CHAPTER IX

CONCERNING KINDS OF DIATONIC-CHROMATIC SCALES

1. The reason why we may call our eighteenth genus diatonic-chromatic is evident from the exponent itself $2^m \cdot 3^3 \cdot 5^2$, as we can see it is the least common exponent which is divisible both by the general diatonic scale $2^m \cdot 3^3 \cdot 5$, and the chromatic scale $2^m \cdot 3^2 \cdot 5^2$ [*i.e.* the least common multiplier] and thus it shows these two kinds of scales taken together. From which it may be allowed to suspect at once how this kind of scale of ours is going to be generally received by musicians, since indeed the musicians too had composed the same kind of music from the old chromatic and diatonic scales.

2. Therefore at first we will consider the tones, which must be present in each kind of our octave. On account of which we may take all the divisors of the numbers $3^3 \cdot 5^2$, which are the following

$$1,\ 3,\ 5,\ 3^2,\ 3\cdot 5,\ 5^2,\ 3^3,\ 3^2\cdot 5,\ 3\cdot 5^2,\ 3^3\cdot 5,\ 3^2\cdot 5^2,\ 3^3\cdot 5^2$$

or as expressed in ordinary numbers :

$$1,\ 3,\ 5,\ 9,\ 15,\ 25,\ 27,\ 45,\ 75,\ 135,\ 225,\ 675.$$

Since the maximum of which shall be 675, the remaining numbers must be multiplied by a power of two of such a kind, that all the ratios lie within the ratio 1 : 2, that is they may be contained within an octave. Therefore these numbers set out in the order of nearby quantities will give the following tones of a single octave

$$512 : 540 : 576 : 600 : 640 : 675 : 720 : 768 : 800 : 864 : 900 : 960 : 1024.$$

3. Therefore 12 tones of this kind will be contained in one octave of our kind, which number indeed agrees with the number of tones of the received diatonic-chromatic scale; but plainly whether the tones shall be the same in each, the intervals will declare. Indeed with our kind the intervals between each neighboring sounds are progressing in this order:

512		720	
	Minor limma		Major semitone
540		768	
	Major semitone		Minor semitone
576		800	
	Minor semitone		Major limma
600		864	
	Major semitone		Minor semitone

640		900	
	Minor limma		Major semitone
675		960	
	Major semitone		Major semitone
720		1024	

Just as which intervals we may observe to agree with the received division of the octave.

4. But just as even now some musicians may disagree about the division of the octave and hence several different kinds thence may be used, yet one must be taken apart from the others, which is seen to be the most approved. Moreover in this the intervals beginning from the tone *F* are progressing thus :

F		*H*	
	Minor limma		Major semitone
Fs		*c*	
	Major semitone		Minor semitone
G		*cs*	
	Minor semitone		Major limma
Gs		*d*	
	Major semitone		Minor semitone
A		*ds*	
	Minor limma		Major semitone
		e	
	Major semitone		Major semitone
H		*f*	

These intervals are taken from Mattheson's book [1731] with the title *Die General-Baβ Schule*.

5. This same account of dividing the octave is seen well enough to be new, as before many years musicians may well be using ratio. but in order that they may arrive at another way, there is no doubt, why the experiences they have come upon using this method may not be the more suitable for producing the true nature of the harmonies. Therefore since this same manner received may differ so very little from the true harmonious nature (for only two have differing intervals and from that a single different tone), otherwise the truth of our principles indeed to prevail well enough from long experience, maybe agreed to be confirming our theories wonderfully well with that same strictness.

6. Therefore the received manner of dividing the octave now by experience alone has risen to so great perfection, so that, where it maybe rendered the most perfect, there shall be no need for any other correction, except that the sound designated by the letter maybe effected to be deeper by a diesis only, which is the difference between a greater and

smaller limma. Moreover with this correction used the most perfect musical genus will be had and the most suitable for producing harmony. So that indeed for the number of tones reached, this genus will contain just as many tones, neither more nor less, than the number the harmony requires; and sides all the tones will hold that same relation between themselves, which is determined from the laws of harmony.

7. Therefore the tones of these kinds of diatonic-chromatic intervals now indeed to be taken into use, but the correct theory, if it maybe had, is shown by the following table. but this table is accustomed to be established in the customary manner of the musicians, while it begins from the tone *C* and progresses to *c;* but we express the tones of the numbers in two ways, both by being resolved into numbers as well as into factors, so that it may be able for the mutual relation and the intervals to be judged more easily from these.

Genus XVIII. Exponent $2^m \cdot 3^3 \cdot 5^2$

Sign of the Tone	Tone		Interval	Interval name	Genus Diatomic-chromatic corrected for the present time.
C	$2^7 \cdot 3$	384			
Cs	$2^4 \cdot 5^2$	400	24:25	Minor semitone	
D	$2^4 \cdot 3^3$	432	25:27	Major limma	
Ds	$2 \cdot 3^2 \cdot 5^2$	450	24:25	Minor semitone	
E	$2^5 \cdot 3 \cdot 5$	480	15:16	Major semitone	
F	2^9	512	15:16	Major semitone	
Fs	$2^2 \cdot 3^3 \cdot 5$	540	128:135	Minor limma	
G	$2^6 \cdot 3^2$	576	15:16	Major semitone	
Gs	$2^3 \cdot 3 \cdot 5^2$	600	24:25	Minor semitone	
A	$2^7 \cdot 5$	640	15:16	Major semitone	
	$3^3 \cdot 5^2$	675	128:135	Minor limma	
H	$2^4 \cdot 3^2 \cdot 5$	720	15:16	Major semitone	
c	$2^8 \cdot 3$	768	15:16	Major semitone	

This tale is a continuation of the kinds of music to be attached to the preceding chapter.

8. Therefore it is understood at once from this table, which ratio any tone may hold to any other. Moreover so that these more distinct ratios maybe viewed, the following table is seen to be attached, in which all the simple ratios of the intervals of the individual tones are contained.

