ment of the knowledge of objects which relate only to the earth.

5th September 1761.

Letter XLVI.—Knowledge of the Longitude, from a Calculation of the Direction, and of the Space passed through.

I now proceed to the longitude; and remark, that on taking a departure, whether by land or water, from a known place, it would be easy to ascertain the spot we had reached, did we know exactly the length of the road, and the direction which we pursued. This might, in such a case, be effected even without the aid of astronomy; and this obliges me to enter into a more particular detail on the subject.

We measure the length of a road by feet; we know how many feet go to a mile, and how many miles go to an arch of one degree upon the globe: thus we are enabled to express in degrees the distance we have travelled.

As to the route or direction in which we travel, it is necessary accurately to know the position of the meridian at every place where we are. As the meridian proceeds in one direction toward the north pole, and in the other toward the south, you have only to draw, on the horizon of the spot where you are, a straight line from north to south, which is called the *meridian line* of that place. All possible care must be taken to trace this meridian line very accurately, and here the heavens must again perform the office of a guide.

You know it is mid-day when the sun is at his greatest elevation above the horizon; or, which is the same thing, the direction of the sun is then exactly south, and the shadow of a staff fixed perpendicularly on a horizontal plane will fall, at that instant, precisely northward. Hence it is easy to comprehend, how an observation of the sun may furnish us with the means of accurately tracing a meridian line, wherever we may be.

Having traced a meridian, every other direction is very easily determined.

Let the straight line NS (Plate IV. *Fig.* 14.) be the meridian, one of the extremities N being directed toward the north, and the other S toward the south. With this meridian let there be drawn at right angles the straight line EW, whose extremity E shall be directed toward the east, and the other extremity W toward the west. Having divided the circle into 16 equal parts, we shall have so many different directions, denominated according to the letters affixed to them; and in case of not pursuing a direction which exactly corresponds with some one of the sixteen, the angle must be marked which that deviating line of direction makes with the meridian NS, or with EW, which is perpendicular to it.

It is thus we are enabled to determine exactly the direction which we pursue in travelling; and so long as we are assured of the length of the way, and of the direction pursued, it will be very easy to ascertain the true place at which we have arrived, and to indicate both its longitude and latitude. We employ for this purpose an accurate map, which contains the point of departure, and that which we have reached; and by means of the scale, which gives the quantity of miles or leagues that go to a degree, it is easy to trace, on such map, the track pursued and completed.

Figure 15. of Plate IV. represents a map, on which are marked from left to right the degrees of longitude, and those of latitude from top to bottom; it is likewise visible on the face of it, that the meridians con-

verge as they approach toward the north, and retire from each other toward the south, as is the actual case on the globe.

This map contains part of the surface of the earth, from the 53d degree of north latitude to the 59th degree; and from the 13th degree of longitude to the 26th.

Suppose, then, I take my departure from the place L, the longitude of which is 16°, and the latitude 57° 20′, and that I proceed in the direction ESE, and have travelled a space of 345 English miles. In order to determine the longitude and latitude of the place I have reached, I draw from the place L the straight line LM, making with the meridian an angle of 67° 30′, the same angle which the direction ESE in the preceding figure makes with NS. Then on that line I take, according to the scale marked on the chart, LM equal to 345 English miles, and the point M shall be the place which I have reached.

I have then only to compare this place with the meridians and parallels traced on the map, and I find that its longitude is 24° nearly; and on measuring more exactly the part of the degree to be added to the 24th degree, I find the longitude of the point M to be 24° 4′. As to the latitude, I observe it to be between the 55th and 56th degree, and by an easy computation I find it to be 55° 25′; so that the latitude of the place M, which I have reached, is 55° 25′, and its longitude 24° 4′.

It has here been supposed that I have invariably pursued the same direction, ESE, from first to last; but if I have from time to time deviated from that direction, I have only to perform the same operation on each deviation, to find the place where I then was; from this I take a fresh departure, and trace my direction till another deviation takes place; and so on, till I reach my object. By these means it is always in my power, whether travelling by sea or land, to ascertain the place I have reached; provided I know exactly, through my whole progress, the direction I pursue, and measure with equal accuracy the length of the way.

We might in this case dispense even with the assistance of astronomy, unless we had occasion for it accurately to determine our direction, or the angle which it makes with the meridian; but the magnetic needle or compass may, in many cases, supply this want.

You must be sensible, however, that it is possible to make a very considerable mistake, both in the computation of the direction, and of the length of the way, especially in very long voyages. How often is it necessary to change the direction in travelling even from hence to Magdeburg? and how is it possible to measure exactly the length of the way? But when we travel by land we are not reduced to this expedient; for we are enabled to measure by geometrical experiments the distance of places, and the angles which the distances make with the meridian of every place; and thus we can determine, with tolerable accuracy, the true situation of all places.

