Preface to the Analysis.

The art of Analysis had been abandoned and left in an uncultivated state ever since the passing of the erudite age of the ancient Greeks, and indeed is the reason for bringing the present work into being. However, *Francois Viete*, that most distinguished of men and an ornament to the French race - by reason of his outstanding skill in the science of Mathematics, has been the first in recent times to make a single-minded assault on the subject in an attempt to restore it in accordance with his own plans. The natural turn of his mind in this direction has been set out in various published investigations, and in his writings the subject matter has been set out in an acute and elegant style, and this testament he has left to posterity. Indeed, while the old Analysis was undergoing this restoration, he also proposed his own modifications, as he considered other matters of importance; and thus not only did he work to restore the analysis of the ancients, but he also augmented and adorned the work himself, as he propounded new properties of his own invention. Initially then, we can show what Viete has himself discovered. The enunciation of the general principles [established by the Greeks and elaborated on by Vieta] have been explained more recently with a little more elaboration by our own most distinguished author *Thomas Harriot*, who has indeed followed Viete in his understanding of Analysis, and who can also be seen to be an outstanding and noteworthy mathematician.

Accordingly, we can run over the development from the beginning of the solution of problems investigated by these Old Masters, by whom the deductions did not pass beyond the bounds of Quadratics. The solution of quadratic equations was the Analysis generally practised at the time, and this was clearly sufficient for them, with the outcomes of their investigations set forth in various of their recorded works. Thus we accept the Mathematical knowledge of these ancient works, and with the benefit of the art of this invention, it must surely be judged that the ways to solve most problems have been enriched. For in solving a given problem, the solution can be established by Analysis in a free and easy manner, and the remainder [associated with a square root] can be made smaller through Analysis, for the demonstration and the construction can be set out in a synthetic manner that can be attached to the problem, with the actual Analytical method suppressed. However, this is a concession granted as a privilege, limited to those ordinary elementary scholars, or as [masters and pupils] themselves say, put in place to make matters plain for the living. But with such things falling to one's lot, by attempting with the Greek Analysis to move on to higher order formulas (such as Cubics especially), there is less hope for an answer to one's prayers for the desired successful solution to these problems; certainly not with the aid of some kind of geometry that one might expect to be a source of help, but which here is seen to be destitute. Neither can help be obtained by recourse to the solid locus (by which name conic sections are understood), nor to the loci that are called curved lines (such as the Helix, Conchoids, Quadrantica, and similar curves), such as the ancient mathematicians were accustomed to have recourse to in order to supplement their art when it failed. But these supplementary delineations for a curve with a given winding nature either have been drawn mechanically by a composite motion, or calculated from further reasoning than that which was assumed in the first place, and upon which they depend for their own generation, and hence everything is incapable of being resolved in a satisfactory manner. Thus the fact is, by making use of these mechanical drawing devices, the desired solution of the problem can hardly be expedient, with hands and eyes getting in the way mechanically. [Thus, lengths are measured from a scale drawing, rather than being the outcome of a calculation]. And the ability of these ancient Greeks to solve problems analytically remained in this state, as long as the profession and study of the art of mathematics flourished among them. ['*The History of Greek Mathematics*', including the works of Pappus, Heron, and Diophantos can be found in that admirable work by Sir Thomas Heath, published by Dover.]

The Greeks at last having been truly vanquished by the armies of the Barbarians, and reduced to slavery, all the Greek literature transmigrated to the Arab scholars. Where, in time through the following ages, on being studied by men of genius, it was greatly improved and enlarged upon. Moreover, many other useful parts of philosophy, and even several of their more obstruse works of genius have come down to us that have been discovered and tracked down. For the Arabic name *Algebra* was itself imposed by the Arabs, (from a few of their foremost writings from that age still extant), plainly the argument should be the study and the most vigorous practise of the art in these works. Moreover, one *Diophantus*, a Greek Analyst surviving from the ancient school of Mathematicians was outstanding, and by whom perhaps a little algebra was invented. Thus, regarding the perfection of or additions made to the analysis discovered by the Greeks, we should acknowledge our debt to the Arabs. [Vieta for religious reasons did not study Arab mathematics; if he had done so, then the course of analysis would have been much furthered. For Viete was in that fortunate position (a), he was a brilliant and resourceful mathematician; (b), he was a Greek scholar of the first order, enabling him to look back into the history of the subject; and (c), his foresight and enthusiasm helped to fashion the shape of mathematics in the coming century.]

