MATHEMATICS.

No. IX.

OPTICS, CATOPTRICS AND DIOPTRICS FROM A SINGLE PRINCIPLE

This can be established as a *primary hypothesis* for these common sciences, by which the direction of all rays of light may be determine geometrically : *The light shining from a point arrives by the easiest of all ways* ; *which first is required to be determined in respect of planar surfaces, truly adapted for concave as well as convex surfaces, by considering the tangential planes of these.* Yet here I have given no account of certain irregularities, perhaps occurring in the generation of colors, and with other unusual phenomena, which are not attended to in practice in optics.

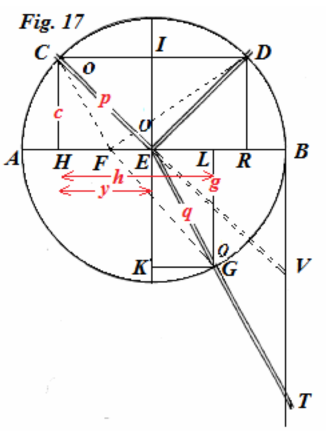


Fig. 17

Hence in *simple optics, the ray directed from the point* C *shining towards the point* E *required to be illuminated arrives* directed by the shortest way, clearly by remaining in the same medium, that is along the *right line* CE.

In Catoptrics the angle of incidence CEA, *and of reflection* DEB *are equal.* For the point shining shall be C, the point illuminated D, the plane mirror AB : the point of the mirror E is sought, reflecting the ray to D. I say it is to be such that the total path, $\mathrm{CE}+\mathrm{ED}$, is made the smallest of all, or less than $\mathrm{CF}+\mathrm{FD}$, if some other point F of the mirror were taken. This will be obtained, if E may be taken such that the angles CEA and DEB shall be equal ; as it is agreed from geometry. *Ptolemy* and other ancients have used this demonstration, just as it can be found in other places, such as in the work of *Heliodorus of Larissa.*

In Dioptrics the sine of the complement, EH and EL, *of the angles of incidence* CEA, *and of refraction* GEB, *always maintain the same ratio, which is reciprocal to the resistance of the mediums.* IE shall be air, EK water, glass, or another medium denser than air; *C* the point shining in air, G the point being illuminated below the water : it is sought, by which path that may shine according to this ; or which shall be the point E on the surface of the water AB, sending a ray emitted from C by being refracted to G. Here E must be taken such, so that the path shall be the easiest of all. Now in different mediums the difficult paths are in a ratio composed both from the lengths of the paths and the resistances of the mediums. The right lines *m* and *n* representing the resistance with regard to the light, the former for air, the latter for water ; the difficulty of the path from C to E, will be as the rectangle subtended by CE and *m* ; from E to G, as the rectangle under EG and *n.* Therefore so that the difficulty of the path shall be the minimum of all the paths CEG, the sum of the rectangles CE by $m+\mathrm{EG}$ by *n* is to be the minimum of all possible, or lesser than CF by $m+\mathrm{FG}$ by *n* ; for some other point F taken besides E. E is sought. Therefore since the points C and G likewise the right line AB, shall be given in place, therefore we will call the given right lines perpendicular to the plane CH, c; and GL, g; moreover HL, *h,* is itself given. But EH sought we will call *y*; EL will be, $h-y$, and CE, $\sqrt{cc+yy}$, which we will call *p*; and EG will be $\sqrt{gg+yy-2hy+hh}$, which we will call *q*. Therefore $m\sqrt{cc+yy}+n\sqrt{gg+yy-2hy+hh}$ (or $mp+nq$) must be the minimum of all possible of the similarly enunciated quantities, and so that it may become such, y is sought. *From my method of maxima and minima,* which above all at this point draws together the noted calculation wonderfully well, in the first place at once by consideration, without just about any calculation it is apparent ; to become *mqy* equals *np* by $h-y$ or *np* to *mq*, as *y* to $h-y$, or the rectangle CE by *n* to the rectangle EG by *m,* to be as EH to EL.

$$[Let\ P = m\sqrt{cc+yy}+n\sqrt{gg+yy-2hy+hh}=mp+nq;$$

$$\frac{dP}{dy}=m\frac{dp}{dy}+n\frac{dq}{dy}=0; \quad ;\frac{my}{\sqrt{cc+yy}}=\frac{n(h-y)}{\sqrt{gg+yy-2hy+hh}};$$

$$m\frac{y}{p}=n\frac{(h-y)}{q}; mqy=np\times(h-y); \text{or } m\sin\text{CEI}=n\sin\text{GEK.}]$$

