** – **20**, transpose 5**R** – **20**, and it becomes –5**R** **+ 20**, in this case, **4R – 6** –5**R** + **20 = 0**, which abbreviated becomes **–1R +14 = 0**, which equation is equal to nothing. Likewise **1♀ – √♀.3♀ – 2 = 3*a***, the left part of which if you transpose to the right, becomes **0 = – 1♀ + √♀.3♀ – 2 + 3*a***, which indeed is called and equation for nothing.

5. If the largest term from the front should have a **–** sign, convert all of the leaders of the minimal terms, and a clearer equation is produced.

As by the example above : If **–1R + 14** is required to be 0, in this case, **–1R +14 = 0**, and it follows that **–1R +14** also will be equal to 0, in this case, **+1R – 14 = 0**. Likewise, in the same manner from **– 1♀ + 3*a* + √♀.3♀ – 2 = 0** it becomes **1♀ –3*a* – √♀.3♀ – 2 = 0**. Likewise from this : **–1R – 1 +** $\frac{32}{1a+1}$ **= 0**, this becomes, **1R +1–** $\frac{32}{1a+1}$ **= 0**.

6. If you divide all the positive maximas of the orders of an equation, and with different roots of the same order, by the unit of the maximum order of positive sign, and by that quotient you divide the whole equation, hence a clearer equation will be produced having its maximum order denoted by one.

Example of the equation, **2𝕮 – 8♀ + 6℞ = 0**: Divide the positive of the maximum order, *viz*. **2𝕮** by **1𝕮** the quotient becomes 2; therefore divide the whole equation by two, and it becomes **1𝕮– 4♀ + 3℞ = 0**.

Likewise the maxima of the positive order of this equation, **3℞ – √♀2♀ – 6 = 0**, are **3℞ – √♀2♀** , which indeed (by Prop. 5 Ch. 4 of this) are powers of the same order, and the order of these is the thing [i.e. the unknown sought]; therefore divide **3℞ – √♀2♀** by **1℞**, or (what is the same) by **√♀1♀**, the quotient becomes **3 – √♀2 ;** divide the whole equation by this quotient (by Prop. 2 Ch. 11 Book I.), and this equation is produced **1℞ –** $\frac{18}{3}$ **–** $\frac{\sqrt{♀72}}{7}$ **= 0**, which shall be some fraction, yet more transparent than before, because the ♀ sign has been taken away.

Likewise a third example, **1℞*a* + 1*a* + 1℞ – 31 = 0**, in which you have in mind to clean up the equation and delete the mixed sign, *viz*. **1℞*a***: Therefore divide **1℞*a* + 1*a*** per **1℞**, (you may wish to accept either in the place of the maximum order,) for example **1℞** may be taken: and thus divide **1℞*a* + 1℞** by **1℞**, the quotient **1*a* +1** emerges, by which divide the whole equation **1℞*a* + 1℞ + 1*a* – 31 = 0**, and this equation comes about, **1℞ +1–** $\frac{32}{1a+1}$ **= 0**, which permits a fraction, yet is clearer than before, because from that the mixed sign that obscured the first equation has now been taken away.

7. If the minimal order of the equation were positive, then divide the whole equation by the unit sign of the sign of the smallest order, and thence a clearer equation comes about having an absolute number in place of the smallest order.
An example, **1𝕮 – 4♀ + 3℞ = 0**, divide which by the unit of the minimum order, *viz*. by **1℞**, it becomes **1♀ – 4℞ + 3 = 0**. Likewise **3♀ – √♀2℞ = 0**, divide this by **√♀1℞**, thence this equation arises **√♀9𝕮 – √♀2 = 0**, of which the final sequence always is a number.

8. If some terms of an equation were true fractions, and into the denominators of these you multiply the whole equation, an equation of whole numbers is produced, and much more clear.

So that, in this equation $\frac{6℞-8♀}{1𝕮+3℞}$ **+ 2 = 0**, there is $\frac{6℞-8♀}{1𝕮+3℞}$, truly a fraction capable of abbreviation ; therefore multiply the whole equation into the denominator **1𝕮 + 3℞**, and it becomes **2𝕮 + 12℞ – 8♀ = 0**.

Likewise hence multiply the equation **1♀ +** $\frac{2}{3}$ **℞ –** $\frac{88}{75}$ **= 0** by 3, at first it becomes **3♀ + 2℞ –** $\frac{264}{75}$ **= 0**, and multiply this again by 75, and it becomes **225♀ + 150℞ – 264 = 0**, which indeed is an equation of whole numbers, and freed from fractions.

9. If there was a single universal root in an equation, you will separate that from the rest of the equation (by Prop. 3), and multiply each side of the equation as many times as the universal sign denotes, and a clearer equation will be produced, for it will have no universal signs.
Example, **2♀ + 3℞ – √♀. 12𝕮 + 4♀♀ +18 = 0**: First by transposition it becomes

2ℨ + 3℞ = √ℨ. 12ℭ + 4ℨℨ +18 ; then the sides may be squared, because the universal sign is **√ℨ**, and the equation becomes **4ℨℨ + 12ℭ + 9ℨ = 12ℭ + 4ℨℨ +18**, and by consequent transposition and abbreviation it becomes **1ℨ = 2**.

Another example: **√ℭ.2℞ – 6 = 3℞** multiply the sides cubically, it becomes **2℞ – 6 = 27ℭ**, otherwise **2℞ – 27ℭ – 6 = 0**.

10. If an equation with two universal roots with similar roots and without any other small parts [*i.e.* terms] were put in place, they are separated by transposition, and multiplied into themselves as many times as the universal sign denotes ; a clearer equation is produced with no universal roots.

So that **√ℨ.2℞ + 5 – √ℨ.3℞ – 4 = 0** may be separated. and it becomes **√ℨ. 2℞ + 5 = √ℨ.3℞ – 4** ; the sides may be squared, and the equation becomes **2℞ + 5 = 3℞ – 4**, and by transposition and abbreviation **1℞ - 9 = 0**.

11. If an equation may depend on only two dissimilar universal roots, the universal roots may be separated, and each side may be multiplied into itself according to the nature of the universal sign of each, and a clearer equation comes about without universal signs.

So that **√ß.3ℨ + 6 – √ℨ.2℞ – 3 = 0** ; first they may be separated by transposition, thus, **√ß.3ℨ + 6 =√ℨ.2℞ – 3** ; then, the sides in the supersolid are squared into themselves, and the equation becomes

32ß – 240ℨℨ + 720ℭ – 1080ℨ + 810℞ – 243 = 9ℨℨ + 36ℨ + 36,

which transposed and abbreviated becomes

32ß – 249ℨℨ +720ℭ – 1l16ℨ + 810℞ – 279 = 0.

12. If two universal square roots were in the equation, with certain other simple numbers or uninomials, the universal signs include signs separate from the remaining terms, and multiply each side into itself separately, and an equation is present with only a single universal root, also to be removed by Prop. 9 of this chapter.

So that this equation:

$\frac{1}{2}$ + √ℨ.48$\frac{1}{4}$ + I℞ – 1ℨ + $\frac{1}{2}$℞ – √ℨ.79 – $\frac{3}{4}$ℨ = 0

may be transposed thus :

√ℨ.79– $\frac{3}{4}$ℨ – √ℨ.48$\frac{1}{4}$ + I℞ – 1ℨ = $\frac{1}{2}$℞ + $\frac{1}{2}$;

then each side may be squared, and it becomes :

127$\frac{1}{4}$ + I℞ – 1$\frac{3}{4}$ℨ – √ℨ.15247 + 316℞– 460$\frac{3}{4}$ℨ – √ℭ + 3ℨℨ = $\frac{1}{4}$ℨ+$\frac{1}{2}$℞ + $\frac{1}{4}$

transpose and abbreviate, and the equation becomes :

√ℨ.15247 + 316℞– 460$\frac{3}{4}$ℨ – 3ℭ + 3ℨℨ = 127 + $\frac{1}{2}$℞ – 2ℨ,

which finally (by Prop. 9) becomes **1ℨℨ + 1ℭ – 47ℨ –189℞ + 882 = 0.**

13. If the equation consists of three universal square roots except for other small terms, then two squares roots are separated by transposition from the rest, and the sides may be squared, and an equation arises with only one universal sign, removed by Prop. 9.

So that the equation shall be **√ꝗ.3R + 2 + √ꝗ.2R – 1 – √ꝗ.4R – 2 = 0** , and it may be separated thus : **√ꝗ.3R –2 + √ꝗ.2R + 1 = √ꝗ.4R + 2** ; the sides may be squared, and it becomes **5R – 1 + 2√ꝗ.6ꝗ – 1R – 2 = 4R + 2**; then by abbreviation it becomes **2√ꝗ. 6ꝗ – 1R– 2 = 3 – 1R**; next (by Prop. 9) it becomes **4.6ꝗ – 1R – 2 = 1ꝗ – 6R + 9**, and finally it becomes **23ꝗ + 2R – 17 = 0**. [There are mistakes in the original working.]

14. If the equation depends on three universal square roots, with a single uninomial or simple term ; the two universals are transposed from the rest, and the sides squared, and an equation arises of two universal roots, to be removed by Prop. 12.

So that the equation may be :

√ꝗ.√ℭ2R + 3 +√ꝗ.3R – 2 –2R – √ꝗ.2ꝗ + 1 = 0,

[*i.e.* $\sqrt{\left(\sqrt[3]{2x+3}\right)+\sqrt{3x-2}}-2x-\sqrt{2x^2+1}=0$;]

it is transposed thus :

√ꝗ.√ℭ2R + 3 +√ꝗ.3R – 2 = 2R + √ꝗ.2ꝗ + 1

[*i.e.* $\sqrt{\left(\sqrt[3]{2x+3}\right)+\sqrt{3x-2}}=2x+\sqrt{2x^2+1}$;]

the sides are multiplied into themselves by squaring, and the equation becomes

√ꝗ. √ℭ3456ꝗꝗ –√ℭ1024R – 24 +√ℭ2R+3R+1 = 6ꝗ+1+√ꝗ. 32ꝗꝗ + 8R

[*i.e.*

$$\left(\sqrt{\left(\sqrt[3]{2x+3}\right)+\sqrt{3x-2}}\right)^2=\left(2x+\sqrt{2x^2+1}\right)^2$$

$$\left(\sqrt[3]{2x+3}\right)+3x-2+2\sqrt{\left(\sqrt[3]{2x+3}\right)\times\sqrt{3x-2}}=6x^2+1+4x\sqrt{2x^2+1};\text{ etc.]}$$

with two universal squares in place, removed by Prop. 12.

15. If the equation depends on four universal square roots except for some lesser terms ; two are separated from two by transposition, and the sides may be squared, and this equation arises of only two universal signs, to be deleted by Prop. 12.

Let this equation be transposed thus :

√ꝗ .5ꝗ –2R –√ꝗ.10 –1R = √ꝗ. 2R+6 +√ꝗ.1ꝗ + 4,

the sides of which are squared, and it becomes :

5q – 3R + 10 –√q.208q – 2C – 80R = 1q + 2R + 10+√q.8C + 24q + 32R + 96,

which with the two universal signs only in place, requiring to be deleted by Prop. 12.

16. If a single most universal sign from one side may be equal to a single most universal sign, or to a single universal, or to a universal and an uninomial, or to simple single terms, or to uninomials and simple terms only from the other side : then multiply the sides into themselves according to the order of the most universal sign, and the universal signs will be removed, with the remaining universal signs removed by the preceding methods.

As in this equation :

√q.10 +√q.5R – 2 = √q.3 +√q.3R + 1,

the most universal sign is equal to the most universal sign, therefore the sides may be squared, and the equation is made **10 +√q.5R –2 = 3 +√q.3R + 1**, or

7+√q.5R – 2 = √q.3R + 1, the universal sign of which you may delete by Prop. 12.

Another example.

Likewise : **√q.3 +√q.2R – 1 = √Cß.5 +√q.3R – 4**

the sides are multiplied into each other in the supersolid cube of the square, and the equation arises [note the positions of the underlying lines]:
18R + 18 +√q.8C –12q+6R – 1+√q.1458R – 729 = 21+3R +√q.300R – 400, or
15R – 3 +√q.8C –12q + 6R– 1 +√q.1458R – 729 =√q.300R – 400, the universal signs of which are unable to be deleted.

Third example.

Likewise **√C.3 +√q.2R – 1 =√C.20 – 4R**, multiply cubically into its sides,

and it becomes **3 + √q.2R – 1 =20 – 4R**, or **√q.2R – 1 = 17 – 4R**, the universal of which you will remove (by Prop. 9). The same is of a similar account.

17. With the same propositions by which it has been said universals are to be deleted, simple irrationals are able to be transposed between rationals, to be multiplied, and to be deleted finally.

As the equation may be **12 –√q1R = 1R**, it may be separated thus by Prop. 9, **12 –1R =√q1R**, and the sides may be multiplied by the square, and the equation becomes **1q – 24R + 144 = 1R**, or **1q – 25R + 144 = 0**, which evidently is rational. And thus what was said in Propositions 9, 10, 11, 12, 13, 14, and 15 about universals, also are said to be understood about simple roots.

18. Which equations can be prepared otherwise, and not prepared by the proposition presented ; for the multiplication of simple irrationals as well as by several exponents can be shown.

As the preceding example **12 –√♀1℞ = 1℞**, by the aforesaid multiplication, returns the equation **1♀ – 25℞ + 144 = 0**, which has two valid exponents, *viz*. 16 and 9, with the actual principal equation, **12 –√♀1℞ = 1℞**, having only a single exponent, *viz*. 9, as will become apparent later. Therefore that principal equation is not prepared by Prop. 17, just as the same shall be prepared better and more simply by the following Prop. 20, as may be said in that place.

19. If some true root of an equation to 0 may be extracted (*viz*. with no remainder), that root will be more succinct [*i.e.* with less solutions], and the equation equal to zero.

So that from the equation **1𝕮 – 6♀ + 12℞ – 8 = 0** extract the true cubic root, *viz*. **1℞ – 2 = 0**, which will be a short and succinct equation.

Likewise extract the square root of the equation **1℞ – √♀36℞ + 9 = 0**, and that will be the true root (by Chap. 8), *viz.* **√♀1℞ – 3 = 0**, which is a more succinct equation.

20. If some root shall be extracted from an equation equal to 0, it will be either formed or reformed (by Prop. 8, Chap. 8 of this book) ; with the sign of the remainders changed, and the square or cube roots, etc. such as may be extracted from the same remainder ; these roots (with the plus and minus signs changed) with the formed and approximate root with signs become equations, neither quadrinomials nor two quadrinomials [quadratics or biquadratics], but equated more succinctly to 0, and completing the roots of the first equation.

Et caetera.

[At this point the manuscript breaks off.]

LIBER TERTIUS.

DE LOGISTICA GEOMETRICA.

CAPUT I.

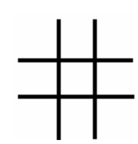
DE NOTATIONE ET NOMINATIONE CONCRETORUM.

PRACEDENTE Libro Arithmeticam docuimus, hic ordine Logistica Geometrica sequitur.
GEOMETRICA ergo dicitur Logistica quantitatum concretarum per numeros concretos. Concretus dicitur omnis numerus quatenus quantitatem concretam et continuam referat.
Ut *3a*, si tres lineas digitales referat sic —— —— —— est numerus discretus: Quum autem tridigitalem lineam concretam et continuam refert, hoc modo, ——+——+—— dicitur numerus concretus, sed hoc improprie et ratione subjecti.

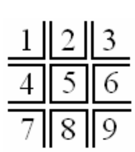
Proprie autem, et per se, concretos numeros dicimus radices numerorum quae nullo numero (sive integro sive fracto) mensurari possunt.

Ut radix bipartiens, seu quadrata, septenarii major est binario, minor ternario, et nulli fracto, in universa fractorum numerorum essentia, aequalis aut commensurabilis reperietur; dicitur ergo concretus numerus proprie. Sic radix tripartiens, seu cubics, denarii numeri, non est numerus discretus, nec numero commensurabilis, sed concretus; et aliae infinitae numerorum radices, quas vulgo surdos et irrationales vocant.

Horum concretorum ortus habetur extrahendo e numeris radices eis non insitas.
Ut cap. 4 Lib. I. et cap. 9 Lib. II. monuimus.
Unde ex diversitate radicum oriuntur diversae notationes et nominations concretorum.
Ut radicem bipartientem septenarii (quam vulgo radicem quadratam septenarii vocant) sic notamus, ⊔7, et pronuntiamus radicem bipartientem septem seu septenarii. Item, radicem cubicam 10, nos proferimus radicem tripartientem decem, et sic scribimus, └10. Item, radicem quadripartientem 11 ita notamus, ⊐11. Item, radicem quintupartientem numeri sic, □ ; sextupartientem sic, ⊏ .



Hanc characterum radicalium varietatem cum suis indicum numeris suppeditat nobis (memoriae gratia) hoc unicum schema, in suas partes distinctum.



Ut in exemplis praecedentibus, ⊔ └ ⊐ □ ⊏ praeposita numeris, suas radices bipartientem, tripartientem, quadrupartientem, quintupartientem, sextupartientem denotabant.
Sicut et;

┐ septupartientem
⊓ octupartientem
┌ noncupartientem ; itemque –
┘° decupartientem
┘‖ undecupartientem
_⊔ duodecupartientem
_⊥_ tredecupartientem
_┘⊐ vel ⊣ quadrudecupartientem
_□ quindecupartientem
_⊏ sedecupartientem
_┐‖ septemdecupartientem
_⊓ octodecupartientem
_┌ novemdecupartientem
⊔° vigecupartientem

⊔‖^{tem}
⊔⊔^{tem}
⊔_^{tem}
⊔⊐ vel ⊔┘^{tem}; et caetera: Itemque
└o 30^{m}
⊔° 40^{m}
□° 50^{m}
⊏o 60^{m}
┐° vel ┐o 70^{m}
⊓o 80^{m}
┌° 90^{m}
┘°° 100^{m}
Et ita in infinitum, more figurarum arithmeticarum.

Geometrici numeri eo quod quantitatem potius nominent quam numerent, ideo vulgo nomina dicuntur.

Nominum alia sunt unius nominis, ut uninomia; alia plurium.

Uninomium est idem quod concretus numerus unicus, sive proprie, sive improprie dictus.

Unde sequitur quod uninomium vel est numerus unicus simplex, vel unica numeri simplicis radix aliquae.

Ut 10 sunt numerus simplex, et a geometris usurpatur pro uninomio. Item ⊔10, L12, ⊐26, et similia sunt radices numerorum, et vere uninomia radicata sigillatim sumpta.

Cumque ita radicatum uninomium sit vel abundantis vel defectivi numeri radix, ejusque index vel par vel impar,–quadrifario hoc casu sequetur, quaedam uninomia esse abundantia, quaedam defectiva, quaedam et abundantia et defectiva, quae gemina dicimus; quaedam tandem nec sunt abundantia nec defectiva, quae nugacia vocamus.

Hujus arcani magni algebraici fundamentum superius Lib. I. cap. 6, jecimus: quod (quamvis a nemine quod sciam revelatum sit) quantum tamen emolumenti adferat huic arti, et caeteris mathematicis, postea patebit.

In uninomiis abundantibus et defectivis, non multum refert an debita copula praeponatur an interponatur; praestat tamen eam praeponere. In uninomiis autem geminis et nugacibus, copula debita est semper interponenda.