Tones	Intervals	Names of intervals
C:Cs	24:25	Minor semitone
C:D	8:9	Major tone
C:Ds	64:75	Minor third less diesis
C:E	4:5	Major third
C:F	3:4	Fourth
C:Fs	32:45	Tritone
C:G	2:3	Fifth
C:Gs	16:25	Minor sixth less a diesis
C:A	3:5	Major sixth
C:	128:225	Minor seventh
C:H	8:15	Major seventh
C:c	1:2	Octave

Tones	Intervals	Names of intervals
Cs:D	25:27	Major limma
Cs:Ds	8:9	Major tone
Cs:E	5:6	Minor third
Cs:F	25:32	Major third with diesis
Cs:Fs	20:27	Fourth with comma
Cs:G	25:36	Tritone
Cs:Gs	2:3	Fifth
Cs:A	5:8	Minor sixth
Cs:B	16:27	Major sixth with comma
Cs:H	5:9	Minor seventh
Cs:c	25:48	Major eighth
Cs:cs	1:2	Octave

Tones	Intervals	Names of intervals
D:Ds	24:25	Minor semitone
D:E	9:10	Minor tone
D:F	27:32	Minor third less comma
D:Fs	4:5	Major third
D:G	3:4	Fourth
D:Gs	18:25	Tritone
D:A	27:40	Fifth less a comma
D:B	16:25	Sixth minor less a diesi
D:H	3:5	Major sixth
D:c	9:16	Minor seventh
D:cs	27:50	Major seventh
D:d	1:2	Octave

Tones	Intervals	Names of intervals
Ds:E	15:16	Major semitone
Ds:F	225:256	Major tone with diaschisma
Ds:Fs	5:6	Minor third
Ds:G	25:32	Major third with diesis
Ds:Gs	3:4	Fourth
Ds:A	45:64	Tritone
Ds:B	2:3	Fifth
Ds:H	5:8	Sixth minor
Ds:c	75:128	Major sixth with diesis
Ds:cs	9:16	Minor seventh
Ds:d	25:48	Major seventh
Ds:ds	1:2	Octave

Tones	Intervals	Names of intervals
E:F	15: 16	Major semitone
E:Fs	8:9	Major tone
E:G	5:6	Minor third
E:Gs	4:5	Major third
E:A	3:4	Fourth
E:B	32:45	Tritone
E:H	2:3	Fifth
E:c	5:8	Minor sixth
E:cs	3:5	Major sixth
E:d	5:9	Minor seventh
E:ds	8:15	Major seventh
E:e	1:2	Octave

Tones	Intervals	Names of intervals
F:Fs	128:135	Minor limma
F:G	8:9	Major tone
F:Gs	64:75	Minor third less diesis
F:A	4:5	Major third
F:B	512:675	Fourth less diaschisma
F:H	32:45	Tritone
F:c	2:3	Fifth
F:cs	16:25	Minor sixth less diesis
F:d	16:27	Major sixth with comma
F:ds	128:225	Minor seventh
F:e	8:15	Major seventh
F:f	1:2	Octave

Tones	Intervals	Names of intervals
Fs: G	15:16	Major semitone
Fs: Gs	9:10	Minor tone
Fs:A	27:32	Minor third less comma
Fs:B	4:5	Major third
Fs:H	3:4	Fourth
Fs:c	45: 64	Tritone
Fs: cs	27:40	Fifth less comma
Fs:d	5:8	Minor sixth
Fs:ds	3:5	Major sixth
Fs:e	9:16	Minor seventh
Fs: f	135:256	Major seventh
Fs: fs	1:2	Octave

Tones	Intervals	Names of intervals
G: *Gs*	24:25	Minor semitone
G:A	9:10	Minor tone
G:B	64:75	Minor third less diesis
G:H	4:5	Major third
G:c	3:4	Fourth
G: cs	18·: 25	Tritone
G:d	2:3	Fifth
G:ds	16:25	Minor sixth less diesis
G:e	3:5	Major sixth
G:f	9:16	Minor seventh
G: fs	8:15	Major seventh
G:g	1:2	Octave

Tones	Intervals	Names of intervals
Gs:A	15:16	Major semitone
Gs:B	8:9	Major tone
Gs:H	5:6	Minor third
Gs:c	25:32	Major third with diesis
Gs: cs	3:4	Fourth
Gs:d	25:36	Tritone
Gs:ds	2:3	Fifth
Gs:e	5:8	Minor sixth
Gs:f	75: 128	Major sixth with diesis
Gs:fs	5:9	Minor seventh
Gs:g	25:48	Major seventh
Gs:gs	1:2	Octave

Tones	Intervals	Names of intervals
A:B	128: 135	Limma minor
A:H	8:9	Major tone
A:c	5:6	Minor third
A:cs	4:5	Major third
A:d	20:27	Fourth with comma
A:ds	32:45	Tritone
A:e	2:3	Fifth
A:f	5:8	Minor sixth
A: fs	16:27	Major sixth with comma
A:g	5:9	Minor seventh
A :gs	8:15	Major seventh
A:a	1:2	Octave

Tones	Intervals	Names of intervals
B:H	15:16	Major semitone
B:c	225:256	Major tone with diaschisma
B:cs	27:32	Minor third less comma
B:d	25:32	Major third with diesis
B:ds	3:4	Fourth
B:be	45:64	Tritone
B:f	675:1024	Fifth with diaschisma
B: fs	5:8	Minor sixth
B :g	75: 128	Major sixth with diesis
B:gs	9: 16	Minor seventh
B:a	135:256	Major seventh
B:b	1:2	Octave

Tones	Intervals	Names of intervals
H:c	15:16	Major semitone
H:cs	9:10	Minor tone
H:d	5:6	Minor third
H:ds	4:5	Major third
H:e	3:4	Fourth
H:f	45:64	Tritone
H:fs	2:3	Fifth
H:g	5:8	Minor sixth
H:gs	3:5	Major sixth
H:a	9:16	Minor seventh
H:b	8:15	Major seventh
H:h	1:2	Octave

8[a]. Therefore all the intervals in this kind are either these consonants themselves, to which these names are imposed, or from so many intervals minimally different from these, which shall be imperceptible to duller ears. As also to be elaborated on exceedingly by musicians, lest any interval may differ from the name designated by less than a minimum interval which is either a comma, diesis, or diaschisma, it follows that practical musicians will themselves have to know our correct rule to be in place. Indeed for the tone *B,* as musicians wish, is allowed to be more acute by a diesis, then the interval *Cs:B* may become the sixth major with a comma and diesis, which two intervals, although small, yet together produce almost a semitone minor, thus so that in this kind of usage the interval *Cs:B* will be had for a minor seventh rather than a major sixth. In a similar manner the minor third interval *B* : *cs* would be diminished by a comma and diesis and thus more like the tone than the third.

9. Moreover we have produced the following table from the preceding, in which equal intervals may be allowed to be seen together in order.