8th September 1761.

Letter XLVII.—Continuation. Defects of this Method.

A method of observing the direction pursued, and the length of the course, seems to be of singular utility in sea voyages, because there we are not under the necessity of deviating from the direction every moment, as in travelling by land; for, with the same wind, we can proceed in the same direction.

Pilots are accordingly very attentive in exactly observing the course of the vessel, and in measuring the progress she has made. They keep an accurate journal of all these observations, at the close of every day, nay still more frequently; they trace on their sea-charts the progress they have made, and thus are enabled to mark on the charts, for every period of time, the point where they are, and of which they consequently know the latitude and longitude. Accordingly, so long as the course is regular, and the vessel is not agitated by a tempest, good pilots are seldom mistaken; but when they are in doubt, they have recourse to astronomical observations, from which they discover the elevation of the pole; and this being always equal to the latitude of the place where they are, they compare it with that which they have marked on the chart, conformably to the computation of their progress. If these are found to coincide, their computation is just; if they discover a difference, they conclude with certainty that some error has been committed in the computation of the distance and of the course; in that case they re-examine both the one and the other more carefully, and endeavour to apply the necessary corrections, in order to make the computation agree with the observation of the height of the pole, or of the latitude, which is equal to it.

This precaution may be sufficient in short voyages, as the errors committed can in these be of no great importance; but in very long voyages, these slight mistakes may accumulate to such a degree, that at last a very gross mistake may be committed, and the place where the vessel actually is may differ considerably from what it was supposed to be on the chart.

I have hitherto gone on the supposition that the voyage proceeded quietly; but should a storm arise, during which the vessel is subjected to the rudest concussions of wind and waves, it is evident that the

computation of distance and course is entirely deranged, and that it is impossible to trace on the chart the progress she has made.

It would be very easy, after this derangement, to ascertain by astronomical observations the latitude of the ship's place; but this would determine only the parallel of that place, and it would remain totally uncertain at what point of the parallel she actually was.

It is necessary, therefore, to discover likewise the longitude of the place, which shows us the meridian under which it is situated; and then the intersection of that meridian with the parallel found, will give the vessel's true place. This will make you sensible of what importance it is to assist mariners, in discovering likewise the longitude of the place where they are.

This necessity is imposed not only from the consideration of the tempests to which navigation is liable; for it is possible, supposing the voyage to proceed ever so quietly, to be grossly mistaken in the computation of both course and distance. Could we suppose the sea to be at rest, it might be possible to invent various methods of ascertaining, with tolerable exactness, the way which the vessel has made; but there are rapid currents in many places of the ocean, which have the resemblance of a river running in a certain direction. Thus it is observed, that the Atlantic ocean has a perpetual current into the Mediterranean sea, through the straits of Gibraltar; and that the ocean between Africa and America has a very considerable current from east to west, so that a voyage to America is performed in much less time than a voyage from America to Europe.

Were such currents constant and well known, we should have considerable assistance toward forming our calculations; but it has been observed, that they are sometimes more, sometimes less rapid, and that they frequently change their direction; which deranges the calculations of the most skilful navigator to such a degree, that it is no longer safe to trust them. We have but too many fatal instances of ships dashed on concealed rocks, and lost, because these were computed to be still at a considerable distance. It was afterwards discovered, when too late, that these calamities had been occasioned by the currents of the ocean, which deranged the calculations of navigators.

In fact, when the ocean has a current which makes it flow like a river, following a certain direction, vessels caught in it are carried away imperceptibly. In a river we clearly perceive that the current is carrying us along, by observing the banks or the bottom; but at sea no land is visible, and the depth is too great to admit of our making any observation from the bottom. At sea, then, it is impossible to discern the currents; and hence so many dreadful mistakes respecting both course and distance. Whether, therefore, we take tempests into the account or not, we are always under the necessity of falling on other methods of ascertaining the longitude of the places where we may arrive; and of the various methods hitherto employed for acquiring this knowledge of the longitude, I now proceed to inform you.

12*th September* 1761.

Letter XLVIII.—Second Method of determining the Longitude, by Means of an exact Time-Piece.

A very sure method of finding the longitude, would be a clock, watch, or pendulum, so perfect, that is to say, which should always go so equally,

and so exactly, that no concussion should be able to affect its motion.

Supposing such a time-piece constructed, let us see in what manner, by means of it, we should be enabled to solve the problem of the longitude. We must return, for this purpose, to the consideration of meridians, which we are to conceive to be drawn through every place on the surface of the globe.

You know that the sun seems to describe every day a circle round the earth, and that, of consequence, he passes successively over all the meridians in the space of twenty-four hours.