Primi casus exemplum est L10, seu (quod per cap. 6 Lib. I. idem est) L +10, est (per cap. 6 Lib. I.) uninomium abundans.
Secundi casus exemplum, L –10, est uninomium defectivum (cap. eodem). Tertii casus exemplum est ⊔10, seu ⊔+10 (quae, ut supra, eadem sunt), significat tam quantitatem

abundantem, quae in se ducta facit +10, quam defectivam quae in se ducta facit etiam +10: Veluti, lucidioris exempli gratia, ⊔9, seu ⊔ + 9, est tam + 3 quam –3; ut superius Lib. I. cap. 6 demonstravimus. Ultimi casus exemplum est ⊔ – 9, quod ex meris nugacibus est, nec quicquam significat quod vel abundet vel deficiat; nam novenarius defectivus nullam habet radicem bipartientem, ut Lib. I. cap. 6 patet.

In nugacibus summopere cavendum est ne copula – minutionis, interponenda, praeponatur.

Ut si, pro ⊔–9 (quae est radix bipartiens minuti novenarii, et absurdum atque impossibile infert), sumpseris –⊔9, quae quantitatem minutam radice bipartiente novenani significat, longe aberrabis: Radix enim bipartiens novenarii hic abundantis (scilicet ⊔9) gemina est, scilicet + 3 et –3, id est, ternarius abundans et ternarus defectivus; et ita quantitas his geminis + 3 et – 3, minuta gemina erit; qui itaque pro ⊔–9 ponit –⊔9, pro absurdo et impossibili, et quantitate nugaci et nihil significante, profert quantitatem geminae seu duplicis significationis; ab hoc ergo, in quo plurimi errarunt, cavendum.

In caeteris uninomiis .(significativis scilicet) idem est copulam inter signum radicale et numerum interponere, sive utrique praeponere: Nec in uninomiis illis valorem mutat, primo vel medio etiam loco vacuo (per cap. 6 Lib. I.) copulam + inserere.

Ut ⊔9, et ⊔+9, et +⊔9, et +⊔+9, idem prorsus significat, videlicet tam +3 quam –3; item ∟27, seu ∟ +27, seu + ∟ 27, seu + ∟ +27, idem valent quod +3 tantummodo; item ∟ –27, seu + ∟ –27, seu – ∟ 27, seu – ∟ +27, idem valent quod –3 tantummodo; item in nugantibus idem est ⊔–9, et +⊔–9, scilicet eandem impossibilitatem implicant; sed cave ne pro ipsis postueris –⊔9, seu –⊔+9, ut praecedente sectione monuunus.

Atque hae sunt affectiones uninomiorum in se; sequuntur uninomiorum ad invicem affectiones.

Sunt itaque uninomia bina, aut invicem commensurabilia aut incommensurabilia.

Commensurabilia sunt, quae se habent ad invicem ut numeri discreti, seu absoluti.

Unde, omnis numerus absolutus omni numero absoluto est commensurabilis. Itemque, uninomia bina consimiliter radicata, quorum alterius numerus simplex, numerum simplicem alterius partitus, reddit numerum tali radice praeditum qualem radicale signum indicat, dicuntur ad invicem commensurabilia in ratione quam indicat radix.

Ut 5 ad 7, quia sunt numeri absoluti, seu rationales, sunt commensurabiles; item, sint bina uninomia consimiliter radicata, ⊔8 et ⊔2, quorum numerus simplex 8, per numerum simplicem 2 partitus, reddit 4; habet autem quaternarius radicem signi ⊔, scilicet bipartientem, estque binarius; sunt ergo ⊔8 et ⊔2 commensurabiles invicem in ratione radicis, scilicet dupla.

Caetera omnis uninomia ad haec irreducibilia, incommensurabilia esse constat.

Ut ∟12 et ⊔3, quia sunt dissimiliter radicata, sunt incommensurabilia; item, ⊔6 et ⊔2 (quamvis sint consimiliter radicata) sunt incommensurabilia, quia 6 per 2 partita producuntur 8, quae carent radice signi ⊔, scilicet bipartiente; at 12 et ⊔4 sunt commensurabilia, quia reducta idem valent quod 12 et 2.

Et caetera.

ALGEBRA JOANNIS NAPERI
MERCHISTONII BARONIS.

LIBER PRIMUS.

DE NOMINATA ALGEBRAE PARTE.

CAPUT I.

DE DEFINITIONIBUS ET DIVISIONIBUS PARTIUM, ET DE VOCABULIS ARTIS.

1. ALGEBRA Scientia est de quaestionibus quanti, et quoti, solvendis tractans.
2. Estque ea duplex, -altera nominatorum, altera positivorum.
3. Nominata sunt, quae a numeris rationalibus, aut irrationalibus, nomen habent.
4. Rationales sunt numeri absoluti, aut numeri partes; de quibus tractat etiam Arithmetica.
5. Irrationales sunt radices numerorum rationalium non habentes radices inter numeros.
6. Atque hae (quod quantitates sunt) in Geometriam etiam spectant.
7. Positiva Algebrae pars est, quae quantitates et numeros latentes per suppositiones fictitias prodit; de qua Libro II. tractabimus.
8. Priorem autem Algebrae partem, de numeris et quantitatibus nominatis, hoc Libro I. docebimus.
9. Suntque nominatorum tres species: Uninomia, plurinomia, et universalia; de quibus ordine agetur.
10. Uninomia sunt, numerus unicus simplex, aut numeri simplicis radix aliqua.
11. Atque radices numerorum diversae sunt; diversis igitur characteribus praepositis, artis et doctrinae gratia, exprimuntur; dicunturque hi characteres signa radicalia.

Ut radix cubica senarii sic scribitur, √C6; item, radix quadrata quinarii sic √Q5 ; et sic de reliquis, ut sequitur:-

√Q	Radix quadrata
√C	Radix cubica.
√QQ	Radix quadrati quadrata.
√β	Radix supersolida.
√QC	Radix quadrati cubica.
√ſβ	Radix secunda supersolida.
√QQQ	Radix quadrati quadrati quadrata.
√CC	Radix cubi cubica.

Et sic de caeteris in infinitum.

12. Radicum et radicalium quaedam sunt simplices :
Ut √Q, √C, √β, √ſβ.
Quaedam multiplices :
Ut √QQ, √QC, √QQQ, √CC, etc.
13. Rursus, radicum et radicalium quaedam sunt quadratinomiae, quae nominantur a quadrato, sive solo, sive secus.

Ut radix quadrata, radix quadrati quadrata, radix quadrati cubica, radix quadrati quadrati quadrata, radix quadrati supersolida, etc.
Quaedam vero sunt exquadratae, in quorum nomine non fit quadrati mentio.
Ut radix cubica, radix supersolida, radix secunda supersolida, radix cubi cubica, radix cubi supersolida, etc.
14. Uninomia bina, eodem radicali signo affecta, consimiliter radicata dicuntur; et quid diverso, dissimiliter.
15. Omnes duo numeri rationales commensurabiles sunt; itemque, uninomia bina consimiliter radicata, quorum alterius numerus simplex, per alterius numerum simplicem divisus, reddit numerum tali radice praeditum qualem radicale demonstrat, dicuntur ad invicem commensurabilia.
Ut 12 ad 2, quia sunt rationales, erunt etiam commensurabiles; itemque sint bina uninomia consimiliter radicata, √Q**8** et √Q**2**, quorum 8 per 2 divisus reddit quaternarium praeditum radice quadrata signa √Q demonstrata, quae est 2; sunt ergo √Q**8** et √Q**2** commensurabilia.

Corollarium.

16. Hinc patet, caetera uninomia ad haec irreducibilia incommensurabilia esse.

Ut √Q6 et √Q2 sunt incommensurabiles, quia 6 divisa per 2 reddunt 3, quae carent radice quadrata; item, √C12 et √Q3, quia sunt dissimiliter radicata, sunt incommensurabilia; at 12 et √Q4 sunt commensurabiles, quia reducti idem valent quod 12 et 2.

CAPUT II.

DE UNINOMIORUM ADDITIONE.

1. Si uninomia bina proposita commensurabilia fuerint, divide majorem numerum absolutum per minorem; quotientis extrahe (per Arithmeticam) radicem qualem radicale demonstrat; huic radici adde unitatem; productum in se due toties quoties radicale demonstrat; inde hoc in numerum absolutum minoris uninomii duc, atque producto praepone pristinum suum radicale: fiet hoc uninomium aequale prioribus binis.
Ut sint uninomia commensurabilia addenda √Q**12** et √Q**3** ; divide 12 per 3, fiunt 4; ex 4 extrahe √Q, fiunt 2; his adde 1, fiunt 3; quae in se duc toties quoties signum Q monstrat, videlicet quadrate, fiunt 9; haec per 3 (numerum scilicet minoris uninomii) duc, fiunt 27; quibus praepone signum pristinum, fiunt √Q**27**, quae est aggregatum utriusque √Q**12** et √Q**3**. Item, √C**24** et √C**3** (per cap. 1 prop. 14) sunt commensurabiles, et (per hanc) additae faciunt. √C**81**. Item, √Q $\frac{2}{3}$ ad √Q $2\frac{2}{3}$ additae producunt √Q**6**.
2. Si uninomia proposita incommensurabilia fuerint, non aliter connectuntur quam interposito hoc signo + , quod augmenti dicitur copula.
Ut sint addendae √C**5** et √Q**3**, fiunt √C**5** + √Q**3** ; quae sic pronuncianda sunt, – radix cubica quinarii aucta radice quadrati ternarii. Item, √Q**6** et √Q**2** additae faciunt √Q**6 +** √Q**2**.

Corollarium.

8. Hinc patet, ex additione uninomiorum incommensurabilium oriri binomia et plurinomia abundantia,**–** sic dicta quod duobus aut pluribus uninomiis, copula **+** conjunctis, constent; de quibus suo loco.

CAPUT III.

DE UNINOMIORUM SUBSTRACTIONE.

1. Si uninomia proposita commensurabilia fuerint, divide numerum uninomii, ex quo substractio fit, per numerum uninomii substrahendi ; quotientis extrahe radicem qualem radicale denotat; ab hac substrahe unitatem; reliquum in se toties duc quoties indicat radicale; productum etiam in numerum uninomii substrahendi duc, atque huic praepone signum radicale pristinum, et fiet inde residuum seu reliquum substractionis priorum uninomiorum.

Exempla.

Sit substrahenda √𝔮**12** ex √𝔮**27**; divide 27 per 12 arithmetice, fient $\frac{9}{4}$, cujus √𝔮 est $\frac{3}{2}$; hinc aufer 1, fiet $\frac{1}{2}$; quod in se quadrate duc, fiet $\frac{1}{4}$ et hanc in 12 duc, fient 3 ; quibus praepone radicale suum, fietque √𝔮3, pro reliquo substractionis √𝔮12 ex √𝔮27. Simili modo, √𝔠24 subducta ex √𝔠81 relinquit √𝔠3 . Item, √𝔮 $\frac{2}{3}$ ex √𝔮6 subducta relinquit √𝔮 $2\frac{2}{3}$.

2. Si uninomia proposita incommensurabilia fuerint, ambo simul scribe postposito uninomio substrahendo, et interposito hoc signo – , quod copula minutionis nuncupatur.

Exempla.

Sit ex √𝔮3 substrahenda √𝔠5, remanent √𝔮3 – √𝔠5, quae sic pronunciatur, radix quadrata ternarii minuta radice cubica quinarii. Item, √𝔮2 ex √𝔮3 relinquunt √𝔮3 – √𝔮2.

Corollarium.

3. Hinc patet, ex substractione uninomiorum incommensurabilium oriri binomiorum et plurinomiorum residua defectiva, seu apotomes; ex uninomiorum enim plurium commixtione per copulam substractionis definitur apotomes.

CAPUT IV.

DE EXTRACTIONE RADICUM EX UNINOMIIS.

1. Si radix extrahenda simplex fuerit, atque in numero absoluto uninomii latitet, eam arithmetice extrahe retento pristino radicali.

Exempla.

Sit radix quadrata extrahenda ex hoc uninomio √c4 *;* quia in 4 latitat radix quadrata, eam extrahe retento pristino radicali, eritque √c2. Item, √c extracta ex √q $\frac{54}{16}$, seu √q $\frac{27}{8}$ erit √q $\frac{3}{2}$.

2. Si vero radix simplex extrahenda in numero absoluto uninomii non deprehendatur, tunc praepone numero absoluto et radicis extrahendae signum et pristinum signum.
Ut sit extrahenda √q ex hac √q**3**, ea erit √qq**3**; item, √c hujus √q**5**, fit **√qc5**.

3. Si autem radix multiplex ex uninomio extrahenda sit, primo multiplicis unam simplicem, inde aliam ex hac, et sic omnes sigillatim (per 1 et 2 hujus), extrahe.
Ut sit extrahenda **√qc** ex hac **√β27**, extrahe hinc (per 2) √q, ea erit **√qβ27**; deinde ex hac extrahe (per 1) vt, ea erit**√qβ3**. Item, √qq hujus √c**16** erit √c**2**. Item, √qq hujus √c**10** erit **√qqc10**.

4. In radicibus extrahendis ex fractionibus, idem est praeponere radicale lineae interpositae, ac si id utrique numeratori scilicet et denominatori praeponeres.
Ut sit ex $\frac{2}{3}$ extrahenda **√qβ**, ea erit $\frac{\sqrt{qc}\,2}{\sqrt{qc}\,3}$, seu √q$\frac{\sqrt{c}\,2}{\sqrt{c}\,3}$, seu optime **√qβ** $\frac{2}{3}$; sunt enim haec omnia penitus eadem.

CAPUT V.

DE REDUCTIONE AD IDEM RADICALE.

Si duo uninomia fuerint dissimiliter radicata, et utriusque numerum absolutum toties in se multiplicaveris, quoties dissimile radicale socii indicat; et utrique producto, per se posito, utrumque radicale praeposueris; ad idem radicale reducentur, salvo valore pristino.

Exemplum.

Sint dum dissimiliter radicatae, √c3 et √q2, reducendae ad idem radicale; multiplica ergo 3 in se quadrate, et 2 in se cubice, fient ex illis 9, ex his 8, quae praepositis utrisque radicalibus fient **√qc9** et **√qc8**, quae consimiliter radicatae sunt, retento etiam valore pristino; idem enim valet **√qc9** quod √c**3**, et idem valet **√qc8** quod √q**2**, ut per cap. 4 Lib. I. patet. Item, √qq**2** et **√qc5** sic reducuntur: multiplica 2 cubice in se, et 5 in se quadrate (in priore enim quadrato sunt similes), fient **√qqc8** et **√qqc25**. Item, √q**6** et 2 sunt √q**6** et √q**4**.

CAPUT VI.

DE MULTIPLICATIONE ET DIVISIONE UNINOMIORUM.

1. Omne uninomium, nulla notatum copula, notari copula augmenti subintelligitur.
Ut √q**10** pro +√q**10** habetur.

2. Eadem copula per eandem multiplicata, aut divisa, producit augmenti copulam; contrariae autem copulae invicem multiplicatae, aut divisae, producunt minutionis copulam.
Exempla sunt inferius.

3. Primo ergo uninomia proposits fiant consimiliter radicata, si non per se, saltem per reductionem; deinde multiplica aut divide numerum per numerum, producti radicem qualem indicat radicale extrahe, per cap. 40; ultimo (per 2 hujus) multiplica aut divide copulas, copulamque productam praefatae radici praepone, fietque inde multiplicationis aut divisionis productum.

Sint uninomia multiplicanda √♀**12** per √♀**3**; duc 12 in 3, fiunt 36, quorum radix quadrata est 6, quae sunt productum. Item, √ℭ**3** per **–** √ℭ**2** ducta facit **–** √ℭ**6** productum, per 2 hujus. Item, **–** √ℭ**3** per √♀**2** multiplicantur, prius facta reductione ad √♀ℭ**9** et √♀ℭ**8**, per cap. 5; inde 9 in 8 ducta faciunt 72, quorum radix ♀ℭ (per cap. 4 sect. 2) erit √♀ℭ**72**, cui praepone copulam –, per 2 hujus productam, fiet **–** √♀ℭ**72**.

Exempla divisionis.

Sit dividenda √♀**l2** per –√♀**3**, fiet, per praemissa, productus quotiens –2. Item, –√ℭ**16** per √ℭ**2** divisa fiet – 2. Item, √♀ℭ**72** per √♀ℭ**9** divisa facit √♀ℭ**8**, alias √♀**2**; ut ex capitis 4 sectione 1 patet.

Corollarium.

4. Hinc patet uninomium quadrate, cubice, aut ad aliquem ordinem multiplicari, cum numerus ejus absolutus in se quadrate, cubice, aut ad illum ordinem multiplicatu, retento pristino radicali.

Ut √♀ **2** cubice ducta fit √♀**8**. Item, √ℭ**3** quadrate ducta fit √ℭ**9**, etc.

Corollarium.

5. Hinc sequitur unum quodvis radicatum in se toties multiplicari, quoties suum radicale quadratinomium indicat, quum copula praecedens et radicale auferuntur; aut non quadratinomium, cum radicale tantum aufertur.

Ut –√ℭ5 in se cubice ducta facit –5. Item √♀6 in se quadrate ducta facit 3. Item, √♀ℭ6 in se quadrate ducta facit √ℭ6, cubice vero, √♀6. Simili modo fit in universalibus radicibus, et in positivis, de quibus postea.

CAPUT VII.

DE PLURINOMIIS.

1. PLURINOMIA sunt, quae pluribus uninomiis copulatis constant.
2. Plurinomiorum alia abundantia dicuntur (cap. 2 sect. 8 descripta), alia defectiva, vulgo residua seu apotomes, quae cap. 3 sect. 3 describuntur.
3. Plurinomia infima sunt ea, quorum uninomia omnia sunt quadratae numerorum radices, cum numero vel sine numero.

Ut √♀3 + √♀5 –√♀2 + 5, item √♀3 + √♀5–√♀2 dicuntur plurinomia infima.
4. Caetera omnia plurinomia dicuntur superiora.

5. Alia aliis magis aut minus plurinomia dicuntur, quanto plura aut pauciora fuerint uninomia.
Ut quadrinomium √q3 + √q5 –√q2 + 5 magis plurinomium hoc trinomio √q3 + √q5 –√q2, quod trinomium rursus magis plurinomium est quam binomium, ut √q6 –√q5.

6. Si dati plurinomii convertas quasdam copulas, non autem omnes, ex abundante facies suum defectivum seu apotomen; aut contra, ex defectivo suum abundans fiet.
Ut **√q5 + √q3 –√q2** sit trinomium abundans, ejus defectivum erit **√q5 – √q3 –√q2**, vel **√q5 – √q3 + √q2** vel **√q3 – √q5 +√q2**, vel **√q3 – √q5–√q2**. Item, sit idem trinomium √q5 + √q3 –√q2 defectivum, ejus abundans erit **√q5 + √q3 +√q2**. Quo exemplo patet, idem plurinomium posse et abundans et defectivum esse, diverso tamen respectu.

7. Si plurinomii dati fuerint bina uninomia commensurabilia ejusdem copulae, ea (per cap.2 s.1) adde, copulamque illam producto praepone, et fiet abbreviatio magis plurinomii in minus.
Sit trinomium hoc **√q12 + √q3 – 2**, cujus **√q12** et **√q3** commensurabiles, et ejusdem copulae, additae facient **√q27**, quae cum – 2 facient abbreviationem ad minus plurinomium, viz. ad **√q27 –2**, quod binomium est.