Minor Seconds

<table>
<tr><td>24:25</td><td rowspan="4">Minor semitone</td><td>15:16</td><td rowspan="7">Major semitone</td></tr>
<tr><td>C:Cs
D:Ds
G:Gs</td><td>Ds:E
E:F
Fs:G
Gs:A
B:H
H:c</td></tr>
<tr><td>128 : 135</td></tr>
<tr><td>F:Fs
A:B</td></tr>
<tr><td></td><td rowspan="3">Minor limma</td><td>25:27</td><td rowspan="2">Major limma</td></tr>
<tr><td></td><td>Cs:D</td></tr>
</table>

Major seconds	
9:10	Minor tone
D:E *Fs:Gs* *G:A* *H:cs*	
8 : 9	Major tone
C:D *Cs:Ds* *E:Fs* *F:G* *Gs:B* *A:H*	
225:256	Major tone with diaschisma
Ds:F *B:c*	

Major thirds	
4:5	Perfect major third
C:E *D:*Fs *E: Gs* *Fs:B* *G:H* *A:cs* *H:ds*	
25:32	Major third with diesis
Cs:F *Ds:G* *Gs:c* *B:d*	

Minor thirds	
64:75	Minor third less a diesis
C:Ds *F:Gs* *G:B*	
27:32	Minor third less a comma
D:F *Fs:A* *B:cs*	
5:6	Perfect minor third
C:E *D:F* *E:G* *G:H* *A:c* *H:d*	

Fourths	
512:675	Fourth less a diaschisma
F:B	
3:4	Perfect Fourth
C:F *D:G* *Ds: Gs* *E:A* *Fs:H* *G:c* *Gs:cs* *B:ds* *H: e*	
20:27	Fourth with comma
Cs:Fs *A:d*	

Tritones	
18:25	Fourth plus minor semitone
D:Gs	
G:cs	
32:45	Fifth less a major semitone
C:Fs	
E:B	
F:H	
A:ds	
45:64	Fourth plus major semitone
Ds:A	
Fs:c	
B:e	
H:f	
25:36	Fifth less major semitone
Cs:G	
Gs:d	

Minor sixths	
16:25	Minor sixth less a diesis
C:Gs	
D:B	
F:cs	
G:ds	
5:8	Perfect minor sixth
Cs:A	
Ds:H	
E:c	
Fs:d	
Gs:e	
A:f	
B:fs	
H:g	

Fifths	
27:40	Fifth less comma
D:A	
Fs:cs	
2:3	Perfect quinta
C:G	
Cs:Gs	
Ds:B	
E:H	
F:c	
G:d	
Gs:ds	
A:e	
H:fs	

Major sixths	
3:5	Perfect major sixth
C:A	
D:H	
E:cs	
Fs:ds	
G:e	
H:gs	
16:27	Major sixth plus comma
Cs:B	
F:d	
A:fs	
75:128	Major sixth plus diesis
Ds:c	
Gs:f	
B:g	

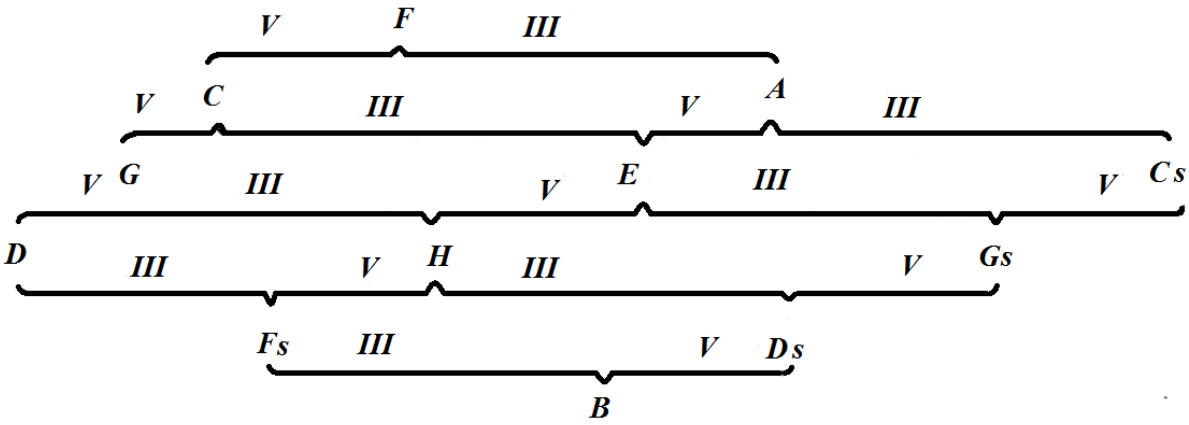
Minor sevenths		*Major sevenths*	
128:225	Major sixth plus minor limma	27:50	Octave less major limma
C:B		*D:cs*	
F:ds		8:15	Octave less major semitone
9:16	Octave less major tone	*C:H*	
D:c		*E:ds*	
Ds:cs		*F:e*	
Fs:e		*G:fs*	
G:f		*A:gs*	
B:gs		*H:B*	
H:a		135:256	Octave less minor limma
5:9	Octave less minor tone	*Fs:f*	
Cs:H		*B:a*	
E:d		25:48	Octave less minor semitone
Gs:fs		*Cs:c*	
A:g		*Ds:d*	
		Gs:g	

10. Therefore from this table the intervals are seen at once, which each two tones are understood to maintain between each other within the octave. Likewise truly also the different kinds are seen of the same name within the intervals, which commonly by the unskilled are taken for equality. Evidently four kinds of semitones are given, three kinds of tones and just as many other minor thirds, etc., to be allowed to be used from the table. But of all the octaves there is a single kind and that contained in the perfect ratio 1 : 2 ; for this interval on account of perfection may scarcely appear to depart from the ratio 1 : 2, provided likewise it may not be affected by the great annoyance to the listener. For indeed where the interval is more able to be perceived with perfection, there the perceived error shall be greater or smaller; but less is perceived from small aberrations in less perfect intervals.

11. But it will be easy to tune musical instruments according to this diatonic-chromatic genus with the aid of a monochord, evidently with the monochord divided up into the same ratios, which the tones must hold amongst themselves in this genus, of which we have examined certain precepts in the first chapter. But anyone who would tune a musical instrument in this way by listening alone without the aid of a monochord, it is required to provide him with these three requisites, so that in the first place the octave interval may be able to be distinguished and able to be formed to be heard alone; also, either by increasing or decreasing the tension, so that in the second place the fifth ratio also 2 : 3 may be tuned, and finally in the third place so that the third major also may prevail to be produced exactly.

