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Book II Section VIII.

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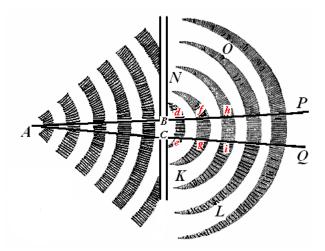SECTION VIII.

Concerning motion propagated by fluids.

PROPOSITION XLI. THEOREM XXXII.

A pressure is not propagated by a fluid along straight lines, except when the particles of the fluid lie in a straight line.

If the particles *a, b, c, d, e* may lie on a right line, some pressure can be propagated directly from *a* to *e* ; but the particle *e* will act on the sideways placed particles *f* and *g* in an oblique manner, and these particles *f* and *g* will not sustain the inflected pressure, unless they are supported by the more distant particles *h* and *k* ; but provided they are supported, they press on supporting particles, and these cannot support the pressure unless they are supported by the further particles *t* and *m*, and they press on these; and thus henceforth indefinitely. Therefore the pressure, that in the first place is propagated to the particles which do not lie along the direction, will begin to branch out, and to be propagated indefinitely obliquely; and after the pressure begins to be propagated obliquely, if it should be incident on more distant particles, which do not lie along the same direction, it will branch out again ; and just as often as it is incident on particles that are not accurately in line, so this will happen. *Q.E.D.*

Corol. If some part of the pressure, propagated from some part of the fluid, may be intercepted by some obstacle, the remaining part, by which it is not intercepted, will branch out in the space behind the obstacle, as that can also be demonstrated thus. From the point *A* the pressure can be propagated in every direction, and thus if it were able to made along right lines, and all that pressure may be intercepted by the obstacle *NBCK* perforated in *BC*, except the conical part *APQ*, which passes through the circular opening
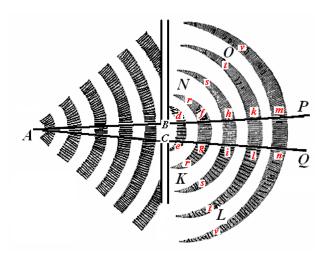
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BC. The cone *APQ* may be separated into frustums by the transverse planes *de, fg, hi* ; and while being propagated within the cone *ABC*, the pressure may act on the further conical frustum *degf* at the surface *de*, and this frustum may act on the nearby frustum *fgih* at the surface *fg*, and that frustum acts on the third frustum, and thus henceforth indefinitely; it is evident (by the third law of motion) that the first frustum *defg* by the reaction with the second frustum *fghi*, will be acting and by pressing with as great a pressure on the surface *fg*, as may act and press on that second surface. Therefore the frustum *degf* is compressed on both sides between the cone *Ade* and the frustum *fhig*, and therefore (by Corol. VI. Prop. XIX.) is unable to maintain its shape, unless it shall be compressed by the same force on both sides. Therefore by the same impulse by which it is pressed on the surfaces *de* and *fg*, it will try to go to the sides *df* and *eg*; and there (since it shall not be rigid, but always a fluid) it will run and expand out, unless surrounding fluid may be near. Therefore by trying to depart, it presses with the same impulse both on the surrounding fluid on the sides *df* and *eg,* as well as on the frustum f*ghi* : and on that account none the less the pressure will be propagated from the sides *df* and *eg* into the spaces *NO* and *KL* and thus hence, so that it may be propagated from the surface *fg* towards *PQ*. Q.E.D.

PROPOSITION XLII. THEOREM XXXIII.

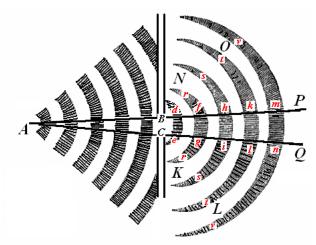*All motion propagated by a fluid diverges from rectilinear motion in an immoveable space.*

    *Case*1. The motion may be propagated from the point *A* through the hole *BC*, and it may go, if it can happen, into the conical space *BCQP*, along right lines diverging from the point *A*. And in the first place we may suppose that this motion shall itself be of waves on the surface of water at rest. And let *de, fg, hi, kl*, &c. be the highest parts of individual waves [crests], with just as many distinct intermediate valleys [troughs] in turn. Therefore because the water in the crests of the waves is higher than in the fluid in the stationary parts *LK, NO*, the same may flow away from the crest terms *e, g, i, l*, etc. *d, f, b, k*, etc.

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hence from there towards *KL* and *NO*: and since it is lower in the troughs of the waves
than in the motionless parts of the waves *KL, NO*; the same may flow from the motionless
parts into the troughs of the waves. By flowing from the first crest of the waves, hence it
will be expanding out into the troughs behind that and may propagate towards *KL* and
NO. And because the motion of the waves from *A* towards *PQ* shall be by a continued
flowing away of the crests into the nearest troughs, and thus the speed of crests shall not
be faster than the speed of the troughs ; and hence the descent of the water from there
towards *KL* and *NO* must be going with the same velocity; hence the expansion of the
waves from there towards *KL* and *NO* will be propagated with the same velocity by which
these waves are progressing from *A* towards *PQ* . And hence the total space from there
towards *KL* and *NO* is occupied by an abundance of spreading crests *rfgr, shis, tklt, vmnv*,
&c.. *Q.E.D.* Thus these can be tested by anyone for themselves in any still water.

   Case 2. We may now suppose that *de, fg, hi, kl*, *mn* may designate pulses propagating
successively from the point *A* through an elastic medium. The pulses may be considered

to be propagated by successive condensations and rarefactions of the medium, thus so that
the most dense part of each pulse may occupy a spherical surface described about the
centre *A*, and equal successive intervals may fall between the pulses. Moreover the lines
de, fg, hi, kl, etc. may designate the densest parts of the pulses, propagating through the
hole *BC*. And because the medium here is more dense than in any quarter towards the
spaces *KL* and *NO,* thus it will expand itself both towards these spaces *KL* and *NO*
situated on each side, as well as towards the intervals of more rare pulses; and with that
understood they will share in the same motion, the rarer always emerge from the region of
the [quiet] intervals and the denser from the region of the [denser] pulses. And because
the progressive motion of the pulses arises from the continual relaxation of the more
dense parts towards the antecedent rarer intervals ; and the pulses with almost the same
speed must relax in the quiet parts of the medium from the different quarters *KL* and *NO* ;
these pulses expand themselves out on each side with almost the same speed into the
unmoved spaces *KL, NO*, by which they may be propagated directly from the centre *A* ;
and thus they occupy the whole space *KLON*. *Q.E.D*. We experience this with sounds,
which are heard either with a mountain in between, or they expand on themselves being
admitted into a room by the window, to be heard in all the corners, not only as reflected

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from the opposite walls, but propagated directly from the window as well, as much as can be judged from the senses.

Case 3. Finally we may suppose that a motion of any kind may be propagated from *A* through the opening *BC* : and because the propagation itself does not happen, unless in so far as the parts of the medium closer to the centre *A* may urge the more distant parts into some excitation, and the parts which are acted on by the fluid, and thus which recede in every direction where they act with a lesser pressure: all the same recede towards the resting parts of the medium, both to the sides *KL* and *NO*, as well as to the anterior parts *PQ*, and with that agreed upon the motion of all, that first has passed through the opening *BC*, will begin to expand, and thence at last from a beginning at the centre, is propagated directly in every direction. *Q. E. D.*

PROPOSITION XLIII. THEOREM XXXIV.
Any body vibrating in an elastic medium will propagate the motion of pulses in every direction; truly in a non-elastic medium it will excite a circular motion.

Case 1. For the parts of a vibrating body by alternately coming and going in turn, in its going will act on and propel the nearby parts of the medium themselves, and on being acted on the same will be compressed and condensed; then by its return the compressed parts themselves recede and expand. Therefore the parts of the medium by the vibrations of the body will in turn go and return, in the image of the vibrating parts of the body : and by that account the parts of this body will disturb these parts of the medium, these by similar agitated vibrations will disturb parts nearby to themselves, and these similarly agitated will disturb the further parts, and thus henceforth indefinitely. And in whatever manner the first parts of the medium by going may be condensed and are relaxed on return, thus the remaining parts as often as they shall go will be condensed, and just as often as they return they expand themselves. And therefore not all will go and return at the same time (for since in turn by maintaining determined distances, they would not be rarefied and condensed in turn) but in turn by approaching where they are condensed, and by receding where they are rarefied, some of these will be going while others will be returning ; and that alternately in turn indefinitely. Moreover the parts going and by so going being condensed, are the pulses on account of their progressive motion, by which they strike obstacles, and therefore the pulses will be propagated successively from any vibrating body along a line ; and thus at around equal distances in turn, on account of the equal intervals of time, in which the body by vibrating excites the individual pulses. And whenever the vibrations of the body may go and return along a certain determined direction, and the pulses thence propagated by the medium still themselves expand laterally, by the preceding proposition; and they will be propagated in both directions from that vibrating body as from a common centre, along almost spherical and concentric surfaces. We have some sort of example of this from waves, which if they are excited by a waving finger, not only do they go to and fro in the direction of the motion of the finger, but, in the manner of concentric circles, they will surround the finger at once and be

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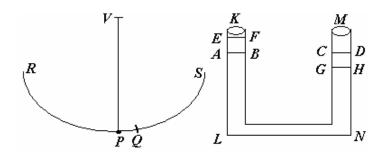
propagated in all directions. For the weight of the waves takes the place of the elastic force.

Case 2. For if the medium shall not be elastic : because the parts of this are unable to be compressed to condense from the vibrations of the shaking parts of the body, the motion will be propagated at once to the parts where the medium can go most easily, that is, to the parts that the shaking body otherwise may leave empty behind it. It is likewise the case with a body projected in some medium. A medium by making way for projectiles, does not recede indefinitely, but by going in a circle, it will go to the space which the body has left behind. Therefore as often as a vibrating body will go into some part, the medium by giving way goes by a circle into the parts that the body has abandoned ; and as often as the body is returned to the first place, the medium will be repelled from there and will return to its first place. And in whatever manner the vibrating body may not be rigid, but flexible in some manner; if yet it may have a given magnitude that remains, because the medium is unable to become disturbed by the vibrations, in what other way can it affect the medium, so that the medium, by receding from the parts where it is pressed, will always go round to the parts which may yield to the same. *Q. E. D*.

Corol. Therefore people must be wandering in their minds who believe that the disturbance of the parts of a flame be conducive to a pressure propagating along right lines, through the surrounding medium. A pressure of this kind cannot be derived from the agitation of the flame alone, but from the expansion of the whole medium.

PROPOSITION 44.   ***THEOREM 35.***

If water alternately rises and falls in turn in [uniform] *pipes with upright legs KL and MN ; and moreover, if a pendulum is made of which the length between the point of suspension V and the centre of oscillation P is equal to half the length of the water in the pipe : then I say that the water rises and falls in time with the oscillations of the pendulum.*

I measure the length of the water along the axes of the pipe and legs, with the same equal height of these ; and I ignore the resistance of the water which arises from the friction with the pipes. *AB* and *CD* designate the mean height of the water in both legs ;

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and when the water in leg KL has risen to the height *EF*, the water in leg *MN* has fallen to the height *GH*. Also, let *P* be the body of the pendulum, *VP* the thread, *V* the point of suspension, *RPQS* is the cycloid that the pendulum describes, of which *P* is the lowest point, and the arc *PQ* is equal to the height *AE*. The force, by which the motion of the water is either accelerated or decelerated in turn, is the excess of the weight of water in the one leg above the weight in the other; thus, when the water in *KL* has risen to *EF*, and in the other leg fallen to *GH*, that force is twice the weight of water *EABF*, and therefore is to the total weight of water as *AE* is to *VP* or *PQ* or *PR*. Also, the force by which the weight *P* at some place *Q* is accelerated or decelerated in the cycloid (from the corol. to prop 51.) is to the total force as this distance *PQ* from the lowest place P, is to the length of the cycloid *PR*. Whereby the equal intervals of the water and the pendulum *AE* and *PQ* describing the motive forces are as the weights to be moved; and thus, if the water and the pendulum are initially at rest, these forces will move the same equally in the same time, and are effective in order that the reciprocal motions can go and return at the same time. *Q. E. D.*

   Corollary 1. Therefore all the oscillations for the rise and fall of the water in turn are isochronous, whether they are made stronger or weaker [i. e. the period of oscillation is independent of the amplitude.]

   Corollary 2. If the whole length of the water in the pipes is $6\frac{1}{9}$ Parisian feet, then the water descends in a time of one second, and rises in the time of one second, and thus henceforth in alternate turns indefinitely. Likewise, the pendulum of length $3\frac{1}{18}$ is oscillating with a time of one second.