8. Si plurinomii dati fuerint bina uninomia commensurabilia diversarum copularum, minus a majore substrahe, (per cap. 3 s. 1) et producto praepone copulam majoris uninomii, et fiet abbreviatio magis plurinomii in minus.
Ut sit trinomium **√q10 + √q2 –√q8**, in quo **√q2** et **– √q8** sunt commensurabiles, et diversis copulis notantur; substracta ergo faciunt **√q2**, cui copulam majoris uninomii praepone, fiet **– √q2**, quae cum **√q10** faciunt abbreviationem ad binomium, viz. **√q10 – √q2**.

CAPUT VIII.

DE ADDITIONE PLURINOMIORUM.

1. PLURINOMIORUM addendorum omnia uninomia simul cum copulis suis in unicum plurinomium connecte; deinde si quae fuerint commensurabilia, ea (per 7 et 8 praecedentis) abbreviato, et inde producitur additionis summa.
Ut sint addenda **√q8 + √q8** ad **4 –√q2**, primo per hanc fient **√q3 + √q8 + 4 – √q2**; deinde, quia +**√q8** et–**√q2** sunt commensurabiles, ideo fiet abbreviatio (per cap. 7 sect. 8) ad **√q3 + √q2 + 4**. Item, sint addenda **√q5+√q3** ad **√q20 – √q12**, ea primo per hanc fient **√q5+√q3 + √q20 –√q12**; deinde, per dictam abbreviationem, erunt **√q45 –√q3**. Item, **√c16 + √q18**, ad **√c2 –√q2**, faciunt **√c54+√q8**. Item, **√c54+√q18 –1**, ad **√q2+√q3**, faciunt **√c54+√q32+√q3 –1**.

Corollarium.

2. Hinc patet, in additione abundantium ad sua defectiva, particulas abundantes et defectivas sese mutuo destruere, particulas vero reliquas duplari.

Ut abundans **12+√Q3** additum ad suam apotomen **12 –√Q3**, facit **12 + √Q3 + 12 –√Q3**, quod idem valet quod 24.

CAPUT IX.

DE SUBSTRACTIONE PLURINOMIORUM.

1. PLURINOMII substrahendi converte omnes copulas, deinde hoc conversum plurinomium addatur (per caput praecedens) ad plurinomium ex qua fieri debuit substractio, et producentur inde substractionis reliquiae.

Ut a binomio **√Q45 –√Q3** substrahendum sit **√Q5+√Q3,** cujus converte copulas, sic, **– √Q5 – √Q3**, hoc ad **√Q45 –√Q3** adde (per cap. praecedens), fiet , **√Q20 –√Q12.** Item, ex √C54 + √Q3 substrahendum sit √C2 –√Q2 remanebit , **√C16 + √Q18.** Item, ex **√C54 +√Q32 + √Q3 –1** sint substrahenda **√Q2 + √Q3,** remanebit **√C54 +√Q18 –1**.

Corollarium.

2. Hinc patet, in substractione defectivi a suo abundante, particulas abundantes seu defectivas duplari, caeteras vero se invicem destruere.

Ut ex abundante **√Q13 +7** sit substrahendum suum defectivum **√Q13 –7,** primo fiet **√Q13 + 7 –√Q13 +7**, inde fiet ex his 14.

CAPUT X.

DE MULTIPLICATIONE PLURINOMIORUM.

1. SINGULA multiplicandi uninomia duc in singula multiplicantis, per cap. 6 ; aggregatum autem (si quae habet commensurabilia) abbrevia, per sect. 7 et 8 cap. 7.

Ut **√Q3 –√C2 + 6** sit multiplicandum, **√Q5 –7** sit multiplicans;

√Q15 –√QC 500+√Q180 –√Q147+√C686 –42

erit productum, undique incommensurabile et ideo inabbreviabile.

Item, sit **√Q8+√Q3 –5** multiplicandum, **√Ql2 –√Q2** multiplicans; **√Q96+√Q36** (alias 6) – **√Q300 –√Q16**(alias – 4) **–√Q6 + √Q50,** quod productum (per sect. 7 et 8 cap. 7) abbreviatum facit **√Q54+2 –√Q300 + √Q50.**

2. In multiplicando abundante per suum defectivum, sufficit partem abundantem per partem defectivam, atque partem utrique communem in se multiplicare; reliquiae enim transversae multiplicationes se invicem destruunt.

Ut √Q7 + √Q5 abundans multiplicandum per **√Q7 –√Q5**, suum defectivum, fiet **7 –5** (alias 2) pro totali producto; etenim transversae multiplicationes, **√Q7** per **–√Q5,** et **√Q7** per **+√Q5**, sunt **–√Q35** et **+√Q35**, quia sa invicem destruunt, utque igitur inutiles.

Corollarium.

3. Si infimum plurinomium abundans in suum defectivum ducatur, producit minus plurinomium infimum.

Ut multiplicetur hoc trinomium infimum abundans , **√♀11 –√♀3 + √♀2,** per suorum defectivorum aliquod, viz. per **√♀1l –√♀3 – √♀2**, producetur inde binomium hoc **12 – √♀132**, quod etiam per suum abundans **12 + √♀132** multiplicatum facit uninomium imo numerum, viz. 12.

Corollarium.

4. Si infimum binomium abundans in suum defectivum ducatur, producitur numerus.

Ut, in jam dicto, si binomium abundans **12 + √♀132** duxeris (per 2 hujus) in suum defectivum **12 – √♀132**, producitur numerus, viz. 12.

Annotandum est, quod binomium irrationale per tale plurinomium multiplicari possit, ut inde numerus rationalis proveniat, hoc modo: Duo nomina cubica duc in sa et invicem, et fiet trinomium abundans, ex abundante binomio, aut defectivum ex defectivo; hoc trinomium, si abundans sit, per binomium defectivum ducatur, aut, si defectivum, per abundans, et proveniet numerus simplex. Alter: Per prop. 2 lib. viii. Euclid., quaere tres quantitates in eo ratione quam habent invicem nomina cubica; aut quatuor quantitates in eo ratione quam habent binomia biquadrata; aut quinque pro supersolidis; et deinde duc ut supra.

Exemplum.

Ex binomio abundante √𝕮6+√𝕮4 fac trinomium **√𝕮36+√𝕮24+√𝕮16**, quod per defectivum **√𝕮6 –√𝕮4** duc, fiunt 2. Item, ex defectivo **√𝕮6 –√𝕮4** fiat trinomium defectivum **√𝕮36 –√𝕮24 + √𝕮16,** quod per binomium abundans **√𝕮6 + √𝕮4** duc, fient 10.

Aliud exemplum.

Ex **√♀♀3 –√♀♀2**, fit quadrinomium **√♀♀27 –√♀♀18 + √♀♀12 –√♀♀8**, quod per **√♀♀3+√♀♀2** duc, fiet 1.

CAPUT XI.

DE DIVISIONE PLURINOMIORUM.

1. SI divisor fuerit uninomium, divide per illud singula dividendi uninomia, per cap. 6, et quotientis uninomia cum copulis productis connecte.

Ut sit dividendum **√q12+√q8** per **√q2**, fient **√q6+2.**
Item, sit dividendum **√q36300 + √q7200 –√q10800 + √q6600 + √q9900** per 12, fiet quotiens √q $\frac{3025}{12}$ +**√q50 –√q75** +√q $\frac{275}{6}$ + √q $\frac{275}{4}$.

2. Si plurinomium fuerit divisor, illudque infimum; ex hoc plurinomio fac (per sect. 3 et 4 capitis 10) numerum simplicem; perque eadem plurinomia per quae multiplicaveras divisorem multiplicabis etiam dividendum; productum per dictum numerum simplicem divide, et reddetur inde quotiens prioris divisoris et dividendi.

Sint 5 dividendae per trinomium hoc infimum **√ql1 –√q3 –√q2** quod si prius in **√q11 –√q3+√q2** duxeris, fiet inde **12 –√q132**; deinde hoc in **12+√q132** duxeris, fient (per 3 et 4 capitis 10) 12; deinde per idem trinomium **√q11-√q3+√q2** multiplica dividendum, viz. 5, fient **√q275 –√q75 + √q50**; hoc rursus multiplica per praefatum binomium **12+√q132,** et fit **√q36300+√q7200 –√q10800 + √q6600 + √q9900** pro novo dividendo, quo, per dicta 12 diviso, provenit quotiens
√q $\frac{3025}{12}$ + **√q50 –√q75 +** √q $\frac{275}{6}$ +√q $\frac{275}{4}$ congruens praefatis 5 divisis per
√q11 –√q3 –√q2.

*Haec sunt emendanda; nam per **6+√ℭ2** fieri potest divisio, ut per omne binomium, ex fine praecedentis capitis.*

3. Si divisor fuerit ex superioribus plurinomiis, vix unquam dividet integrum dividendum sine reliquiis; atque igitur inter superscriptum dividendum et subscriptum divisorem linea ducatur more fractionum arithmetices.

Ut sint **10 –√q3** dividenda per **6 + √ℭ2**, non aliter fiet quam interlineali divisione hoc situ $\frac{10-\sqrt{q}3}{6+\sqrt{\mathfrak{c}}2}$, quae sic pronuntiantur, **10 –√q3** divisa per **6+√ℭ2.**

Corollarium.

4. Hinc patet, ex divisione per superiora plurinomia oriri plurinomia irrationalia fracta.

CAPUT XII.

DE EXTRACTIONE RADICUM EX PLURINOMIIS.

1. PLURINOMIORUM quaedam radices perspicuae sunt, quaedam obscurae. Perspicuas dicimus, quae non sunt magis plurinomia quam ea quorum sunt radices.
2. Obscuras autem radices appellamus, quae plurimis uninomiis et radicibus plurinomiorum confuse plerumque scatent.
3. Si binomii infimi oblati fuerit extrahenda radix quadrata; ex differentia quadratorum utriusque uninomii radicem quadratam extrahe, quam ad majus uninomium adde et ab eodem substrahe, si scilicet commensurabilia sunt (alioquin enim erit radix quaesita obscura), et ab horum dimidiis educ radices quadratas (per cap. 4), has binas radices connecte copula qua prius binomium, et erit hoc binomium radix quadrata perspicua prioris binomii.

Exemplum.

Sit extrahenda radix quadrata hujus binomii defectivi **3–√q5** ; quadrata uninomiorum sunt 9 et 5, quorum differentia est 4, radix quadrata hujus differential est 2, commensurabiles ad 3; ea igitur 3 et 2 adde, fient 5; et etiam substrahe 2 ex 3, restat 1; ex dimidiis 5 et 1 educ radices quadratas, fient √q $\frac{5}{2}$ et √q $\frac{1}{2}$, quas connecte copula pristina, fientque √q $\frac{5}{2}$ –√q $\frac{1}{2}$ radix quadrata hujus binomii 3–√q5. Item, sit extrahenda radix quadrata ex **√q48 – 6**; radix quadrata differentiae quadratorum est **√q12**, quae addita et substracta ad et a **√q48**, facit **√q108**, et **√ql2**, quarum radices dimidiorum copulatae faciunt **√qq27 – √qq3** quaesitam. Item, **√q24+√q18** habet radicem quadratam perspicuam hanc √qq $\frac{27}{2}$ +√qq $\frac{3}{2}$.
4. Caeterorum omnium plurinomiorum radices qualescunque pro obscuris habentur.

Exemplum.

√q48+√q28 caret radice perspicua, quia √q differentiae quadratorum, quae est **√q20**, non est commensurabilis ad **√q48,** cum (per praemissam 3) deberet esse commensurabilis. Item, radix quadrata vel cubica hujus **√c33+1** obscura est; et sic de omnibus, praeter binomia infima jam dicta.

5. Radices autem obscurae non alter extrahuntur quam praeponendo signum radicale radicis cum periodo ante plurinomium oblatum, idque radicale, cum periodo sequente, universalis radicis signum dicitur; indicat enim universi plurinomii sequentis radicem. Ut, sit extrahenda radix quadrata hujus **√q48+√q28**; praepone huic binomio hoc radicale √q cum periodo hac, fietque inde **√q.√q48+√q28,** quae sic pronuntiantur, radix quadrata universalis radicis quadratae 48 auctae radice quadrata 28; significatur enim √q48 jungi cum radice quadrata 28 in unam summam; ejusque totalis summae radicem quadratam capiendam. Item, sit extrahenda radix cubica hujus **√c3 +√q2 – 1**, ea erit **√c.√c3 +√q2 – 1**.

Corollarium.

6. Hinc patet, ex radicum obscurarum extractione, radices universales oriri.

CAPUT XIII.

DE FRACTIONIBUS IRRATIONALIBUS.

1. QUAE in fractionibus rationalibus fieri praecipit Arithmetica, haec in irrationalibus fractionibus per Algebram perfice.

In fractionibus autem irrationalibus plurinomiis, operamur per Arithmeticam, quatenus sunt fractiones; et per Algebram, quatenus plurinomiae et irrationales sunt. Ut, sint $\frac{\surd q\,3+2}{\surd c\,3}$ dividenda per $\frac{\surd q\,5}{\surd c\,2-1}$ quod, in rationalibus per Arithmeticam, fieret per transversam multiplicationem utriusque numeratoris per utriusque denominatorem seorsum; haec ergo multiplicatio per Algebram fiat et producetur $\frac{\surd qc\,108+\surd c\,16-\surd q\,3-2}{\surd qc\,1125}$ quotiens optatae divisionis.

Item, sint addenda $\frac{\surd q\,3+2}{\surd c\,3}$ ad $\frac{\surd q\,5}{\surd c\,2-1}$ quae et multiplicari ex transverso, et recto denominatores, ut sint ejusdem denominationis, praecipit Arithmetica; algebraice ergo sic multiplicentur, et fient unius primo denominationis sic $\frac{\surd qc\,108+\surd c\,16-\surd q\,3-2}{\surd c\,6-\surd c\,3}$ pro uno, et $\frac{\surd qc\,1125}{\surd c\,6-\surd c\,3}$ pro altero; deinde per Algebram, jubente Arithmetica, adde numeratores retento communi illo denominatore, fietque $\frac{\surd qc\,1125+\surd qc\,108+\surd c\,16-\surd q\,3-2}{\surd c\,6-\surd c\,3}$ productum additionis.

CAPUT XIV.

DE UNIVERSALIUM RADICUM ADDITIONE ET SUBSTRACTIONE.

1. UNIVERSALES adduntur copula augmenti, et substrahuntur copula minutionis interpositis.
Sint addenda $\surd q\,10 + \surd q\,2$ ad $\surd q.\,8 - \surd q\,3$, interpone copulam + , fietque $\surd q.10 + \surd q\,2+\surd q.8-\surd q\,3$. Item, substrahatur $\surd q.8 - \surd q\,3$ **ex** $\surd q.10 + \surd q\,2$, interpone copulam **–** , fietque $\surd q.10 + \surd q\,2 - \surd q.8 - \surd q\,3$.

2. Si radix quadrata universalis, binomii infimi defectivi, ad radicem quadratam universalem sui abundantis addatur, aut a radice quadrata universali sui abundanti auferatur, per copulas **+** et **–** , productum (per cap. 16 sect. 5 sequens) erit abbreviandum.

Ut ex additione $\surd q\,10 - \surd q\,2$ ad $\surd q.10 + \surd q\,2$ producetur, $\surd q.10 + \surd q\,2 + \surd q\,10 - \surd q\,2$; quod per cap. 16 sect. 5 abbreviabile est.

CAPUT XV.

DE UNIVERSALIUM DIVERSORUM AD IDEM SIGNUM REDUCTIONE.

1. MULTIPLICA utrumque plurinomium universalis toties in se quoties dissimile universale socii indicat, et productum universali utriusque signabis.

Ut sint reducenda √qq**.2 – √c3,** et √qc.**7+√q2**; duc plurinomium **2 – √c3** in se cubice, et **7 + √q2** in se quadrate, fient **5+√c1944 – √c5184** et **51 + √q392,** quibus praepone √q commune, una cum q et c dissimilibus, fientque √qqc.
5+√c1944 – √c5184 et √qqc.**51 + √q392** reducta ad idem universale, viz. √qqc.

2. Eadem ratione, universalia cum particularibus ad idem radicale reducuntur.
Ut 3 et √q**.18 + √q243,** fiunt √q.**9** et √q**.18+√q243.**
Item, √q.**13+√q20** et **2+√q3**, fiunt √q.**13+√q20,** et √q**.7+√q48.**

Corollarium.

8. Hinc patet, uninomium signari universali quod particulari radicali idem est.
Ut √q**.9** et √q**9** eadem sunt. Item, √c**5** et √c**.*5***; haec dum puncto notantur universalia dicuntur.

CAPUT XVI.

DE MULTIPLICATIONE ET DIVISIONE UNIVERSALIUM.

1. Si universalis per universalem multiplicanda aut dividenda fuerit, primo fiant (per cap. 15) ejusdem signi universalis.
2. Deinde, deletis (saltem mente) signis universalibus, fiat more uninomiorum et plurinomiorum multiplicatio et divisio.
3. Ultimo, praepone producto, vel quotienti, signum universale pristinum, cum copula (per cap. 6 sect. 2) debita praecedente.

Sint ejusdem signi universalis √c**.5+√q2** et √c**.4 – √q3** ad invicem multiplicanda: Duc ergo **5+√q2** per **4 – √q3**, producentur **20 + √q32 – √q75 – √q6**, quibus praepone √c. vel **+** √c. fient √c**.20+√q32 – √q75 – √q6**. Item, √q**.4 + √c2** sint ducenda in 3 seu in √q**9**, seu (per cap.15 sect.3) in √q.**9**: Duc ergo **4+√c2** in **9**, fient **36+√c1458**, per cap.6 et cap.10. His praepone √c. vel +√c. fient √c**.36+√c1458**. Item, √c**.10+√q2** ductum in **–** √q**.10 – √q2** facit –√q.**98**, seu **–** √q**98**.

4. Si universalis quadrata, copula + praenotata, sit ducenda in eandem universalem, copula – praenotatam; praenotatam copulam et radicale universale dele, et reliquiarum copulas in contrarias converte, et orietur inde multiplicationis productum.
Ut **+√q.2 – √q3**, per **–√q.2 – √q3** ducta, facit **+√q3 – 2.**

5. Verum, si universale quadratum in se ducendum sit, deleto signa universali cum copula praenotata, orietur multiplicationis productum.
Ut **√q.10 + √q2** multiplicetur in se, producetur **10+√q2**.

6. Si autem plures per plures universales multiplicandae fuerint, aut dividendae, quod producitur ex unica per unicam, totum, habebit illud unicum signum universale praepositum.
Ut **√c.10+√q5+√q.8 – √q3** multiplicentur per **√q.3 + √q6 – √q. 4 – √q7**, hoc modo: Reduc **√c.10+√q5** cum **√q.3+√q6** ad idem universale, fiet illa, **√qc.105+√q2000**, haec autem, **√qc.81+√q6534**; has invicem duc, fientque (per hoc caput,)
√qc.8505 + √q13068000 + √q13122000 + √q72037350;
quae producuntur ex unica universali in unicam ducta. Habet ergo hoc totum productum commune illud universale signum **√qc** ei praepositum. Simili modo duc **3 + √q6** per **8 – √q3**, eique suum universale, viz. **+√q.** praepone, fiet
+√q.24 – √q27 + √q384 – √q 18, pro secunda parte producti. Tertio,
reduc **– √q.4 – √q7** cum **√c.10+√q5** ad idem universale, fient **–√qc.148 – √q21175**, et **√qc.105+√q2000** ; has invicem duc, fientque
– √qc.15540 –√q233454375 + √q43808000 – √q42350000, pro tertia parte producti. Quarto, duc **8 – √q3** per **4 – √q7**, et producto praepone **–√q.** fiet
√q. 32– √q48 – √q448 + √q21, pro quarta parte producti. Quarum quatuor partium quaelibet fit ex ductu unicae tantum universalis in unicam; quare quaelibet comprehenditur sub unico signo universali, fitque totum productum,

√qc.8505+√q13068000 + √q13122000 + √q72037350 + √q.24 – √q27 + √q384 – √q18 – √qc.15540 – √q 233454375 + √q 43808000 – √q42350000 – √q.32 – √q448+√q21.