12. Thus anyone able to operate with so much skill in hearing alone without the aid of a monochord, may approach the tuning of musical instruments in the following order. Initially the tone *F* must be established , just as the circumstances demand, and from that all the tones will be had designated by the same letter. Then in the second place he will form its fifth *c* above, and the third major *A*, and all the remaining tones designated by the same required first letter. In the third place from the tone *C* , he will form its fifth *G* and a major third *E* above, which tone *E* likewise will be the fifth of the tone *A*, and from *A* he will form its major third *cs* . Fourthly, from the tone *G*, he will form the fifth *d* and likewise the major third *H*, from E truly also the major third *Gs* , which also will be the tone of its fifth *Cs* . Fifthly from *H* he finds the fifth *fs* and the major third *ds*, or from *Gs* he will be able to form also *ds*. Finally, the fifth of *Ds* itself will give the tone *B* , and with this agreed for the whole octave the whole instrument will be tuned correctly.

13. Moreover this whole process of tuning may be understood more distinctly from this added figure.



Therefore since the tones *E, H, Gs, Fs, Ds* may be determined in two ways, both by fifths as well as by thirds, from this not to be disregarded aid will be obtained in the tuning of instruments, since an error, which perhaps may be committed, shall be perceived and corrected at once.

14. But to what extend the daily music experienced pertains mainly to this genus of perfect music, with which the excellence of this music is seen in abundance, yet also much luck is required to be attributed, so that it will have arrived there. Then indeed in the diatonic genus both tones as well as semitones are understood to be present, they are allowed to construct a more perfect genus, if the individual tones may be divided into two parts and within these tone interval distances new tones may be inserted, so that whatever

tone may be contiguous to a nearby semitone, may perhaps provide separations acceptable to the senses.

15. And in this matter not only the musicians fantasies are satisfied but also the harmonies, while they have decided to interpolate between such tones, which not only are consistent with harmony, but also will agree perfectly well with the genus of the music. Therefore this rather happy discovery they must still refer to chance rather than to a knowledge of true harmony ; for the case occurs, so that the genus of the true diatonic-chromatic shall be prepared thus, in that both 12 tones as well as contiguous semitones may be held separated from each other in turn.

16. But thus more can be elicited from that, because many musicians considered true music to consist rather in the equality of the intervals than in simplicity of these. Therefore these, so that they may satisfy more than harmonies between each other, without doubt divide the octave interval into twelve equal parts and following this division to constitute the 12 customary tones. But in this situation more will be confirmed there, because in this agreement all the intervals may become equal and on this account any music you please without the need of any other alteration may be allowed to be played in all the said ways and may be moved from the natural manner into some other. In which opinion indeed they are least deceived; but with this agreed they have not noticed this to be taken from every mode of the music.

17. So that which may become more apparent, we will show and expression for the individual tones both of our kind of diatonic-chromatic scale as well also of this kind from equal logarithms, so that it may be able to be judged at once from the difference of the intervals ; moreover we may put the logarithm of the tone $F = 0$.

Tones	Natural Genus	Equal Genus	Difference
F	0,000000	0,000000	0,000000
Fs	0,076815	0,083333	+ 0,006518
G	0,169924	0,166666	− 0,003258
Gs	0,228819	0,250000	+0,021181
A	0,321928	0,333333	+ 0,011405
B	0,398742	0,416666	+ 0,017924
H	0,491852	0,500000	+ 0,008148
c	0,584962	0,583333	− 0,001629
cs	0,643857	0,666666	+ 0,022809
d	0,754886	0,750000	− 0,004886
ds	0,813780	0,833333	+ 01019553
e	0,906890	0,916666	+ 0,009776
f	1,000000	1,000000	0,000000

Therefore it is evident the difference between the same tones of each kind everywhere may be greater than a comma, so that the harmony may be somewhat disturbed. Indeed the Fifth and the Fourth may differ a little from the natural order, without doubt scarcely the tenth part of a diaschisma, but the major and minor third disagree much more, by which however no more than the harmony for the Fifth and the Fourth disagree. Finally on account of no rational ratio of the tones apart from the octave, this kind of harmony is considered to be highly contradictory, even if duller ears scarcely may perceive a difference.

18. Moreover other musicians retained unchanged the remaining tones of the diatonic kind, the said chromatic tones they did not doubt by their choice to define nothing with regard to the harmony. Music of this kind thus was produced in England not long ago, in which both the major and minor tones were cut into two almost equal parts, yet of which the lower surpasses the greater, truly each is defined in a super-particulate ratio [*i.e.* a whole number plus a fraction]. Which is seen following the originator Pythagoras, who only allowed super-particulate ratios for effecting the harmony in music: for between two tones he had inserted the distance of a major tone the one deeper to the tone in the ratio 17 : 16, truly the other more acute holding the ratio 17 : 18 . Indeed which division may be agreed as of little consequence to the harmony, as may be agreed well enough from what arises.

19. Therefore the eighteenth genus has been set out, called the diatonic-chromatic, indeed thus received into use at this time, so that in that all the general rhythmical measures in that may become accustomed to be used. But this genus has before others this significant property, that all the intervals taken between its two neighbors are presented to be heard almost equal ; from which without inconvenience whatever melodies can be sung, whether semi-tonal or tonal, or in whatever interval, whether higher or lower, as one would wish. This cannot eventuate in other kinds, in which a greater inequality of the intervals is present. But before we may establish the general rules of composition of this genus, we will consider other kinds, follow in the account of the order which itself we have treated here .

CAPUT IX

DE GENERE DIATONICO-CHROMATICO

1. Quod genus nostrum decimum octavum diatonico-chromaticum appellemus, ratio ex ipso exponente $2^m \cdot 3^3 \cdot 5^2$ est manifesta, quippe qui est minimus communis dividuus exponentium generis diatonici $2^m \cdot 3^3 \cdot 5$ et chromatici $2^m \cdot 3^2 \cdot 5^2$ ideoque haec duo genera coniuncta exhiet. Ex quo statim suspicari licet hoc nostrum genus cum nunc a Musicis recepto genere conveniens fore, si quidem Musici quoque istud genus ex veterum chromatico et diatonico composuerunt.

