Chapter I.

Concerning the Nature and Propagation of Sound.

§ 1. The explanation of sound by the old philosophers was very obscure and confused, so much can be understood from their writings that have come down to us. Some were of the opinion, like Epicures [341 - 270 B. C.], that sound emanated from a pulsating body rather like the flow of a river; while others with the foremost of the Latin writers, believed with Aristotle, [384 - 322 B. C.] that sounds were formed from the breaking of the air which arose from the more violent collisions of bodies. Among the more recent commentaries, Honoré Fabri [1608 - 1688], and Descartes [1596 - 1650], discovered that sound consisted of tremors or vibrations of the air, but their reasoning concerning these vibrations were equally confused. Newton [1643 - 1727], with the sharpest of minds, considered the matter with more care, and undertook to set forth an explanation especially for the propagation of sound, truly with much more success. A determined effort has been made [by me] to grasp the difficult matters involved in an understanding of the nature of sound, which are set out in the two chapters of this dissertation. In the first chapter it becomes apparent, after some careful thought, what the nature of sound really is, and how it is propagated from one place to another. Moreover, in the following chapter, three sources of sound are to be considered.

§ 2. However, before this work on sound is undertaken, certain facts relating to air in the generation of sound are first to be related. I regard air as consisting of small globules, in a state of compression from the incumbent atmospheric weight, and this compression is relieved to a great extent with elevation, as the force of compression diminishes with height, so that the particles can restore themselves to their natural state. Thus, the weight of the air above compresses the air below, and the air globules are not allowed to be extended. The elastic force of compression on the air globules is equal to the weight of the atmosphere; on account of which one can measure this force by experiment, which truly is equal to the maximum weight of the atmosphere present. This weight is equal to a column of mercury of height 2460 scruples or thousandths of Rhenish feet [One Rhenish foot is equal to 313.8355 mm], and I will always adhere to these measurements in the following text; if the atmosphere has a smaller weight, equal to a column of mercury of height 2260 scruples, then this too can be taken as equivalent to the elastic force of the air [at a greater altitude]. Indeed, the weight of the air has been determined with the aid of pneumatic pumps; and the ratio of the specific gravity of quicksilver to the specific gravity of the warmest air has been observed to be in the ratio 12000 to 1; while for the coldest air the ratio is around 10000 to 1.

§ 3. If we consider one of a series of air globules to be compressed more than the rest, then that globule will dilate according to the law discussed above, while the surrounding globules become compressed by the force acting on them from the dilation of the single globule, which in turn compress others further away, as the globules scattered at a distance experience a little of the [original] compression. And by this line of reasoning the sound is transferred to other places. But, concerning the motion by which the globule considered expandes out, after coming to rest relative to the others, it then returns suddenly and is unable to be confined, as it has been extended excessively; hence it is again compressed with respect to the other globules, yet again excessively. Thus each one of the not too distant globules is itself dilated in this way by the trembling motion of that first globule considered, and in this manner each globule is constrained to move. [Thus, the physical idea of a central source consisting of an air globule executing an S.H.M. is presented, with a time delay or phase shift for neighbouring globules ; there is no physical argument presented for the reality of such globules, which are a convenient figment of the imagination.] But such a vibration of the globules of air nearby ought not to occur for globules of a very small size, and which hence depend on an indefinitely short time for a single oscillation; therefore innumerable oscillations or undulations with a finite period are to be given out by a globule in the manner prescribed, since truly the motion of any such globule of continually decreasing size cannot happen. Moreover, a finite time is required for perception by our senses, and it is not possible for sound to consist of a vibratory motion of that kind in the air.

§ 4. Then at last the sound is produced by the same globule, from the force exerted on other globules, with finite intervals placed between those allowed to have denser compressions. It is of course required in order to produce the sound, that the same globule is alternately contracted and relaxed, and indeed the time for these oscillations should not to be indefinitely small, but finite, in order that the number of these vibrations or oscillations for a given time can be determined. [Note: Mersenne, in his *Harmoniae* (1635) had already set out tables of frequencies associated with musical scales, and determined the speed of sound experimentally.] Of course the number of pulses arriving on the ear from an organ note in a given finite time can be expressed numerically.

§ 5. With the time now noted for which the sound is present, it is easy to explain the differences of sounds; here I will only distinguish between the principal kinds. Generally there are loud and soft sounds. A sound is loud or violent when the compressions of the air globules are stronger, and a sound is soft or small when these compressions are weaker. When the sound made by the oscillating globule is propagated by the communication of the compressions with each of the globules placed around it, the number of these increases in the ratio of the square of the distances from the place of origin, and the strength of the sound decreases in the inverse square ratio of the distances, unless perhaps the sound is augmented from elsewhere. The distinction between notes of low and high tones lies in the nature of the maximum duration of the movement. The case for low notes occurs when the vibrations of the air globules follow each other in turn more slowly, or for a given time the undulations are sent out less frequently. Moreover, the note is of a higher tone when the vibrations have shorter delays placed between them, in order that more oscillations are carried out in the same time. Hence the notes, with respect to low and high notes, are in the ratio of the number of oscillations made in a given time interval.

§ 6. The maximum distinction of the cause of the sound is between the high and low notes. It is low [or bass in music] when the vibrations of the air globules follow each other more slowly, or when in a give time fewer undulations are produced. But the sound is high [or treble] the briefer vibrations of which have smaller delays between them, and thus more oscillations are carried out in the same time. And hence the tones, with respect to bass and treble notes, are between each other in the ratio of the number of oscillations made in a given time.

§ 7. A sound is also either simple or composite. A simple sound [or note] is one in which the vibrations have equal distances between each other, and they are of equal strength. A composite sound is constructed from many simple sounds produced at the same time, and this sets up either concordant or discordant sounds. Concordant notes [or chords] are perceived as being produced by simple sounds or notes maintaining the simplest ratio between the components, such as two as in the octave, or as one and a half as in the musical fifth, etc. On the other hand, the dissonant [or discordant] sounds have their components in a more abstruse ratio, such as two superimposed fractions as in a three tone.

[The interested but uninformed reader may wish to hone up on the physics of music scales; a good introduction is found in a rather dated book : *Analytical Expermental Physics*, by Ference, Lemon, & Stevenson (1956) U. Chicago, Ch. 33]

§ 8. Now we may observe the propagation of sound with a little more attention, that can be done with some consistency. For the distance which a give sound can travel across in a given time can be found from the theory set out above : for the minutes and seconds of the hour are found by observation to be the same for all sounds, either loud or soft, low or high tones , to be carried through a given distance; and in fact these sounds always move forwards with the same speed. In order that this shall be so, this question may be asked : during the time the globule of air is compressed, what distance does it get thrust forwards ? This question can be answered without difficulty from the rules governing the communication of the motion and from the nature of air. Indeed a way can be found, but I omit doing this, as I prefer not work with imagined quantities. I put in its place what results are found from physical measurments.

§ 9 In order that I can consider the problem in a general way, the specific gravity of mercury to air is put in the ratio n to 1; the height of mercury in the barometer is equal to k, the length of a pendulum is f, from which it is a pleasant task to measure the time taken for the sound to travel a distance a from the pendulum's oscillations. From these denominated factors, I can find the ratio of the time for one oscillation of the pendulum f to the time for the sound to travel a distance a to be as 1 to $\frac{a}{4\sqrt{nkf}}$

[Following Newton, we assume the pendulum is one that follows the arc of a cycloid, in which case it executes a simple harmonic motion (shm) whatever the amplitude, and the period T_p of this oscillation is related to the length *f* of a simple pendulum by $T_p = 2\pi\sqrt{\frac{f}{g}}$. On the other hand, the speed of sound according to Newton $v = \sqrt{\frac{P}{\rho}} = \sqrt{gA}$, where A is the the height of the homogeneous atmosphere. Hence, the time T_{air} for the sound to travel a distance *a* is given by $T_{air} = a/v = a/\sqrt{gA}$. Hence,
$T_p / T_{air} = 2\pi\sqrt{\frac{f}{g}} \times \sqrt{gA}/a = 2\pi\sqrt{fA}/a = 2\pi\sqrt{nkf}\ /a$. Here *A* is the height of the homogeneous atmosphere which is $\rho_{air}\times g \times A$, or $\rho_{Hg}\times g \times k$, from which $A = nk$. Newton was the first person who had a grasp of the mechanical nature of wave motion to the extend that he was able to produce a formula for the speed of sound *c* in air; that this formula does not predict exactly the correct value for *c* does not detract in the least from the method, but only reflects the lack of understanding at the time of how heat was involved in the gas laws. Thus, the raising and lowering of the temperature in the air due to compressions and rarefactions are considered adiabatic as they happen so quickly. These effects were accounted for by Laplace a hundred years later by inroducing an extra factor γ into Newton's formula for *c* ($\gamma = c_p/c_v$, the ratio of the specific heats of air at constant pressure and temperature). Thus, in Euler's day, these temperature related effects were still unknown. We now look at Euler's formula.]