   Corollary 3. Moreover with the length of the water increased or diminished, the time of reciprocation is increased or diminished in the ratio of the square root of the length.

   [The Manometer as a S. H. M. Oscillator : In modern terms, if *A* is the cross-sectional area of the pipe, *l* the length of water in the pipe, and ρ the density of the water, then if *x* is the extension AE of one arm of the manometer from the equilibrium level *AB*, and - *x* or *DH* is the depression of the other level, or vice versa, then the mass of water accelerated is *Alρ,* while the unbalanced force is 2ρ*Agx;* hence, from Newton's Second Law of motion,
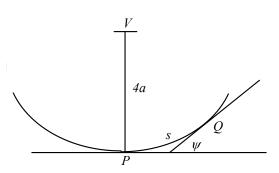
   $Al\rho\ddot{x} = -2\rho Agx, or\ \ddot{x} = -(2g/l)x = -\omega^2 x$. Hence, the period of the oscillation is given by

$T = 2\pi\sqrt{\frac{l/2}{g}}$. In the case where the period of the whole oscillation is 2 seconds, note that Newton has a habit of referring to half periods - in his example 1 second - then the length of water is approximately 2 m.

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The Inverted Cycloidal Pendulum as a S. H. M. Oscillator :

We will save some time by merely quoting the formula for the length of arc *s* of an

inverted cycloid, which is of course a rectifiable curve - and hence was part of its fascination for early workers - in terms of the tangent angle ψ at some point Q: $s = 4a\sin\psi$, where $4a$ is the length of the thread of the equivalent simple pendulum VP (the cycloid can be considered as generated by a point on a circle of radius *a* rolling along a horizontal line at a vertical distance $2a$ above the x- axis.) An equivalent S. H. M. is a small bead of mass *m* to slide on a wire in the shape of the inverted cycloid without friction. Thus, the unbalanced force on the bead due to gravity acting down the slope at Q is $mg\sin\psi$, and we can set $m\ddot{v} = -mg\sin\psi$. That is,

$\frac{d^2s}{dt^2} = -g\sin\psi = -\frac{g}{4a}s.$ Hence, setting $4a = l/2$ insures equality of the periods, and both

motions are independent of the amplitude, though Newton has set these or the half periods

equal in his experiment as this was presumably more expedient : $\frac{T_{manometer}}{T_{pendulum}} = \frac{\sqrt{l/2}}{\sqrt{4a}} = 1$, on

setting $4a = l/2$. Note that one does not have to contend with the factors 2π and g, on taking a ratio in this way, and the whole or half periods can be used with impunity.
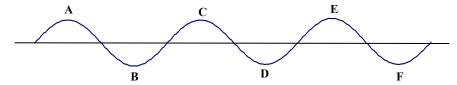
PROPOSITION XLIV. THEOREMA XXXVI.

The velocity of waves varies as the square root of the wavelength.

This theorem follows from the construction of the following proposition.

PROPOSITION XLVI. PROBLEM X.

To find the speed of waves.
    A pendulum is set up, the length of which between the point of suspension and the centre of oscillation, is equal to the length of the waves : and during the time the

pendulum performs single oscillations, by advancing the same amount, the waves progress to almost their own width.

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The transverse width of the waves measured I call the length of the waves, which lies between either the deepest valleys or the highest peaks. *ABCDEF* designates the surface of the still water, with successive waves rising and falling; *A, C, E,* &c. are the peaks of the waves, & *B, D, F*, &c. are the intervening valleys. And since the motion of the waves is by the water successively ascending and descending, thus the parts *A, C, E*, &c. of this surface which now are the highest, soon will become the lowest; & the driving force of the motion, by which the highest parts will descend & the lowest parts ascend, is the weight of the elevated water ; this alternate rising and falling is analogous to the reciprocal motion of the water in the pipes, and the same laws governing the time will be observed : & therefore (by prop. XLIV) if the distances between the peaks of the waves *A, C, E* & and the troughs *B, D, F* are equal to twice the length of the pendulum, then the highest parts *A, C, E,* in the time of one oscillation avoid the troughs, & in the time of another oscillation have ascended again. [Recall that the manometer always has a peak and a trough for the maximum displacements, and therefore corresponds to half a wavelength.] Therefore between the passage of individual waves there will be the time of two oscillations; that is, the wave describes its own width in the time that pendulum oscillates twice ; but the pendulum that oscillates in time with the wave is four times as long, and thus oscillates once, equal in time with the length of the waves. Q.E.I.

Corol. 1. Therefore waves which are $3\frac{1}{18}$ feet long, progress a distance equal to their own width in a time of one second (around 1 m/s); and thus in a time of one minute from the start run through a distance of $183\frac{1}{3}$ feet, & in the space of an hour 11000 ft approximately.

Corol. 2. And the speed of the long waves is increased or diminished in the ratio of the square root of the width.

From the hypothesis, thus waves are considered to have part or the water either ascending straight up or descending straight down (as in the manometer) ; but the up and down motion of the water shall more truly be described in a circle, and likewise I emphasize that the time derived from this proposition can only be defined approximately.

[Notes : There may be some confusion as to what Newton means by the time of an oscillation - the word itself just means a swing, of course. However, in the experiment the pendulum bob is released as a peak of the wave train passes, and reaches the position of the peak of the passing wave again at the end of its forward swing, which occupies at this instant the position of the preceding peak at the start of the pendulum's motion. Newton asserts that the pendulum which achieves this synchronous behavior has the same length as the distance between the peaks. The previous experiment with the manometer tube, which is a sort of standing wave generator of water waves with a 'free end', has a wave or pulse that travels a distance set to some half wavelength by the length *l* (for small amplitudes) in the time the pendulum completes its forward motion, as the water in the legs interchange peaks and troughs. Now, by analyzing the s. h. m. of the pendulum and the manometer, we find the periods are equal when $4a = l/2$, or a half-wavelength *l* corresponds to a pendulum twice as long as that used at present, and the period needed for a whole wavelength is thus four times as long as the original pendulum. The original pendulum is *l*/2 or λ/4, so that the reasoning is correct : a pendulum of length λ is required

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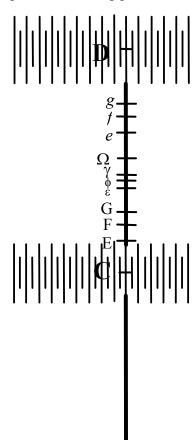Translated and Annotated by Ian Bruce.   Page 680

to be synchronous with the waves.   However, the period of such a simple pendulum is 2 seconds, and hence we conclude that Newton is talking about single swings when he considers oscillations of 1 second.

As mentioned in Cor. 2, there is augmentation or diminution of the waves as they proceed, as they do not all travel with the same speed, and dispersion is taking place. Hence, it is more appropriate to consider the group velocity of the pulses of waves, rather than the phase velocity - which cannot be measured in any case - and the group velocity is responsible for the transfer of energy down the channel. Thus, from his experimental measurements, Newton had observed that the length of the pendulum λ executed its forward swing in a time T given by $\pi\sqrt{\frac{\lambda}{g}}$, for
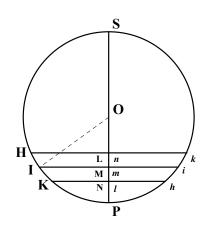
which $v = \lambda / T = \frac{1}{\pi}\sqrt{\lambda g} \sim 1m/s$ as $\lambda \sim 1$m and g ~ 10m/s^2. This quantity we would identify as the group velocity . We cannot read much more into Newton's experiments, as he has not furnished details of the physical dimensions of the channel and pipes apart from the total length of the axis, factors upon which the rate of transmission depends. Nevertheless, the main ideas are essentially correct, and at the time, he was a man in a hurry, and he had sown the seeds for further development.]

PROP. XLVII. THEOR. XXXVII.

For pulses propagating through the fluid, the individual particles of the fluid are oscillating in the shortest reciprocal motion, always accelerating and decelerating according to the law of the pendulum.

Let *AB, BC,* and *CD*, &c. designate the positions of equally spaced successive pulses [*i. e.* such as progressive sound waves of a given wavelength λ of *AB*.] ; the motion of the pulses is propagated from *A* towards *B* along a line *ABC* in the region ; *E, F, G* are three physical points of the quiescent medium on the line *AC,* situated at equal distances from each other; *Ee, Ff, Gg* are equal lengths in turn [of the maximum amplitudes] through which in short time intervals, by the individual reciprocal motions, these points *E, F,* and *G* move to and fro ; *ε, φ, γ* are some intermediate locations of the same points in the medium; *EF* and *FG* are small

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physical sections or incremental parts of the medium placed between these points, &
which in succession are translated into the positions $\varepsilon\varphi$ and $\varphi\gamma$, & then *ef* and *fg*. The line
PS is drawn equal to the line *Ee* [in the lower diagram]. PS is bisected in *O*, and with
centre *O* & length *OP,* a small circle *SIPi* is described. In this circle, the whole
circumference represents the time of one complete vibration, together with its
proportional intermediate parts. Thus, in order that some time such as *PH* or *PHSh* can be
compared with the time of the complete oscillation, if a perpendicular *HL* or *hl* is dropped
on *PS*, then *Eε* is taken to be equal *PL* or *Pl*, at this instant the physical point *E* is to be
found at ε in the moving fluid. According to the law of the pendulum, any point *E* in the
fluid moves from the equilibrium value *E* to the maximum displacement *e* through ε, &
returns to E again through ε, where each vibration has the same degrees of acceleration
and retardation [at intermediate points], so that the oscillation is completed in step with
the oscillation of a pendulum [i.e. any particle such as *E* executes s. h. m. from its
equilibrium point in the fluid; the actual words in Newton's explanation have been
augmented occasionally to reinforce the reader's understanding, as Latin is a little skimpy
at times]. This must be the case since the individual physical points of the medium are
disturbed in this way by such a motion [as in the analogous case of the water waves].
Hence we establish a medium in which such a motion is produced in some manner, and
we observe what may then follow. On the circumference *PHSh,* the equal arcs *HI* and *IK*
or *hi* and *ik* [of a traveling wave or pulse] are taken in the same ratio to the total
circumference as the equal lines *EF* and *FG* have to the total length of the pulse interval
BC, and the perpendiculars *IM* and *KN* or *im* and *kn* can be dropped, as the points *E, F*
and *G* are disturbed in turn by the same motion, & their whole vibrations meanwhile are
carried out from the sum of the oscillations as the pulse is transferred from *B* to *C*. Thus,
if *PH* or *PHSh* is the time of the motion starting from the initial point *E*, then similarly *PI*
or *PHSi* is the time of the motion starting from the initial point *F*, and again *PK* or *PHSk*
is the time of the motion starting from the initial point *G* [An extended pulse passed
through the increments E, F, and G in turn, then the angles are in proportion to the times
as the are length $s = OP \times \Delta\theta = OP \times \omega\Delta t$, where ω is the angular frequency]. Hence, *Eε,*
Fφ and *Gγ* will be respectively equal to the lengths *PL, PM* and *PN* themselves in the
movement away from equilibrium position , or to *Pl, Pm* and *Pn* themselves in the return.
From which $\varepsilon\gamma$ or *EG + Gγ - Eε* leads to *EG - LN* being equal to the incremental pulse
width in the movement away from equilibrium. But $\varepsilon\gamma$ is the width or the expansion
[contraction really] of the part of the medium EG when it is transferred to the location $\varepsilon\gamma$;
& therefore the expansion of that part in the outward motion is to the mean expansion as
EG - LN is to *EG* ; and moreover in the return journey, the ratio is as *EG + ln* to *EG*.
[EG - LN is the contracted length $\varepsilon\gamma$; and thus (EG - LN)/EG is 1 - ΔV/V, as Newton
goes on to demonstrate. Again, this is needed to make the outgoing contraction into an in
going expansion on the return leg of the journey, taken to be (EG + *ln*)/EG or 1 + ΔV/V]

  Whereby the ratio *LN* to *KH* shall be as *IM* to the radius *OP* [This involves
differentiation : see following note.], & *KH* to *EG* as the circumference *PHShP* to *BC, i.
e.*, if *V* is put in place for the radius of another circle with the circumference set equal to
the pulse interval *BC*, then the ratio becomes as [the amplitude] *OP* to [the wavelength λ
or] *V* ; & from the equality *LN* to *EG* as *IM* to *V* : the expansion of the part *EG* or of the