2. Primo igitur sonos investigabimus, qui in quaque generis nostri octava inesse debent. Quamobrem sumemus numeri $3^3 \cdot 5^2$ omnes divisores, qui sunt sequentes

$$1,\ 3,\ 5,\ 3^2,\ 3\cdot 5,\ 5^2,\ 3^3,\ 3^2\cdot 5,\ 3\cdot 5^2,\ 3^3\cdot 5,\ 3^2\cdot 5^2,\ 3^3\cdot 5^2$$

seu in numeris ordinariis

$$1,\ 3,\ 5,\ 9,\ 15,\ 25,\ 27,\ 45,\ 75,\ 135,\ 225,\ 675.$$

Quorum euro maximus sit 675, reliqui per huiusmodi potestates binarii debebunt multiplicari, ut omnes intra rationem 1 : 2, hoc est intra intervallum diapason contineantur. Dabunt ergo hi numeri iuxta quantitatis ordinum dispositi sequentes sonos unius octavae

$$512 : 540 : 576 : 600 : 640 : 675 : 720 : 768 : 800 : 864 : 900 : 960 : 1024.$$

3. In huius ergo nostri generis una octava continentur 12 soni, qui quidem numerus cum recepti generis diatonico-chromatici numero sonorum convenit; num autem plane iidem in utroque sint soni, intervalla declarabunt. In nostro quidem genere intervalla inter quosque sonos contiguos hoc ordine progrediuntur:

512		720	
	Limma minus		Hemitonium maius
540		768	
	Hemitonium maius		Hemitonium minus
576		800	
	Hemitonium minus		Limma maius
600		864	
	Hemitonium maius		Hemitonium minus
640		900	

	Limma minus		Hemitonium maius
675		960	
	Hemitonium maius		Hemitonium maius
720		1024	

Quae intervalla quomodo cum recepta octavae divisione conveniant, videamus.

4. Quamvis autem Musici etiamnunc circa octavae divisionem dissentiant pluresque diversi modi hinc inde usurpentur, tamen prae aliis in Musicorum scriptis unum deprehendi, qui maxime proatus videtur. In hoc autem intervalla a sono *F* notato incipiendo ita progrediuntur:

F		*H*	
	Limma minus		Hemitonium maius
Fs		*c*	
	Hemitonium maius		Hemitonium minus
G		*cs*	
	Hemitonium minus		Limma maius
Gs		*d*	
	Hemitonium maius		Hemitonium minus
A		*ds*	
	Limma minus		Hemitonium maius
		e	
	Hemitonium maius		Hemitonium maius
H		*f*	

Haec intervalla sunt desumta ex MATTHESONI Liro *Die General-Baβ Schul* inscripto.

5.lsta octavae dividendae ratio satis nova esse videtur, cum ante plures annos Musici alia ratione sint usi. Quod autem ad allatum modum pervenerint, dubitandum non est, quin experientia deprehenderint hunc modum ad harmoniam producendam magis esse idoneum. Cum igitur iste modus receptus a vero genere harmonico tam parum discrepet (duo enim tantum habent intervalla dissidentia unicumque sonum *B* differentem), veritas nostrorum principiorum, alias quidem satis evicta, isto tam stricto theoriae nostrae cum longa experientia consensu mirifice confirmatur.

6. Receptus ergo octavam dividendi modus iam ad tantam perfectionem sola experientia est evectus, ut, quo perfectissimus reddatur, alia correctione non sit opus, nisi ut solus sonus littera *B* signatus diesi tantum, quae est differentia inter limma maius et minus, gravior efficiatur. Hac autem correctione adhibita habebitur genus musicum perfectissimum et ad harmoniam producendam aptissimum. Quod enim ad numerum sonorum attinet, tot continebit hoc genus sonos, nec plures nec pauciores, quam quot

harmonia requirit; atque praeterea omnes soni inter se eam ipsam tenebunt relationem, quae ex legibus harmoniae determinatur.

7. Soni ergo eorumque intervalla generis diatonico-chromatici usu nunc quidem recepti, sed theoria correcti, se habebunt, ut sequens tabula repraesentat. Adornata autem est tabula haec more Musicorum consueto, dum incipit a sono *C* et progreditur ad *c;* sonos autem duplici modo numeris expressimus, tum solutis tum in factores resolutis, quo facilius de eorum mutua relatione et intervallis iudicari possit.

Genus XVIII. Exponens $2^m \cdot 3^3 \cdot 5^2$

Signa Sonorum	Soni		Intervalla	Nomina Intervalla	Genus Diatomico-chromaticum hodiernum correctum.
C	$2^7 \cdot 3$	384			
Cs	$2^4 \cdot 5^2$	400	24:25	Hemitonium minus	
D	$2^4 \cdot 3^3$	432	25:27	Limma maius	
Ds	$2 \cdot 3^2 \cdot 5^2$	450	24:25	Hemitonium minus	
E	$2^5 \cdot 3 \cdot 5$	480	15:16	Hemitonium maius	
F	2^9	512	15:16	Hemitonium maius	
Fs	$2^2 \cdot 3^3 \cdot 5$	540	128:135	Limma minus	
G	$2^6 \cdot 3^2$	576	15:16	Hemitonium maius	
Gs	$2^3 \cdot 3 \cdot 5^2$	600	24:25	Hemitonium minus	
A	$2^7 \cdot 5$	640	15:16	Hemitonium maius	
B	$3^3 \cdot 5^2$	675	128:135	Limma minus	
H	$2^4 \cdot 3^2 \cdot 5$	720	15:16	Hemitonium maius	
c	$2^8 \cdot 3$	768	15:16	Hemitonium maius	

Haecque tabula est continuatio generum musicorum praecedenti capiti annexae.

8. Ex hac ergo tabula statim cognoscitur, quamnam rationem teneat quisque sonus ad quemliet alium. Hae autem rationes quo distinctius ob oculos ponantur, sequentem tabulam apponere visum est, in qua omnia intervalla simplicia singulorum sonorum ad singulos continentur.