[Thus, the original proof of the law of refraction presented here can be reduced to Fermat's Principle of the least action or time, if each part of the path of the ray is divided by the speed of light in that medium, and the minimum time found by differentiation of their sum; the method developed by Fermat was derived originally from a technique that has come to be known as the Method of Adequality, on which numerous people have written, and which resembles that used by Newton for finding a rate of change of a displacement, velocity, etc. See e.g. *Historia Mathematica* (10 part 1; 1983), K. Andersen for further details.]

Therefore with equal CE and EG put in place, *n* will be the resistance of the water with regard to light, to the resistance of the air *m*, as EH the sine of the complement of the angle of incidence CEA in air, to EL the sine of the complement of the angle of refraction GEB in water ; or the sines of the complement will be in the reciprocal ratio to the resistance of the mediums, which was being brought forth. And thus if EL may be found even in one example to be two thirds of EH, it will be in all the others, wherever C and G may be taken, the first in air, the second in glass. If E were in air, and G under water, EL will be around $\frac{3}{4}$ of EH.

[Thus, L.'s resistance *m* can be viewed as the refractive index of air, with the resistance *n* becomes the refractive index of the other medium, so that L. minimizes the path length for rays travelling from C to G, and the relative refractive index is *n/m*, the reciprocal of the ratios quoted above.]

Therefore we have reduced all the proven experimental laws of rays to pure geometry, and the calculation, with a single principle used, with an assumed final cause, if you may consider the matter correctly: for neither does the ray going out from C determine, how it may be able to arrive most easily to the point E, D, or G, nor how it may be referred by these points to that one ; but the Creator of things thus created light, so that from its nature that most beautiful event could arise. And thus they err greatly, which I shall not say more seriously, who reject with Descartes the final causes in physics ;

[The final cause, in Leibniz's philosophy, according to Wikipedia, is the end or goal, which guides one to take the necessary actions to obtain it. For that there needs to be an intelligence capable of conceiving the end, and realizing that certain actions must be taken to achieve the goal. Descartes and his followers in these days did not recognize the finality of natural causes as a guiding principle of investigation, no more than modern science does to-day.]

since still, besides the wonder of divine wisdom, they present for us the most beautiful principle requiring to be found and the properties of these things too, which within nature has not yet been recognised very clearly by us, so that with the efficient use from nearby causes, and machines, which the founder requiring to produce these effects, and he has held out to obtaining his ends, we may prevail to explain. Indeed we understand hence the old meditations into these things too and they are not to be held in contempt, with what may be considered today. For it seems to me to be definitely plausible, the greatest Geometers, *Snell*, and *Fermat*, most versatile in the geometry of the ancients, adapted the method, which the ancients used in Catoptrics to lead on to Dioptrics. Certainly *Snell's* theorem, which *Isaac Voss* brings forth [in his *De Lucis*…. without the mathematical details] from *Snell's* the three unpublished books on Optics, I suspect, to have been found as well by an almost similar method (although not to so great extent I suppose, as what we have used here, with the aid of the calculus). [Isaac Voss (1618- 1689) was a Dutch scholar and manuscript collector, who wrote a short work on optics pub. in 1662 : *De Lucis Natura et Proprietate*.]

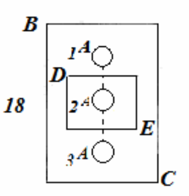
Indeed, I will show Snell's Law follows at once from our work thus. The circle CBG, with centre E may be described with radius EC or EG, the tangent of this at B produced crosses CE at V, and EG in T. The eye shall be at C, the object, which may be seen at T under water, and the point T will appear to be at V, because we may consider the right line CEV to be seen by us, since we may be actually still seeing along the fracture CET, it is apparent EV is the secant of the angle of incidence CEA, or for that to be equal to the angle VEB ; & ET is the secant of the angle of refraction GEB. But from a noted proposition of trigonometry, [the ratio of] the secants are reciprocally as the sines of the complement, therefore directly EV is to ET, as EL to EH, or (by our theorem) as m to n. Therefore with the eye C present in the other medium, as the object T, the apparent ray EV in the medium of the object (water), is to the ray in the medium of the object (water) truly ET, as the resistance m (of the air) for the medium of the eye to the resistance n (of the water) to the medium of the object. Because the ratio shall be the same always, with the same mediums in place, therefore the ratio between the true ray ET, and the apparent ray EV, will be same always, as it was in Snell's theorem.

