The Description of the Wonderful Canon of Logarithms,

and the use of which not only in Trigonometry, but also in all Mathematical Calculations, most fully and easily explained in the most expeditious manner.

By the author and discoverer
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[Preliminary dedicatory material not included.]

ON THE AMAZING CANON OF LOGARITHMS.

Preface.

Since nothing is more tedious, fellow mathematicians, in the practice of the mathematical arts, than the great delays suffered in the tedium of lengthy multiplications and divisions, the finding of ratios, and in the extraction of square and cube roots– and in which not only is there the time delay to be considered, but also the annoyance of the many slippery errors that can arise: I had therefore been turning over in my mind, by what sure and expeditious art, I might be able to improve upon these said difficulties. In the end after much thought, finally I have found an amazing way of shortening the proceedings, and perhaps the manner in which the method arose will be set out elsewhere: truly, concerning all these matters, there could be nothing more useful than the method that I have found. For all the numbers associated with the multiplications, and divisions of numbers, and with the long arduous tasks of extracting square and cube roots are themselves rejected from the work, and in their place other numbers are substituted, which perform the tasks of these rejected by means of addition, subtraction, and division by two or three only. Since indeed the secret is best made common to all, as all good things are, then it is a pleasant task to set out the method for the public use of mathematicians. Thus, students of mathematics, accept and freely enjoy this work that has been produced by my benevolence Farewell.

ON LOGARITHMS.

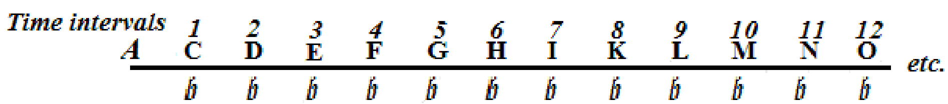
By which all the sines, tangents, and secants,
are set out for you from great labour and prolixity;
And which this little table of Logarithms, gentle reader,
Gives to you all at once, without great labour.

[The original translator of Napier's *Demonstratio*, Edward Wright, made fundamental contributions to the art of navigation, and in 1610 published a book : *Certain errors in Navigation detected and corrected*. He was reputedly a lecturer in navigation at Gresham College around this time. See, Vol. 17 of the *Encyclopædia Britannica,* 9th Ed., p. 254. He undertook to translate Napier's work, as it obviously had application to navigation, but unfortunately died before the translation was published; this was done by his son, with the help of Henry Briggs, who was the Professor of Geometry there at that time.]

BOOK 1.

Chapter 1.
Concerning Definitions.

Def. 1. ***A line is said to increase uniformly, when the point describing it progresses through equal intervals in equal moments or intervals of time.***

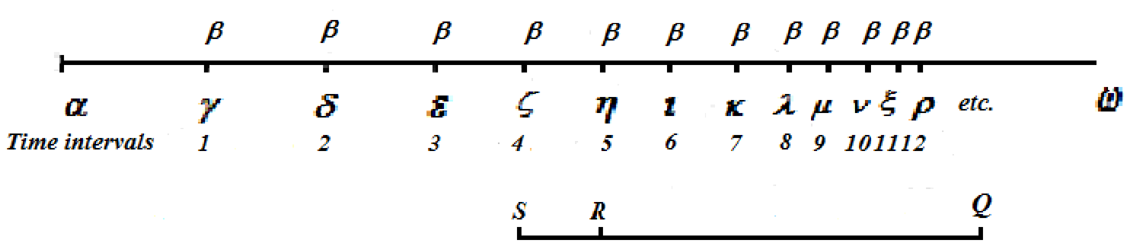


There is a point A, from which a line can be drawn by the flow [*i.e.* regular motion] of another point B, and hence in the first moment [or interval of time] B flows from A to C. In the second moment, from C to D. In the third moment from D to E; and thus henceforth indefinitely, describing the line A C D E F etc. by the equal intervals AC, CD, DE, EF, and with the rest equal successively, and described in equal intervals of time. This line can be said to increase equally by the definition treated above.

Cor. ***From this it is necessary that equally differing quantities are produced by equally differing increments of time.***

Since in the above figure, in a single moment B has progressed from A to C, and in three moments from A to E. Thus in six moments, B has progressed from A to H, and in eight moments from A to K. Moreover, the differences of these moments of time, one and three, and of the other six and eight, obviously are equal to two. Thus also, as above, there are equal differences of these quantities, AC and AE, CE ; and of these AH and AK, HK

Def. 2. ***A line is said to decrease proportionally in becoming shorter, when the point describing the line in equal moments of time, continually cuts off segments in the same ratio to the length of the line left, from which they are being cut off.***



For example. Let the line of the whole sine αω be required to be diminished proportionally: the point by its running motion diminishing that in the ratio β. Finally the ratio of the individual segments to the lines from which they are cut shall be as QR to QS [lower diagram ; the original text has q.r to q.s.]. Whereby QS is cut in the same ratio by the point R (by VI.10 Eucl.) as αω [α, ω in the original, which we have changed as with the rest] is cut by γ, and thus β, crossing from α to γ in the first moment cuts αγ from αω, with the line or the sine γω remaining . Moreover from this length γω, β cuts a similar section in the ratio QR to QS in the next moment of time, which is γδ, with the sine δω remaining. From which hence in the third moment, the segment δω is cut in the same ratio β by ε, with the remaining line or sine εω. From which similarly for a fourth moment, the segment εζ is cut (by the flow β), with the remaining sine ζω. From this ζω, in the fifth moment the segment ζη is cut in the same ratio β [p. 3] with the remaining sine ηω , and thus henceforth indefinitely. Thus I say here that the line of the whole sine (from the first definition) is to decrease proportionally to the sine ηω, or to any other final segment in which β stops, and thus for the others.

[Another number line is constructed with an indefinite number of segments, the length of each segment of which is in a fixed ratio less than one to the preceding segment, and the first segment has length one. The line is called the whole sine as the sines of the arcs or angles between zero and 90^0 are to be displayed upon it in the tables; it can be thought of as a line of length 10,000,000 units, which is the number of subdivisions chosen by Napier for his tables. How these numbers are obtained is the task of the *Constructio*.]

Cor. ***Hence by decreasing in equal time intervals, it is necessary that the lines are left in the same ratio of proportionality.***

Which indeed is the proportion of the above sines that are continually made shorter, αω, γω, δω, εω, ζω, ηω, ιω, κω, &c. and of the segments of the cuts of these αγ, γδ, δε, εζ, ζη, ηι, ικ, & κλ. By necessity also the proportions of the sines of the remainders, obviously γω, δω, εω, ζω, ηω, ιω, κω, & λω, [are to remain in the same proportion], as is apparent from Euclid V.19. & VII.11.

Def. 3. ***Surd magnitudes, or quantities that cannot be described by numbers, are said to be defined by numbers that are nearby, that are a little greater than the surd, but which are not different from the true values of the surds by as much as one unit* [in the final place].**

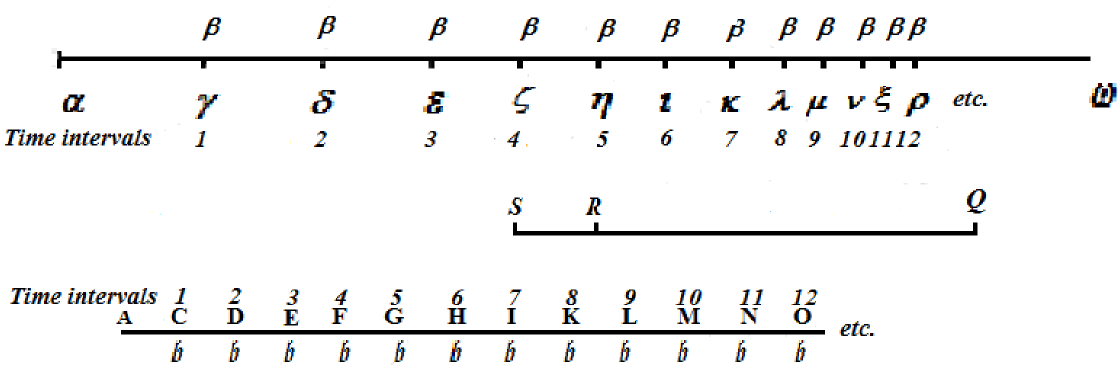
So that the semi-diameter or the rational whole sine shall be the number 10,000,000. The sine of 45 degrees is the square root taken from the square 50,000,000,000,000, which is a surd or unexplainable number , and which is included between the smaller limit 7071067 and the larger limit 7071068. And thus from either or these, the number does not differ by one. Therefore that surd sine of 45 can be said to be defined and explained by the nearest numbers, which are the whole numbers 7071067 and 7071068, and which are defined with the fractions ignored. And indeed with fractions of unity removed from the magnitudes of the numbers no sensible error emerges.

Def. 4. ***Motions are synchronous that are made together and in the same lengths of time.*** As in the above since B shall be moved from A to C in the same time in which β is moved from α to γ; the lines AC and αγ will be said to be described by a synchronous motion.

Def. 5. & postulate.
When some motion can be given that is either slower or faster, by necessity it follows that for any motion, some other motion can be given at an equal speed (that we define to be neither slower nor faster).

Def. 6. ***Hence the Logarithm of any sine is the number that very nearly defines the line that has increased equally, in the same time that the line for the sine of the whole has decreased proportionally into that sine, and with each motion understood to be synchronous, and starting with the same speeds.***

[p. 4.]



[The 'very nearly' phrase indicates that very short time intervals are preferred for increased accuracy, and hence we are dealing with the first order finite difference equivalent of a differential equation, which indicates an intuitive understanding by Napier both of the limiting process and of instantaneous rates of change.]

An example:

As an example, both the above diagrams are repeated, and B is moved everywhere with the same equal velocity always, or with a velocity equal to that with which β initially moves when it is at α. Then in the first moment B proceeds from A to C, and in the same time β proceeds from α to γ, proportionally : the number defining AC is the logarithm of the line or the sine, γω. Then in the second moment of time, B is moved from C to D, and in the same moment β has moved proportionally from γ to δ , and the number defining AD is the logarithm of the sine δω. Thus in the third moment, B proceeds uniformly from D to E, and in the same interval β has moved proportionally from δ to ε, and AE is the number defining the logarithm of the sine εω. Likewise in the fourth interval, B proceeds to F, and β to ζ , and the number AF is the logarithm of the sine ζω. And likewise by keeping the same order, the number AG is the logarithm of the sine ηω (from the definition handed down from above); AH is the log. of ιω; AI the log. of the sine Κω; AK the log.of the sine λω, and so on indefinitely.

Cor. ***Thus the logarithm of the whole sine 10000000 is zero or 0: and as a consequence, the sines of numbers greater than the whole sine have logarithms less than zero.***

Indeed since it is apparent from the definition that with the sines decreasing from the whole sine, the logarithms increase from zero; therefore, on the other hand, for numbers (that we still call sines) increasing to the whole sine, obviously to 10000000, by necessity the logarithms have decreased to 0 or nothing. And as a consequence of numbers that increase beyond the total sine 10000000, (which are now called secants or tangents, and which are no longer called sines), the logarithms are less than zero. [p. 5.]