Soni	Intervalla	Nomina Intervallorum
C:Cs	24:25	Hemitonium minus
C:D	8:9	Tonus maior
C:Ds	64:75	Tertia minor diesi minuta
C:E	4:5	Tertia maior
C:F	3:4	Quarta
C:Fs	32:45	Tritonus
C:G	2:3	Quinta
C:Gs	16:25	Sexta minor demta diesi
C:A	3:5	Sexta maior
C:B	128:225	Septima minor
C:H	8:15	Major seventh
C:c	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
Cs:D	25:27	Limma maius
Cs:Ds	8:9	Tonus maior
Cs:E	5:6	Tertia minor
Cs:F	25:32	Tertia maior cum diesi
Cs:Fs	20:27	Quarta cum commate
Cs:G	25:36	Tritonus
Cs:Gs	2:3	Quinta
Cs:A	5:8	Sexta minor
Cs:B	16:27	Sexta maior cum commate
Cs:H	5:9	Septima minor
Cs:c	25:48	Septa maior
Cs:cs	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
D:Ds	24:25	Hemitoniam minus
D:E	9:10	Tonus minor
D:F	27:32	Tertia minor commate minuta
D:Fs	4:5	Tertia maior
D:G	3:4	Quarta
D:Gs	18:25	Tritonus
D:A	27:40	Quinta demto commate
D:B	16:25	Sexta minor demta diesi
D:H	3:5	Sexta maior
D:c	9:16	Septima minor
D:cs	27:50	Septa maior
D:d	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
Ds:E	15:16	Major semitone
Ds:F	225:256	Tonus maior cum diaschismate
Ds:Fs	5:6	Tertia minor
Ds:G	25:32	Tertia maior cum diesi
Ds:Gs	3:4	Quarta
Ds:As	45:64	Tritonus
Ds:B	2:3	Quinta
Ds:H	5:8	Sexta minor
Ds:c	75:128	Sexta maior cum diesi
Ds:cs	9:16	Septima minor
Ds:d	25:48	Septa maior
Ds:ds	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
E:F	15: 16	Hemitonium maius
E:Fs	8:9	Tonus maior
E:G	5:6	Tertia minor
E:Gs	4:5	Tertia maior
E:A	3:4	Quarta
E:B	32:45	Tritonus
E:H	2:3	Quinta
E:c	5:8	Sexta minor
E: cs	3:5	Sexta maior
E:d	5:9	Septima minor
E:ds	8:15	Septima maior
E:e	1:2	Octava

Soni	lntervalla	Nomina Intervallorum
F:Fs	128:135	Limma minus
F:G	8:9	Tonus maior
F:Gs	64:75	Tertia minor diesi minuta
F:A	4:5	Tertia maior
F:B	512:675	Quarta demto diaschismate
F:H	32:45	Tritonus
F:c	2:3	Quinta
F:cs	16:25	Sexta minor demta diesi
F:d	16:27	Sexta maior cum commate
F:ds	128:225	Septima minor
F:e	8:15	Septima maior
F:f	1:2	Octava

Soni	lntervalla	Nomina Intervallorum
Fs: G	15:16	Hemitonium maius
Fs: Gs	9:10	Tonus minor
Fs:A	27:32	Tertia minor commate minuta
Fs:B	4:5	Tertia maior
Fs:H	3:4	Quarta
Fs: c	45: 64	Tritonus
Fs: cs	27:40	Quinta demto commate
Fs:d	5:8	Sexta minor
Fs: ds	3:5	Sexta maior
Fs:be	9:16	Septima minor
Fs: f	135:256	Septima maior
Fs: *fs*	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
G: *Gs*	24:25	Hemitonium minus
G:A	9:10	Tonus minor
G:B	64:75	Tertia minor diesi minuta
G:H	4:5	Tertia maior
G:c	3:4	Quarta
G: cs	18·: 25	Tritonus
G:d	2:3	Quinta
G:ds	16:25	Sexta minor demta diesi
G:e	3:5	Sexta maior
G:f	9:16	Septima minor
G: fs	8:15	Septima maior
G:g	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
Gs:A	15:16	Hemitonium maius
Gs:B	8:9	Tonus maior
Gs:H	5:6	Tertia minor
Gs: c	25:32	Tertia maior cum diesi
Gs: *cs*	3:4	Quarta
Gs:d	25:36	Tritonus
Gs:ds	2:3	Quinta
Gs :e	5:8	Sexta minor
Gs:f	75: 128	Sexta maior cum diesi
Gs:fs	5:9	Septima minor
Gs:g	25:48	Septima maior
Gs:gs	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
A:B	128: 135	Limma minus
A:H	8:9	Tonus maior
A:c	5:6	Tertia minor
A :cs	4:5	Tertia maior
A:d	20:27	Quarta cum commate
A:ds	32:45	Tritonus
A:e	2:3	Quinta
A:f	5:8	Sexta minor
A: *fs*	16:27	Sexta maior cum commate
A:g	5:9	Septima minor
A :gs	8:15	Septima maior
A: *a*	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
B:H	15:16	Hemitonium maius
B:c	225:256	Tonus maior cum diaschismate
B: *cs*	27:32	Tertia minor demto commate
B:d	25:32	Tertia maior cum diesi
B :ds	3:4	Quarta
B:be	45:64	Tritonus
B:f	675:1024	Quinta cum diaschismate
B: *fs*	5:8	Sexta minor
B :g	75: 128	Sexta maior cum diesi
B:gs	9: 16	Septima minor
B:a	135:256	Septima maior
B:b	1:2	Octava

Soni	Intervalla	Nomina Intervallorum
H:c	15:16	Hemitonium maius
H:cs	9:10	Tonus minor
H:d	5:6	Tertia minor
H:ds	4:5	Tertia maior
H:e	3:4	Quarta
H:f	45:64	Tritonus
H:fs	2:3	Quinta
H:g	5:8	Sexta minor
H:gs	3:5	Sexta maior
H:a	9:16	Septima minor
H:B	8:15	Septima maior
H:h	1:2	Octava

8[a]. Omnia ergo intervalla in hoc genere vel sunt ipsae illae consonantiae, quibus haec nomina sunt imposita, vel tantum intervallis minimis ab his differunt, quae crassioribus auribus sint imperceptibilia. Quod cum etiam a Musicis summopere intendatur, ne ullum intervallum a nominato plus quam minimo intervallo differat, hoc est vel commate vel diesi vel diaschismate, ipsi Musici practici agnoscere debebunt correctionem nostram iure esse factam. Namque sono *B,* ut Musici volunt, diesi acutiore admisso, tum intervallum *Cs :B* foret sexta maior cum commate et diesi, quae duo intervalla, etsi minima, hemitonium minus tamen coniunctim fere conficiunt, ita ut in hoc usitato genere intervallum *Cs :B* pro septima minore potius quam pro sexta maiore haberetur. Simili modo foret *B* : *cs* tertia minor commate et diesi minuta ideoque tono quam tertia similior.

9. Ex praecedente autem tabula formavimus sequentem, in qua intervalla aequalia in ordine coniunctim posita conspicere licet.

