The Canons of the Equations are derived or deduced from their own original forms: set out in the order prescribed, generated [through multiplication] from their own original constituted binomial roots [meaning that each term has two numbers or symbols, such as a - b, a - c, etc].

Aequationum Canoricarum ab originalibus suis derivatio sive deductio: praemissa ipsarum originalium e radicibus binomiis per genesim constitutarum ordinata hac descriptione.

Quadratics.

1. . . . .
$$\left.\begin{array}{r} a - b \\ a - c \end{array}\right| = \begin{array}{l} aa - ba \\ \quad - ca + bc \end{array}$$

2.. . . . . . .
$$\left.\begin{array}{r} a - b \\ a + c \end{array}\right| = \begin{array}{l} aa - ba \\ \quad + ca - bc \end{array}$$

3.. . . . . . .
$$\left.\begin{array}{r} a + b \\ a + c \end{array}\right| = \begin{array}{l} aa + ba \\ \quad + ca + bc \end{array}$$

Cubics.

1. . . . .
$$\left.\begin{array}{r} a - b \\ a - c \\ a - d \end{array}\right| = \begin{array}{l} aaa - baa + bca \\ \quad - caa + bda \\ \quad - daa + cda - bcd \end{array}$$

2.. . . . . . .
$$\left.\begin{array}{r} a - b \\ a - c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad daa - cda + bcd \end{array}$$

3. . . . .
$$\left.\begin{array}{r} a + b \\ a + c \\ a - d \end{array}\right| = \begin{array}{l} aaa + baa + bca \\ \quad caa - bda \\ \quad - daa - cda - bcd \end{array}$$

4. . . . .
$$\left.\begin{array}{r} a + b \\ a + c \\ a + d \end{array}\right| = \begin{array}{l} aaa + baa + bca \\ \quad caa + bda \\ \quad daa + cda + bcd \end{array}$$

The reciprocals of Cubics [i.e. two equal and opposite roots].

5\. . . . . $$\left.\begin{array}{c} aa - bc \\ a - d \end{array}\right| = aaa - daa - bca + bcd.$$

6\. . . . . $$\left.\begin{array}{c} aa + bc \\ a - d \end{array}\right| = aaa - daa + bca - bcd.$$

7\. . . . . $$\left.\begin{array}{c} aa - bc \\ a + d \end{array}\right| = aaa + daa - bca - bcd.$$

8\. . . . . $$\left.\begin{array}{c} aa + bc \\ a + d \end{array}\right| = aaa + daa + bca + bcd.$$

Three kinds of canonical cubic equations are established from the roots of the original equation.

Let $r - a = q$.

9.Therefore . . $$\left.\begin{array}{c} r - a \\ r - a \\ r - a \end{array}\right| = rrr - 3.rra + 3.raa - aaa = +qqq$$

Let $r + a = q$.

10.Therefore . . $$\left.\begin{array}{c} r + a \\ r + a \\ r + a \end{array}\right| = rrr + 3.rra + 3.raa + aaa = +qqq$$

Let $a - r = q$.

11\. Therefore . . $$\left.\begin{array}{c} a - r \\ a - r \\ a - r \end{array}\right| = aaa - 3.raa + 3.rra - rrr = +qqq.$$

Biquadratics.

1\. . . . . $$\left.\begin{array}{c} a - b \\ a - c \\ a - d \\ a - f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad - faaa + bfaa - bcfa \\ \qquad\quad + cfaa - bdfa \\ \qquad\quad + dfaa - cdfa + bcdf \end{array}$$

2\. . . . . $$\left.\begin{array}{c} a - b \\ a - c \\ a - d \\ a + f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa - bcdf \end{array}$$

3. . . . .

$$\left.\begin{array}{c} a+b \\ a+c \\ a+d \\ a-f \end{array}\right| = \begin{array}{l} aaaa + baaa + bcaa \\ \qquad + caaa + bdaa \\ \qquad + daaa + cdaa + bcda \\ \qquad - faaa - bfaa - bcfa \\ \qquad\qquad - cfaa - bdfa \\ \qquad\qquad - dfaa - cdfa - bcdf \end{array}$$