Thus the logarithms of sines, which are always greater than zero, we call abundant, and either the sign +, or nothing at all is written down before the number. Moreover, the logarithms less than zero we call defective, and note down before such numbers the sign –.

[Thus, positive and negative numbers are accepted; this was unusual at the time but necessary. Vietè, for example, only found the positive roots of equations, and did not attach a meaning to negative roots, usually. In this translation we use the usual terms positive and negative numbers rather than the more expressive terms bountiful, abundant, deficient, etc., to express them, and which Napier was amused to use.]

A Caution.

There is indeed the freedom from the start to have assigned zero or 0 to the logarithm of any sine : but it is better to have applied it to the total sine before the rest; since at any later time it will produce the least trouble for us in the addition and subtraction of this logarithm which is used most often in any calculation. Indeed as the remainder of the sines of numbers less than the total sine are used less frequently : therefore the logarithms of these we put as positive [abundant]; truly the rest are considered as negative [defective]: although there is the choice to make them otherwise initially.

[The use of the term total sine occurs simply because in Napier's Tables, the numbers in proportion from 0 to 10,000,000 are related to sines; they can of course refer to quantities of any kind involved in calculations. We note that these tables proceed from known defined logarithms to the ratios of the proportional lengths that need to be calculated.]

CH. II.

Concerning the Propositions of Logarithms.

Prop.1. ***The logarithms of proportional quantities or numbers are equally spaced.***

On account of the proportionality of the sines, obviously γω to εω, is in the same proportion as ιω to λω, of which the respective logarithms are the numbers defining AC, AE, AG, & AK (as is apparent from Def. 6.); moreover the difference of AC and AE is CE : and the difference of AH and AK is HK. But from Def. 1, and its Corollary, CE and HK are equal : therefore the logarithms of the sines mentioned above are equidistant; and thus the same is true for everything in proportion.

For logarithms have acquired these properties [here called *affections*] and conditions [here called *symptoms*, or things that fall together; these words were not used exclusively in Napier's day as now to express human feelings or the state of one's health, as people were then in closer touch with the original Latin or Greek words, from which the English words arose] from the manner in which they have originated, and it is necessary that quantities derived from them keep these properties. For from their generation and development they have been imbued with this condition or affection, and this law is prescribed for them, in order that they shall be equidistant [p. 6.], when the sines or amounts of these are in proportion (since from the definition of logarithms, and from the motion of these it is apparent, and in the construction of logarithms, when it shall become more apparent) ; hence the logarithms of proportional quantities are equidistant.

Propos. 2. ***With the logarithms of three numbers in proportion, the third is equal to twice the second diminished by the first.***

Since, by Prop.1, the difference of the logarithms of the first and the second is equal to the difference of the logarithms of the second and the third, that is, the second less the first is equal to the third minus the second : Thus, by adding the second to both, the side of the equation comes about that twice the second or double the second less the first is equal to the third, that was to be approved.

Propos. 3. ***Of the logarithms of three numbers in proportion, double the second or mean is equal to the sum of the extremes.***

From Prop. 2. preceding, double the second less the first is equal to the third. To both sides of the equation add the first, then it arises that twice the second is equal to the sum of the first and the third, that is to the sum of the extremes. Q. e. d.

Propos. 4. ***Of the logarithms of four numbers in proportion, the sum of the second and the third minus the first is equal to the fourth.***

Since by Prop.1, from the logarithms of four numbers in proportion, the second minus the first is equal to the fourth minus the third, and to both sides of the equation, add the third, then the second and the third less the first is equal to the fourth. Q. e. d.

Propos. 5. ***Of the logarithms of four numbers in proportion, the sum of the middle numbers (obviously of the second and the third) is equal to the sum of the extremes, clearly the first and the fourth.***

By the preceding Prop.4, the second and the third minus the first is equal to fourth : and to both sides of the equality add the first, and the equality becomes the second plus the third is equal to the fourth plus the first. Q. e. d.

Propos. 6. ***Of the logarithms of four proportional numbers, the triple of either middle number is equal to the sum of the furthest extreme and the double of the nearer number.***

By the second Prop., double the second or mean less the first is equal to the third : and by the third Prop., double this, that is, [p. 7] four times the second less twice the first, is equal to the sum of their extremes, clearly the fourth plus the second. Now if from each side of the equality you take away the second, it becomes the triple of the second less the double of the first is equal to fourth; again to this equality of the sides add the double of the first, and there arises the triple of the second that is equal to the fourth plus double the first: which we had undertaken to show.

[These propositions show how the multiplication and division of numbers in proportion can be replaced by the addition and subtraction of their logarithms.]

Notes.

Up to this point we have explained the origin and properties of logarithms : truly there is a need for the explanation of the places where they are to be used in a calculation, or in an arithmetical method [such as extracting roots]. But since in the first place we have presented the whole canon of logarithms and their sines of the quadrant to the individual minutes, thus we will pass over the explanation of the construction of the logarithms to a time more suited, and so we hurry to these tables, in order that we may have a first experience of the usefulness of logarithms, the other matters may be more pleasing to tackle after this, or perhaps may be less displeasing by being suppressed in silence. Indeed I await the judgment and censure of the learned men concerning these tables, before advancing the rest to be published, perhaps rashly, to be examined in the light of envious disparagement.

CH. III.
Comprising a Description of the Table of Logarithms, and of its Seven Columns.

Section. 1. ***The first column expresses the arcs increasing from 0 to 45 degrees: and it is understood that also it expresses the remaining parts of these arcs in the semicircle.***

Section. 2. ***Moreover column seven expresses the arcs from the quadrant* [i. e. 90^0] *decreasing to 45 degrees : and it is understood that it also expresses the remaining parts of these arcs in the semicircle.***

Section. 3. ***Thus the arcs of the other column are the complementary arcs from the corresponding regions of the angles.*** [p.8]

4. ***And in the first column the smaller acute angle of any plane right-angled triangles are expressed.***
5. ***Moreover in the seventh column, the larger acute angle of any plane right-angled triangles are put in place from the corresponding region of the angle in the first column.***
6. ***In the second column there are the sines of the arcs of the first column.***
7. ***And these are the sines for which the smaller leg is subtending the smaller angle of the right-angled triangle, the base or hypotenuse of which is the total sine.***
8. ***In the sixth column there are the sines of the arcs of the seventh column.***
9. ***And these are the sines of the larger leg subtending the larger angle of the same right-angled triangle, of which clearly the hypotenuse is the total sine.***
10. ***Thus for every right-angled plane triangle there is an equiangular and similar triangle in the corresponding row of the table, composed from the total sine, and wih the sines in the second and sixth columns.***
11. ***The third column contains the logarithms of the sines of the arcs on the left-hand side.***
12. ***Which are the logarithms of the proportion of the shorter side of the right-angles triangle with the same hypotenuse.***
13. ***And likewise these are the logarithms of the sines of the complementary arcs on the right-hand column, which we call antilogarithms.*** **[This is not the modern meaning of the term antilogarithm.]**
14. ***The fifth column contains the logarithms of the sines of the right-hand arcs.***
15. ***Which are the logarithms of the proportion of the greater leg of the right-angled triangles with the same hypotenuse.***
16. ***Likewise, these are the antilogarithms of the sines of the left-hand arcs, or the logarithms of the complementary angles.***
17. ***And finally the fourth or middle column contains the differences between the logarithms of the third and fifth columns. Thus this column is two-fold, and can be considered to be positive or negative.***
18. ***The positive differences are those which arise from the subtraction of the logarithms of the fifth column from those of the third column.***
19. ***The negative differences are those which arise from the subtraction of the logarithms of the third column from those of the fifth column : and which hence are less than zero.***
20. ***The positive differences are called the differentials of the numbers of the left-hand arcs.***
21. ***And these are the logarithms of the proportion of the shorter leg of the right-angled triangle to the longer leg of the same.*** [p. 9]
22. ***And likewise they are the logarithms of the tangents of the left-hand arcs.***
23. ***Moreover the negative differences are called the differentials of numbers of the right-hand arcs.***
24. ***And they are the logarithms of the proportion of the longer leg of the right-angled triangle to the shorter leg of the same.***
25. ***And likewise they are the logarithms of the tangents of the right-hand arcs.***

26. Also every left-hand arc, to the remaining semicircle, is called the arc of the complement of the arcs of the right-hand sines, logarithms, and negative differentials.

27. And on the other hand, every right-hand arc, to the semicircle of the remainder, is called the arc of the complement of the left-hand arcs of the sines, logarithms, and positive differentials.

Notes.

28. Here it is to be noted, if you have made the logarithms of the third column negative (obviously prefixed by the — sign,)then they make the logarithms of the hypotenuses, or of the secants of the arcs or the right-hand seventh column. [Recall that since the logarithm of whole sine is zero, forming the secant inverts the sign of the right-hand logarithm of the complementary angle. Likewise there is an inversion in the following sections.]

29. And these also become the logarithms of the proportion of the hypotenuse of the right-angled triangle to the shorter leg.

30. And if you make the logarithms of the fifth column negative, they make the logarithms of the hypotenuses or of the secants of the left-hand first column negative.

31. These also become the logarithms of the proportion of the hypotenuse of a right-angled triangle to the larger side of the same. Truly for a knowledge of the right-angled triangles to be compared, only the sines, and the arcs and logarithms of these with the differentials are required : but for spherical right-angled triangles, without the sines, only the arcs, and the logarithms and differentials are sufficient : thus we have excluded the hypotenuses or secants, and the ratios that follow from the table : just as we wish to ignore the sines themselves in spherical triangles. Nevertheless we will show you as we proceed that it is possible, if it should please you, for all these to be put into use readily enough in the rectilinear use, but truly with spherical triangles minimally.
[p. 10.]

CH. IV.
Considering the uses of the tables, and of the numbers in the tables.

Section. 1. ***The logarithms of sines, tangents, and secants found precisely in their tables are to be given with the same precision.***

By sections 11 and14 of Ch. 3, for a given sine found in the second or in the seventh column of our table, the logarithm of this is found on the same line in the third or fifth column. Therefore the logarithms of the sines of the tables are obtained exactly. Moreover, from the numbers found in these tables for the tangents or secants, the arcs can be found. Truly with the arcs known, the logarithms of the tangents or their differences with the sign, can be shown from the middle column our table, by Sections 22 and 25. And the logarithms of the secants inverted in columns three and five, yet with the sign — put in front of these, by Sections 28 and 30. Therefore the logarithms of sines, tangents, and secants of the table can be found.

[These values are corrected, rather than Napier's values, which are in error in the last few places; note that the sines of angles greater than 45 degrees proceed from the right hand side of the tables upwards.]

Examples of sines.

Degree 44

min.	*Sine*	*Logarithm*	*Differential +/-*	*Logarithm*	*Sine*	*min.*
0	**6946584**	**3643351**	**349137**	**3294214**	**7193398**	**60**

…Degree 45

The sine is 6946584. I seek the logarithm. I can find that sine precisely in the second column corresponding to the arc 44 degrees and 0 minutes, and in the third column of the same line standing at 3643349, there is the logarithm of the sine that I sought.