§ 10. If *a* and *k* are measured in scruples, but in place of *f* is put 3166 [a scruple is ~ 0.34mm; hence f ~ 1076mm, giving a time of 2 seconds for a complete swing, or 1 second for a swing from one side to the other], will give the value $\frac{a}{4\sqrt{3166nk}}$, the number of seconds in which the sound should be propagated a distance *a*. For the length of the pendulum with a time of one oscillation [*i. e.* half-swing] is indeed 3166 scrup. Thus, with the distance *a* solved for the time, [$v = a/t =$] $\frac{a}{4\sqrt{3166nk}}$ /*a*, the distance that sound travels out in a time of one second will be $4\sqrt{3166nk}$ scrup. [Thus v = 4 × √ (3166 × 12000 × 2460) = 1200000 scrup. or 415 m/s. If we put π in place of 4 in the formula for the half-swing, we get 326 m/s, which is near the correct value.]

§ 11. Thus these conclusions follow on. The speed of sound remains the same if *nk* remains the same too, if the density of the air and the pressure are in proportion, the sounds are carried with the same speed. For one may know that there is no difference to the senses nor speed for sound in air of the maximum compression than for sound moving in air with the maximum rarefaction. Hence sound on the summits of mountains ought to travel with the same speed as sound within the valleys, except for other causes to be explained soon that might be added.

§ 12. By increasing the factor *nk*, the speed of sound should be increased. Hence with the density of air remaining the same or decreasing a little, but with the pressure increased, then the speed of sound will be greater ; but truly from the contrary with the density of the air increasing, and with the pressure remaining the same or decreasing a little, the sound is slowed down. And hence collecting these things together, since both the density or weight and pressure of the air surrounding the earth are subject to various changes, the speed of sound is constantly changing also. Hence the maximum speed of sound will be [found] on the hottest days with a clear sky, or with the most careful measurements made from barometers and thermometers at the highest levels of elevation. Truly with the harshest cold and the fiercest storm, the speed of sound should be a minimum, that which may come about by measurements from the liquids in barometers and thermometers present in the lowest places.

§ 13. Hence the maximum speed of sound is found, if 12000 is put in place of *n* and 2460 scrup., in place of *k* as thus the distance traveled by sound in a time of one second is $4\sqrt{3166.12000.2460} = 1222800$, *i. e.* the maximum speed of sound following my theorem should be 1222 Rhenish feet per second. Indeed the minimum speed of sound is obtained by putting 10000 for *n* and 2260 for *k*, as thus the distance passed through in one second is $4\sqrt{3166.10000.2260} = 1069600$, or 1069. Hence the distance which sound passes through in one second ought to be contained between the limits 1222 and 1069.

§ 14. If those numbers are brought together with experimental trials, they are found to be in excellent agreement with those, that confirms my theory. For FLAMSTED made observations [J. Flamsteed (1646 - 1719) circa 1675 had determined the speed of sound in air in Greenwich with Ed. Halley (1656 - 1742).] and DERHAM [W. Derham (1657 - 1735), Experiments and observations concerning the motion of sound ..., Philosophical Transactions (London) 26, 1708, no. 313, p. 1] with the most accurate experiments set up found that sound traveled through 1108 feet in one second, which is a number placed almost midway between the limits found. If now we consider what NEWTON has to say in [I. Newton, *Philosophiae naturalis principia mathematica* Book II prop. 50, scholium; 2nd Ed, Cambridge 1713, p. 342 - 344; see also this website on Newton] in the *Phil*. Book. II, Section VIII, he found that sound passes through a distance in one second (with his reasoning reduced to our way of talking) of $\frac{p}{d}\sqrt{3166nk}$ Rhenish scruples with $\frac{d}{p}$ denotining the ratio of the diameter to the periphery of a circle, *i. e.* that is approximately 7 : 22. And thus his expression is less than our one, if indeed NEWTON introduced the factor $3\frac{1}{7}$ to $\sqrt{3166nk}$, however, I will adhere to 4 in place of this number. [Newton actually compared the times with a pendulum and so did not introduce π.]

§ 15. Hence this is not so wonderful, since the most acute NEWTON found the exceedingly small distance that sound will reach in a second ; that he could not determine greater than 947 feet, which certainly is a huge discrepancy [really only ~ 11% out] from that distance which is found by experiment ; but he reports in order to confirm his method, that the descrepancy is shared by the impurities mixed with pure air that slow it down. Indeed air is corrupted by vapours, but the force of the pressure is always equal to the atmospheric weight, and the weight of the air to the senses too does not change. It is not possible to be always maintaining the change in the speed of sound in tems of these factors, and neither does the size of the molecules of the air make any difference to these things.

CHAPTER II.

CONCERNING THE PRODUCTION OF SOUND.

§ 16. It is a well-known requirement in the production of sound, for the theory set forth in the previous chapter, that the vibrations have to be re-applied in some manner, in order that the globules of air can have separate alternate contractions and expansions for short time intervals. I have been able to infer three kinds of vibratory motion from the three ways in which sound is generated in the first place. Whereby a few words should be said here concerning the three diverse ways in which sounds can be produced. Moreover, I refer to as the first kind of sound, the sound of stringed instruments, drums, bells, musical instruments under the control of the tongue, etc., all well-known sounds, which have their origin in the vibration of a solid body. To be referred to as the second kind of sound, are sounds such as thunder, bombardment and the snapping of twigs, and any in which a body is set in some more violent state of motion, all nevertheless are sound arising from the sudden restitution of compressed air, and as a stronger percussion of the air. Finally, the third kind of sound I add are the sounds of wind instruments such as the flute, and I will spend some time in explaining the nature of these with care, since no one up to the present has given anything of substance concerning these instruments.

§ 17. As far as all the sounds of the first kind are concerned, so much I know, that clearly all sounds initially were referred to and considered to be generated by the vibrations of some solid body, and no other kind of sound considered possible; but the falseness of this idea will soon be shown, when I set forth the two remaining ways in which sound can be produced. But now, concerning the manner in which sounds are produced, the first has been examined with greater care. Indeed for the present I need only consider how sounds of different kinds arise from strings, as the other sounds of this type can easily be deduced from this example. I examine closely how the exact tension in the string can be obtained by being extended around the column of the instrument in order that the force in the string is allowed to be measured accurately.

§ 18. Before all else, it is to be noted that strings give rise to sounds with the same equal ratio of low or high pitch, whatever the applied pulsating force, although it is possible to have a huge difference in the ratio of the strength to weakness of the force; indeed the intensity of the sound is proportional to the speed with which the string beats the air, and sounds are equally intense if the air is struck with the same force. Wherefore, since musical sounds of either low or high in pitch should be of equal strength, in order that a pleasant harmony is produced, this is given earnest attention in the making of musical instruments, in order that equall sounds with the same ratio of intensity or robustness are produced , which should be obtained following the rules, which indeed have been found now from much gross trial and error practise by the most recent craftsmen, the truth of which indeed is perfectly apparent from what follows, which have been diligently observed.
I. *The lengths of the strings are in the reciprocal ratio of the notes, i. e. the number of vibrations sent out in a given time.*
II. *The thickness of the strings or the transverse cross-sections are also in the reciprocal ratio of the notes, if the same material of course may be said to be used; if indeed this is not so, then the ratio of the densities has to be included with the ratio of the widths.*
These rules can be applied for the construction of wind instruments such as the flutes, where in place of the length of the strings is to be taken the length or height of the flute and in place of the thickness of the strings the internal width of the flute.