4. . . . .

$$\left.\begin{array}{c} a-b \\ a-c \\ a+d \\ a+f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \qquad - caaa - bdaa \\ \qquad + daaa - cdaa + bcda \\ \qquad + faaa - bfaa + bcfa \\ \qquad\qquad - cfaa - bdfa \\ \qquad\qquad + dfaa - cdfa + bcdf \end{array}$$

5. . . . .

$$\left.\begin{array}{c} a+b \\ a+c \\ a+d \\ a+f \end{array}\right| = \begin{array}{l} aaaa + baaa + bcaa \\ \qquad + caaa + bdaa \\ \qquad + daaa + cdaa + bcda \\ \qquad + faaa + bfaa + bcfa \\ \qquad\qquad + cfaa + bdfa \\ \qquad\qquad + dfaa + cdfa + bcdf \end{array}$$

6.. . . . . .

$$\left.\begin{array}{c} aaa - cdf \\ a-b \end{array}\right| = aaaa - baaa - cdfa + bcdf$$

7. . . . . .

$$\left.\begin{array}{c} aaa - cdf \\ a+b \end{array}\right| = aaaa + baaa - cdfa - bcdf$$

8. . . . . .

$$\left.\begin{array}{c} aaa + cdf \\ a-b \end{array}\right| = aaaa - baaa + cdfa - bcdf$$

9. . . . . .

$$\left.\begin{array}{c} aaa + cdf \\ a+b \end{array}\right| = aaaa + baaa + cdfa + bcdf$$

Some other kinds of Biquadratics derived from the original equations.

10. . . . . . .

$$\left.\begin{array}{c} b - a \\ c - a \\ df - aa \end{array}\right| = \begin{array}{l} bcdf - bdfa + dfaa + baaa \\ \qquad - cdfa - bcaa + caaa - aaaa \end{array}$$

11. . . . . . .

$$\left.\begin{array}{c} b - a \\ c - a \\ df + aa \end{array}\right| = \begin{array}{l} bcdf - bdfa + dfaa - baaa \\ \qquad - cdfa + bcaa - caaa + aaaa \end{array}$$

12. . . . . . .

$$\left.\begin{array}{c} b + a \\ c + a \\ df - aa \end{array}\right| = \begin{array}{l} bcdf - bdfa + dfaa - baaa \\ \qquad + cdfa - bcaa - caaa - aaaa \end{array}$$

13. . . . . . .

$$\left.\begin{array}{c} b - a \\ c + a \\ df - aa \end{array}\right| = \begin{array}{l} bcdf + bdfa - dfaa + baaa \\ \qquad - cdfa + bcaa - caaa - aaaa \end{array}$$

14. . . . . . .

$$\left.\begin{array}{c} b + a \\ c - a \\ df - aa \end{array}\right| = \begin{array}{l} bcdf - bdfa - dfaa + baaa \\ \qquad + cdfa - bcaa - caaa + aaaa \end{array}$$

15. . . . . . .

$$\left.\begin{array}{c} b + a \\ c + a \\ df + aa \end{array}\right| = \begin{array}{l} bcdf + bdfa + dfaa + baaa \\ \qquad cdfa + bcaa + caaa + aaaa \end{array}$$

16. . . . . . .

$$\left.\begin{array}{c} bc - aa \\ df - aa \end{array}\right| = \begin{array}{l} bcdf - dfaa \\ \qquad - bcaa + aaaa \end{array}$$

17. . . . . . .

$$\left.\begin{array}{c} bc - aa \\ df + aa \end{array}\right| = \begin{array}{l} bcdf - dfaa \\ \qquad bcaa - aaaa \end{array}$$

18. . . . . . .

$$\left.\begin{array}{c} bc + aa \\ df + aa \end{array}\right| = \begin{array}{l} bcdf + dfaa \\ \qquad bcaa + aaaa \end{array}$$

The derivation of the quadratic series of canons.