Degree 43

min.	*Sine*	*Logarithm*	*Differential +/-*	*Logarithm*	*Sine*	*min.*
50	**6925630**	**3673561**	**407356**	**3266205**	**7213574**	**10**

…Degree 46

Likewise, the sine is 7213574, and the logarithm is sought. Here the sine is found that corresponds to the arc 46 degrees and 10 minutes, and nearby to this is 3266204, the logarithm sought of this number.

Examples of tangents.

The logarithm of the tangent 2186448 is sought. The arc 12 degrees and 20 minutes corresponds to this tangent and the corresponding logarithm sought lies in the middle column of our table, or the positive difference 15203064.

Degree 12

min.	*Sine*	*Logarithm*	*Differential +/-*	*Logarithm*	*Sine*	*min.*
20	**2135988**	**15436559**	**15203069**	**233490**	**9769215**	**40**

Likewise if you seek the logarithm of the tangent 4573629, you hit it in the table of tangents the arc of which is 77 degrees and 40 minutes; and the same difference of this arc in our table, however negative, is obviously — 15203064. [p. 11.]

20	**2135988**	**15436559**	**15203069**	**233490**	**9769215**	**40**

…Degree 77

Examples of secants.

To the secant 18118009, there corresponds in the table of secants of arcs, 56 degrees and 30 minutes, and it is agreed that — 5943212 is the negative reciprocal for this arc in our table, the logarithm of the secant 18118009. Thus for the secant 13118337 you find the logarithm — 2714255, and for the secant 3960592 you strike the logarithm – 3336533. [For secants, a right-angles triangle is taken with the side adjacent to the angle set to the value 10,000,000, which as we know, has the logarithm zero. Hence the log of the secant is the negative of the log of the complementary angle.]

Degree 33

min.	*Sine*	*Logarithm*	*Differential +/-*	*Logarithm*	*Sine*	*min.*
30	**5519370**	**5943214**	**4126626**	**1816588**	**8338858**	**30**

Degree 56

2. ***To estimate the logarithms of given numbers of sines, tangents, and secants not found in tables.***

For a given number similar to a value in the tables, if it has be given as the original number × 10, × 100, × 1000, × 10000, × 100000, or × 100000, then you can look in the second or in the sixth column of our tables; or if you prefer, in the tables of tangents or of secants: and the arc of this number is noted. Indeed the logarithm of this number may be elicited from our table, and is that you seek, yet bear in mind either by remembering the number of places noted to be expressed, or the number of figures to be multiplied by. For example if the logarithm of the number 137 is sought, and which is not found in the tables : then you will find among the sines the numbers 14544, 136714 and 1371564, and among the tangents 13705046, and among the secants truly the number13703048, which is the most alike of all to the given number, provided the final or right-most five figures are understood to be erased. Hence the logarithm (by the preceding section, or by sections 28 and 30 of Ch.3) of the secant 13703048 , of the arc 43 degrees and 8 minutes is sought, and — 3150332 is found, which is taken for the logarithm of the given number 137. Also, it is to be taken into account, by having recorded or by remembering, that the final five figures are still to be taken away, expressly marked out in this manner : — 3150332 — 00000. [We can think of this as being like the modern index notation $\times 10^{-5}$, but bear in mind that 5 digits must be cut from the number to which the log. corresponds.] Similarly if the above expression of the logarithm of the number 137 is sought through the tangent 13705046, from the arc 53 degrees and 53 minutes of that tangent, the logarithm — 3151790 of that tangent 13705046 is found in the middle column (by sect. 25), which since it exceeds the given 137, five places or figures, thus — 3151790 — 00000 is the logarithm of the given number 137. Yet this exact logarithm is as much less than the true value, by as much as the number 13705046 is different [p. 12.] from the number 13700000, or to the other given for × 100000 [the secant]; but this error surpasses the number by $\frac{5046}{100000}$ parts of unity. If finally you should seek the logarithm of the number 137 from the above sine 3705046 , this logarithm (by this section and sect.11 of Ch.3.) is taken to be 19866327— 0000. Nor should the + sign appear from the operation, when the number represented by the given figures, of the quantity required, exceeds the sine of a number of figures similar to that, since this rarely happens; [in modern terms we have an exponential decay, with the sought numbers on the y-axis, and their logarithms along the x-axis; usually, the number for which the logarithm is sought is larger than those taken from the table, and hence an amount has to be taken from the logarithm to find the more accurate value of the logarithm; occasionally, however, a logarithm that is too small arises, corresponding to a sine in the tables that is too large, and in this case, an amount has to be added on to the logarithm for better agreement.] For, if the logarithm of the number or the discrete quantity 232702 is sought, you will find the sine 23271 in the table, of all numbers the most similar to this number, but one digit short for the required figure. Therefore by finding the logarithm of this in the table (by section 11. Ch. 3), which is 60631284, with one cipher being put in place with the + sign, and 60631284 + 0 is made the estimate for the logarithm of the number sought 232702. But the best way of estimating all the logarithms is that by which they have been made : concerning that more elsewhere.

3. ***Hence, as the logarithms in the above first section are presented simple and pure: thus in the preceding section with zeros put in place, the logarithms have emerged impure.***

4. ***The logarithms of the same sign to be added, that is the sum of both with the common sign to be shown.***

For by adding — 56312 to — 73495 there comes about — 129807. Likewise by adding 4216 to + 5392, 9608 is produced. Thus 3219 —00 to 4360 —000 makes 7579 —00000.

5. ***When logarithms of opposite signs are added, it is the difference of these with the sign of the larger number that is to be shown.***

For by adding — 210 to 332, + 122 is produced. Likewise by adding—210 to + 192 — 18 is produced. Thus —210 + 000 added to 332 — 00 gives 122 + 0. Likewise —210. — 000 to 192 + 00 gives —18 — 0.

6. ***Of two logarithms, the one negative and the other properly called the positive of that: since both the number and the ciphers are common or the same : truly they have all the positive or contrary negative sign within.***

As the positive number is 56312, so the negative number is — 56312. Likewise the positive number is 56312 — 00, and the negative is — 56312 + 00. Thus the positive number is 56312 + 00, the negative is — 56312 — 00.

7. ***To subtract a positive number is to add the negative of this number.*** [p. 13.]

For to subtract the positive number 56312 from — 73495 will be the same, when the negative of that number, which (by 6) is — 56312 is added to the same, — 73495, and they make (set out by 4.) — 129807. Thus to subtract 56312 + 00 from — 73495 — 00 is the same as adding — 56312 + 00 to — 73495 — 00, which becomes (by 4. & 5. preceding) — 129807— 00000.

8. ***To subtract a negative number is to add the positive of this number.***

For to subtract the negative number — 4216 from + 5392 is the same as to add 4216 to 5392, and (by 4) 9608 is made. Thus it is the same to subtract — 4216 + 00 from 5392 + 0, since by addition they make (by 4 above) — 129807. Thus to subtract 56312 from — 73495 — 00 is the same as to add 4216. — 00 to 4216. + 00, and 9608 — 0 is produced.

9. ***The logarithm of any number can be increased or diminished as far as to hold the original value by adding or subtracting one the following logarithms, 23025842 + 0 or 46051684 + 00, or 69077527 + 000, or 92103369 + 0000, or 115129211 + 00000, which I can save with the initial value for the number. In short with no significant difference.*** [These are multiples of the first logarithm, which corresponds to multiplying the original number by 0.1 a number of times, thus decreasing or increasing the number of significant figures by one, each time this is done; Napier's tables gives *e* the value 2.71828288267020 ; the underlined places are in error. Thus, one presumes for the first time in the history of mathematics, a set of numbers had emerged from which one could calculate *e*.

Thus, using the true values, the last two places are in error above :

power of ten	Napier logarithm
1.0000000000E+06	2.3025850930E+07
1.0000000000E+05	4.6051701860E+07
1.0000000000E+04	6.9077552790E+07
1.0000000000E+03	9.2103403720E+07
1.0000000000E+02	1.1512925465E+08
1.0000000000E+01	1.3815510558E+08
1.0000000000E+00	1.6118095651E+08

It is a feature of the exponential function that a change in the argument gives rise to the same fractional change in the function whereever it occurs, and vice versa : thus, for the function to drop to a tenth of its initial value, the argument of the exponential must increase by 2.3025850930E+07 Thus the idea of half-life in radioactivity, etc, where it does not matter what the initial magnitude is, it always decreases by the same fraction in the same time interval. Hence, to find the logarithm of some number with less than 7 places, add the appropriate number of zeros to give the number 7 figures, then find the logarithm of the number closest to this in the tables, and finally take the logarithm corresponding to the power of ten added on.]

For let the logarithm be 39156 — 0, if to which you have added one of these logarithm, as for example 23025842 + 0, then the logarithm becomes 23064998 for a larger number, but with the same value again which is 39156 — 0. And if the quantity or the value of this number, of this logarithm 39156 — 0 (by the following sections 12 and13 of this section) is 9960920, from which you take the furthest final figure, as — 0 noted, and it becomes 996092. But the value of the number corresponding to this logarithm 23064998 (by sections 12 and 13 of this) is also 996092. The same as before. [Essentially, we are moving backwards and forwards through the tables, losing or picking up extra digits as we go.]

An example of diminishing the logarithm.

Let the logarithm to be diminished be 25451769, from which if you take 23025842 + 0, then 2425927 — 0 is left of the value of the previous 25451769. For the value of the number that corresponds to the pure and simple logarithm 2425927 is ten times the value of the other of these logarithms [p. 14]. Therefore the values are in turn equal [In the sense that the significant figures are the same, and only a zero has been added]. Indeed there is no other significance in adding the logarithm 23025842 + 0, other than that the value of the number, to which the logarithm has been added, has been divided by ten, and to this tenth part a single cipher is to be added [to the logarithm, to indicate it is not pure by this single digit or the original number corresponding to the pure log.]: truly the subtraction signifies the same value of the logarithm from which a tenfold has been taken away, and from which a single cipher is to be thrown away, and the value remains the same as before. Thus 46051684 + 00 added to the logarithm signifies a hundredth part of the original value and two ciphers are adjoined [indicating two figures dropped from the end of the number], and taking this logarithm away increases the original number by a factor of hundred, with two ciphers to be rejected, and thus for the rest expressed above.

[Thus, for numbers and their logarithms in the tables, the value of a rounded number is to be calculated from the logarithm in the table for the full number, and then the result for the required number of places found by adding the appropriate log for that power of 10.]

10. ***Thus, if you should have added some number of zeros to a small logarithm, or from a logarithm increased by zeros, you subtract some zeros, then if from the said logarithms you subtract all the ciphers, then from an impure logarithm there is produced the pure logarithm for the number of the same value.***

As in the first example above, let the impure logarithm be 39156 — 0, that is to be freed from its cipher and negative sign. Therefore add to this 23025842 + 0, and thus, as above, the pure number 23064998 of the first value. Thus from the impure logarithm 63584468 + 00 if you take away the logarithm 46051684 + 00 all the ciphers are obviously taken away, and the pure logarithm 17532784 is left, and of the same value as the that previous impure logarithm.