7. Unde fit quod radice quadrata universali binomii infimi abundantis aucta aut minuta radice quadrata sui defectivi, producto in se ducto praeposueris √q., fiet inde ejusdem valoris plurinomium minus et abbreviatum.
Ut si **√q.l0+√q2+√q.l0 – √q2** in se duxeris, et √q. praeposueris, fiet **√q.20+√q392** aequale ad **√q.10 + √q2 + √q.10 – √q2** eoque brevius. Pari ratione ex **√q.10+√q2 – √q.10 – √q2**, fit **√q.20 – √q892**.

8. Exempla divisionis universalium sunt eadem plurinomia quae cap.11 praecedente scribuntur; si modo eorum divisoribus dividendis et quotientibus signum universale praeposueris.
Ut ex hac et cap.11 sect.1, **√c. √q12+√q8** per **√qc 2**, seu quod idem est per **√c. √q2** divisa, reddunt quotientem **√c.√q6 + 2**. Item, ex hac et cap. 11 sect. 2, **√q.5** divisa per **√q.√q11 – √q3 – √q2** reddit quotientem **√q.√q $\frac{3025}{12}$ +√q50 – √q75 + √q $\frac{275}{6}$ + √q $\frac{275}{4}$** .
Et sic de caeteris.

CAPUT XVII.

DE RADICUM UNIVERSALIUM EXTRACTIONE.

1. PLURINOMII oblati radicem sine respectu universalis signi per cap. 12. extrahe; et huic radici praepone pristinum suum signum universalitatis.

Ut sit extrahenda radix quadrata ex √ℭ.**3 – √Q5**, ea (per cap. 12 sect. 3) erit √Q $\frac{5}{2}$ **–** √Q $\frac{1}{2}$, cui per hanc praepone suum universale √ℭ. fietque radix quaesita √ℭ.√Q $\frac{5}{2}$ **–** √Q $\frac{1}{2}$. Item, radix cubica hujus √Q.√ℭ**3+√Q2** erit (per hanc et cap. 12, sect. 5) **√Qℭ.√ℭ3+√Q2**. Item, radix quadrati cubica hujus **√ß.7 – √Q48**, primo ejus radix quadrata erit **√ß.2 – √Q48** ; deinde hujus radix cubica erit **√ℭß.2—√Q3**, pro radice quaesita.

2. Si ex pluribus universalibus copulatis, aut ex universalibus copulatis cum uninomiis radicem extraxeris, ea dicetur universalium universale ; totique debet signum universale radicis extrahendae praeponi, lineaque per totum duci.

Ut sit extrahenda radix quadrata hujus **5+√ℭ2 – √Q.3 – √Q2**, ea extrahitur praeponendo signum universale radicis, una cum linea ducta hoc modo:

√Q.5 +√ℭ2 – √Q.3 – √Q2.

Corollarium.

3. Hinc sequitur in universalibus effectum universalis signi tantum extendi quantum linea protracta; et si nulla ducatur linea effectus universalis signi in sequens universale signum desinit ab eaque intercipitur.

Ut per

√Q.60 +√Q16 – √Q.6 – √Q4

significatur totius **60 +√Q16 – √Q.6 – √Q4** radicem quadratam capiendam, eaque est **√Q62** ; at si abesset linea, hoc modo, **√Q.60 +√Q16 – √Q.6 – √Q4**, tunc prioris **√Q.** effectus et vis per **60+√Q16** tantum extenditur, et posterioris **√Q.** vis per reliquum, viz. per **6 – √Q4** extenditur. Idem ergo valet

√Q.60 +√Q16 – √Q.6 – √Q4,

quod √Q62; atqui **√Q.60 +√Q16 – √Q.6 – √Q4** idem est quod 6:
Et similiter in similibus.
Haec de irrationalibus dicta sufficiunt, licet et aliae sint irrationalium species: Ut enim per extractionem radicum ex numeris non habentibus radices oriuntur uninomia (quae prima parte hujus docuimus), et ex additione et substractione uninomiorum incommensurabilium oriuntur plurinomia (de quibus secunda parte hujus tractavimus), et per extractionem radicum obscurarum ex plurinomiis oriuntur universalia (de quibus hac tertia et ultima parte hujus tractavimus). Sic etiam ex universalibus oriuntur universalium universalia, et ex his rursus alia ad infinitum universalissima: Quorum artem si aliquando in usum cadat, quod rarissime accidit, facillime ex praecedentibus colliges.

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DE POSITIVA SIVE COSSICA ALGEBRAE PARTE.

CAPUT I.

DE DEFINITIONIBUS ET DIVISIONIBUS PARTIUM, ET DE VOCABULIS ARTIS.

1. POSITIVAM Algebrae partem, per suppositiones fictas, veram quantitatem verumque numerum quaesitum patefacere diximus, Lib. I. cap. 1, sect. 7.

2. Positiones etiam, sive suppositiones, sunt notulae quaedam fictae unitate notatae, quas loco ac vice quantitatum ac numerorum ignotorum addimus, substrahimus, multiplicamus aut dividimus.

3. Positiones autem, et positionum notulae, tot sunt diversae et dissimiles quot diversos, dissimiles, ignotosque numeros aut quantitates complectitur quaestio.

Quarum, exempli gratia, figurae et nomina sunt 1℞, quae una prima positio dicitur, 1a, quae unum *a*, sive una secunda positio dicitur; 1*b*, unum *b*, sive una tertia positio; 1*c*, unum *c*, sive una quarta positio; et sic per alphabetum.

4. Hae positionum notulae (eo quod pro omnis rei numero et mensura incognito ponuntur) vulgari nomine RES dicuntur, suntque primae ordine.

5. Quadratum est productum ortum ex harum rerum aliqua in se ducta, estque secundum ordine.

Ut 1℞ in se ducta facit unum primum quadratum, quod sic scribitur 1♀. Item, 1***b***♀ in se ductum facit 1***b***♀, quod unum *b* quadratum dicitur. Item, l*a* per 1*a* ductum facit l*a*♀, quod unum *a* quadratum dicitur; et sic de caeteris.

6. Cubus est qui ex ductu rei cujusvis in suum quadratum oritur ; estque ordine tertius.
Ut 1℞ ducta in 1♀ facit unum cubum, qui sic scribitur 1𝕮.
Item, l*a* per l*a*♀ ductum facit l***a***𝕮, qui pronuntiatur sic, unus *a* cubus. Item, 1***b*** per 1***b***♀ ductum facit 1***b***𝕮, etc.

7. Quadrati quadratum est quod ex ductu rei cujusvis in suum cubum provenit; estque ordine quartum.
Ut 1℞ ducta in 1𝕮 producit unum quadrati quadratum, quod sic scribitur 1♀♀. Item, 1***a*** per l***a***𝕮 facit l***a***♀♀, quod sic pronuntiatur, unum *a* quadrati quadratum. Sic l*b*♀♀, l*c*♀♀, etc.

8. Supersolidus est qui ex ductu rei cujusvis in suum quadrati quadratum provenit; estque ordine quintus.
Ut 1℞ ducta in l♀♀ facit 1*ß*, scilicet unum supersolidum.

Item, l*a* per l*a*ꝙꝙ facit 1***a*ß**, quod pronuntiatur unus *a* supersolidus.
Sic de 1***b*ß**, et 1*c***ß**, etc.

Corollarium.

9. Hinc patet alios ex aliis oriri ordines in infinitum progredientes.

Ut 1**R** ducta per 1**ß** facit 1ꝙℭ, qui ordine sextus est. Item, 1**R** per 1ꝙℭ facit 1**ſß**, qui secundus supersolidus dicitur, estque ordine septimus. Caetera ex tabella subsequenti contemplari licebit, in qui supponimus exempli gratia 1**R** valere 3, l***a*** valere 2, et 1***b*** valere 4, quibus datis caeterorum ordinum valores necessario sequentur, ut inferius :

Numeri ordinum 0	Characteres et exempla ordinum primae positionis.	Characteres et ordinum exempla secundae positionis.	Characteres et ordinum exempla tertiae positionis.	&c.
1	1R 3	1***a*** 2	1***b*** 4	
2	1ꝙ 9	1***a***ꝙ 4	1***b***ꝙ 16	
3	1ℭ 27	1***a***ℭ 8	1***b***ℭ 64	
4	1ꝙꝙ 81	1***a***ꝙꝙ 16	1***b***ꝙꝙ 256	&c.
5	1ß 243	1*a*ß 32	1***b***ß 1024	
6	1ꝙℭ 729	1***a***ꝙℭ 64	1***b***ꝙℭ 4096	
7	1ſß 2187	1***a***ſß 128	1***b***ſß 16384	
8	1ꝙꝙꝙ 6561	1***a***ꝙꝙꝙ 256	1***b***ꝙꝙꝙ 65536	&c.
9	1ℭℭ 19683	1***a***ℭℭ 512	1***b***ℭℭ 262144	
10	1ꝙß 59049	1***a***ꝙß 1024	1***b***ꝙß 1048576	
11	1ſſß 177147	1***a***ſſß 2048	1***b***ſſß 4194304	
12	1ꝙꝙℭ 531441	1***a***ꝙꝙℭ 4096	1 ***b***ꝙꝙℭ 16777216	
13	1ſſſß 1594323	1***a***ſſſß 8192	1***b***ſſſß 67108864	
&c.	&c.	&c.	&c.	&c.

10. Positivi dicuntur numeri quicunque rationales, vel irrationales, signis positivorum ordinum notantur.
Ut **6R**, vel **5*a***, vel ***7b*ℭ**, vel √ꝙ**6*b***, vel √ℭ**7*a*ꝙ**, positivi numeri dicuntur. Interdum etiam nomen positivi pro numero quovis capitur.

11. Simplex dicitur quivis numerus positivus unicus solus, aut solitarie sumptus.
Ut ***6a*** est simplex. Item, √ꝙ**3ℭ**. Item, √ꝙl***ab***.

12. Compositus dicitur qui ex pluribus simplicibus qui signis pluris vel minoris copulantur constat.
Ut **6*a* + √ꝙ3ℭ**. Item, 5**R – 2√ꝙ**. Item, **√ꝙ30ℭ + 3*a* – 4R*b*.**

13. Purus dicitur simplex qui, post unicum uninomium habet unius tantum positionis signum conscriptum.
Ut **5aꝙ**. Item, **3ℭ**. Item, **√ꝙ2*c*ℭ**, etc., puri dicuntur.

14. Mistus dicitur simplex qui post unicum uninomium, habet diversarum positionum signa conscripts.

Ut **5♀*ac*, 2℞*ac*, √♀1*ab*, √ℭ1*a*♀*b*ß*c***, et similes infiniti, misti dicuntur; de quorum origine inferius cap 5, sect.2 et 3, tractabitur.

15. Simplices rationales sunt qui numeros rationales habent praepositos signis positionis et ordinis. Irrationales autem qui irrationales numeros praepositos habent.

Ut **6*a***, item **5♀*a*♀**, item **2℞**, rationales sunt; atqui **√♀6*a***, item **√ℭ5♀*b*ß**, item **√♀ℭ7*b*ß**, sunt irrationales.

16. Similiter radicati dicuntur simplices quorum signa radicalia aut nulla aut similia sunt; dissimiliter autem contra quorum radicalia dissimilia sunt.

Ut **2♀** et 3***a***, item **√♀3℞** et **√♀5ℭ**, item **√ℭ6** et **√ℭ2*ab***, sunt similiter radicati; atqui **√♀3ℭ** et **√ℭ3ℭ**, item **√ß1*a*** et **5℞*b***, etc. sunt dissimiliter radicati.

17. Ejusdem positionis sunt bini simplices notati characteribus omnimode ejusdem positionis, licet non ejusdem ordinis.

Ut 2**℞** et **√♀5ß**, item **3℞*a*♀** et **√ℭ2♀*a*ℭ**, etc. sunt ejusdem positionis; at **2℞** et √♀1*a*, item 3**℞***a*♀ et2**℞**, sunt diversarum positionum.

18. Ejusdem ordinis actu dicuntur simplices similiter radicati, quorum signa etiam ordinis eadem sint, licet non sint ejusdem positionis.

Ut **3*a*** et ***2b***, item **√♀2ℭ** et **√♀5ℭ**, item **2℞*a*** et **5℞*b***, item **√♀2℞*a*** et **√♀3*bc***, sunt ejusdem ordinis actu; at **2℞*a*♀** et **3*b***, item **√♀2℞*a*♀**, et **√♀3*b***, item √♀2**℞*a*** et **√♀3℞*a*♀**, sicut et alia ejus generis sunt diversorum ordinum. Quae vero sunt ejusdem ordinis potentia, et quomodo potentia in actum reducatur, inferius cap. 4 sect. 5 dicetur.

19. Commensurabiles sunt duo simplices ejusdem positionis et ejusdem ordinis actu, quorum uninomia (sepositis signis positionis et ordinis) fuerint commensurabilia.

Ut **8ℭ** et **2ℭ** sunt commensurabiles, quia 3 et 2 sunt commensurabiles, per cap. 1 sect. 15 Lib. I; item **√♀12℞** et **√♀3℞** sunt commensurabiles, quia **√♀12** et **√♀3** sunt commensurabiles, per cap. 1 sect. 14 Lib. I; sic **√♀2℞*a*♀** et **√♀3℞*a*♀**, et similes omnes.

CAPUT II.

DE ADDITIONE ET SUBSTRACTIONE POSITIVORUM.

1. Si positivi addendi vel substrahendi minuendique simplices fuerint atque commensurabiles, tunc uninomia utriusque adde, vel minus a majore substrahe et producto sive residuo postpone signa positiva pristina.

Ut sint addendae **3℞** ad **2℞**, fient **5℞**; item, **4♀** ad **3♀** fient **7♀**; item, **6*a*ℭ** ad **9*a*ℭ** fient **15*a*ℭ**; item, **√♀2ℭ** ad **√♀8ℭ** fient **√♀18ℭ**, quia **√♀2** addita ad **√♀8** facit **√♀18**, per cap. 2 sect. 1 Lib. I. Sic **√♀2℞*a*♀** ad **√♀8℞*a*♀** facit **√♀18℞*a*♀** ; item, $\frac{\sqrt{♀}2ℭ}{3}$ ad $\frac{\sqrt{♀}8ℭ}{3}$ facit

√Q242C/15, quia eorum uninomia, per cap. 2 sect. 1 Lib. I, et cap. 13 Lib. I, reducta ad eandem denominationem
faciunt , √Q50/15 et √Q72/15 et addita faciunt √Q242/15. Item, sint substrahendae 3R ex 5R, remanent 2R; item, **3*b*QQ** ex **8*b*QQ**, remanent **5*b*QQ**; item, √C**3ß** ex √C**192ß** relinquunt √C**81ß**, per hanc, et cap. 3 sect. 1 Lib. I. Sic √C**3Q*a*Q** ex √C**192Q*a*Q** relinquunt √C**81Q*a*Q** .

2. Si simplices incommensurabiles fuerint, interpone copulam + in additione, et copulam – in substractione.
Ut sint addendae **3R** ad **2Q** fient **2Q + 3R**; item, **4Q** ad **2*a*Q** fient **4Q + 2*a*Q**; item, √Q**5C** ad √Q**10C** fient √Q**10C** + √Q**5C**; item 5***a*Q*b*** ad **7*a*Q*b*** sunt **7*a*Q*b* + 5*a*Q*b***. Item, sint substrahenda **3Q**ex **2*a*Q**, remanent **2*a*Q–3Q**; item, **2*a*** ex **3*a*Q**, remanent **3*a*Q – 2*a***; item, √Q**3Q** ex √C**12Q**, remanent √C**12Q – **√Q**3Q** ; sic √Q**2*a*** ex √Q**2*ab*** fiunt √Q**2*ab* – **√Q**2*a*.**

Corollarium.

3. Hinc patet ex simplicium incommensurabilium additione et substractione compositos oriri.
Quod ex superioribus exemplis constat quorum producta compositi sunt.

4. Adduntur autem, substrahuntur, et abbreviantur compositi eisdem regulis quibus plurinomia cap. 8 et 9, et cap. 7 sect. 7 et 8, Lib.I. Et quae illic de plurinomiis et uninomiis dicuntur, hic de compositis et simplicibus subintelligantur.
Ut sint addenda √Q**2C*b*Q +3Q – 2R + 1** ad **5C+**√Q**8C*b*Q – 4Q + 3*a* – 6** : ea primo (per cap. 8 sect. 1 Lib. I.) copulato, et fient √Q**2C*b*Q + 3Q – 2R + 1 + 5C +**√Q**8C*b*Q – 4Q + 3*a* – 6**; deinde ea (per cap. 7 sect. 7 et 8 Lib. I.) abbreviata faciunt √Q**18C*b*Q – 1Q – 2R – 5 + 5C + 3*a***.

Exemplum substractionis.
Ex hoc novissimo producto √Q**18C*b*Q – 1Q – 2R – 5 + 5C + 3*a***
substrahantur haec √Q**2C*b*Q + 3Q – 2R + 1**: primo (per cap. 9 sect. 1) converte copulas et simplices copulato, fientque
√Q**18C*b*Q – 1Q – 2R – 5 + 5C + 3*a* – **√Q**2C*b*Q – 3Q +2R + 1**; ultimo, haec abbreviato, et fient √Q**8C*b*Q – 4Q – 6+ 5C+ 3*a***, ut superius.

CAPUT III.

DE RADICUM EX SIMPLICIBUS EXTRACTIONE.

1. OMNE signum purum tales et tot habet in se radices insitas, quales et quot sint ejus signi characteres, et praeter eas nullas.

Ut **0♀ℭ** habet in se radicem quadratam, item cubicam, item denique quadrati cubicam, et nullam praeterea aliam.

2. Omne signum mixtum tales et tot habet radices insitas, quales et quot fuerint in singulis diversis suis positionibus characteres communes repetitae, et nullas praeterea alias.

Ut **0♀ℭ*ßa*♀♀ℭ** habet insitas radices quadratam, quia tam in prima quam secunda positione ejus reperitur signum ♀, item cubicam eadem ratione, item denique quadrati cubicam; et praeterea nullam, veluti nec supersolidam, quia *ß* non reperitur inter signa secundae positionis ejus exempli, nec quadrati quadratam, quia ♀♀ non reperitur inter signa prioris positionis ejusdem exempli.

3. Ex signo puro radicem insitam extrahere, est numerum ordinis signi puri per numerum ordinis qualitatis radicis dividere, et quotientis signum ordinis notare.

Ut sit extrahenda radix cubica ex **0♀ℭ**, numerus ordinis ♀ℭ est 6, quae divisa per numerum ordinis cubici, viz., per 3, fit quotiens 2, cujus signum ordinis est ♀; fit ergo **0♀** radix cubica hujus **0♀ℭ**; sic ejusdem **0♀ℭ** radix quadrata est **0ℭ**; item ejusdem radix quadrati cubica est **0R**.

4. Ex signo mixto radix aliqua insita extrahitur quum (per praemissam) ex suis singulis diversis positionibus radix talis extrahitur.