First Proposition.

The canonical equation $\begin{array}{l} aa - ba \\ \quad + ca \end{array} = +bc$

is deduced from the original equation $\left.\begin{array}{l} a - b \\ a + c \end{array}\right| = \begin{array}{l} aa - ba \\ \quad + ca - bc \end{array}$

For if a is put equal to b, $a = b$, $a - b = 0.$

Therefore with $a = b$, $\left.\begin{array}{l} a - b \\ a + c \end{array}\right| = 0.$

But the eqn. generated is : $\left.\begin{array}{l} a - b \\ a + c \end{array}\right| = \begin{array}{l} aa - ba \\ \quad + ca - bc \end{array}$

Therefore : $\begin{array}{l} aa - ba \\ \quad ca - bc \end{array} = 0.$

Therefore : $\begin{array}{l} aa - ba \\ \quad + ca \end{array} = +bc,$

which is the proposed equation.

Therefore the proposed canonical equation is deduced from the original equation, by setting b equal to a itself, as has been stated.

Proposition 2.

The canonical equation

$$\begin{array}{l} aa - ba \\ \quad - ca \end{array} = - bc$$

is deduced from the original

$$\left.\begin{array}{l} a - b \\ a - c \end{array}\right| = \begin{array}{l} aa - ab \\ \quad - ac + bc \end{array}$$

by placing b or c to be equal to a itself.

[Note that only positive roots are considered in Propositions 1 and 2 and thereafter: the initial product is equated to zero by setting the variable quantity equal to the positive root, resulting in one or both factors becoming zero, in which case the whole product is equal to zero, Def. 15 is then applied.]

For if $a. = b,\ a - b = 0.$

Or if $a. = c,\ a - c = 0.$

Therefore from $a. = b$ or $c,$ $\left.\begin{array}{l} a - b \\ a - c \end{array}\right| = 0.$

But the eqn. generated is :

$$\left.\begin{array}{l} a - b \\ a - c \end{array}\right| = \begin{array}{l} aa - ba \\ \quad -ca + bc \end{array}$$

Which is the original equation designated here.

Hence . . . . $\begin{array}{l} aa - ba \\ \quad - ca + bc \end{array} = 0.$

Hence . . . . $\begin{array}{l} aa - ba \\ \quad ca \end{array} = - bc,$ which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original equation, by setting b or c itself equal to $a.$

The derivation of the cubic series of canons.

Proposition 3.

The canonical equation

$$\begin{array}{l} aaa - baa - bca \\ \quad caa - bda \\ \quad daa + cda = +bcd, \end{array}$$

is deduced from the original

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad + caa - bda \\ \quad + daa + cda - bcd, \end{array}$$

by placing b equal to a itself:

For if $a. = b, \ a - b = 0.$

Therefore from $a = b,$

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = 0.$$

But the eqn. generated is :

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad + caa - bda \\ \quad + daa + cda - bcd \end{array}$$

Which is the original equation designated here.

Hence . . . .

$$\begin{array}{l} aaa - baa - bca \\ \quad caa - bda \\ \quad daa + cda - bcd = 0. \end{array}$$

Hence . . . .

$$\begin{array}{l} aaa - baa - bca \\ \quad caa - bda \\ \quad daa + cda = +bcd, \end{array}$$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting b equal to a, as was declared.

Proposition 4.

The canonical equation
$$\begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad + daa - cda \end{array} = -bcd,$$

is deduced from the original eqn.
$$\left.\begin{array}{c} a - b \\ a - c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad + daa - cda + bcd, \end{array}$$

by placing b or c equal to the root a itself.