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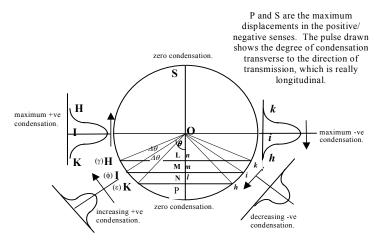Translated and Annotated by Ian Bruce.   Page 682

physical point F at the location $\varepsilon\gamma$ to the mean or quiescent expansion, as that part has in its first position EG, as V - IM to V in going, and as $V + im$ to V on returning. From which the elastic force of the point F at the position $\varepsilon\gamma$ is to the mean elastic force of this at the position EG, as $\frac{1}{V-IM}$ to $\frac{1}{V}$ in going, and on returning truly as $\frac{1}{V+im}$ ad $\frac{1}{V}$. And by the same argument the elastic forces of the physical points E and G on going, are in the ratios $\frac{1}{V-HL}$ and $\frac{1}{V-KN}$ to $\frac{1}{V}$; and the difference of the forces to the mean or quiescent elastic force of the medium, as $\frac{HL-KN}{VV-V\times HL-V\times KN+HL\times KN}$ to $\frac{1}{V}$. That is, as $\frac{HL-KN}{VV}$ to $\frac{1}{V}$, or as HL - KN to V, if (on account of the narrow limits of the vibrations) we may suppose HL and KN to be indefinitely smaller quantities than V. Whereby when the quantity V is given, the difference of the forces is as HL - KN, that is as OM (on account of the proportionals HL - KN to HK, & OM to OI or OP, with HK & OP given) ; i.e. if Ff is bisected in Ω, as $\Omega\varphi$. And by the same argument the difference of the elastic forces of the physical points ε & γ, in the return of the small physical line $\varepsilon\gamma$ is as $\Omega\varphi$. But that difference (i.e, the excess of the elastic force at the point ε over the elastic force at point γ) is the force of the medium which is introduced for the small physical line $\varepsilon\gamma$ to be accelerated in returning and retarded in going; & therefore the acceleration force on the small physical line $\varepsilon\gamma$, is as its distance from the mean position of the vibrations Ω. Hence the time for the straight line motion PI is explained (by prop. XXXVIII. Book. I) ; & the part $\varepsilon\gamma$ of the medium is moved according to the prescribed law, that is, by the law for the oscillations of a pendulum : and the reasoning is the same for all the line increments from which the whole medium is composed. *Q. E.D.*

   Corol. Hence it is apparent that the number of pulses propagating is the same as the number of vibrations of the trembling body, without change in their number in progressing. For the incremental line in the medium $\varepsilon\gamma$, when first to its original situation returns remains at rest, and henceforth will not move, except either by the impulse of a trembling or oscillating body, or from the impulse of a pulse which is propagating from such a body, when it sets off a new movement. The medium will therefore be quiescent when the starting pulses from the vibrating body cease to be propagated.

Notes on Prop. 48 :

We enlarge on Newton's ideas a little from a modern perspective, but relate to his derivation as much as possible. First, we need to explain his diagram accompanying the trajectory diagram

P and S are the maximum displacements in the positive/ negative senses. The pulse drawn shows the degree of condensation transverse to the direction of transmission, which is really longitudinal.

P and S are the positions where the amplitude and pressure gradient are at maximum values, and the condensation is zero; as the phase angle increases, the condensation grows to a maximum at 90^0, while the amplitude and pressure gradient go to zero, and subsequently these quantities revert again at S to their P conditions. The return stroke sees the dilatation go through the same sequence.
The left-hand sketches show positive density or pressure changes for the outgoing air, while the right-hand ones show negative changes for the returning air. The projection of HIK on to the PS axis shows the passage of the condensation along PS, and the returning dilatation, which are a maximum along the diameter at I and *i*.

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for a ray of sound in one dimension, based on his ideas. According to the diagram opposite, the points *P* and *S* represent the positions of the maximum amplitude χ of the s.h.m. associated with the point *F*. In Newton's words, the time for the motion is spread around the circumference of the circle; in the course of the motion, the matter in the incremental length *EFG* is physically moved as a whole to some intermediate distance Fφ relative to the reference frame of still air, and a wave of compression passes through the element to give the incremental length εγ , finally to come to rest again momentarily at *f* with no compression. The pulse is considered to move from the positive displacement χ of the s. h. m. to the negative displacement in the diagram. Thus, *R* = *OP* = χ rotates clockwise. The times of arrival at *e, f*, and *g* can also be recorded, as are the times of the return of the air to its instantaneous position of maximum displacement : the matter that left first returns first, so *H* → *h*, etc, in the intermediate section. The actual rest position of the air in the absence of waves is at Ω, while *P* and *S* are the points of instantaneous rest for a continuous wave. Newton considers the projection of the maximum compression *HIK* on to the line *PS* as the contraction or expansion of the element at the same point of its motion to and fro : thus, there is no compression or condensation at *P* and *S*, while the maximum compression/expansion occurs at *O*, and a wave of compression/ expansion of some lesser amount but in the same ratio passes along the element *PS* at other times. In addition, by taking ratios at the same out and in positions on the cycle, he is able to proceed without using the bulk modulus, that we now consider as part of the modern theory.
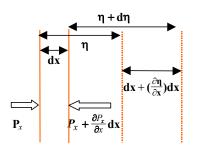
   Modern ideas: Initially, we consider the boundary conditions placed on the elemental oscillators. Each incremental length acts as an s.h.m. oscillator, each driven by or coupled to the one before, and driving the next one in a chain of oscillators, which we assume to be in one dimension. Each oscillator has the same amplitude χ, and all vibrate with the same angular frequency ω as the wave, which we assume to be continuous and of a single frequency; also, each increment has an in-built constant phase factor φ(*x*) depending on its location: when *t* = 0, these phases construct a harmonic wave between the crests of the wavelength λ. Thus, the increment associated with the point *E* has two components of phase that add/subtract to give the total phase : There is the time related phase of the form (2π/*T*).*t* = ω*t*, and for a given quiescent position, there is the constant distance related phase angle for the oscillator, that we can call e. g. φ(*E*) = (2π/λ).*BE* = *k.BE*. The other equally separated quiescent oscillator points *F* and *G* have similar distance phases φ(*F*) = (2π/λ).*BF* and φ(*G*) = (2π/λ).*BG* associated with them. The passage of the wave is the transmission of regions of constant phase from oscillator to oscillator. This region of constant phase is driven forwards in the mathematical model by an argument of the form *kx* - ω*t* = constant, or ω*Δt* = *kΔx*, resulting in a phase velocity *v* = *Δx/Δt* = ω/*k*. There are some details of Newton's model that are inconsistent with the modern theory, even at this kinematic level. We have noted already that he describes the air as being at rest at P and S; although this is true, it is not in its quiescent condition as there is a maximum pressure difference across the element here in accordance with s. h. m. principles, and the air element is actually at its true length (*i. e.* in the absence of the sound wave)when it passes the origin of the oscillation at its maximum speed at the half-way point Ω, when there is no pressure difference across the ends of the element. Recall that for s. h. m. the

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acceleration is proportional to the negative displacement : hence, there is a maximum acceleration and force at the maximum displacement, and zero acceleration and force at zero displacement. The model succeeds in producing the s. h. m. equation with the correct ω and wave speed ω/k.

   Before leaving this narrative, we shall briefly give a modern derivation of the wave equation [following Pain, *The Physics of Vibrations and Waves*, Ch. 5 (Wiley)]. The

motion of an undisturbed infinitesimal element of air of original thickness Δx and unit area under the influence of a sound wave in one dimension is considered. The element as a whole is displaced a distance η, and expanded by an amount $(\partial\eta/\partial x)dx$, as shown in the diagram :

The increase in the volume is $\frac{\partial\eta}{\partial x}dx$, while the change in the volume per unit volume is $\frac{\partial\eta}{\partial x}$ or

the dilatation dv/v. The quantity $d\rho/\rho$ inverse to the dilitation is known as the condensation. Meanwhile, the net force exerted on the element to the right in the compression or expansion due to the pressure gradient is $-\frac{\partial P_x}{\partial x}.dx$; hence, by Newton's

Second Law, $-\frac{\partial P_x}{\partial x}.dx = \rho_0 dx.\frac{\partial^2\eta}{\partial t^2}$. There is now a need to relate the pressure gradient to the changes in the volume : this is usually done by means of the bulk B modulus for the substance. The change in the volume per unit volume is proportional to the impressed pressure,
or $dp = - B. dv/v$, where B is the constant of proportionality, and the negative sign is necessary as an increase in pressure results in a decrease in the dilatation, or change in

volume per unit volume. In the present case, $dv/v = \frac{\partial\eta}{\partial x}$, and hence $\frac{\partial P_x}{\partial x} = -B.\frac{\partial^2\eta}{\partial x^2}$. From

which it follows that $B\frac{\partial^2\eta}{\partial x^2} = \rho_0 \frac{\partial^2\eta}{\partial t^2}$ and $c^2\frac{\partial^2\eta}{\partial x^2} = \frac{\partial^2\eta}{\partial t^2}$, where $c^2 = B/\rho_0$. This is the

conventional wave equation for sound waves in a gas: Newton does not derive this equaton; instead, he derives the equation for the s. h. m. of an elemental section of air,

resulting in $-k^2c^2\eta = -\omega^2\eta = \frac{\partial^2\eta}{\partial t^2}$ for a sinusoidal motion. Now, compression or expansion

of a gas results in heating or cooling; Newton was unaware of the adiabatic nature of sound waves, and used essentially the isothemal form of B or P/ρ_0, rather than the correst adiabatic form $\gamma P/\rho_0$; thus his value for c was out by $\sqrt{\gamma}$, where γ is the ratio of the specific heats of the gas at constant pressure to constant volume, and depends on the nature of the gas - internal degrees of freedom, etc. It would take us too far away from Newton's work to consider this matter further, though of course it is developed in books on thermodynamics.

   There are inevitably problems associated with understanding what the words actually mean when comparing Newton's model with the actual model for sound waves that we have briefly outlined above; thus, the word 'expansion' can mean either the volume or the

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change in volume - the various translations suffer from this ambiguity, and one must proceed with caution. We are not in the business of correcting Newton's model, which would be a great travesty as well as a meaningless exercise, but are merely trying to understand what his thoughts might have been as he developed his ideas.

 So we return to Newton's argument:

Subsequently, to make further progress with the Proposition, use has to be made of calculus. Initially, two useful ratios are evaluated. Pressure - modified volumes represented by the lengths *OL, OM, ON* are related to $\Delta V/V$, the dilatation; and the angular phase of the rotating radius *OP* of the s. h. m. is related to the linear phase of the wave.

Thus, *OL, OM*, and *ON* are given by : $\chi\cos\omega(t +\delta t)$, $\chi\cos\omega t$, and $\chi\cos\omega(t - \delta t)$, leading to

$LN/KH = [\chi\cos\omega(t +\delta t) -\chi\cos\omega(t - \delta t)]/2\chi\omega\delta t \rightarrow -\sin\omega t$, the limiting value of the ratio;

or equivalently, $\dfrac{LN \text{ or } NL}{KH} = \dfrac{\chi\cos(\vartheta - \Delta\vartheta) - \chi\cos(\vartheta + \Delta\vartheta)}{\chi.2\Delta\vartheta}(\rightarrow -\sin\vartheta, \text{ as } \Delta\vartheta \rightarrow 0) = \dfrac{IM}{\chi}$;

[This first differentiation gives the rate of change of *LN* or ΔV, the decrease in the volume, and $LN/EG = \Delta V/V$, the fractional change in the volume.]

and $\dfrac{KH}{EG} = \dfrac{circum.PHSbP}{BC} = \dfrac{circum.PHSbP}{circle \text{ with } circum.BC} = \dfrac{OP}{V} = \dfrac{\chi}{(\lambda/2\pi)}$ relates the phases.

[Thus, In the time the line *OP* turns through a certain small angle, the pulse advances a certain amount along *PS*. The original circle with radius *OP* describes the time variation or the phase of the oscillation at some fixed point, while the second circle with radius $V_r = \lambda/2\pi$, rather than *V* that we use for the volume for a little while *EG* describes the displacement variation of the phase of the oscillation at some fixed time. Hence,

$\dfrac{KH}{EG} = \dfrac{OP}{V_r} = \dfrac{\chi.2\pi}{\lambda}$ or $\dfrac{EG}{\lambda} = \dfrac{2\chi\Delta\vartheta}{\chi.2\pi} = \dfrac{2.\Delta\vartheta}{2\pi}$: thus, a path difference of *EG* corresponds to a

phase difference of $2.\Delta\theta$ as required by the coupled oscillators discussed above.