§ 19. When the string is vibrating, it strikes the air globules which are nearby and since they unable to recede, they are compressed ; but with the enduring oscillatory motion the air globules suffer continual new compressions, thus the note arises. And thus the air strikes the ear or the tympanic membrane of the ear as often as the string returns. Indeed the number of percussions of any note carried to the ear for a given time can be found of course by finding the numberof oscillations of the string emitting the note in the same time. Moreover, my solution, which with the solutions of Cl. Cl. D. D. JOH. BERNOULLI [JOH. BERNOULLI (1667 - 1748), *Meditationes de chordis vibrantious,* Commt. acad. sc.Petrop. 8 (1728), 1732, p. 13; Opera omnia, Lausannae et Genevae 1742, t. III, p. 198] and BROOK TAYLOR [(1685 - 1731), *Methodus incrementorum directa et inversa,* Londini 1715, p. 26] are in exact agreement, is the following:

§ 20. Let the weight holding the string $= p$, the weight of the string $= q$, and the length of the string $= a$, from which three given quantities the number of vibrations to be found in a given time can be proposed. I find the number of oscillations set forth in a second to be :

$$\frac{22}{7}\sqrt{\frac{3166\,p}{aq}},$$

where a should be measured in scruples. [Recall that the period of the pendulum measuring single seconds for a single swing from left to right of vice versa is given by $T_p = \pi\sqrt{\frac{f}{g}}$, where f is 3166 scruples. Hence, the time of one second corresponds to $T_p = \pi\sqrt{\frac{3166}{g}}$. If the tension in the string is p as a force, the density of the string is q/a as a mass per unit length, then the speed of transverse wave in the string is given by $v = \sqrt{\frac{pa}{q}}$, while the time to travel a length a is given by $t = a/v = a/\sqrt{\frac{pa}{q}} = \sqrt{\frac{qa}{p}}$. If the reciprocal is the frequency of the associated sound wave, then the frequency is $\sqrt{\frac{p}{qa}}$. If p is given as a mass, then the associated force is pg, and hence the frequency is given by $\sqrt{\frac{pg}{qa}}$ while the associated period is $T_{sound} = \sqrt{\frac{qa}{pg}}$. This period compared with the pendulum is

$T_{sound}/T_p = \sqrt{\frac{qa}{pg}}/\pi\sqrt{\frac{3166}{g}} = \frac{1}{\pi}\sqrt{\frac{qa}{pg}} \times \sqrt{\frac{g}{3166}} = \frac{1}{\pi}\sqrt{\frac{qa}{p.3166}}$. The frequency of the sound is hence

$\pi\sqrt{\frac{3166p}{qa}}$ as Euler has shown.] Since the number of vibrations of a sound is in proportion to this number, sounds sent forth by different strings are amongst themselves in the ratio $\sqrt{\frac{p}{aq}}$, *i. e.* the frequencies of the sounds are in a square root ratio composed in direct proportion to the weight of the tension, and indirectly to the length and weight of the string. I do not deduce more particular consequences from this, but I will inquire into the nature of known sounds, and the number of vibrations answering to these formulae from the experiment that is set up by me in the end.

§ 21. If I take a copper string with the density of that kind of substance, which is indicated in §18, of length 980 scruples, which has a weight of $\frac{49}{175000}$ pounds, and I have stretched this with a weight of $\frac{11}{4}$ pounds, the sound pulses from which are to be taken in agreement with that instrument in the choral mode, as they say, to be fitted [*i. e.*, by someone singing the same tone], which to the musical is heard as $\overline{ds}$. Hence therefore, one is permitted to count up, how often that sound is heard, and hence how may vibrations of the other sound in the given time are heard from the mechanical device being struck ; indeed in the given general formula, if 980 is substituted in place of a , in place of p $\frac{11}{4}$ pounds, in place of q $\frac{49}{175000}$, then the sound $\overline{ds}$ is found to have 579 vibrations per second and since $\overline{ds}$ shall be to $\bar{c}$ as 6 to 5 [a minor third], the sound $\bar{c}$ has 466 vibrations [per second] and thus at the minimum C is 116 vibrations [per second].

§ 22. To this method of sound production, reference is made to the sounds produced by the tongue as well as by elastic plates [valves] inserted in a tube in a strong airflow, though both these methods are related also to the third manner of sound production ; for indeed they are each related to the other. Mechanisms of this kind are to be seen in the various compressed air pipes such as trumpets and bugles, as in the imitation of the human voice in song, all of these instruments must have air blown into them in order that they can produce sound. The compressed air, by itself seeking to pass out, uncovers the tongue from the opening, like a valve, but holds that opening to an excessive extent, so that the tongue acts as a valve and closes again by stretching to its former state, which is thus opened anew, in order that the air passing through can be given a vibratory motion. Indeed it is unavoidable, with the air flow produced by uniform breathing, that this kind of valve finally comes to rest and the sound ceases; but this warning applies only to the wind instruments themselves, for if the air is expelled from bellows [as in an organ], it strikes the openings of the mechanism in a non-uniform manner, and with the help of an inserted valve the wind in the tube is carried along and the vibration continues.

§ 23. Clearly the human voice is produced in the same way; indeed the epiglottis holds in place the seat of the tongue in the organ of speech, the vibration of which is maintained by the passage of the air ascending through the rough windpipe. Besides, the vibratory motion of the air escaping from the end of the rough windpipe is changed in the cavity of the mouth in a number of ways, by which the low and high-pitched tones of the voice can be produced, and different vocal effects are formed, which with the help of the tongue, lips, and the pharynx provide sounds with consonants So with the nose, for it is apparent that the air in continuous vibration coming from the epiglottis can leave by the nose too; but with regard to the various high and low pitched notes which can be produced in this way, they are unable to give rise to either clear speech and neither can consonants be preserved, which are thus different from sounds produced by the mouth.

§ 24. But these sounds sent forth by the vibrations of the tongue, unless they are strengthened by pipes or tubes, are soon to become very weak, as is to be seen in the vibrations of thin plates [in contact with the mouth, one imagines], where hardly anything is perceived by the ear. But the miracle is the way in which these sounds are amplified in tubes, and also the human voice by the mouth cavity, not in the least being the manner sounds of low and high notes sent into a pipe undergo a great change. Truly this is not the place to sow the seeds [for further study] of how sound waves are intensified and reflected within pipes; there would need to be a work with a special title, in which that material could be set out more carefully, and in which the miraculous amplification of sound by megaphones could also be established, as well as the principle of the echo and many other phenomena; but the task of carefully considering that material more carefully is not yet complete, and concerning what is held in other writings, a lot of things that I have examined in these sources are very confused and in the greater part false.

§ 25. By the second class of sounds I refer to these sounds which arise either from the sudden release of a noteworthy quantity of compressed air or which strike the air with a greater force. The air in the latter mode is also compressed, since it attempts not to leave the place to the vibrating body, from where the air left has again expanded. Thus the cause of the sounds belonging to the second class is the restitution of the air to its state before being compressed. It is apparent that the sound arises from that restoraton, the air which was compressed itself expands too and hence again contracts and so on, with which undulation of the air shall be, as the smallest of the air globules too, which of course make up the mass of the air, participate in that vibratory motion and in consequence lead the sound forwards; it is to be noted, that if a greater body of air is compressed, a deeper sound is produced, and if it is smaller, the sound will be sharper. Truly sounds produced in this way are not able to endure for a long time, but from what is left they must stop, because the vibratory motion of the air set up a long way off is lost at once.

§ 26. Therefore all the causes prevail for the production of suitable sound, whereby either the air is compressed for the sound to be sent, or truly it is thus compressed again in order that the expansion can be stopped. Whereby all the swifter motions of bodies in the air ought to emit sound; indeed from the motions of bodies the air on account of its own inertia is unable to go freely and is compressed, and again by expanding itself the vibratory motion of the air is induced in the smallest of air globules, to produce a suitable sound. Thus sounds flow from the stronger vibrations of branches, as from bodies moving more swiftly. Also the origin of the sounds of the breeze and of the wind arise from this font; indeed the air from the preceeding is compressed as for that following example, as from a hard body.

§ 27. Easily the loudest of the sounds which arise from the relaxation of suddenly compressed air are those associated with artillery and thunder. Moreover various experiments set up with sulphur and saltpetre powder prove that the cause of these immense sounds is the restoration of compressed air to its former state; since it is found that the air there is in a state of maximum compression when the powder is set alight and ways are found for it to emerge, so that it can burst forth with the maximum force. Since moreover from the saltpetre and fiery dust, and the many constituent clouds and vapours that are present, is it not a miracle that a fire arises from that material, and the astonishing sounds that then reverberate.

§ 28. The sounds of flutes constitute the third kind of sounds. The explanation of the nature of these sounds has been astonishingly twisted by scrutineers at any time. Most have thought that with the inflation of the pipes the smallest particles of the inner surface are struck and the vibratory motion is set up, thus the internal surface of the pipes restores the vibration by inflation and enables the oscillations to be communicated with the air; but how that explanation is consistant with the laws of nature and of motion, they themselves may examine. I myself am certainly unable to conceive this, all the same it is possible to explain how pipes of different heights can give rise to differences of sounds of the same amplitude ; for indeed if the internal particles can bring about a motion at any time, then I am unable to see how, for tubes of different heights, it is possible to have different kinds of oscillations. In next to no time I can decide by setting up an experiment even with a single pipes, how it is possible to explain the theory.