Ut sit extrahenda radix cubica ex **0♀ℭ*ßa*♀♀ℭ**: primo, (per praecedentem) ex **0♀ℭ*ß*** extrahatur radix cubica, eaque erit **0♀*ß***; deinde extrahatur (per eandem) radix cubica ex ***a*♀♀ℭ**, eaque erit ***a*♀♀**; unde et tota radix cubica hujus **0♀ℭ*ßa*♀♀ℭ** erit **0♀*ßa*♀♀**. Sic ejusdem exempli radix quadrata erit haec, **0ℭ*ßa*♀ℭ**; item ejusdem radix quadrati cubica erit **0*ßa*♀**.

5. Si simplicis totius radix quaevis extrahenda sit, ejusque simplicis non solum absolutus numerus complectatur talem radicem, sed et signum positivum ejus talem habeat (per sect. 1 et 2, hujus) sibi insitam; tunc extrahe radicem illam ex numero, eique ascribe radicem illam signi retento priori (si quod fuit) radicali.

Ut sit extrahenda radix cubica ex **64♀ℭ**: primo, radix cubica numeri absoluti erit 4, deinde radix cubica signi ♀ℭ erit ♀, per 1 hujus; tota ergo radix cubica horum **64♀ℭ** erit **4♀** : item eorundem **64♀ℭ** radix quadrata erit 8**ℭ**: item eorundem radix quadrati cubica erit 2**R**: simili modo et radix quadrata hujus √ℭ**9♀*a*♀*ß***
est √ℭ**3R***aß*.

Corollarium.

6. Hinc fit quod simplices habentes et in numero et in signis positivis talem radicem insitam, qualem suum vel totale vel particulare radicale indicat, abbreviantur delendo radicale illud, et extrahendo (per praecedentem) radicem illam ex reliquo.

Ut, sit ille simplex √♀𝕮**4**♀*ß*, qui sic abbreviatur; dele particulare radicale ♀, remanet √𝕮4♀*ß*, cujus (per praemissam) extrahe radicem talem, viz., quadratam, eaque erit √𝕮**2***ß* pro abbreviationis producto, idem valente quod √♀𝕮**4**♀*ß*. Item, √♀𝕮**64**♀𝕮 sic abbreviatur; dele totale radicale √♀𝕮, remanet **64**♀𝕮, cujus (per praemissam) extrahe talem radicem, viz., quadrati cubicam, eaque erit 2**R**, quae idem valet ac √♀𝕮**64**♀𝕮

7. Si simplicis (cujus radix aliqua sit extrahenda) et numero absoluto et signo positivo talis radix. non fuerit insita, tunc toti simplici praepone signum radicale radicem illam denotans.

Ut sit extrahenda radix quadrata hujus **4**𝕮, ea sit √♀**4**𝕮 ; item, radix cubica 4𝕮 sit √𝕮**4**𝕮*;* item, radix. cubica hujus √♀**3R** erit √♀𝕮**3R**; item, radix quadrata horum **4**♀***a*** erit √♀**4**♀***a***.

CAPUT IV.

DE SIMPLICIUM IN SE MULTIPLICATIONE, ET DE REDUCTIONE.

1. MULTIPLICARE signum purum in se quadrate vel cubice, vel ad alium ordinem, est utriusque ordinis numeros invicem ducere, et producti signum ordinis notare.

Ut sit **0**♀ (cujus numerus ordinis est 2) multiplicandum in se cubice (cujus cubi numerus ordinis est 3 :) duc ergo 2 in 3, producuntur 6, quorum signum ordinis est ♀𝕮, est ergo **0**♀𝕮 cubus hujus **0**♀; item, **0**♀ supersolide in se ductum facit **0**♀*ß*; item, **0R** in se quadrati-cubice ducta facit **0**♀𝕮.

2. Multiplicare signum mixtum in se ad aliquem ordinem, est signa singularum positionum in se ad illum ordinem (per praemissam) ducere.

Ut sit **0***ß**a*♀ in se quadrati-cubice ducenda: primo (per praecedentem 1) duc **1***ß* in se quadrati-cubice, fiet **0**♀𝕮*ß*, deinde duc ***a***♀ in se quadrati-cubice, fiet ***a***♀♀𝕮, et per consequens totum **0**♀𝕮*ß**a*♀♀𝕮 erit quadrati cubus hujus **0***ß**a*♀.

3. Si ergo totum simplicem in se multiplicare volueris quadrate, vel cubice, vel ad alium ordinem, primo tam ejus numerum absolutum arithmetice, quam ejus signum (per 1 et 2 hujus) in se ad illum ordinem duc, deinde producti radicem talem extrahe (per cap. 3 sect. 5 et 7) qualem suum radicale (si quod sit) denotat.

Ut sint **3**𝕮 in se quadrate multiplicandi: duc ergo 3 in se quadrate, per Arithmeticam, et 𝕮 in se quadrate (per 1 hujus), fient **9**♀𝕮 pro vero quadrato trium cuborum. Item, **2**♀♀ in se cubice ducta faciunt **8**♀♀𝕮. Item, √𝕮**3R** in se quadrate ducta facit √𝕮**9**♀. Item, sit

√ꝗ2℞*a*ꝗ*c* in se quadrate ducenda; primo quadrentur 2 et **℞*a*ꝗ*c***, fientque (per 2 hujus) **4ꝗ*a*ꝗꝗ*c*ꝗ**, cujus extrahatur radix quam radicale indicat, viz., quadrata, ea erit (per cap. 3 sect. 5) 2℞*a*ꝗ*c* pro vero quadrato hujus √ꝗ2℞*a*ꝗ*c*, quae quidem √ꝗ2℞*a*ꝗ*c* etiam facilius (per cap. 6 sect. 6 Lib. I.) deleto radicali quadratur.

4. Si simplices dissimiliter radicati, ad similia radicalia sint reducendi, uniuscujusque partem rationalem toties (per praecedentem) in se duc quoties caeterorum dissimilia omnia radicalia indicant, et unicuique producto per se posito, cuncta dissimilia radicalia, una cum communi et simili (si quod sit) radicali, praepone.

Ut √𝕮**3℞** et √ꝗ**2℞** sic reducuntur: duc **3℞** in se quadrate, et **2℞** in se cubice, fientque **9ꝗ** et **8𝕮**; quibus utrumque radicale dissimile praepone, fientque √ꝗ𝕮**9ꝗ** et √ꝗ𝕮**8𝕮**, ejusdem nempe radicalis, valoris autem ejusdem cujus √𝕮**3℞** et √ꝗ**2℞** : Item, √ꝗ**6ꝗ** et **2℞** sic reducuntur; duc **2℞** in se quadrate, quia in priore est hoc radicale √ꝗ, at **6ꝗ** non multiplicantur, quia 2℞ carent radicali; fiunt ergo **6ꝗ** et **4ꝗ**, quibus illud unicum radicale praepone, fientque √ꝗ**6ꝗ** et √ꝗ**4ꝗ**, ejusdem radicalis et praeterea ejusdem ordinis actu: Item, √ꝗꝗ**2*b*ꝗ** et √ꝗ𝕮**3*ac*** sic reducuntur; duc **2ꝗ*b*** in se cubice, et **3*ac*** in se quadrate, (quia eorum radicalia differunt in cubo et altero quadratorum, et in altero quadratorum conveniunt), fiunt **8ꝗ𝕮*b*𝕮** et **9*a*ꝗ*c*ꝗ**, quibus praepone illud commune radicale √ꝗ, una cum. dissimilibus ꝗ et 𝕮, fient √ꝗꝗ𝕮**8ꝗ𝕮*b*𝕮** et √ꝗꝗ𝕮**9*a*ꝗ*c*ꝗ**, ejusdem radicalis, et valoris pristini.

Exemplum plurium duobus reducendorum.

Sint haec tria √ꝗ**2ꝗ**, √𝕮**3℞**, et √ꝗꝗ**l*a*** reducenda ad idem radicale: duc primo **2ꝗ** in se quadrati – cubice, non autem quadrati quadrati cubice (quia in altero quadratorum convenit primum cum ultimo, non autem in reliquo quadrato), fiet ergo primum, cum. suis radicalibus dictis, √ꝗꝗ𝕮**64ꝗꝗ𝕮**; secundum autem simili ratione ductum quadrati-quadrate, faciet suum productum, cum radicalibus debitis, √ꝗꝗ𝕮**81ꝗꝗ**; tertium denique ductum cubice tantum et adhibitis suis radicalibus fiet √ꝗꝗ𝕮1***a*𝕮**; quae quidem tria sunt ejusdem jam radicalis; et sic de reliquis.

Corollarium.

5. Hinc sequitur quod quidam simplices dissimiliter radicati ejusdem ordinis potentia dicti, per reductionem fiunt ejusdem ordinis actu; quidam autem non, videlicet quorum alterum. majoris, alterum minoris ordinis fiunt.

Ut in superioribus exemplis √ꝗ**6ꝗ**, et **2℞** sunt ejusdem ordinis potentia, quia reducti constituunt √ꝗ**6ꝗ** et √ꝗ**4ꝗ**, qui (per cap. 1 sect. 18 Lib. II.) sunt ejusdem ordinis actu ; at √𝕮3℞ et √ꝗ**2℞** reducti constituunt √ꝗ𝕮**9ꝗ** et √ꝗ𝕮**8𝕮**, quorum alterum, viz., √ꝗ𝕮**8𝕮** est altioris seu majoris ordinis, alterum vero √ꝗ𝕮**9ꝗ** est minoris.

CAPUT V.

DE POSITIVORUM MULTIPLICATIONE GENERALI.

1. MULTIPLICARE signa ejusdem positionis invicem est numeros ordinum eorum signorum addere, et signum ordinis producti numeri notare.

Ut sit multiplicandum **0*a*♀** per **0*a*ℭ**, numeri ordinum ***a*♀** et **aℭ** sunt 2 et 3, quae addita faciunt 5, quorum signum ordinis est ***a*ß** est ergo **0*a*ß** productum multiplicationis **0*a*♀** in **0*a*ℭ**. Sic **0*b*** ductum per **0*b*ℭ**, facit **0*b*♀♀**. Item **0♀** per **0ß** fiet **0ßß**.

2. Multiplicare signa pura diversarum positionum invicem, est ipsa signa simul connexa scribere, praeposito semper signo primae positionis (si quod sit) ante reliqua.

Ut sit multiplicandum **0*a*♀** per **0*b*ℭ**, producetur **0*a*♀*b*ℭ**, cujus pronuntiatio, quum numerum habet ut **6*a*♀*b*ℭ**, est haec, **6*a*♀** ducta per **1*b*ℭ**. Item, **0*a*♀** per **0ℭ** non producit **0*a*♀ℭ** sed **0ℭ*a*♀**, praeposito signo primae positionis; quod quidem **0ℭ*a*♀** sic pronuntiatur, tot seu nulli cubi primae positionis ducti in unum quadratum secundae. At contra, **0*a*♀ℭ** ex toto est secundae tantum positionis (ut ex Tabula cap.1 hujus patet), et sic pronuntiatur, unum *a* quadrati cubicum.

3. Hinc sequitur, ex signis puris diversarum positionum invicem ductis produci mixta.

Ut in exemplo superiore, purum **0*a*♀** per purum **0*b*ℭ** ductum producit **0*a*♀ *b*ℭ**, per praecedentem; quod (per cap. 1 sect. 14) est mixtum.

4. Mixta etiam, quatenus communicant invicem positiones, per sectionem primam hujus multiplicantur; quatenus autem sunt diversarum positionum, multiplicantur per secundam hujus.

Ut **0℞*a*♀** per **0ℭ*a*** ductum facit **0♀♀*a*ℭ** ; ductis nempe (per 1 hujus) similibus positionibus invicem, viz. **0℞** per **0ℭ**, et ***a*♀** per ***a***, fient **0♀♀** et **0ℭ**; et rursus **♀♀** ductum per ***a*ℭ** fit (per 2 hujus) **0♀♀*a*ℭ**. Item, **0℞*a*♀** per **0ℭ*b*** ductum facit **0♀♀*a*ℭ*b*** ; quia **0℞** per **0ℭ** facit **0♀♀**, et **0♀♀** per ***a*♀** facit (per 2 hujus) **0♀♀*a*♀**, et **0♀♀*a*♀** per **0*b*** facit (per eandem 2) **0♀♀*a*♀*b***.

5. Si simplices invicem ducendi sint: primo, fiant consimiliter radicati, saltem per reductionem (cap. 4 sect. 4); deinde, duc utriusque numeros absolutos invicem; tertio, duc ad invicem signa positiva per jam dicta; quarto, numeri signique producti radicem talem extrahe qualem indicat suum radicale, (per cap. 3) et huic tandem praepone debitam copulam, (per cap. 6 Lib. I.)

Ut **2*a*♀** per **5*a*ℭ** ducta faciunt **10*a*ß**, quia 2 per 5 ducta faciunt 10, et ***a*♀** per ***a*ℭ** facit ***a*ß**. Sic **2*a*♀** per **5ℭ**ducta faciunt **10ℭ*a*♀**. Item, **√♀ 2℞*a*♀** per **√♀ 8℞** ducta facit **4℞*a*** productum; quia ductis numeris invicem, et signis invicem, fiunt **16♀*a*♀**, quorum radix quam indicat radicale, viz. radix quadrata, est **4℞*a***. Item, sint ducenda **2℞*a*** per **– √♀3*ab***; primo per reductionem fiant **√♀ 4♀*a*♀,** et **– √♀3*ab***; deinde ductis numeris invicem, et signis positivis invicem, fiunt **12♀*a*ℭ*b***, quorum radix quadrata cum debita copula est

– √♀12♀*a*ℭ*b*, pro multiplicationis producto.

6. Si compositi ad invicem ducendi sint; singulos multiplicandi simplices duc in singulos multiplicantis; productum autem per commensurabilium additionem et substractionem abbrevia, modo, quo plurinomia ducuntur et abbreviantur, per cap. 7, sect. 7 et 8; et cap. 10, sect. 1 Lib. I.

Ut sint multiplicanda **2*a*♀ +√♀3R – 2R*a* – 4** per **√♀12ℭ + 2♀**, duc simplices singulos in singulos, fientque **√♀48ℭ*a*♀♀ +6♀ – √♀48ß *a*♀ – √♀192ℭ + 4♀*a*♀ + √♀12ß – 4ℭ*a* – 8♀**, quae inde per abbreviationem fiunt :
√♀48ℭ*a*♀ – 2♀ – √♀48ß *a*♀ – √♀192ℭ + 4♀*a*♀ + √♀12ß – 4ℭ*a* .

CAPUT VI.

DE SITU ET COLLOCATIONE SIMPLICIUM COMPOSITI.

1. INTERVALLUM ordinum est differentia inter ordines simplicium ejusdem radicalis, qua numerus majoris ordinis exsuperat numerum minoris ordinis proxime sequentis.

Ut in hoc composito **√ℭ3R – √ℭ2♀**, intervallum est 1; quia numerus ordinis **R** est 1, et numerus ordinis **♀** est 2, quorum differentia est 1. Item in hoc similiter radicato **√♀3ℭ + √♀2R**, intervallum inter **√♀3ℭ** et **√♀2R** est 2, quia differentia inter numeros ordinum eorum est 2. Sic intervallum inter **√♀2R** et **√♀5** est 1, quia enim numerus ordinis **R** est 1, et numerus ordinis numeri simplicis est 0; eorum ergo differentia seu intervallum erit 1.

2. Simplex fictus alicujus ordinis est nihil, seu 0, ornatum signis radicalibus et positivis illius ordinis.

Ut simplex fictus ordinis cubici est **0ℭ**. Item simplex fictus ordinis quadrati est **0♀**. Item simplex fictus pro radice quadrati aliquorum cuborum est **√♀0ℭ**. Item simplex fictus ponendus pro radice cubica supersolidorum est **√ℭ0ß**.

3. Intervalla redduntur eadem, cum (per 1, 2, et 3 Lib. VII. Euclidis,) maxima communis mensura dividens illa capitur, atque intervallo hujus mensurae a maximo ordine ad minimum progrediuntur simplices veri, aut (ubi veri desunt) ficti.

Ut in hoc superiore exemplo, **√♀3ℭ + √♀2R – √♀5** sunt (per 1 hujus,) duo diversa intervalla, viz. 2 et 1, quorum 2 et 1 maxima communis mensura, ex Euclide Lib. VII. prop. 1, est unitas: hac ergo unitate tanquam intervallo a maximo ordine, viz. a **√♀3ℭ** progredere ad minimum, viz. **√♀5**: hoc modo, substrahe a numero ordinis hujus **√♀3ℭ** unitatem, fit **√♀0♀** fictus simplex, quia verus deest. Item ex numero ordinis hujus **√♀0♀** aufer 1, fit ordo hujus **√♀2R** ; ex cujus denique numero ordinis aufer 1, fit ordo minimus, viz. hujus **√♀5**. Sic ergo collocabis simplices praefati exempli **√♀3ℭ + √♀0♀ + √♀2R – √♀5**, et omnia intervalla erunt eadem. Item in hoc **1ℭ – 3R– 6** maxima etiam mensura communis est 1, per quam unitatem fit haec progressio **1ℭ + 0♀ – 3R– 6**, et erunt intervalla eadem. Item in hoc **√ℭ1♀ℭ – √ℭ3R + √ℭ8** erunt intervalla diversa, viz. 5 et 1, quorum unitas adhuc erit communis mensura. Sic ergo (facta per substractionem unitatis progressione,) collocabis

ordines
√ℭ1♀ℭ + √ℭ0ß + √ℭ0♀♀ + √ℭ0ℭ + √ℭ0♀ – √ℭ3R + √ℭ8.
Item in hoc **√ℭ2ʃʃß + √ℭ3ß + √ℭ10R** erunt intervalla 6 et 4, et communis mensura 6 et 4 est binarius; facto igitur progressu per binarium, hic erit situs, **√ℭ2ʃʃß + √ℭ0ℭℭ + √ℭ0ʃß + √ℭ3ß + √ℭ0√ℭ – √ℭ10R**, quorum intervalla eadem sunt, viz. binario constant.

4. Ut ergo simplices compositorum recte collocentur; primo, fiant omnes consimiliter radicati, per cap. 4, sect. 4; deinde, quae ejusdem sunt ordinis simul ponuntur, et qui majores sunt ordines minoribus anteponuntur; tertio, omnia intervalla fiant (per praecedentem) eadem : ultimo, simplices abbreviabiles (si libet,) abbreviare poteris per hujus cap. 8. sect. 6.

Ut **2– √ℭ3R + 1♀ + √ℭ2♀ℭ** facta ejusdem radicalis, fient **√ℭ8– √ℭ3R + √ℭ1♀ℭ + √ℭ2♀ℭ** ; deinde, praepositis majoribus ordinibus, et simul positis eis qui ejusdem sunt ordinis, fient **√ℭ2♀ℭ + √ℭ1♀ℭ – √ℭ3R + √ℭ8**; tertio, factis intervallis eisdem fient **√ℭ2♀ℭ + √ℭ1♀ℭ + √ℭ0ß + √ℭ0♀♀ + √ℭ0ℭ + √ℭ0♀ – √ℭ3R + √ℭ8**; ultimo, haec (si libet) abbreviabis, fientque **√ℭ2♀ℭ + 1♀ + √ℭ0ß + √ℭ0♀♀ + 0R + √ℭ0♀ – √ℭ3R + 2**. Item eisdem rationibus **1♀ + √♀2R – 3** recte sic collocantur, **1♀ + √♀0ℭ + 0R + √♀2R – 3**: et sic de caeteris.