Or, $v = \omega/k = (2\Delta\theta/\Delta t)/(2\pi/\lambda) = (2\Delta\theta/\Delta t).V_r$; hence $EG = v.\Delta t = 2.\Delta\theta . V_r = $ arc $HK.V_r/OP$]

Since $\dfrac{LN}{KH}.\dfrac{KH}{EG} = \dfrac{LN}{EG} = \dfrac{IM}{OP}.\dfrac{OP}{V_r} = \dfrac{IM}{V_r}$, or $\dfrac{\Delta V}{V} = \dfrac{\text{compression at } \varepsilon\gamma}{\text{quiescent volume at F}} = \dfrac{LN}{EG} = \dfrac{IM}{V_r} [= \dfrac{-\chi\sin\omega t}{\lambda/2\pi}]$, it

follows that the excursion compression of EG at $\varepsilon\gamma$ to the quiescent volume at F is in the

ratio $\dfrac{V - \Delta V}{V} = \dfrac{\varepsilon\gamma}{EG} = \dfrac{EG - LN}{EG} = 1 - \dfrac{LN}{EG} = \dfrac{V_r - IM}{V_r}$, while the return expansion ratio is :

$\dfrac{V + \Delta V}{V} = \dfrac{EG + \ln}{EG} = 1 + \dfrac{\ln}{EG} = \dfrac{V_r + im}{V_r}$.

[The common reader may wish to refer to the work by S. Chandrasekhar at this point (Newton's *Principia* for the Common Reader (1995); Oxford. p. 586) : this author has not attempted, as we have attempted, to actually link up Newton's derivation with modern theory, but has presented this theory from a Newton - friendly point of view. The other

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authors of interest, Cohen & Whitman: Newton The *Principia*. (California), and Cajori :
Newton's *Principia* (U. Cal.) have not given any explanation of Newton's theory of sound,
and have only presented an English version of the Latin text, with all its vagaries. Cajori
in his translation, even goes to the extent of re-arranging the labels on the phase diagram,
thus changing something which is correct into something which is incorrect !]

   Newton now sets out to construct what is essentially the second order differential
equation describing the s. h. m. of an elemental volume of air in the presence of a sound
wave of constant frequency.

Now, if the elastic force varies inversely as the expansion or volume, then

$$\frac{el.\,force\,at\,\varepsilon\gamma}{quiescent\,el.\,force\,at\,F} = \frac{V}{V - IM} \text{ , essentially Boyle's Law, where we revert to Newton's } V =$$

$\lambda/2\pi$ rather than V_r. On the return, $\dfrac{el.\,force\,at\,\varepsilon\gamma}{el.\,force\,at\,F} = \dfrac{V}{V + im}$. A similar argument applies for

the ratio of the elastic forces (or pressures) acting on the volume increments E and G :

$\dfrac{el.\,force\,at\,\varepsilon\phi}{el.\,force\,at\,E} = \dfrac{V}{V - HL}$ and $\dfrac{el.\,force\,at\,\phi\gamma}{el.\,force\,at\,G} = \dfrac{V}{V - KN}$ respectively. It follows that the difference

of these elastic forces to the mean elastic force, which is the same at E, F, and G, is given
by :

$$\frac{el.\,force\,at\,\varepsilon\phi - el.\,force\,at\,\phi\gamma}{quiescent\,el.\,force\,at\,F} = \frac{V}{V - HL} - \frac{V}{V - KN} = \frac{V(HL - KN)}{(V - HL)(V - KN)} \sim \frac{V(HL - KN)}{V^2} = \frac{HL - KN}{V} \,.$$

Note that second order quantities are ignored, as they vanish in the limit.
Now,

$$\frac{(HL - KN)}{HK} = \frac{R(sin(\vartheta + \varDelta\vartheta) - sin(\vartheta - \varDelta\vartheta))}{2R\varDelta\vartheta} \to cos\,\vartheta \text{ , i.e. } \frac{OM}{OI} \text{ or } \frac{OM}{OP} \text{ , as } \varDelta\vartheta \to 0 \,.$$

Hence:

$$\frac{el.\,force\,at\,\varepsilon\phi - el.\,force\,at\,\phi\gamma}{quiescent\,el.\,force\,at\,F} = \frac{HL - KN}{V} = \frac{HK\,cos\,\vartheta}{V} = \frac{HK.OM}{OP.V} \,;$$

or, $$\frac{el.\,force\,at\,\phi\gamma - el.\,force\,at\,\varepsilon\phi}{2.\chi.\varDelta\vartheta} = -\left[\frac{quiescent\,el.\,force\,at\,F}{\chi.(\lambda/2\pi)}\right].\Omega\varphi = -const.\,\Omega\varphi.$$

Now, $\dfrac{2\varDelta\vartheta}{2\pi} = \dfrac{\varepsilon\gamma}{\lambda}$ or $2\varDelta\vartheta = \varepsilon\gamma.\dfrac{2\pi}{\lambda} \sim \varDelta x.\dfrac{2\pi}{\lambda}$. Hence,

$$\rho\varDelta x.\frac{d^2x}{dt^2} = \frac{d(el.\,force)}{dx}\varDelta x = -\varDelta x.\left[\frac{quiescent\,el.\,force\,at\,F}{(\lambda/2\pi)^2}\right].\Omega\varphi = -const.\,\Omega\varphi.$$

Hence, calling P the pressure or the quiescent elastic force at F, and canceling the $\varDelta x$
which is synonymous with $\varepsilon\gamma$, while setting $\Omega\phi = x$, we find that

$$\frac{d^2x}{dt^2} = -\frac{P}{\rho.(2\pi/\lambda)^2}.x = -\frac{v^2}{k^2}x = -\omega^2 x, \text{ as required for s.h.m. In which case, } v = \sqrt{\frac{P}{\rho}}. \text{ Newton did}$$

not pursue his differential equation to this logical conclusion for some reason, and was
content to note that the equation defined the same kind of motion as the cycloidal
pendulum, although he immediately proceeds to use the above formula for the speed. His
final proposition is to present an extreme case involving the s. h. m. of a cycloidal

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pendulum to corroborate his formula ; one gets the impression he was over-impressed with this particular kind of s. h. m., rather than seeing it rather as just another example of this kind of motion, and that he was not entirely convinced with his derivation of this proposition.

PROPOSITION XLVIII.   *THEOREM XXXVIII.*

The speeds of pulses propagating in an elastic fluid are in the ratio composed from the direct proportion of the square root of the elastic force or pressure and the inverse proportion of the square root of the density ; but only if the same elastic force is supposed for the same proportional condensation [i.e. the gases obey Boyle's Law].

Case. 1. [Newton's explanation of why pulses or waves of differing intensities travel at the same speed in a medium.]
   If the media are homogeneous, & the distances between the pulses in these media are equal amongst themselves, but the motion [i. e. the sound] is more intense in one medium than in the other, then the contractions and expansions of the analogous parts are as the same motions [i.e. one has a larger amplitude than the other]. The proportion of these intensities cannot be measured with accuracy. However, unless the contractions and dilatations are of greatly differing intensities, there will be no sensible error, and thus these can be used [for the measurement of physical quantities] with accuracy. But the elastic motive forces are in the ratio of the contractions & dilations ; & the velocities of the equal parts likewise generated are in the ratio of the forces [as the forces act for the same lengths of time]. Hence equal & corresponding parts of corresponding pulses are coming and going by contracting and dilating in their proportional intervals, with velocities which are in the ratio as the intervals, likewise are carried out : & therefore pulses, which in the time of one oscillation are made to progress a distance of one width, & always follow into the place of the nearest proceeding pulse, on account of the equality of the distances, with equal velocity can proceed in either medium.
[We note that the amplitude cancels in the derivation presented in the previous theorem, but see notes below.]

Case 2. [Newton's explanation of why all wavelengths travel at the same speed in the same medium.]
   But if the distances between the pulses or the widths are greater in one medium than the other, then we can put corresponding parts in place , and the proportional widths of the individual pulses that come and go can be described [i.e. we can compare the ratio of the wavelengths or pulse widths]: and the contractions and dilations of [each of] these are equal. Thus if the media are homogeneous, then these elastic motive forces by which the reciprocating motion is driven are also equal [i.e. the amplitudes of the pressure fluctuations are the same]. But the masses to be moved by these forces are in the same ratio as the widths of the pulses; & the corresponding wavelengths of the pulses as they come and go are in the same ratio. But the time of one complete reciprocal motion is composed from the ratio of the square root of the mass & the square root of the interval, and thus as the interval. [Thus, the oscillation time T is proportional to the width of the

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pulses of wavelength λ, or λ α T or 1/f] . But pulses perform their reciprocal motion or s.h.m in a time equal to the passing of one width, that is, the space and the time intervals are proportionals that advance in step, and hence the velocities are equal. [See notes below.]

Case 3. [The ratio of the speeds in differing media.]
    Therefore all the pulses travel at the same speed in media with the same density and elastic forces. But if either the density or the elastic force of the medium is increased, since the motive force is increased in the ratio of the elastic force, & the matter to be moved is increased in the ratio of the density ; the time, by which the same motions are driven from the previous situation, will be augmented in the ratio of the square root of the density, and diminished in the square root ratio of the elastic force. And therefore the velocity of the pulses will be in the ratio composed from the ratio of the inverse of the square root of the density & directly in the ratio of the square root of the elastic force. Q.E.D.
    This proposition is more apparent from the following construction.

Notes on Prop. 48 :
Case 1. We are to imagine two media with the same pressure and density, and waves of some wavelength travel at the same speed in each medium. However, pulses with different intensities in the two media are considered. We try to understand why the speeds of the pulses are equal in the two media and independent of the amplitude of the oscillations, using s.h.m. Using the pendulum analogy of s.h.m, the ratio of the elastic motive forces on corresponding elements is the same as the ratio of the amplitudes, and since these forces act for the same lengths of time on the corresponding elements, during which time the pulses move forwards a distance equal to the inter-pulse displacement, then the velocity of propagation is the same. [However, the ratio of the maximum velocities of the elements is proportional to the ratio of the maximum displacements, as there is a greater pressure associated with the more intense pulses. There is hence a distinction to be drawn between the phase velocity and the maximum speed associated with the s.h.m.]
Case 2. We are to imagine two media with the same pressure and density, and thus at least waves of one wavelength travel at the same speed in each medium. However, pulses with different widths or wavelengths in the two media are considered, each wavelength of constant width in its medium. We try to understand why the speeds of the pulses are nevertheless equal in the two media using s.h.m ideas. The s.h.m motion of the small incremental widths have the same amplitude in each case, otherwise the waves will have differing amplitudes and intensities. However, the longer wavelength requires more of the elementary oscillators, and the pressure differences across each of the incremental widths is thus less for the longer wavelength, as the total pressure fluctuation is the same for both wavelengths, which supplies the motive force on the element. Newton, however, does not consider the individual elements as such at this stage, but focuses his attention on the ratio of the whole condensed or rarefied pulses at some point, and returns to his pendulum idea of s.h.m. In this case, he considers the period *T* of an s.h.m to depend on the mass *m* to be

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moved, and the force constant *k* to effect the motion, for which $T = 2\pi\sqrt{\frac{m}{k}}$. By simply adding the number of elementary incremental oscillators, the masses to be moved for the long and short wavelengths λ_L and λ_S are in the ratio $\frac{m_S}{m_L} = \frac{\lambda_S}{\lambda_L}$. Now, regarding the force constant *k,* or the spring constant of the air, it can be taken as proportional to the excess pressure p, and inversely as the wavelength λ; or, $k = \frac{p}{\lambda}$. Hence, the periods for the short

and the long waves are as : $\frac{T_S}{T_L} = \frac{\sqrt{\frac{m_s}{k_s}}}{\sqrt{\frac{m_L}{k_L}}} = \frac{\sqrt{\frac{m_s}{m_L}}}{\sqrt{\frac{k_S}{k_L}}} = \sqrt{\frac{\lambda_S}{\lambda_L}} \times \sqrt{\frac{\lambda_S}{\lambda_L}} = \frac{\lambda_S}{\lambda_L}.$ Or, the frequencies vary

inversely as the ratio of the wavelengths, as required.

 Case 3. The speed of propagation *v* of a pulse in a medium with elastic force or pressure *p* and density ρ is given by $v = \sqrt{\frac{p}{\rho}}$, to be finally proved in the next Proposition. Thus, if the density is increased from ρ_1 to ρ_2, for the same pressure, then the associated speeds are in the ratio $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$; while if the pressure is increased from p$_1$ to p$_2$, for the same density, then $\frac{v_1}{v_2} = \sqrt{\frac{p_1}{p_2}}$. If both pressure and density increase in the same ratio, then there is no change in the speed of the pulses. In general, if both pressure and density are allowed to change, but in different ratios, (as for example, for media such as ideal gases at different temperatures), then $\frac{v_1}{v_2} = \sqrt{\frac{p_1/\rho_1}{p_2/\rho_2}} = \sqrt{\frac{p_1}{p_2}}.\sqrt{\frac{\rho_2}{\rho_1}}$, which is Newton's comment.]