§ 29. Moreover, since I will resolve this matter, at first the structure of flutes is to be considered, and then what indeed is going on inside while air is blowing through them has to be considered with great care. For flutes are pipes or tubes, for which beyond the junction of the tubes there is cavity for the mouth [the embrouchure or blow hole] in order to receive suitable air blown in, which is placed towards the end of the tube and which changes the air flow by striking a sharp edge on the inner side of the hole on the side of the tube. This finally allows air from the mouth to be sent in through the hole in the tube, and the air is allowed to move slowly back along the inner surface of the tube [Euler thus seems to believe that reflected air moves back along the inner surface of the tube slowly, rather than reflected sound]; if the pipe is made in this way, sound is emitted on air being blown in, that is readily apparent, for if some tube on its own is blown into by the mouth, so that the air in the tube can move slowly along on the inner surface, as it also sends out the same sound as the flute. The internal surface of the tube should be hard and go to the left, so that the air can neither intrude, nor be given a place in that extended tube [to the right] where it is possible to expand [i. e., standing waves are established in the left-hand part of the tube, the small space to the right

acting as a sort of buffer, evens out sounds of different frequencies], wherefore the pipes should be prepared from tubes with a closed end and smooth rigid sides.
[Note however that the flute acts almost like a tube open at both ends, due to the presence of the blow hole; thus, if all the note holes are closed, then the sound produced will have a wavelength approximately twice the length of the instrument. The closed end now has a stopper, and its exact position is important for tuning.]

§ 30. We may now see what may eventuate in the tube while air is blown in, what vibration of the air it is possible to retain, or by what ratio the air moving slowly in the tube in the said manner can be retained by holding the vibration. It is clear, for the air entering the flute, with the understanding that the air is going to be compressed along the length of the tube; where since it will expand by itself again, truly greatly, again it will be compressed by the weight of the atmosphere, as the vibratory motion in the tube is produced, which vibration is the reason for the sound nearby. Thus truly the reason for the sound produced by flutes has been found, but the reality and truth of this will become abundantly well-known; but to get to the heart of the matter, the first consideration is the manner by which the motion can produce that vibration.
[We may note that the player has a lot to do with the resonance, by feeding in air at the correct speed via the blow - hole to maintain the oscillation; it is the pressure difference introduced by this air that is responsible for the pressure variations in the tube, and there is a pressure node at each end; the atmospheric pressure is brought into the scheme of things as it determines the speed of sound, as Newton (almost) demonstrated.]
§ 31. An air column in a pipe itself undulates following the amplitude of expansions and contractions in the same manner as strings, and thus I can consider that same air column as a bundle of air strings with the tension given by the weight of the atmosphere. But although the weights stretching the strings can try to pull them apart, here indeed the direct opposite effect is obtained, with the air in such a column made narrower by the weight of the atmosphere, nevertheless the analogy is a legitimate one; indeed the weight of the atmosphere exerts the same effect on an air column, which a weight does stretching the strings, if indeed we compare both sides, we have here the weight of the atmosphere, and there the weight of the strings being stretched, and the greatly extended strings are again drawn together. But in place of ordinary strings, since they can emit sound pulsating from one point, for these air strings with a pulse made from one point, the whole air column may be unable to vibrate on account of the discontinuity of the part, for likewise the whole length ought to be pulsating, which shall be the case in flutes, where the air moving slowly through the whole air length in the tube is content to be compressed.
[Thus, if a tuning fork is struck and placed in an organ pipe, the pipe may not vibrate as a whole, depending on the resonant frequencies of the fork and the pipe involved; on the other hand, a plucked string will always resonate according to its tension, length, line density, etc This may be the sort of situation Euler has in mind. In any case, the analogy is a rather loose one.]

§ 32. In order that the oscillations of the air in flutes can be found, or the number of waves that can be set in motion for some pipe according to our previous argument, to the air in the pipe from the weight of the atmosphere has the same weight as the tension in the chords in both cases ; and this will be found by considering the oscillations in § 21. Let the length of some string, *i. e.* of some pipe $= a$; p (the weight holding the string) = to the weight of the atmosphere or column of mercury in a barometer, *i. e.* from the minimum 2260 to the maximum 2460 scrup.; q (the weight of the string) moreover will be = to the weight of the air in the tube. Again the specific gravity of mercury to air will be $= n : 1$ and k the height of the mercury in the barometer; p to q is in the ratio composited from n to 1 and k to a or p to q is as nk to a.

§ 33. Thus by putting these proportions nk and a in expression § 20 in place of p and q the number of vibrations per second of the sound emitted is found to have the value $\frac{22}{7a}\sqrt{3166}nk$. Thus it is apparent, since n and k may change with the season, that the sound also is to be changed, it is apparent that with an increase in nk the pitch will rise, and with a decrease the pitch will fall. Therefore the sounds of flutes will be sharpest pitch with the maximum heat, and the air the least dense, but to be lowest pitch with the maximum cold and the most dense. This difference of sounds is especially observed by musicians and organists. But since all flutes have the same change in place equally, the melody is not changed.

[From § 20, we have the speed $v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{\rho_{Hg} gk}{\rho_{air}}} = \sqrt{ngk}$. The fundamental frequency $f = v/\lambda = v/2a = \frac{\sqrt{ngk}}{2a}$; now, the pendulum that measures the seconds is a means of determinig g, the acceleration due to gravity, and in this case, with the units used, this amounts to $T = 1 = \pi\sqrt{\frac{3166}{g}}$; or $\sqrt{g} = \pi\sqrt{3166}$. Hence, $f = \frac{\sqrt{ngk}}{2a} = \frac{\pi\sqrt{3166nk}}{2a}$; the frequency in Euler's formula is twice this amount, corresponding to the first harmonic.]

§ 34. In order that the number of oscillations of flutes can be expressed by numbers, if the frequency of the sound is desired with the liquids in barometers put in position at the greatest height, for *n* put 12000 and for *k* put 2460 scruples; then the number of oscillations sent forth in one second from a flute *a* scruples long is found to be $= \frac{22}{7a}\sqrt{3166 \cdot 12000 \cdot 2460} = \frac{960771}{a}$.

Truly for the greatest cold of the most savage season for the digits of liquid barometers and thermometers, put 10000 for *n* and for *k* put 2260 scruples, the number of oscillations sent forth per second is found to be equal to $= \frac{22}{7a}\sqrt{3166 \cdot 10000 \cdot 2260} = \frac{840714}{a}$.

§ 35. Hence the ratio is therefore apparent, whereby flutes sent out sounds in reciprocal proportion with their lengths and where the width makes no difference, and indeed also where the material of flutes does not introduce different sounds. Though neither the width nor the material of the body of the flute results in any change in the raising or lowering of the pitch, nevertheless these contribute much affection and charm to the flute ; moreover the width of the tube is the basis of the strength of the sound, for a wider tube gives a stronger sound also ; the width of flutes is well-known to be analogous to the density of strings. And as not every string is suitable for the production of any sound, truly as lower notes are required so the density of the strings becomes greater, thus also it prevails for flutes that the taller these are, so also a greater width is requires.

§ 36. For the whole number ratio of the notes which vary between themselves as 8 to 9, under different conditions of blowing the air, the same flute can give rise to both maximum and minimum tones. Let the flute be 4 feet long, which is used to produce the note C as in the singing manner [i. e. middle C]; and there will be vibrations per second of this which are at most 240 and at least 210; which agrees well enough with these, which we found before, when we considered the action of strings; there indeed the number of oscillations per second for the note C was found to be 116, thus it is apparent that the note C of flutes is nearly an octave higher than that note C of the string. Because they stand a whole octave apart, they are commonly taken as being equal, which is not to be wondered at, since concerning notes, the most difficult to judge are those which are heterogenous or discordant, or else they are in unison; now truly it is sufficient to have an octave or in short with two or more octaves set apart, since in one octave leap I can find these dissonant sounds or discrepancies, which confirms my theory well enough.

§ 37. Up to this stage we have been concerned with the sounds produced by pipes, but the opening of a cylindrical pipe should be understood, where the exit for the air blown in from the top of the tube lies open. For since the tube opening above is covered [i. e. the end of the tube beyond the blow hole], the air blown in from above is unable to escape, and thus it is necessary to go back, in order that it can emerge from the lower opening. Thus it shall be the case that so much more of the length is made free with the air reflected from the upper stoppered end of the tube to the other opening before reaching the open exit. And by following the air in the tube a string of twice the length as it were has to be considered, obviously strings that bend at the joint have to be considered Thus the sound is gathered by the closed end of the tube to be sent forth in the same way as for a tube twice as long [one open at both ends], or it emits sound an octave lower than if it were open.

[There seems to be a mistake here, as both the string and the open-ended pipe support a fundamental wavelength equal to twice the length of the string or pipe; this error is due to the factor of $^1/_2$ missing in the previous formula, which results in an octave shift.]

Whereby such pipes do not sent out sounds everywhere of the same wholeness, *i. e*. the sounds may be converging or diverging, likewise even from the part of the cover of the above pipe.

I put this work forward to be examined with the distinguished canditates.