Now, the sun is said to pass *over*, or *through* a given meridian, if a straight line drawn from the sun to the centre of the earth C (Plate IV. *Fig.* 12.), pass precisely through that meridian. If, therefore, in the present case, the line drawn from the sun to the centre of the earth pass through the meridian BLMA, we would say that the sun was in that meridian, and then it would be mid-day to all the places situated under this meridian; but under every other, it would not be mid-day at that precise instant; it would there be before noon or after it every where else.

If the meridian BNA is situated to the eastward of the meridian BMA, the sun, in making his circuit from east to west, must pass over the meridian BNA before he reaches the meridian BMA; consequently, it will be mid-day under the meridian BNA earlier than under the meridian BMA: when, therefore, it shall be mid-day under this last meridian, mid-day under every other meridian to the eastward will be already past, or it will be afternoon with them. On the contrary, it will be still forenoon under every meridian, say BDA, situated to the westward, as the sun cannot reach it till he has passed over the meridian BMA.

And as the motion of the sun is regular and uniform, and he completes his circuit of the globe, that is 360 degrees, in twenty-four hours, he must every hour describe an arch of 15 degrees. When, therefore, it is noon at Berlin, and at every other place situated under the same meridian, noon will be already past under meridians situated to the eastward; and more particularly still under the meridian situated 15 degrees to the eastward of that of Berlin, it will already be one o'clock; under the meridian 30 degrees eastward, two o'clock; under that of 45 degrees, three o'clock afternoon, and so on. The contrary will take place under meridians situated to the westward of that of Berlin; when it is noon there, it will be only eleven o'clock forenoon under the meridian 15 degrees to the westward, ten o'clock under the meridian of 30, nine o'clock under the meridian of 45 degrees westward, and so on; a difference of 15 degrees between two meridians always amounting to an hour of time.

To elucidate still more clearly what has now been remarked, let us compare the two cities Berlin and Paris. As the meridian of Berlin is 11° 17′ 15″ to the eastward of that of Paris, reckoning an hour to 15 degrees, this difference of 11° 17′ 15″ will give 44 minutes and 29 seconds of time, or three quarters of an hour nearly. When, therefore, it is midday at Paris, it will be 44 minutes and 29 seconds after mid-day at Berlin; and reciprocally, when it is mid-day at Berlin, it will only be 15 minutes and 31 seconds after eleven o'clock at Paris; so that it will not be noon at this last city till 44 minutes and 29 seconds afterwards. Hence it is evident, that the clocks at Berlin should always be faster than those of Paris, and that this difference ought to be nearly 44 minutes and 29 seconds.

The difference between the meridians of Berlin and Magdeburg is nearly 1° 40′; Berlin therefore is to the eastward of Magdeburg; and this difference reduced to time gives 6 minutes and 40 seconds, which the clocks of Berlin ought to indicate more than that of Magdeburg. Consequently, if it is just now noon at Magdeburg, and the clocks there, which I suppose well regulated, point to XII., the clocks at Berlin should, at the same instant, indicate 6 minutes and 40 seconds after XII., that is, noon there is already past.

Hence you see, that in proportion as places differ in longitude, or as they are situated under different meridians, well regulated time-pieces ought not to point out the same hour at the same instant, but the difference ought to be a whole hour when that of the longitude is 15 degrees.

In employing a time-piece, then, for ascertaining the longitude of the places through which we pass, it would first be necessary to regulate it exactly at some place where we actually were. This is done by observing the instant of noon, that is, the instant when the sun passes over the meridian of that place; and the time-piece ought then to point precisely to XII. It ought afterwards to be adjusted in such a manner, that always after a revolution of 24 hours, when the sun returns to the meridian, the index, after having made two complete circuits, should again point exactly to XII. If this is carefully observed, such well regulated time-pieces will not coincide in different places, unless these be situated under one and the same meridian; but if they are situated under different meridians, that is, if there be a difference of longitude, the time indicated by the clock or watch, at the same moment, will likewise be different; at the rate of one whole hour of time for every 15 degrees of longitude.

Knowing, then, the difference of time indicated by well regulated time-pieces, at different places, and at the same instant, we are enabled exactly to compute the difference of longitude at these two places, reckoning always 15 degrees for an hour, and the fourth part of a degree for a minute.

15th September 1761.

LETTER XLIX.—CONTINUATION, AND FARTHER ELUCIDATIONS.

YOU will be less surprised at the difference of time which well regulated time-pieces must indicate under different meridians, when you recollect, that while it is noon with us, there are countries toward the east where the sun is already set, and that there are others toward the west where he is but just rising. It must therefore be already night with the one, and still morning with the other, at the same instant that it is noon with us. You know, besides, that with our antipodes, who are under the meridian diametrically opposite to ours, it is night, while it is day with us; so that our noon corresponds exactly to their midnight.

It will be an easy matter, after these elucidations, to show how an exact time-piece may assist us in discovering the difference of meridians, or that of the longitude, at different places.