PROPOSITION XLIX.   PROBLEM XI.

To find the velocity of pulses for a given density and elastic force of medium.

    We can imagine the medium to be compressed by the incumbent weight of the air in the manner of our air ; and let *A* be the height of the homogeneous medium which is equal to the incumbent weight [from the quiescent point], and the density of which is the same as that of the compressed medium, in which the pulses are propagated. Moreover, a pendulum is considered to be set up, the length of which is *A* between the point of suspension and the centre of oscillation [thus, the radius of the generating circle of the cycloid is *A*/2, and the whole arc length is 2*A*.] : and in the time that the pendulum executes a complete to and fro oscillation, a pulse will travel a distance equal to the circumference of the circle described by the radius *A*. [Referring back to Prop. 47, and to the phase diagram; note that PS = λ/2.]
    For in agreement with what has been stated in proposition XLVII, if some physically narrow region *EF* is pushed to the limits of the oscillatory motion which are located at *P* and *S*, with the vibrations within the space PS to be described by the single element, [thus, the motion of the single increment occupies the whole amplitude or is responsible for the entire s. h. m.] then the elastic force [or pressure] is itself equal to the weight of the air,

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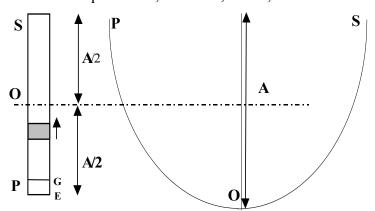Translated and Annotated by Ian Bruce.   Page 690

and it will perform these individual vibrations in the same time that the oscillations can be performed on the cycloid, the whole perimeter of which is equal to the length *PS* [= 2*A*]: since equal forces can push small bodies [of equal mass] through equal distance in the same time. Whereby as the times of oscillations are in the square root ratio of the lengths of pendulums, and the length of the pendulum is equal to half of the whole arc of the cycloid ; the time of one vibration in the air to the time of the oscillation of the pendulum, the length of which is *A*, is in the ratio of the square root of the length $\frac{1}{2}PS$ (or *PO*) to the length *A*.

But the elastic force, by which the incremental length *EG* is forced*,* present in its extreme places *P* and *S*, is (as in the demonstration of proposition XLVII) to the total force of this as *HL - KN* to *V*,
that is (as the point *K* thus falls in *P*) as HK to V: and that total force, that is the incumbent weight by which the incremental line *EG* is compressed, is to the weight of the elemental line as the altitude *A* for the weight of the incumbent air to the incremental line of length *EG*; and thus from the equality, the force by which the incremental line *EG* is pushed in its locations *P* and *S,* is to the weight of the incremental line as *HK × A* to *V × EG*, or as *PO ×A* to *VV*, for *HK* is to *EG* as *PO* to *V*. Whereby with the times, for which equal bodies are pushed through equal distances, are reciprocally in the square root ratio of the forces [Essentially at^2 is constant, or Ft^2/m is constant, giving $t \propto 1/\sqrt{F}$.], the time of one vibration will be, from the force exerted by the pressure, to the time of the vibration, for the force due to the weight [in the pendulum case], in the square root ratio *VV* to *PO × A,* and thus to the time of the oscillation of the pendulum of which the length is *A* in the square root ratio *VV* to *PO ×* A and the square root ratio *PO* to *A* jointly; that is, in the ratio all together as *V* to *A*. [The starting point P or S of the motion is chosen; there is no difference for other points in the motion as the displacement cancels; see the note.] But in the time of one vibration composed from the to and fro motion, the pulse by proceeding makes its own length *BC* [for λ = 2 × PS. Therefore the time, in which the pulse travels through the distance *BC*, is to the time of one oscillation composed from the to and fro motion of the pendulum, as *V* to *A*, that is, as *BC* to the circumference of the circle of

which the radius is *A*. Moreover the time, in which the pulse travels through the distance *BC*, is to the time by which it travels the length of this equal circumference, in the same ratio; and thus for the time of such an oscillation the pulse travels a distance equal

to the circumference of this circle. *Q.E.D.*

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Notes on Proposition 49 :

Newton initially considers the height of an atmosphere of uniform density that results in the pressure observed at ground level. We note in passing that this height *h* is related to the height *k* of mercury in a barometer according to the elementary rule $\rho_{air}h = \rho_{Hg}k$, or

$h = (\rho_{Hg} / \rho_{air})k$, or the ratio of the specific gravity of mercury to air times by the length of the mercury column, as you would expect if you is not too concerned about finer details. Newton calls this height *A*.

Let us set up the oscillating atmosphere envisaged :
The height of the homogeneous atmosphere is *A*, and *O* is taken as the half-way point. A relatively small segment of air of quiescent length *EG* is part of the oscillating air mass of amplitude *OS = OP*. The situation at some intermediate stage ascending is shown at on the left-hand side of the diagram L. The explanation relies on Prop. 47 :

$\dfrac{el.\,force\,at\,\varepsilon\phi - el.\,force\,at\,\phi\gamma}{quiescent\,el.\,force\,at\,F} = \dfrac{unbalanced\,force\,on\,element\,EG}{quiescent\,pressure} = \dfrac{HL - KN}{V}$. In the present case, the

element is at an extreme position, in which case KN is zero, and the length HL is

approximately equal to the arc length HK; hence, $\dfrac{Unbalanced\,force\,on\,element\,EG}{quiescent\,pressure} \rightarrow \dfrac{HK}{V}$.

Also, the ratio

$\dfrac{incumbent\,force\,or\,pressure\,on\,EG\,at\,P}{weight\,of\,line\,EG} = \dfrac{\rho gA}{\rho g \times EG} = \dfrac{A}{EG}$. Hence,

$\dfrac{Unbalanced\,force\,on\,element\,EG}{quiescent\,pressure} \times \dfrac{quiescent\,pressure}{weight\,of\,EG} = \dfrac{HK}{V} \times \dfrac{A}{EG}$. Again, from Prop.47 :

$$\rho \Delta x . \frac{d^2 x}{dt^2} = -\frac{d(el.\,force)}{dx}\Delta x = -\Delta x \left[\frac{quiescent\,el.\,force\,at\,F}{(\lambda / 2\pi)^2}\right].\Omega\varphi = -const.\,\Omega\varphi,$$

we have the ratio of the unbalanced force to the mass of the element :

$$\rho EG . \frac{d^2 x}{dt^2} / \rho . EG = -\frac{d(P)}{dx} / \rho = -\left[\frac{\rho gA}{V^2}\right].x / \rho = -const.\;\;x \rightarrow -\left[\frac{gA}{V^2}\right].OP = -\left[\frac{gA}{V^2}\right].OP$$

$$= \frac{HK \times Ag}{V \times EG};\;\text{hence}: \frac{d^2 x}{dt^2} = -\left[\frac{Ag}{V^2}\right].x\;\text{for the air mass; while for the pendulum,}\;\frac{d^2 x}{dt^2} = -\frac{g}{A}.x.$$

Thus, the period of oscillation of the air $T_{air} = 2\pi\sqrt{\dfrac{V^2}{A.g}}$, while the period of the cycloidal

pendulum is given by : $T_{pen.} = 2\pi\sqrt{\dfrac{A}{g}}$. Hence. $T_{air} / T_{pen.} = \sqrt{\dfrac{V^2}{A^2}} = \dfrac{V}{A} = \dfrac{\lambda}{2\pi A}$. Note that

Newton always uses ratios, so there is no need to worry about constant factors such as *g* and 2π that have to be inserted in absolute measurements and calculations; indeed the use of π as a ratio had not been introduced at this time.

Corollary 1. The velocity of the pulses is the same as that which weighty bodies acquire by falling under the acceleration of gravity, and in their case through half the height *A*. For the time in this case, for the velocity to be acquired by falling, the pulse travels through the space equal to the whole interval *A*; and thus the time of the oscillation composed from one coming and going the space traveled through is equal to the circumference of the circle *A* described. Indeed the ratio of the time to fall to the time of the oscillation is as the radius of the circle to the circumference of the same.

Note on *Cor*. 1 : For the time for a body to fall a vertical distance *A*/2 is given by

$T_{body} = \sqrt{\dfrac{A}{g}}$, while the time for the pulse to perform half an oscillation and travel from P to

S is given by $T_{air}/2 = \pi\sqrt{\dfrac{V^2}{A.g}}$; hence, $T_{body} : T_{air}/2 = \sqrt{\dfrac{A}{g}} : \pi\sqrt{\dfrac{V^2}{A.g}} = \dfrac{A}{\pi V} = \dfrac{A}{\lambda/2} = 1$. Also, the

pendulum bob in these times traverses a space in the ratio

$T_{air}/T_{pen.} = \sqrt{\dfrac{V^2}{A^2}} = \dfrac{V}{A} = \dfrac{\lambda}{2\pi A} = \dfrac{2A}{2\pi A}. = \dfrac{1}{\pi} = \dfrac{diameter}{circumference}.$

Corollary 2 : Hence since that height A shall be directly proportional to the elastic force and in inverse proportion to the density of the fluid; the velocity of the pulse will be in the ratio composed from the square root of the inverse of the density and in direct proportion to the square root of the elastic force.

Note on *Cor*. 2 : If the periodic time *T* for a complete oscillation is inversely proportional to √*A*, and
$P = \rho g A$, then the velocity is proportional to 1/*T* or $\sqrt{(P/\rho)}$. As Chandrasekhar points out, this theorem was probably added by Newton as he was not entirely satisfied with Prop. 47, which he did not work through to its conclusion. However, this author makes claims for what Newton has done which bear little resemblance to reality - there is a distinct lack of sophisticated mathematical machinery in Newton's work, although the intuitive ideas are there. The mathematical structure describing phase velocity was not in place at the time, and Newton's work presumably set this theory in motion. We may note in passing its use of an extreme amplitude of around 8000m for the height of the isotropic atmosphere to give a known pressure, as Newton sought known numbers to use in his equation as a check, with which one could associate a wave with a period of some 25 seconds, more in the realms of internal gravitational waves in the atmosphere than sound waves.

PROPOSITION L.   PROBLEM XII.

To find the lengths of the pulses.

The number of vibrations in a given time need to be found for the body which is exciting the pulses. The distance which the pulses are able to traverse in this time is divided by this number, and the fraction of the length found is the width of one pulse. *Q. E. I.*

Note : Thus the well-known result for the phase velocity $v = f\lambda$ comes into being.

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Scholium.

The most recent propositions consider the motion of light and of sound. Indeed light is propagated in straight lines, without interaction (by prop. XL & XLII.). Hence, since sounds arise from the vibrations of bodies, they are nothing other than pulses propagated in the air, by prop. XLIII. This is confirmed from the vibrations which they cause in bodies presented to them, but only if they are strong and deep, such as the sounds of small drums. For quicker and shorter vibrations are more difficult to be excited. But it is well known also that any sounds can interact with the strings of musical instruments and excite vibrations. This is also confirmed from the velocity of sound. For since the specific gravity of rainwater and quicksilver are in turn as 1 to $13\frac{2}{3}$ roughly, and where the height

of mercury in a Barometer reaches a height of 30 English inches, the specific gravity of air and of rainwater are in the ratio 1 to 870 roughly; the specific gravity of air and quicksilver are as 1 to11890. Then since the height of quicksilver is 30 inches, the height of the air in a uniform atmosphere, which our air is subject to in compression, is 356700 inches, or 29725 English feet. This is the height that we have called A in the construction of the above problems. The circumference of the circle described by a radius of 29725 feet is 186768 feet. And since it is well-known that a pendulum $39\frac{1}{5}$ inches long results in

a complete oscillation in a time of two seconds, then a pendulum 29725 feet long or 356700 inches ought to complete a similar oscillation in a time of $190\frac{3}{4}$ seconds. Therefore

in that time the sound should progress a distance of 186768, and thus in a time of one second sound should travel 979 feet.

In the computation presented here, no account is made for other effects, such as the density of solid particles in the air, through which the sound certainly is propagated. Since the weight of air is to the weight of water as 1 to 870, and salts are nearly twice as dense as water ; if the particles of air are put to be roughly the same density as the particles of water or salt, and the rareness of air arises from the intervals between the particles: then the diameter of the air particles will be as the interval between the centres of the particles, as 1 to 9 or 10 roughly, and to the interval between the particles as 1 to 8 or 9. Hence to the 979 feet in the above calculation, one may add $\frac{979}{9}$ or 109 feet roughly,

to the distance that sound travels in a time of one second, on account of the density of particles in the air : & thus the distance that sound travels in a time of one second is made to be roughly 1088 feet.