In the same manner the ratio EL and EH of the sines of the complements of the angles of refraction and of incidence [note that *L*. regards the angles of the rays to the surface rather than to the normal as the angles of incidence and refraction], since for us it shall be the reciprocal of the resistance of the medium, will be the same always: which is the *Cartesian* theorem, although regarding the resistances of the mediums to be different from ours, indeed perceived in the contrary sense of *Descartes.*

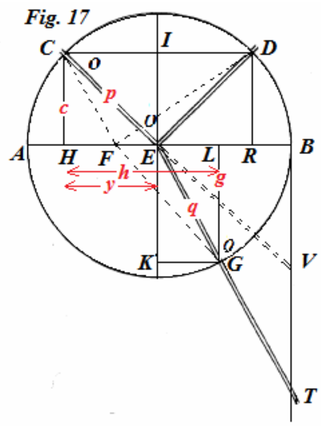
[Recall that in one such model *Descartes* presented in his *Dioptrique*, in which light was represented by a ball brushing against a piece of woolen cloth as it moved past, from which he assumed the light was transmitted faster in the denser medium, or the one with the greater resistance; note that *L*. has made no assumptions about the time of flight, but talks instead about the ease of transmission, and his speed changes are contrary to those of Descartes.]

Whereby, it cannot be doubted that *Descartes* saw the *Snellian* theorem when he was in Batavia, nor also with *Thomas Spleiss* agreeing with this conclusion, observed in his most versatile studies concerning this matter ; for it is noted the established names of the authors themselves have been omitted, and to the example he [*Descartes*] brings forth of the vortices of the worlds, to which *Giordano Bruno* and *Johann Kepler* thus might have raised a finger, so that finally that name itself [*i.e. Snell*] may be seen to be lacking. It arises, that this *Cartesian* theorem can be shown to have fallen into deep divisions by its own initiative: indeed because the ray CE was seen to be conveyed from the air into the water, there to be refracted at EG, and thus towards the perpendicular EK, and thus to be restored as that more similar ray, of which the action is stronger evidently towards the perpendicular; it is supposed that less resistance is to be found in water or in glass than in air. Yet from which on supposing the contrary, which is much more in agreement with reason, by using our principle of the easiest way, it may arrive at the same conclusion. From which *Fermat* deduced correctly, *Descartes* had not returned the true account of his theorem. Similarly also, when he tries to illustrate his explanation, it is not very suitable. In *fig.* 18 there shall be a small ball A on a polished table BC running at the position $_1$A : in the middle it runs into a part of the table DE raised by a woolen covering, there it will run slower at $_2$A. Therefore in the same manner consider glass, or some other solid body to hinder the rays of light less than air, which shall be more with more fibrous. But (as I may be silent on that matter, and the parts of water to be soft enough following *Descartes* himself) it suffices to consider the globule, where from $_2$A, with the wool DE crossed over, again it arrives at the part of the table put at $_3$A, where it does not recover its former speed, which it had at $_1$A, before it encountered the wool : yet since the ray of light advances again from the more resistant medium into the less resisting medium, similar to the first, it recovers the former state, and with the first and last of the two mediums put in place, the surfaces (of that emitting and of this receiving) are parallel planes, the direction it may receive by the latter refraction shall be parallel to that, which it had before earlier.