Supposing me possessed of such an excellent time-piece, which, once exactly regulated, shows me every day the precise time it is at Berlin, so that whenever it is noon at Berlin, it points precisely to XII.: supposing farther, that it goes so regularly, that once adjusted, I have no farther occasion to touch it, and that its motion is not to be deranged either by the shaking of a carriage, or the agitation of a vessel on the ocean, or by any concussion whatever to which it may be exposed.

Provided thus with a time-piece of this description, I set out to travel, whether by land or by sea; perfectly assured, that go where I will, its motion will be steady and uniform, as if I had remained at Berlin: it will every day point to XII. at the very moment it is noon at Berlin, and that wherever I may happen to be. On this journey, I arrive first at Magdeburg: there I observe the sun when he passes the meridian, and this happens when he is exactly south; and it being then noon at Magdeburg, I consult my time-piece, and observe it points to 6 minutes and 40 seconds after XII.: whence I conclude, that when it is noon at Magdeburg, noon at Berlin is already past, and that the difference is 6′ 40″ of time, which correspond to 1° 40′ of distance; therefore the meridian of Magdeburg is to the westward of that of Berlin. The longitude of Berlin, therefore, being nearly 31° 7′ 15″, the longitude of Magdeburg will be 1° 40′ less, that is, it will be 29° 27′ 15″.

I thence proceed to Hamburgh, accompanied by my time-piece, which I never touch; and there observing when it is noon by the sun, for I cannot depend on the public clocks which there announce the hour, I find my time-piece already announces 13′ 33″ after XII.; so that at Berlin noon is passed 13′ 33″ when it is exactly noon at Hamburgh: hence I conclude, that the meridian of Hamburgh is 3° 23′ 15″ to the westward of that of Berlin; reckoning 15° to an hour, that is one degree for every four minutes of time: accordingly, I find that 13′ 33″ of time give 3° 23′ 15″ of distance for the difference of the meridians. The longitude of Hamburgh will be, of course, 27° 44′.

At Hamburgh I go to sea, still accompanied by my time-piece, and after a long voyage I arrive at a place where, waiting for noon, the moment of which I ascertain by observing the sun, I find that my time-piece indicates only 58′ 15″ after X.; so that then it is not yet noon at Berlin, and the difference of time is 1 hour 1′ 45″, from which I conclude, that the place at which I have arrived is to the eastward of Berlin; and as one hour gives 15 degrees, one minute of time 15′, and 45 seconds of time 11′ 15″, the difference of the meridians will therefore be 15° 26′ 15″. I find, then, that I am at a place to the eastward of Berlin, whose longitude is greater than that of Berlin by 15° 26′ 15″; now the longitude of that city being nearly 31° 7′ 15″, the longitude of the place where I am must be 46° 33′ 30″. Thus I have discovered under what meridian I now am; but I am still uncertain as to the point of the meridian. In order to ascertain this I have recourse to astronomical observations, and find the height of the pole to be precisely 41°. Knowing likewise that I am still in the northern hemisphere, as I have not passed the equator, I discover that I actually am at a place whose latitude is 41° north, and longitude 46° 33′ 30″. I take therefore my globe or maps, and trace the meridian whose longitude is 46° 33′ 30″; I look for the place whose latitude is 41°, and at the point of intersection I find I have got to the city of Constantinople, without having occasion to apply for information to any person whatever.

Thus, at whatever place of the globe I may arrive, possessed of a time-piece so exact, I am able to ascertain the longitude of it; and then an observation of the height of the pole will show me its latitude. All that remains, therefore, is to take the terrestrial globe, or a good map, and it will be easy for me to ascertain where I am, however unknown to me the country may in other respects be.

It is much to be regretted, that artists of the greatest ability have hitherto been unsuccessful in the construction of time-pieces, such as I have described, and such as the case requires. We meet with a great many very good pendulum machines, but they go regularly only when fixed in undisturbed situations; the slightest concussion is apt to derange their motion; they are therefore totally useless in long sea voyages. It is obvious that the pendulum, which regulates the motion, is incapable of resisting the shocks to which it is exposed in navigation. About ten years ago, however, an English artist pretended that he had constructed a time-piece proof against the motion of a ship at sea, and that after having tried it a long time together in a carriage on the road, it was impossible to perceive the slightest derangement; on which the inventor claimed and received part of the parliamentary reward proposed for the discovery of the longitude, and the rest was to be paid after it had been put to the proof of a long voyage. But since that time we have heard no more of it; from which it is to be presumed that this attempt too has failed, like many others which had the same object in view.*

19*th September* 1761.

Letter L.—Eclipses of the Moon, a Third Method of finding the Longitude.