5. Hinc fit quod compositorum simplices recte, per praecedentem, collocati, ordinem servant proportionalem; singuli scilicet intermedii simplicis quadratum ejusdem erit ordinis (potentia. saltem,) cujus fuerit productum quod fit ex proxima praecedente in proxime subsequentem ducto.

Ut in hoc ultimo exemplo, praecedente

i ii iii iiii v
1♀ + √♀0ℭ + 0R + √♀2R – 3, quadratum secundi est **0ℭ**, et duc **1♀** in **0R**, fit etiam **0ℭ**. Item quadratum tertii erit 0♀, et duc **√♀0ℭ** in **√♀2R**, fit etiam **0♀**, seu (quod idem est) **√♀0♀♀**. Item quadratum quarti erit **2R**, et duc **0R** in **3**, fit **0R**; atque **2R** et **0R** sunt ejusdem ordinis: unde in similibus ordines semper proportionales dicuntur.

6. Hinc etiam sequitur (quum minores ordines posterius collocentur) numerum simplicem et absolutum (quia est nullius positivi ordinis) ultimo omnium loco collocandum.

7. In compositis mixtis vel plurium positionum, quot fuerint positiones diversae, tot aggregati positionum characteres ponuntur pro rebus, seu rerum ordine, ac tot etiam rerum characteres in se, et invicem ducti, et aggregati, faciunt ordinem integrum quadratorum, ac singuli illi characteres rerum in singulos hos quadratorum ducti, et aggregati, faciunt integrum ordinem cuborum, et sic deinceps in infinitum.

Ut in **1♀ – 1R + 1*a*♀ + 1*a* – 18**, duae sunt positiones diversae, viz. **R** et *a;* igitur – 1**R** + 1*a* pro ordine uno, viz. rerum ponitur. Item duc (per cap. 5,) **– R + *a*** in se, fient **+♀ – 0R*a* + ♀,** atque igitur **1♀ – 0R*a* + 1*a*♀,** licet sint tres simplices, efficiunt unicum tantum ordinem, quadratorum nempe. Est ergo illius compositi hic situs rectus 1♀ **– 0R*a*** + 1*a*♀ + 1*a* – 18. Simili ratione ordo cubicus hujus exempli esset hic.

CAPUT VII.

DE DIVISIONE.

1. DIVIDERE signum purum majus per minus signum, ejusdem positionis, est signum ordinis intervalli eorundem post numerum simplicem, aut post ejus numeratorem, collocare.

Ut sit dividendum **0*a*𝕮** per **0*a*♀** signum intervalli est ***a*℞**, quod post locum saltem numeri simplicis colloca, vel post ejus numeratorem, ut libet, fitque **0*a*℞**, vel $\frac{0a\text{℞}}{0}$, pro quotiente divisionis. Item **0ß** per **0𝕮** fit quotiens **0♀**, vel $\frac{0\text{♀}}{0}$.

2. Dividere signum purum minus per majus ejusdem positionis, est signum intervalli eorundem post numeri simplicis fracti denominatorem collocare, et quotiens semper erit fractio.

Ut sit dividendum **0*a*♀** per **0*a*𝕮**; signum intervalli est *a*, quod post locum saltem denominatoris numeri simplicis colloca, fitque $\frac{0}{0a\text{℞}}$ pro quotiente fracto. Item **0𝕮** per **0ß** divisum facit quotientem $\frac{0}{0\text{♀}}$.

3. Dividere signum purum per alterius positionis signum, est signum dividendum superscribere, (viz. post numeratorem numeri simplicis fracti,) et signum dividens subscribere, (viz. post ejusdem denominatorem,) et quotiens semper erit fractio.

Ut sit **0*a*𝕮** per **0♀** dividendum, fit quotiens $\frac{0a\text{𝕮}}{0\text{♀}}$. Item sit **0♀** dividendum per **0*a*𝕮**, fit quotiens $\frac{0\text{♀}}{0a\text{𝕮}}$.

Corollarium.

4. Hinc sequitur, in mixtis divisionem debere fieri per sect. 1 et 2, quatenus communicant positiones; quatenus autem sunt diversarum positionum, per sect. 3.

Ut sit **0𝕮*a*♀** dividendum per **0♀*a***; divide (per sect. 1) **0𝕮** per **0♀**, fit **0℞**. Item ***a*♀** per ***a***, fit a*;* fit ergo totus quotiens **0℞*a***. Item **0𝕮*a*♀** per **0ß*b*** dividitur hoc modo, **0♀** per **0ß** dividitur (per sect. 2,) et fit $\frac{0}{0\text{♀}}$, Item, ***a*♀** dividitur per ***b*** (per sect. 3) et fit $\frac{a\text{♀}}{b}$; fit ergo pro toto quotiente $\frac{0a\text{♀}}{0\text{♀}b}$. Item sit **0*b*♀*c*** per **0*bc*♀** dividendum, fit quotiens $\frac{0b}{0c}$.

5. Ex omni integro fit fractio ejusdem valoris subscribendo unitatem, et interponendo lineam.

Ut 5 sunt numerus integer, et ex eo fit $\frac{5}{1}$ fractio. Item **√𝕮7** est integra, et ex ea fit $\frac{\sqrt{\text{𝕮}}7}{1}$ fractio.

6. Si ergo simplex per simplicem dividendus fuerit; primo, fiant consimiliter radicati, (per cap. 4, sect. 4;) deinde, divide numerum simplicem dividendi per numerum simplicem divisoris; tertio, (per jam dicta,) divide signum positivum dividendi per signum divisoris;

quarto, numeri signique pro quotiente producti radicem talem extrahe, qualem indicat suum radicale, (per cap. 3,) et huic tandem praepone debitam copulam, per cap. 6 Lib. 1.

Ut **12*b*ℭ** sint dividenda per 3***b*♀** divide 12 per 3, fiunt 4 ; et divide ***b*ℭ** per ***b*♀**, fit ***b;*** erit ergo totus quotiens **4*b***. Item sint **√♀20ß** dividenda per **√♀8ℭ**; divide numerum per numerum, fit $\frac{5}{2}$, et divide signum per signum, et fit ♀. Ex his ergo $\frac{5}{2}$♀, seu $\frac{5♀}{2}$, seu $2\frac{1}{2}$♀, (quae per 1hujus eadem sunt,) extrahe radicem quadratam; ea est $\frac{√♀5♀}{√♀2}$, seu √♀$\frac{5}{2}$♀, seu √♀2$\frac{1}{2}$♀ pro quotiente; suntque haec eadem, per cap. 4, sect. 4 Lib. I. Item, e converso, sit dividenda **√♀8ℭ** per **√♀20ß**, erit quotiens $\frac{√♀2}{√♀5♀}$, seu rectius √♀$\frac{2}{5}$♀. Item, sit dividenda **– √♀12♀*a*ℭ*b*** per **√♀3*ab*** : divide numeros et signa ut dictum est, fiet **4♀*a*♀**, quorum radix quam indicat radicale, viz. quadrata, est **– 2℞*a*** pro quotiente, cum sua debita copula. Item, sint **4℞*a*** dividenda per **– √♀2℞*a*♀** : primo, fiant ejusdem radicalis, sic, **√♀16♀*a*♀** et **– √♀2℞*a*♀**; deinde, illius et numerum et signum per hujus divide, et fit **8℞**, quorum radix quadrata, cum debita copula, est **– √♀8℞** pro quotiente.

7. Hic observandum, quod si quotiens signorum positivorum fuerit (per 2 et 3 hujus) fractio, et quotiens numeri absoluti fuerit integer, ex hoc integro fieri debet (per 5 hujus) fractio.

Ut si sint **12ℭ** dividenda per **3ß** *;* divide numerum per numerum, fiunt 4, numerus integer; et divide signum per signum, fit fractio, nempe $\frac{0}{0♀}$, seu $\overline{♀}$; atque igitur ex integris 4, seu $\frac{4}{1}$ et fiet totus quotiens $\frac{4}{1♀}$. Item sint **15*b*♀*c*** per **5*bc*♀** dividenda, fit quotiens numeri divisionis 3 integer, et quotiens signorum $\frac{0b}{0c}$, seu $\frac{b}{c}$ quae fractio est, atque igitur ex 3 fiat fractio $\frac{3}{1}$ et fiet totus quotiens $\frac{3b}{1c}$, et non $3\frac{b}{c}$ nec $\frac{3b}{0♀}$.

8. Si compositus per simplicem dividendus fuerit, divide (per jam dicta) quamvis particulam simplicem compositi per hunc simplicem divisorem, et quotientis simplices debitis copulis connecte.

Ut sint **12♀ – √♀2♀ + 6** dividenda per **2℞** ; divide **12♀** per **2℞**, fiunt **6℞***;* item divide **–**
– √♀2♀ per **2℞**, fiunt **– √♀**$\frac{1}{2}$; denique divide **+ 6** per **2℞**, fiunt $\frac{+3}{1℞}$; quibus copulatis fit totus quotiens **6℞ – √♀**$\frac{1}{2}$ + $\frac{3}{1℞}$.

(Si per compositum unius ordinis sit dividendum quippiam &c., ut sit dividendum per **6℞ – √♀3♀** , fac ut cap. 11, sect. 2 Lib. I.)

9. Si compositus per compositum plurium ordinum dividendus fuerit : primo, utriusque compositi simplices situ recto (per cap. 6 sect. 4) collocentur; deinde simplicem maximi ordinis dividendi per simplicem maximi ordinis divisoris divide, (per 6 hujus,) et producetur simplex primus quotientis; per hunc multiplica totum divisorem, productum ex toto dividendo aufer, reliquias nota, deleto caetero dividendo. Ex his reliquiis conficito alium dividendum, ex quo, eodem quo prius modo, alium quotientis simplicem, aliasque forsan reliquias, producito. donec tandem aut nihil relinquitur dividendi, aut reliquiae saltem paucioribus constent ordinibus proportionalibus quam divisor; quibus peractis,

collige et connecte singulas dictas particulas quotientis cum copulis suis, et fiet quotiens totalis, observatis etiam reliquiis ultimis si quae sint.

Ut sint **1♀♀ + 71♀ + 120 –154℞ –14ℭ** dividenda per **6 +** 1♀ **– 5℞** : primo, (per cap. 6,) recte collocentur hoc situ:—

1♀♀ –14ℭ +71♀ –154℞ +120 (1♀
1♀ – 5℞ +6

Deinde divide **1♀♀** per **1♀**, fit quotiens **1♀**, quod apud hemicyclum nota, ut supra; per hunc quotientem duc totum divisorem, fit inde **1♀♀ –5ℭ +6♀**, quae ex toto dividendo substrahe; relinquentur **–9ℭ +65♀ –154℞ +120**; haec igitur nota, deletis caeteris hac forma :—

	–9ℭ	+65♀			
1♀♀	– 14ℭ	+ 71♀	– 154℞	+ 120	(1♀
1♀	– 5℞	+ 6			
	1♀	– 5℞	+6		

Has reliquias per eundem divisorem modo quo prius divide, et fiet hic situs :—

		+20♀			
	–9ℭ	+65♀	– 100℞		
1♀♀	– 14ℭ	+ 71♀	– 154℞	+ 120	(1♀– 5℞
1♀	– 5℞	+ 6	+ 6	+ 6	
	1♀	– 5℞	+ 5℞		
		1♀			

Has denique reliquias eodem quo prius modo divide, et fiet hic situs :—

		+20♀			
	–9ℭ	+65♀	– 100℞		
1♀♀	– 14ℭ	+ 71♀	– 154℞	+ 120	(1♀– 5℞ + 20
1♀	– 5℞	+ 6	+ 6	+ 6	
	1♀	– 5℞	+ 5℞		
		1♀			

Quotiens ergo totalis est **1♀– 5℞ + 20**, et novissimae reliquiae nullae supersunt.

Aliud exemplum unico typo expressum, in quo **1ℭ –11♀ + 40℞ – 36** dividenda per **1℞ – 4**

```
                          +12
                  +12℞
           – 7♀
   1ℭ     –11♀   – 40℞   –36   (1♀–7℞ + 12 pro quotiente totali.
   1℞     – 4
           1℞     – 4
           1℞     – 4
```

Et jam ultimo supersunt 12, pauciores habentes ordines quam divisor **1℞– 4.**

10. Si dividendus non fuerit integre seu totaliter (viz. sine residuo ultimo,) divisibilis per divisorem, tunc subscribendo divisorem sub residuo fiat fractio, quam apud quotientem colloca, aut si mavis sub toto dividendo subscribe divisorem.

Ut in superiore exemplo, erit quotiens verus **1♀–7℞ + 12** 12/(1℞ - 4) aut si mavis (1ℭ –11♀ + 40℞ –36)/(1℞ - 4) ; fitque hoc quia divisor ille nequit dividendum integre, et sine residuo dividere.

CAPUT VIII.

DE RADICUM EX COMPOSITIS EXTRACTIONE.

1. SI compositi radix quadrata fuerit extrahenda., compositus ille primo recte (per cap. 6 sect. 4) collocetur; deinde, ex maximi ordinis simplice vel simplicibus (per cap. 3 sect. 5 et 7,) radicem quadratam extrahe, quam apud hemicyclum pro quotiente constitue, et illa simplicia maximi ordinis dele, nullo, si possis, relicto residuo. Secundo, per totius quotientis duplum divide primam partem compositi remanentem (per cap. 7 sect. 9), nullo, si poteris, relicto residuo. Hujus divisionis quotientem novum post praefatum quotientem scribe, ejusque novi quadratum ex dicta parte superstite aufer, notatis reliquiis. Hocque secundum opus tertia, quarto, saepiusque repete, donec tandem aut nullae supersint reliquiae, et tunc totus quotiens cum copulis suis erit vera radix insita quaesita, aut si aliquae quam paucissimae supersint reliquiae, tunc dictus quotiens radix proxima dicitur, et non vera.

Ut sit ex composito hoc **√♀4ℭ +1♀ – √♀576℞ + 144 – 23℞** extrahenda radix quadrata : primo, recte locentur simplices, hoc situ:—

1♀ + √♀4ℭ – ♀ + √♀4ℭ – 23℞ – √♀576℞ + 144

deinde ex primo, viz. ex **1♀** extrahe radicem veram, ea erit **1℞** pro quotiente, et relinquentur caetera, hoc situ : —

1♀ **+ √♀4𝕮 – 23℞ – √♀576℞ + 144 (1℞ ;**

secundo, per quotientis duplum, viz. per **2℞** seu **√♀4♀** divide primam partem reliqui, viz. **√♀4𝕮**, fiet novus quotiens **+ √♀1℞** et reliquiae **– 23℞ – √♀576℞ + 144**, ex quibus aufer hujus novi quotientis quadratum, quod est **1℞**, restabunt reliquiae et quotiens hoc situ:—

– 24℞
1♀ + √♀4𝕮 – 23℞ – √♀576℞ + 144 (1℞ + √♀1℞
+√♀4♀

Adhuc repete hoc secundum opus, nempe per quotientis duplum, viz. per **2℞ + √♀4℞** divide partem primam reliquiarum superiorum, viz.

– 24℞ – √♀576℞,

fiet novissimus quotiens **–12**, et reliquiae **+ 144**, ex quibus aufer novissimi quotientis quadratum, viz. **–144**, et nihil relinquitur, ut hoc situ patet:—

– 24℞
1♀ + √♀4𝕮 – 23℞ – √♀576℞ + 144 (1℞ + √♀1℞–12
+√♀4♀ + 2℞ + √♀1℞

Unde patet quod **1℞ + √♀1℞–12** sunt vera radix quadrata insita hujus compositi suprascripti, quia nihil post extractionem restat.

Aliud exemplum.

Ex **√𝕮4♀ – 8 – √𝕮16℞** sit extrahenda radix quadrata: primo, fiat hic situs:—

√𝕮4♀ – √𝕮16℞ – 8 (√𝕮2℞

Secundo, fiat hic situs :-

– 9
√𝕮4♀ – √𝕮16℞ – 8 (√𝕮2℞ – 1
– √𝕮16℞

Unde patet quod **√𝕮2℞ – 1** est proxima et non vera radix quadrata hujus compositi **√𝕮4♀ – 8 – √𝕮16℞**, quia restant reliquiae, viz. **– 9**.

Exemplum plurium positionum.

Ex **1♀ + 2℞*a* +1*a*♀** + **1℞ + 1a –110** sit extrahenda radix quadrata. Primo, fiat hic situs, (per cap. 6 prop. 7) :—

1♀ + 2℞*a* +1*a*♀ + **1℞ + 1a –110 (1℞**

Secundo, fiat hic situs :—

1♀ + 2℞*a* +1*a*♀ + 1℞ + 1a –110 (1℞+1*a*
+ 2℞

Tertio, fiat hic situs :-

–110 $\frac{1}{4}$
1♀ + 2℞*a* + 1*a*♀ + **1℞ + 1a –110 (1℞+1*a* +** $\frac{1}{2}$ pro radice proxima.
+ 2℞ 2℞ + 1a

Aliud exemplum plurium positionum.
Sit ex **1♀ + 6℞*a* – 7** extrahenda radix quadrata. Primo, fiat hic situs (per cap. 6 prop. 7) :—

–9*a*♀ – 7
1♀ + 6℞*a* +0*a*♀ – 7 (1℞

Secundo, fiat hic situs :—

–9*a*♀
1♀ + 6℞*a* +0*a*♀ – 7 (1℞ + 3a pro radice proxima.
2℞

Exemplum difficile unius positionis.
Sit ex **1♀ – √♀8♀ – 6℞+ 8 + √♀32** extrahenda radix quadrata.
Primo, fiat hic situs :—

1♀ – √♀8♀ – 6℞ + 8 + √♀32 (1℞

Secundo, fiat hic situs :—

– 3 – √♀8
1♀ – √♀8♀ – 6℞ + 8 + √♀32 (1℞– √♀2 – 3 radice proxima.
+ 4√♀4♀ + 6℞

Sextum exemplum.

Ex **1♀ – 0℞*a* + 1*a*♀ – 1℞ + 1*a* – 18** sit extrahenda radix quadrata.
Primo, fiat hic situs, per cap. 6 prop. 7 :—

1♀ – 0℞*a* +1*a*♀ – 1℞ + 1*a* –18 (1℞

Secundo :—

+2℞*a*
1♀ – 0℞*a* +1*a*♀ – 1℞ + 1*a* –18 (1℞–1*a*
+2℞

Tertio, fiat hic situs :—

+2℞*a* **–18 $\frac{1}{4}$**
1♀ – 0℞*a* 1*a*♀ – 1℞ + 1*a* –18 (1℞–1*a*–$\frac{1}{2}$
+2℞ **+2℞ – 2*a***

2. Si compositi radix cubica fuerit extrahenda, compositus ille primo recte (per cap. 6 sect. 4) collocetur; deinde ex maximi ordinis simplice (per cap. 8 sect. 5 et 7) radicem cubicam extrahe, quam apud hemicyclum pro quotiente constitue, et illum simplicem maximi ordinis dele. Secundo, per tria quadrata totius quotientis divide primam partem non deletam compositi (per cap. 7 sect. 9), deletis divisis, nota reliquias. Hujus divisionis quotientem novum post primum quotientem scribe, ejusque novi tria quadrata ducta in primum antecedentem quotientem ex dictis reliquiis aufer, et ex eisdem aufer cubum novi, reliquiis notatis. Hocque secundum opus iterum atque iterum repete, donec tandem aut nullae supersint reliquiae, et tunc totus quotiens cum copulis suis erit vera radix insita quaesita; aut si aliquae quam paucissimm supersint reliquiae, tunc dictus quotiens dicitur radix proxima, et non vera.