[Some of the latter material presented by Euler on flutes is suitable for someone with an interest in the history of the flute: a good starting point for such a person to look is in Vol. 23 of the 8th Edition (1878) of the *Encylopaedia Britannica*, p.519 onwards. We may note that in Euler's day, the flute was in a period of transition to one or other of its modern forms, and had a small U shaped pipe with the mouth piece at the top end, as well as having joints to insert tubes of differing lengths to accomodate several octaves as required. The end cover was a cork which could be removed to discharge condensation, and its position was critical for tuning. Those considered at an earlier date by Mersenne in his *Harmoniae* were straight, and resembled recorders in construction, I think.] The modern theory of the flute and other musical instruments can be found on the web at the site of the Physics Department of the University of N.S.W. in Australia, which is well worth reading.

Attachments.

1. *The systems of the body and the preformed harmonies of the mind, from which the actions of the mind and body in turn assert themselves, is not consistent with the truth.*
[In other words, even with a sane mind in a sound body, that is not enough to stop you making a fool of yourself!]
2. *The attractive force of NEWTON is the most suitable way for all phenomina of celestial bodies to be explained. And I believe beyond doubt the idea put forward that all bodies by their own mutual attraction draw together.*
3. *From thw position of the centre of the earth (which moreover is truly far away and alien) any bodies are to be attracted by the inverse square of their distance, and a hole is to be drilled through the centre of the earth. It is asked, for a stone sent down the hole, what will happen, when it reaches the centre, whether it will either remain there permanently, or progress away from the centre without pausing and return to us again soon from the centre of the earth. I confirm that the latter is the case..*
4. *The strengths [i.e. kinetic energies] of moving bodies are composed in the ratio of their masses to the first power and of their speeds to the second power.*
5. *A sphere is descending rotating on an inclined plane, in the absence of all resistance, the speed that it can acquire from falling perpendicularly through the same height, is found to be much less. Indeed it will be in the ratio to that acquired when a body falls normally in the ratio* $\sqrt{5}$ to $\sqrt{7}$.
6. *Masts on ships should not be exceedingly high, so that the force of the wind does not capsize the ship. Moreover we can put the mast equipped with sails exceedingly high, as surely the ship is knocked over by the given wind. I say, that if wider sails are attached, in order that the force driving the ship forwards, can be stronger, to be less liable to upset the ship. And always, whatever shall be the height of the mast, the width of the sails of these can be increased, in order that even in the strongest wind, it will not be possible to report the loss of the ship.*

That's Enough !

Caput I.

De
Natura et Propagatione Soni.

§ 1.
Obscura admodum atque confusa fuit vererum Philosophorum soni explicatio, quantum ex scriptis eorum nobis relictis intelligi potest. Alii, cum Epicuro sonum, istar fluminis ex corporibus sonoris pulsatis emanare statuerunt. Alii autem & præprimis interpretes ARISTOTLIS latini cum illo naturam soni posuerunt in fractione æris, quæ oritur ex collisione vehementior corporum. Inter recentiores HONORATUS FABRI atque CARTESIUS invenerunt sonum consistere in æris tremore, de isto autem tremore pariter

confuse sentiebant. Acutissimus NEUTONUS, hanc rem accuratius expendere atque exponere aggressus est, præcipue soni propagationem explicando, verum parum feliciori successu. Arduam ergo hanc de sone materiam, istac in dissertatione tractare, atque pro viribus dilucidare constitui, duobus capitibus eam comprehenendo. Priore hoc capite scilicet perpendetur, quare sonus consistat, & quomodo ab uno loco ad alium propagetur. In posteriore autem, tres sonum producendi : considerabuntur.

§ 2. Antequam autem ipsius soni tractationem aggrediar, quædam de aëre, utpote soni subiecto præmittenda sunt. Aërem concipio constantem ex globulis in parvis, compressis ab incumbente pondere atmosphærico & tanto gaudentibus elaterio, ut semota vi comprimente sese queant in statum naturalem restituere. Cum itaque pondus aëris superioris inferiorem comprimat, prohibeatque ne globuli aërei extendantur, vis globulorum aëreorum elastica æquatur ponderi atmosphæræ; quocirca eam experimentis definire licet. æqualis nempe est, maximo existente pondere atmosphæræ ; columnæ Mercuriali altæ 2460 scrupula seu millesimas pedis Rhenani [1 pes Rhenanus est 3138355 mm], quam mensuram in posterum semper adhibebo; sin autem atmosphæra minimo pondero gavisa fuerit, æquivalens deprehenditur vis aëris elastica columnæ Mercuriali altitudinis 2260 scrupulorum. Quin etiam pondus aëris ope antliæ pneumaticæ determinatum est ; gravitas enim specifica argenti vivi se habere observata est ad gravitatem specificam aëris, maximo calore, ut 1200 ad 1, & summo frigore, ut 10000 ad 1 circiter.

§ 3. Si concipiamus in serie globulorum aëreorum unum reliquis magis compressum, ille sui iuris factus dilatabitur, globulos circumiectos quaqua versus impellendo ad compressionem in illos effundendo, qui ulterius alios impellent, ut globuli procul dissiti aliquantillum compressionem sentiant. Atque hac ratione sonus in alia loca transfertur. Cum autem motus, quo globulus ille se expandit, postquam in æqualem cum ceteris statum redierit subito cohiberi nequeat, nimium is extendetur; unde a reliquis rursus comprimetur, denuo tamen nimis ; ut itaque motu tremulo unusquisque ab illo primo non nimis dissitus globulus modo se dilatet , modo se rursus contrahat. Iste autem tremor globulorum aëris in instante cessare debet ob globulorum infinite exiguam magnitudinem, & inde dependens infinite breve unius oscillationis tempus, edendæ igitur essent ab huiusmodi globulo tempore finito oscillationes seu undulationes innumeræ, quod vero ob motus cuiusvis globuli continuam diminutionem fieri nequit. Quum autem ad sensum in nobis excitandum tempus requiratur finitum, in isto aëris motu tremulo sonus consistere nequit.

§ 4. Tum demum oritur sonus, cum idem globulus a vi aliena, intervallis interpositis finitis, crebriores patitur compressiones; requiritur scilicet ad sonum excitandum, ut idem globulus alternatim contrahatur atque relaxetur, verum tempora harum oscillationum non infinite parva esse debent, sed finita, ut numeros vibrationum seu oscillationum illarum data tempore determinari queat; numerus scilicet pulsuum in auris organum dato tempore finito illidentium tantus esse debet, ut numeris exprimi possit.

§ 5. Cognito iam tempore, in quo sonus consistit, facile erit explicare sonorum diversitates, hic nonnisi primarias adducam. Distribuitur vulgo sonus in magnum & parvum. Magnus est vel vehemens, cum compressiones globulorum æreorum sunt validiores, sonus vero debilis vel parvus est, cum compressiones illæ debiliores sunt. Quum sonus globulo tremulo facto propagetur communicatione compressionis cum globulis undequaque circumpositis, horum autem numerus crescat in ratione duplicata distantiarum a loco originis, decrescet soni vehementia in ratione distanarium duplicata inversa, ni forte sonus aliunde augmenta accipiat.

§ 6. Maximi momenti soni distinctio est in gravem atque acutum. Gravis est, cum vibrationes globulorum aëriorum tardius se invicem insequuntur, seu cum dato tempore rariores eduntur undulationes. Acutus autem est sonus cuius vibrationes breviores interpolitas habent morulas, ut adeo plures eodem tempore peraguntur oscillationes. Et hinc soni, respectu gravis & acuti, sunt inter se in ratione numeri oscillationum dato tempore factarum.

§ 7. Sonus etiam est vel simplex vel compositus, Simples sonus est, cuius vibrationes æqualiter inter se sunt distances æquales fortes. Compositus constat pluribus sonis simplicibus simul sonantibus, hic constituit vel consonantiam, vel dissonantiam. Consonantia percipitur sonis simplicibus componentibus rationem servantibus simpliciorem v. g. duplam ut in diapason, vel sesquialteram ut in diapente &c. Dissonantiæ autem sunt, cum ratio sonorum componentium magis est abstrusa v. g. superbipartiens septimas, quemadmodum in tritono.

§ 8. Contemplemur iam soni propagationem aliquantum attentius, id quod non incongrue fiet. id ex Theoria supra jacta computetur spatium, quod sonus tempore dato pervadere potest, v. g. minuto horæ secundo, observatum enim est sonos omnes, sive magnos sive parvos, sive graves sive acutos, eodem tempore per datum spatium ferri, nec non eos perpetuo eadem velocitate promoveri. Ut illud præstetur, quærendum est, quanto tempore globulus æreus compressus compressionem ad datam distantiam protrudere queat. Id quod ex regulis communicationis motus & contemplatione naturæ æris haud difficulter erui potest; ipsum quidem inveniendi modum, ut evitem iconismos; omitto, quod autem inde resultat, appono.