Therefore the original Analysis of the Greeks, the same in short that had been left by them with imperfections, was passed on with the basic principles via the Arab hand to ours safe and sound, and there it was to remain until the time was ripe for further development. The Italians *Cardan* and *Tartaglia*, celebrated as the outstanding Mathematicians of an earlier age, and greatly learned in the art of algebra, in work based on certain geometrical foundations, (greatly vying for the glory of the discovery between themselves), had endeavoured to demonstrate clearly how a class of cubic equations could be resolved, but only according to certain precise conditions. The form of the cubic with reduced roots were very perplexing, and still had not been resolved. Because of the conditions imposed, it must be said that theirs was not a general and absolute solution. After the discovery of the solutions of these cubics, other cubic equations were re-examined to have their solutions hammered out; the Belgian *Stevinus* has set out, all of this material in a general and optimal manner in his *Arithmetica,* with great diligence. He has proposed for resolution, in the first place, the equations of the family of cubics with a condition, which has been solved by his own natural primary method (for the resolution of which it is permitted by the basic supposition that the immediate extraction of the roots is possible), [i.e. a formula can be used rather than a recursive scheme]. In the second place, cubic equations of forms that can be reduced to the condition of the first form that he has reduced could also be resolved. In the third place, bi-quadratics too that are reducible to the first kind of cubics, likewise can be resolved. Of the rest, as with the cubics so with the bi-quadratics, for families that do not satisfy the required conditions, (which it is the greater part of a multitudinous total) they are insoluble, to the detriment of the greatest art, by their silent condemnation. And so here the progression of this Italian invention came to an end, not so much by our ignorance as by the nature of the thing itself being pre-determined.

But nevertheless Viete came along, that great architect of Analysis who, with the addition of various Supplements, Investigations, and with the assistance of Angular Sections, had tried in all kinds of ways and by which, finally by the ingenuity of his constructions, he was able to overcome this invincible anomaly in the art of analysis. It seems moreover that the end of the difficulty was produced not long after his own preface to his proceeding work; marked with frustration at the approach of the Geometers, and by standing his ground, happily he devised numerous of his own discoveries from the Arithmetician mode in the *Exegeticen*. When at last he had discovered his method of solving cubics, he was filled with pride by his general solution to the problem: *There is*, he was able to faithfully observe, *no insoluble problem*. [Nullum non problema solvere.] For by that art, all other orders and forms of equations in general, by a uniform and infallible method, could be resolved, each according to its own natural order. For truly, the solutions had finally been accomplished for the resolution of these problematic equations. Viete therefore, with the immense insight of his *Exegetices,* for the resolution of equations to an observed power, by ministering to the possibility of a universal solution existing for these problems, wished to make clear that magnificent form for solving this kind of proposition. Let the *Exegetices* occupy the first place in the story of this discovery by Viete, in the restoration of works from their conception, that are of outstanding worth, and that have been collected together.

The other part of his discovery remains within the framework of mathematics: with the title *Logistices Speciosae Introductum*, which permits a restoration [*i.e.* root extraction] by Analyses essentially with a smaller amount of working than the number-rich *Exegetice* touches on; nevertheless, the first longer method with general use may surpass the other, and should not be considered the lesser of the two methods. Indeed any of the ancients would have spared no expense to read his *Logistice Speciosa,* the most esteemed of his works, and would have recognised some of his incredible ability in these matters mathematical expounded briefly and lucidity, for he gravitated towards the words and style of these ancient mathematicians, with whose work and language he was an expert. Therefore with the addition of these two works, the *Logistice Speciosa* and with the *Numerosa Exegetice*, (of which in the ancient writings no vestige of any kind is extant) the enriching art of Viete shall endure; this new idea, as it has been said, has to be understood not only for its merit in the restoration of the old, but also because it has made a giant leap forwards to better things.