To these you may add the effect of vapours hidden in the air, since they are of a different tone and elastic nature they may or may not participate in the motion of sound that is propagated through the air. But from these quiet sources, the motion is propagated more quickly than by the air alone, and that in the ratio of the square root of the lesser matter. For if the atmosphere is made up from ten parts air and one part of vapor, the speed of sound is faster in the ratio of the square root of 11 to 10, or altogether around the ratio 21 ad 20, than if the sound is propagated by eleven parts of pure air : and thus the motion of the air found above is increased in this ratio. From which the speed of sound is agreed upon to be 1142 feet in one second.

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Thus these ought to have an effect in springtime and autumn, when the air is rarefied by the temperate heat and the pressure is increased. In wintertime, when the air is condensed by the cold, and its pressure is lowered, the speed of sound should be less in the square root ratio of the densities, while in summertime in turn, the speed should be increased.

Moreover, it is agreed upon by experiment that the distance gone in a time of one second is more or less *London* feet 1142, and truly 1070 *Parisian* feet.

With the speed of sound recognised, the intervals between the pulses can also become known. Certainly Sauveur has found from measurements made in his experiments, that an open pipe, the length of which is more or less five *Parisian* feet, send forth a sound of the same tone as the sound of strings which are vibrating at a rate of a hundred times in one second. Thus, there are more or less one hundred pulses in a space of 1070 *Parisian* feet, which sound travels through in a time of one second ; hence a single pulse takes up a space of $10\frac{7}{10}$ *Parisian* feet, that is, around twice the length of the tube. Thus, it is the same for pulses of all lengths from the sounds produced by tubes, for they are equal to twice the lengths of the open ended tubes.

Again since sounds stop with the motion of the vibrating body when we stand nearby, but not for a long time when we stand a long way from the source of the sound, which is apparent from the corollary to proposition XLVII of this book. Moreover why sounds are greatly increased in volume by deep sounding trumpets is apparent from these principles. Indeed the reciprocal motion of all recurring individual pulses is usually increased by the source of the vibration. Moreover, the motion of sound is impeded in trumpets, to be emitted later and louder, and therefore a new individual motion is returned later more loudly. And these are the main phenomena associated with sound.

[Thus, Newton remained unaware of the true source of the error in his analysis; this was eventually corrected by Laplace when the effect of heat on a gas was much better understood, and the adiabatic form of the gas law was applied, rather than Boyle's Law.]

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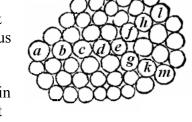
SECTIO VIII.

De motu per fluida propagato.

PROPOSITIO XLI. THEOREMA XXXII.

Pressio non propagatur per fluidum secundum lineas rectas,nisi ubi particulae fluidi in directum iacent.
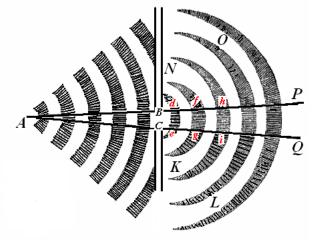
Si iaceant particulae *a, b, c, d, e* in linea recta, potest quidem pressio directe propagari ab *a* ad *e* ; at particula *e* urgebit particulas oblique positas *f* & *g* oblique, & particulae illae *f* & *g* non sustinebunt pressionem illatam, nisi fulciantur a particulis ulterioribus *h* & *k* ; quatenus autem fulciuntur, premunt particulas fulcientes , & hae non sustinebunt pressionem nisi fulciantur ab ulterioribus *t* & *m* easque premant, & sic deinceps in infinitum. Pressio igitur, quam primum propagatur ad particulas quae non in directum iacent, divaricare incipiet & oblique propagabitur in infinitum; & postquam incipit oblique propagari, si inciderit in particulas ulteriores, quae non in directum iacent, iterum divaricabit; idque toties, quoties in particulas non accurate in directum iacentes inciderit. *Q.E.D.*

Corol. Si pressionis, a dato puncto per fluidum propagatae, pars aliqua obstaculo intercipiatur, pars reliqua, qua non intercipitur, divaricabit in spatia pone obstaculum, id quod sic etiam demonstrari potest. A puncto *A* propagetur pressio quaquaversum, idque

si fieri potest secundum lineas rectas, & obstaculo *NBCK* perforato in *BC* intercipiatur ea omnis, praeter partem coniformem *APQ*, quae per foramen circulare *BC* transit. Planis transversis *de*, *fg, hi* distinguatur conus *APQ* in frusta; & interea dum conus *ABC*, pressionem propagando, urget frustum conicum ulterius *degf* in superficie *de*, & hoc frustum urget frustum proximum *fgih* in superficie *fg*, & frustum illud urget frustum tertium, & sic deinceps in infinitum, manifestum est (per motus legem tertiam, quod

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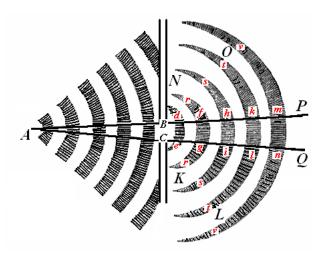
fructum primum *defg*, reactione frusti secundi *fghi*, tantum urgebitur & premetur in superficie *fg*, quantum urget & premit frustum illud secundum. Frustum igitur *degf* inter conum *Ade* & frustum *fhig* comprimitur utrinque, & propterea (per Corol. VI. Prop. XIX., figuram suam servare nequit, nisi vi eadem comprimatur undique. Eodem igitur impetu quo premitur in superficiebus *de, fg*, conabitur cedere ad latera *df, eg*; ibique (cum rigidum non sit, sed omnimodo fluidum excurret) ac dilatabitur, nisi fluidum ambiens adsit, quo conatus iste cohibeatur. Proinde conatu excurrendi, premet tam fluidum ambiens ad latera *df, eg* quam frustum f*ghi* eodem impetu : & propterea pressio non minus propagabitur a lateribus *df, eg* in spatia *NO, KL* hinc inde, quam propagatur a superficie *fg* versus *PQ. Q.E.D.*

PROPOSITIO XLII. THEOREMA XXXIII.
Motus omnis per fluidum propagatus divergit a recto tramite in spatia immota.

   Cas.1. Propagetur motus a puncto *A* per foramen *BC*, pergatque, si fieri potest, in spatio conico *BCQP*, secundum lineas rectas divergentes a puncto *A*. Et ponamus primo quod motus iste sit undarum in superficie stagnantis aquae. Sintque *de, fg, hi, kl*, &c. undarum singularum partes altissimae, vallibus totidem intermediis ab invicem distinctae. Igitur quoniam aqua in undarum iugis altior est quam in fluidi partibus immotis *LK, NO*, defluet eadem de iugorum terminis *e, g, i, l*, &c. *d, f, b, k*, &e. hinc inde versus *KL & NO*: & quoniam in undarum vallibus depressior est quam in fluidi partibus immotis *KL, NO*; defluet eadem de partibus illis immotis in undarum valles. Defluxu priore undarum iuga, posteriore valles hinc inde dilatantur & propagantur versus *KL & NO*. Et quoniam motus undarum ab *A* versus *PQ* sit per continuum defluxum iugorum in valles proximos,

ideoque celerior non est quam pro celeritate descensus; & descensus aquae hinc inde versus *KL & NO* eadem velocitate peragi debet; propagabitur dilatatio undarum hinc inde versus *KL & NO* eadem velocitate qua unde ipse ab *A* versus *PQ* recta progrediuntur. Proindeque spatium totum hinc inde versus *KL & NO* abundis dilitatis *rfgr, shis, tklt*, *vmnv*, &c. occupabitur. *Q.E.D.* Haec ita se habere quilibet in aqua stagnante experiri potest.

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Cas. 2. Ponamus iam quod *de, fg, hi, kl*, *mn* designent pulsus a puncto *A* per medium elasticum successive propagatos. Pulsus propagari concipe per successivas condensationes & rarefactiones medii, sic ut pulsus cuiusque pars densissima sphericam occupet superficiem circa centrum *A* descriptam, & inter pulsus successivos aequalia intercedant intervalla. Designent autem lineae *de, fg, hi, kl*, &c. densissimas pulsuum partes, per foramen *BC* propagatas. Et quoniam medium ibi densius est quam in spatiis hinc inde versus *KL & NO*, dilatabit sese tam versus spatia illa *KL, NO* utrinque sita, quam versus pulsuum rariora intervalla; eoque pacto rarius semper evadens e regione intervallorum ac densius e regione pulsuum, participabit eorundem motum. Et quoniam pulsuum progressivus motus oritur a perpetua relaxatione partium densiorum versus antecedentia intervalla rariora ; & pulsus eadem fere celeritate sese in medii partes quiescentes *KL, NO* hinc inde relaxare debent ; pulsus illi eadem fere celeritate sese dilatabunt undique in spatia immota *KL, NO*, qua propagantur directe a centro *A* ; ideoque spatium totum *KLON* occupabunt. *Q.E.D*. Hoc experimur in sonis, qui vel monte interposito audiuntur, vel in cubiculum per fenestram admissi sese in omnes cubiculi partes dilatant, inque angulis omnibus audiuntur, non tam reflexi a parietibus oppositis, quam a fenestra directe propagati, quantum ex sensu iudicare licet.

Cas. 3. Ponamus denique quod motus cuiuscunque generis propagetur ab *A* per foramen *BC* & quoniam propagatio ista non fit, nisi quatenus partes medii centro *A* propiores urgent commoventque partes ulteriores , & partes quae urgentur fluidae sunt, ideoque recedunt quaquaversum in regiones ubi minus premuntur: recedent eaedem versus medii partes omnes quiescentes, tam laterales *KL & NO*, quam anteriores *PQ*, eoque pacto motus omnis, quam primum per foramen *BC* transiit, dilatari incipiet & inde tanquam a principio & centro, in partes omnes directe propagari. *Q. E. D*.

<div align="center">PROPOSITIO XLIII. THEOREMA XXXIV.</div>

Corpus omne tremulum in medio elastico propagabit motum pulsuum undique in directum; in media vero non elastico motam circularem excitabit.

*Case*1. Nam partes corporis tremuli vicibus alternis eundo & redeundo, itu suo urgebunt & propellent partes medii sibi proximas, & urgendo compriment easdem & condensabunt; dein reditu suo sinent partes compressas recedere & sese expandere. Igitur partes medii corpori tremulo proximae ibunt & redibunt per vices, ad instar partium corporis illius tremuli: & qua ratione partes corporis huius agitabant hasce medii partes, hae similibus tremoribus agitatae agitabunt partes sibi proximas, eaeque similiter agitatae agitabunt ulteriores, & sic deinceps in infinitum. Et quemadmodum medii partes primae eundo condensantur & redeundo relaxantur, sic partes reliquae quoties eunt condensabuntur, & quoties redeunt sese expandent. Et propterea non omnes ibunt & simul redibunt (sic
enim determinatas ab invicem distantias servando, non rarefierent & condensarentur per vices) sed accedendo ad invicem ubi condensantur, & recedendo ubi rarefiunt, aliquae earum ibunt dum aliae redeunt; idque vicibus alternis in infinitum. Partes autem euntes & eundo condensatae, ob motum suum progressivum, quo feriunt obstacula, sunt pulsus , & propterea pulsus successivi a corpore omni tremulo in directum propagabuntur ; idque

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aequalibus circiter ab invicem distantiis, ob aequalia temporis intervalla, quibus corpus tremoribus suis singulis singulos pulsus excitat, Et quanquam corporis tremuli partes eant & redeant secundum plagam aliquam certam & determinatam, tamen pulsus inde per medium propagati sese dilatabunt ad latera, per propositionem precedentem, & a corpore illo tremulo tanquam centro communi, secundum superficies propemodum sphaericas & concentricas, undique propagabuntur. Cuius rei exemplum aliquod habemus in undis, quae si digito tremulo excitentur, non solum pergent hinc inde secundum plagam motus digiti, sed, in modum circulorum concentricorum, digitum statim cingent & undique propagabuntur. Nam gravitas undarum supplet locum vis elasticae.