§ 9 Sit (ut rem generaliter complectar) gravitas merurii specifica ad æris gravitatem ut *n* ad 1, altitudo mercurii in barometro = *k*, longitudino penduli = *f*, secundum cuius oscillationes tempus, quo sonus per intervallum *a* transmittitur, dimetiri lubet. Hisce factis denominationibus, ego invenio, quod tempus unius oscillationis penduli, *f*, se habeat ad tempus propagationis soni per intervallum *a* ut 1 ad $\frac{a}{4\sqrt{nkf}}$.

§ 10. Si *a* et *k* determinentur in scrupulis, loco *f* autem ponatur 3166, indigitabit hic valor $\frac{a}{4\sqrt{3166nk}}$, quot minutis sucundis sonun per intervallum a propagari debeat. Est enim longitudo penduli singulis minutis secundis oscillantis scrup. 3166. Cum itaque distantia *a* absolvatur tempore $\frac{a}{4\sqrt{3166nk}}$, erit distantia, ad quam sonus uno minuto secundo diffunditur scrup. $4\sqrt{3166nk}$.

§ 11. Unde haec fluunt consectaria. Manente *nk* eodem celeritas soni eadem quoque erit adeoque, si fuerint densitates aëris elasticitatibus proportionales, soni eadem celeritate provehentur, scilicet in aëre quam maxime compresso sonus ad sensum non celerius quam in aëre maxime rarefacto promovetur. Et hinc sonun in summis montibus eadem velocitate progredidebet, qu in imis vallibus, nisi aliae causae accesserint mox exponendae.

§ 12. Crescente facto nk soni celeritas augeri debet. Densitate ergo aëris manente vel minuta, elaterio autem aucto soni celeritas maior erit; sin vero e contrario aëris densitas crescat, elaterio manente vel minuto, sonus retardabitur. Atque hinc colligitur, cum aëris tellurem cingentis et pondus seu densitas et vis elastica variis obnoxia sit mutationibus, soni velocitatem subinde quoque variari. Maxima ergo soni celeritas erit maximo calore coeloque sudo seu accuratius liquoribus in barometro et thermometro ad summam altitudinem elevatis. Acerbissimo vero frigore et saevissima tempestate celeritas soni minima esse debet, id quod evenit liquoribus in barometris et thermometris in infimis locis existentibus.

§ 13. Maxima ergo soni celeritas reperietur, si ponatur loco *n* 12000 et loco *k* 2460 scrup., ut adeo spatium uno secundo a sono percursum reperiatur scrup. $4\sqrt{3166.12000.2460} = 1222800$, i. e. sonus maxima celeritate pervadere debet secundum istam meam Theoriam intervallo minuti secundi 1222 pedes Rhenanos. Minima vero soni celeritas habebitur ponendo pro *n* 10000 et pro *k* 2260, ut adeo spatium secundo emensum sit scrupulorum

$$4\sqrt{3166.10000.2260} = 1069600,$$

seu 1069 pedum. Distantia ergo, ad quam sonus secundo dispergi debet, continetur inter hos limites 1222 et 1069 ped.

§ 14. Si ista cum experientia conferantur, egregie cum ea consentire reperientur, id quod meam methodum confirmabit. Observarunt enim FLAMSTEDIUS [J. Flamsteed (1646 - 1719) in observatorio ab ea a. 1675 in oppido Greenwich instituto una cum Ed. Halley (1656 - 1742) soni celeratem in aëre determinaverat.] et DERHAMIUS [W. Derham (1657 - 1735), Experimenta et observationes de soni motu aliisque ad id attinentibus, Philosophical Transactions (London) 26, 1708, no. 313, p. 1] accuratissime institutis experimentis sonum tempore minuti secundi percurrere 1108 pedes, qui numerus fere medium tenet inter limites inventos. Si iam condideremus, quae NEUTONUS [I. Newton, *Philosophiae naturalis principia mathematica* lib. II prop. 50, scholium; editio secunda, Cantabrigiae 1713, p. 342 - 344] hac de re habet Phil. lib. II sectione VIII, invenit ille pro distantia, quam sonus minuto secundo percurrit (ad nostrum loquendi modum eius ratiocinio reducto) scrup. Rhenani $\frac{p}{d}\sqrt{3166nk}$ denotante $\frac{d}{p}$ rationem diametri ad peripheriam, i. e. quam proxime 7 : 22. Est itaque eius expressio nostra minor, si quidem NEUTONUS $\sqrt{3166}nk$ ducat in $3\frac{1}{7}$, ego autem loco huius numeri adhibeam 4.

§ 15. Hinc ergo mirum non est, quod acutissimus NEUTONUS nimis exiguam inveniat distantiam, ad quam sonus secundo minutio pertingit; maiorem eam non determinat quam 947 ped., quae sane ingens est discrepantia ab illa distantia, quae experimentis erat inventa; quod autem ad confirmationem methodi affert, tribuendo istam discrepantiam impuritati aëris, mera est tergiversatio. Utcumque enim aër vaporibus sit infectus, vis eius elastica aequalis semper est ponderi atmosphaerico pondusque aëris inde ad sensum quoque non mutator. His vero obtinentibus soni celeritas mutationem ullam perpeti non potest. Nec magnitudo molecularum aërearum quicquam ad rem facit.

CAPUT II

DE PRODUCTIONE SONI

§ 16. Ad producendum sonum requiritur, ut aër eo, quem capite praecedente exposui, modo tremulus reddatur, scilicet ut globuli aëris habeant contractiones atque expansiones finitio tempore a se invicem separatas. Huiusmodi tremulum motum aëris triplici modo diverso, imprimis ex triplici sonorum genere concludere potui. Quocirca isto in capite de tribus diversis sonum producendi modis verba erunt facienda. Refero autem ad genus sonorum primum sonus chordarum, tympanorum, campanarum, instrumentorum lingulis instructorum, etc., omnes scilicet sonos, qui originem suam debent corpori solido contremiscenti. Ad secundum genus referendi sunt soni tonitrus, bombardarum atque virgarum et quorumvis corporum vehementius commotorum, omnes nimirum soni orti a subitanea restitutione aëris compressi, ut et validiore percussione aëris. Tertio generi autem annumero sonos tibiarum, quorum naturam, cum nemo hactenus quicquam solidi hac de re dederit, diligentius expendam.

§ 17. Ad primum sonorum genus hactenus omnes, quantum scio, cunctos plane sonos referebant arbitrabanturque nullum sonum nisi a corpore solido contremiscente exoriri posse; falsitas autem huius sentientiae mox ob oculos ponetur, cum duos reliquos sonum producendi modos explicaturus ero. Nunc autem modus, quo soni excitantur, primus accuratius perpendendus est. Verum in praesentiam nonnisi, cum reliqua facile eo reduci queant chordas, quomodo et quales edant sonos, contemplabor. Ad quod exactius obtinendum chordas ut pondere tensas considero, cum alias circumvolutione circa columnam extenduntur, ut accurate vim chordam extendentem metiri liceat.

§ 18. Ante omnia observandum est chordas easdem aequales ratione gravis et acuti edere sonos, quacunque vi pulsentur, licet ingens esse possit discrepantia ratione vehementiae et debilitatis; soni enim vehementia est ut celeritas, qua chorda aërem percutit sonique aeque fortes sunt, si aër eadem vi impellitur. Quocirca, cum soni musici tam graves quam acuti aequaliter fortes esse debeant, ut dulcis harmonia habeatur, in fabricatione instrumentorum musicorum probe in id incumbendum est, ut soni ratione fortitudinis seu roboris aequales edantur, ad quod obtinendum sequentes regulae, que quidem a recentioribus artificibus ex multiplici praxi crasse iam erutae sunt, quarum vero veritas ex sequentibus perfecte patibit, diligenter observandae sunt.

I. *Chordarum longitudines sint in reciproca ratione sonorum, i. e. numeri vibrationum dato tempore edendarum.*

II. *Chordarum crassitudines seu sectones transversae sint quoque in ratione reciproca sonorum, si scilicet eiusdem materiae chordae in usum vocentur; sin vero minus, tum cum ratione crassitudinis densitatis ratio inversa coniungenda est.*

Ad instrumenta tibiis instructa regulae istae quoque applicari possunt, sumendo ibi loco longitudinis chordarum longitudinem seu altitudinem tibiarum et loco crassitudinis chordarum amplitudinem tibiarum internam.