Secundae minores

24:25	Hemitonium minus	15:16	Hemitonium maius
C:Cs *D:Ds* *G:Gs*		*Ds:E* *E:F* Fs: *G*	
128 : 135	Limma minus	*Gs:A* *:H*	
F:Fs *A:*		*H:c*	
		25:27	Limma maius
		Cs:D	

Secundae maiores		*Tertiae maiores*	
9:10	Tonus minor	4:5	Tertia maior perfecta
D:E *Fs:Gs* *G:A* *H:cs*		*C:E* *D:*Fs *E: Gs* *Fs:*	
8 : 9	Tonus maior	*G:H* *A:cs*	
C:D *Cs:Ds* *E:Fs* *F:G* *Gs:* *A:H*		*H:ds*	
		25:32	Tertia maior cum diesi
		Cs:F *Ds:G* *Gs:c* *:d*	
225:256	Tonus maior cum diaschismate		
Ds:F			
B:c			

Tertiae minores	
64:75	Tertia minor diesi minuta
C:Ds	
F:Gs	
G:B	
27:32	Tertia minor commate minuta
D:F	
Fs:A	
B:cs	
5:6	Tertia minor perfecta
C:E	
D:F	
E:G	
G:H	
A:c	
H:d	

Quartae	
512:675	Quarta diaschismate minuta
F:	
3:4	Quarta perfecta
C:F	
D:G	
Ds: Gs	
E:A	
Fs:H	
G:c	
Gs:cs	
:ds	
H: e	
20:27	Quarta cum commate
Cs:Fs	
A:d	

Tritoni	
18:25	Quarta cum hemitonio minore
D:Gs	
G:cs	
32:45	Quinta hemitonio maiore minuta
C:Fs	
E:B	
F:H	
A:ds	
45:64	Quarta cum hemitonio maiore
Ds:A	
Fs:c	
B:e	
H:f	
25:36	Quinta hemitonio maiore minuta
Cs:G	
Gs:d	

Sextae minores	
16:25	Sexta minor diesi minuta
C:Gs	
D:B	
F:cs	
G:ds	
5:8	Sexta minor perfecta
Cs:A	
Ds:H	
E:c	
Fs:d	
Gs:e	
A:f	
B:fs	
H:g	

		Sextae maiores	
Quintae		3:5	Sexta maior perfecta
27:40	Quinta commate minuta	*C:A*	
D:A		*D:H*	
Fs:cs		*E:cs*	
2:3	Quinta perfecta	*Fs:ds*	
C:G		*G:e*	
Cs:Gs		*H:gs*	
Ds:B		16:27	Sexta maior cum commate
E:H		*Cs:*	
F:c		*F:d*	
G:d		*A:fs*	
Gs:ds		75:128	Sexta maior cum diesi
A:e		*Ds:c*	
H:fs		*Gs:f*	
		:g	

Septimae minores		*Septimae maiores*	
128:225	Sexta maior cum limmate minore	27:50	Octava limmate maiore minuta
C:B		*D:cs*	
F:ds		8:15	Octava hemitonio maiore minuta
9:16	Octava tono maiore minuta	*C:H*	
D:c		*E:ds*	
Ds:cs		*F:e*	
Fs:e		*G:fs*	
G:f		*A:gs*	
B:gs		*H:B*	
H:a		135:256	Octava limmate minore minuta
5:9	Octava tono minore minuta	*Fs:f*	
Cs:H		*B:a*	
E:d		25:48	Octava hemitonio minore minuta
Gs:fs		*Cs:c*	
A:g		*Ds:d*	
		Gs:g	

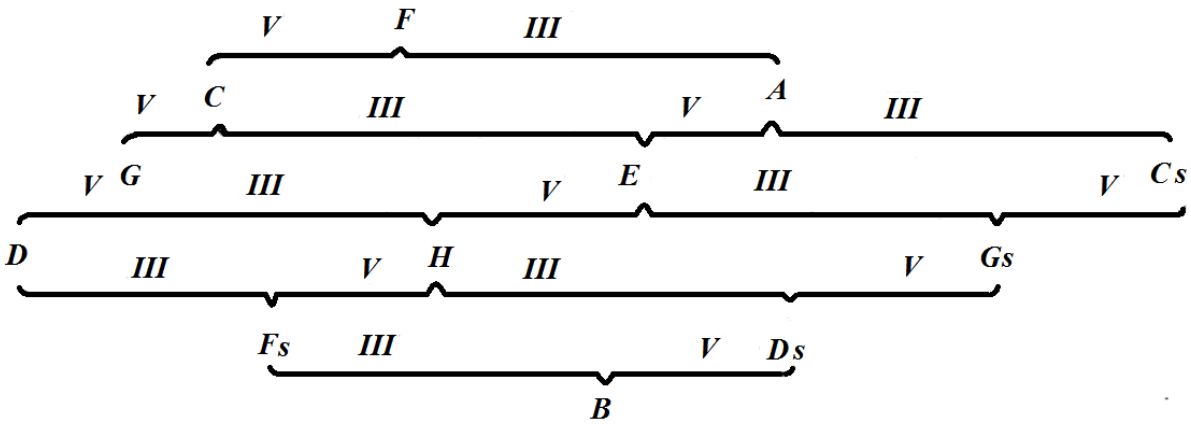
10. Ex hac igitur tabula statim conspiciuntur intervalla, quae duo quique soni intra octavae intervallum comprehensi inter se tenent. Simul vero etiam perspicitur differentia ingens inter intervalla eiusdem nominis, quae vulgo ab imperitioribus pro aequalibus habentur. Hemitoniorum scilicet quatuor dantur species, tres tonorum totidemque tertiarum minorum etc., uti ex tabula intelligere licet. Octavarum autem omnium unica est species eaque perfecta ratione 1 : 2 contenta; hoc enim intervallum propter perfectionem vix aberrationem a ratione 1 : 2 pati posset, quin simul auditus ingenti molestia

afficeretur. Namque quo perfectius perceptuque facilius est intervallum, eo magis sensibilis fit error vel minimus; minus autem sentitur exigua aberratio in intervallis minus perfectis.

11. Instrumenta autem musica ad hoc diatonico-chromaticum genus ope monochordi facile attemperari poterunt, monochordo scilicet iisdem rationibus secando, quas soni inter se tenere debent, cuius quidam operationis praecepta capite primo tradidimus. Qui autem solo auditu ad hunc modum instrumenta musica attemperare voluerit, eum tribus istis requisitis praeditum esse oportet, ut primo intervallum octavam distinguere et solo auditu efformare possit, secundo ut quintam quoque ratione 2 : 3 contentam et tertio denique ut tertiam maiorem chordis vel intendendis vel remittendis exacte producere valeat.