Case 2. Quod si medium non sit elasticum : quoniam eius partes a corporis tremuli partibus vibratis pressae condensari nequeunt, propagabitur motus in instanti ad partes ubi medium facillime cedit, hoc est, ad partes quas corpus tremulum alioqui vacuas a tergo relinqueret. Idem est casus cum casu corporis in medio quocunque proiecti. Medium cedendo proiectilibus, non recedit in infinitum; sed in circulum eundo, pergit ad spatia quae corpus relinquit a telgo. Igitur quoties corpus tremulum pergit in partem quamcunque, medium cedendo perget per circulum ad partes quas corpus relinquit ; & quoties corpus regreditur ad locum priorem, medium inde repelletur & ad locum suum priorem redibit. Et quamvis corpus tremulum non sit firmum, sed modis omnibus flexile, si tamen magnitudine datum maneat, quoniam tremoribus suis nequit medium ubivis urgere, quin alibi eidem simul cedat; efficiet ut medium, recedendo a partibus ubi premitur, pergat semper in orbem ad partes quae eidem cedunt. *Q. E. D.*

Corol. Hallucinantur igitur qui credunt agitationem partium flammae ad pressionem, per medium ambiens, secundum lineas rectas propagandum conducere. Debebit eiusmodi pressio non ab agitatione sola partium flammae, sed a totius dilatatione derivari.
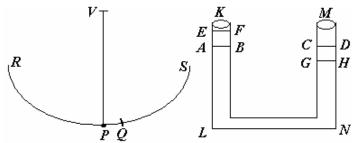
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PROPOSITION XLIV. THEOREMA XXXV.

Si aqua in canalis cruribus erectis KL, MN vicibus alterius ascendat & descendat; construatur autem pendulem cuius longitudo inter punctum suspensionis & centrum oscillationis aequetur semissi longitudines aquae in canali : dico quod aqua ascendet & descendet iisdem temporibus quibus pendulum oscillatur.

Longitudinem aquae mensuro secundum axes canalis & crurum, eandem summae horum axium aequando ; & resistentiam aquae, quae oritur ab attritu canalis, hic non considero. Designent igitur *AB, CD* mediocrem altitudinem aquae in crure utroque ; & ubi aqua in crure *KL* ascendit ad altitudinem *EF*, descenderit aqua in crure *MN* ad altitudinem

GH. Sit autem *P* corpus pendulum, *VP* filum, *V* punctum suspensionis, *RPQS* cyclois quam pendulum describat, *P* ejus punctum infimum, *PQ* arcus altitudini *AE* aequalis. Vis, qua motus aquae alternis vicibus acceleratur & retardatur, est excessus ponderis aquae in alterutro crure supra pondus in altero,ideoque, ubi aqua in crure *KL* ascendit ad *EF*, & in crure altero descendit ad *GH*, vis illa est pondus duplicatum aquae *EABF*, & propterea est ad pondus aquae totius ut *AE* seu *PQ* ad *VP* seu *PR*. Vis etiam qua pondus *P* in loco quovis *Q* acceleratur & retardatur in cycloide (per Corol. Prop LI.) est ad eius pondus totum ut eius distantia *PQ* a loco infimo *P*, ad cycloidis longitudinem *PR*. Quare aquae & penduli, aequalia spatia *AE, PQ* describentium vires motrices sunt ut pondera movenda; ideoque, si aqua & pendulum in principio quiescunt, vires illae movebunt eadem aequaliter temporibus aequalibus, efficientque ut motu reciproco simul eant & redeant. *Q. E. D.*

Corol. 1. Igitur aquae ascendentis & descendentis, sive motus intensior sit sive remissior, vices omnes sunt isochronae.

Corol. 2. Si longitudo aquae totius in canali sit pedum *Parisiensium* $6\frac{1}{9}$: aqua tempore minuti unius secundi descendet, & tempore minuti alterius secundi ascendet; & sic deinceps vicibus alternis in infinitum . Nam pendulum pedum $3\frac{1}{18}$ longitudinis tempore minuti unius secundi oscillatur.

Corol. 3. Aucta autem vel diminuta longitudine aquae, augetur vel diminuitur tempus reciprocationis in longitudinis ratione subduplicata.

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PROPOSITION XLIV. THEOREMA XXXVI.

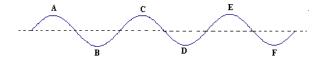*Undarum velocitas est in subduplicata ratione latitudinum.*

Consequitur ex constructione propositionis sequentis.

PROPOSITION XLVI. PROBLEMA X.

Invenire velocitatem undarum.

Constituatur pendulum cuius longitudo, inter punctum suspensionis & centrum oscillationis, aequetur latitudini undarum : & quo tempore pendulum illud oscillationes singulas peragit, eadem undae progrediendo latitudinem suam propemodum conficient.
Undarum latitudinem voco mensarum transversam, quae vel vallibus imis,vel summis culminibus interjacet. Designet *ABCDEF* superficiem aquae stagnantis, undis successivis ascendentem ac descendentem; sintque *A, C, E*, &c. undarum culmina, & *B, D, F*, &c. valles intermedii. Et quoniam motus undarum sit per aquae successivum ascensum & descensum, sic ut ejus partes *A, C, E*, &c. quae nunc altissimae sunt, mox fiant infimae; & vis motrix, qua partes altissimae descendunt & infimae ascendunt, est pondus aquae elevatae; alternus ille ascensus & descensus analogus erit motui reciproco aquae in canali, easdemque temporis leges observabit : & propterea (per Prop. XLIV) si distantiae inter undarum loca altissima *A, C, E* & infima *B, D, F* aequentur duplae penduli longitudini; partes altissimae *A, C, E*, tempore oscillationis unius evadent infimae, & tempore oscillationis alterius denuo ascendent. Igitur inter transitum undarum singularum tempus

erit oscillationum duarum; hoc est, unda describet latitudinem suam, quo tempore pendulum illud bis oscillatur ; sed eodem tempore pendulum , cuius longitudo quadrupla est, ideoque aequat undarum latitudinem, oscillabitur semel. *Q.E.I.*
Corol. 1. Igitur undae, quae pedes Parisienses $3\frac{1}{18}$ latae sunt, tempore minuti unius secundi progrediendo latitudinem suam conficient ; ideoque tempore minuti unius primi percurrent pedes $183\frac{1}{3}$, & horae spatio pedes 11000 quamproxime.
Corol. 2. Et undarum majorum velocitas augebitur vel diminuetur in subduplicata ratione latitudinis.
Haec ita se habent ex hypothesi quod partes aquae recta ascendunt vel recta descendunt; sed ascensus & descensus ille verius fit per circulum, ideoque tempus hac propositione non nisi quamproxime definitum esse affirmo.

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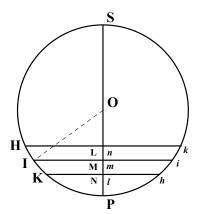Translated and Annotated by Ian Bruce.   Page 701
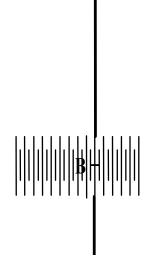
PROP. XLVII.   THEOR. XXXVII.

Pulsibus per fluidum propagatis, singulae fluidi particulae, motu reciproco brevissimo euntes & reeuntes, accelerantur semper & retardantur pro lege oscillantis penduli.

Designent *AB, BC, CD*, &c. pulsuum successivorum aequales distantias ; *ABC* plagam motus pulsuum ab *A* versus *B* propagati *; E, F, G* puncta tria physica medii quiescentis in recta *AC* ad aequales ab invicem distantias sita *; Ee, Ff, Gg* spatia aequalia per brevia per quae puncta illa motu reciproco singulis vibrationibus eunt & redeunt ; *ε, φ, γ* loca quaevis intermedia eorundem punctorum ; & *EF, FG* lineolas physicas seu medii partes lineolas punctis illis interjectas, & successive translatas in loca *εφ, φγ, & ef, fg*. Rectae *Ee* aequalis ducatur recta *PS*. Bisecetur eadem in *O*, centroque *O* & intervallo *OP* describatur circulus *SIPi*. Per hujus circumferentiam totam cum partibus suis exponatur tempus totum vibrationis unius cum ipsius partibus proportionalibus; sic ut completo tempore quovis *PH* vel *PHSh*, si demittatur ad *PS* perpendiculum *HL* vel *hl*, & capiatur *Eε* aequalis *PL* vel *Pl*, punctum physicum *E* reperiatur in ε. Hac lege punctum quodvis *E*, eundo ab *E* per ε ad *e*, & inde redeundo per *ε* ad *E*, iisdem accelerationis ac retardationis gradibus vibrationes singulas peraget cum oscillante pendulo.

Probandum est quod singula medii puncta physica tali motu agitari debeant. Fingamus igitur medium tali motus a causa quacunque cieri, & videamus quid inde sequatur.

In circumferentia *PHSh* capiantur aequales arcus *HI, IK* vel *hi, ik*, eam habentes rationem ad circumferentiam totam quam habent aequales rectae *EF, FG* ad pulsuum intervallum totum *BC*, Et demissis perpendiculis *IM, KN* vel *im, kn* ; quoniam puncta *E, F, G* motibus similibus successive agitantur, & vibrationes suas integras ex itu & reditu compositas interea peragunt dum pulsus transfertur a *B* ad *C*; si *PH* vel *PHSh* sit tempus ab initio motus puncti *E*, erit *PI* vel *PHSi* tempus ab initio motus puncti *F*, & *PK* vel *PHSk* tempus ab initio motus puncti G; & propterea *Eε, Fφ, Gγ* erunt ipsis *PL, PM, PN* in itu punctorum, vel ipsis *Pl, Pm, Pn* in punctorum reditu, aequales respective. Unde *εγ* seu *EG + Gγ - Eε* aequalis erit *EG - LN*, in reditu autem aequalis EG + *ln*. Sed *εγ* latitudo est

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seu expansio partis medii *EG* in loco $\varepsilon\gamma$; & propterea expansio partis illius in itu est ad ejus expansionem mediocrem, ut *EG - LN* ad *EG* ; in reditu autem ut *EG + ln* seu *EG + LN* ad *EG*. Quare cum sit *LN* ad *KH* ut *IM* ad radium *OP*, & *KH* ad *EG* ut circumferentia *PHShP* ad *BC*, id est, se ponatur *V* pro radio circuli circumferentiam habentis aequalem intervallo pulsuum *BC*, ut *OP* ad *V* ; & ex aequo *LN* ad *EG* ut *IM* ad V : erit expansio partis EG punctive physici F in loco εγ ad expansionem mediocrem, quam pars illa habet in loco suo primo EG, ut *V - IM* ad V in itu, utque *V + im* ad *V* in reditu. Unde vis elastica puncti *F* in loco $\varepsilon\gamma$ est ad vim ejus elasticam mediocrem in loco *EG*, ut $\frac{1}{V-IM}$ ad $\frac{1}{V}$ in itu, in reditu vero ut $\frac{1}{V+im}$ ad $\frac{1}{V}$. Et eodem argumento vires elasticae punctorum physicorum *E* & *G* in itu, sunt ut $\frac{1}{V-HL}$ & $\frac{1}{V-KN}$ ad $\frac{1}{V}$; & virum differentia ad medii vim elasticam mediocrem, ut $\frac{HL-KN}{VV-V\times HL-V\times KN+HL\times KN}$ ad $\frac{1}{V}$. Hoc est, ut $\frac{HL-KN}{VV}$ ad $\frac{1}{V}$, sive ut *HL - KN* ad *V*, si modo (ob angustos limites vibrationum) supponamus *HL* & *KN* indefinite minores esse quantitae *V*. Quare cum quantitas *V* detur, differentia virium est ut *HL - KN*, hoc est (ob proportionales *HL - KN* ad *HK*, & *OM* ad *OI* vel *OP*, dataque *HK* & *OP*) ut *OM* ; id est, si *Ff* bisecetur in Ω, ut $\Omega\varphi$. Et eodem argumento differentia virium elasticarum punctorum physicorum ε & γ, in reditu lineolae physicae εγ est ut $\Omega\varphi$. Sed differentria illa (id est, excessus vis elasticae puncti ε supra vim elasticam puncti γ) est vis qua interjecta medii lineola physica $\varepsilon\gamma$ acceleratur in itu & retardatur in reditu ;& propterea vis acceleratrix lineolae physicae $\varepsilon\gamma$, est ut ipsius distantia a medio vibrationis loco Ω. Poinde tempus (per prop. XXXVIII. lib. I) recte exponitur, id est, lege oscillantis penduli : estque par ratio partium omnium linearium ex quibus medium totum componitur. *Q. E.D.*

   Corol. Hinc patet quod numerus pulsuum propagatorum idem sit cum numero vibrationum corporis tremuli, neque multiplicatur in eorum progressu. Nam lineola physica $\varepsilon\gamma$, quamprimum ad locum suum primum redierit, quiescet; neque deinceps movebitur, nisi vel ab impetu corporis tremuli, vel ab impetu pulsuum qui a corpore tremulo propagantur, motu novo cieatur. Quiescet igitur quamprimum pulsus a corpore tremulo propagari desinunt.