Fig. 18



Yet the manner may be considered, by which *Descartes* explains reflection and equally the refraction of light, according to the imitation of the motion of some other body, worthy by its ingenuity, and it is not to be rejected but only requires to be corrected. Just as concerning reflection, in the first place that itself will be required to be explained, why some ball such as I, incident along the perpendicular IE in the plane AB, may be reflected thence; for we may consider some bodies as soft, and not to be reflected equally. The true cause of this reflection is elasticity, either of the globe, or of the plane, or of each ; for the body does not fall on a plane without elasticity, as we have seen by making a membrane tense, or a bladder inflated, with a pebble falling on it, and indeed it goes off by so much more when it is hit stronger ; and by that greater force reconstituting itself to that speed, which it had incident, by which it had come, and thrown back with that same speed. Indeed *Descartes* was very unwilling in his own time to consider some other explanation of reflection, as appears from his letters : yet today it has been placed beyond doubt from reasoning and experiments. Therefore when the globe may arrive at C from E by the right line CE in *fig*.17. and with the motion henceforth composited from two motions, to the horizontal as CI, or HE, by which it arrives at IE from CH ; and to the perpendicular, as CH, or IE, by which it arrives at HE from CI ; and with each beginning from C, ending at E ; and of that arriving at E by trying to continue horizontally towards IE from CH in the right line CI, or HE, not placed opposite but shall be parallel to the surface AB, therefore equally the speed and the direction of the horizontal motion will be retained unimpaired, and how great a time it has spent, so that it might come from CH to IE, also may be spent, as it may come from EI to RD; with equal intervals put between CH and IE, likewise EI and RD. Truly the perpendicular motion, by which it comes from CI to HE, with the speed retained, will be turned in the opposite direction, so that it may spend just as great an amount of time, by which it may revert from ER to ID, since therefore ER shall be equal EH, and RD to CH itself. Triangles CHE and DRE will be similar and equal, and thus the angle DEB is equal to the angle CEA. All these may become clearer, if we imagine a ruler CID parallel to the surface AB with the parallelism maintained by CH and DR themselves, to impinge on AB at HR, while meanwhile the globe is carried along with the rule itself CI from C to I, from which the whole composite motion of the globe actually will be along the diagonal CE; but the ruler CID rebounds from the solid surface AB, with the same speed, and it will return in the same way as it came; and with an equal time it will return again to CID, while meanwhile the globe by continuing within the ruler with the same motion, it has gone on with the speed, and thus in the same time arrives at D from I, along lD, equal to CI itself, for with the same speed remaining in equal times equal distances are run through, and thus the globe from its composite motion from its own motion above the ruler, along ID, and by the motion of the ruler itself along EI, clearly by returning from HR to CD, will be brought from E to D along the right line ED.



For explaining refraction, a medium is considered that is seen to be more resisting to light (yet without being opaque), which hinders the spreading of the light more, or the distribution through more parts of the medium, and that medium can be said to be less illuminated, for the nature of light itself depends on the diffusion.

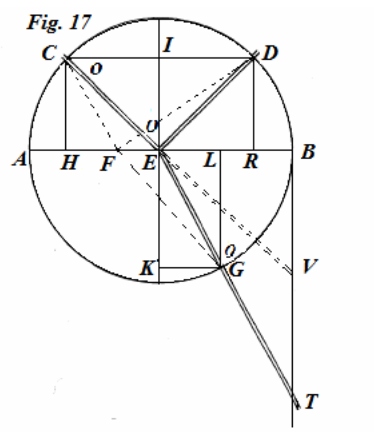
On the other hand, how much more light will affect the parts of a medium equally, which it illuminates, or where the light will communicate more of its strength to the illuminated places, with several insensitive particles, there the medium will be more illuminable, and less resistant to the light. From which the parts of the medium affected more by the light are solid and small, or they leave less space between themselves for other heterogeneous matter not permitting light, there the medium will be said to be illuminated more. Truly it is agreed from the principles of mechanics, the same blow inflicted on several bodies at the same time, to give a smaller individual force, than if one of these were knocked against ; therefore it arises, so that in a medium with more resistance to the diffusion of light, or allowing fewer parts to be affected, the individual parts may be affected much stronger than in parts to be illuminated more, but more weakly, and with the impressed impetus fainter. Now by taking the motion of the globule in place of the ray, and by considering the globule arriving at E from G, to be incident there in the medium, which may delay its speed or impetus if it pleases, in the three on two proportion, therefore if in the time of a single scruple [*i.e.* an arbitrary short time] it may come from G to E in the first medium KE, likewise in the new medium EI in the time of one and a half scruples it may arrive at C from E, by putting EC and GE to be equal, wherever C shall be last C. But since regarding the first entry of the globule into the new medium EI, for the horizontal speed at G K, LE and the parallels to be acted on, or at the point E itself, the separating surface of the two media AB has not opposed the entry at the first instant (for that motion acts only on the horizontal LEH), (by considering the globule indefinitely small in the likeness of a point, and as the rays are accustomed to be viewed as if without width) at once the inclination of the line ACE is required to be determined : therefore such that at once it is to be assumed at the beginning, that with respect to the horizontal motion the velocity may remain the same, and the initial amount of the advance into the new medium has been assumed it may remain such there. Therefore the globule, which, while it may go first into the medium from G to E, has resolved the interval GK in the horizontal direction in a scruple of time, or LE (between GL and KE) is now one and a half scruples of time, so that it must go from E to C, the interval EH will be absolved in the same horizontal direction, or IC (taken between EI and HC) which must be three halves of the former LE ; because with the same velocity remaining to the horizontal, (as the moment of refraction is not changed) , the distances shall be as the times, therefore EH is to EL, *in the direct ratio* of the times, or with the inverse ratio of the speeds, or the *reciprocal of the resistances* we have shown indeed in the case of light diffused from a resisting medium with the velocity impeded, or the resistive forces increased ; and with its ability to diffuse to weaken in certain regions : On the other hand the strength of a ray, and thus the direction to recover, when it enters into a medium again, where the diffusion is less, and more rays are expended being impelled into fewer parts, which recovery of *Decartes* with his tapestry, or to be explained by comparison with some other wooly body, is not possible, as we have warned above.