For if $b = a$, $a - b = 0.$
Or if $c = a$, $a - c = 0.$

Therefore by placing $a = b$, or $a = c$
$$\left.\begin{array}{c} a - b \\ a - c \\ a + d \end{array}\right| = 0.$$

But the eqn. generated is :
$$\left.\begin{array}{c} a - b \\ a - c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad + daa - cda - bcd, \end{array}$$

Which is the original equation designated here, hence it follows that :
$$\begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad daa - cda + bcd \end{array} = 0.$$

Hence . . . .
$$\begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad daa - cda \end{array} = -bcd,$$
which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting b or c equal to a, as was declared.

Proposition 5.

The canonical equation
$$\begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda \end{array} = bcd,$$

is deduced from the original
$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda - bcd, \end{array}$$

by placing b or c or d equal to the root a itself.

For if $b. = a,\ a - b = 0.$
Or $c. = a,\ a - c = 0.$
Or $d = a,\ a - d = 0.$

Therefore by placing b or c or d $= a$
$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = 0.$$

But the eqn. generated is :
$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad -caa - bda \\ \quad -daa - cda - bcd, \end{array}$$

which is the equation originally designated here.

Hence . . . .
$$\begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda - bcd \end{array} = 0.$$

Hence . . . .
$$\begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad -daa - cda \end{array} = +\, bcd,$$
which is the proposed equation.

Therefore the proposed canonical equation is deduced from the original, by setting b or c or d equal to a. As was declared.

***The derivation of the recurring cubic form* [or order].**

Proposition 6.

The recurring equation $aaa - baa + cda = bcd,$

is deduced from the original

$$\left.\begin{array}{r} a - b \\ aa + cd \end{array}\right| = aaa - baa + cda - bcd$$

by placing b equal to a itself.

For if $b = a, \quad a - b = 0.$

Therefore if $a. = b,$ $\left.\begin{array}{r} a - b \\ aa + cd \end{array}\right| = 0.$

But the eqn. generated is : $\left.\begin{array}{r} a - b \\ aa + cd \end{array}\right| = aaa - baa + cda - bcd$

Which is the original equation designated here.

Hence . . . . $aaa - baa + cda - bcd = 0.$

Hence . . . . $aaa - baa + cda = bcd,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting b equal to a. As was declared.

Proposition 7.

The recurring equation $aaa + baa - cda = +bcd,$

is deduced from the original $\begin{array}{c|} a+b \\ aa-cd \\ \hline \end{array} = aaa + baa - cda - bda$

by placing $cd = aa.$

For if $cd = aa$, $aa - cd = 0.$

Therefore $cd. = aa$ $\quad \begin{array}{c|} a+b \\ aa-cd \\ \hline \end{array} = 0.$

But the eqn. generated is : $\begin{array}{c|} a+b \\ aa-cd \\ \hline \end{array} = aaa + baa - cda - bcd$

Which is the original equation designated here.

Hence . . . . $aaa + baa - cda - bcd = 0.$

Hence . . . . $aaa + baa - cda = + bcd,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting $cd = aa.$ As was declared.

Proposition 8.

The recurring equation $aaa - baa - cda = - bcd,$

is deduced from the original $\left.\begin{array}{r} a - b \\ aa - cd \end{array}\right| = aaa - baa - cda + bcd$

by placing $b = a.$ or $cd = aa.$

For if $b = a\ ,\ a - b = 0.$

or . . . $cd = aa\ ,\ aa - cd = 0.$

Therefore by placing $b = a.$ or $cd. = aa,$ $\left.\begin{array}{r} a - b \\ aa - cd \end{array}\right| = 0.$

But the eqn. generated is : $\left.\begin{array}{r} a - b \\ aa - cd \end{array}\right| = aaa - baa - cda + bcd$

Which is the original equation designated here.

Hence . . . . $aaa - baa - cda + bcd = 0.$

Hence . . . . $aaa - baa - cda = - bcd,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting

$b = a$ or $cd = aa.$ As was declared.

The derivation of the Biquadratic Canon.

Proposition 9.