§ 19. Quando chorda oscillatur, aëreos globulos ferit, qui, cum in instanti cedere nequeant, comprimuntur;durante autem oscillatorio motu globuli aërei continuo novas patiuntur compressiones, unde sonus oritur. Aër itaque toties ferit aurem seu tympanum auris, quoties chorda redierit. Adeoque reperiri poterit numerus percussionum uniuscuiusque soni dato tempore aurem invectarum, investigando scilicet numerum oscillationum chordae sonum illum edentis eadem tempore. Mea solutio autem, quae cum solutionibus Cl. Cl. D. D. JOH. BERNOULLI [JOH. BERNOULLI (1667 - 1748), *Meditationes de chordis vibrantious,* Commt. acad. sc.Petrop. 8 (1728), 1732, p. 13; Opera omnia, Lausannae et Genevae 1742, t. III, p. 198] atque BROOK TAYLOR [(1685 - 1731), *Methodus incrementorum directa et inversa,* Londini 1715, p. 26] exacte conspirat, est haec.

§ 20. Sit pondus chordam tendens = *p*, pondus chordae = *q* et longitudo chordae = *a*, ex quibus tribus datis numerus vibrationum dato tempore inveniendus proponitur. Invenio ego pro numero oscillationum uno minuto secundo editarum

$$\frac{22}{7}\sqrt{\frac{3166\,p}{aq}},$$

ubi *a* determinari debet in scrupulis. Huic numero cum sonus proportionalis sit, soni a diversis chordis editi erunt inter se ut $\sqrt{\frac{p}{aq}}$, i. e. soni sunt in ratione subduplicata composita ex ponderis tendentis directa et reciproca longitudinis et ponderis chordae. Plura hinc magis particularia consectaria non deduco, sed inquiram in naturam sonorum cognitorum atque numeros vibrationum illis respondentes ex experimento a me hunc in finem instituto.

§ 21. Sumsai chordam aëream ex eius crassitiei genere, quod No. 18 indigitatur, longitudinis 980 scrup., quae ponderabat $\frac{49}{175000}$ libr., eamque tetendi pondere $\frac{11}{4}$ libr., qua pulsa deprehendi sonum convenisse cum eo, in instrumento chorali modo, ut aiunt, adaptoto,qui musicis audit $\overline{\overline{ds}}$. Hinc ergo licebat supputare, quoties iste sonus et proin quivis alius dato tempore auditus organum feriat; in data enim formula generali, si substituatur loco *a* 980, loco *p* $\frac{11}{4}$ libr., loco *q* $\frac{49}{175000}$, sonus $\overline{\overline{ds}}$ minuto secundo habere invenietur vibrationes 559 et cum sit $\overline{\overline{ds}}$ ad $\bar{c}$ ut 6 ad 5, habebit sonus $\bar{c}$ 466 et proinde infimum C 116 vibrationes.

§ 22. Ad hunc modum productionis soni quoque referendi sunt soni a lingulis seu laminis elasticis tubo insertis inflatione venti editi, quanquam quoque ex parte ad tertium modum pertineant; ad utrumque enim pertinent. Huiusmodi machinas videre est in variis organis pneumaticis tubarum, buccinarum, ut et hominum cantus imitantibus, quae instrumenta omnia inflari debent ad id, ut sonum edant. Ventus, transitum sibi quaerendo, aperit lingulam instar valvulae, nimium autem eam aperiendo tendit, ut rursus valvula retrocedat, in priorem statum tendendo quae proinde denuo aperitur, ut ita motu tremulo aërem transeuntem inficiat. Necesse quidem esset, ut vento aequabiliter flante valvula tandem quiesceret sonusque cessaret; ad hoc autem cavendum ventus ipse, praeterquam quod per se, dum e follibus propellitur, non aequabiliter orificia machinarum impetat, ope valvulae tubo ventum deferenti insertae tremulus redditur.

§ 23. Eodem plane modo vox humana generatur; lingulae enim locum in organo loquelae obtinet epiglottis, quae tremula redditur ab aëre per arteriam asperam ascendente. Tremulus iste motus aëris egredientis cum in capite arteriae asperae tum in cavitate oris variis modis immutatur, ex qup vox gravis atque acuta inflectitur variique vocales formantur, qui soni ope labiorum, linguae atque faucium consonantibus exornantur. Quin et naso, cum aëri ab epiglottide tremulo reddito exitus per nasum quoque pateat, varii soni respectu gravis et acuti edi possunt, qui autem a sonis oris in eo differunt, quod nec vocalibus distincte interstingui nec consonantibus condiri possint.

§ 24. Isti autem soni a lingulis tremulis editi, nisi in tubis confirmentur, admodum debiles essent, ut percipi vix possent, quemadmodum observare est in lamina contremiscente, ubi nil fere auribus percipitur. Mirum autem in modum soni isti intenduntur in tubis atque vox humana in ore, nec non quoad gravitatem atque aciem ingens mutatio huiusmodi sonis infertur in tubo. Verum de hisce soni intensionibus ac inflexionibus hic non est locus fusius disserere; peculiari opus esset capite, quo ista materia accuratius expenderetur, ubi explicanda quoque veniret mirifica soni in tubis stenterophonicis amplificatio, ut et doctrina de Echo pluraque alia; sed istam materiam accuratius perpendere nondum vacavit, et quae in aliorum scriptis continentur, quantum ex iis perspexi admodum confusa sunt et maximam partem falsa.

§ 25. Ad secundum sonorum classem retuli eos sonos, qui oriuntur vel notabili aëris quantitate compressa subito dimissa vel validiore aëris percussione. Posteriori modo aër quoque comprimitur, cum corpori verberanti loci cessionem denegare conetur, unde aër iterum sibi relictus sese expandit. Causa itaque sonorum ad secundam classem perinentium est restitutio aëris antea compressi. Istam vero restitutionem sonum generare debere exinde patet, quos aër compressus sese dilatando nimium expandat et proinde iterum contrahatur et ita porro, qua undatione aëris sit, ut quoque minimi aërei globuli, quippe qui aëris massam componunt, motum istum tremulum participent atque per consequens sonum producant; ubi notandum, ut, quo maior aëris copia sit compressa, eo graviorem edi sonum, ac quo minor ea sit, eo acutiorem. Huiusmodi vero soni diu durare nequeunt, sed e vestigio cessare debent, quia aër, motum in longe dissita loca diffundendo, motum tremulum statim amittit.

§ 26. Omnes ergo causae, quae aërem vel iam compressum dimittere vel vero comprimere, ita tamen, ut se statim relaxare possit, valent, ad sonum producendum aptae sunt. Quocirca omnes velociores corporum motiones in aëre sonum edere debent; motis emim corporibus aër ob propriam inertiam liberrime cedere nescius comprimitur rursusque se dilatando motum tremulum globulis aëreis minimis inducit, ad sonum producendum aptum. Hinc fluunt soni vehementius vibratarum virgarum, ut et omnium velocius motorum corporum. Soni quoque flatuum atque ventorum ex hoc fonte originem ducunt; aër enim praecedens ab insequaente etiam comprimitur, quemadmodum a corpore duro.

§ 27. Soni, qui oriuntur aëre iam compresso subito relaxato, facile validissimi sunt tormentorum atque tonitrui. Horum autem immensorum sonorum causam esse restitutionem aëris compressi comprobant varia experimenta pulvere pyrio atque nitro instituta, quandoquidem repertum sit aërem inibi quam maxime esse condensatum, cui inflammatione nitri viae exitum ei praebentes adaperiuntur, ut maximo impetu erumpere queat. Cum autem ex materia nitrosa et pulvis pyrius, et multi vapores nubes constituentes constent, mirum non est, ignem ista materia concipiente, tam stupendos inde resultare sonos.

§ 28. Tertium sonorum genus constituunt soni tibiarum. Horum sonorum explicatio quovis tempore naturae scrutatores mirum in modum torsit. Plerique existimavere inflatione tibiarum minimas internae superficiei particulas impelli atque ad motum tremulum sollicitari, ut ita interna tibiarum superficies inflatione tremens reddatur faciatque oscillationes cum aëre communicandas; sed quomodo ista explicatio cum legibus naturae et motus consistant, ipsi inquirant. Ego sane concipere nequeo, quomodo duntaxat differentia sonorum tibiarum diversae altitudinis non mutata earum amplitudine exinde exponi possit; quare enim particulae internae, si unquam motum concipiant, pro diversa altitudine tuborum diversimode oscillari debeant, videre non possum, brevi vix arbitror vel unicum tibiis institutum experimentum ex ista theoria explicari posse.

§ 29. Ut autem veram huius rei explicationem nanciscar, primum tibiarum structura, et quaenam in illis, dum inflantur, eveniant, accuratius perpendenda sunt. Sunt tibiae seu fistulae tubi, quibus infra iunctum est peristomium cavum aëri recipiendo aptum, quod versus tubum in crenam desinit directe oppositam lateri cuidam internae tubi superficiei, eum in finem, ut aër peristomio inflatus per fissuram in tubum secundum eius longitudinem irruat, rependo super superficie tubi interna; si fistula hoc modo constructa sit, sonum inflata edit, uti facile patet, si quivis tubus peristomo destitutus ita quoque infletur, ut aër in tubum super superficie interna repat, tum enim sonum etiam sicut tibiia edit. Interna autem tubi superficies dura laevisque esse debet, ne aëri irruenti cedere, nec illi i tubo contento locum sese expandendi dare possit, quocirca fistulae ex tubis ad latera clausis atque rigidis interneque non scabris parari debent.