Ut, sit sequentis compositi extrahenda radix cubica, qui sic primo recte collocetur:—

1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192℞ + 64 (1♀

viz. extrahitur ex **1♀ℭ** radix cubica, quae est **1♀**, quod pro quotiente ponitur. Secundo, per tria quadrata quotientis, viz. per **3♀♀**, divide **12ß**, fiet novus quotiens **+ 4℞** deletis **+ 12ß** , hoc situ :—

1♀ℭ + 12ß + 60♀♀ + 160ℭ + 240♀ + 192℞ + 64 (1♀ + 4℞

+ 3QQ

Deinde, hujus novi quotientis **+ 4R** tria quadrata, viz. **48Q**, duc in priorem quotientem, viz. in **1Q**, fient **48QQ**, quae aufer ex **60QQ** &c., restant **+ 12QQ + 96C + 240Q + 192R + 64**, ex quibus etiam reliquiis aufer cubum horum +4**R**, qui est **64C**, restant **+12QQ + 96C + 240Q + 192R + 64**, hoc situ :—

		+ 12QQ	**+ 96C**				
1QC	**+ 12ß**	**+ 60QQ**	**+ 160C**	**+ 240Q**	**+ 192R**	**+ 64**	**(1Q + 4R**
	+ 3QQ	**+ 48QQ**	**+ 64C**				

Tertio, repete secundum opus, viz. per tria quadrata quotientis, quae sunt **3QQ + 24C + 48Q**, divide primam partem dictarum reliquiarum, fient quotiens et reliquiae ut infra :—

		+ 12QQ	**+ 96C**	**+ 48Q**			
1QC	**+ 12ß**	**+ 60QQ**	**+ 160C**	**+ 240Q**	**+ 192R**	**+ 64**	**(1Q + 4R + 4**
	+ 3QQ	**+ 48QQ**	**+ 64C**				
		+ 3QQ	**+ 24C**	**+ 48Q**			

Deinde, hujus novissimi quotientis tria quadrata, viz. 48, duc in totum antecedentem quotientem, viz. in **1Q + 4R**, fient **48Q + 192R**, quae ex reliquiis illis, viz. ex **48Q + 192R + 64** aufer, restant **+ 64**, ex quibus aufer etiam cubum novissimi quotientis, qui est **+ 64**, et nihil restat, ut sequitur:—

		+ 12QQ	**+ 96C**	**+ 48Q**			
1QC	**+ 12ß**	**+ 60QQ**	**+ 160C**	**+ 240Q**	**+ 192R**	**+ 64**	**(1Q + 4R + 4**
	+ 3QQ	**+ 48QQ**	**+ 64C**				pro radice
		+ 3QQ	**+ 24C**	**+ 48Q**			cubica vera.
				+ 48Q	**+ 192R**	**+ 64**	

Aliud exemplum.

Sit sequentis compositi extrahenda radix cubica, qui sic prima collocetur :—

1C – 10Q + 31R – 30 (1R

Secunda, sic disponatur :—

– $\frac{7}{3}$R + $\frac{190}{27}$

1ℭ	**– 10℥**	**+ 31℞**	**– 30**	(**1℞ – 3$\frac{1}{3}$** pro radice cubica proxima, non
	+ 3℥	**+ $\frac{100}{3}$ ℞**	**– $\frac{1000}{27}$**	autem veri, propter reliquias extantes.

3. Si compositi radicem veram extrahere volueris, qui tamen radicem veram insitam non habuerit, sed proximam tantum, ei composito praepone signum universale, et fiet inde radix obscura vera.

Ut ex secundo exemplo quadratorum praefato, viz. ex **√ℭ4℥ – √ℭ16℞ – 8** sit extrahenda radix vera quadrata, ea erit **√℥.√ℭ4℥ – √ℭ16℞ – 8**

Item, sit ex **1ℭ–10℥ + 31℞ – 30** extrahenda radix cubica vera, ea erit **√ℭ.1ℭ–10℥ + 31℞ – 30.**

4. Radices autem quadrati quadratas, supersolidas, et caeteras superiores, tum quia rarissimi sunt usus, tum quia ex dictis considerari possunt, omittimus.

Ut si sit extrahenda radix quadrati quadrata, eam per regulam cubi sic emendatam extrahe. Primo, ' pro radicem cubicam extrahe,' lege, 'radicem quadrati quadratam extrahe.'· Secundo, pro 'tria quadrata,' lege, 'quatuor cubos.' Tertio, pro 'tria quadrata ducta in 'primum antecedentem quotientem,' lege, ' sex quadrata ducta in quadratum primi antecedentis quotientis et quatuor cubos novi ductos in primum antecedentem quotientem,' etc. Quarto, pro ' cubum novi,' lege, 'quadrati quadratum novi.' Et sic emendata regula ad quadrati quadratam radicem extrahendam inserviet.

At vero si pro supersolida radice extrahenda regulam emendare volueris, pro' cubicam,' lege, 'supersolidam,' et pro' tria quadrata,' lege, 'quinque quadrati quadrata,' et pro' tria quadrata ducta in primum antecedentem quotientem,' lege, 'decem quadrata ducta in cubum primi antecedentis quotientis, et decem cubos novi ductos in quadratum antecedentis, et quinque quadrati quadrata novi in primum antecedentem quotientem,' etc. Et pro' cubum novi,' lege, ' supersolidum novi;' et simili modo, ad omnes superiores radices extrahendas, poterint constitui regulae.

Exemplum regulae quadrati quadratae.

1*a*℥℥	**+ 4*a*ℭ*b***	**+ 6*a*℥*b*℥**	**+ 4*b*ℭ*a***	**+ 1*b*℥℥**	(**1*a* + 1*b*** pro vera
	+ 4*a*ℭ	**+ 6*b*℥*a*℥**	**+ 4*b*ℭ*a***	**+ 1*b*℥℥**	radice cubica.

Exemplum regulae supersolidae.

1*a*ß	**+ 5*a*℥℥*b***	**+ 10*a*ℭ*b*℥**	**+ 10℥*b*ℭ**	**+ 5*b*℥℥*a***	**+ 1*b*ß**	(**1*a*+1*b*** vera radix
	+ 5*a*℥℥	**+ 10*b*℥*a*ℭ**	**+ 10 *b* ℭ*a*℥**	**+ 5*b*℥℥*a***	**+ 1*b*ß**	supersolida.

5. Patet itaque ex praemissis quod aliquae extractionum reliquiae nulla habent signa positiva, et hae reliquiae totae formales dicuntur; aliae reliquiae habent, et hae totae informales dicuntur.

Ut in exemplis 2,3, et 4 extractionis quadratae, reliquiae omnes priorum operum sunt informales, eorundem autem exemplorum novissimae reliquiae sunt numerus; atque ideo formales dicuntur,

Exemplis vero 5 et 6 quadratae, et 2 cubicae reliquiae omnes, tum primae tum novissimae, eo quod positivis scatent, informales dicuntur.

Informatium reliquiarum quaedam sunt formabiles, quaedam reformabiles, quaedam prorsus deformes et irreformabiles.

Ut exemplis proxime subsequentibus patebit.

6. Formabiles sunt reliquiae cum quibus secunda pars regulae extractionis exerceri possit, reliquias inde nullas, aut prioribus minus informales reddentes. Ipsumque opus secundae partis regulae extractionis

Conformatio dicitur.

Ut in exemplis omnibus superioribus, et quadrati et cubi, reliquiae omnes praeter novissimas dicuntur formabiles, quia per solam secundam partem regulae extractionis conformantur, et reliquae novissimae inde minus informales exsurgent.

7. Reformabiles sunt reliquae quas si diviseris per compositum aliquod aequale nihilo (seu per aequationem ad 0), et hinc extantes recentiores reliquias per aliam atque aliam ad 0 aequationem, si opus sit, diviseris; extabunt tandem reliquae aut nullae, aut formales, aut formabiles, illaeque aequationes Reformatrices vocabuntur, et ipsum opus dividendi Reformatio dicetur.

Exemplum.

Ex **1♀ – 0℞*a* + l*a*♀ – 1℞ + l*a* – 18** extrahatur radix quadrata proxima, (quod supra, exemplo 6, fit,) et erit radix **1℞ – l*a* –** $\frac{1}{2}$, et reliquiae erunt informales, viz. **+ 2℞*a* –18** $\frac{1}{4}$; detur autem, exempli gratia, compositum hoc **1℞*a* + 1℞– l*a* –10**, quod nihilo aequetur; per hoc divide illas reliquias, et exsurgent reliquiae **–2℞ + 2*a*** + **1** $\frac{3}{4}$, quae, quia per prop. 6 hujus sunt formabiles; ideo reliquae **+ 2℞*a* – 18** $\frac{1}{4}$ dicuntur reformabiles, et compositum **1℞*a* + 1℞– l*a* –10** reformatrix, et opus ipsum reformatio dicentur.

Aliud exemplum.

Item ex **1♀ + 4℞*a* + l*a*♀ – 4℞*b* – 4*ab*+4*b*♀+4℞ + 4*a* – 8*b* – 61** extrahatur radix quadrata, eaque erit **1℞+1*a* – 2*b* + 2**, et reliquae erunt **+ 2℞*a* – 65** informales. Detur autem aequatio ad 0, quae sit **1℞*a* – 1*ab* – 1*b* – 5**, per quam divide illas reliquias, et exsurgent reliquae **+2*ab* + 2*b* – 55**, quae, quia nec formales nec formabiles sunt, debent per aliam aequationem ad 0 dividi, exempli gratia, per **2*ab* – 3℞ – 3*a* + 8*b* – 21**, et exsurgent reliquae **3℞+3*a* – 6*b* – 34**, quae, quia formabiles sunt (respectu, viz. praefatae radicis proximae, viz. **1℞ + 1*a* –2*b* +2**, ideo et reliquae **2℞***a* **– 65**, et reliquae **2*ab*+2*b* – 55** reformabiles dicuntur, atque et compositum **1℞ – 1*ab* – 1*b* – 5**, et compositum **2*ab* – 3℞ – 3*a* + 8*b* – 21** sunt reformatrices aequationes.

8. Ut igitur reliquae informales fiant formales, conformabiles conformabis (per 6 prop. hujus); et reformabiles (per 7,) reformabis, et reliquias omnium novissimas notabis, quae si aut nullae aut formales fuerint, bene est, tunc enim quotientes omnes conformationum copulandae et abbreviandae sunt, et erunt radix proxima reformata; quotientes vera reformationum inutiles semper, et spernendae sunt.

Ut exempli penultimi radix quadrata proxima erat **1℞ – 1*a* –** $\frac{1}{2}$, et reliquae **+ 2℞*a* – 18**$\frac{1}{4}$, quia reformabiles sunt (per prop. 7) per reformatricem suam **1℞*a* + 1℞– l*a* –10** reformato, et spreto quotiente exsurgent reliquiae **– 2℞ + 2*a* –** 1$\frac{3}{4}$, quae quia formabiles sunt (per 7) conformato, et exsurgent reliquae formales notandae, viz. **+**$\frac{3}{4}$ et quotiens conformationis **–1** cum radice praefata copulatus et abbreviatus fiet **1℞ – 1*a* – 1**$\frac{1}{2}$ pro radice proxima reformata.

Item exempli ultimi reliquae erant primo **2℞*a* – 65**, quas per suam reformatricem, viz. **1℞ – 1*ab* – 1*b* – 5** reformato, et spreto quotiente exsurgent reliquae **2*ab* + 2*b* – 55** ut prop. 7 diximus; quae rursus reformato per aliam reformatricem (ut ibidem monuimus,) viz. per **2*ab* - 3℞ – 3*a* + 8*b* – 21**, exsurgent spreto quotiente reliquiae **3℞ + 3*a* – 6*b* – 34** quas, quia formabiles sunt (per 6 prop.) ad suam radicem proximam, viz. ad **1℞ +1*a* – 2*b* + 2**, conformabis, et erit tota radix proxima reformata **1℞ +1*a* – 2*b* +** 3$\frac{1}{2}$, et reliquae novissimm notandae sunt $-\frac{169}{4}$ sive **42**$\frac{1}{4}$ formales.

9. At si defectu reformatricium post ultimam conformationem exstent reliquiae informales, hae deformes aut irreformabiles appellantur.

Ut, novissimae reliquiae exempli 5 et 6 quadratae, et 2 cubicae extractionis (si nullae occurrant reformatrices,) dicuntur deformes et irreformabiles.

10. Deformium reliquiarum et suarum radicum duae sunt species, singulares et plurales, quarum singulares sunt eae deformes quae habent unum aliquod simplex et purum positivum, aut mixti positivi particulam unam in radice, seu quotiente cui non fuerit aliud simile vel ejusdem positionis, nec in quotiente seu radice nec inter reliquias.

Ut radix quadrata hujus **1♀+ 6℞*a* – 7** est **1℞ + 3*a***, et reliquiae sunt **– 9*a*♀ – 7**, quae ideo singulares dicuntur quia in eis non sunt plures positivi primae positionis uno, qui est **1℞**. Item radix quadrata hujus **1♀*a*♀ – 6℞a – 1*a* + 8 = 0** est **1℞*a* – 3**, et reliquiae sunt **–l*a* – 1**, in quibus signum **℞** non saepius quam semel reperitur.

11. Plurales dicuntur radices suaeque reliquiae, quum uniuscujusque positionis plures simplices in radice vel inter reliquias reperiuntur.

Ut radix cubica proxima hujus **1𝕮 – 9♀ + 36℞ – 80 = 0** est **1℞ – 3**, et reliquiae erunt **+9℞ – 53**. In quibus primae positionis signum **℞** bis reperitur.

Item radix quadrata proxima hujus **1♀ + 1*a*♀ – 1℞ + 1*a* – 18** (deficiente reformatrice,) erit **1℞*a* –1*a* –** $\frac{1}{2}$, et reliquae erunt **2℞*a* – 18**$\frac{1}{4}$, ut supra exemplo 6 docuimus, in quibus primae positionis duae sunt simplices, viz. **1℞** et 2**℞*a****;* item secundae positionis totidem, viz. **1*a*** et 2**℞*a***.

12. Sunt itaque radicum quatuor formae. Prima est verarum radicum, secunda est formalium, tertia singularium, quarta pluralium; quarum extrahendarum usum inferius docebimus.

CAPUT IX.

DE AEQUATIONIBUS ET SUIS EXPONENTIBUS.

1. AEQUATIO est positivorum incertorum valorum cum aliis sibi aequalibus collatio, ex qua positionis valor quaeritur.

Ut si quis pro numero quaesito aut quantitate quaesita ponens **1℞**, ejus valorem ignorans, postea per hypothesin quaestionis deprehendens **3℞** aequari ad 21, conserat tres res cum suis aequalibus 21, ea aequalitatis collatio dicitur aequatio; et hinc infertur rem unam seu unam positionem valere 7.

2. Inter aequationis partes invicem aequales interjicitur linea duplex, quae signum aequationis dicitur.

Ut **3℞ = 21**, quae sic pronuntiantur, tres res aequales viginti uni. Item **℞ = 7**, quae pronuntiantur, una res aequalis ad septem.

3. Aequationum aliae unius tantum sunt positionis, aliae plurium positionum.

Unius tantum positionis, ut **l*a*♀+8*a* = 10**: Plurium positionum, ut **2♀ – l*a* = 6**.

4. Item aequationum aliae rudes, quae ad minores terminos, magisque perspicuous et succinctos reduci possunt, aliae perfectissimae dicuntur, quae e contra sunt maxime perspicuae et succinctae.

Ut, **3℞ = 21** est aequatio rudis, quia in perfectissimam, viz. in **℞ = 7** reduci possit. Item, **5*a*♀ = 20** est aequatio rudis, quia in perfectiorem, viz. in **1*a*♀ = 4** reduci possit. Sed et **1a♀ = 4** rudis est, quia adhuc in perfectiorem, imo perfectissimam, viz. in **l*a*** = **2** reduci possit, arte quam inferius tractabimus. Item **12♀ +3*a* = 6** rudis est aequatio, quia in perfectiorem **4♀ + la = 2** reduci possit.

5. Item aequationum aliae simplices, aliae quadratae, aliae cubicae, aliae superiores: quarum simplices sunt quae duobus ordinibus tantum constant.

Ut **3℞** = **27**, seu **1℞ = 9**; item **5*b*♀ = 20**, simplices aequationes dicuntur.

6. Simplicium aequationum aliae sunt reales, quae sunt rerum aequalium ad numerum; aliae radicales, quae sunt quorundam quadratorum, cuborum, vel aliorum superiorum ad numerum aequationes.

Reales, ut **3℞ = 21**, seu **1℞ = 7**. Item **l*a* = 8**. Item **2℞ =√♀3 –** 1. Radicales, ut **2♀ = 3**. Item **3ℭ** = 24. Item **1*a*ß** = √ℭ**9**, etc.

7. Aequatio quadrata est quae tribus proportionalibus ordinibus constat.
Ut **2♀ + 3℞ = 4**, seu **3℞ = 2♀ – 4**. Item **l*a*♀ℭ– l0 = 3*a*♀**.
Item **12 – √♀1℞ = 1℞**.

8. Aequatio cubica est quae quatuor proportionalibus ordinibus constat.
Ut **1ℭ – 9ℨ = 24 – 26℞**. Item **l ℭ+ 0ℨ – ℞2 = 4**. Item
l*a*ℨℭ – 2*a*ℨ = 4 est aequatio cubica, quia (per prop. 4 cap. 6) sic collocata
l*a*ℨℭ + **0*a*ℨℨ – 2*a*ℨ = 4**, constat quatuor ordinibus.

9. Aequatio quadrati quadrata est quae quinque; supersolida, quae sex; quadrati cubica, quid septem, proportionalibus ordinibus constant:
Et sic de reliquis superioribus in infinitum.
Quadrati quadrata, ut **2ℨℨ – 28ℭ + 142ℨ = 308℞ – 240**. Supersolida, ut **1*b*ℨß – 4*b*ℨℨℨ +l*b*ℨℭ – 3*b*ℨℨ – l*b*ℨ = 12**. Quadrati cubica, ut **l*a*ℨℭ – 8*a*ß + 2*a*ℨℨ – 6*a*ℭ + l*a*ℨ = l*a* + 6**.

10. Aequatio illusiva est ea quae impossibile asserit, et siquis impossibile quaerit in aequationem illusivam cadet ejus responsum.
Ut **1℞ = 3℞** est aequatio illudens, siquidem impossibile est quicquam suo triplo aequari. Item **1ℨ = 4℞ – 5** est aequatio illudens, siquidem nullum quadratum possit aequari quatuor rebus seu radicibus suis, ablatis quinque; ut inferius patebit.

11. Expositio est reductio rudis aequationis ad perfectissimam et realem aequationem, et pars aequationis realis quae uni rei requatur dicitur Exponens, eaque quaestionem solvit.
Ut quum haec rudis **3℞ = 21** reducitur ad hanc perfectissimam **1℞ = 7**, exponens utriusque aequationis erit 7, quia uni rei (viz. ad **1℞**) aequatur. Item haec rudis **5ℨ = 20** reducitur ad hanc perfectiorem **1ℨ = 4**, deinde ad hanc perfectissimam et realem **1℞ = 2**, quod quidem opus reductionis dicitur expositio, et 2 exponens, quia uni rei aequatur: quaestionem vera exponente solvi postea docebimus.