No. IX.

UNICUM OPTICAE, CATOPTRICAE
ET DIOPTRICAE PRINCIPIUM

Ex Actis. Erud. Lips. 1682.

Hypothesis primaria his scientiis communis, ex qua omnis radiorum lucis directio Geometrice determinatur, haec constitui potest : *Lumen a puncto radiante ad punctum illustrandum pervenit via omnium facillima* ; *quae determinanda est primum respectu superficierum planarum, accommodatur vero ad concavas, aut ad convexas, considerando earum planas tangentes.* Non tamen hic rationem habeo quarundam irregularitatum, fortasse locum habentium in generatione colorum, aliisque phaenomenis extraordinariis, quae in praxi Optica non attenduntur.



Hinc in *Optica simplici, radius directus a puncto radiante* C *ad punctum illustrandum* E *pervenit* via brevissima directa, eodem nempe manente medio, id est in *linea recta* CE.

In Catoptrica angulus incidentiae CEA, *& reflexionis* DEB *sunt aequales.* Sit enim punctum radians C, illustrandum D, speculum planum AB : quaeritur punctum speculi E, radium ad D reflectens. Dico id esse tale, ut tota via, $\mathrm{CE}+\mathrm{ED}$ fiat omnium minima, seu minor quam $\mathrm{CF}+\mathrm{FD}$, si nimirum aliud quodcunque speculi punctum F fuisset assumtum. Hoc obtinebitur, si E sumatur tale, ut anguli CEA & DEB sint aequales; ut ex Geometria constat. Hac demonstratione usi sunt *Ptolemaeus*, & alii veteres, extatque tum alibi, tum apud *Heliodorum Larissaeum.*

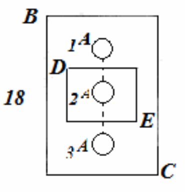
In Dioptrica sinus complementi, EH & EL, *angulorum incidentiae* CEA, & *refractionis* GEB, *servant eandem semper rationem, quae est reciproca resistentiae mediorum.* Sit IE aer, EK aqua, vel vitrum, vel aliud medium densius aere; punctum radiam in aere *C*, punctum illustrandum sub aqua G : quaeritur, qua via radiet illud ad hoc; seu quod sit punctum E in superficie aquae AB, radium a C emissum refringendo mittens ad G. Hoc E sumi debet tale, ut via sit omnium facillima. Jam in diversis mediis viae difficultates sunt in composita ratione, & longitudinis viarum, & resistentiae mediorum. Sint rectae *m* & *n* repraesentantes resistentiam respectu luminis, illa aeris, haec aquae; erit difficultas viae a C ad E, ut rectangulum sub CE & *m* ; a E ad G, ut rectangulum sub EG & *n.* Ergo ut difficultas viae CEG sit omnium minima, debet summa rectangulorum CE in $m+\mathrm{EG}$ in *n* esse omnium possibilium minima, seu minor quam CF in $m+\mathrm{FG}$ in *n* ; sumto puncto F alio praeter E quocunque. Quaeritur E. Cum ergo puncta C & G item recta AB, data sint positione, ideo rectas datas ad planum perpendiculares, vocabimus, CH, c; & GL, g; ipsam autem HL, *h*, etiam datam. At quaesitam EH vocabimus *y*, erit EL, $h-y$, & CE, $\sqrt{cc+yy}$, quam vocabimus *p*; & EG erit