§ 30. Videamus iam, quid in fistula, dum inflatur, eveniat, quod aërem tremulum reddere possit, seu qua ratione aër dicto modo in tubum reptans aërem in tubo contentum tremulum reddere queat. Manifestum est, aëre in tibiam ingrediente, comprehensum in illa aërem secundum longitudinem compressum iri; quo cum sese rursus expandat, verum nimis, comprimetur rursus a pondere premente atmosphaerico, ut ita motus tremulus in tubo producatur, qui tremor causa soni proxima est. Atque sic detecta est causa sone tibiarum vera, cuius autem realitas et veritas abundius innotescet; penitius autem prius considerandus est modus, quo motus ille tremulus prodicitur.

§ 31. Columna aërea in fistula sese secundum amplitudinem expandendo ac contrahendo more chordarum undat atque idcirco istam columnam considerabo ut fasciculum chordarum aërearum tensarum a pondere atmosphaerico. Licet autem pondera chordas tendentia eas divellere conentur, hic vero directe contrarium obtineat, cum columna illa aëres a pondere atmosphaerico coarctetur, nihilo tamen minus analogia legitima est; eundem enim pondus atmosphaerae exerit in columnam aëream effectum, quem pondera tendentia in chordas, si quidem utrinque, ibi pondus atmosphaerae, hic pondera tendentia, chordas nimium extensa rursus comprimunt. Loco autem, quod chordae ordinariae unico in puncto pulsatae sonum edant, chordae illae aëreae, cum pulsu unico in puncto facto totae ob discontinuationem partium contremiscere nequeant, simul per integram longitudinem pulsari debent, id quod sit in tibiis, ubi aër irrepens per totam longitudinem aërem in tubo contentum comprimit.
§ 32. Ad inveniendum itaque oscillationes aëris in tibiis seu ad determinandum numerum undulationum cuiusvis fistulae res eo redit, ut aër in tubo habeatur pro chorda tensa utrinque a pondere atmosphaerico; atque hoc intuitu oscillationes reperientur ex § 21. Sit scilicet longitudo chordae, i. e. tubi, = a; erit p (pondus chordam tendens) = ponderi atmosphaerico seu columnae mercurii in barometro, i. e. ad minimum 2260 ad maximum 2460 scrup.; q (pondus chordae) autem erit = ponderi aëris in tubo. Sit rursus ratio gravitatum specificarum mercurii et aëris = n : 1 et k altitudo mercurii in barometro. Erit p ad q in ratione composita n ad 1 et k ad a seu erit p ad q ut nk ad a.

§ 33. Posita iam in expressione § 20 loco p et q eorum proportionalibus nk et a reperietur pro numero vibrationum uno minuto secundo editarum iste valor $\frac{22}{7a}\sqrt{3166nk}$. Unde patet, cum n et k pro diversa tempestate mutentur, sonum quoque mutari, scilicet crescente nk ille sit acutior, decrescente autem nk sit gravior. Erunt ergo soni tibiarum acutissimi maximo calore et aëre ponderosissimo, gravissimi autem maximo frigore aëreque levissimo. Quae differentia sonorum egregie quoque observatur a musicis atque organariis. Quia autem ista mutatio in omnibus tibiis aequabiliter locum habet, harmonia non mutatur.

§ 34. Ut numeris exprimatur numerus oscillationum tibiarum, ponantur, si desideretur sonus liquoribus in barometris et thermometris ad maximam altitudinem consistentibus, pro n 12000 atque pro k 2460 scrup.; reperietur numerus oscillationum minuto secundo editarum in tibia a scrup. longa

$$= \frac{22}{7a}\sqrt{3166.1200.2460} = \frac{960771}{a}.$$

Maximum vero frigus saevissimamque tempestatem indigitantibus liquoribus barometrorum ac thermometrorum, ponendo pro n 10000 et pro k 2260 scrup., reperietur numerus oscillationum minuto secundo editarum aequalis

$$= \frac{22}{7a}\sqrt{3166.1000.2260} = \frac{840714}{a}.$$

§ 35. Hinc ergo ratio patet, quare tibiae edant sonos longitudinibus reciproce proportionales et quare amplitudino ad rem nihil faciat, immo etiam, quare materia tibiarum nullam sono diversitatem inferat. Quanquam autem nec amplitudo nec materia tuborum quicquam ad soni gravitatem seu aciem immutandam conferat, tamen haec ad affectionem atque suavitatem multum contribuit; illa autem amplitudo tubi fundamentum est vis soni, ut, quo amplior sit tubus, eo fortior quoque sit sonus; amplitido scilicet in tibiis analogia est crassitiei in chordis. Et quemadmodum non quaevis chorda ad quemvis sonum edendum apta est, verum ad graviorem crassior requiratur, ita etiam in tibiis istud locum obtinet, ut, quo altiores eae sint, eo maior amplitudo requiratur.

§ 36. Cum ratio sonorum tonum integrum a se invicem distantium sit ut 8 ad 9, eadem tibia pro diversa aëris conditione sonos ad summum plus quam tono discrepantes edere potest. Sit tibia 4 pedes longa, quae adhibetur ad sonum C in chorali modo edendum; et erunt eius vibrationes secundo minuto editae ad summum 240 atque ad minimum 210; id quos satis convenit cum iis, quae antea invenimus, ubi de chordis actum fuit; ibi enim numerus oscillationum soni C repertus fuit 116, unde patet sonum tibiarum C esse quam proxime octava superiorem eo sono C in chordis. Quod autem tota octava distent, vulgo tamen pro aequalibus habeantur, mirandum non est, cum de sonis heterogeneis difficillimum sit iudicare, an unisoni sint; num vero octava vel prorsus duabus aut pluribus octavis distent, sufficit, quod unica saltem octava eos sonos discrepantes repererim, id quod meam theoriam satis confirmat.

§ 37. Quae hucusque de sono fistularum allata sunt, intelligi debent de fistulis cylindricis apertis, ubi aëri inflato supra in supremo tubo exitus patet. Cum autem tubus supra tectus fuerit, aer inflatus supra egredi nequit, ideoque eum retrogredi necesse est, ut ad orificium inferius emergat. Unde fit, ut quasi ad operculum supra tubum reflectat alterumque tantum spatii absolvat, antequam exitus ei patet. Et per consequens aër in tubo tanquam chorda duplo longior est consideranda, quippe chordae ut complicatae concipi debent. Unde colligitur fistulam tectam sonum edere eundem cum aperta duplo longiore, seu edet sonum octava graviorem aperta.

Quales autem edant sonos fistulae non ubivis eiusdem amplitudinis, i. e. vel convergentes vel divergentes, item fistulae supra ex parte saltem tectae, Clar. Competitoribus examinandum propono.

ANNEXA.

1. *Systema corporis et animae harmoniae praestabilitae, quo actiones corporis et animae minime a se invicem dependere asseruntur, veritati non consentaneum est.*

2. *Vis attractiva NEUTONI aptissimus cuncta corporum coelestium phaenomena explicandi modus est. Et idea extra dubium positum esse credo corpora omnia ex sua natura se mutuo trahere.*

3. *Posito, centrum telluris (quod autem a vero longe est alienum) quaevis corpora attrahere i reciproca ratione duplicata distantiarum, terramque per centrum esse perforatam. Quaeritur,lapide per foramen demisso, quid eveniret, cum ad centrum perveniret, utrum ibi vel quiescens permansurus vel protinus ultra centrum progressurus, an vero a vestigio ad nos ex centro reversurus esset. Postremum ego affirmo.*

4. *Vires corporum motorum sunt in ratione composita ex simplici massarum et duplicata celeritatum.*

5. *Globus super plano inclinato rotando descendens, abstrahendo ab omni resistentia ea celeritate quam ex eadem altitudine perpendiculariter cadendo acquireret, multo minorem nanciscetur. Erit enim illa ad istam normaliter cadendo acquisitam ut* $\sqrt{5}$ *ad* $\sqrt{7}$.

6. Mali in navibus nimis alti esse non debent, ne venti vis navem subvertat. Ponamus autem malum velis instructum nimis altum, ut scilicet navis vento data certo prosterneretur. Dico, latiora vela si appenderentur, ut vis navem propulsans fortior esset, minus fore navem subversioni obnoxiam. Et semper, quantumvis malus altus sit, latitudo velorum eiusque augeri poterit, ut nequidem vehementissimus ventus navi damnum afferre possit.

TANTUM.