The *Exegetice Numerosa* is presented here, brought together from the notes of our own Thomas Harriot; it has a form indeed not from the first deliberations of Viete, but transformed thus by Harriot for posterity, as if in some way with the invention of the *Exegetices* by Viete a new vision of the Analysis has been made, and with the re-examination of the *Exegetices* by Harriot, a new Vieta himself appears, certainly new and much more expedient. It is observed that it is much easier to make progress using the new method when compared with the practise of the old. [The interested reader may wish to consult note 275 on page 280 of Jacob Kline concerning this: *Greek Mathematical Thought and the Origin of Algebra*; Dover publication.]

But as the reformation of the *Exegetice*s itself was perfected, it was also necessary that the original form of Viete's *Logistices* had to be entirely changed. For indeed, with the preceding explanatory notes on an exercise for the example Viete proposed, it was necessary to make intelligent use of the new discipline, as indeed the original form was later been found to have disadvantages in ordinary use.

Harriot therefore has used a letter notation only, for the desired elements either individually or in some combination, for the purpose of carrying out a calculation or [train of] reasoning. Indeed with the opportunity for this change, the *Logistices Speciosae* to some extent troublesome before and less well adjusted for practical usage, is in short made easy and clear on being brought to its present condition, and has been set out with many examples presented and expounded upon. Certainly Harriot, on the strength of these *Logistices*, toiled with dexterity at the reformation of the *Exegetice*s with two rules of his own invention. Firstly, he has constructed certain equations generated from binomial roots, which he calls Canons. These are applied as products to the general equations, if any ambiguous [*i.e.* double] roots should be concealed within the general [equations], through canons of equal significance invented by the most ingenious method, these are detected and determined. Secondly, when the *Exegetice*s *numerosa* itself is turned to, he has deduced certain kinds of polynomials from the proposed species for the resolution of the equation; which in like manner he calls Canons. For the equations are in truth themselves resolved by the continued application of these canons or rules, and the progress by the labours of Analysis are guided from first principles to the conclusion, in an easy manner and with certitude. For *Harriot,* by means of this single artifice, indeed the first person to make use of this invention which was his own discovery (by him considered the most useful of all mathematical tools), can truly be considered to have brought the *Exegeticen Numerosa* to a state of absolute perfection. And these are surely the things that have been completely worked through by our Author burning the midnight oil in his reformation of the *Exegetice*s, indeed said here summarily, but which in the following tract are expounded with the maximum of Analysis, and diligently explained in great detail.

PRAEFATIO AD ANALYSTAS.

Artis Anaylticae, cuius causa hic agitur, post eruditum illud Graecorum saeculum antiquatae iamdiu & incultae iacentis, restitutionem *Franciseus Vieta*, Gallus, vir clarissimus, & ob insignem in scientiis peritiam, Gallicae gentis decus, primus singulari consilio & intentato antehac conamine aggressus est atque ingenuam hanc animi sui intentionem per varios tractatus, quos in argumenti huius elaboratione eleganter & acute conscripsit, posteris testatam reliquit. Dum vero ille veteris Analytices restitutionem, quam sibi proposuit, serio molitus est, non tam eam restitutam, quam propriis inventionibus auctam & exornatam, tanquam novam & suam, nobis tradidisse videtur. Quod generali conceptu enuntiatum paulo fusius explicandum est; ut, ostenso eo quod primum a Vieta in instituto suo promovendo actum est, quid postea ab authore nostro doctissimo Thoma Harrioto,qui illum in certamine isto Analytico sequutus est, praestitum sit, melius innotescere possit.