11. ***If to the negative logarithm for some number you should add some greater logarithm from the above list of logarithms in section 9, then a positive logarithm for the same number will arise with a positive logarithm.***

As to the logarithm —28595270 —0000 add one from section 9 for a greater number, e. g. 46051684 +00 and hence the logarithm becomes 17456414 — 00 of the same value, and positive.

12. ***You can exhibit the sines, tangents, secants or the numerical values of any logarithms found in our table of logarithms, by Ch. 3, Sect.11.14.22.25.28.30. , whether they are pure or impure.***

As for the logarithm of 36 degrees and 40 minutes, which is 5155724 in the third column, there corresponds the sine of this, 5971586 in the second column : and for the negative of this — 5155724 there corresponds the secant of 53 degrees and 20 minutes in the table of secants, 16745970 [p. 15.] Likewise, for the logarithm of the difference 2950794, and for the negative of this — 2950794 there corresponds the tangent 13432331, obviously of 53 degrees and 20 minutes. Thus, for the logarithm 2204930 in the fifth column, the value of the number in the sixth column, 8021232, obviously is the sine of 53 degrees and 20 minutes, obviously for the negative of the same — 2204930, the value of the number is the secant of the number 12466913, agreeing with the arc for 36 degrees and 40 minutes.

Examples of impure logarithms.

It is required to find the value of the impure logarithm 97796 — 0, to which there corresponds the sine for the number 9902681 in our table [complement of 8^0], from which take away the right-hand figure, (as — 0 shows) , and it becomes 990268, the value sought that corresponds to the logarithm 97796—0. [The actual value of the logarithm of the number 990268 is found by adding 23025842 to 97796, giving 23123638]. Again, for the logarithm 25451769 + 00 the value is 78459100, since for the pure logarithm 25451769 in our table there corresponds the sine 784591[,we simply multiply by 100]. Likewise for the logarithm — 349136 —00 in the fourth column, to be found at 46 degrees, the corresponding value is 103553, since the tangent of 46 degrees is 10355302 [The logarithm corresponding to 103553 is found by adding 46051684 to — 349136, giving 45702548]. Thus of the logarithm —6350305 —00, to be found in the third column at 32 degrees, the value is 188708, since the secant of the complement of 32 degrees obviously 58 degrees, is 18870800, the two right-hand figures 00 of which are to be deleted on account of the —00 adjoined to the logarithm.

13. ***To estimate the numeral values of given logarithms not found in our table.***

For common geodesic measurements and many more it is sufficient, to accept for the value of the number, the given value of the nearest logarithm in the table. Truly if you want to get closer to the mark, increase or diminish the given logarithm for the number held, according to section 9 of this Chapter, with the former value kept, until either there is found in the table, or in some other little table, a value close enough to the value set out, is that sought. E. g. , the value of this logarithm 23149721 + 0, to which there is not found a satisfactory value in the tables, or one close enough. Truly if from that logarithm you have taken away [p. 16.] 23025842 + 0, there is left 123879, to which under 81 degrees is found the satisfactory and nearby number 123881, and the sine of this is 9876883, found by the former section, 23149721 + 0 is the value of the logarithm proposed, that was required.

Note.

For this section, and the second section of this chapter, we want to give some advice about the logarithms of given numbers, and on the other hand, the numerical values of the numbers corresponding to given logarithms (when they cannot be found in the table); all of which are produced with the greatest accuracy by using the method by which all are made or resolved : that is, for a given sine in the table you can descend by via geometric means proportionally until you reach to the next smaller sine that you can find nearby in the table of sines ; while similarly from the table of logarithms you can descend through all the arithmetic means in agreement to the corresponding arithmetic mean, and of these means the last will be the logarithm of the first. On the contrary by resolution, you can descend from a given logarithm via arithmetic means to the next smaller value in the table, and from the value of this in the table, similarly, you can descend also through all the congruent geometric means, and of these the final will be the numerical value of the first of the these logarithms. [As the geometric means decrease from left to right, and the arithmetical means decrease from right to left.] Truly which equally-spaced arithmetic mean agrees with a corresponding geometric mean from continued proportion is difficult to say. Whereby concerning these (God willing), we will defer until such time as a fuller understanding can be given, when we set about an explanation of how the table of logarithms is constructed.

CH. V.
Concerning the fullest use of Logarithms, and of their expedient practice.

Problem 1. ***From the logarithms of three proportions, from the given middle logarithm and from the one extreme, to give the other extreme and the proportion of this; or to give the arc by a single doubling and subtraction.*** [p. 17.]

Since by the second proposition of Ch.2 : twice the middle (obviously of the logarithm) with the other extreme taken is equal to the remaining extreme, thus from twice the mean logarithm given take the logarithm of the extreme given, and the logarithm of the extreme sought is left: to which in the third, fourth, or fifth column of the table there is found the arc that corresponds in the first or seventh column : moreover the sine is found either in the second or in the sixth column : and the secants or tangents for the extremes sought are found in these tables according to Ch.3, sections 1, 2, 6, 8, 11,14, 22, 25, 28, 30.

An Example.

10000000 is given for the first proportion, and 7071068 for the second, the third is sought. In common practice the square of the middle term is found, and that is to be divided by the first, [to give the other extreme term]. But more easily by doubling the logarithm 34655735 of the middle term, and from this double, which is 6931470, by taking the logarithm of the first (which is 0) : and so there remains 6931470, the logarithm sought: of which the arc is 30 degrees, and the sine 5000000. (obviously for the proportional sought) that you can find next to the logarithm. Therefore 10000000, 7071068 and 5000000 are the three numbers in proportion, the final of which we have found by doubling and subtracting alone, that we has promised to find. Likewise for the two proportions, first 10562556, and secondly 7660445, or even the logarithms of these can be given : —547302 and 2665149. The third can be obtained thus : from the double of 5330298 take —547302, and (by sect. 8 of Ch.4) the logarithm 5877600 is produced of the sine of 33 degrees and 45 minutes, of which the sine 55555702 is the third sought for the proportion.

Prob.2. ***From the logarithms of three proportions, from the given logarithms of the extremes, to give the middle logarithm and the proportional for this and the arc, from one addition and division by two.***

Since by sect. 3 of Ch. 2, double the middle logarithm is equal to the sum of the logarithms of the extremes, and thus by adding the logarithms of the extremes, the sum is to be divided in two, and the emerges the logarithm of the middle logarithm, and hence the middle arc can be noted from the columns [p. 18.] and sections as above.

For the sake of an example.

The extremes 10000000 and 50000000 are given and the mean proportion is sought. That is commonly acquired by multiplication the one given by the other, and the square root is to be extracted from the product. Truly we can do this more easily thus. We can add the given logarithms of the extremes, of the first 0 and of the last 6931470, and the sum is 6931470, which is halved, and it becomes 3465735, the logarithm of the wanted middle. Hence the middle number 7071068, and the arc of this 45 degrees, is obtained for the above mentioned middle term. Likewise, the extremes 10562556 and 5555702 are given, and the logarithms of these are —547302 and 5877600. The sum of the addition of these is 5330298, by sect.5 of Ch.4, that we can divide by two, and the logarithm is 2665149, and the arc of this is 50 degrees: and the sine, or the mean or middle proportion sought, is 7660445, which has been found by addition and division by two alone.

Prob.3. ***Form the logarithms of four proportional numbers, from three given, or from the arcs of these, the fourth logarithm is to be found, and the sine and the arc of this by one addition and subtraction.***

In this problem what is sought for the fourth can always be set up as the ratio the given first has to the second is equal to the ratio the third has to that sought. And since thus the sum of the constituents from the logarithms of the second and the third less the logarithm of the first is equal to the logarithm of the fourth sought which is equal to the fourth logarithm by sect.4. of Ch.2. Thus, add the logarithms of the second and third, and hence take away the logarithm of the first, and the logarithm of the fourth sought comes about : and the arc of this.

For the sake of an example.

Let the ratio be : 7660445 to 9848078 thus as 50000000 to the fourth, which we seek. This can be done in the usual way by taking the second and the third as a product, and by dividing by the first. But you can do this thus in an easier way, if you add the logarithms of the second153088 and the third 6931469, that becomes 7084557: and from this you take the logarithm of the first, which is 2665149, and there is left 4419408, the logarithm of the fourth, the sine of which is 6427876 and is the desired fourth part, and the arc of this sine is 40 degrees. Likewise it comes about if, (with the sines removes) only with the arcs of the three given 50 degrees, [p. 19.] 80 degrees, and 30 degrees. And if from the logarithm of the arcs of 80 degrees and 30 degrees by taking the logarithm of the arc of 50 degrees there is left the logarithm of 40 degrees. And thus the arc of 40 degrees can become known from the sines, or from the multiplication and division of these, as we promised in the beginning.

Another example.

Let the tangent or positive number of 43 degrees be to the sine 57 degrees, thus as the positive tangent of 35 degrees to the fourth unknown sine , thus we find the sine with the arcs of the sines and tangents ignored. We add the differential logarithm of 35 degrees, obviously 3563784 found in the middle column to the logarithm of 57 degrees, obviously 17593572 located in the fifth column : with the result 5323156. We take away the differential of 43 degrees which is 698698, and the remainder is 4624458. The logarithm of the fourth proportional number (the sine of course) with being found in the third column by sect.11 of Ch. 3, you will find it in the first column next to 39 degrees and 2 minutes, which is nearly the arc sought of the fourth proportional, or the sine ignored.

The arc is found from this ratio of the proportions, without the sines, tangents, secants, or any proportional numbers.

Certainly this short method helps to measure out the angles of plane triangles, and brings together the greater part of spherical trigonometry, as will become apparent in its own place.

Prob.4. ***For four given numbers in continued proportion with their extremes or arcs given, to find the middle terms in some way, or the arcs of these ; with a simple repartition introduced for the arduous extraction of the cube root.***

Since with the logarithms of these, the triple of either middle quantity is equal to the sum of the more distant extreme and to double the nearer proportional number, by Prop. 6 of Ch. 2. Thus, add double of the other extreme logarithm the remaining extreme, and divide the result by three, and the logarithm of the middle term before the nearest extreme is found [p. 20.], and in the same way the other middle proportion is found. [Thus, if the logarithms of the equally spaced proportions satisfy : $\ell_4 - \ell_3 = \ell_3 - \ell_2 = \ell_2 - \ell_1$, then $3\ell_2 = 2\ell_1 + \ell_4$.] As for example: Let the first extreme be 4029246 and the last truly 10562556. The middle proportions are sought, which you thus find without the extraction of cube roots. The logarithms of the given proportional numbers are 9090051 and , and add to the double of this 18180102 thus add —547302, and the sum is 17632800, which on division by three gives 5877600. Of the sine 55555702, this is the logarithm, and this is the first middle proportional number sought. Likewise in the same manner, add the double of this —547302, that is —1094604, to that 9090051, and 7995447 is produced, which divided into three equal parts gives 2665149. The logarithm, the sine of which 7660445, is the last middle proportion sought. Therefore the four numbers in continued proportion are 4029246, 5555702, 7660445, and 10562556.

Another example.