12. Qui igitur tanta auditus sollertia pollet, is sequenti ordine temperationem instruinenti musici aggrediatur. Primo figat sonum *F*, prout circumstantiae postulant, ex eoque habebit omnes sonos eadem littera signatos. Deinde formet eius quintam *c* tertiamque maiorem *A* habebitque omnes reliquos sonos iisdem litteris signatos per requisitum primum. Tertio ex sono *C* formet eius quintam *G* tertiamque maiorem *E*, qui sonus *E* simul erit quinta soni *A*, atque ex *A* quoque formet eius tertiam maiorem *cs* . Quarto ex sono *G* formet quintam *d* itemque tertiam maiorem *H*, ex E vero quoque tertiam maiorem *Gs* , qui sonus quoque erit quinta ipsius *Cs* . Quinto ex *H* faciat *fs* quintam et *ds* tertiam maiorem, seu ex *Gs* poterit quoque formare *ds* . Denique quinta ipsius *Ds* dabit sonum *B* hocque pacto sumendis octavis totum instrumentum erit rite attemperatum.

13. Totus autem hic temperationis processus ex adiecta hic figura distinctius percipietur.



Cum ergo soni *E, H, Gs, Fs, Ds* et duplici modo, tum per quintas, tum per tertias determinentur, ex hoc non contemnendum obtinebitur susidium in temperandis instrumentis, cum error, qui forte sit commissus, statim percipi et corrigi queat.

14. Quamvis autem hodierna musica ad hoc musicum genus perfectum experientia potissimum pertigerit, ex quo huius musicae praestantia abunde perspicitur, tamen etiam fortunae multum est tribuendum, quod eo pervenerint. Dum enim in genere diatonico tum tonos tum hemitonia inesse deprehenderunt, genus magis perfectum construere sunt arbitrati, si singulos tonos in duas partes secarent et intra quaeque intervalla tonum distantia sonos novos intersererent, quo quosque sonos contiguos hemitonio latiori saltem sensu accepta distantes obtinerent.

15. Hocque in negotio non solum phantasiae sed etiam harmoniae litarunt, dum tales sonos interpolare decreverunt, qui cum harmonia non tantum consisterent, sed etiam genus musicum satis perfectum constituerent. Hanc igitur quamvis felicem inventionem potius tamen fortunae acceptam referre debent quam verae harmoniae cognitioni; casu enim accidit, quod genus diatonico-chromaticum genuinum ita sit comparatum, ut in eo tum 12 soni tum quique contigui hemitonio a se invicem distantes contineantur.

16. Hoc autem eo magis ex eo elucet, quod plures Musici putaverint veram musicam potius in aequalitate intervallorum consistere quam in eorum simplicitate. Hi igitur, ut sibi magis quam harmoniae satisfacerent, non dubitaverunt intervallum diapason in duodecim partes aequales dissecare atque secundum hanc divisionem sonos 12 consuetos constituere. In hoc autem instituto eo magis confirmabantur, quod hoc pacto omnia intervalla fiant aequalia atque hanc ob rem quodvis opus musicum sine ulla alteratione in omnibus ita dictis modis liceat modulari et ex genuino modo in quemque alium transponere. In qua quidem sententia minime falluntur; sed hoc pacto ex omni modo harmoniam tolli non animadverterunt.

17. Quod quo clarius appareat, singulos sonos tum nostri generis diatonico-chromatici tum etiam huius generis aequabilis logarithmis expressos exhibebimus, quo statim de discrepantia intervallorum iudicari possit; ponemus autem logarithmum soni $F = 0$.

Soni	Genus genuinum	Genus aequabile	Differentiae
F	0,000000	0,000000	0,000000
Fs	0,076815	0,083333	+ 0,006518
G	0,169924	0,166666	-0,003258
Gs	0,228819	0,250000	+0,021181
A	0,321928	0,333333	+ 0,011405
B	0,398742	0,416666	+ 0,017924
H	0,491852	0,500000	+ 0,008148
c	0,584962	0,583333	-0,001629
cs	0,643857	0,666666	+ 0,022809
d	0,754886	0,750000	-0,004886
ds	0,813780	0,833333	+ 01019553
e	0,906890	0,916666	+ 0,009776

f	1,000000	1,000000	0,000000

Perspicuum igitur est inter sonos eosdem utriusque generis differentiam commate passim esse maiorem, quo harmonia non parum turbatur. Quintae quidem et quartae parum a genuinis discrepant, vix nimirum decima diaschismatis parte, sed tertiae maiores et minores multo magis aberrant, quibus tamen non minus quam quintis et quartis harmonia constat. Denique ob nullam sonorum rationem rationalem praeter octavas hoc genus harmoniae maxime contrarium est censendum, etiamsi hebetiores aures discrepantiam vix percipiant.

18. Alii autem retentis sonis generis diatonici invariatis reliquos chromaticos dictos suo arbitrio nullo ad harmoniam habito respectu definire non dubitaverunt. Huiusmodi genus musicum non ita pridem in Anglia prodiit, in quo tam tonus maior quam minor in duas partes fere aequales secatur, quarum tamen inferius maius est superiori, utrumque vero ratione superparticulari definitur. Qua in re auctor PYTHAGORAM secutus videtur, qui solas rationes superparticulares in musicam ad harmoniam efficiendam admittendas iudicavit: ita inter sonos tonum maiorem distantes inserit sonum ad graviorem rationem 17 : 16, ad acutiorem vero rationem 17 : 18 tenentem. Quae quidem divisio quam parum harmoniae consentanea sit, satis ex allatis constat.

19. Expositum igitur est genus decimum octavum, diatonico-chromaticum dictum, usu hoc quidem tempore ita receptum, ut omnes omnino modulationes in eo fieri soleant. Habet autem hoc genus prae aliis hanc insegnem proprietatem, ut omnia in eo sita intervalla ad sensum fere aequalia existant; unde non incommode quaevis melodiae vel hemitonio vel tono vel quolibet intervallo sive acutiores sive graviores cantari possunt. Id quod evenire non posset in alio genere, in quo maior intervallorum inaequalitas inest. Antequam autem regulas componendi generales ad hoc genus accommodemus, alia genera considerabimus, hoc ipsum, quod tractavimus, ratione ordinis sequentia.