$\sqrt{gg+yy-2hy+hh}$, quam vocabimus *q*. Debet ergo $m\sqrt{cc+yy}+n\sqrt{gg+yy-2hy+hh}$ (seu $mp+nq$) esse omnium possibilium similiter enuntiandarum quantitatum minima, & ut talis fiat, quaeritur *y*. *Ex mea methodo de maximis & minimis,* quae super omnes hactenus notas calculum mirifice contrahit, primo statim obtutu, sine ullo propemodum calculo patet ; fore *mqy* aequal. *np*, $h-y$ seu *np* ad *mq*, ut *y* ad $h-y$, seu rectangulum CE in *n* ad rectangulum EG in *m*, ut EH ad EL. Ergo positis CE & EG aequalibus, erit, *n* resistentia aquae respectu luminis, ad *m* resistentiam aeris, ut EH sinus complementi anguli incidentiae CEA in aere, ad EL sinum complementi anguli refractionis GEB in aqua ; seu sinus complementi erunt in reciproca resistentiae mediorum ratione, quod afferebatur. Itaque si EL in uno exemplo, vel experimento reperiatur esse duarum tertiarum ipsius EH, erit, & in aliis omnibus, ubicunque C & G sumantur, illud in aere, hoc in vitro. Si E in aere, & G sub aqua, erit EL $\frac{3}{4}$ ipsius EH circiter.

Reduximus ergo omnes radiorum leges experientia comprobatas ad puram Geometriam, & calculum, unico adhibito principio, sumto a causa finali, si rem recte consideres: neque enim radius e C egrediens consultat, quomodo ad punctum E, vel D, vel, G pervenire quam facillime possit, neque per se ad ipsa refertur ; sed Conditor rerum ita creavit lucem, ut ex ejus natura pulcherrimus ille eventus nasceretur. Itaque errant valde, ne quid gravius dicam, qui *causas finales* cum *Cartesio* in Physica rejiciunt ; cum tamen praeter admirationem divinae sapientiae, pulcherrimum nobis *principium* praebeant *inveniendi* earum quoque rerum proprietates, quarum interior natura nondum tam clare nobis cognita est, ut causis efficientibus proximis uti, machinasque, quas conditor ad effectus illos producendos, finesque suos obtinendos adhibuit, explicare valeamus. Intelligimus etiam hinc veterum meditationes in his quoque rebus non ad eo esse contemnendas, ac hodie quibusdam videtur. Nam mihi valde verisimile sit, summos Geometras, *Snellium,* & *Fermatium,* versatissimos in Veterum Geometria, methodum, qua illi usi erant in Catoptricia, ad Dioptricam traduxisse. *Snellii* sane theorema, quod ex ejus tribus Opticae libris ineditis affert V. Cl. *Isaacus Vossius,* suspicor simili fere methodo (licet non tanta opinor, quanta hic usi sumus, calculi facilitate) repertum tum fuisse. Ex nostro enim statim sequi, sic ostendam. Circulus CBG, centro E radio EC, vel EG describatur, hujus in B tangenti productae occurrant CE in V, & EG in T. Sit oculus in C, objectum, quod sub aqua videtur in T, & punctum T apparebit esse in V, quia per lineam rectam CEV videre nobis videmur, cum tamen videamus revera per fractam CET, patet EV esse secantem anguli incidentiae CEA, vel ei aequalis VEB ; & ET esse secantem anguli refractionem GEB. Sunt autem, ex nota trigonometriae propositione, secantes reciproce ut sinus complementi, ergo directe EV ad ET, ut EL ad EH, seu (per theorema nostrum) ut *m* ad *n.* Ergo oculo C existente in alio medio, quam objectum T, erit radius in medio objecti (aqua) apparens EV, ad radium in medio objecti (aqua) verum ET, ut *m* resistentia (aeris) medii oculi ad *n* resistentiam (aquae) medii objecti. Quae ratio cum semper sit eadem, iisdem manentibus mediis, ideo ratio inter radium verum ET, & apparentem EV, semper eadem erit, quod erat theorema *Snellianum.*