Let the given extreme proportions be 14142135 and 5000000, the logarithm of the secant of this found in our table of secants is — 3465735, and truly the logarithm of 5000000 is 6931470, to the double of this, 13862940, add —3465738, and the sum is 10397205, that divided by three, and for which the logarithm is + 3465735, the logarithm of the lesser middle proportion closest to 5000000, that is 7071068. Thus, with double — 3465735, which is —6931470, add + 6931470, and hence it becomes 0 or zero, that on being divided by three also gives 0, the sine of this is the value 10000000 for the remaining larger middle proportional number. Thus the four numbers in continued proportion are 14142135, 10000000, 7071068, and 50000000.

CONCLUSION.

From these demonstrations, the learned can judge how great a benefit is bestowed by these logarithms : since by the addition of these for multiplication, by subtraction for divisional, by division by two for the extraction of square roots, and by three for cube roots, and for other prostaphæresces **[the use of certain trigonometric identities to replace a multiplication by an addition or subtraction, as used by J. Burgi and others.],** ***all the heavier work of calculation is avoided; and we have given some examples of this first in this book. But in the book following we shall set out their proper and particular use in that noble kind of Geometry, that is called Trigonometry.***

End of the first book.

MIRIFICI LOGARITHMORUM

Canonis description,

Ejus usus, in utraque Trigonometria, ut etiam in omni Logistica
Mathematica, Amplissimi, Facillimi & expeditissimi explicatio.

Autore ac Inventore
JONANNE NEPERO.
Baron Merchistonii, etc. Scoto.

IN MIRIFICIUM
Logarithmorum Canonem

Praefatio.

Quum nihil sit (charissimi mathematum cultores)mathematicae praxi tam molestum, quodque; Logistas magis remoretur, ac retardet, quam magnorum numerorum multiplications, partitiones, quadrataeque ac cubicae extractiones, quae praeter prolixitatis tædium, lubricis etiam erroribus plurimum sunt obnoxiæ : Coepi igitur animo revolvere, qua arte certa & expedita, possem dicta impedimenta amoliri. Multis subinde in hunc finem perpensis, nonnulla tandem inveni præclara compendia, alibi fortasse tractanda : verum inter omnia nullum hoc utilius, quod una cum multiplicationibus, partitionibus, & radicum extractionibus arduis & prolixis, ipsos etiam numeros multiplicandos, dividendos, & in radices resolvendos ab opera rejicit, & eorum loco alios substituit numeros, qui illorum munere fungantur per solas additiones, subtractiones, bipartitiones, & tripartitiones. Quod quidem arcanum, cum (ut cætera bona) sit, quo communius, eo melius: in publicum mathematicorum usum propalare libuit. Eo itaque librerè fruamini (matheseos studiosi) & qua à me profectum est benevolentia, accipite. Valete.

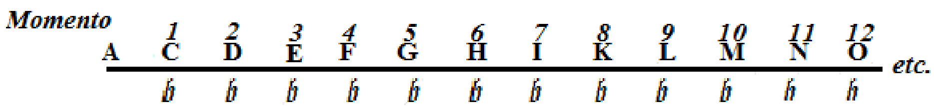
IN LOGARITHMOS.

Qua tibi cunque sinus, tangents atque secants;
Prolixo praestant, atque labore gravi;
Absque labore gravi, & subito tibi, Candide Lector,
Hac Logarithmorum parva tabella dabet.

LIBER 1.

Caput 1.
De Definitionibus.

Def. 1. ***Linea æqualiter crescere dicitur, quam punctus eam describens, æqualibus momentis per æqualia intervalla progreditur.***

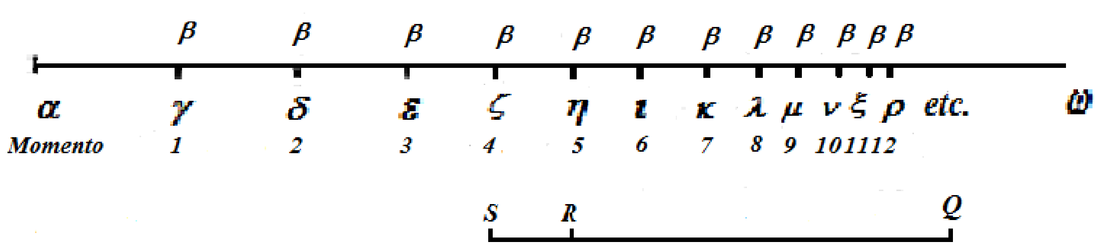


Sit punctus A, à quo ducenda sit linea fluxu alterius puncti, qui sit B, fluat ergo primo momento B ad A in C. Secundo momento a C in D. Tertio momento a D in E, atque ita deinceps in infinitum describendo lineam ACDER, &c. intervallis AC, DC, DE, EF, & cæteris deinceps aequalibus, & momentis aequalibus descriptis, dicetur haec linea per definitionem superius traditam aequaliter crescere.

Corollarium. ***Unde hoc incremento quantitates æqui-differentes temporibus æqui-differentibus producti est necesse.***

Ut in superiori schemate unico momento B ab A in C, & tribus momentis ab A in E progressum est. Sic sex momentis ab A in H, & octo momentis ab A in K. Sunt autem illorum momentorum unius & trium, & horum sex & octo differentias aequales, scilicet duorum. Sic etiam erunt quantitatum illarum AC, & AE, & harum AH, & AK differentiae CE, & HK, aequales, atque differentes ergo, ut supra.

2. Def. ***Linea proportionaliter in breviorem decrescere dicitur, quum punctus eam transcurrens aequalibus momentis, segmenta abscindit ejusdem continuo rationis ad lineas a quibus abscinduntur.***



Exempli gratia. Sit linea sinus totius α, ω proportionaliter minuenda. Sit punctus transcursu suo eam minuens β. Sit denique ratio segmentorum singulorum ad lineas a quibus abscinduntur, ut q.r ad q.s. Qua ergo ratione secatur q.s. in r. eadem ratione (per 10.6 Eucl.) secetur α, ω in γ, atque sic β, transcurrens ab α in γ primo momento ab α, ω abscindat α, γ relicta linea seu sinu γ, ω . Ab hac autem γ, ω procedens β secundo momento abscindat simile segmentum quale est q.r. ad qs, quod sit γ, δ, reliquo sinu δ, ω. A quo proinde tertio momento abscindat β simili ratione segmentum δ, ε relicto sinu ε, ω. A quo similiter quarto momento abscindatur (fluxu β) segmentum ε ζ, relicto sinu ζ, ω. Ab hoc ζ, ω, quinto momento abscindat β eadem ratione [p. 3] segmentum ζ, η relicto sinu η, ω (ex praemissa definitione) proportionaliter decrescere in sinuum η, ω, aut in alium quemvis ultimum in quo sistit β, & sic in aliis.

Cor. ***Unde hac æqualibus momentis decremento, ejusdem etiam rationis proportionales lineas relinqui est necesse.***

Quae enim superius est continua proportio sinuum minuendorum, αω, γω, γω, δω, εω, ζω, ηω, ιω, κω, &c. atque segmentorum ab eis abscissorum αγ, γδ, δε, εζ, ζη, ηι, ικ, & κλ. Eadem erit necessario etiam sinuum relictorum proportio, scilicet , γω, δω, εω, ζω, ηω, ιω, κω, & λω, ut ex 19.prop.5. & 11.prop.7.Eucl. patet.

3.Def. ***Quantites surdæ, seu numero inexplicabiles, numeris quàm proximè definiri dicuntur, quum numeris majusculis, qui à veris surdarum valoribus unitate non differant, definiuntur.***

Ut sit semi-diameter seu sinus totus rationalis numerus 10,000,000 erit sinus 45. graduum radix quadrata 50,000,000,000,000, quae surda seu irrationalis & numero inexplicabilis est, atque inter terminos 7071067. minorem, & 7071068. majorem includitur. Ab horum itaque utrovis non differt unitate. Surdus igitur sinus ille 45. graduum quam proximè dicitur definiri & explicari, quum per numeros integros7071067. vel 7071068. neglictis fractionibus definitur. In magnis etenim numeris ex fragmentis unitatis spretis nullus error sensibilis emergit.

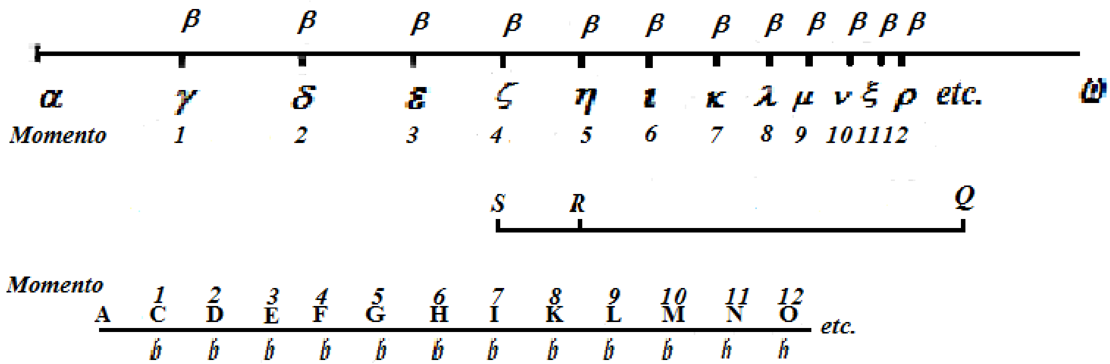
4.Def. ***Synchroni motus sunt, qui simul & eodem tempore fiunt.***

Ut in superioribus esto quod B. moveatur ab A in C. eodem tempore quo β movetur ab α in γ. dicentur rectae AC, & αγ synchrono motu describi.

5.Def. & post. ***Quum quolibet motu & tardior & velocior dari possit, sequetur necessario cuique motui aequivelocem (quem nec tardiorem, nec velociorem definimus) dari posse.***

6.Def. ***Logarithmus ergo cujusque sinus, est numerus quam proxime definiens lineam, quae aequaliter crevit, interea, dum sinus totius, linea proportionaliter in sinum illum descrevit, existente utroque motu synchrono, atque initio aequiveloce.***

[p. 4.]



Exempli gratia repetantur ambo superiora schemata & moveatur B. semper & ubique eadem seu aequali velocitate qua coipit moveri β initio quum est in α. deinde primo momento procedat B. ab A in C, & eodem momento procedat β ab α in γ, proportionaliter : erit numerus definiens AC logarithmus lineae, seu sinus γω. Tum secundo mometo promoveatur B à C in D, & eodem momento promoveatur proportionaliter β à γ in δ , erit numerus definiens AD logarithmus sinus δω. Sic tertio momento procedat aequaliter B à D in E, & eodem momento promoveatur proportionaliter β à δ in ε erit numerus definiens AE logarithmus ipsius sinus εω. Item quarto momento procedat B in F, & β in ζ , erit numerus AF Logarithmus sinus ζω. Atque eodem continuò servato ordine erit (ex definitione superius tradita) numerus AG logarithmus sinus ηω. AH log. sinus ιω. A log.sinus Kω. AK log.sinus λω, & ita in infinitum.