The canonical equation
$$\begin{array}{l} aaaa + baaa - bcaa \\ \quad +caaa - bdaa \\ \quad +daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad +cfaa - bdfa \\ \qquad +dfaa + cdfa = + bcdf \end{array}$$

is deduced from the original
$$\left.\begin{array}{r} a - b \\ a + c \\ a + d \\ a + f \end{array}\right| = \begin{array}{l} aaaa + baaa - bcaa \\ \quad + caaa - bdaa \\ \quad + daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa + cdfa - bcdf \end{array}$$

By placing $b = a.$

$a - b = 0.$

Therefore by placing $b = a$

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \\ a + f \end{array}\right| = 0.$$

But the eqn. generated is :

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \\ a + f \end{array}\right| = \begin{array}{l} aaaa + baaa - bcaa \\ \quad + caaa - bdaa \\ \quad + daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa + cdfa \quad - bcdf \end{array}$$

Which is the original equation designated here.

Hence . . . .

$$\begin{array}{l} aaaa + baaa - bcaa \\ \quad + caaa - bdaa \\ \quad + daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa + cdfa \quad - bcdf = 0. \end{array}$$

Hence . . . .

$$\begin{array}{l} aaaa + baaa - bcaa \\ \quad + caaa - bdaa \\ \quad + daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa + cdfa = bcdf, \end{array}$$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting

$b = a.$ As was declared.

Proposition 10.

The canonical equation

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa = - bcdf \end{array}$$

is deduced from the original

$$\left.\begin{array}{l} a - b \\ a - c \\ a + d \\ a + f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa + bcdf \end{array}$$

by placing $b = a.$ or $c = a.$

For if $b. = a, \quad a - b = 0.$

Or . . . . $c = a, \quad a - c = 0.$

Therefore by placing b *or* $c = a$

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \\ a + f \end{array}\right| = 0.$$

But the eqn. generated is :

$$\left.\begin{array}{l} a - b \\ a - c \\ a + d \\ a + f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa + bcdf \end{array}$$

Which is the original equation designated here.

Hence . . .

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa - bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa + bcdf = 0. \end{array}$$

Hence . . .

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa - bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa = - bcdf, \end{array}$$

which is the proposed equation . Therefore the proposed canonical equation has been derived from that originally designated, by setting $b = a.$
As was declared.

Proposition 11.

The canonical equation

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa = + bcdf \end{array}$$

is deduced from the original

$$\left.\begin{array}{l} a - b \\ a - c \\ a - d \\ a + f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa - bcdf \end{array}$$

by placing b or c or $d = a$.

For if $b = a$, $a - b = 0$.

or . . . $c = a$, $a - c = 0$.

or . . . $d = a$, $a - d = 0$.

Therefore by placing b or c or $d = a$ $\left.\begin{array}{l} a - b \\ a - c \\ a - d \\ a + f \end{array}\right| = 0.$

But the eqn. generated is :

$$\left.\begin{array}{l} a - b \\ a - c \\ a - d \\ a + f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa - bcdf \end{array}$$

Which is the original equation designated here.

Hence . . . .

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa - bcdf = 0. \end{array}$$

Proposition 12.

The canonical equation

$$\begin{array}{llll} aaaa & -\,baaa & +\,bcaa & \\ & -\,caaa & +\,bdaa & \\ & -\,daaa & +\,bfaa & -\,bcda \\ & -\,faaa & +\,cdaa & -\,bcfa \\ & & +\,cfaa & -\,bdfa \\ & & +\,dfaa & -\,cdfa \;=\; -\,bcdf \end{array}$$

is deduced from the original

$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a-f \end{array}\right| = \begin{array}{llll} aaaa & -\,baaa & +\,bcaa & \\ & -\,caaa & +\,bdaa & \\ & -\,daaa & +\,bfaa & -\,bcda \\ & -\,faaa & +\,cdaa & -\,bcfa \\ & & +\,cfaa & -\,bdfa \\ & & +\,dfaa & -\,cdfa \;+\,bcdf \end{array}$$

by placing b or c or d or f $= a$.