PROPOSITIO XLVIII.   THEOREMA XXXVIII.

Pulsuum in fluido elastico propagatorum velocitates sunt in ratione composita ex subduplicata ratione vis elasticae directe & subduplicata ratione densitatis inverse ; si modo fluidi vis elastica ejusdem condensationi proportionalis esse supponatur.

Cas. 1. Si media sint homogenea, & pulsuum distantiae in his mediis aequentur inter se, sed motus in uno medio intensior sit : contractiones & dilationes partium analogarum erunt ut iidem motus. Accurata quidem non est haec proportio. Veruntamen nisi contractiones & dilatationes sint valde intensae, non errabit sensibiliter, ideoque pro physice accurata haberi potest. Sunt autem vires elasticae motrices ut contractiones & dilatationes ; & velocitates partium aequalium simul genitae sunt ut vires. Ideoque aequales & correspondentes pulsuum correspondentium partes itus & reditus suos per spatia contractionibus & delatationibus proportionalia, cum velocitatibus quae sunt ut

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spatia, simul peragent : & propterea pulsus, qui tempore itus & reditus unius latitudinem suam progrediendo conficiunt, & in loca pulsuum proxime praecedentium semper succedunt, ob aequalitatem distantiarum, aequali cum velocitate in medio utroque progredientur.

Cas. 2. Sin pulsuum distantiae seu longitudines sint majores in uno medio quam in altero ; ponamus quod partes correspondentes spatia latitudinibus pulsuum proportionalia singulis vicibus eundo & redeundo describant : & aequales erunt earum contractiones & dilatationes. Ideoque si media sint homogenea, aequales erunt etiam vires illae elasticae motrices quibus reciproco motu agitantur. Materia autem his viribus movenda est ut pulsuum latitudo ; & in eadem ratione est spatium per quod singulis vicibus eundo & redeundo moveri debent. Estque tempus itus & reditus unius in ratione composita ex ratione subduplicata materiae & ratione subduplicata spatii, atque ideo ut spatium. Pulsus autem temporibus itus & reditus unius eundo latitudines suas conficiunt, hoc est, spatia temporibus proportionalia percurrunt; & propterea sunt aequiveloces.

Cas. 3. In mediis igitur densitate & vi elastica paribus, pulsus omnes sunt aequiveloces. Quod si medii vel densitas vel vis elastica intendatur, quoniam vis motrix in ratione vis elasticae, & materia movenda in ratione densitatis augetur ; tempus, quo motus iidem peragantur ac prius, augebitur in subduplicata ratione densitatis, ac diminuetur in subuplicata ratione vis elasticae. Et propterea velocitas pulsuum erit in ratione composita ex ratione subduplicata densitatis medii inverse & ratione subduplicata vis elasticae directe. Q.E.D.

Haec propositio ulterius patebit ex constructione sequentis.

PROPOSITIO XLIX.   *PROBLEMA XI.*

Datis medii densitate & vi elastica, invenire velocitatem pulsuum.

Fingamus medium ab incumbente pondere promore aeris nostri comprimi; sitque *A* alitudo medii homogenei, cuius pondus adaequet pondus incumbens, & cuiis densitas eadem sit cum densitate medii compressi, in quo pulsus propagantur. Constitui autem intelligatur pendulum, cuius longitudo inter punctum suspensionis & centrum oscillationis sit *A* : & quo tempore pendulum illud oscillationem integram ex itu & reditu compositam peragit eodem pulsus eundo conficiet spatium circumferentiae circuli radio *A* descripti aequale.

Nam stantibus quae in Propositione XLVII constructa sunt, si linea quaevis physica *EF*, singulis vibrationibus describendo spatium *PS*, urgeatur in extremis itus & reditus cuiusque locis *P* & S, a vi elastica quae ipsius ponderi aequetur; peraget haec vibrationes singulas quo tempore eadem in cycloide, cuius perimeter tota longitudini *PS* aequalis est, oscillari posset : id adeo quia vires aequales aequalia corpuscula per aequalis spatia simul impellent. Quare cum oscillationum tempora sint in subduplicata ratione longitudinis pendulorum, & longitudo penduli aequetur dimidio arcui cycloidis totius; foret tempus vibrationis unius ad tempus oscillationis penduli, cuius longitudo est *A*, in subduplicata ratione longitudinis $\frac{1}{2}PS$ seu *PO* ad longitudinem *A*. Sed vis elastica, qua lineola physica

EG, in locis suis extremis *P, S* existens, urgetur, erat (in demonstratione propositionis XLVII) ad eius vim totam elasticam ut *HL - KN* ad *V*, hoc est (cum punctum *K* iam

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incidat in *P*) ut *HK* ad *V*: & vis illa tota, hoc est pondus incumbens, quo lineola *EG* comprimitur, est ad pondus lineolae ut ponderis incumbentis altitudo *A* ad lineolae longitudinem *EG*; ideoque ex aequo, vis qua lineola *EG* in locis suis *EG* ut *PO* ad *V*. Quare cum tempora, quibus aequalia corpora per aequalia spatia impelluntur, sint reciproce in subduplicata ratione virium, erit tempus vibrationis unius, urgente vi illa elastica, ad tempus vibrationis, urgente vi ponderis, in subduplicata ratione *VV* ad *PO* × *A*, atque ideo ad tempus oscillationis penduli cuius longitudo est *A* in subduplicata ratione *BB* ad *PO* × A. & subduplicata ratone *PO* ad *A* conjunctim; id est, in ratione integra *V* ad A. Sed tempore vibrationis inius ex itu & reditu compositae, pulsus progrediendo conficit latiudinem suam *BC*. Ergo tempus, quo pulsus percurrit spatium *BC*, est ad tempus oscillatonis unius ex itu & reditu compositiae, ut *V* ad *A*, id est, ut *BC* ad circumferentiam curculi cuius radius est *A*. Tempus autem, quo pulsus percurret spatium *BC*, est ad tempus quo percurret longitudinem huic circumferentiae aequalem, in eadem ratone; ideoque tempore talis oscillationis pulsus percurret longitudinem huic circumferentiae aequalem. *Q.E.D.*

Corol. 1. Velocitas pulsuum ea est, quam acquirunt gravia aequaliter accelerato motu cadendo, & casu suo describendo dimidium altitudinis A. Nam tempore casus huius, cum velocitate cadendo acquisita, pulsus percurret spatium quod erit aequale toti altitudini A; ideoque tempore oscillationis unius ex itu & reditu compositae percurret spatium aequale circumferentiae circuli radio A descripti : est enim tempus casus ad tempus oscillationis ut radius circuli ad ejusdem circumferentiam.

Corol. 2 Unde cum altitudino illa A sit ut fluidi vis elastica directe & densitas eiusdem inverse ; velocitas pulsuum erit in ratione composita ex subduplicata ratio densitatis inverse & subduplicata ratione vis elasticae directe.

PROPOSITIO L.   PROBLEMA XII.

Invenire pulsuum distantias.

Corporis, cuius tremore pulsus excitantur, inveniatur numerus vibrationibus dato tempore. Per numerum illum dividatur spatium quod pulsus eodem tempore percurrere possit, & pars inventa erit pulsus unius latituto. *Q. E. I.*

Scholium.
Spectant propositiones novissimae ad motum lucis & sonorum. Lux enim cum propagatur secundum lineas rectas, in actione sola (per prop. XL & XLII.) consistere nequit. Soni vero propterea quod a corporibus tremulis oriantur, nihil aliud sunt quam aëris pulsus propagati, per prop. XLIII. Confirmatur id ex tremoribus quos excitant in corporibus objectis, si modo vehementes sint & graves, quales sunt soni tympanorum. Nam tremores celeriores & breviores difficilius excitantur. Sed & sonos quosvis, in chordas corporibus sonoris unisonas impactos, excitare tremores notissimum est. Confirmatur etiam ex velocitate sonorum. Nam cum pondera specifica aquae pluvialis & argenti vivi sint ad invicem ut 1 ad $13\frac{2}{3}$ circiter, & ubi mercurius in *Barometro* altitudinem attingit digitorum

Anglicorum 30, pondus specificum aëris & aequae pluvialis sint ad vicem ut 1 ad 870 circiter; erunt pondera specifica aëris & argenti vivi ut 1 ad 11890. Proinde cum altitudo argenti vivi sit 30 digitorum, altitudo aëris uniformis, cuius pondus aërem nostrum subjectum comprimere posset, erit 356700 digitorum, seu pedum Anglicorum 29725. Estque haec altitudo illa ipsa quam in constructione superioris problematis nominavimus A. Circuli radio 29725 pedum descripti circumferentia est pedum 186768. Et cum pendulum digitos $39\frac{1}{5}$ longum oscillationem ex itu & reditu compositam tempore minutorum duorum secundorum, uti notum est, absolvat; pendulum pedes 29725 seu digitos 356700 longum oscillationem consimilem tempore minutorum secundorum $190\frac{3}{4}$ absolvere debebit. Eo igitur tempore sonus progrediendo conficiet pedes 186768, ideoque tempore minuti unius secundi pedes 979.

Caeterum in hic computo nulla habetur ratio crassitudinis solidarum particularum aëris, per quam sonus utique propagatur in instanti. Cum pondus aëris sit ad pondus aquae ut 1 ad 870. & sales sint fere duplo densiores quam aqua; si particulae aëris ponantur esse ejusdem circiter densitatis cum particulis vel aquae vel salium, & raritas aëris oriatur ab intervallis particularum : diameter particulae aëris erit ad intervallum inter centra particularum, ut 1 ad 9 vel 10 circiter, & ad intervallum inter particulas ut 1 ad 8 vel 9. Proinde ad pedes 979, quos sonus tempore minuti unius secundi juxta calculum superiorem conficiet, addere licet pedes $\frac{979}{9}$ vel 109 circiter, ob crassitudinem particularum aëris : & sic sonus tempore minuti unius secundi conficiet pedes 1088 circiter.

His adde quod vapores in aëre latentes, cum sint alterius elateris & alterius toni, vix aut ne vix quidem participant motum aëris veri quo soni propagantur. His autem quiescentibus, motus ille celerius propagibitur per solum aërem verum, idque in subduplicata ratione minoris materiae. Ut si atmosphaera constet ex decem partibus aëris veri & una parte vaporum, motum sonorum celerior erit in subduplicata ratione 11 as 10, vel in integra circiter ratione 21 ad 20, quam si propagaretur per undecim partes aëris veri : ideoque motus sonorum supra inventus , augendus erit in hac ratione. Quo pacto sonus, tempore minuti unius secundi, conficiet pedes 1142.

Haec ita se habere debent tempore verno & autumnali, ubi aër per ca;orem temperatum rarescit & ejus vis elastica nonnihil intenditur. At hyberno tempore, ubi aër per frigus condensatur, & ejus vis elastica remittitur, motus sonurum tardior esse debet in subduplicata ratione densitatis; & vicissim aestivo tempore debet esse velocior.

Constat autem per experimenta quod soni tempore minuti unius secundi eundo conficiunt pedes *Londinenses* plus minus 1142, *Parisienses* vero 1070.

Cognita sonorum velocitate innotescunt etiam intervalla pulsuum. Invenit utique D. Sauveur, factis a se experimentis, quod fistula aperta, cujus longitudo est pedum *Parisiensium* plus minus quinque, sonum edit ejusdem toni cum sono chordae quae tempore minuti unius secundi centies recurrit. Sunt igitur pulsus plus minus centum in spatio pedum *Parisiensium* 1070, quo sonus tempore minuti unius secundi percurrit; ideoque pulsus unus occupat spatium pedum *Parisiensium* $10\frac{7}{10}$, id est, duplam circiter longitudinem fistulae. Unde versimile est quod latitudines pulsuum, in omnium apertarum fistularum sonis, aequentur duplis longitudinibus fistularum.

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Porro cur soni cessante motu corporis statim cessant, neque diutius audiuntur ubi longissime distamus a corporibus sonoris, quam cum proxime absumus, patet ex corollario propositionis XLVII libri huius. Sed & cur soni in tubis stentorophonicis valde augentur ex allatis principiis manifestum est. Motus enim omnis reciprocus singulis recuribus a causa generante augeri solet. Motus autem in tubis dilatationem sonorum impedientibus, tardius amittitur & fortius recurrit, & propterea a motu novo singulis recursibus impresso magis augetur. Et haec sunt praecipua phaenomena sonorum.