Cor. ***Unde sinus totius 10000000. nullum seu 0 est logarithmus : & per consequens, numerorum majorum sinus toto logarithmi sunt nihilo minores.***

Quum enim ex definitione pateat quod à sinu toto descescentibus sinibus, à nihilo accrescant logarithmi, ideò contra crescentibus numeris (quos adhuc sinus vocamus) in infinum totum, scilicet in 10000000. decrescant, in 0. seu nihilum logarithmi est necesse. Et per consequens numerorum crescentium ultra sinum totum 10000000. (quos secantes aut tangentes, & non amplius sinus vocamus) logarithmi erunt minores nihilo. [p. 5.]

Itaque logarithmos sinuum, qui semper majores nihilo sunt, abundantes vocamus, & hoc signo +, aut nullo praenotamus. Logarithmos autem minores nihilo defectivos vocamus, praenotantes eis hoc singu -.

Admonitio.

Erat quidem initio liberum cuilibet sinui, aut quantitati nullum seu 0, pro logarithmo attribuisse : sed praestat id prae caeteris sinui toti accommodasse ; ne unquam in posterum vel minimam molestiam parturiret nobis additio & subtractio ejus logarithmi in omni calculo frequentissimi. Caeterùm etiam quia sinuum & numerorum sinu toto minorum frequentior est usus : eorum igitur logarithmos abundantes ponimus : aliorum vero defectivos, etsi contrà fecisse initio liberum erat.

CAP. II.
De Logarith. propositionibus.

Propos. 1. ***Proportionalium numerum, aut quantitatum, æqui-differentes sunt Logarithmi.***

Ut proportionalium sinuum, scilicet γω, qui se habet ad εω, ut ιω ad λω, Logarithmi respectivè sunt numeri definientes AC, AE, AG, & AK, (ut per def. 6. patet.) differunt autem AC, & AE differentia CE : atque AH, & AK differentia HK. Sunt autem ex 1. def. & suo corollario CE, & HK aequales : aequi-differentes igitur sunt Logarithmi praefatorum sinuum proportionalium. Et ita in omnibus proportionalibus.

Nam quas affectiones & symptomata Logarithmi ab ortu & genesi suo acquisiverint, eas inposterum retineant, est necesse. Ab ortu autem & in genesi suà imbuuntur hac affectione, & haec lex illis praescribitur, ut sint [p. 6.] aequi-differentes, quum eorum sinus seu quantitates sint proportionales (prout ex def. Logarithmi, & utriusquae motus patet, & in constructione logarithmorum amplius aliquando patebit.) proportionalium ergo quantitatum aequi-differentes sunt Logarithmi.

Propos. 2. ***Ex trium proportionalium Logarithmis, duplum secundi seu medii minutum primo, aequatur tertio.***

Quum, per prop.1. differentia logarithmorum primi & secundi aequetur differentiae logarithmorum secundi & tertii, id est, secùndus minus primo aequetur tertio minus secundo : Ideo addito secundo ad utrumque aequationis latus proveniet bis secundus seu duplum secundi minus primo aequale tertio, quod erat probandum.

Propos. 3. ***Ex trium proportionalium Logarithmis, duplum secundi seu medii minutum aequatur aggregato extremorum.***

Ex praecedente prop. 2. duplum secundi minutum primo aequatur tertio. Utrique aequalium laterum adde primum, & exurget duplú secundi aequale primo & tertio, id est, aggregato extremorum, quod erat demonstrandum.

Propos. 4. ***Ex quatuor proportionalium Logarithmis, aggregatum secundi & tertii minutum primo, aequatur quartio.***

Quum per 1. prop. ex quatuor proportionalium logarithmis, secundis minutus primo, aequetur quarto minus tertio, utrique aequalitatis lateri, adde tertium, & fient secundus & tertius minuti primo aequales quarto, quod erat propositum.

Propos. 5. ***Ex quatuor proportionalium Logarithmis, aggregatum mediorum (secundi, scilicet, & tertii) aequatur aggregato extremorum, primi videlicet, & quarti.***

Per prop. 4 praecedentem, secundus & tertius minuti primo erant aequales quarto : utrique aequalitatis lateri adde primum, & fiet secundus, plus tertio aequaalis quarto, plus primo, quod demonstrandum erat.

Propos. 6. ***Ex quatuor proportionalium Logarithmis, triplum alterutrius mediorum aequatur aggregato extremi remoti, & dupli vicini.***

Per secundam prop. duplum secundi seu medii minutum primo est aequale tertio : & per tertiam prop. duplum hujus, [p. 7] quod est, quadruplum secundi minutum duplo primi, aequabitur aggregato suorum extremorum, videlicet quarto plus secundo. Iam si ab utroque aequalitatis latere subduxeris secundum, fiet triplum secundi minutum duplo primi aequale quarto. hujus rursus aequalitatis lateribus adde duplum primi, & exurget triplum secundi aequale quarto plus primi duplo : quod probandum suscepimus.

Admonitio.

Huc usque logarithmorum genesin & symptomata explicavimus : quo verò calculo, quave logisticæ methodo habeantur, hoc loco explicandum foret. Sed quia ipsum canonem integrum, ejusque logarithmos omnes cum suis sinibus ad singulas quadrantis minutias primas exhibemus, ideo in tempus magis idoneum doctrinam constructionis logarithmorum transilientes, ad eorum usum properamus, ut praelibatis prius usu, & rei utilitate, cetera aut magins placeant posthac edenda, aut minus saltem displiceant silentio sepluta. Præsolor enim eruditorum de his judicium & censuram, priusquam cæteræ in lucem tererè prolata lividorum detrectationi exponantur.

CAP. III.
Descriptionem complectens tabulæ logarithmorum, & septem ejus columnarum.

Sectio. 1. ***Prima columna est expressè arcuum ab 0. in 45. Gra. crescentium: & subintelligitur esse etiam suorum ad semicirculum reliquorum.***

Sectio. 2. ***Septima autem columna est arcuum à quadrante in 45. gradum decrescentium: & subintelligitur esse etiam suorum ad semicirculum reliquorum.***

Sectio.3. ***Unde alterius columnæ arcus, sunt arcuum alterius è regione respondentium complementa.*** [p.8]

4. Atque in prima exprimitur omnis trianguli rectilinei rectanguli angulus acutus minor.

5. In septima autem ei è regione collacatur ejusdem rectanguli angulus acutus major.

6. In secunda columnæ sunt finus arcuum primæ columnæ.

7. Suntque hi crus minus subtendens minorem angulum rectanguli, cujus basis, seu hypotenusa est sinus totus.

8. In sexta columa sunt sinus arcuum septimæ columnæ.

9. Suntque hi crus majus subtendens majorem angulum ejusdem rectangulis, cujus scilicet hypotenusa est sinus totus.

10. Unde omni triangulo rectilineo rectangulo sit æquiangulum & simile ex sinu toto, & sinu secundæ columnæ, & sinu sextæ ei è regione respondente.

11. Tertiæ columna continet Logarithmos arcuum, & sinuum sinistrorum.

12. Qui sunt Logarithmi proportionis cruris minoris rectanguli ad ejusdem hypotenusam.

13. Itemque hi sunt arcuum, & sinuum dextrorum Logarithmi complementorum, quos antilogarithmos appellamas.

14. Quinta columna continet Logarithmos acruum, & sinuum dextrorum.

15. Qui sunt Logarithmi proportionis cruris majoris rectanguli ad ejusdem hypotenusam.

16. Itemque hi sunt arcuum & sinuum sinistrorum antilogarithmi, seu Logarithmi complimentorum

17. Quarta denique seu mediæ columna continet differentias inter logarithmos tertæ columnæ, & quintæ. Unde duplex est hæc columna, Abundans & defectivæ.

18. Abundantes, sunt differentie, quæ oriuntur ex subtractione logarithmorum tertiæ à logarithmii tertiæ.

19. Defectivæ verò, sunt differentiæ ortæ ex subductione logarithmorum tertie à logarithmis quinitæ : quæ ideo sunt minores nihilo.

20. Differentiæ abundantes dicuntur differentiales numeri, arcuum sinistrorum.

21. Suntque logarithmi proportionis minoris cruris rectangulis ad eiusdem crus maius. [p. 9]

22. Itemque sunt Logarithmi fecundorum, sive tangentium arcuum sinistrorum.

23. Differentiæ autem defectivæ dicuntur numeri differentiales arcuum dextrorum.

24. Suntque Logarithmi proportionis majoris cruris rectanguli ad ejusdem crus minus.

25. Itemque sunt Logarithmi facundorum, sive tangentium arcuum dextrorum.

26. Omnis etiam arcus sinister, ejusque ad semicirculum reliquus, dicitur arcus complementi arcuum, sinuum, & Logarithmorum dextrorum, atque differentialium defectivorum.

27. Et contra, omnis arcus dexter, ejusque ad semicirculum reliquus, dicitur arcus complementi arcuum, sinuum, & Logarithmorum sinistrorum, atque differentialium abundantium.

Admonitiones.

28. Hic notandum est, si Logarithmos tertiæ columnæ defectivos feceris (præposito scilicet— signo,)fient Logarithmi hypotenusarum, sive secantium arcuum dextrorum septimæ columnæ.

29. Et hi etiam fient Logarithmi proportionis hypotenusæ rectanguli ad ejusdem crus minus.

30. Et si Logarithmos quintæ columnæ defectivos feceris, fient Logarithmi hypotenusarum, sive secantium arcuum sinistrorum primæ columnæ.

31. Fient etiam hi Logarithmi proportionis hypotenusæ rectanguli ad ejusdem crus majus. Verùm quia ad rectilineorum scientiam comparandam, soli sinus, eorumque arcus, & logarithmi cum differentialibus : ad sphæricorum autem, spretis sinibus, soli arcus, & earum logarithi, & differentiales sufficiunt : ideo hypotenusas & fœcundos tabula exclusimus : sicuti & sinus ipsos in sphæricis negligi volumus. Ostendemus tamen obiter te posse (se libuerit)eis omnibus satis expeditè in rectilineis uti, in sphæricis verò minimè. [p. 10.]

CAP. IV.
De usu tabulae, & numerorum eius.

Sectio. 1. ***Sinuum, tangentium, & secantium præcise in tabulis suis repertorum, Logarithmos non minus præcisè dare.***

Per sect. 11. & 14. cap. 3. reperto sinu dato in secunda, aut septima columna nostræ tabulæ, repertietur ejus Logarithmus in ejusdem lineae tertia vel quinta columna. Habentur igitur sic exactè sinuum tabulatorum logarithmi. Tangentium autem & secantium numeris in suis tabulis repertis habentur arcus. Ex arcubus verò cognitis nostra tabula exhibet tangentium logarithmos seu differentiales cum signis suis in media columna per sect. 22. & 25. Et secantium logarithmos reciprocè in tertia & quinta columna, præposito tamen his — signo per sect. 28 & 30. Habentur igitur sinuum, tangentium, & secantium tabulatorum logarithmi.

Exempla sinuum.