For if $b = a$, $a - b = 0$.

or if . . $c = a$, $a - c = 0$.

or if . . . $d = a$, $a - d = 0$.

or if . . . $f = a$, $a - f = 0$.

Therefore by placing b, or c, or d $= a$

$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a-f \end{array}\right| = 0.$$

But from the original this is

$$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a-f \end{array}\right| = \begin{array}{llll} aaaa & -\,baaa & +\,bcaa & \\ & -\,caaa & +\,bdaa & \\ & -\,daaa & +\,bfaa & -\,bcda \\ & -\,faaa & +\,cdaa & -\,bcfa \\ & & +\,cfaa & -\,bdfa \\ & & +\,dfaa & -\,cdfa \;+\,bcdf \end{array}$$

Which is the original equation designated here.

Hence . . . .

$$\begin{array}{llll} aaaa & -\,baaa & +\,bcaa & \\ & -\,caaa & +\,bdaa & \\ & -\,daaa & +\,bfaa & -\,bcda \\ & -\,faaa & +\,cdaa & -\,bcfa \\ & & +\,cfaa & -\,bdfa \\ & & +\,dfaa & -\,cdfa \;+\,bcdf \;=\; 0. \end{array}$$

Hence . . . .

$$\begin{array}{llll} aaaa - baaa & + bcaa & & \\ - caaa & + bdaa & & \\ - daaa & + bfaa & - bcda & \\ - faaa & + bfaa & - bcfa & \\ & + cfaa & - bdfa & \\ & + dfaa & - cdfa & = - bcdf. \end{array}$$

which is the proposed equation . Therefore the proposed canonical equation is deduced from the original, by setting b or c or d or f $=$ $a.$ As was declared.

The derivation of the recurring biquadratic form.

Proposition 13.

The recurring equation $aaaa - baaa + cdfa = bcdf,$

is deduced from the original $\left.\begin{array}{r} aaa + cdf \\ a - b \end{array}\right| = aaaa - baaa + cdfa - bcdf$

by placing b to be equal to a itself.

For if $b = a,\ \ a - b = 0.$

Therefore by placing $a. = b$ $\left.\begin{array}{r} aaa + cdf \\ a - b \end{array}\right| = 0.$

But the eqn. generated is : $\left.\begin{array}{r} aaa + cdf \\ a - b \end{array}\right| = aaaa - baaa + cdfa - bcdf$

Which is the original equation designated here.

Hence . . . . $aaaa - baaa + cdfa - bcdf = 0.$

Hence . . . . $aaaa - baaa + cdfa = +bcdf,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting $b = a.$ As was declared.

Proposition 14.

The recurring equation $aaaa + baaa - cdfa = + bcdf,$

is deduced from the original $\left.\begin{array}{r} aaa - cdf \\ a + b \end{array}\right| = aaaa + baaa - cdfa - bcdf$

by placing *cdf* to be equal to *aaa* itself.

For if $cdf = aaa, \quad aaa - cdf = 0.$

Therefore by placing $aaa - cdf = \left.\begin{array}{r} aaa - cdf \\ a + b \end{array}\right| = 0.$

But the eqn. generated is : $\left.\begin{array}{r} aaa + cdf \\ a - b \end{array}\right| = aaaa - baaa - cdfa - bcdf$

Which is the original equation designated here.

Hence . . . . $aaaa + baaa - cdfa - bcdf = 0.$

Hence . . . . $aaaa + baaa - cdfa = +bcdf,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting $cdf = aaa.$ As was declared.

Proposition 15.

The recurring equation $aaaa - baaa - cdfa = - bcdf$,

is deduced from the original

$$\left.\begin{array}{r} aaa - cdf \\ a - b \end{array}\right| = aaaa - baaa - cdfa + bcdf$$

by placing $cdf = aaa$ or $b = a$.

For if $b = a,\ a - b = 0.$

or, ... $cdf = aaa,\ aaa - cdf = 0.$

Therefore by placing $b = a$, or $cdf = aaa$ $\left.\begin{array}{r} aaa - cdf \\ a - b \end{array}\right| = 0.$

But from the original it is $\left.\begin{array}{r} aaa - cdf \\ a - b \end{array}\right| = aaaa - baaa - cdfa + bcdf$

Which is the original equation designated here.