Eodem modo ratio EL, & EH sinuum complementi angulorum refractionis, & incidentiae, cum nobis sit reciproca resisentiae mediorum, semper eadem erit: quod est theorema *Cartesianum,* licet de resistentia mediorum diversa nostris, imo contraria senserit *Cartesius.* Quare non abs re Cl. *Spleissius* Vir in his quoque studiis versatissimus animadverso hoc consensu conclusionum, dubitat annon *Cartesius,* cum in Batavis esset, viderit theorema *Snellianum;* notat enim solenne ipsi suisse praeteriae nomina autorum, & exemplum affert mundanorum vorticum, ad quos *Jordanus Brunus,* & *Johannes Keplerus* ita digitum intenderint, ut tantum istud vocabulum ipsis defuisse videatur. Accedit, quod *Cartesius* theorema hoc suum proprio marte demonstraturus in magnas incidit salebras : quoniam enim videbat radium CE ex aere in aquam delatum, ibi refringi in EG, & ita versus perpendicularem EK, adeoque illi radio similiorem reddi, cujus fortior est actio, nempe perpendiculari ; suspicatus est eum minus resistentiae invenire in aqua, vel in vitro, quam in aere. Cum tamen supponendo contrarium, quod multo magis rationi consentaneum est, adhibito nostro facillimae viae principio, ad eandem conclusionem perveniatur. Unde recte collegit *Fermatius, Cartesium* non reddidisse veram theorematis sui rationem. Similitudo quoque, qua illustrare conatur explicationem suam, parum apta est. Sit in *fig.* 18 globulus A super tabula polita BC procurrens in loco 1A: is in medio occursu offendat partem tabulae DE tapete inflatam, ubi curret tardius in 2A. Eodem igitur modo putat vitrum, aliudve corpus solidum minus morari radios lucis, quam aerem, qui sit magis villosus. Sed (ut taceam, & aquae partes satis molles esse secundum ipsum *Cartesium)* sufficit considerare globulum, ubi ex 2A, tapete DE superato, rursus ad partem tabulae positam in 3A pervenit, ibi non recuperare priorem celeritatem, quam habebat in 1A, antequam tapetem offenderet : cum tamen radius lucis ex medio magis resistente in medium minus resistens, primo simile, rursus ingressus, priorem statum recuperet, & posito duorum mediorum similium, primi & ultimi, superficies (illius emittentem, hujus recipientem) esse planas parallelas, Directionem recipiat per refractionem posteriorem illi parallelam, quam habuit ante priorem.

Fig. 18



Videtur tamen modus, quo *Cartesius* reflexionem pariter, ac refractionem lucis, ad imitationem motus aliorum corporum explicat, ingenio ejus dignus, nec rejiciendus, sed tantum emendandus. Nam quod ad reflexionem attinet, prius explicandum ipsi erat, cur globus aliquis ut I, secundum perpendicularem IE incidens in planum AB, inde reflectatur; videmus enim, aliqua corpora, ut mollia, non aeque reflecti. Cujus reflexionis vera causa est Elastrum, vel globi, vel plani, vel utriusque ; nam cedet non nihil planum Elasticum, ut videmus facere membranam tensam, vel vesicam inflatam, lapillo incidente, & quidem cedet tanto magis, quanto fortior erit ictus; eoque majore vi se se restituens, id, quod inciderat, ea qua venerat via, & celeritate rejicit. Quanquam enim *Cartesius* hanc reflexionis explicationem jam suo tempore a quibusdam alia, tam, ferre noluerit, ut patet ex Epistolis : hodie tamen rationibus, atque experimentis extra dubium posita est. Cum ergo globus veniat ex C ad E recta CE in *fig.* 17. motuque proinde composito ex duobus, horizontali ut CI, vel HE, qua venit ex CH ad IE, & perpendiculari, ut CH, vel IE, qua venit ex CI ad HE; utroque incipiente a C, terminante in E ; & ejus in E venientis conatui horizontali ex CH versus IE in recta CI, vel HE, non opponatur, sed parallela sit superficies AB, ideo celeritatem pariter, & directionem motus horizontalis illibatam retinebit & quantum temporis insumsit, ut veniret ex CH ad IE, insumet etiam, ut veniat ex EI ad RD; positis intervallis inter CH, & IE, item EI, & RD aequalibus. Verum motus perpendicularis, quo venit ex CI ad HE, retenta celeritate, in contrariam directionem vertetur, ut temporis tantumdem insumat, quo revertatur ex ER ad ID, cum ergo sit ER aequalis ipsi EH, & RD ipsi CH, Triangula CHE, DRE similia, & aequalia erunt, adeoque angulus DEB angulo CEA aequalis. Omnia haec clariora erunt, si fingamus regulam CID superficiei AB parallelam servato parallelisimo per CH, DR, ipsi, AB impingere in HR, dum interea globus in ipsa regula CI fertur a C ad I, unde revera totus compositus globi motus erit in diagonali CE; regula autem CID repercussa a superficie solida AB, eadem celeritate, & via qua venerat redibit; & aequali tempore iterum veniet CID, dum interea globus continuato in regula motu eadem, celeritate perrexit, ac proinde eodem tempore pervenit ex I ad D, per lD, aequalem ipsi CI, nam manente eadem celeritate aequalibus temporibus aequalia spatia percurruntur, itaque globus motu composito ex proprio super regula, per ID, & motu ipsius regula per EI, redeuntis scilicet ex HR in CD, feretur ex E in D per rectam, ED.