From want of the exquisite time-piece, of which I have endeavoured to give you an idea, the eclipses of the moon have hitherto been considered as the

* The success of these attempts has been very great. The chronometers of Harrison, Earnshaw, Arnold, &c. in England, and those of M. Breguet in Paris, and of M. Jurgensen at Copenhagen, enable the navigator to determine his longitude at sea with an extraordinary degree of accuracy. —Ed.

most certain method of discovering the longitude; but these phenomena present themselves so rarely, that we have it not in our power to employ them so often as occasion requires.

You know that the moon is eclipsed when it passes into the shadow of the earth: it is possible, then, to observe the moment when the moon begins to enter into the shade, and when she has emerged; the one is denominated the beginning of the eclipse, and the other its end; and when both are observed, the mean time betwixt them is denominated the middle of the eclipse. The moon is sometimes wholly immerged in the shadow of the earth, and remains for some time invisible; this we call a total eclipse, during which we may remark the moment when the moon entirely disappears, and that when she begins to emerge; the former is called the beginning of total darkness, and the latter the end of it. But when a part only of the moon is obscured, we call it a partial eclipse; and we can remark only the moment of its beginning and ending. You know likewise that eclipses of the moon can happen only at the full, and that but rarely.

When, therefore, an eclipse of the moon is observed at two different places situated under different meridians, the beginning of the eclipse will be clearly seen at both, and at the same instant; but the time-pieces at these different places will by no means indicate the same hour, or any other division of time exactly the same: I mean well regulated time-pieces, each of which points precisely to XII. when it is noon at that place. If these places are situated under the same meridian, their time-pieces will no doubt indicate the same time at the beginning and at the end of the eclipse. But if these two meridians are 15 degrees distant from each other, that is, if the difference of their longitude be 15°, the time-

pieces must differ a complete hour from the beginning to the end of the eclipse; the time-piece of the place situated to the eastward will indicate one hour more than the other: the difference of 30° in longitude will occasion that of two hours in the time indicated by well regulated clocks or watches; and so on, according to the following table:

DIFFERENCE OF LONGITUDE.		DIFFERENCE OF LONGITUDE.	
Of Degrees.	Of Time.	Of Degrees.	Of Time.
15°	1 Hour.	105°	7 Hours.
30	2	120	8
45	3	135	9
60	4	150	10
75	5	165	11
90	6	180	12

If, therefore, the difference of the longitude were 150°, the time-pieces would differ ten hours from the beginning to the end of the eclipse.

Thus, when the same eclipse is observed at two different places, and the moment of its commencement is exactly marked on the time-pieces at each, it will be easy to calculate, from the difference of the time indicated, the difference of longitude between the two places. Now, that where the time is more advanced, must be situated more toward the east, and consequently its longitude greater, as longitude is reckoned from west to east.

By such means, accordingly, the longitude of the principal places on the globe have been determined, and geographical charts are constructed conformably to these determinations. But it is always necessary to compare the observations made in a place, the longitude of which was not already known, with those which had been made in a known place, and to wait the result of that comparison. Were I to arrive, then, after a long voyage, at an unknown place, and an opportunity presented itself of there observing an eclipse of the moon, this would, in the first instance, afford me no assistance toward the discovery of the longitude of that place; I could not, till after my return, compare my observation with another made in a known place, and thus I should learn too late where I was at that time. The grand point in request is, How am I at the moment to acquire the necessary information, that I may take my measures accordingly?

Now, the motion of the moon being so exactly known, it is possible to attain this satisfaction; for we are thereby enabled not only to calculate beforehand all future eclipses, but to ascertain the moment of the beginning and end, according to the time-pieces of a given place. You know that our Berlin almanacks always indicate the beginning and the end of every eclipse visible at that city. In the view, then, of undertaking a long voyage, I can furnish myself with a Berlin almanack; and if an opportunity presents itself of observing an eclipse of the moon at an unknown place, I must mark exactly the time of it, by a time-piece accurately regulated by the sun at noon, and compare the moments of the beginning and end of the eclipse with those indicated in the almanack, in order to ascertain the difference between the meridian of Berlin, and that which passes through the place where I am.

But beside the rarity of eclipses of the moon, this method is subject to a farther inconvenience; we are not always able to distinguish with sufficient accuracy the moment of the beginning and end of the eclipse, which comes on so imperceptibly, that a mistake of several seconds may very easily be committed. But as the mistake will be nearly the

same at the end as at the beginning, we calculate the middle point of time between the two moments observed, which will be that of the eclipse; and we afterwards compare this with that which is indicated by the almanack for Berlin, or for any other known place.

If the almanack for next year should not be published when I set out on my voyage, or supposing it to last more years than one, there are books containing the eclipses calculated for several years to come.

22d September 1761.

Letter LI.—Observation of the Eclipses of the Satellites of Jupiter, a Fourth Method of finding the Longitude.