Gr. 44

min.	*Sinus*	*Logarithmi*	*Differentia +/-*	*Logarithmi*	*Sinus*	*min.*
0	**6946584**	**3643351**	**349137**	**3294214**	**7193398**	**60**

…Gradus 45

Sinus 6946584. Logarithmum quæro. Sinum illum praecisè reperio in secunda columna respondentem arcui 44. Gr. 0. m. & in eadem linea tertæ columnæ adstat 3643349. suis logarithmus, quem quæsivi.

Gr. 43

min.	*Sinus*	*Logarithmi*	*Differentia +/-*	*Logarithmi*	*Sinus*	*min.*
50	**6925630**	**3673561**	**407356**	**3266205**	**7213574**	**10**

…Gradus 46

Item sinus 7213574. quæratur logarithmus. Sinus hic invenietur respondens arcui 46. Gr. 10. m. & ei vicinus 3266204. logarithmus ejus quæsitus.

Exempla tangentium.

Quæratur tangentium 2186448. logarithmus. Huic tangenti in sua tabula respondet arcus 12. Gr. 20. m. & huic arcui in media columna tabulæ nostræ respondet logarithmus, seu differentialis abundans 15203064.

Gr. 12

min.	*Sinus*	*Logarithmi*	*Differentia +/-*	*Logarithmi*	*Sinus*	*min.*
20	**2135988**	**15436559**	**15203069**	**233490**	**9769215**	**40**

quæsitus. Item si tangentis 4573629. logarithmum quæveris, offendes in tabula tangentium ejus arcum 77. Gr. 40. m. hujusque arcus in tabula nostra differentialem eandem, defectivam tamen, scilicet — 15203064. [p. 11.]

20	**2135988**	**15436559**	**15203069**	**233490**	**9769215**	**40**

…gradus 77

Exempla secantium.

Secanti 18118009. respondet in tabula secantium arcus 56. gr. 30 m. & huic arcui in tabula nostra convenit reciprocè defectivus — 5943212. logarithmus secantis 18118009. suprascripti. Sic secantis 13118337. invenies logarithmum — 2714255. & secantis 13960592. offendes logarithmum –3336533.

Gr.
33

min.	*Sinus*	*Logarithmi*	*Differentia +/-*	*Logarithmi*	*Sinus*	*min.*
30	**5519370**	**5943214**	**4126626**	**1816588**	**8338858**	**30**

…Gradus 56

2. ***Numerorum datorum, & in tabulis sinuum, tangentium, & secantium non repertorum, logarithmos æstimare.***
Numerum dato simillum, sive is fuerit dati decuplus, centuplus, millecuplus, 10000^{plus}, 100000^{plus}, aut 1000000^{plus}, quære in secunda, aut sexta columna tabulæ nostræ, aut si mavis, in tabulis tangentium, aut secantium : & hujus arcum nota. ejus enim logarithmus è tabula nostra elicitus, est quem quæris. mente tamen reservando, aut memoriæ gratiâ notis exprimendo numerum locorum, seu figurarum multiplicitatis. Ut si quæratur logarithmus numeri 137. in tabulis non reperti : reperies inter sinus numeros 14544. 136714. & 1371564. & inter tangentes 13705046. inter secantes verò numerum 13703048, qui est omnium dato simillimo, dummodo ejus ultimæ vel dextimæ quinque figuræ deleri subintelligantur. hujus ergo secantis 13703048. & sui arcus 43. gr. 8. m. logarithmus (per præced. aut per sect. 28 & 30. cap. 3) quæratur, & invenietur — 3150332. qui pro logarithmo dati numeri 137. etiam habetur : recordando tamen ultimas quinque figuras abscindendas esse, aut memoriæ gratia, expressè hoc modo signadas — 3150332.— 00000. Similiter si tangentem 13705046. superiùs expressum quaesiveris logarithmum numeri 137. ex tangentis illius arcu 53. gr. 53. m. invenietur (per sect. 25) in media columna — 3151790. logarithmus illius tangentis 13705046. qui quia excedit 137. datum quinque locis seu figuris, ideo — 3151790. — 00000. erit logarithmus numeri dati 137. Tanto tamen minus exactus est hic logarithmus, quanto magis 13705046. est dissimilis [p. 12.] numero 13700000. seu céties millecupio dati. sed hic error partes $\frac{5046}{100000}$ unitates nó exuperat. Si tandé per sinum suprascriptum 13705046. quæsiveris logarithmu dati 137. is (per hanc, & 11. sect. cap. 3.) deprehendetur esse 19866327—0000. Nec secus operandum erit signo + quando numerus figurarum datæ quantitatis excedit numerum figurarum sinus ei simillimi, quod rarò contingit, ut, si quæratur numeri seu discretæ quantitatis 232702. logarithmus, invenies in tabula sinum 23271. ei omnium simillimum, sed unica deest huic figura. Hujus ergo logarithmo tabulo (per sect. 11. cap. 3) reperto, qui est 60631284. adjiciatur unica cyphra signo + interposito, & fiet 60631284 + 0. pro logarithmo numeri 232702. quæsito. Sed modus logarithmos æstimandi omnium optimus est, quo primò creati sunt : de quo alibi.

3. ***Unde, ut superiore prima sectione logarithmi simplices, & puri exhibentur : ita hac præcedente appositis cyphris impuri emergunt.***

4. ***Similium signorum logarithmos addere, est aggregatum utriusque cum signo communi exhibere.***

Ut ex additione — 56312. ad — 73495. provenient — 129807. Itemque addito 4216. ad + 5392. producuntur 9608. Sic 3219 —00. ad 4360 —000. faciunt 7579 —00000.

5. ***Dissimilium signorum logarithmos addere, est differentiam eorum cum signo majoris numeri exhibere.***
Ut ex additione — 210. ad 332. producitur + 122. Item ex additione—210. ad + 192. producitur —18. Sic —210. +000. ad 332 —00. sunt 122 + 0. Item —210. —000. ad 192 + 00. sunt —18 — 0.

6. ***Duorum logarithmorum, hic illius defectivus, illo autem hujus abundans propriè dicitur: cum & numerum, & cyphras communes seu eosdem : signa verò omnia + & — penitus contraria habeant.***
Ut abundantis 56312. defectivus est — 56312. Idem abundantis 56312. — 00. defectivus est — 56312. + 00. Sic abundantis 56312. + 00. defectivus est — 56312. — 00.

7. ***Abundantem subtrahere, est ejus defectivum addere.*** [p. 13.]
Ut subtrahere abundantem 56312. ex — 73495. idem erit, quod addere illius defectum, qui (per 6.) est — 56312. ad eundem — 73495. fientque (per 4. præmissam) — 129807. Sic subtrahere 56312. + 00. ex — 73495. — 00. est idem quod addere — 56312. + 00. ad — 73495. — 00. fiuntque (per 4. & 5. præcenentes) — 129807— 00000.

8. ***Defectivum subtrahere est ejus abundantem addere.***
Ut subtrahere defectivum — 4216. ex + 5392. est idem quod addere 4216. ad 5392. & (per 4.) producere est 9608. Sic idem est subtrahere — 4216. + 00. ex 5392. + 0. quod addere fientque (per 4. præmissam) — 129807. Sic subtrahere 56312. ex — 73495. — 00. est idem quod addere 4216. — 00. ad 4216. + 00. & producere 9608 — 0.

9. ***Logarithmum numero - tenus augere vel minuere salvo valore pristino, est ad illum addere, aut ab eo subtrahere quemvis ex logarithmis sequentibus, scilicet 23025842 + 0.vel 46051684 + 00.vel 69077527 + 000.vel 92103369 + 0000. vel 115129211 + 00000. nihil prorsus significantibus.***
Ut sit Logarithmus 39156 — 0. cui si addideris illorum quemvis, ut exempli gratia, 23025842 + 0. fiet inde 23064998. major numero, valore autem prorsus idem que 39156. — 0. Namque hujus 39156 — 0. logarithmi, quantitas seu valor numeralis (per 12. & 13. sect.seq.hujus) est 9960920. à quibus deme unicam figuram ultimam, prout — 0.notat, & fiet 996092. Illius autem Logarithmi 23064998. valor numeralis (per 12. & 13. hujus) est etiam 996092. idem qui prius.

Exemplum minutionis.

Sit Logarithmus 25451769. minuendus, à quo si subduxeris 23025842 + 0. relinquitur 2425927 — 0. ejusdem valoris, cujus prior hic 25451769. Nam simplicis & puri Logarithmi 2425927. valor est decuplus valoris utrius [p. 14.] eorum. Sunt ergo valores invicem aequales. Nihil enim aliud significat additio Logarithmi 23025842 + 0, quam quod valor numeri cui additur, sit decupartiendus, & huic decimæ parti cyphræ unicæ sit adjicienda : subtractio verò ejusdem significat valorem logarithmi à quo subtrahitur decuplari, & ab hoc decuplo cyphram unicam abiici. remanet itaque in utrâque pristinus valor. Sic 46051684 + 00. additus significat ad centisimam partem valoris duas cyphras adjiei : & substractus, quod à centuplo duæ cyphræ rejiciantur : & sic de reliquis suprà expressis.

10. ***Si itaque ad logarithmum minutum aliquot cyphris addideris, aut à logarithmo aucto cyphris subtraxeris aliquem ex logarithmis suprascriptis totidem cyphrarum, producitur ex impuro logarithmus purus ejusdem valoris.***
Ut in superiore primo exemplo sit logarithmus impurus 39156 — 0. purgandus à cyphræ sua & — signo. adde ergo illi 23025842 + 0, fiet inde, ut supra 23064998. logarithmus purus pristini valoris. Sic à logarithmo 63584468 + 00. impuro si substraxeris 46051684 + 00. totidem scilicet cypharum, relinquetur logarithmus 17532784. purus, & ejusdem valoris, cujus prior ille impuris.

11. ***Si ad logarithmum numero defecivum addideris, aliquem ex supra dictis logarithmis nonæ sectionis numero majorem, proveniet logarithmus ejusdem valoris numero abundans.***
Ut ad logarithmum —28595270 —0000. adde quemvis ex numeris nonæ sectionis numero majorem. v. g. 46051684 +00. & fiet inde 17456414 — 00. ejusdem valoris, & numero abundans.

12. ***Logarithmorum in tabula nostra numero-tenus inventorum sinus, tangentes, secantes seu numerales valores quoscunque exhibere poteris, per cap. 3, sect.11.14.22.25.28.30. sive sint puri, sive impuri.***
Ut logarithmo 36. graduum & 40. minutorum, qui est 5155724. in tertia columna, respondet suus sinus 5971586. in secunda : & ejus defectivo — 5155724. respondet in tabulæ secantium 16745970. secans 53.gr.20.m. [p. 15.] Item logarithmo differentiali 2950794. & ejus defectivo — 2950794. respondet tangens 13432331. graduum scilicet 53. & 20. minut. Sic logarithmi 2204930. in quinta columna numeralis valor est in sexta columna 8021232. sinus scilicet gr.53. & 20.m. & ejusdem defectivi scilicet — 2204930. numeralis valor est secans 12466913. conveniens gradibus 36. & 40.min.

Exemplum impurorum.