12. omnis aequatio praeter illusivam habet saltem unicum exponens, validum sive invalidum.
Hoc postea docebimus, hic praemonuisse sufficit.

13. Exponentia valida sunt ea quae per se posita copula **+** notantur, et semper sunt majora nihilo. Exponentia vero invalida sunt quae per se posita copula **–** notantur, et haec minora sunt nihilo.
Ut in hac aequatione **1℞ = 7**, septcm sint exponens validum, quia (per prop. 1 cap. 6 Lib. I.) copula **+** notari subintelligitur.
At in hac reali aequatione **1℞ = – 7**, exponens contraria ratione invalidum dicitur, quia copula **–** notatur sic, **– 7**, estque nihilo minus.

14. Exponentium alia et numero solo et quantitate solo, alia tantum numero solo, alia tantum quantitate sola, alia partim hac partim ilio, alia neutro, exprimi possunt.
De his, suisque exemplis, latius per ordinem, capitibus 11, 12, 13, dicetur.

15. Omnis aequationis portio ditioni unius anterioris copulae subdita Minima dicitur, quotcunque copulas et terminos habuerit; copulaque anterior et praedominans dicitur Ductrix; Caeterae vero copulae Intermediae dicuntur.

Ut in hac aequatione **1ℭ – 3 + √♀2 +** $\frac{3℞ - 4}{1♀ + 1}$ **– √♀.6 + √♀1℞ = 0**,
in qua **1ℭ** dicitur minima, et **+** dicitur ejus ductrix copula.
Item 3 dicitur minima, et **–** ejus ductrix. Item **√♀2** dicitur minima, et **+** ejus ductrix. Item $\frac{3℞ - 4}{1♀ + 1}$ dicitur minima, et **+** ejus ductrix, quia in totam fractionem extenditur ejus vis. Caeterae vero copulae hujus fractionis intermediae dicuntur. Item **√♀.6+ √♀1℞** dicitur minima, et copula **–** ejus ductrix, quia in aggregatum valorem totius universalis radicis extenditur ejus vis, et reliqua copula **+** intermedia dicitur.

CAPUT X.

DE GENERALI AEQUATIONUM PRAEPARATIONE.

1. PRAEPARATIO est reductio aequationum rudium ad perfectiores, quas postea ad perfectissimas reales reducit expositio.

Ut **5*a*♀ = 20** prius praeparantur, et fient **1*a*♀=4**: deinde exponuntur, et fient **1*a* = 2**. Quibus modis praeparantur jam dicetur; quibus vero exponuntur postea patebit.

2. Praeparantur et perspicuae redduntur aequationes rudes quinque modis; transpositione, abbreviatione, divisione, multiplicatione, et extractione.

Quorum modorum regulae et exempla sequuntur.

3. Si minimam ex una parte aequationis in contrariam transferas, illique contrariam copulam ductricem praeposueris, erunt partes (ut antea.) aequales, et Transpositio dicitur.

Ut, ex hujus aequationis **4℞ – 6 = 5℞ – 20** posteriore parte, si transposueris – 20 in priorem partem aequationis, copula ejus mutata, hoc situ, **4℞ – 6 + 20 = 5℞** : Item adhuc si transposueris **4℞**, fient **– 4℞** hoc situ, **– 6 + 20** =5**℞** – **4℞**. Item aequationis hujus **1♀ – √♀.3♀ – 2 = 3*a***, si transposueris **–√♀.3♀ – 2**, erit **+ √♀.3♀ – 2**, hoc situ, **1♀ = 3*a* + √♀.3♀ – 2** ; et si adhuc etiam transposueris **3*a***, erunt – **3*a***, hoc situ, **+ 1♀ – 3*a* = √♀.3♀ – 2** et partes oppositae aequales sunt, ut antea fuerant.

4. Si omnes minimas alterius partis aequationis (per praemissam) in contrariam partem transponas, totum compositum aequabitur nihilo, et dicitur aequatio ad nihil; debetque haec aequatio (per prop. 4 cap. 2 hujus) abbreviari.

Ut, in exemplo suprascripto **4℞ – 6 =** 5**℞** – **20**, transpone 5**℞** – **20**, eruntque –5**℞ + 20**, hoc situ, **4℞ – 6** –5**℞** + **20 = 0**, quae abbreviata efficiunt **–1℞ +14 = 0**, quae aequatio ad nihil est.
Item **1♀ – √♀.3♀ – 2 = 3*a***, cujus partem sinistram si dextrorsum transponas, fiet **0 = – 1♀ + √♀.3♀ – 2 + 3*a***, quae quidem dicitur aequatio ad nihil.

5. Si positivus maximus a fronte habeat copulam **–** , converte omnes omnium minimarum ductrices, et producetur aequatio magis perspicua.

Exempla ut supra: Si **–1℞ + 14** requetur ad 0, hoc situ, **–1℞ +14 = 0**, per consequens **–1℞ +14** aequabitur etiam ad **0**, hoc situ, **+1℞ – 14 = 0**. Item, eodem modo ex **– 1𝔔 + 3*a* + √𝔔.3𝔔 – 2 = 0** fiet **1𝔔 –3*a* – √𝔔.3𝔔 – 2 = 0**. Item ex hac **–1℞ – 1 +** $\frac{32}{1a+1}$ **= 0** fiet haec, **1℞** +1– $\frac{32}{1a+1}$ **= 0**.

6. Si aequationis positivos omnes maximi ordinis per unitatem signatam signis positivis et radicalibus ejusdem ordinis diviseris, et per quotientem diviseris totam aequationem; hinc producetur aequatio perspicua habens maximum ordinem unitate notatum.
Exemplum aequationis, **2ℭ – 8𝔔 + 6℞ = 0**: Divide positivum maximi ordinis, viz. **2ℭ** per **1ℭ** fiet quotiens 2; per duo igitur divide totam aequationem, fientque **1ℭ– 4𝔔 + 3℞ = 0**.

Item hujus aequationis, **3℞ – √𝔔2𝔔 – 6=0**, positivi maximi ordinis sunt **3℞ – √𝔔2𝔔**, qui quidem (per prop. 5 cap. 4 hujus) sunt ejusdem ordinis potentiae, eorumque ordo est rerum; divide ergo **3℞ – √𝔔2𝔔** per **1℞**, sive (quod idem est) per **√𝔔1𝔔**, fiet quotiens **3 – √𝔔2 ;** per hunc quotientem (per prop. 2 cap. 11 Lib. I.) divide totam aequationem, fietque haec aequatio **1℞ –** $\frac{18}{3}$ **–** $\frac{\surd𝔔 72}{7}$ **= 0**, quae quamvis fractio sit, tamen magis perspicua est quam prius, eo quod signum 𝔔 aufertur.

Item tertium exemplum, **1℞*a* + 1*a* + 1℞ – 31 = 0**, in quo sit tibi animo expurgare et delere signum mixtum, viz. **1℞*a***: Divide ergo **1℞*a* + 1*a*** per **1℞**, (utram volueris loco maximi ordinis acceptare,) exempli gratia acceptetur **1℞**: divisa itaque **1℞*a* + 1℞** per **1℞**, orietur quotiens **1*a* +1**, per quem divide totam aequationem **1℞*a* + 1℞ + 1*a* – 31 = 0**, fiet aequatio haec, **1℞** +1– $\frac{32}{1a+1}$ **= 0**, quae licet fracta, tamen magis perspicua est quam prius, eo quod signum mixtum quod prius obscurum erat jam aufertur.

7. Si aequationis minimus ordo fuerit positivus, tunc per unitatem signatam signis minimi ordinis divide totam aequationem, et proveniet inde aequatio perspicua habens numerum absolutum loco minimi ordinis.
Exemplum, **1ℭ – 4𝔔 + 3℞ = 0**, quae divide per unitatem minimi ordinis, viz. per **1℞**, fiet **1𝔔 – 4℞ + 3 = 0**. Item **3𝔔 – √𝔔2℞ = 0**, haec divide per **√𝔔1℞**, fiet inde haec aequatio **√𝔔9ℭ – √𝔔2 = 0**, quarum ultima series semper est numeri.

8. Si particulae aliquae aequationis sint verae fractiones, inque earum denominatores duxeris totam aequationem, producetur aequatio integra, et plerumque magis perspicua.

Ut, in hac $\frac{6℞-8𝔔}{1ℭ+3℞}$ **+ 2 = 0**, sunt $\frac{6℞-8𝔔}{1ℭ+3℞}$, vere fractio licet abbreviabilis ; duces ergo totam aequationem in denominatorem **1ℭ + 3℞**, fientque **2ℭ + 12℞ – 8𝔔 = 0**.

Item hanc aequationem **1𝔔 +** $\frac{2}{3}$ **℞ –** $\frac{88}{75}$ **= 0** duc per 3, fiet primo **3𝔔 + 2℞ –** $\frac{264}{75}$ **= 0**, haecque rursum per 75 duc, fientque **225𝔔 + 150℞ – 264 = 0**, quae quidem aequationes integrae sunt, et expertes fractionum.

9. Si in aequatione fuerit radix universalis unica, eam a reliquo aequationis (per prop. 3) separabis. et utrumque aequationis latus in se duces toties quoties signum universale denotat, et producetur aequatio magis perspicua, nulla enim habebit universalia signa. Exemplum, **2ʒ + 3℞ – √ʒ. 12ℭ + 4ʒʒ +18 = 0**: Primo per transpositionem fiant **2ʒ + 3℞ = √ʒ. 12ℭ + 4ʒʒ +18** ; deinde latera quadrentur, quia signum universale est **√ʒ**, fientque **4ʒʒ + 12ℭ + 9ʒ = 12ℭ + 4ʒʒ +18**, et per consequens transposita et abbreviata facient **1ʒ = 2**.

Aliud exemplum: **√ℭ.2℞ – 6 = 3℞** ducantur cubice latera, fient **2℞ – 6 = 27ℭ**, alias **2℞ – 27ℭ – 6 = 0**.

10. Si aequatio duabus radicibus universalibus consimiliter radicatis absque ullis aliis minimis constiterit. transpositione separentur, et in se multiplicentur quoties denotat universale signum; producitur aequatio perspicua nullius universalis radicis.

Ut **√ʒ.2℞ + 5 – √ʒ.3℞ – 4 = 0** separentur. et fient **√ʒ. 2℞ + 5 = √ʒ.3℞ – 4** ; quadrate multiplicentur. et fient **2℞ + 5 = 3℞ – 4**, et per transpositionem et abbreviationem **1℞ - 9 = 0**.

11. Si aequatio constet duabus solis dissimiliter radicatis universalibus, separentur universalia, et ducatur utrumque latus in se ad utriusque signi universalis dissimilis qualitatem, proveniet aequatio perspicua sine universalibus.

Ut **√β.3ʒ + 6 – √ʒ.2℞ – 3 = 0** ; prius transpositione separentur, sic, **√β.3ʒ + 6 =√ʒ.2℞ – 3** ; deinde, latera in se quadrati supersolide ducantur, fientque **32β – 240ʒʒ + 720ℭ – 1080ʒ + 810℞ – 243 = 9ʒʒ + 36ʒ + 36**, quae transposita et abbreviata fient **32β – 249ʒʒ +720ℭ – 1l16ʒ + 810℞ – 279 = 0**.

12. Si in aequatione fuerint duae universales radices quadratae, cum allis quibusdam simplicibus aut uninomiis, universales ambas copulatas a caeteris separa, et utrumque latus in se quadrate duc, et fiet aequatio constans unica tantum universali radice, delenda etiam per prop. 9 hujus.

Ut haec aequatio

$$\tfrac{1}{2} + \underline{\sqrt{ʒ}.48\tfrac{1}{4} + \text{I℞} - 1ʒ} + \tfrac{1}{2}\text{℞} - \sqrt{ʒ}.79 - \tfrac{3}{4}ʒ = 0$$

sic transponantur,

$$\sqrt{ʒ}.79 - \tfrac{3}{4}ʒ - \underline{\sqrt{ʒ}.48\tfrac{1}{4} + \text{I℞} - 1ʒ} = \tfrac{1}{2}\text{℞} + \tfrac{1}{2};$$

deinde quadretur utrumque latus, fientque

$$127\tfrac{1}{4} + \text{I℞} - 1\tfrac{3}{4}ʒ - \sqrt{ʒ}.15247 + 316\text{℞} - 460\tfrac{3}{4}ʒ - \sqrt{ℭ} + 3ʒʒ = \tfrac{1}{4}ʒ + \tfrac{1}{2}\text{℞} + \tfrac{1}{4}$$

transpone et abbrevia, fietque aequatio,

$$\sqrt{ʒ}.15247 + 316\text{℞} - 460\tfrac{3}{4}ʒ - 3ℭ + 3ʒʒ = 127 + \tfrac{1}{2}\text{℞} - 2ʒ,$$

quae tandem (per prop. 9) fient **1ʒʒ + 1ℭ – 47ʒ –189℞ + 882 = 0.**

l3. Si aequatio constet tribus radicibus universalibus quadratis absque allis minimis, duae quadratae a reliqua per transpositionem separentur, lateraque quadrentur, et proveniet aequatio unius tantum universalis, per prop. 9 delendae.

Ut sit aequatio **√♀.3℞ + 2 + √♀.2℞ – 1 – √♀.4℞ – 2 = 0** separentur sic, **√♀.3℞ –2 + √♀.2℞ + 1 = √♀.4℞ + 2** ; quadrentur latera, et fient **5℞ – 1 + √♀.6♀ – 1℞ – 2 = 4℞ + 2**; deinde fient per abbreviationem **√♀. 6♀ – 1℞– 2 = 3 – 1℞**; postea (per prop. 9) fient **6♀ – 1℞ – 2 = 1♀ – 6℞ + 9**, et tandem fient **5♀ + 5℞ – 11 = 0**, alias **1♀ + 1℞ – 2**$\frac{1}{5}$ **= 0**.

14. Si aequatio constet tribus universalibus quadratis, cum unico uninomio aut simplici; transponantur dum universales a reliquis, lateraque quadrentur, et proveniet aequatio duarum universalium radicum, per prop. 12 delendarum.

Ut sit aequatio

√♀.√ℭ2℞ + 3 +√♀.3℞ – 2 –2℞ – √♀.2♀ + 1 = 0

transponantur sic,

√♀.√ℭ2℞ + 3 +√♀.3℞ – 2 = 2℞ + √♀.2♀ + 1

multiplicentur latera in se quadrate, fientque

√♀. √ℭ3456♀♀ –√ℭ1024℞ – 24 +√ℭ.2℞+3℞+1 = 6♀+1+√♀. 32♀♀ + 8℞

constantia duobus universalibus quadratis, per prop. 12 delendis.

15. Si aequatio constet quatuor universalibus quadratis absque allis minimis; binae a binis per transpositionem separentur, lateraque quadrentur, et proveniet aequatio duarum tantum universalium, per prop. 12 delendarum.

Sit aequatio haec sic transposita,

√♀ .5♀ –2℞ –√♀.10 –1℞ = √♀. 2℞+6 +√♀.1♀ + 4,

cujus latera quadrentur, fientque

5♀ – 3℞ + 10 –√♀.208♀ – 2ℭ – 80℞ = 1♀ + 2℞ + 10+√♀.8ℭ + 24♀ + 32℞ + 96,
quae duabus universalibus tantum constant, per prop. 12 delendis.

16. Si universalissima unica ex uno latere aequetur universalissimae soli, sive universali soli, sive universali et uninomio aut simplici unicis, sive uninomiis et simplicibus tantum ex altero latere: tunc duc in se latera ad signorum universalium qualitatcs, et signa universalissima delebuntur, caeteris universalibus delebilibus per praecedentia delendis.
Ut in aequatione hac

√ꝗ.10 +√ꝗ.5℞ – 2 = √ꝗ.3 +√ꝗ.3℞ + 1,

universalissima universalissimae aequatur; latera ergo quadrentur, fientque **10 +√ꝗ.5℞ –2 = 3 +√ꝗ.3℞ + 1**, sive **7+√ꝗ.5℞ – 2 = √ꝗ.3℞ + 1**, quorum universalia per prop. 12 delebis.

Aliud exemplum.

Item hujus **√ꝗß.3 +√ꝗ.2℞ – 1 = √ℭß.5 +√ꝗ.3℞ – 4**

ducantur latera in se quadrati cubice supersolide, fientque **18℞ + 18 +√ꝗ.8ℭ –12ꝗ+6℞ – 1+√ꝗ.1458℞ – 729 = 21+3℞ +√ꝗ.300℞ – 400**, sive **15℞ – 3 +√ꝗ.8ℭ –12ꝗ + 6℞– 1 +√ꝗ.1458℞ – 729 =√ꝗ.300 – 400**, quorum universalia deleri nequiunt.

Tertium exemplum.

Item hujus **√ℭ.3 +√ꝗ.2℞ – 1 =√ℭ.20 – 4℞**, duc cubice in se latera, fientque **3 + √ꝗ.2℞ – 1 =20 – 4℞**, sive **√ꝗ.2℞ – 1 = 17 – 4℞**, quorum universale (per prop. 9) delebis. Eadem est similium ratio.

17. Eisdem propositionibus quibus universales deleri dictum est, possunt et simplices irrationales inter rationales transponi, multiplicari, et tandem deleri.

Ut sit aequatio **12 –√ꝗ1℞ = 1℞**, per prop. 9 separentur, sic, **12 –1℞ =√ꝗ1℞**, et multiplicentur quadrate latera, fientque **1ꝗ – 24℞ + 144 = 1℞**, sive **1ꝗ – 25℞ + 144 = 0**, quae prorsus rationales sunt. Quae itaque, propositionibus 9, 10, 11, 12, 13, 14, et 15, dicuntur de universalibus, eadem de simplicibus radicatis etiam dici intelligantur.

18. Quaee aliter praeparari possunt aequationes, per propositionem ne praeparentur praemissam; multiplicatio enim irrationalium simplicium plerumque plura exponentia debito exhibet.

Ut praecedens exemplum **12 –√ꝗ1℞ = 1℞**, per praemissam multiplicatum, reddit aequationem **1ꝗ – 25℞ + 144 = 0**, quae duo habet valida exponentia, viz. 16 et 9, cum revera ipsa principalis aequatio, **12 –√ꝗ1℞ = 1℞**, habeat unicum exponens tantum, viz. 9, ut postea patebit. Illa igitur aequatio principalis per prop. 17 ne praeparetur, dummodo eadem per prop. 20 subsequentem melius et simplicius praeparari possit, ut ibidem dicetur.

19. Si aequationis ad 0 extrahatur radix aliqua vera (viz. relicto nihilo), radix illa erit magis succincta, et ad 0 aequatio.

Ut ex aequatione **1ℭ – 6ꝗ + 12℞ – 8 = 0** extrahe radicem cubicam veram, viz. **1℞ – 2 = 0**, quae erit abbreviata et succincta aequatio.

Item aequationis **1℞ – √♀36℞ + 9 = 0** radicem quadratam extrahe, eaque erit vera (per cap. 8), viz. **√♀1℞ –3 = 0**, quae est magis succincta aequatio.

20. Si aequationis ad 0 extracta radix aliqua sit, aut formalis aut (per prop. 8 cap. 8 hujus) reformata; reliquiarum copulam converte, et earundem radices quadratas vel cubicas, etc. quales ex reliquo extrahe; has radices (conversis copulis) cum radice proxima et formali copulato, fient aequationes, et unica, non quadratinomia vel duae quadratinomiae, ad 0 magis succinctae, priorisque aequationis exponentia complectentes.

Et caetera.