Eclipses of the sun may likewise assist in ascertaining the longitude, but in a way that requires more profound research, because the sun is not immediately obscured; it is only the interposition of the body of the moon which obstructs the transmission of his rays to us, as when we employ a parasol to shelter us from them, which does not prevent others from beholding all their lustre. For the moon conceals the sun only from part of the inhabitants of the earth; and an eclipse of the sun may be clearly visible at Berlin, while at Paris there is no interception of his light.

But the moon is really eclipsed by the shadow of the earth; her own light is diminished or extinguished by it: hence the eclipses of the moon are seen in the same manner, wherever she is above the horizon at the time of the eclipse.

It cannot have escaped your penetration, that if there were other heavenly bodies which from time to time underwent any real obscuration, they might be employed with similar success as the eclipses of the moon in ascertaining the longitude. The satellites of Jupiter, which pass so frequently into the shadow of their planet, that almost every night one or other of them is eclipsed, may be ranked in the number of these, and furnish us with another excellent method of determining the longitude. Astronomers accordingly employ it with great success.

You know that Jupiter has four satellites, which make their revolutions round him, each in his own orbit, as represented in the annexed figure (PLATE IV. *Fig.* 1.), by circles described round Jupiter. I have likewise represented the sun in this figure, in order to exhibit the shadow AOB behind the body of Jupiter. You see the first of these satellites, marked 1, on the point of entering into the shadow; the second, marked 2, has just left it; the third, 3, is still at a great distance, but approaching to it; and the fourth, 4, has left it a considerable time ago.

As soon as one of these satellites passes into the shadow, it becomes invisible, and that suddenly; so that at whatever place of the globe you may happen to be, the satellite which was before distinctly visible disappears in an instant. This entrance of a satellite into the shadow of Jupiter is denominated *Immersion*, and its departure from the shadow *Emersion;* when the satellite, which had for some time been invisible, suddenly re-appears.

The immersions and emersions are equally adapted to the determination of the longitude, as they take place at a decided instant; so that when such a phenomenon is observed at several places of the globe, you must find, in the time indicated by the time-pieces of each, the difference which exactly corresponds to the difference of the distance of their meridians. It is the same thing as if we observed the beginning or the end of an eclipse of the moon; and

the case is then involved in no difficulty. For some time past, we have been able to calculate these eclipses of the satellites of Jupiter, that is, their immersions and emersions; and we have only to compare the time observed, with the time calculated for a given place, say Berlin, in order to conclude, at once, the distance of its meridian from that of our capital.

This method is accordingly practised universally in travelling by land; but the means have not yet been discovered of profiting by it at sea, where, however, it is of still greater importance for a man to know with certainty where he is. Were the satellites of Jupiter as visible to the naked eye as the moon is, this method would be attended with no difficulty, even at sea; but the observation cannot be made without a telescope of at least four or five feet in length—a circumstance which presents an insurmountable obstacle.

You well know that it requires some address to manage, even at land, a telescope of any length, to direct it toward the object which you wish to contemplate, and to keep it so steady as not to lose the object; you will easily comprehend, then, that a ship at sea, being in a continual agitation, it must be almost impossible to catch Jupiter himself; and if you could find him, you would lose him again in a moment. Now, in order to make an accurate observation of the immersion or emersion of one of the satellites of Jupiter, it is absolutely necessary that you should have it in your power to look at him steadily for some time together; and this being impossible at sea, we are to all appearance constrained to abandon this method of determining the longitude.

This inconvenience, however, may be remedied two ways; the one by the construction of telescopes six inches long, or less still, capable of discovering clearly the satellites of Jupiter; and there can be no doubt that these would be more manageable than such as are four or five feet in length. Artists are actually employing themselves with success in bringing telescopes of this sort to perfection; but it has not yet been proved whether or not it will require as much address to point them to the object, as those which are longer.

The other way would be to contrive a chair, to be used on ship-board, which should remain fixed and motionless, so as not to be affected by the agitation of the vessel. It does not seem impossible that a dexterous mode of balancing might effect this. In fact, it is not long since we read in the public prints, that an Englishman pretended that he had constructed such a chair, and therefore claimed the prize proposed for the discovery of the longitude.* His claim was well founded, if he indeed constructed the machine, as it would be possible, by means of it, to observe at sea the immersions and emersions of the satellites of Jupiter, which are undoubtedly very much adapted to the making of this discovery; but for some time past no farther mention has been made of it. From the whole, you must have perceived how many difficulties attach themselves to the discovery of the longitude.

26th September 1761.

Letter LII.—The Motion of the Moon, a Fifth Method.