Sit logarithmi impuri 97796 — 0. inquirendus valor. huic numero-tenus respondet in tabula nostra sinus 9902681. à quo aufer dextimam figuram (prout —0. indicat) & fient 990268. valor logarithmi 97796—0. quæsitus. Sic logarithmi 25451769 + 00. valor est 78459100. quia logarithmo 25451769. puro respondet in tabula nostra sinus 784591. Item logarithmi — 349136 —00. in quarta columna, apud graduum 46.graduum reperti, valor erit 103553. quia tangens 46. graduum est 10355302. Sic logarithmi —6350305 —00. in tertia columna apud graduum 32. reperti, valor est 188708, quia secans complementi 32.graduum, scilicet 58. graduum, est 18870800. cujus duæ ultimæ & dextemæ figuræ 00. delendæ sunt propter —00. annexa logarithmo.

13. ***Logarithmorum datorum, in tabula nostra non repertorum numerales valores æstimare.***
Ad vulgares Geodesias sufficit plerumque, logarithmi tabulati propinquioris dato, numeralem valorem pro dati accipere. verum si propius ad metam accedere desideras, logarithmum datum per nonam hujus numero-tenus auge, vel minue salvo valore pristino,

donec aut in tabula reperiatur, aut alicui tabulato satis similis devenerit, & hujus logarithmi valor per præmissam inventus, est quem quæris. ut exempli gratia, quæratur valor hujus logarithmi 23149721 + 0. cui in tabula non reperitur similis vel satis propinquus. verùm si ab illo subduxeris [p. 16.] 23025842 + 0. relinquetur 123879. cui sub 81.gradu reperietur satis propinquus & similis 123881. cujus sinus 9876883. per præmissam inventus, est valor oblati logarithmi 23149721 0.quæsitus.

Admonitio.

Pro hac sectione, & secunda hujus monitum volumus, numerorum datorum logarithmos, & contra logarithmorum datorum numerales valores (ubi non reperiuntur in tabula) ommium accuratissimè exhibiri per modum ipsum quo creantur, aut resolvuntur logarithmi, que est ,ut à sinu dato per media Geometricè proportionalia descendas, donec in proximè minorem sinum tabulatum perveneris : similiter ab hujus logarithmo tabulato descendas etiam per totidem media Arithmetica congrua, & horum ultimus erit illorum primi logarithmus : & contra per resolutionem, ut à logarithmo dato per media Arithmetica in Logarithmum tabulatum proximè minorem descendas, & ab hujus valores tabulato similiter etiam descendas per totidem media Geometrica & congrua : & horum ultimus erit numeralis valor illorum Logarithmorum primi. Vèrum quæ æqui-differentia Arithmetica cuique continuatæ proportioni Geometricæ conveniat & sit congrua, exquirere non est mediocris ingenii. Quare de his (Deo aspirante) ubi de Logarithmis condendis & creandis agetur, amplius aliquando differemus.

CAP. V.
De amplissimo Logarithmorum usu, & expedita per eos praxi.

Problema 1. ***Ex trium proportionalium Logarithmis, dato logarithmo medio & altero extremo, reliquum extremum ejusve proportionalem; vel arcum, per unicam duplationem & subtractionem dare.*** [p. 17.]

Quum per secundam prop.cap.2.duplum medii (scilicet logarithmi) minutum altero extremorù æquetur reliquo, ideo à duplo medii logarithmi dati aufer logarithmum extremi datum, & relinquetur logarithmus extremi quæsiti: cui in tertia, quarta, aut quinta columna tabulæ inventæ respondet arcus in prima & septima : sinus autem in secunda, aut sexta: & sui secantes aut tangentes in tabulis suis per cap.3.sect.1.2.6.8.11.14.22.25.28.30. pro extremo quæsito habentur.

Exemplum.

Dentur 10000000 primù proportionale, & 7071068. secundum, quæratur tertium. Id vulgò exquiritur medium in se quadratè multiplicando, & hoc quadratum per primum dividendo. Sed nos facilius medii logarithmum 34655735. duplando, & ab hoc duplo, quod est 6931470. logarithmus primi (qui est 0) auferendo : & ita restat 6931470. logarithmus quæsitus: cujus arcum 30. graduum, & sinum 5000000. (scilicet proportionale quæsitum) juxta eum invenies. Sunt ergo 10000000. 7071068.5000000. tria proportionalia, quorum ultimum sola duplatione, & subtractione acquisivimus, quod polliciti sumus. Item duo proportionalia 10562556. primum, & 7660445. secundum, aut saltem eorum logarithmi —547302. & 2665149. dentur. Tertium sic habebis. Ab huius duplo 5330298. aufer —547302, & (per 8.sect.cap.4) producitur logarithmus 5877600. 33.graduum.& 45.min.cuius sinus 55555702. est tertium proportionale quæsitum.

Prob.2. ***Ex trium proportionalium logarithmis, datis logarithmis extremis, medium, ejusque proportionale, & arcum per unicum additionem & bipartitionem dare.***

Quum per sect. 3.cap.2. duplum Logarithmi medii æquetur aggregatio extremorum, ideo extremorum Logarithmos adde: productum bipartire, & emerget Logarithmus medii: atque inde medium arcus innotescit [p. 18.] in columnis, & per sectiones, ut supra.

Exempli gratia.

Dentur extrema 10000000 & 50000000. quæratur medium. Id vulgò acquiritur multiplicando data illa invicem, & producti radicem quadratant extrahendo. Verùm nos sic facilius. Datos extremorum Logarithmos, 0. primi, & 6931470. ultimi addimus, & aggregatum 6931470. bipartimur, fietque 3465735. optatos medii Logarithmus. Unde ipsum medium 7071068. & ejus arcus 45.gr.ratione supradicta habentur. Item sint extrema data 10562556. & 5555702. eorum Logarithmi —547302. & 5877600. Horum additorum summa est 5330298. per sect.5.cap.4, quam bipartimur, & sit 2665149. Logarithmus, & ejus arcus 50 graduum: Et sinus, seu medium proportionale quæsitum est 7660445. sola additione, & bipartitione inventum.

Prob.3. ***Ex quatuor proportionalium Logarithmis, datis tribus, eorumve arcubus, invenire quartum Logarithmum, ejusque sinum, & arcum per unicam additionem, & subtractionem.***

In hoc problemate quæsitum semper pro quarto statuimus ita ut datòrum primum se habeat ad secundum, ut tertium ad quæsitum. Quumque ita constitutorum aggregatum ex Logarithmis secundi & tertii minutii Logarithmo primi æquetur quarti Logarithmo per 4.sect. cap.2. Ideo Logarithmos secundi & tertii adde, & hinc aufer Logarithmù primii, & proveniet Logarithmus quarti quæsiti: & inde ipsum quartum, & ejus arcus.

Exempli gratia.

Sit ut 7660445. ad 9848078. ita 50000000. ad quartum, quod quærimus. Hoc vulgus acquirit ducendo secundum in tertium, & dividendo per primum. Tu autem sic facilius Logarithmos secundi 153088 & tertii 6931469. addes, fiet 7084557: à quo auferes Logarithmum primi, qui est 2665149, & relinquetur 4419408, Logarithmus quarti: cujus sinus 6427876. est ipsum quartum desideratum, & ejus arcus est 40.graduum. Idem proveniret si (spretis sinibus) solum darentur tres sui arcus 50. [p. 19.] gr. 80.gr.& 30.gr. Namque ex Logarithmis arcuum 80.gr.& 30.gr. ablato Logarithmo 50.gr. remanebit Logarithmus 40.gr. Et ita ipse arcus 40.gr. innotescet absque sinibus, eorumve multiplicatione aut divisione, prout initio polliciti sumus.

Aliud exemplum.

Sit ut tangens seu foecundus numerus 43.gr. ad sinum 57.gr., ita foecundus seu tangens 35.gr. ad sinum quartum tacitum, cujus arcum neglectis & spretis tam sinibus quam tangentibus, sic inveniemus. Logarithmum differentialem 35.gr.scilicet 3563784. in media columna inventum ad Logarithmum 57.gr.videlicet 17593572. in quinta columna locatum addimus : à producto videlicet 5323156. differentialem 43.gr.qui est 698698. subducimus, & relinquitur 4624458. Logarithmus quarti (sinus scilicet,) quo in tertia columna per 11.sect.cap.3. reperto, reperies juxta eum in prima columna 39.grad.2. minut. serè qui est arcus quæsitus quarti proportionalis seu sinus spreti.

Hac ratione proportionalium arcus, absque eorum sinibus, tangentibus, secantibus, aut proportionalibus quibuscunque acquiruntur.

Quod certè compendium ad triangulorum planorum angulos dimetiendos, & ad universam sphæricorum Trigonometriam conducit plurimum: ut suo loco patebit.

Prob.4. ***Quatuor continuè proportionalium datis extremis eorumve arcubus, mediorum quodvis, eorumve arcuum quemvis invenire, inducta simplici tripartione pro ardua cubicæ radicis extractione.***

Quum in horum Logarithmis, triplum cujusque medii æquetur aggragato extremi remoti & dupli vicini, per prop.6.cap.2. Ideo duplum Logarithmi extremi alterutrius ad Logarithmum extremi reliqui adde, & productum tripartire, & proveniet Logarithmus medii [p. 20.] priori extremo proximi, & eodem modo alterum medium. Ut exempli gratia: Sint extrema, primum 4029246. ultimum verò 10562556. Quæruntur media, quæ absque extractione radicis cubicæ sic invenies. Datorum Logarithmi sunt 9090051. & —547302 ad illius duplum 18180102. adde hunc, & fiet 17632800. qui tripartitus producit 5877600. Logarithmum, cujus sinus 55555702, est prius medium quæsitum. Item simili modo ad hujus —547302, duplum, quod est —1094604. adde illum 9090051. & producetur 7995447, qui tripartitus producit 2665149, Logarithmum,cujus sinus 7660445. est posterius medium etiam quæsitum. Quatuor itaque proportionalia continua sunt 4029246.5555702.7660445.& 10562556.

Aliud exemplum.

Sint extrema data 14142135 & 5000000. illius in tabula secantium inventi logarithmus in tabula nostra est — 3465735. hujus verò 5000000. logarithmus est 6931470. cujus duplo 13862940. adde —3465738.fiet 10397205. quem tripartire, & sit + 3465735. logarithmus medii proportionalis minori extremo 5000000. proximi, quod est 7071068. Sic duplo — 3465735. quod est —6931470. adde + 6931470. & fiet inde 0. seu nihil, quod tripartitù etiam reddit 0. cujus sinus & valor est 10000000. pro reliquo & majore medio. Quatuor itaque, hæc continuè proportionalia sunt 14142135. 10000000. 7071068. 50000000.

CONCLUSIO.

Ex his prælibatis judicent eruditi quantum emolumenti adferent illis Logarithms : quandoquidem per eorum additionem multiplicatio, per subtractionem divisio, per bipartione extractio quadrata, per tripartitionem cubica, & per alias facilis prostaphæresces omniæ graviorœ calculi opera, evitantur: cujus rei specimen generale hoc priore libro exhibuimus. Sequente autem de eorundem proprio & particulari usu innobili. illa Geometriæ specie, qua Trigonometria dicitur, tractaturi summus.

Finis prioris libri.