Hence . . . . $aaaa - baaa - cdfa + bcdf = 0.$

Hence . . . . $aaaa - baaa + cdfa = - bcdf,$

which is the proposed equation .

Therefore the proposed canonical equation is deduced from the original, by setting $b = a$, or $cdf = aaa$. As was declared.

Notes.

The derivation of the orders of the equations of the biquadratic canons from eight of the original kinds are described; finally, namely for numbers 10, 11, 12, 13, 14, 15, 16, 17 of the preceding, it has been sufficient to show these by example.

Aequationum biquadratici ordinis canonicorum derivatio ab octo originalium speciebus ultimo descriptis, scilicet 10. 11.12.13.14.15.16.17. superiorum exemplo satis manifesta est.

For indeed the derivation of the canons from these originals are ignored, namely no. 3 for Quadratics; nos. 4 & 8 for Cubics; and nos. 5, 9, & 18 for Biquadratics, as the substitution of negative roots are unable to be made as they are not useful, as it were.

Canonicarum vero derivationes ab his originalibus, scilicet, 3. Quadratica; 4 & 8 Cubica; 5.9. & 10. Biquadratica, cum absque radicum privativarum suppositione fieri nequeant, tanquam inutiles, negliguntur.

Besides these three special equations of the cubic kind that can be generated from the roots of the equation, the remainder can be generated symbolically from the arranged order of the roots from the derived canons referred to here. *Clearly.*

Praeter tres illae cubici generis aequationes speciales ex radicibus aequatis generatae, relicta formali radicum ordinatione, generationis symbolo, pro canonicis derivatis huc referri possunt. Videlicet.

Let . . . . $b - a = c.$

Hence . . . . $aaa - 3.baa + 3.bba = - ccc + bbb.$

Let . . . . $b + a = c.$

Hence . . . . $aaa + 3.baa + 3.bba = + ccc - bbb.$

Let . . . . $a - b = c.$

Hence . . . . $aaa - 3.baa + 3.bba = + ccc + bbb.$

A summary of the derivations of the canonical equations demonstrated in this second section.

Quadratics.

1. . . . . $\begin{array}{l} aa - ba \\ + ca = + bc \end{array}$

2. . . . . $\begin{array}{l} aa - ba \\ - ca = - bc \end{array}$

Cubics.

1. . . . . $\begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda = + bcd \end{array}$

2. . . . . $\begin{array}{l} aaa - baa + bca \\ - caa - bda \\ + daa - cda = - bcd \end{array}$

3. . . . .
$$\begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda \;=\; + bcd \end{array}$$

4. . . . . $aaa - baa + cda \;=\; + bcd$

5. . . . . $aaa + baa - cda \;=\; + bcd$

6. . . . . $aaa - baa - cda \;=\; - bcd$

Biquadratics.

1. . . . .
$$\begin{array}{l} aaaa - baaa - bcaa \\ \quad + caaa - bdaa \\ \quad + daaa - cdaa - bcda \\ \quad + faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa + cdfa \;=\; + bcdf \end{array}$$

2. . . . .
$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - cdaa + bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad - cfaa - bdfa \\ \qquad + dfaa - cdfa \;=\; - bcdf \end{array}$$

3. . . . .
$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad - cfaa + bdfa \\ \qquad - dfaa + cdfa \;=\; + bcdf \end{array}$$

4. . . . .
$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad - faaa + bfaa - bcfa \\ \qquad + cfaa - bdfa \\ \qquad + dfaa - cdfa \;=\; - bcdf \end{array}$$

5. . . . . $aaaa - baaa + cdfa \;=\; + bcdf$

6. . . . . $aaaa + baaa - cdfa \;=\; + bcdf$

7. . . . . $aaaa - baaa - cdfa \;=\; - bcdf$