The heavens furnish us with one resource more for discovering the longitude without the assistance of telescopes, in which astronomers seem to place

* The invention here alluded to was Irwin's Marine Chair, which was tried at sea, but it was not found to answer the purpose of the inventor.—Ed.

the greatest confidence. It is the moon, not only when eclipsed, but at all times, provided she be visible; an unspeakable advantage, considering that eclipses are so rare, and that the immersions and emersions of the satellites of Jupiter are of such difficult observation; there being a considerable time every year during which the planet Jupiter is not visible to us, whereas the moon is almost constantly in view.

You must undoubtedly have already remarked, that the moon rises every day almost three quarters of an hour later than the preceding, not being attached to one fixed place relatively to the stars, which always preserve the same situation with respect to each other, though they have the appearance of being carried round by the heavens, to accomplish every day their revolution about the earth. I speak here according to appearances; for it is the earth which revolves every day round its axis, while the heavens and the fixed stars remain at rest; while the sun and planets are continually changing their place relatively to these. The moon has likewise a motion abundantly rapid from one day to another, with relation to the fixed stars.

If you were to see the moon to-day near a certain fixed star, it will appear to-morrow at the same hour at a considerable distance from it toward the east; and the distance sometimes exceeds even 15 degrees. The velocity of her motion is not always the same, yet we are able to determine it very exactly for every day; by which means we can calculate before-hand her true place in the heavens for every hour of the day, and for any known meridian, say that of Berlin, or Paris.

Suppose, then, that after a long voyage I find myself at sea, in a place altogether unknown, what use can I make of the moon, in order to discover the longitude of the place where I am? There is no difficulty with respect to the latitude, even at sea, where there are means abundantly certain for ascertaining the height of the pole, to which the latitude is always equal. My whole attention, then, will be directed to the moon; I will compare her with the fixed stars which are nearest, and thence calculate her true place relatively to them. You know there are celestial globes on which all the fixed stars are arranged, and that celestial charts are likewise constructed, similar to geographical maps, on which are represented the fixed stars which appear in a certain quarter of the heavens. On taking, then, a celestial chart on which the fixed stars to which the moon is near are marked, it will be an easy matter to determine the true place where the moon at that time is; and my watch, which I have taken care to regulate there, from an observation of the moment of noon, will indicate to me the time of my lunar observation. Then, from my knowledge of the moon's motion, I calculate for Berlin, at what hour she must appear in the same place where I have seen her. If the time observed exactly correspond with the time of Berlin, it will be a demonstration that the place where I am is precisely under the meridian of Berlin, and that consequently the longitude is the same. But if the time of my observation is not that of Berlin, the difference will give that which is between the meridians; and reckoning 15 degrees for every hour of time, I compute how much the longitude of the place I am at is greater or less than that of Berlin: the place where time is more advanced has always the greater longitude.

This is an abstract of the manner of determining longitude by simple observations of the moon. I remark, that the happiest moments for successfully performing this operation, and for accurately determining the moon's place, are, when a fixed star happens to be concealed behind her body; this is called *Occultation*, and there are two instances favourable to observation, that when the moon in her motion completely covers the star, and that when the star re-appears. Astronomers are particularly attentive to catch these instants of occultation, in order to calculate from them the moon's true place.

I foresee, however, an objection you will probably make respecting the time-piece with which I suppose our navigator provided, after having maintained the impossibility of constructing one that shall be proof against every agitation of a ship at sea. But this impossibility respects only such time-pieces as are expected to preserve a regular motion for a long time together, without the necessity of frequent adjustment; for as to the observations in question, a common watch is quite sufficient, provided it go regularly for some hours, after having been carefully adjusted to the noon of the place where we are; supposing a doubt to arise, whether we could calculate from it the succeeding evening or night, at the time we observe the moon, the stars likewise will afford the means of a new and accurate adjustment. For as the situation of the sun with relation to the fixed stars is perfectly known for any time whatever, the simple observation of any one star is sufficient to determine the place where the sun must then be; from which we are enabled to calculate the hour that a well regulated time-piece ought to indicate. Thus, at the very instant of making an observation by the moon, we are enabled likewise to regulate our time-piece by the stars; and every time-piece is supposed to go regularly for so short a space.

29th September 1761.

Letter LIII.—Advantages of this last Method; its Degree of Precision.

This last method of finding the longitude, founded on lunar observations, seems to merit the preference, as the others are subjected to too many difficulties, or the opportunities of employing them occur too seldom to be useful. And you must be abundantly sensible, that success depends entirely on the degree of precision attained in forming the calculation, and that the errors which may be committed would lead to conclusions on which we could place no dependence. It is of importance, therefore, to explain what degree of precision we may reasonably hope to attain in reducing this method to practice, founded on the considerable change which the moon undergoes from one day to another in her position. It may be affirmed, that if the moon's motion were more rapid, it would be more adapted to the discovery of the longitude, and would procure for us a higher degree of precision. But if, on the contrary, it were much slower, so that we could scarcely discern any change of her position from day to day, we could derive very little, if any, assistance from her toward the discovery of the longitude.