Quare ut rem ab initio repetamus; Veteres illos Artifices, in Problematum solutionibus investigandis, quorum deductiones ordinis Quadratici limites non excedunt, Analyticen communiter exercuisse, in variis ipdorum monumentis tum effectu manifestum, tum diserte ab ipsis significatum est. Vnde scientias Mathematicas quas ab illis accepimus, artis huius inventricis beneficio, quamplurimis acessionibus locupletatas suisse, pro certo existimandum est. Nam Problemate processu Analytico ad solutionis statum deducto, liberum & facile eis fuit, facto per Analysis vestigia regressu, demonstrationem synthetice construere, constructamque, suppressa Analysi, Problemati attexere. Sed privilegium hoc eis intra communium Elementorum terminos, sive (ut ipsi loquuntur) in loco plano versantibus, concessum tantummodo fuit. Cum autem tentata Analysi in sublimiorum ordinum formulas (ut cubicas praesertim) incidere eis obtigit; votis suis minus prospere succedente solutione, ne omni artis subsidio ad eam forma aliqua Geometrica prestandam destituti vederentur; vel ad locos solidos (quo nomine sectiones Conicas intellexere) vel ad locos quos lineares vocarunt, (ut sunt Helice, Chonoides, Quadrantica, & huius generis similia) tanquam ad postulata artis defectuosae supplementa, confugere solebant. Sunt autem supplementa ista delineationes quaedam tortuosae per motus compositos mechanice descriptae, calculi aut ratiocinii ulterioris, quam quod ex praesupposita ipsarum genesi immmediate dependet, omnino incapaces. Unde factum est, ut earum adhibito adminiculo , desiderata problematis solutio manus & oculi officio organice tantum expedienda erat. Atque, in huiusmodi statu haesit veterum Graecorum in problematis solvendis facultas Analytica, quamdiu artium Mathematicarum studium & professio apud eos floruit.

Deducta vero tandem Barbarorum armis Graecia, & in seruitutem redacta, universa Graecorum literatura ad Arabum scholas transmigravit. Ubi, per succedentis saeculi tempora, gentis ingeniosae studiis summopere exculta & amplificata est. Quanquam autem in aliis philosophiae partibus multa quidem utilia, atque; nonnulla atiam abstrusiora solerti eorum indagine inventa, ad nos pervenerint; ac tametsi Arabicum ipsius *Algebrae* nomen ab eis impositum, (praeter scripta eorum paucula in eo genere extantia) artis apud eos studium & praxim viguisse argumentum euidens sit; unus tamen *Diophantus* Analysta Graecus, ex antique Mathematicorum familia superstes, obstat, quo minus vel Algebrae inventionem, vel quicquam, quod ad Analyticen perficiendam vel augendam faciat, Graecorum inventis superadditum, Arabibus acceptum referre teneamur.

Pristina igitur Graecorum Analytice, eodem prorsus quo ab ipsis relicta est imperfectionis statu, per Arabum manus intacta ad nostra usque tempora devoluta permansit. Dum Cardanis & Tartaglia, Itali, celebres superioris aetatis Mathematici, & Algebrae studiosissimi fundamento quoda Geometrico innixi, (de inventionis gloria magnospere inter se discertantes) arte, ad Cubici ordinis aequationes apodictice resolvendas promovere conati sunt; casus nonnullos conditionatos accurate quidem, sed forma Binomiis radicalibus admodum perplexa, resoluendo. De conditionatis dictum est, quia resolutionis fundamentum hoc generale & absolutum non est. Post hos alii inventum istud eorum ad incudem revocarunt, inter quos Stevinus Belga in Arithmetica suo generali omnium optime & diligentissime materiam hanc pertractauit. Primo, aequationum Cubice generis, quae natura & conditione sua primaria resolubiles sunt, (quarum scilicet resolutio ex supposito fundamento immediate extrui potest) resolutionis modum proponit. Secundo, aequationum Cubicarum formas illas quae conditione sua ad primarias reducibiles sunt, reducit & resolvit. Tertio, Biquadraticas quoque, ad primarias cubicas reducibiles reductas, itidem resolvit; reliquis, tam Cubici quam Biquadratici generis non conditionatis, (quae totius multitudinis pars magna est) pro irresolubilibus, maximo artis detrimento, tacita exclusione damnatis. Atque, hic Italici huius inventi progressus & terminus fuit, non tam ignorantia nostra quam ipsa rei natura praefinitus.