Pro refractione explicanda considerandum est, medium magis resistens lumini (sine opacitate tamen) illud esse videri, quod magis impedit lumenis diffusionem, seu distributionem per plures medii partes, idque dici poterit minus illuminabile, luminis enim natura se diffundere nititur. Contra, quanto magis lumen aequabiliter partes medii, quod illuminat, afficies, aut quo pluribus insensibilibus loci illuminati particulis suam vim communicabit, eo medium magis erit illuminabile, minusque lumini resistet. Unde quo partes medii lumine affectae magis sunt solidae, & exiguae, aut minus inter se pro alia heterogenea materia a lumine nihil patiente, spatii relinquunt, eo magis illuminatum esse dicetur medium. Verum constat ex mechanicis principiis, eumdem ictum pluribus corporibus simul impressum, minorem singulis vim dare, quam si uni eorum fuisset inflictus ; ideo fiet, ut in medio magis resistente diffusioni luminis, seu secundum pauciores partes affecto, singulae partes tanto fortius afficiantur ; in magis illuminabili plures, sed debilius, impetuque impresso languidiore. Assumendo jam motum globuli loco radii, & ponendo globulum ex G venientem in E, ibi in medium incidere, quod celeritatem ejus, seu impetum retardet, proportione, si lubet, sesquialtera, ergo si tempore unius scrupuli venit a G ad E in priore medio KE, idem in novo media EI tempore unius scrupuli & dimidii veniet ab E in C, posito EC & GE esse aequales, ubicunque demum sit C. Sed cum celeritati horizontali, in G K, LE & parallelis exercitae, sub primum ingressum globuli in novum medium EI, seu in ipso puncta E, superficies mediorum separatrix AB non obstiterit (hanc enim motus horizontalis LEH tantum radit) in ipso autem primo ingressus momento, seu in puncto E, (globulum considerando indefinite parvum instar puncti, ut & radii solent quasi sine latitudine spectari) statim determinanda sit lineae ACE inclinatio : ideo talis statim initio assumenda est, ut respectu motus horizontalis eadem maneat velocitas, & qualis initio ingressus in novum medium, assuma est, talis in eo manet. Ergo globulus, qui, dum iret in medio priore a G ad E, absolverat scrupulo temporis horizontali directione intervallum GK, vel LE (inter GL, KE) is nunc scrupulo uno, & dimidio temporis, quo ire debet ab E ad C, absolvet in eadem horizontali directione intervallum EH, vel IC (comprehensum inter EI, & HC) quod debet esse sesquialterum prioris LE ; quia eadem manente velocitate horizontali, (quam momentum refractionis non immutat,) spatia sunt ut tempora, est ergo EH ad EL, i*n ratione* directa temporum, seu reciproca celeritatum, seu reciproca celeritatum, seu *reciproca resistentiarum* ostendimus enim in casu luminis a resistentia medii diffusionem impediente velocitatem, seu impetum pro resistentia crescere ; & pro majore diffundendi sui facilitate in singulis partibus languescere : Contra radium vim suam, atque adeo directionem recuperare, cum iterum in medium venit, ubi minor diffusio est, pluresque radii in pauciores partes impellendas impenduntur, quam recuperationem *Cartesius* sua tapetis, vel alterius corporis villosi comparatione explicare, ut supra monuimus, non potuisset