Let us suppose, then, that the moon changes her place among the fixed stars a space of 12 degrees in 24 hours; she will, in that case, change it one degree in two hours, and half a degree, or 30 minutes in an hour: if we were to commit a mistake in observing the moon's place, of 30 minutes, it would be the same thing as if we observed the moon an hour earlier or later, and we should commit a mistake of one hour in the conclusion respecting the difference of the meridians. Now, one hour's difference in the meridians corresponds to 15 degrees in their longi-

tude; consequently, we should be mistaken 15 degrees in the longitude itself of the place we look for; which would undoubtedly be an error so enormous, that it were almost as well to know nothing about it; and a simple computation of the distance and the direction, however uncertain, could not possibly lead to a mistake so very gross. But a man must have gone to work in a very slovenly manner, to commit a mistake of 30 minutes respecting the moon's place; and the instruments which he employed must have been very bad, a thing not to be supposed.

Nevertheless, however excellent the instruments may be, and whatever degree of attention may have been bestowed, it is impossible to keep clear of all error; and he must have acquitted himself very well indeed, who has not committed the mistake of one minute in determining the moon's place. Now, as it changes half a degree, or 30 minutes, in one hour, it will change one minute of distance in two minutes of time. When, therefore, the mistake of the moon's place amounts to no more than one minute, the mistake in the difference of meridians will amount to two minutes of time. And one hour, or 60 minutes, being equivalent to 15 degrees of longitude, there will result from it an error of half a degree in the longitude; and this point of precision might be sufficient for every purpose, were it but attainable.

I have hitherto supposed our knowledge of the moon's motion to be so perfect, that, for a known meridian, we could determine the moon's true place for every moment without an error; but we are still very far short of that point of perfection. Within these twenty years, the error in this calculation was more than six minutes; and it is but lately that the ingenious *Professor Mayer* of Gottingen, pursuing the track I had pointed out to him, has succeeded so far as to reduce this error to less than a minute. It may very easily happen, then, that in the calculation likewise, the error of one minute may be committed; which, added to that of a minute committed in the observation of the moon's place, will double that which results from it respecting the longitude of the place where we are; and, consequently, it may possibly amount to a whole degree: it is proper farther to remark, that if the moon in 24 hours should change her relative situation more than 12 degrees, the error in the longitude would be less considerable. The means may perhaps be discovered of diminishing still farther the errors into which we are liable to fall, in the observation and in the calculation; and then we should be able to ascertain the longitude to a degree, or less. Nay, we ought not to despair of attaining a still higher degree of precision. We have only to make several observations, which can be easily done by remaining several days together at the same place. It is not to be apprehended, in that case, that all the conclusions should be equally defective; some will give the longitude sought too great, others too small, and by striking a medium between all the results, we may rest assured that this longitude will not be one degree removed from the truth.

The English nation, generously disposed to engage genius and ability in this important research, has proposed three prizes for ascertaining the longitude, one of L. 10,000, one of L. 15,000, and one of L. 20,000. The first of these is to be bestowed on the person who shall determine the longitude to a degree, or about it, so as to give perfect assurance that the error shall not exceed one degree at most. The second is to be given to him who shall discover a method still more exact, so that the error shall never exceed two-thirds of a degree, or 40 minutes. The highest prize is destined to the man who shall

ascertain the longitude so exactly that the error shall never exceed half a degree, or 30 minutes; and a higher degree of precision is hardly to be expected. No one of these prizes has hitherto been allotted: I do not take into the account the gratification bestowed on the artist who pretended to it from his construction of perfect time-pieces. *Mr. Mayer* is at this moment claiming the highest, and I think he is entitled to it.*

3*d October* 1761.

Letter LIV.—On the Mariner's Compass, and the Properties of the Magnetic Needle.

You are by this time sufficiently informed respecting the discovery of the Longitude: I have had the pleasure of explaining the various methods which have been employed for the determination of it.

The first, and most natural, is carefully to observe the quantity of space which we have gone over, and the direction in which we moved; but the currents and tempests to which sea voyages are exposed, render this method impracticable.

The second requires the construction of a time-piece so perfect as to go always uniformly, notwithstanding the agitation of a ship at sea; which no artist has hitherto been able to accomplish.

The third is founded on the observation of the eclipses of the moon, which would completely answer every purpose, were not opportunities of employing it too rare, and least in our power when the necessity may be most urgent.

* The widow of Professor Mayer received from the British Parliament a reward of L. 3000 Sterling; and Euler himself received L. 300 for furnishing the theorems on which Mayer's Tables are founded. The latter received also a reward from the French Government, and gained several prizes for his improvement of the Lunar Theory.—Ed.