Prodiit autem tandem *Vieta*, magnus ille in Analytices architectus, quo quum, variis adhibitis Supplementorum , Recognitionum, atque Angularium sectionum subsidiis, omnia tentasset quibus, tanquam ingenii sui machinis, invictam hanc artis Analyticae anomaliam superaret; haud longe tamen ultra praefatum antecessorum suorum terminum rem provexisse videtur; donec frustra tenatis Geometricis, in Arithmeticp genere insistens, Exegeticen suam numerosam feliciter excogitavit. Qua demum inventa, fastuosum illud & universale problema suum, *Nullum non problema solvere*, fidenter asseverare potuit. Est enim Ars illa, ad omnes omnium ordinum & formarum aequationes generali, uniformi ac infallibili methodo resolvendas, ab ipsa natura ordinata. Quum vero problematum solutiones aequationum resolutionibus finaliter perficiantur; *Vieta* idcirco, immensa *Exegetices* huius in aequationibus resolvendis potestate perspecta, universalem problematum solutionem illius ministerio possibilem existimans, magnifica huiusmodi enunciationis forma Problema illud insignite voluit. Exegetices huius inventum, eorum quae a *Vieta*, ad opus restitutorium ab ipso conceptum, collata sunt, dignitate praecipuum, narrationis ordine primum esto.

Restat alterum ipsius inventum in scholam Mathematicam, titulo Logistices Speciosae introductum : quae licet Analytices restitutionem minus essentialiter quam *Exegetice* numerosa, attingat; tamen, cum naturae prioritate, ac proinde usus generalitate, illam longe superet, non minoris aestimanda est. Veteres sane *Logistice* hac speciosa non sine maximo dispendio caruisse, agnouerit quisquis incredibilem illius commoditatem in materia Mathematica compendiose & dilucide tractanda, prae verbosa veteris stili gravitate, expertus fuerit. Quoniam igitur duobus hisce auctariis, Logistice Speciosa atque *Exegetice* numerosa, (quarum in veterum scriptis ne vestigium quidem ullum extat) *Vietam* artem locupletasse constet; novam eam potius, ut dictum est, magna saltem ex parte fecisse, quam vererem restituisse, non immerito censendus est.

Exegerice ista numerosa est quam hic proferimus, e *Thoma Harrioti* nostri schediasmatis depromptam; non quidem ut primis Vieta Exegetices inventione Analyticen novam quodammodo fecisse visus fueri, *Harriotum Exegetices* recognitione ipsum *Vietam* novum, nove certe ac multo magis expedito & ad usum facto habitu conspiciendum produxisse facile iudicaverint, qui utrisque institutionis formas ad praxem revocatas comparaverint.

Ad *Exegetices* autem reformationem istam perficiendam, Logistices quoque *Vietaeam* formam prius mutatam esse omnino necessarium ei fuit. Quam enim Vieta notis interpretatis exercendam praecepto & exemplo proposuit, licet ad novae disciplinae intelligentiam utilis esse potuit, ad ordinariam tamen praxim incommoda postea reperta est.

Harriotus igitur sola literali notatione, Elementis scilicet vel simplicibus vel utcunque combinatis, pro calculi aut ratiocinii exigentia usus est. Opportuna quidem hac mutatione, *Logistices* speciosae aliquatenus molestam antea & minus concinnam praxim ad summam tum facilitatem tum perspicuitatem redactam esse, multiplicibus praedentis tractationis exemplis patefactum est. Logistices certe huius dexteritate fretus *Harriotus*, *Exegetices* reformationem duobus precipue inventis suis molitus est. Primo, aequationes quasdam e radicibus Binomiis generatas constituit, quas Canonicas appellat. Harum ad aequationes communes facta applicatione, siqua radicum ambiguitas commmunibus subsit, per Canonicarum istarum aequipollentiam invento admodum ingenioso detegitur ac determinantur. Secundo, quum ad ipsam Exegetices numerosae praxim deventum est, species quasdam polynomias ex ipsis aequationum resolutioni propositarum speciebus deducit; quas item Canonicas vocat. Sunt enim revera ipsius resolutionis Canones sive regulatrices quarum uniformiter continuata applicatione, operis Analytice processus a principio ad finem tanta facilitate ac certitudine dirigitur, ut Harriotus unica huius artificii. prae caeteris huius generis inventis suis, inventione, *Exegeticen* numerosam (artem Mathematicarum omnium instrumentariam, atque eo nomine utilissimam) ad absolutam perfectionem redegisse vere existimandus sit. Atque haec fere sunt quae ab Authore nostro in *Exegetices* reformatione elucubranda peracta sunt, fummatim quidem dicta, sed quae in sequenti tractatu maximo Analystarum commodo particulatim & diligenter explicata sunt.